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Modeling the Credit Risk of Mortgage Loans: A Primer

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Abstract

This paper presents a simple version of the application of option based pricing models to mortgage credit risk. The approach is based on the notion that default can be viewed as exercising a put option, and that the place to look in modelling default is the extent to which the option is in the money (the extent to which the borrower has negative equity in the property) and, given that, the incentive, e.g., a trigger event and inability to withstand it, to exercise the option. The main focus is on how the probability of default can be estimated and how the default risk can be priced. The analysis considers both "first principles" and specific analysis about U. S. default experience.

I. INTRODUCTION

This paper presents a simple version of the now standard approach to modelling and pricing mortgage credit risk. The approach is based on the notion that default can be viewed as exercising a put option, and that the place to look in modelling default is the extent to which the option is in the money (the extent to which the borrower has negative equity in the property) and, given that, the incentive, e.g., a trigger event and inability to withstand it, to exercise the option. The main focus is on how the probability of default can be estimated and how the default risk can be priced. The analysis considers both "first principles" and specific analysis about U. S. default experience.

There are three themes regarding credit risk:

1. Borrower default can be compared to exercising an option to exchange the house for the mortgage. It is of course a "sloppy" option; it is costly to borrowers who exercise it. Nonetheless, the option approach does appear to be useful. It suggests, for instance, that borrower equity, or lack of it, is an important predictor of default It also suggests an asymmetry present in all option models: when property values go up the lender benefits very little, but when they go down the lender is at risk for large losses.
2. Recent ability to collect large amounts of data has shown that other factors, especially credit history, are important determinants of mortgage default and that the effects of these factors on default can be estimated. These factors can be interpreted as proxies for "trigger events" that make giving up a negative equity house an optimal (or unavoidable) decision
3. Geographic diversification is an important tool for managing the credit risk of a portfolio of mortgages.

II. CREDIT RISK: SOME THEORY

It is not possible to predict default very accurately at the level of the individual loan, but it is possible to analyze it, understand its determinants and attach probabilities to it, so that it can be priced and to some extent controlled. For instance, we can understand how a decline in property value can be a factor in causing default even if we cannot predict which properties will decline in value, and we can estimate probabilities of property value decline. We can, then, view the problem in probabilistic terms; that is, we can estimate the probability of default. The option framework helps with this.

Understanding the Determinants of Default

The option-based approach leads to a relationship between homeowner equity and default cost, which comes from two notions:

1. Borrowers are unlikely to default if they have equity in the property. They will do what they can to raise money to protect their investment, and they will sell the property and keep the equity rather than turn it over to the lender.
2. Even if they do default with positive equity, the lender is likely to recover cost after selling the property.

Hence, focusing on negative equity and thinking of default behaviour as akin to exercising an option is a good way to begin, because it is only in states of the world with negative equity (states where a “rational” borrower might choose to exercise the option) that default is a serious problem to the lender. Of course, there is more to default than just negative equity. Most analysts and researchers believe that a good first approximation to default behaviour is that default comes from the intersection of three events:

1. Negative equity.
2. A “trigger event” such as illness or job loss.
3. Lack of resources to get over the trigger event.

Detailed analysis of *how* these interact (e.g., there are probably occasions where equity is so far negative that borrowers default without a trigger event and/or they choose not to survive a trigger event even if they have the resources) is generally not possible with most data sets. So analysts generally must be satisfied with proxies for these factors and *ad hoc* empirical models.

Recent research suggests that a reasonable predictor of trigger events is the borrower’s credit history. It appears to be the case from this research that credit history and equity are both very strong predictors of the probability of default, but there is no good way of predicting *which* borrowers will default.

Option-Based Models

It is clear that mortgage borrowers do not exercise their options in the same “ruthless” way that owners of financial options exercise their options. In part this is because the exercise of the option, defaulting on the loan, has extra costs for mortgage borrowers. In particular, it usually involves moving out of the house and finding a new one, and it affects borrowers' credit rating. What the option-based model does suggest is that borrower equity is important. If a borrower has sufficient equity (enough to cover selling costs), then selling the house is likely to be better for the borrower than is defaulting on the loan. In terms of option jargon, we should not expect people to exercise options when they are "out of the money". While we can say that out of the money options will not be exercised, we cannot say very precisely when an in-the-money option will be exercised because of the problem of not being able to observe the detailed calculations that borrowers make about the benefits and costs of default.

