THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THE THEORY OF EJECTORS

Henry H. Hicks, Jr.

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in The
University of Michigan
1960

February, 1960

IP-419

ACKNOWLEDGMENTS

The author of this dissertation wishes to recognize the encouragement, assistance, patience, and forbearance of the many people with whom he has been associated while doing this work.

The mention of the doctoral committee: Prof. J. L. York, chairman, Professors R. B. Morrison, L. O. Case, J. J. Martin, and R. Tek, have all been most helpful in technical and personal council. Special recognition is due to Professor York for his efforts in smoothing administrative problems, and to Professor Morrison for his assistance in the understanding of gas dynamic concepts.

On the administrative side, the author wishes to mention Mrs. Thelma Dyer of the Chemical Engineering Department for her always friendly and cheerful assistance. Much credit is also due to Ray Carroll and the staff of the Industry Program of the College of Engineering for their efforts in publishing this dissertation.

Finally the author wishes to mention his wife, Jeanne, who has been willing to put up with the "grind" for so long.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF ILLUSTRATIONS	v
LIST OF SYMBOLS	vii
I. INTRODUCTION	1
II. CRITERION OF PERFORMANCE FOR THE EJECTOR	6
III. INTERNAL PROCESSES IN THE EJECTOR	11
A. Sources of Irreversibility	11
B. Analysis of Ejectors Operating with Identical Fluids.	12
1. Pressure, Velocity, Temperature Relations for a Single Gas	12
2. The Case for Which the Induced Fluid has the Higher Stagnation Temperature	14
(a) An Operating Cycle for the Ejector	24
3. The Case for which the Driving Fluid has the Higher Stagnation Temperature	26
(a) Equalization of Temperature	29
(b) Equalization of Velocity	31
4. Resume of Ejector Design for Identical Fluids	43
5. The Constant Volume Process with Heat Addition	45
C. Analysis of Ejectors Operating with Different Fluids.	48
D. Gas Dynamics and Experimental Investigation	54
1. The Ejector with Closed Suction	58
2. The Ejector with Partially Throttled Suction	80
3. Unthrottled Suction Ejector	93
IV. RESUME	104
APPENDIX I. TEMPERATURE IRREVERSIBILITY	107
APPENDIX II. EFFECT OF HEAT ADDITION ON THE STAGNATION PRESSURE OF A FLOWING FLUID	109

TABLE OF CONTENTS CONT'D

		Page
APPENDIX III.	THE RELATIONS GOVERNING TEMPERATURE EQUALIZATION OF TWO QUANTITIES OF AN IDENTICAL GAS	110
APPENDIX IV.	FLUID SHEAR EFFECTS	113
APPENDIX V.	ONE-DIMENSIONAL FLOW WITH HEAT ADDITION (CONSTANT PRESSURE PROCESS)	116
APPENDIX VI.	HEAT ADDITION IN ONE-DIMENSIONAL FLOW (CONSTANT VOLUME PROCESS)	117
REFERENCES		119
BIBLIOGRAPHY		120

LIST OF ILLUSTRATIONS

Figure | Page

[redacted]

LIST OF ILLUSTRATIONS CONT'D

Figure		Page
18	Wall Static Pressure for the Unthrottled Suction Ejector	97
19	Stagnation Pressure Traverse for the Unthrottled Suction Ejector	98
20	Schlieren Photos of Pressure Shocks in the Mixing Tube, Unthrottled Suction Ejector	100
21	Schlieren Photos of Pressure Shocks in the Diffusor, Unthrottled Suction Ejector	103
22	Fluid Shear in Two-Dimensional Flow	114

LIST OF SYMBOLS

A	Area of stream normal to the flow
C_p	Heat capacity at constant pressure
C_v	Heat capacity at constant volume
M	Mach Number
$\overline{M}$	Molecular weight
P	Power applied to the stream
Q	Heat (Quantity)
R	Specific gas constant
$\overline{R}$	Universal gas constant
S	Entropy
T	Temperature
V	Velocity
W	Work
a	Local velocity of sound
g	Acceleration due to gravity
h	(Static Enthalpy of Fluid)
k	Ratio of specific heats for induced fluid when different than the driving stream
p	Pressure
q	Heat (rate)
t	Time
v	Specific volume
$\overline{v}$	Total volume
w	Weight flow

γ	Ratio of specific heats for driving fluid
τ	Intensity of fluid shear

Subscripts

c	Indicates common value for both fluids
d	Driving fluid
i	Induced fluid
m	Combined fluids
o	Stagnation or total value

Superscript

*	Refers to choking condition $(M = 1)$.

I. INTRODUCTION

One of the paradoxical aspects of the ejector is the way the simplicity of its intuitive concept contrasts with the difficulty of its mathematical analysis. The inducing of motion into a slow moving stream by the pulling effect of a fast moving stream is a concept that is easy to grasp, but the precise analysis of this process, so that flow ratios or pressure increases can be predicted, is difficult. The analysis is difficult because it must consider the principles of fluid dynamics as well as the heat and work processes occurring between the two streams. In practical design, the mixing rate of the two streams plays a part. As a result, there is at the present time no satisfactory solution to the design problem of an ejector. There is not even a generally accepted criterion of performance.

It is observed that the efficiency of an ejector is rather poor as compared to that of a reversible engine-reversible compressor combination. Actually, this depends to some extent on how the matter is viewed. There is a fundamental characteristic of the ejector to be considered, which is, that the motivating or driving fluid must supply the energy to compress both itself and the induced fluid to the final delivery pressure. For the engine-compressor combination, it is not necessary that the fluids be mixed, and there is generally a lower pressure available for the exhaust of the engine than the delivery pressure of the compressor. This, of course, improves the performance. When the ejector is compared in performance with an engine-compressor combination working

between the same pressure limits, then the performance of the ejector appears more favorable.

Ejector analysis currently rests on one-dimensional fluid flow theory. No attempt is made to consider the internal processes occurring in the device. To this extent at least, the analysis is incomplete and since the internal processes largely determine the entropy change of the fluids, no analysis which ignores these processes can give a precise prediction of performance. Another failing of the present analysis is that it does not show clearly the effect that changes in operating conditions have on the performance of a particular design.

In brief, the present scheme of analysis[(1)*] centers around the mixing tube of the ejector which is assumed to be of constant cross-sectional area. The basic equations of one-dimensional fluid dynamics are written for the conditions around the tube. These are:

Conservation of matter $w_d + w_i = w_m = \frac{A_m V_m}{v_m}$

Conservation of energy $w_d h_d + w_i h_i + \frac{V_d^2}{2g} = w_m \left(h_m + \frac{V_m^2}{2g}\right)$

Conservation of momentum $\frac{w_d V_d}{g} + p_c A_m = \frac{w_i + w_d}{g} V_m + p_m A_m$

This assumes in the momentum equations, that the induced stream has low velocity and contributes little to entering momentum.

Since the velocity enters into these equations as the square as well as the first power, the simultaneous solution of these equations is of quadratic form.

The solution will generally result in two sets of values for p, v, and V at the outlet of the mixing tube, corresponding to two states of

---

* This reference is fairly typical of the method.

entropy. One solution will be for supersonic flow and the other will be for the subsonic flow that corresponds to the same total energy. Elrod[1] has suggested that the subsonic state is more probable since it represents the state of higher entropy. Actually, there is no way to tell which state exists since the internal process is not analyzed. What appears to be the case is that the attempt to simplify the analysis by the adoption of a constant-area mixing process, imposes unnecessary restrictions on the possible outlet states. For example, if heat transfer must occur from one stream to another, the minimum increase in entropy of the cold stream is attained when the heat is added at the highest temperature of the cold stream consistent with the requirements of the dynamical process. There is a comparable statement for the maximum decrease in entropy of the hot stream. Neither of these optimum requirements are necessarily consistent with a constant-area mixing process.

Considerable attention has been given to the effect that molecular weight of the working fluids has on jet compressor performance. Work and Haedrick[3], in 1938, performed an extensive set of experiments using a variety of combinations of driving fluids and induced fluids. They were able to show that molecular weight indeed plays a part in ejector performance, and they were able to derive some empirical relations useful for design work. A clear picture of the way molecular weight enters the process is not apparent from this investigation, however. Another paper by Holton[4] with discussions by various contributors, considers the effect of molecular weight and presents means for using data taken from one ejector operating with given fluids and applying this data to

predicting the performance of the same ejector when operating with different fluids.

Some attention has also been given to the effect of temperature of the working fluids on the ejector performance. Holton and Schulz[5] performed experiments with steam-steam and steam-air ejectors in which the temperatures of the induced fluids were varied. This work is summed up in two charts which show the effect of heating of the induced stream on the flow ratio. Flow ratios were decreased by heating the induced stream. No theoretical explanation for this observation is offered.

One of the requirements for the one-dimensional flow analysis to be valid is that all properties of the stream at any particular cross-section, be uniform over that cross-section. Whenever the study of the process in the ejector considers the mutual effect of two streams in contact but moving at different velocities, the analysis is no longer one-dimensional. It may be two-dimensional or three-dimensional. Goff and Coogan[2] have applied two-dimensional flow analysis based on the work of Tollmein[6], Kuethe[7], and others to the ejector analysis. Their analysis leads to a fairly accurate prediction of flow ratio for air-air ejectors working at subsonic velocity, but is probably not valid for high velocity jets due to the energy assumptions involved. Nevertheless, their contention that the rational analysis of the ejector must consider the two-dimensional aspects of the problem is well founded, and it appears that the eventual perfection of the theory must consider the multidimensional flow process.

