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Housing and the Economy: After the Short Run

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Abstract

Much of the attention regarding the role of housing and the economy has been concerned with traditional macroeconomic business cycle problems, such as the role of housing as a stabilizer or destabilizer in the macro economy in the "short run." Here the focus is on the periods beyond that. In particular, housing is a capital good, a part of the capital stock, which is a substitute for other uses of capital, and the paper focuses on adjustment of housing and other capital after the short run. A central point of the paper is that the medium and long term impacts of policy, and other, shocks on housing can be quite different from the short run effects. Short run impacts can be reversed in the medium run. For instance, the currently most popular and largest form of tax subsidy for housing, not taxing imputed rent, will in initially lead to more housing and less business capital, but after the initial shock business capital growth will accelerate and eventually return to its initial level, but the housing stock will be higher.

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1. Introduction

Much of the attention regarding the role of housing and the economy has been concerned with traditional macroeconomic business cycle problems, such as the role of housing as a stabilizer or destabilizer in the macro economy.[1] This analysis is typically "short run" in the sense that it applies to a period of time when markets have not adjusted in a way to provide equilibrium levels of employment. Of course, the short run can sometimes last for several years. Here the focus is on the periods beyond that, in terms of the adjustment process, if not in terms of calendar time. In this paper the short run can be thought of as following the usual macroeconomic short run in the sense that it is assumed to take place after prices have cleared all markets and also after portfolios have adjusted, but before the stock of capital has changed. The medium run is the period during which the capital stock changes and the long run is when the level of wealth has fully adjusted to its long run level.

The focus here is on housing as a capital good, a part of the capital stock, which is a substitute for other uses of capital. Housing constitutes over 25% of the physical capital stock in most countries, and it competes with other forms of capital (business or government), which should affect resource allocation and growth. A central point of the paper is that the medium and long term impacts of policy, and other, shocks on housing can be quite different from the short run effects. What looks like a policy that will stimulate business capital at the expense of housing capital in the short run can stimulate housing in the medium run can end up stimulating both housing and business capital in the long run. That is, short run impacts can be reversed in the medium run.

The paper shows that the currently most popular and largest form of tax subsidy for housing, not taxing imputed rent, can in the short run lead to more housing and

[1] For a recent survery see Leung (2004).

less business capital, but in the medium run it will reverse the trend in business capital, and in the long run business capital will return to its initial level (but the housing stock will still be higher). A cut in business taxes, which in the short run would benefit business capital at the expense of housing, will, because it stimulates wealth accumulation, increase both housing and business capital over time, leading to more of both in the long run. Results would be different for a consumption tax; a switch to a consumption tax would eliminate the mortgage interest deduction and lead to less housing in the short run, but it would lead to more saving and more of both housing and business capital in the long run.

What follows is intended to warn the reader that the role of tax distortions in housing and capital accumulation decisions is more complicated than has been indicated in previous research. This can be couched in terms of the usual income (or wealth) and substitution effect distinction. Tax policy such as favourable tax treatment for housing (a lower tax rate) has a substitution effect in the sense that it alters the relative cost of capital for housing *vs.* business capital. But it also has an income or wealth effect because lower tax rates simulate capital formation and income, which tends to increase both housing and business capital. In the short and medium run it is assumed to be easier to substitute one type of capital for another (portfolio adjustment) than it is to increase wealth via saving. So the short and medium run effects are identified with substitution effects, and the longer run effects with wealth or income effects.

An elegant presentation of the notion that tax breaks for housing have crowded out business investment is given in Feldstein (1982), which emphasizes the role of taxation of nominal capital gains and historic cost depreciation as well as homeowner deductions as reasons for the tax system distorting capital allocation decisions. A similar analysis of tax distortions in housing is given in Hendershott and Hu (1983). Van Order (1990) focuses on long run implications of policy changes. This paper follows a simple version of that paper and focuses on dynamics as well as the long run. The model used here adapts Sidrauski's (1969) classic model of growth.

The point of departure in this paper is that analyses like Feldsteins's operate on only the one margin, the substitution effect: given a stock of capital or saving rate, the question posed is how much is allocated to business capital and how much to housing. Given the fixed-capital assumption it is easy to see that tax penalties to business capital raise business's costs of capital, diminish business investment, and promote housing, and tax breaks for housing will mean less business capital. Hence, tax penalties to housing might be optimal; two wrongs can make a right.