Hence, the option-based model is really quite flexible. A simple version of the model is that the probability of default is the probability of negative equity times the probability of a trigger event times the probability of not having sufficient resources to fall back on. It says that equity should matter and should be included as a key explanatory variable in every model (and it enters in the asymmetric option exercise manner), but it is also consistent with a wide variety of other factors, if they are plausible proxies for the trigger events. To a very large extent trigger events actually used in estimating default models depend on the data available.

Formally, we can estimate

(1) $d(t) = p \bullet f(x,t)$

where *d(t)* is the probability (during some small time period at time *t*) that the loan will default, *p* the probability of negative equity, and *f(x)* is some function of a wide range of trigger variables measured by the vector x and t is the time expired since origination. Most research uses historical data to fit equations of this form, and many important developments in default research have been achieved from this line of research.[1]

A particular variant, which is commonly used, is the "hazard" model, which takes the form:

(2) $d(t) = a(t)exp(bx)$

where a(t) is a baseline time trend, *x* is a vector of explanatory variables including the probability of negative equity and *b* is the vector of coefficients giving the effects of the *x* variables on *d(t).* This is a particularly useful way of setting up the model because of its multiplicative. For each item in *x* the corresponding *exp(b)* gives a "multiplier" for the effect of change in *x* on *d(t)*. This is, for instance, useful in the sample models presented below where the elements of *x* are "categorical" variables, meaning they indicate whether a variable is in some group, for instance if the loan to value ratio is less than 80%. In that case the *exp(b)* gives the multiplier relative to some baseline for being in that category (as opposed to, say, having LTV less than 80%. A multiplier of 4 (*exp(b)*=4) would mean that loans with LTVs above 90% were, other things equal, 4 times as likely to default as those with LTVs less than 80%. Two versions of models like this are given below.

A Framework

We begin with the initial value of the property and the loan balance, which for simplicity is taken to be constant over time. The ratio of the initial loan balance to the initial value is called the Loan to Value Ratio or LTV. It is related to the down-payment ratio, DP, by

[1] There are of course lots of complications. For instance, the likely *extent* of negative equity, if it is negative, should matter as well as simply the probability. For instance, there should probably a series of probabilities: for equity close to zero, slightly negative, strongly negative etc.

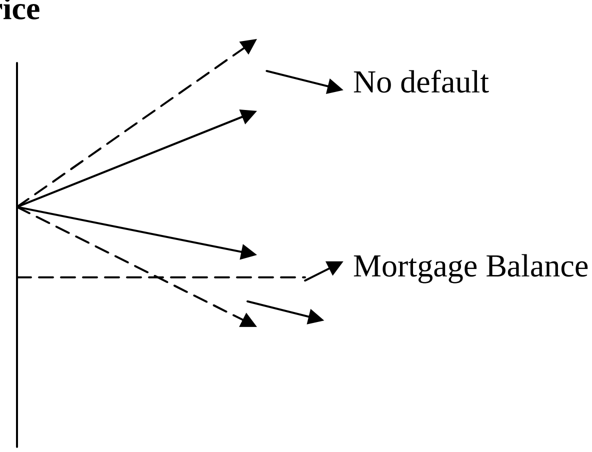
$DP = 1 - LTV$

where DP is the ratio of the down payment to initial property value. It is common to speak in terms of LTV rather than DP, but both ratios can be used to convey the same information, how much equity the borrower has in the house at the time the loan is originated.

A simple depiction of the process of property value's evolution over time is contained in the follow diagram. The assumption that prices go either up or down with some probability.

FIGURE 1: THE UPS AND DOWNS OF PROPERTY PRICES

Price



Possible Default

Time

Lenders do not know whether values will go up or down, but they can use data from the past to estimate the probability of ups or downs and the likely strength of the moves. Arrows in the figure depict possible price trends. The solid arrows are modest moves, and the dashed ones represent strong moves. The lender needs to know the steepness of the arrows and the probabilities of increases or decreases. These will vary by location. For instance, in the U.S. it is generally the case that California has stronger moves both up and down than does Michigan, but in California the probability of an increase has generally been higher, albeit more volatile.