This brief backgroundon jector theory is admittedly far from complete, but it is presented in order to fix a base from which a new start may be made. In searching about for a possible approach, there is a temptation to take the general equations of fluid dynamics (the Navier-Stokes relation) and to attempt to apply these equations to the ejector process, but an examination of the background of these equations reveals that no general solution to them has been found, and those special cases where solutions are known are hardly applicable to ejector theory. At the present time, therefore, this does not appear to be a hopeful approach.

One of the things that seems to stand out in an examination of the present ejector theory, is that there is little considerations given to basic thermodynamics. For example, it is well known that if two gases are mixed together there is generally an increase in entropy of the system that is characteristic, and does not depend on other processes that may be occurring in the gases.

It is planned, therefore, to try and analyze the ejector process in such a way that the effect of processes of this type are more easily recognizable, so that their contributions to the overall process may be comprehended and evaluated.

Since the attempt is primarily to develop basic ideas, the analysis will be based on perfect gas theory. In general, it will be assumed that the flow is in frictionless ducts, that one-dimensional flow assumptions are satisfied where used, that the fluids are gases and the Gibbs-Dalton law applies.

II. CRITERION OF PERFORMANCE FOR THE EJECTOR

In order to establish a criterion of performance for the ejector, a limiting process is needed which will define the ultimate performance in a manner analogous to the way the isentropic process defines the ultimate performance of a turbine-compressor combination. In searching for this limiting process, certain characteristics of the ejector are observed. It is apparent, for example, that the output of the device consists of the two streams (driver and induced) mixed together at the same temperature and pressure.

What is needed is a reversible process which will operate on the driving fluid, starting at its stagnation state and transforming it to the delivered stagnation state. The process must also operate similarly on the induced fluid. All transfers of heat and work must be internal to the process, since the process is assumed adiabatic and no shaft work is involved. In principle, the manner in which the heat and work transfers are made is immaterial. These transfers could be made in either a static or dynamic system as long as it was possible to retain the reversible processes for each stream.

Such a process may be devised by allowing each fluid (driver and induced) to undergo an isentropic change of state followed by an isothermal change of state, Figure 1. Concerning each fluid, certain work and heat quantities will be involved in its particular process. These quantities must balance for the two fluids.

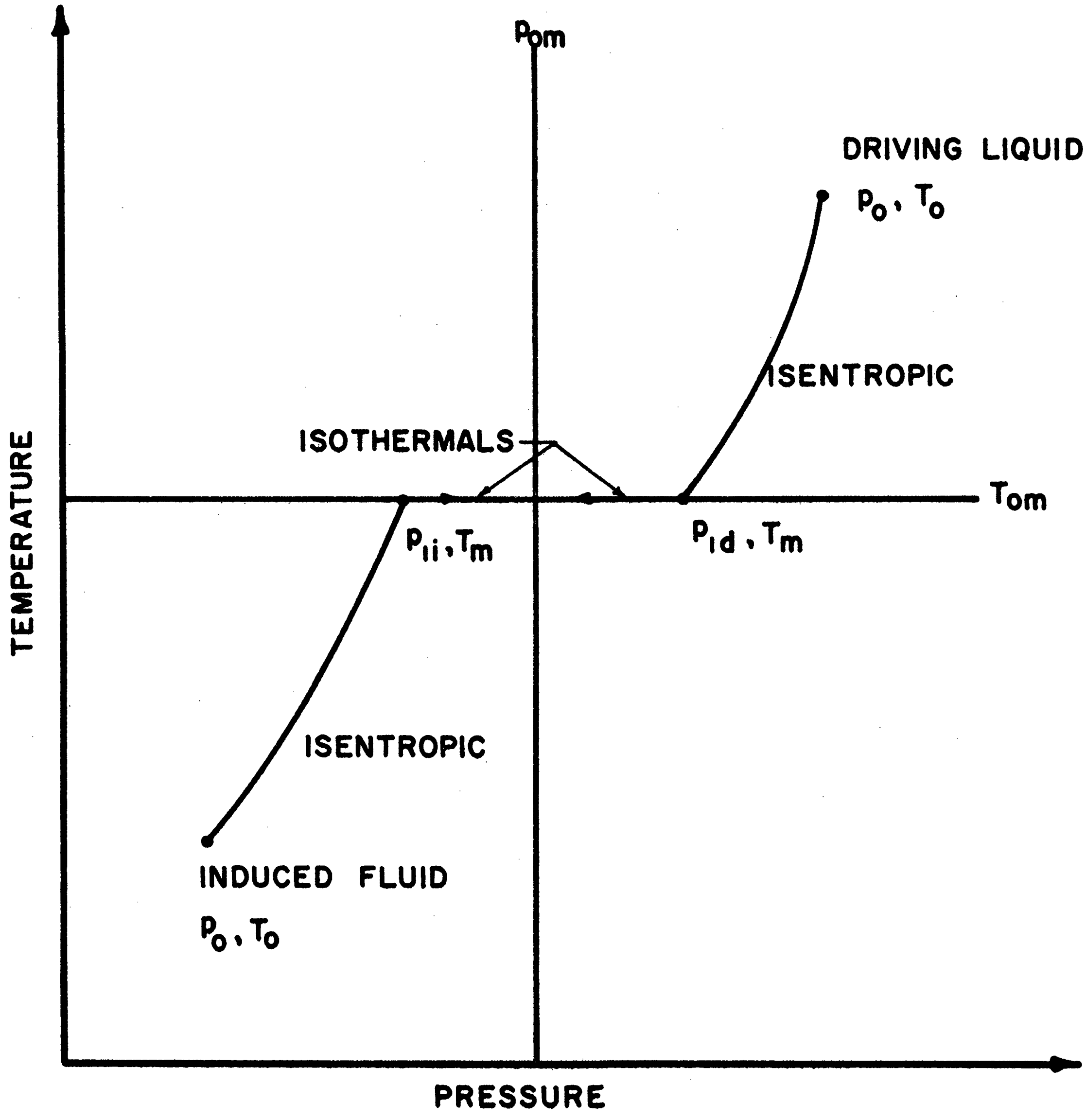


Figure 1. The Limiting Process for Ejectors.

Considering the driving fluid, the following may be written.

Isentropic process

$$Q = 0$$

$$\Delta S = 0$$

$$W = w_d C_{pd}(T_{od} - T_m)$$

Isothermal process

$$Q = W = w_d R_d T_m \ln \frac{p_{1d}}{p_m}$$

$$\Delta S = w_d R_d \ln \frac{p_{1d}}{p_m}$$

For the induced fluid:

Isentropic process

$$Q = 0$$

$$\Delta S = 0$$

$$W = w_i c_{pi}(T_{oi} - T_m)$$

Isothermal process

$$Q = W = w_i R_i T_m \ln \frac{p_{1i}}{p_m}$$

$$\Delta S = w_i R_i \ln \frac{p_{1i}}{p_m}$$

T_m may be immediately determined by the application of the first law, as follows:

$$w_d C_{pd} T_{od} + w_i C_{pi} T_{oi} = w_m C_{pm} T_{om}$$

but

$$w_m = w_d + w_i \quad \text{and} \quad C_{pm} = \frac{1}{w_d + w_i}\left(w_d C_{pd} + w_i C_{pi}\right)$$

$$\therefore \quad T_{om} = \frac{w_d C_{pd} T_{od} + w_i C_{pi} T_{oi}}{w_d C_{pd} + w_i C_{pi}} \qquad (1)$$

But $T_m = T_{om}$ for the isothermal process and the intermediate pressures p_{1d}

and p_{1i} may be found by the application of the isentropic relationship using the value of T_m from above.

$$p_{1d} = p_{od} \left(\frac{T_m}{T_{od}} \right)^{\gamma/\gamma-1}$$

$$p_{1i} = p_{oi} \left(\frac{T_m}{T_{oi}} \right)^{k/k-1}$$

Since the only heat effects occur in the isothermal processes and since $Q_d + Q_i = 0$, the following may be written

$$w_i R_i T_m \ln \frac{p_{1i}}{p_{om}} + w_d R_d T_m \ln \frac{p_{1d}}{p_{om}} = 0$$

Substituting isentropic values for p_{1i} and p_{1d} gives

$$w_i R_i \ln \frac{p_{oi} \left(\frac{T_m}{T_{oi}} \right)^{k/k-1}}{p_{om}} + w_d R_d \ln \frac{p_{od} \left(\frac{T_m}{T_{od}} \right)^{\gamma/\gamma-1}}{p_{om}} = 0$$

Which, with some manipulation, becomes

$$p_{om} = \left[p_{oi} \left(\frac{T_m}{T_{oi}} \right)^{k/k-1} \right]^{\frac{w_i R_i}{w_i R_i + w_d R_d}} \left[p_{od} \left(\frac{T_m}{T_{od}} \right)^{\gamma/\gamma-1} \right]^{\frac{w_d R_d}{w_i R_i + w_d R_d}} \tag{2}$$

The heat effects in the isothermal processes are equal, but opposite for each fluid so that

$$Q_d + Q_i = 0$$

$$\frac{Q_d}{T_m} + \frac{Q_i}{T_m} = 0 \tag{3}$$

$$\Delta S_d + \Delta S_i = 0$$

Therefore, there is a change in entropy for each fluid. These changes are equal in amount, but opposite in sign. The entropy change for the system (driver and induced) is zero.