But the other margin, between current and future consumption, matters. Kau and Keenan (1983) present a model along lines similar to Feldstein's but with two margins, because they assume an interest-elastic savings rate.[2] However, their model, like Feldstein's, is not based on explicit modelling of household behaviour, i.e. utility maximization, so that while it can be reconciled with what follows, it, like the other models, is on a less firm foundation for purposes of policy evaluation. Slemrod (1982) presents a utility-maximizing, general equilibrium model that it is designed to answer policy questions. It is in some ways similar to the model here; but it assumes a fixed stock of wealth, as in Hendershott and Hu, and so it does not analyze the second margin.

What follows analyzes this using the two margins and focusing on the dynamics of the adjustment process in an intertemporal utility-maximizing framework. The model is deliberately simple. There are no costs of shifting capital from one use to another, and there is no population growth or depreciation. The model is intended to serve as a stripped down counter example.

2. The Model

This section applies a very simple version of a standard of growth model to a

[2] Feldstein has, of course considered models like the Kau-Keenan Model, with interest-elastic savings, in the context of overall saving in one commodity model. Those models can be made consistent with what follows; however, his discussion of housing versus business capital cannot.

world with two types of investment, business capital, *k*, which produces the single good that is produced in the economy. This good can be consumed, as the consumption good, *c*, or it can be converted in to the other investment good, housing, *h*. To simplify the model it is assumed that conversion of the good into housing is costless, so that *k*, *c* and *h* are perfect substitutes even in the short run. There is a single representative household that maximizes the integral of an instantaneous utility function discounted at a constant rate over an infinite time horizon. It is assumed that the labor market clears, and labor supply is taken as fixed, so that the only factor of production is capital, and growth comes entirely from capital accumulation.

Because of the constant discount rate the model implies infinitely interest elastic wealth accumulation in the long run, and a reversion of (long term) interest rates to a mean that is given by household time preferences, the “subjective’ discount rate, ρ. That is, if the interest rate, *i* is greater than the subjective discount rate, wealth accumulation goes on *i* reverts to ρ. While this implies an infinitely elastic long-run demand for wealth, it does not imply anything in particular about the interest elasticity of saving, which, among other things, depends on the convexity of the utility function. Summers, (1982) discusses some aspects of saving behavior from this sort of utility function and argues that it is consistent with reality.

Formally, the household, at time *t*, maximizes

$$\int_t^\infty e^{-\gamma s} u(c,h) ds, \qquad (1)$$

where *c* is the flow of nondurable consumer goods, and *h* is the per capita stock of housing, which is assumed to be proportional to its service flow, and *u*( ) is concave. All housing is owner-occupied, there is no depreciation of either *k* or *h*,[3] and there is no population growth, depreciation or technological change. Households can also hold

[3] Because there is only one bracket housing will be either all owner-occupied or all rental. I assume the former because of the tax advantages on which this paper focuses. Rental housing could be included, but that would require having a different depreciation rate for it and adding at least one more tax bracket.

or issue bonds. These bonds are risk free, homogeneous, and can be used by household, businesses, or the government. The net stock of bonds is the government's net indebtedness, which is constrained by the government's budget constraint. Households (and businesses) pay two sorts of taxes, one is an income tax, θ, on current output of the single good and the other is a tax, τ, on the imputed rent, r, on housing.

Households face two sorts of constraints: a flow constraint, which is the budget constraint, and a stock or balance sheet constraint. The budget constraint is given by:

$$v + (1-\theta) f(k) + (1-\theta) ib + \tau r h = c + s \tag{2}$$

where

v = real per capita lump-sum transfers from the government,

c = the amount of nonhousing (nondurable) consumption,

s = savings

i = the interest rate,

r = imputed rent per unit of housing

b = net holding of real bonds per capita,

θ = the tax rate on ordinary income,

τ = the tax on imputed rent

k = the real stock of nonhousing or business capital,

h = the stock of housing

$f(k)$ = the (well-behaved) production function,

and saving is given by

$$s = Da \tag{3}$$

Combining these gives

$$Da = v + (1-\theta) f(k) + (1-\theta) ib + \pi r h - c \tag{4}$$

The household wealth constraint is:

$$a = k + h + b. \tag{5}$$

It is assumed that capital goods are costlessly shiftable, so that, for instance, households can acquire capital goods by converting existing housing. They can also buy capital goods from other households, which constitutes equity funding.