In Figure one, in neither of the upward sloping arrow cases are default losses likely because house price increases in either case. It is the downward sloping arrows that raise problems. The less steep of the two arrows is less likely to be associated with default because while value (or price) did fall it did not fall be enough to make equity negative. In the steep arrow case default is more likely. Note that given the trend (in this case flat), the more volatile are price moves (the steeper are the arrows) the more likely is default.

It is easy to see that the average trend also matters. If for instance increases happen 60% of the time, then the frequency with which negative equity situations occur will be smaller. We should not be surprised that most of the time California has had low default rates (relative to Michigan), because of its strong average level of house price appreciation, but because of its volatility every once in a while (like the early 1990s) it has quite high default rates.

Next we consider a more formal model that includes both default behaviour and pricing. We continue with the simple model, assuming that prices are as likely to go up as down, but we extend it over several periods. We start out with house prices equal to 100, and then trace possible levels and their probabilities over three periods.

FIGURE 3

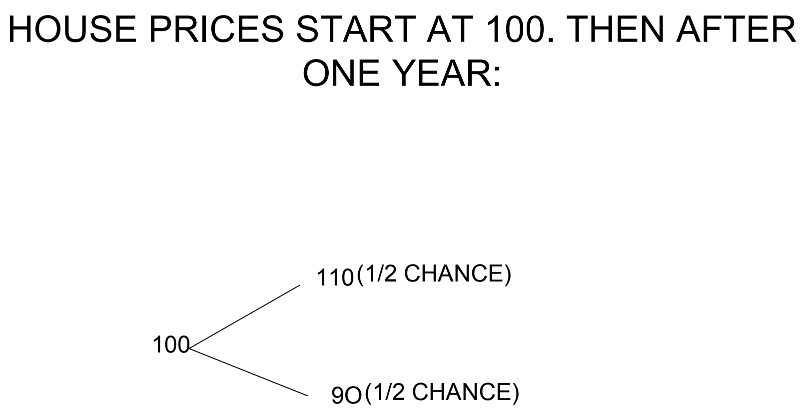


FIGURE 4

AFTER TWO YEARS

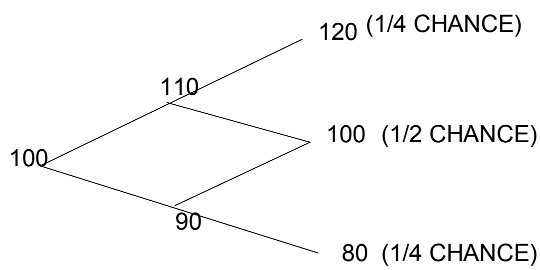
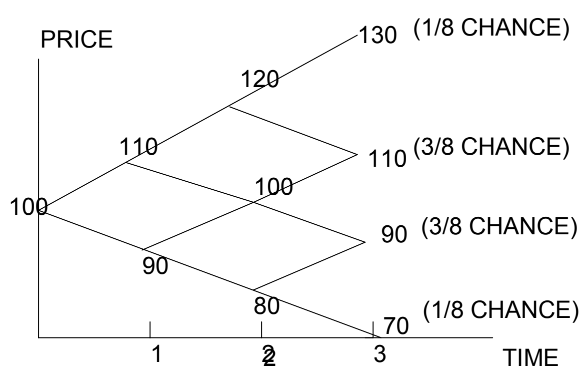


FIGURE 5

AND AFTER THREE YEARS



Version 1: Ruthless Case: Default as a Frictionless Put Option[2]

Now let us consider default losses and pricing for a simple mortgage that that lasts three periods and pays interest only, with a coupon payment of $5 per period and a “balloon” payment of 95 at the end of the period. Then the down payment is 5%, and LTV=.95. For simplicity of calculation we ignore the time value of money (no discounting) and assume that all traders are risk neutral, so that they all discount expected values at the risk free rate, which is zero. Hence, we can calculate expected losses over time and the value of the mortgage as the expected present value of its cash flows. We begin with a benchmark model where borrowers are utterly ruthless wealth-maximizers with no transaction or other cost (other than losing the property) to defaulting.

Borrowers are ruthless in the sense that at every occasion when they have a choice (in this case at each payment date) they make the choice (either make the payment or default and give up the property) that maximizes their wealth, and there are no costs to defaulting other than losing the property.[3] So the borrower can be treated as having a put option, and standard option pricing techniques can be used to value this option.