A limiting process has been devised which brings the two fluids to the same state. They are as yet unmixed. If the fluids are now allowed to mix by the removal of a partition, the resulting mixture will be unchanged in pressure and temperature. If, however, the mixing had been allowed to occur reversibly, i.e., through the agency of a diffusion engine, some work would have been obtained which could be applied to increasing the final pressure p_{om}.

This process is a limiting process for all heat engines, but since the ejector characteristically mixes the fluids irreversibly, then the process without diffusion is the one which applies to the ejector.

The pressure p_{om} defined by Equation (2), is the ultimate pressure which may be obtained by any ejector working with the flow quantities w_d and w_i at the stagnation states specified. It may be that the final pressure p_{om} is to be specified. In this case, the process defines the limiting flow ratio which may be obtained. This is:

$$\frac{w_i}{w_d} = -\frac{R_d}{R_i} \frac{\ln \frac{p_{od}}{p_{om}} \left(\frac{T_m}{T_{od}}\right)^{\gamma/\gamma-1}}{\ln \frac{p_{oi}}{p_{om}} \left(\frac{T_m}{T_{oi}}\right)^{k/k-1}} \tag{2'}$$

III. INTERNAL PROCESSES IN THE EJECTOR

A. Sources of Irreversibility

The precise solution of an ejector design is certain to be difficult inasmuch as this device represents a most complex case of the application of fluid dynamics. In addition to the usual problems of fluid dynamics, there are the additional factors of heat and work processes relative to the two streams. The mixing of the two streams is a further complicating factor. Because of this, it is unlikely that a highly precise solution to the problem can be found, but there is reason to believe that a considerable improvement can be made.

On considering practical ejectors it is generally noted that their measured operating performance falls far short of that predicted by the limiting process. (Values of 20-30 per cent of the limiting flow ratio are representative.) Since this is true, it is evident that irreversible processes occur in the ejector. Sources of irreversibility can be listed as follows:

1. Temperature differences in the two streams.
2. Composition differences in the two streams.
3. Velocity differences in the two streams.
4. Friction in the flow duct.
5. Hydrodynamic shock in the flow.

The first three of these listed sources may be considered inherent characteristics of the ejector, and will be examined in this analysis. Friction, while it has an effect on performance, is not inherently a characteristic

of the process. The question of shock will be deferred until later.

B. Analysis of Ejectors Operating with Identical Gases

1. Pressure, Velocity and Temperature Relations for a Single Gas

Before the ejector proper is considered it will be well to review and develop some of the flow relations for a perfect gas. For the flow process, the following may be written

$$w = C_p \Delta T = C_p T_0\left[1-\left(\frac{p}{p_0}\right)^{\gamma-1/\gamma}\right] = \frac{\gamma}{\gamma-1} R T_0\left[1-\left(\frac{p}{p_0}\right)^{\gamma-1/\gamma}\right] = \frac{\gamma}{\gamma-1} p_1 v_0\left[1-\left(\frac{p}{p_0}\right)^{\gamma-1/\gamma}\right]$$

Since the work may be applied to the gas in the form of kinetic energy, there is of the several relationships, the following:

$$KE = \frac{V^2}{2g} = \frac{\gamma}{\gamma-1} p_0 v_0\left[1-\left(\frac{p}{p_0}\right)^{\gamma-1/\gamma}\right] = C_p T_0\left[1-\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

From this relationship between p and V it is noted that if p becomes equal to zero, there is a maximum velocity attained by the gas which is,

$$V_l = \sqrt{2g\, C_p\, T_0}$$

Also writing the isentropic temperature relations

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\gamma-1/\gamma}$$

it is noted that when p becomes equal to zero, T also is equal to zero.

Thus, the pressure-temperature velocity relations of the gas may be plotted. When these relations are overlaid on the same chart, curves of the type shown in Figure 2 are obtained. It is noted that starting with a gas having stagnation conditions p_0, T_0, the limiting velocity is attained when p and T are both equal to zero. Furthermore, changing the stagnation pressure of the gas to p_0' does not affect the limiting velocity of the gas, as this is determined solely by T_0.

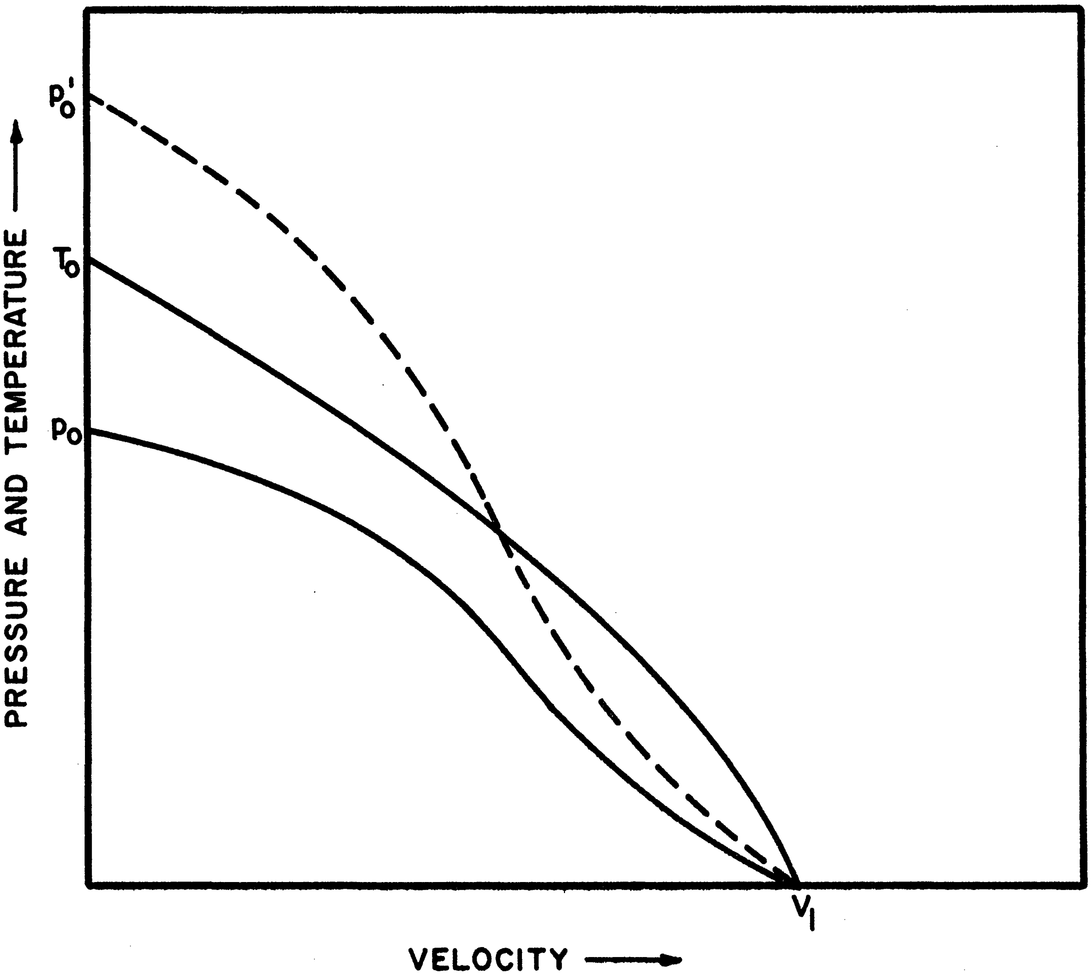


Figure 2. Isentropic Pressure, Temperature, and Velocity Relations for a Perfect Gas.

2. The Case for Which the Induced Gas has the Higher Stagnation Temperature

With this background on gas flow properties, the analysis can now proceed to a case involving an ejector. It is assumed that both the driving and induced gases are expanded through nozzles and the outlets from these nozzles flow parallel into a mixing tube. The conditions where the induced gas has the higher stagnation temperatures will be the first case considered.

(The driver is always taken as the gas having the higher stagnation pressure.) Under these assumptions, the induced stream will have a greater limiting velocity than the driving stream. The p-T-V relations for the two streams without mixing are shown graphically in Figure 3. It is noted at once that there is a common pressure, p_c, to which the two streams may be expanded which result in their having the same velocity, V_c. Under these conditions, the irreversibility due to velocity difference is eliminated, leaving temperature difference as the remaining source of irreversibility.

The various relations are based on the concept of bringing the two streams to a common pressure, as this is a necessary condition at the beginning of the combining process.