It should be clear from the simple setup of the model that two well known theorems hold:

1. *Ricardian Equivalence*. That is, whether the government funds its purchases with debt or taxes does not matter because the market will take account of the present value of the taxes used to pay of the debt. In this case we might as well make the simplest assumption, that the government balances its budget every period.
2. *The Miller-Modigliani irrelevance theorem*. That is, the debt equity structure of the firm does not matter for investment decisions. In that case we can just as well omit bonds and interest from the model, thinking only of a sort of "imputed" interest rate.

This means that we do not have to worry much about the details of how either governments or households fund their purchases as long as they obey the usual budget constraints. Households maximize (1) subject to (4) and (5). This is a standard optimal control problem, which is dealt with more rigorously in the appendix. Here we consider a more intuitive approach. There are two sorts of decisions: portfolio allocation, given the level of wealth, a, and a saving, or wealth accumulation, decision. Consider the latter decision. We begin by defining a "shadow price" λ. for wealth.

This is defined as the benefit in terms of discounted future utility of giving up a unit of the consumption good now. The basic condition for balancing current and future utility is that the household be indifferent between consuming a unit of the good now (u_c) or saving it and consuming the extra output later (λ). Then a necessary condition is that

$$\lambda = u_c \tag{6}$$

The value of λ is the discounted value of the future utility, at any point t in time, from adding a unit of a to the capital stock k in order to produce more future consumption of c, taking account of the fact that future income will be taxed, or that:

$$e^{\rho t}\lambda = \int_s^\infty e^{-i(s-t)} u_c[(1-\theta) f'(k)]ds. \tag{7}$$

Differentiating this and rearranging terms gives

$$D\lambda - \rho\lambda = -u_c(1-\theta) f'(k) \tag{8}$$

which describes the evolution of λ. over time.

Conditions for portfolio allocation are that capital be allocated to the point where it's after tax marginal product equals the interest rate or

$$(1-\theta)i = (1-\theta) f'(k) \tag{9}$$

or

$$i = f'(k) \tag{10}$$

and housing is expanded until the marginal utility from housing equals the opportunity costs of giving up some k and also paying the tax on housing or

$$u_h = \pi r\lambda + (1-\theta) f'(k)\lambda \tag{11}$$

The level of bonds is a residual

Other conditions are the constraints, (4) and (5), and the transversality condition that

$$\lim_{t\to\infty} \lambda e^{-\rho t} = 0 \tag{12}$$

The latter is an infinite horizon generalization of the condition that at the end of the program nothing valuable be left behind. If it did not hold, then in (7) a finite amount of sacrifice now (u_c times the amount of consumption foregone) could generate an infinite amount of future (discounted) benefit.

Equilibrium is given by combining the above with the government budget constraint. To simplify matters it is assume that the government always balances its budget and that *b*=0. As is discussed above, this is actually not much of a restriction because this model with perfect foresight clearly exhibits Ricardian equivalence. Because the model exhibits a very simple version of the Modigliani-Miller irrelevance theorem, that debt vs. equity decisions are irrelevant, so it is easiest to take the simplest assumption that investment is entirely equity funded and let *f'(k)* be synonymous with the interest rate.

Combing these assumptions with the government's budget constraints we can characterize the dynamics of the economy by;

$$D\lambda - \rho\lambda = -\lambda(1-\theta) f'(k) \tag{13}$$

$$Da = f(k) - c \tag{14}$$

$$\lambda = u_c \tag{15}$$

$$u_h = (\tau + (1-\theta)) f'(k)\lambda \qquad (16)$$

and

$$a = k + h \qquad (17)$$

The last three equations can be used to solve for the three variables that can be freely chosen at every moment, c, k and h as a function of *a* and λ. Assuming these functions exist they can be given by *c(a, λ), k(a, λ),* and *h(a, λ),* respectively. Then the system that depicts household wealth accumulation decisions can be compressed into two differential equations in *a* and λ:

$$D\lambda - \rho\lambda = -\lambda(1-\theta) f'(k(a,\lambda)) \qquad (18)$$

$$Da = f(k(a,\lambda)) - c(a,\lambda) \qquad (19)$$

The details of this system and a linear approximation to it are given in the appendix.