Then at each point in time the borrower will decide on which is greater: the value of the property or the value of the mortgage, and the borrower will default if the latter exceeds the former. Note that this requires knowing the *value*, as opposed to the balance, of the mortgage. And value will have to take account of future strategy; the borrower might postpone defaulting now in order to keep the option to default later. This complicates the problem. As is often the case with other formal option models it requires looking first at the last strategy, where there is no future strategy to worry about, and working back to the present in order to calculate the expected present value of the mortgage’s cash flow.

The decision process is depicted in Figure 6. In the last period the choice is whether to make a payment of 100 (the 95 balance plus the $5 interest) or turn over the property. In

[2] For a much more developed survey of these sorts of models see Kau et al (1995) and hendershott and Van Order (1987).
[3] It is straightforward to introduce simple types of default costs into the model.

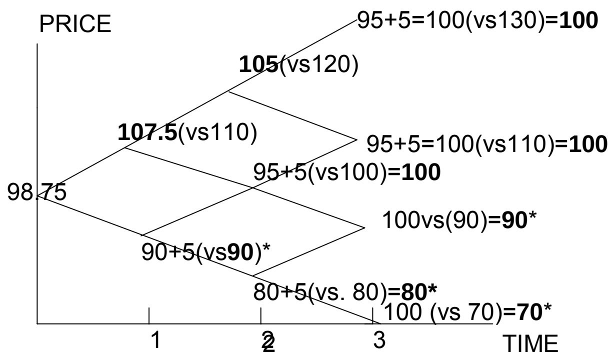
the two top cases where the property value is greater than 100 the answer is to make the payment, but in the other two it is to default.

One period earlier the borrower will have the choice of turning over the property at the then known value or making the $5 payment and accepting ensuing liability, which will have value equal to the expected value of the two possibilities in the final period. For instance, with property value at 80, it will pay to default because the liability incurred by making the payment is $5 plus the value (80) of the incurred liability next period is bigger. The borrower will default now rather than waiting until next period because of the coupon payment.

Figure 6 traces all these decisions back to the present. Values in parenthesis are the values of the property and values outside parenthesis are the present values of the payment. The chosen value is in bold face, and the default choice is denoted by an asterisk. Solving backward, we see that the value of the loan at origination is 98.75. This is equal to the expected present value of cash flows, assuming borrowers are ruthless in their strategies. It is higher than the par value (because of the coupon payment) but lower than what the value would be, 110, if all the payments were sure to be made. Hence, the present value of the default option is 11.25 (110-98.75). It is possible, with repeated solving of the above, to find a (lower) coupon that makes the value of the mortgage 95, or par. That coupon rate would be the default premium in the mortgage rate.

Figure 6. A RUTHLESS MODEL

Discount Rate=0; Coupon Rate =5%
Loan =95. At each node you chose the strategy that maximizes value (property value minus value of debt)



This is a neat model because everything is endogenous; the probability of default, loss severity and the value of the default option all come from wealth maximization and the process for property value, and nothing else. It is also a complicated model, primarily because of the backward solving technique, which becomes computationally tedious as the number of "state" variables increases.[4] Note, however, that the market can help the borrower make choices, by pricing new mortgages. Suppose there is a complete market in the sense that the borrower can take out a new loan without fear of adverse credit rating if he/she has defaulted, and the borrower knows the rate, in this case a coupon of $5. Given the information that borrowers are ruthless and will default whenever market value exceeds property value, traders will solve the pricing problem in the same way as depicted in the figure. If the borrower just looks at the *market* value of the loan vs property value, then it won't be necessary to go through the above computations; the traders in the market will do it!

Version 2: Option-Based Case

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[4] Here the only state variable is property value, but interest rates could also enter, as could "trigger" type variables that force the owner to move, etc.

The Version 1 model probably overstates the degree of ruthlessness by borrowers. There are high transitions costs, such as a ruined credit rating, of defaulting and other reasons why borrowers won't default in the manner depicted in the figure, and putting these into an explicitly wealth-maximizing, backward solving model like the above is likely to be very complicated.. So while it is a good benchmark the ruthless model is unlikely to be very predictive. In the real world we should expect default to be less frequent than the ruthless model suggests. On the other hand there are costs of selling property (foreclosure costs, fees to realtors etc.) that make the cost given default higher than is depicted in the model. Here a simple *option-based* version that takes account of this is developed.