Concerning the temperature difference of the two streams when they are at p_c and V_c, it is of interest to note that at other conditions where the velocities are equal (but not the pressures) the temperature difference is the same as at p_c and V_c. This is because the energy necessary to attain a given velocity comes at the expense of the temperature of the fluid. Equal velocities, therefore, exact equal changes in temperature of the fluid, thus

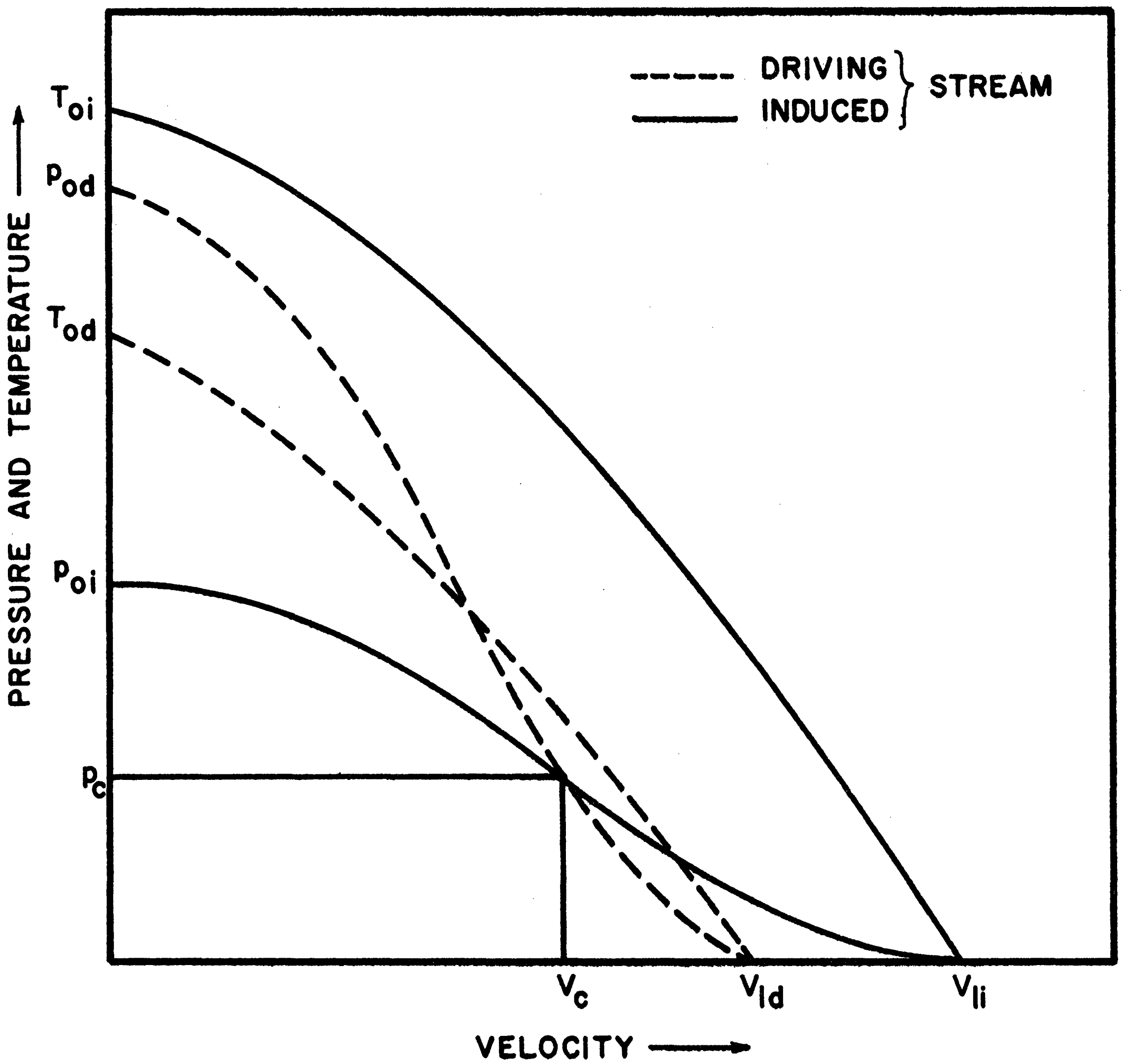


Figure 3. Isentropic p-T-V relations for Identical Gases.

maintaining a constant difference in temperature.

The temperature differences related to the condition where the streams are at equal pressures but not equal velocities are of greater significance to the jet compressor analysis. In this case, the difference in temperature of the two streams is not constant, but converges as the common pressure is reduced. It can be shown (Appendix I) that the irreversibility due to temperature differences following this relation is constant and is independent of p_c. Therefore, the problem of minimizing total irreversibility lies with reducing the difference in velocity. For the example being studied, the velocity difference can be reduced to zero.

It is of significance to note at this point how pumping occurs in a process where the two streams have equal velocities. In effect, there is an increase in density of the induced stream as it mixes with the colder driving stream. As this occurs without change in the overall pressure and volume of the streams, there is no change in velocity.

The total temperature of the driving stream has been increased and that of the induced stream has been decreased. This will lead to a reduction of the stagnation pressure of the driving stream and to an increase of the stagnation pressure of the induced stream (Appendix II). This is the required condition for compression of the induced stream.

There is a unique set of conditions for the case of like fluids and that is where the stagnation temperatures of the driving stream and the induced stream are the same. In this case, if p_c is taken to be zero pressure, then both streams attain the same limiting velocity at zero

temperature. Under these conditions, there are no inherent irreversibilities in the process. In the entire spectrum of ejector operating conditions, this appears to be the only case where entirely reversible operations may be attained, excluding flow losses.

For the case under consideration, where the induced stream has the higher stagnation temperature, the relations between the pressure for equalizing velocities and the stagnation states can be developed.

$$\frac{V_d^2}{2gC_p} = T_{od}\left[1-\left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{V_i^2}{2gC_p} = T_{oi}\left[1-\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Since the velocities are to be equalized $V_i = V_d$, and

$$T_{od}\left[1-\left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}}\right] = T_{oi}\left[1-\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

from which

$$p_c = \left[\frac{T_{oi} - T_{od}}{\dfrac{T_{oi}}{p_{oi}^{\frac{\gamma-1}{\gamma}}} - \dfrac{T_{od}}{p_{od}^{\frac{\gamma-1}{\gamma}}}}\right]^{\gamma/\gamma-1} \qquad (3)$$

Where the induced stream has the higher stagnation temperature, p_c will always be found to be real and less than p_{oi}. For the general case of stagnation states p_c may be $> p_{oi}$ or p_c may be imaginary. These latter cases have no significance to this case of the analysis.

At this point, it is necessary to consider in detail, the process regulating the future state of the two streams after they have been brought separately to the same pressure and velocity. For this purpose they will

be assumed to mix in a constant area duct without wall friction or wall heat transfer. By these assumptions the conservation equations may be written as follows:

Material $$K V_c = \frac{A_m V_m}{v_m}$$

Impulse-Momentum $$\frac{K V_c}{g A_m}\left(V_c - V_m\right) = \left(p_m - p_c\right) A_m$$

Energy $$\frac{A_m}{K} p_c + \frac{V_c^2}{2g} = p_m v_m + \frac{V_m^2}{2g}$$

where $K = \left(\frac{A_d}{v_d} + \frac{A_i'}{v_i'}\right)$

$A_m = A_d + A_i'$

$J = \gamma/\gamma - 1$

m = subscript relating to completely mixed streams.

i = subscript relating to induced stream.

d = subscript relating to driving stream.

These relations may be solved for p_m, V_m, v_m and give the following results,

+ Case

$$v_m = \frac{A_m}{K}$$

$$V_m = V_c$$

$$p_m = p_c$$

− Case

$$v_m = \frac{A_m}{(2J-1)K}\left(1 + \frac{2Jg A_m p_c}{K V_c^2}\right)$$

$$V_m = \frac{V_c}{(2J-1)}\left(1 + \frac{2Jg A_m p_c}{K V_c^2}\right)$$

$$p_m = \frac{2(J-1) K V_c^2}{(2J-1)\, g A_m} - \frac{p_c}{(2J-1)}$$

where the (+) case represents the result obtained from the positive value of the radical in the quadratic solutions and the (−) case accordingly. These two cases of course represent the subsonic and supersonic solutions

of the possible flow at the mixed station of the duct. Under the assumed conditions the (+) case is the proper choice to be made for the following reasons. In the first place, the (+) case will be either subsonic or supersonic. If the (+) case is subsonic, then the (-) case will represent a supersonic solution which would require a spontaneous decrease of entropy of the two streams entering the process which contravenes the second law of thermodynamics and so, is assumed impossible. On the other hand, if the (+) case is supersonic, then the (-) case represents a subsonic solution. The assumptions of this subsonic solution as by Elrod[1] is unduly pessimistic as under the assumption of frictionless flow, the only process which could produce it is hydrodynamic shock, which as will be seen later, can be avoided by proper subsequent treatment of the mixed flow.

Thus for this special case where the induced and driving streams may be brought to a common pressure and velocity before the mixing process is begun, a simple set of relations determine the state after mixing. Also inasmuch as there is no fluid shear between the two streams during the mixing, the only increase in entropy is due to the irreversible heat transfer between the two streams and according to Appendix I this is a constant entropy increase whether or not the velocities of the two streams are equalized before mixing.

The condition of the two streams before mixing where they have equal velocity and pressure thus appears to be the best possible condition for ejector performance where the two streams are of identical gas with the induced stream having the higher stagnation temperature.

After the best possible properties of the mixed stream are determined, it is possible by assuming subsequent isentropic flow to determine the conditions possible in the final delivered stagnation state.