At a particular time, t, *a* is fixed and only changes gradually, but *λ*, a price, is not fixed. As is shown in the appendix for arbitrary λ the system given by (18) and (19) is explosive. But for the tranvsersality condition to hold exp(-ρt) λ must converge. As is typical in models like this and as is show in the appendix for a linearized version of the above (in the neighbourhood of the long run equilibrium), there is one choice of λ that will make the (12) hold, so that the above converges. That means that the system can be represented by:

$$Da = F(a) \qquad (20)$$

where *F(a)* converges to some long run level. This process for *a* determines the time paths of *c*, *k* and *h*.

The next section begins the analysis of the dynamics of this convergence by analyzing the long run equilibrium to which it converges.

3. Long Run Equilibrium

The long run is characterized by the model with the time derivatives set equal to zero. Equilibrium requires that *Da* be zero:

$$f(k) = c\,. \tag{21}$$

It also requires that $D\lambda$ be zero. If we set $D\lambda$=0 and substitute the long-run equilibrium conditions into the household first-order conditions we have:

$$\rho = (1-\theta)i = (1-\theta)f', \tag{22}$$

and

$$u_h / u_c = \rho + \tau f' \tag{23}$$

Observations:

(1) The solution is recursive in the long run, with *f'* and *k* determined entirely in (22), which says that the after-tax marginal product of capital must equal the real, after-tax rate, which just equal ρ .

(2) Then *c* is determined by (21) and *h* by (23). The long run level of *a* is the sum of the long run levels of *k* and *h*.

(3) The tax on imputed rent does not affect the long run level of *k* or *c*, but it does affect housing, from (23).

(4) A cut in the income tax, θ, raises both *h* and *k* in the long run.

(5) Long-run equilibrium in a simple lump-sum tax model follows simply from the above with θ and τ set equal to zero, and it implies that

$$u_h / u_c = \rho \,, \tag{24}$$

$$f' = \rho \,, \tag{25}$$

$$f(k) = c \,. \tag{26}$$

It is straightforward to show that the solution to the 'lump-sum' model is optimal in the first-best sense of maximizing utility subject only to technological constraints.[4]

4. **Dynamics**

The dynamics of this model are simple. Adjustment is a combination of discrete portfolio changes, shifts in *k vs h*, followed by gradual adjustment of *a*, which induces gradual adjustment of *k, h* and c. It is shown in the appendix that a linear approximation to the process for *a* is given by

$$Da = (\rho - \sqrt{\rho^2 - 4\Delta}) / 2)(a - a^*) \tag{27}$$

where

$$\Delta = \tau f' a_3 a_2 / a_1 - 1 / u_{cc} (a_2 (1 - a_2 / a_1) < 0 \tag{28}$$

and

$$a_1 = u_{hh} + (1-\theta) f'' < 0 \tag{29}$$

$$a_2 = (1-\theta)\lambda^* f'' < 0 \tag{30}$$

$$a_3 = (1-\theta + \tau) f' > 0 \tag{31}$$

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[4] This is useful as a benchmark, but not especially helpful in terms of policy because it does allow for there being a reason to make the transfers, v, or in a not too different model, provide public goods, making distortionary taxes necessary.

and a^* is the long run level of a.

It is clear that $\rho - \sqrt{\rho^2 - 4\Delta})/2) < 0$, so that a does indeed converge to a^*.

Given the initial level of a, the adjustment of a to a^* is smooth. The levels of h, k and c are all smooth functions of a and parameters of the model like the tax rates.