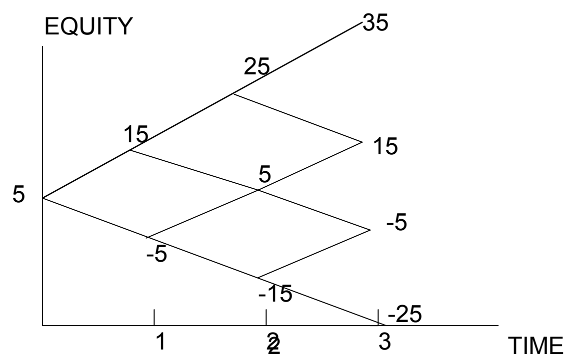
New assumptions are:

- Ignore the coupon payment and measure "equity" as property value vs mortgage book value (in this case it is fixed at 100).
- Borrowers never default when they have positive equity
- When equity is negative they default 25% (probability of a "trigger event") of the time and losses per loan are negative equity + 10 (for selling costs)

We begin by using Figure five and the information that the loan is for 95, and there is no amortization to calculate the movement of equity over time. This is depicted in Figure 7.

FIGURE 7

EQUITY OVER TIME

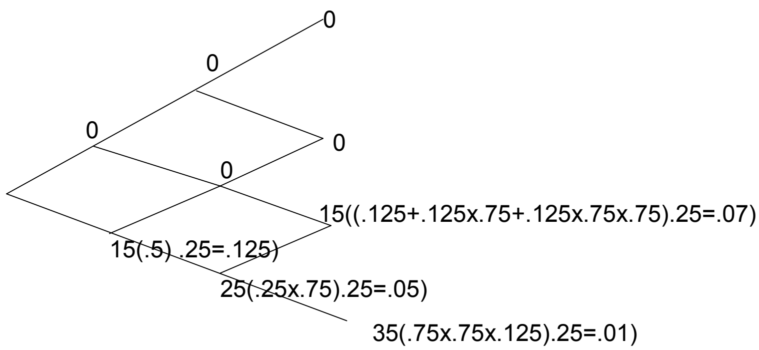


The places where equity is negative are the candidates for default. The assumption is that default has a 25% chance of happening in those states. This along with the assumption about loss per default allows us to calculate expected losses at each node and expected loss at point of origination. This is depicted in Figure 88.

FIGURE 8

EXPECTED LOSSES OVER TIME (Including a $10 selling cost)

- PROBILITY OF DEFAULT AT THAT NODE IS IN PARENTHESIS



Then:

- Expected loss (undiscounted and rounded)=
  - 15x.125 + 25x.05+ 15x.07 + 35x.01 = 4.4
- The value of the mortgage is 110 (total promised payments (95 plus 15 in coupon) minus 4.4 or 105.6. This is less than in the ruthless case.
- The probability of ever defaulting is .228 (.125+.05+.07+.01).
- Both would be smaller if:
  - Lower LTV
  - Smaller dispersion of prices
  - Upward trend in prices

The asymmetry of the situation needs to be noted; it is where the “optionness” of the model comes in to play. Strong property value increases do not help lenders much (borrowers just continue making payments), but strong decreases hurt because they can are a factor in default. The probability (.25 in this case) of defaulting, given negative equity is typically estimated from historical data, and where possible will vary with measurable variables such as credit history, income etc.

Default Factors

The model suggests four important factors in predicting default:

1. The initial LTV
2. Price volatility
3. Price trend
4. Vulnerability to trigger events.

III. TWO SAMPLE EMPIRICAL MODELS`

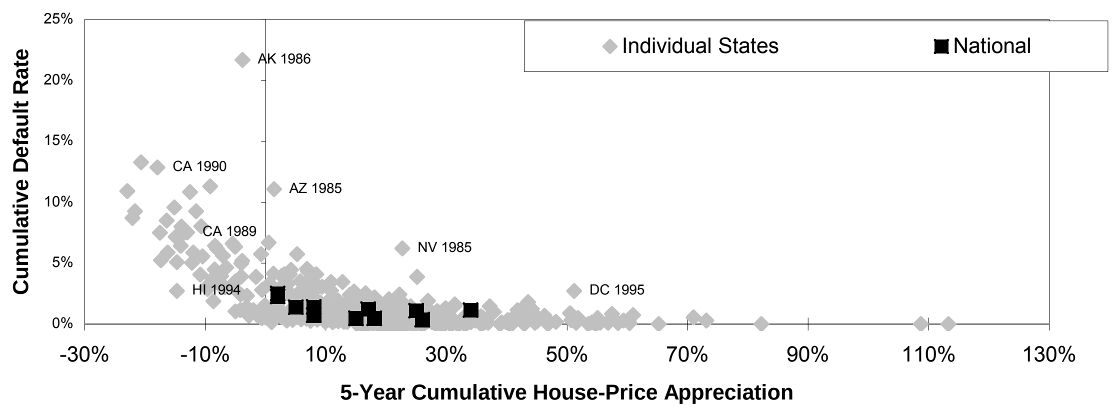
This section provides two examples of empirical models using the hazard framework in equation. In both cases the model estimates the hazard, which is the probability of defaulting in the current period conditional on having survived to this period, as a function of explanatory variables. The first uses a data sample that is narrow in terms of available explanatory variables (it does not for instance have borrower credit history) but which covers a long time period during which the economy went through several cycles. The focus in that model is on borrower equity: the initial loan to value ratio and the strength of the economy. The second has a broader range of data (it does have credit history) but is more recent and covers only one part of a cycle, which was largely expansionary. It shows the role of both equity and "trigger events," as proxied by borrower credit history, and vulnerability to a shock, as proxied by wealth and credit burden.

Model I. Equity and Property Appreciation

Before turning to the formal models, it is useful to look at the following picture, Figure 9, which depicts default experience by state (and nationally) for a fixed (.79-.81) initial loan to value ratio, by state house price appreciation (the grey diamonds) and for the nation as a whole (the black squares). The data come from loans purchased by Freddie Mac from

1985 through 1995 and followed for their first seven years. The horizontal axis depicts cumulative house price changes over the seven years and the vertical axis depicts cumulative foreclosure rates. The grey diamonds represent experience of a particular state-origination year. For instance, the AK diamond represents the experience of loans originated in Alaska in 1986.

FIGURE 9: Default Probability vs. House-Price Appreciation
State/Origination Year and National/Origination Year Cohorts (1985-1995)
80% Loan-to-Value, 30-Year Fixed-Rate Home-Purchase Mortgage



The scatter looks as we would expect it to look. States with rapid property appreciation had low foreclosure rates, and those with price declines had big ones. As the option model predicts, the results are asymmetric: when property values fall default accelerates and lenders lose money, but when they go up they simply cluster at zero. An important thing to note is the large differences in experience across states. The U.S. experience in general has been one of small national recessions but occasionally large regional recessions. Bad times, with low house price growth, have a strong effect on default, and there has been a great deal of regional variation in house price growth, but much less nationally. Now we turn to more rigorous statistical analysis.

The first empirical model (see Van Order (1990)) is a simple variant of models that focus on the role of equity and downpayment in predicting default. It uses Freddie Mac data on

750,000 loans originated from 1976 through 1983, which is a particularly interesting time because the economy went through both a major expansion (especially in terms of house price growth) in the late 1970s and a major contraction in the early 1980s. So it should be a reasonable test of the role of equity and option-based factors. The model estimates a hazard model like (2) above, with the probability of defaulting on the left hand side and: *a(t)* time expired since origination, original loan to value ratio (LTV) and the year of origination, which is a simple proxy for the state of the economy (1976 was a very good year, but 1981 was a bad (recession) one) all on the right hand side.[5] Next to each coefficient for origination year is the average house price growth over the subsequent two years. The variables are categorical and their coefficients have the multiplier characteristics discussed above.

The following table gives basic results of the estimates in the form of the multipliers (the level of *exp(bx))*relative to a "baseline" mortgage, which in this case is a loan with an LTV at .80 or below originated in 1979 (about an average year during the sample). The model shows both how default moves with LTV and economic conditions. For instance, a loan with an LTV greater than or equal to (mostly equal to) .95 will default about 8 times as often as one with an LTV at .80 or below. Furthermore, one originated in 1981, a recession year, will, given LTV default about 2.5 times more frequently than one originated in 1979 and about 25 times more frequently than one originated in 1976, which was a boom year in the middle of sharply rising property values.[6]

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[5] In this and in the next model default means that the borrower actually lost the property and Freddie took it over. In other contexts default can mean in violation of the contract, which could simply mean that the borrower is "delinquent" i.e., is behind on payments.