Having computed p_c for which the velocity of the two stream is V_c, the stream temperatures before mixing are given by the isentropic relations

$$T_d = T_{od}\left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_i = T_{oi}\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}$$

The equalization temperature after mixing is given by

$$T_m = \frac{w_d T_d + w_i T_i}{w_d + w_i} = \frac{w_d T_{od}\left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}} + w_i T_{oi}\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}}{w_d + w_i}$$

The stagnation pressure of the mixed flow is given by the isentropic flow relations

$$p_{om} = p_c\left[\frac{V^2}{2g C_p T_m} + 1\right]^{\gamma/\gamma-1} \qquad (4)$$

The limiting flow ratio pertaining to a selected final pressure, p_{of}, can be derived from the isentropic flow relation in the form

$$\frac{V_c^2}{2g C_p} = T_m\left[\left(\frac{p_{om}}{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

If the value of T_m above is substituted into this equation and the result solved for w_i/w_d, the following is obtained

$$\frac{w_i}{w_d} = \frac{T_{od}\left[\left(\frac{p_{om}}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}}\right] - \frac{V_c^2}{2g C_p}}{T_{oi}\left[\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{p_{om}}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}\right] + \frac{V_c^2}{2g C_p}} \qquad (5)$$

As an example of the use of this relation, consider two fluids whose states are as follows:

Driving Fluid - Air, 400°R, 100 Psia - Stagnation

Induced Fluid - Air, 600°R, 10 Psia - Stagnation

If the delivered pressure p_{of} is chosen to be 100 psia, then it is obvious that zero quantity of induced stream can be handled and the flow ratio must be zero.

For the conditions stated, according to Equation (3)

$$p_c = \left[\frac{T_{oi} - T_{od}}{\dfrac{T_{oi}}{p_{oi}^{\frac{\gamma-1}{\gamma}}} - \dfrac{T_{od}}{p_{od}^{\frac{\gamma-1}{\gamma}}}}\right]^{\gamma/\gamma-1} = \left[\frac{600-400}{\dfrac{600}{(1440)^{.286}} - \dfrac{400}{(14400)^{.286}}}\right]^{3.5}$$

p_c = 0.935 psia

Also by the isentropic flow relation,

$$\frac{V_c^2}{2g\,c_p} = T_0\left[1 - \left(\frac{p_c}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

applied to the induced stream,

$$\frac{V_c^2}{2g c_p} = 297\ ^\circ R$$

With p_{om} specified as 100 psia and using Equation (5)

$$\frac{w_i}{w_d} = \frac{400\left[1^{.286} - 0.00935^{.286}\right] - 297}{600\left[0.0935^{.286} - 10^{.286}\right] + 297} = 0$$

If p_{om} had been specified as 50 psia, the limiting flow ratio is w_i/w_d = 0.205. If, on the other hand, the process were totally reversible, i.e., no irreversible temperature equalization, the final pressure for a flow ratio of 0.205 would be 70 psia instead of 50 psia.

This is found by applying the relation for the reversible process (Equation 1) to the assumed conditions. For convenience let $w_i = 0.205\ w_d$ and since $C_{pd} = C_{pi}$, Equation (1) reduces to

$$T_m = \frac{w_d T_{od} + w_i T_{oi}}{w_d + w_i} = \frac{w_d (400) + 0.205 w_d (600)}{w_d + 0.205\ w_d} = 443\ ^\circ R$$

and by Equation (2)

$$p_{om} = \left[p_{oi} \left(\frac{T_m}{T_{oi}} \right)^{\gamma/\gamma-1} \right]^{\frac{w_i}{w_i + w_d}} \left[p_{od} \left(\frac{T_m}{T_{od}} \right)^{\gamma/\gamma-1} \right]^{\frac{w_d}{w_i + w_d}}$$

$$p_{om} = \left[1440 \left(\frac{433}{600} \right)^{3.5} \right]^{0.17} \left[14400 \left(\frac{433}{400} \right)^{3.5} \right]^{0.83} = 10060\ psfa$$

$$= 70\ psia$$

To continue with the example as started,

where p_{od} = 100 psia, p_{oi} = 10 psia, T_{od} = 400°R, T_{oi} = 600°R and from which the optimum pressure of mixing p_c, has been calculated as 0.935 psia, it will be noted that both streams are in supercritical flow, and the rule of converging-diverging nozzles must be followed. By assigning an absolute flow rate to the ejector, and taking the specified final stagnation pressure of 50 psia as before, which results in the determined maximum flow ratio w_i/w_d = 0.205, there results the following: for the nozzle throat conditions.

$$p_{td} = \left(\frac{2}{\gamma+1} \right)^{\gamma/\gamma-1} p_{od} \qquad = 0.52\ p_{od} = 52 \text{ psia}$$

$$p_{ti} = \left(\frac{2}{\gamma+1} \right)^{\gamma/\gamma-1} p_{oi} \qquad = 0.52\ p_{oi} = 5.2 \text{ psia}$$

$$T_{td} = T_{od}\left(p_{td}/p_{od}\right)^{\gamma-1/\gamma} = 400\left(0.52\right)^{0.286} = 332\ ^{\circ}R.$$

$$T_{ti} = T_{oi}\left(p_{ti}/p_{oi}\right)^{\gamma-1/\gamma} = 600\left(0.52\right)^{0.286} = 497\ ^{\circ}R.$$

$$v_{td} = \frac{R\,T_{td}}{p_{td}} = \frac{(53)(322)}{(52)(144)} = 2.35\ ft^3/lb.$$

$$v_{ti} = \frac{R\,T_{ti}}{p_{ti}} = \frac{(53)(497)}{(5.2)(144)} = 35.2\ ft^3/lb.$$

$$V_{td} = \sqrt{2g\,C_p\,T_{od}\left[1-\left(p_{td}/p_{od}\right)^{\gamma-1/\gamma}\right]} = 905\ ft/sec.$$

$$V_{ti} = \sqrt{2g\,C_p\,T_{oi}\left[1-\left(p_{ti}/p_{oi}\right)^{\gamma-1/\gamma}\right]} = 1110\ ft/sec$$

Take w_i = 1.0 lb/sec from which w_d = 4.87 lb/sec. so that,

$$A_{td} = \left(\frac{w_d v_d}{V_d}\right)_t = \frac{(4.87)(2.35)}{905} = 0.012\ ft^2 = 1.73\ in^2$$

$$A_{ti} = \left(\frac{w_i v_i}{V_i}\right)_t = \frac{(1.00)(35.2)}{1110} = 0.032\ ft^2 = 4.60\ in^2$$

Similarly at the entrance to the mixing tube, and using p_c = 0.935 psia the areas are found to be

A_d = 0.1065 ft^2	=	15.3 sq in	
A_i = 0.064 ft^2	=	9.2 sq in	
Total		24.5 sq in	

In this particular case, the mixing tube is a constant area tube having an area of 24.5 sq in. The area may be constant since temperature equalization is the only necessary process. This results in a single set of outlet states from the mixing tube.

The analysis so far is compatible with the observations of Holton and Schulz[5] where decreased flow ratio (induced to driver) was obtained on heating the induced stream. With the best designed ejector, there is an increased temperature irreversibility as the induced stream is heated to even higher temperature.

(a) An Operating Cycle for the Ejector

At this point it is possible to devise a relationship which with some logic can be termed an ejector cycle. The cycle only applies to the case of identical fluids with the induced stream having the higher stagnation temperature. A diagram is constructed (Figure 4) similar to Figure 3 which shows p-V-T relations for the two fluids. First the p-V-T relations for the driving fluid and the induced fluid are entered on the diagram using the isentropic relations,

$$V^2 = 2g\,c_p\,T_0\left[1 - \left(p/p_0\right)^{\gamma-1/\gamma}\right]$$

$$T = T_0\left(p/p_0\right)^{\gamma-1/\gamma}$$

When this has been done, the point where velocities and pressures are equal is located. This is p_c, V_c. Next select some pressure between p_{od} and p_{oi} that is to be the delivery pressure. Since this point and the point p_c, V_c lie on the isentropic p-V curve for the mixture of the two streams, the limiting velocity $V_{\ell m}$ of the mixture is determined. This gives T_{om}, the stagnation temperature of the mixture since $(w_d+w_i)T_{om} = w_dT_{od} + w_iT_{oi}$