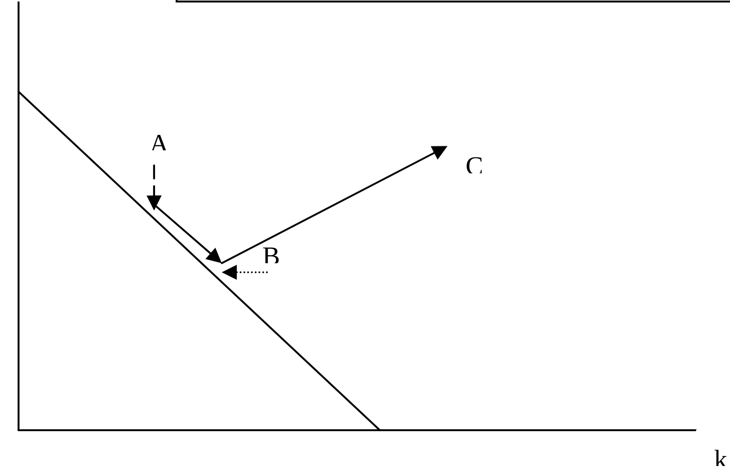
At a moment in time a is fixed and does not make discrete jumps, but the portfolio variables k and h are free to move discretely as long as they satisfy $a= h+k$. The functions $C(a)$, $H(a)$ and $K(a)$, which determine c, h and k as a function of a, are described in the appendix.

Consider the case of an income tax cut. In the short run, with the level of a fixed, it decreases the cost of capital for businesses relative to housing and clearly gives an advantage to business capital. The impact effect of a tax cut is a move along the wealth constraint*:* $a=h+k$, which is line with slope of minus 45 degrees. The size of the shift will be determined by the static conditions (15), (16) and (17). Figure one shows the jump in k and h via the arrow from the initial point at A to B.

But it was shown above that this has the long run effect of increasing both h and k. So the jump will be followed by a gradual increase in wealth, a, toward a^* with smooth growth in both k and h according to $K(a)$ and $H(a)$. This is depicted in Figure One by the movement from B to C. The quick move to B followed by a gradual move to C is an artefact of assuming instantaneous adjustment of the existing stocks of housing and capital. Incorporating costs of adjustment would lead a more gradual but similar adjustment.

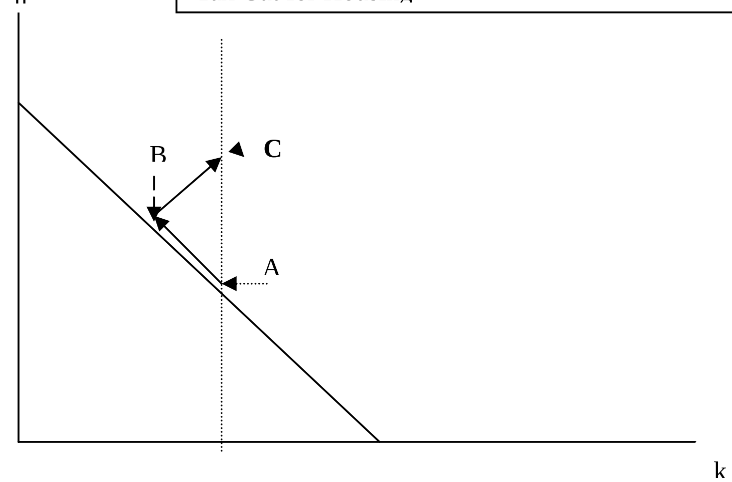
Hence, the dynamics after the short run actually reverse the initial effects of the tax rate change.

Figure One: Dynamics of Housing and Capital After an Income Tax Cut



Something similar happens in the case of a decrease in the tax on imputed rent. In the short run it gives housing an advantage and leads to more housing and less business capital. This is depicted by the arrow from A to B in Figure Two. However, it was shown above that this leaves the long run level of *k* unaffected, As a result the dynamic response is given by the arrow form B to C: a higher level of housing in the long run, but the same level of *k*. The lower tax on housing stimulates saving, which after the initial portfolio shift causes both *h* and *k* to grow

Figure Two: Dynamics of Housing and Capital After a Tax Cut for Housing



5. Policy issues

A full treatment of welfare issues would, of course, require a full intertemporal treatment, adjustment costs and all, as well as a consideration of the optimal level of expenditure.