[6] The model *assumes* that the multipliers are independent, so that a 95 LTV originated in 1981 is about 72 times more likely to default than is a below 80 in 1976, but that assumption may not be accurate for big differences.

TABLE 1

EFFECTS OF LTV AND ORIGINATION YEAR ON ANNUAL DEFAULT RATES (Subsequent two years average house price growth in parenthesis. 1979 and LTV of 0.80 Scaled to Unity)

LTV CLASS	**EFFECT (MULTIPLIER)**
≤80	1.0
81-90	3.9
91-94	5.7
≥ 95	8.1
ORIGINATION YEAR	
1976 (12%)	0.1
1977 (10%)	0.2
1978 (4%)	0.5
1979 (0.6%)	1.0
1980 (-0.4%)	1.9
1981 (1.4%)	2.5
1982 (2.5%)	2.1
1983 (4%)	1.4

See Van Order (1990). Source for house price growth is Freddie Mac Conventional Mortgage House Price Index.

Hence, the evidence here and in other analysis (e.g. see Stegman et al. for a survey) suggests that default does indeed vary strongly with LTV and economic fluctuations. Because the data set does not include things like credit history of the borrowers it cannot tell us much about these rates vary across different borrower types.

Model II

This model (See Van Order and Zorn) uses a larger and more set of Freddie Mac Loans, over 2 million loans originated between 1993 and 1995 and followed through 1999. The data set lacks the sweep of the first set; in particular it lacks a national recession, although some of the regions did have recessions. The set is, however, richer in information about the borrower. For instance it has the borrower's credit history, as measured by "FICO" score[7]

Explanatory variables are: FICO (which is a proxy for a history of trigger events; higher FICO means a better credit history), initial LTV, the ratio of borrower debt payment to income (a proxy for the ability to survive a trigger event), loan amount (which given LTV is really a measure of house value and can be considered a proxy for borrower wealth), and loan purpose, purchase or refinance. Results are depicted in Table two.

[7] FICO stands for Fair Isaac Corporation. They estimate credit scores from a range of data, primarily the borrower's ability to make payments on credit cards and other consumer debt. Use in mortgages credit risk is rather recent, largely over the last ten to fifteen years. As is the case in the model here it ahs turned out to be predictive for both mortgage default and foreclosure.

TABLE 2: DEFAULT MODEL ii *
(Controls for age, origination year and state not shown)

Variable	Coefficient	Multiplier
FICO <620	2.35 (..03)	4.8
FICO 620-679	1.69 (.03)	2.3
FICO 680-720	0.84 (.04)	1.0
FICO >720	0	0.4
LTV <70	-2.96 (.06)	0.2
LTV 71-80	-1.40 (.03)	1.0
LTV 81-90	-0.64 (.03)	2.3
LTV 91-95	0	4.1
DEBT% 0-30	-0.37 (.03)	0.7
DEBT% 31-36	-0.15 (.03)	0.9
DEBT% >37	0	1.0
LOANAMT 0-76K	0.66 (.03)	1.9
LOANAMT 76-125K	0.28 (.03)	1.3
LOANAMT >125K	0	1.0
PURPOSE= PURCHASE	-0.56 (.03)	1.8
PURPOSE= REFI	0	1.0

*Standard errors are not depicted. All variables are statistically significant..

The model works as expected. Both equity (LTV) and trigger events (proxied by FICO) are important, as are loan amount and loan purpose (refinance loans are riskier). Debt burden is statistically significant (with such a large data set everything is statistically significant) but not very important. Note the element of "layering;" a loan with a high LTV and low FICO is 4.8 times 4.1 or almost 20 times as likely to default as a baseline loan.[8]

IV. PRICING

Models like these have great potential for use in estimating default probabilities and pricing and in analyzing "what if" situations, like what would happen in a particularly severe downturn (high LTV loans originated in a year like 1981 will have much higher default costs than low LTV loans originated in a good year like 1976).

Models I and II can lead directly to a "pricing matrix." For instance, in Model I we could assume that each of the eight origination years is equally likely, a very simple assumption, but absent a full data set (which, for instance, is likely to be the case in many emerging markets) perhaps the best that can be done. To get to expected costs we need more assumptions about default severity and about a baseline default profile. Here it is assumed that the baseline scenario in Model I is that a loan in the below 80% LTV category has a 1% chance of ever defaulting. It is also assumed that the loss per default I present value terms is (not very realistically) constant in present value at 25% of the loan balance. Then if we assume each year is equally likely to be repeated, price by LTV can be expressed by the very simple matrix given by the last row in Table 3.