$$\frac{w_i}{w_d} = \frac{T_{om} - T_{od}}{T_{oi} - T_{om}}$$

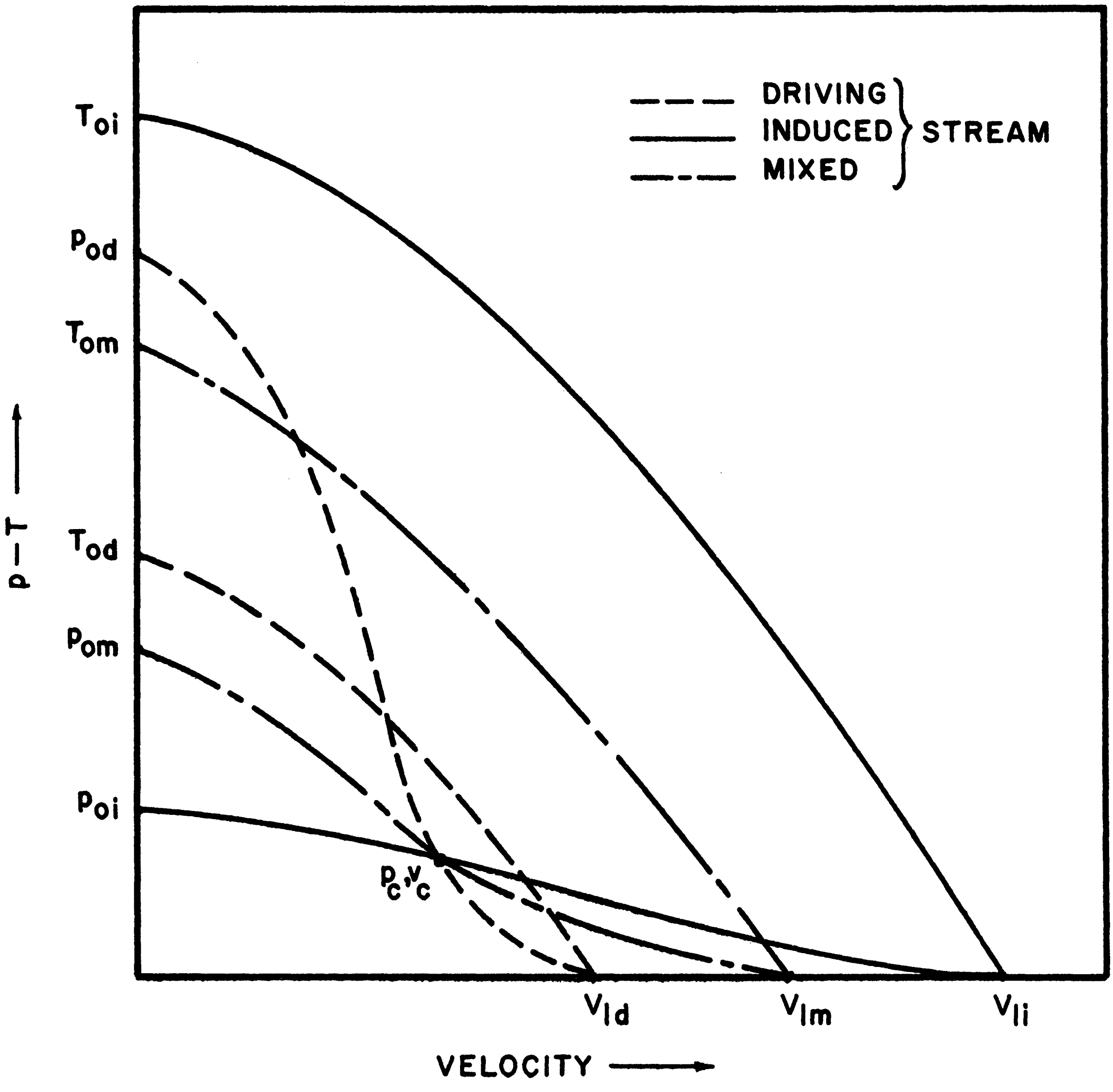


Figure 4. The Ejector Cycle for Identical Gases.

Conversely, w_i/w_d may be chosen and the procedure reversed to find p_{om}.

3. The Case for Which the Driving Fluid has the Higher Stagnation Temperature

In this case, it is impossible to eliminate either temperature or velocity irreversibility and means must be developed to evaluate the performance under these conditions. It will still be assumed that the two streams are brought to a common pressure, p_c, by isentropic processes since there appears to be no possible advantage to be gained by doing otherwise.

It is shown in Appendix I that the pressure losses due to temperature irreversibility are constant, and that the means to improve performance is in minimizing the losses due to velocity differences in the two streams. In general, these differences will be minimized at the limiting velocities of the two fluids, but practically this would not be attempted since the flow losses would overcome the advantage to be gained at minimum velocity difference.

In all cases, the basic problem can be stated in terms of the increase of entropy in the ejector process. If this can be determined, then the final delivered state of the stream is fixed. By assuming reversible processes, it is assured that the overall increase in entropy of all participating substances is zero, and a final delivery state is determined. This is what was done earlier in the analysis to arrive at the limiting process. For irreversible processes, the entropy will increase in a degree dependent on the actual process, and so the analysis must always return to a consideration of the process itself for an ultimate answer.

What then is the actual process of an ejector? Up to now, cases have been considered in which the process is simplified by the suppression of velocity differences in the two streams. In the general case, however, there are two streams at a common pressure but at different velocities and temperatures meeting in more or less parallel flow and mutually interacting. The nature of this interaction is of importance to the problem. Experience and analysis have developed the idea of turbulent flow resulting from this interaction. In turbulent flow are identified the process of fluid shear, the concept of translational kinetic energy, and vortex kinetic energy. Modern analyses have also identified vibrational or acoustical energy as associated with turbulent flow. The resulting picture is thus one of complex fluid motion in which heat transfer or temperature equalization is taking place. There is little hope that this model of the process can ever be expected to serve as a basis for a solution to the problem. If a simplification is to be sought, it must be in a model that describes the predominant process in the ejector.

As an example of a process, imagine a purely elastic collision occurring between the two streams. This might be accomplished by introducing the two streams into a common duct in the manner of tandem packages instead of parallel streams. In this case, the energy of collision could be retained in the packages as acoustical energy. This energy would, of course, begin to degrade into thermal energy, but this degradation or the major part of it might be delayed until recompressions were taking place in the diffusor of the ejector. Thus, heat would be added to the stream at constantly increasing pressures, which process would tend toward lower entropy in the delivered state and a higher value of p_{om}.

This is an interesting process, and might serve as a basis for significant improvement in ejector operations, but it does not appear to be representative of the process in the ejector that is presently used. The process probably occurs to some extent but a consideration of the turbulent state precludes any significant effects.

On the other hand, pure fluid shear may be taken as the predominant interaction between the two streams. The computation of the heat generated in a fluid element due to simple fluid shear may be made fairly readily (Appendix IV), but the resultant increase in entropy of the element due to the added heat depends on the process followed during heat addition. In particular cases, this process might involve constant volume or constant pressure, but the general process will be described in terms of the flow equations and the boundary conditions. Therefore, the evaluation of the entropy increase requires a detailed analysis of the ejector process and imposes essentially the same difficulties as the evaluation of the stagnation pressure which is actually the property to be determined.

It appears then, that for the present, some compromise with accuracy must be taken in order to obtain a practical solution to the general ejector process.

What will be done is to break down the ejector process into separate operations each of which follows a relatively simple process that is subject to practical manipulation. By doing this, certain information values are lost which contribute to the accuracy of the solution. This is essentially what happens when the ejector process is analyzed from terminal conditions only. In the proposed scheme, however, not as many

information values will be lost, because the analysis will consider at least partially the internal process in the ejector.

Since in general, the inlet flow at the mixing section consists of two streams at equal pressure, but at different velocity and temperature, the first operation may consist of temperature equalization. This must be done by heat transfer without mixing as the velocity equalization is to be withheld until later.

(a) Equalization of Temperature

A constant pressure process will be selected for the temperature equalization. It happens that for this process, a particularly simple set of relations are involved. The following terms describe the conditions for a constant pressure process without wall friction or shock.

$$V_2 = V_1$$

$$p_2 = p_1$$

$$T_2 = T_1 + Q/c_p$$

$$A_2 = A_1\, T_2/T_1$$

These relations are derived in Appendix V. According to this process, the velocities and pressures of the two streams are unaffected by the temperature equalization.

The equalization temperature may be found as follows.

$$T_m = T_{1i} + \frac{Q_i}{c_p w_i}$$

$$T_m = T_{1d} + \frac{Q_d}{c_p w_d}$$

However, since heat only flows from one stream to the other, $Q_d + Q_i = 0$.

and $$c_p w_i (T_m - T_{1i}) + c_p w_d (T_m - T_{1d}) = 0$$

From which $$T_m = \frac{w_i T_{1i} + w_d T_{1d}}{w_i + w_d}$$

Also $$A_{2d} = A_{1d} \frac{T_m}{T_{1d}}$$

$$A_{2i} = A_{1i} \frac{T_m}{T_{1i}}$$

The result of the application of this process is that the two streams have been brought to a common temperature, T_m, without change in pressure or velocity of either stream. There will generally be an area change for each stream and also a change in the total area of both streams.

It is to be noted that the process chosen for equalization of temperature is not the only possible process. It is not necessarily the optimum process for minimum increase in entropy of the system. Actually, the only thing that the jet pump process demands is that the pressures of the two streams be approximately equal at any point on the flow axis. However, the process is probably nearly optimum, for it is necessary that the pressures of the two streams either decrease simultaneously or increase simultaneously.

If there is a pressure increase, then Q is being added to one stream at increasing pressure and removed from the other stream at increasing pressure. In the first case, this tends toward a minimum increase in entropy, and in the latter case, it tends toward a minimum decrease in entropy. On the other hand, if the process is one of decreasing pressure, the heated stream tends toward a maximum increase in entropy and the cooled stream tends toward a maximum decrease in entropy.

Thus, the two processes are compensatory with respect to entropy change and the entropy of the system tends to stay constant. The constant pressure process lies between the extremes noted here.

(b) Equalization of Velocity

At this point, the next operation is the equalization of velocity. This operation might be pictured in an actual ejector as involving shear on both streams. Even this simplified assumption involves great difficulties in the solution of the conservation equations, so a different approach is used. It is assumed that the friction between the two streams acts as a body force instead of as a boundary force. It is, furthermore, assumed that the heat of friction is withheld until the velocities are equalized by the body forces, after which the heat is added to the combined streams. It is known that reversible work may be extracted from a flowing stream without altering its static condition. It is assumed that work is so extracted from the driving stream by a body force that represents the friction force between the two streams.