The results of the model depend on the details of the tax structure. In particular the tax structure above is a combination of an income tax on output and a consumption tax on imputed rent. If we had consumption taxes on both *c* and *h* the model would be different and necessary conditions for household equilibrium with θ applied to *c* rather than *f(k)* would be

$$D(\lambda e^{-\rho t}) = -H_a = -l \quad (32)$$

$$\lambda f'(k) = l \quad (33)$$

$$u_c = (1+\phi)\lambda \quad (34)$$

$$u_h = \tau r\lambda + l \quad (35)$$

$$Da = v + f(k) + (1-\theta)ib - \tau rh - (1-\theta)c \quad (36)$$

$$a = k + h \quad (37)$$

Rearranging this with b=0 and $r = f'$ gives

$$D\lambda - \rho\lambda = -\lambda f'(k) \quad (38)$$

$$Da = v + f(k) - \tau f' h - c \quad (39)$$

$$u_c = (1+\theta)\lambda \tag{40}$$

$$u_h = (1+\tau) f'\lambda \tag{41}$$

$$a = k + h \tag{42}$$

In this model there is no distortion in the saving decision in the long run.[5] (We have $\rho = f'(k)$). The distortion would be from θ and τ being different. The dynamics would be similar: a change in, say, business taxes would lead to short run change in the *k-h* mix, followed by an adjustment to long run levels of k and h, but these levels of c and h would not change.

Consumption taxes have a times been opposed by pro housing groups on the grounds that they eliminate the advantage of mortgage interest deduction (and not taxing imputed rent). The model suggests that housing will decline in the short run but that the effect will be overwhelmed by more saving and wealth accumulation in the long run. This is because long run levels of *k* and *c* would increase, which means that f' and u_c decrease, which from (41) means that u_h must decrease, which implies that *h* must increase.

The model cannot say a lot about optimal policy. First it does not discuss what v is used for. The assumption above has been that changes in tax rates are absorbed by changes in v, but that may not be optimal. A way of looking at optimal policy might be to hold v constant and force changes in one tax to be financed by changes in the other. Second, the paper does not discuss welfare implications of the timing of change; in an infinite time horizon model that is likely to be important. The results are intended only to remind the reader of the importance of dynamics and the second margin.

[5] Of course there would be a distortion to the labor-leisure decision, if that were in the model.

6. Concluding Comments

The paper provides a very simple model of housing and business capital dynamics. The theme is that there can be important differences between the short run impact of changes in things like tax rates and the longer run dynamics. For instance, cutting taxes on business capital can increase both housing, h, and the nondurable consumer good, c, even though it leads to less h in the short run, because it would increase wealth over time. The seemingly sensible proposal to tax imputed rent would not affect business capital, k, in the long run, but it would decrease housing and total wealth. In the short run it would lead to an increase in k, but this would be reversed over time.

This would be different for a consumption tax because the long run distortions to saving are missing.

APPENDIX

Households maximize (1) subject to (4) and (5). This is a standard optimal control problem. We begin by setting up the Hamiltonian

$$H = e^{-\rho t}u(c,h) + \lambda(v + (1-\theta)f(k) + (1-\theta)ib - \tau rh - c) + l(a - h - b - k)$$

Necessary conditions are

$$D(\lambda e^{-\rho t}) = -H_a = -l$$

$$\lambda(1-\theta)f'(k) = l$$

$$u_c = \lambda$$

$$u_h = \tau r\lambda + l$$

$$i(1-\theta)\lambda = l$$

$$Da = v + (1-\theta)f(k) + (1-\theta)ib - \tau rh - c$$

$$Da = v + (1-\theta)f(k) + (1-\theta)ib - \tau rh - c$$

$$a = k + h + b$$

and the transversality condition (12).

The interest rate must satisfy

$$i = f'(k)$$

and it plays no essential in solving the model. So we can continue by assuming that b=0.

An equilibrium condition is that

$$v = \theta f(k) + \tau r h$$

so that

$$Da = f(k) - c$$

and in equilibrium imputed rent per unit (cap rate) equals the return on the alternative investment

$$r = i = f'$$

and we must satisfy the portfolio condition;

$$a = k + h$$

Rearranging the household first order conditions with $b=0$ gives

$$D\lambda - \rho\lambda = -\lambda(1-\theta) f'(k)$$

$$Da = v + (1-\theta) f(k) - \tau f' h - c$$

$$u_c = \lambda$$

$$u_h = (1-\theta+\tau) f' \lambda$$

$a = k + h$

The last three can be solved for c, h and k as functions of a and λ. Linearizing this around long run levels, denoted by asterisks, assuming that *u(c,h)* is separable and concave and measuring variables as deviations from their long run levels we have