[8] This is being forced into this particular model because of the multiplicative structure of the model. Whether or not the effects, of e.g., FICO and LTV, really are multiplicative can be tested by adding interactive terms (by FICO and LTV) and testing for the significance of the interactive terms. In general the multiplicative is a reasonable approximation.

Table 3: Default Cost Using Model I

Default Cost (as per cent of mortgage balance)	LTV<=80	81-90	91-94	>=95
1976	0.025	0.10	0.14	0.20
1977	0.05	0.20	0.29	0.41
1978	0.13	0.49	0.71	1.01
1979	0.25	0.98	1.43	2.02
1980	0.48	1.85	2.71	3.85
1981	0.63	2.44	3.56	5.06
1982	0.53	2.05	2.99	4.25
1983	0.35	1.36	2.00	2.83
Average	0.30	1.18	1.73	2.46

The bottom row gives breakeven prices by LTV.

Modellers often have mere sophisticated models of property value, in the form of a probability distribution of house prices over time. In that case the main pricing tool is "Monte Carlo" pricing models, which involve repeated simulation from the distribution of property values to calculate expected values. That is, we can draw randomly, one period at a time, from the distribution and get a particular pattern of default over time. We can calculate present value of losses along this particular path. We can then draw repeatedly from the distribution, calculating present values for each draw. We then take an average across these samples and use that as an estimate of expected present value of loss. This is an "up front," breakeven" premium (that might be charged by a mortgage insurer), which corresponds to the 4.4% in Figure 8. This is a much easier problem to solve than the backward solving approach, especially when there are several variables driving default.

Model II can also be used to generate a pricing matrix with FICO and LTV. Assume that the base case is a loan with LTV between 71 and 80 and FICO between 680 and 720, and those loans have a 1% chance of ever defaulting and a 25% severity rate. This generates the following matrix:

Table 4: Default Cost Using Model II

Default Cost (as per cent of mortgage balance)	LTV <70	LTV 71-80	LTV 81-90	LTV 91-95
FICO <620	0.24	1.20	2.76	4.92
FICO 620-679	0.12	0.58	1.32	2.36
FICO 680-720	0.05	0.25	0.58	1.02
FICO >720	0.02	0.10	0.23	0.41

Note the very wide range of prices, which reflects the layering referred to above. This model assumes that an average year is the base case, an assumption that should be improved upon by modelling house price and FICO processes and developing a corresponding Monte Carlo model.

V. MANAGING THE RISKS OF A PORTFOLIO OF MORTGAGES

The above discussed the risks and pricing of individual mortgages. This is not the same as the risk facing a mortgage lender with a portfolio of mortgages. In particular, the risk to a lender refers to the risk of the lender's overall portfolio, not of the individual loans in it. A portfolio of assets that are individually risky but uncorrelated with one another could be quite safe if the portfolio is large. That is, a diversified portfolio of mortgages might have quite different behaviour from that of individual loans or a pool of loans that are highly correlated (e.g., concentrated in a particular region).

To illustrate this point take another look at Figure 9. Again, it presents results from loans purchased by Freddie Mac from 1985 through 1995 and followed through seven years. In particular, note the black squares, which depict the same things as the grey diamonds, cumulative default vs. price appreciation for the same origination years, but by the country as whole rather than individual states. Note again the large differences in experience across states. The U.S. experience in general has been one of small national recessions but occasionally large regional recessions. The picture tells the story of diversification. The nationally diversified portfolio has a much smaller dispersion and much less risk.

VI. COMMENTS

The above presented a sketch of the basics of mortgage default modelling and pricing, using very simple option-based techniques. Properly understood they can take us a long way in understanding credit risk. The second statistical model is a simplified version of what current credit scoring models look like. The more complicated models are mainly extended versions of that model with more data and explanatory variables as well as more careful attention to the "buckets" into which the variables are put. The pricing models are primarily generalizations of the Monte Carlo models sketched out above. In the U.S. the diversification benefits discussed above come naturally in our current national system of markets. That was not always the case in the past, for instance when Savings and Loans were, by regulation, largely forced to be local. It is not the case in most emerging markets, where lenders will have to work hard to attain a reasonable amount of diversification.

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