It is further assumed that reversible work is applied to the induced stream through this force. Therefore:

Power from driving stream $= FV_d$

Power applied to induced stream $= -FV_i$

Power to friction $= -F(V_d - V_i)$

$$\Sigma P = FV_d - FV_i - F(V_d - V_i) = 0$$

It is assumed that the energy to friction is set aside from the process for later application.

From consideration of the stream geometry

$$W_d = F dx$$

$$P_d = F\left(\frac{dx}{dt}\right)_d = F V_d$$

$$w_i = -F dx$$

$$P_i = -F\left(\frac{dx}{dt}\right)_i = -F V_i$$

Therefore, in the time dt

$$d(KE)_d = F V_d dt = \frac{w_d}{g} V_d \, dV_d$$

$$\frac{dV_d}{dt} = \frac{Fg}{w_d}$$

Similarly

$$\frac{dV_i}{dt} = -\frac{Fg}{w_i}$$

So that

$$\frac{dV_i}{dV_d} = -\frac{w_i}{w_d}$$

This result signifies that for the operation chosen, the final velocity of the two streams is determinable on a simple momentum exchange basis, or

$$V_m = \frac{w_d V_d + w_i V_i}{w_d + w_i}$$

$$KE_d = w_d \frac{V_d^2}{2g}$$

$$KE_i = w_i \frac{V_i^2}{2g} \tag{6}$$

$$KE_m = (w_d + w_i)\frac{V_m^2}{2g} = \frac{(w_d V_d + w_i V_i)^2}{2g(w_d + w_i)}$$

The energy to friction is given by

$$Q = KE_d + KE_i - KE_m = \frac{1}{2g}\left(\frac{w_d w_i}{w_d + w_i}\right)\left(V_d - V_i\right)^2 \qquad (7)$$

By this process both streams are not brought to a common velocity, V_m, and since they were already at equal temperatures and pressures, they may mix without further change. There are, of course, additional area changes in the velocity equalization process. These are:

$$\left.\begin{aligned} A_{3d} &= A_{2d}\,\frac{V_{2d}}{V_{3d}} \\ A_{3i} &= A_{2i}\,\frac{V_{2i}}{V_{3i}} \end{aligned}\right\}$$

by continuity, since v is unchanged for each stream.

By the velocity equalization process, the two streams may be assumed to have lost their identity and may be further treated as one stream.

The next operation is to account for the heat of friction gnerated in the velocity equalization process. This energy was considered as set aside for later application. In order to understand how this heat should be handled, it is necessary to consider the process of heat addition to a stream flowing in a duct. Two cases present themselves. These are for V_m supersonic and V_m subsonic. As the supersonic case is likely to present the most difficulty and is also most likely to occur in ejector operation, it will be considered first.

Supersonic Case of Heat Addition--It is shown in Appendix II that the addition of heat to a flowing stream causes a reduction of the stagnation pressure of that stream. This is detrimental to the performance of the

ejector and since the heat must be added to the stream at this point in the analysis the rule will be followed that the minimum detrimental effect on stagnation pressure is realized when the heat is added at the highest attainable static pressure. This means for the supersonic stream that the heat addition should accompany a compression process. This in general calls for a decrease in the area of the duct. There is, however, a limiting factor in this direction. This is the phenomena of choking. In the case at hand, both thermal and area choking may occur. Area choking is the more common concept and is the condition existing in the throat of a nozzle in critical flow. It is well known that under these conditions, no change in mass flow through the nozzle may occur except as a result of alteration of the up stream state of the fluid, or through changes in the nozzle area. Thermal choking is similar in effect, but is thought of in terms of heat addition to a fluid flowing in a constant area duct. When the flow is supersonic, the addition of heat causes a decrease in velocity and an increase in pressure and temperature of the fluid. This process, if carried far enough, will cause the fluid to attain some velocity, after which if additional heat is added some adjustment in upstream conditions must take place. If the upstream stagnation states are unchanged, then the continued addition of heat, after sonic velocity is attained, will result in a reduction in the mass flow rate through the tube. In the case at hand, it is entirely possible that the heat to be added is sufficient to produce thermal choking in a constant area duct. This would be the case where the ejector design resulted in badly mismatched velocities for the two streams before mixing. In this case, the only recourse is to increase the area of the duct to avoid choking.

Since choking in a supersonic stream is a pressure increasing process, it is appropriate that the process approach choking as nearly as possible while the heat is being added. This will result in the minimum decrease in stagnation pressure of the stream, without altering the upstream condition. The next step in the analysis is, therefore, the development of the quantitative relations governing area and thermal choking.

For analyses of this type, the dimensionless parameter M** has proven useful and convenient, where:

$M = \frac{V}{a}$

V = Velocity of stream.

a = Local velocity of sound in the stream.

Since $a = \sqrt{g\gamma RT}$

$$V^2 = M^2a^2 = M^2 g\gamma RT$$

The analysis begins with the equation of continuity (conservation of mass) which is stated for a constant area duct as:

$$w = \frac{V_1 A}{v_1} = \frac{V_2 A}{v_2} \quad \text{or}$$

$$\frac{w}{A} = \frac{V_1}{v_1} = \frac{V_2}{v_2} = \text{Const.}$$

$$\therefore \frac{V^2}{v^2} = \text{Const.}$$

Substituting $V^2 = M^2 g\gamma RT$ gives

$$\frac{M^2 g \gamma RT}{v} = \text{Const.}$$

and using $v = RT/p$, the equation becomes

$$\frac{M^2 g \gamma RT}{\left(\frac{RT}{p}\right)^2} = \text{Const.}$$

**Mach Number

or

$$\frac{M^2 p^2}{T} = Const.$$

from which

$$\frac{dp}{p} = \frac{1}{2}\frac{dT}{T} - \frac{dM}{M} \tag{8}$$

The conservation of momentum for a constant area duct may be expressed as

$$p_1 A + \frac{w}{g} V_1 = p_2 A + \frac{w}{g} V_2 = Const$$

and since w = VA/v

$$pA + \frac{AV^2}{vg} = Const.$$

substituting $V^2 = M^2 g\gamma RT$ and $v = RT/p$ into the above ~~gies~~

$$p + \frac{M^2 g\gamma RT}{g\frac{RT}{p}} = Const.$$

which reduces to:

From which

$$p(1+\gamma M^2) = Const.$$

$$\frac{dp}{p} = -\frac{2\gamma M dM}{\gamma M^2 + 1} \tag{9}$$

Eliminate dp/p from Equations (8) and (9), and

$$\frac{dT}{T} = \left(\frac{1}{M} - \frac{2\gamma M}{\gamma M^2+1}\right) 2dM = \left[\frac{1-\gamma M^2}{1+\gamma M^2}\right]\frac{dM^2}{M^2} \tag{10}$$

Turning to the energy relationship

$$\frac{V^2}{2g} = c_p(T_0 - T)$$ and using $V^2 = M^2 g\gamma RT$

$$T_0/T = 1 + \frac{\gamma-1}{2} M^2$$ from which

$$\frac{dT_0}{T_0} = \frac{dT}{T} + \frac{\gamma-1}{2\left(1+\frac{\gamma-1}{2}M^2\right)} dM^2 \tag{11}$$

Substituting the value dT/T from Equation (10) into Equation (11) gives

$$\frac{dT_o}{T_o} = \left(\frac{1-\gamma M^2}{1+\gamma M^2}\right)\frac{dM^2}{M^2} + \frac{\gamma-1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\,dM^2$$

which reduces to

$$\int_{T_o^*}^{T_o}\frac{dT_o}{T_o} = \int_{M=1}^{M=M}\frac{1-M^2}{(1+\gamma M^2)\left(1+\frac{\gamma-1}{2}M^2\right)}\,\frac{dM^2}{M^2} \qquad (12)$$

This equation is integrated by parts between the limit To* which is the stagnation temperature associated with thermal choking and corresponds to M = 1, and To which is associated with M = M, and gives

$$\frac{T_o}{T_o^*} = \frac{2(\gamma+1)\left(1+\frac{\gamma-1}{2}M\right.}{(1+\gamma M^2)^2}\,M^2$$

By selecting values of M between 0 and ∞ the corresponding values of To/To* may be determined and arranged in tabular form or may be presented as a chart, Figure 5, for $\gamma = 1.4$.

A similar analysis leads to the dimensionless parameter

$$T/T^* = \left(\frac{\gamma+1}{1+\gamma M^2}\right)M^2$$

and

$$p/p^* \quad \left(\frac{\gamma+1}{1+\gamma M^2}\right)$$

which are also entered in Figure 5, for $\gamma = 1.4$.

Figure 5 contains all the essential information needed to handle problems in thermal choking. The use of this chart will be demonstrated in a subsequent example.