$$u_{cc}c = \lambda$$

$$u_{hh}h - (1-\theta+\tau)f''\lambda^{*}k = (1-\theta+\tau)f'\lambda$$

$$k + h = a$$

Combining terms it can be shown that this implies

$$D\lambda = (\rho - (1-\theta)f'(k) + a_3/a_1)\lambda - a_2(1-a_2/a_1)a \equiv b_{11}\lambda + b_{12}a$$

$$Da = -(a_3^2/a_1 + 1/u_{cc})\lambda + ((1-\theta)f(k) - a_2a_3/a_1)a \equiv b_{21}\lambda + b_{22}a$$

where

$a_1 = u_{hh} + (1-\theta)f'' < 0$

$a_2 = (1-\theta)\lambda^{*}f'' < 0$

$a_3 = (1-\theta+\tau)f' > 0$

The solution to this is of the form

$\lambda = k_1 h_{11}e^{r1t} + k_2h_{12}e^{r2t}$

$a = k_1h_{21}e^{r1t} + k_2h_{22}e^{r2t}$

where r_1 and r_2 are the solutions to

$(b_{11}-r)(b_{22}-r)-b_{21}b_{12}=0$

the h's are the characteristic vectors of the "b" matrix and the k's come from the initial conditions. The k_i are determined by the initial conditions and are given by

$\lambda(0)=k_1 h_{11} + k_2 h_{12}$

$a(0)= k_1 h_2 + k_2 h_{22}$

The level of *a(0)* is given, but $\lambda(0)$ is a price and is determined by model. In particular it is chosen in such a way as to make the transversality condition (11) hold.

The solution for the r's is given by

$$r = (\rho \pm \sqrt{\rho^2 - 4\Delta})/2$$

where

$$\Delta = \tau f' a_3 a_2 / a_1 - 1/u_{cc}(a_2(1-a_2/a_1) < 0$$

Hence, there are two real roots, one negative and the other greater than ρ. For arbitrary initial levels of initial levels of λ and *r*, the time paths of λ and *r* will almost always diverge to the path given by the positive root. Assume that the positive root is r_2. The only way that the transversality condition can be satisfied is by choosing the initial level of λ so that k_2 is zero. This can be done by choosing the λ(0), given a(0), such that

$\lambda(0)=k_1 h_{11}$

$a(0)= k_1 h_{21}$

or

$\lambda(0)=a(0)\, h_{11}/h_{21}$

This means that the time path of a is determined by the negative root, and

$$Da = (\rho - \sqrt{\rho^2 - 4\Delta}) / 2)(a - a^*)$$

which in turn generates the paths of *c*, *k* and *h* by substituting $a(0)\ h_{11}/h_{21}$ for λ in $c(a, \lambda)$, $k(a, \lambda)$, and $h(a, \lambda)$.

These new functions: *C(a), K(a),* and *H(a)* are continuous functions of *a* and other parameters,

.

References

Feldstein, M., 1982, Inflation, tax rules and accumulation of residential and non-residential capital, Scandinavian Journal of Economics 84, no.2, 293-311.

Hendershott, P.H. and S. Hu, 1983, The allocation of capital between residential and non-residential uses: Taxes, inflation and capital market constraints, Journal of Finance 38, no. 3.

Kau, J.B. and D. Keenan, 1983, Inflation, taxes and housing: A theoretical analysis, Journal of Public Economics 21, 93-104.

Leung, Charles. “Macroeconomic snd Housing: A Review of the Literature.” Journal of Housing Economics, 13 (2004). 249-267

Sidrauski, M., 1969, Rational choices and Patterns of Growth, Journal of Political Economy 77, no. 4, 575-585.

Slemrod, J., 1982, Tax effects on the allocation of capital among sectors and among individuals: A portfolio approach, NBER Working paper no 951, Oct.

Summers, L.H., 1982, Tax policy, the rate of return and savings, NBER working paper no.995.

Van Order, R., 1990, Housing, Inflation and Capital Allocation, Journal of Public Economics 42 (1990) 387-398. North-Holland