The problem of area choking can be handled in the same way, but is less complex and can, therefore, be managed easily with algebraic

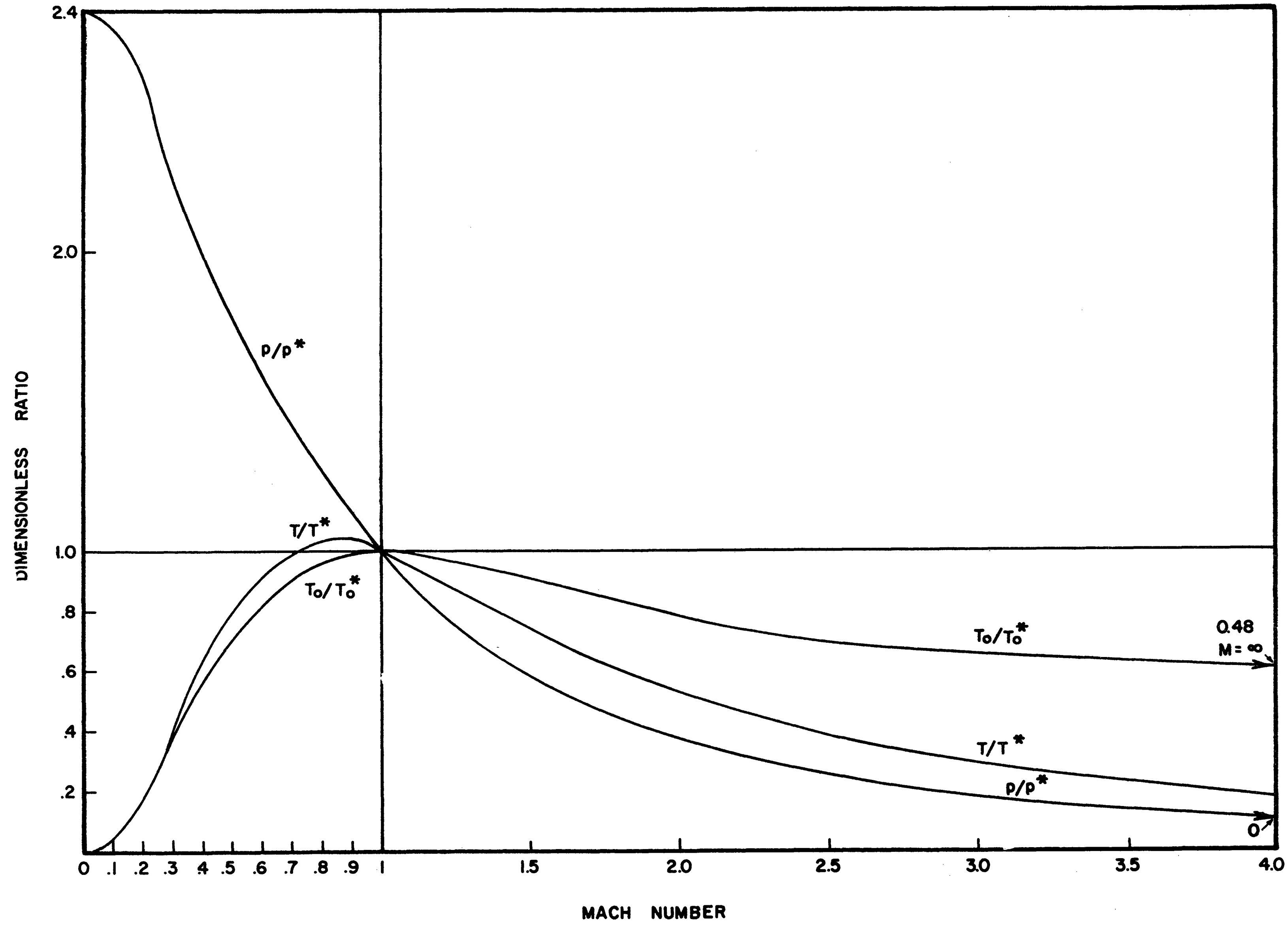


Figure 5. Dimensionless Parameters T_o/T_o^*, T/T^* and p/p^* as Function of Mach Number, for Gas $\gamma = 1.40$.

relations. Begin with the energy relation

$$\frac{V^2}{2g} = C_p T_0 \left[1 - \left(p/p_0\right)^{\gamma-1/\gamma}\right]$$

and substitute $V^2 = M^2 g\gamma RT$

$$\frac{M^2}{2}\gamma RT = C_p T_0 \left[1 - \left(p/p_0\right)^{\gamma-1/\gamma}\right]$$

but $To = T\ (p_o/p)^{\gamma-1/\gamma}$ so that,

$$\frac{M^2}{2}\gamma RT = C_p T \left[\left(p_0/p\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

which reduces to

$$\frac{p_0}{p} = \left[\frac{M^2}{2}(\gamma-1) + 1\right]^{\gamma/\gamma-1}$$

from which the relation may be found

$$\frac{p_2}{p_1} = \left[\frac{\frac{M_1^2}{2}(\gamma-1) + 1}{\frac{M_2^2}{2}(\gamma-1) + 1}\right]^{\gamma/\gamma-1} \qquad (13)$$

A similar analysis gives

$$\frac{T_2}{T_1} = \left[\frac{\frac{M_1^2}{2}(\gamma-1) + 1}{\frac{M_2^2}{2}(\gamma-1) + 1}\right] \qquad (14)$$

Thus, if the condition of a stream is known in state 1 (i.e., w_1, V_1, T_1, p_1) from which M,v,A, etc., may be determined, an isentropic process by area change to a different Mach number, M_2, results in new values of p and T determinable by Equations (13) and (14), and from which V_2, v_2, A_2, etc., may be computed. To produce area choking, M_2 is set at the value, $M_2 = 1$.

At this point, the problem of what to do with the energy of friction can be resolved. The last step in the analysis was the equalization of velocity which gave rise to the friction energy term

$$Q = \frac{1}{2g} \frac{w_d w_i'}{w_d + w_i'} \left(V_d - V_i' \right)^2 \tag{7}$$

At this point the fluids were at identical states and the two streams could be considered as one for further processes. Area changes had been accounted for, so that the area of the combined stream is also known.

Suppose then, that as an example the state of the combined stream is as follows:

Area - 1 ft^2 γ - 1.40

T - 600°R   R - 53

P - 1000 psfa

V - 2000 ft/sec

which also requires

M = 1.67

v = 31.8 ft^3/lb

w = 62.8 lb/sec

Assume that the heat of friction Q amounts to 20 Btu/lb or 15560 ft lb/lb. From Figure 5, and M = 1.67, it is found that To/To* = 0.862 also

$$T_0 = T\left[\frac{\gamma-1}{2} M^2 + 1\right] = 600\left[\left(\frac{1.40-1.00}{2}\right)(1.67)^2 + 1\right] = 935\,°R$$

$\therefore$ To* = 935/.862 = 1084°R

To* represents the stagnation temperature of the flow when it has been brought to thermal choking, and since changes in To are associated with the change in enthalpy in a flow process, the following holds

Δ To = Q/C_p

Therefore, the heat added to the stream to produce thermal choking is

$$Q = C_p \Delta To = C_p(To^* - To) = C_p(1084-935°)$$

$$Q = 30 \text{ Btu} = 28000 \text{ ft lb}$$

It was assumed, however, that only 20 Btu of heat was available for addi tion to the stream. This will not produce thermal choking, so that in order to approach choking, part of the process must be area choking.

Again, note that To for the entering stream is 935°R. The addition of Q = 20 Btu to the stream will raise the stagnation temperature to

$$To^* = 935 + 20/C_p = 935 + 83 = 1018°R$$

This temperature is designated To* because it is desired that it correspond to thermal choking. Thus,

$$To/To^* = 935/1018 = 0.918$$

which determines M from Figure 5. M = 1.45. This is the Mach number that the stream must have that with the additions of Q = 20 Btu will produce thermal choking.

The area change must, therefore, take the stream from its inlet condition M = 1.67 to M = 1.45. Equations (13) and (14) are the connecting equations between these two states. Use of Equation (13) gives

$$p_2 = 1000 \left[\frac{\frac{1.67^2}{2}(1.40-1)+1}{\frac{1.45^2}{2}(1.40-1)+1} \right]^{\frac{1.40}{1.40-1}}$$

$$p_2 = 1384 \text{ psfa}$$

Equation (14) gives

$$T_2 = 600 \times 1.097 = 657°R$$

From which

$$a_2 = \sqrt{g\gamma RT_2} = 1400 \text{ ft/sec}$$
$$V_2 = Ma = (1.45)(1400) = 1814 \text{ ft/sec}$$
$$v_2 = RT_2/p_2 = 25.2 \text{ ft}^3/\text{lb}$$
$$A_2 = wv_2/V_2 = 0.875 \text{ ft}^2$$

This means that a reduction in area of the duct from 1.00 ft^2 to 0.875 ft^2 with the addition of Q = 20 Btu/lb will just produce choking in the flow. The state of the stream will thus be as follows.

$$M = 1.0$$
$$A = 0.875 \text{ ft}^2$$
$$p^* = p_2/0.610 = 1384/0.610 = 2270 \text{ psfa}$$
$$T^* = T_2/0.785 = 657/0.785 = 837°R$$
$$V = 1410 \text{ ft/sec}$$

The final step in the ejector process is the reduction to the stagnation state, of the stream obtained by application of thermal and area choking. This may be done by making use of the isentropic flow relations,

$$p_0 = p\left[\frac{V^2}{2gC_pT} + 1\right]^{\gamma/\gamma-1}$$

and $To = T(p_o/p)^{\gamma-1/\gamma}$

Applying these relations to the example just studied gives:

$$p_0 = p^*\left[\frac{(1410)^2}{(2)(32)(188)(837)} + 1\right]^{3.5} = 2270(1.875) = 4250 \text{ psfa}$$

$$To = T^*(4250/2270)^{0.286} = 837(1.197) = 1000°R$$

<u>Subsonic Case of Heat Addition</u>--This case is most likely to be encountered where the purpose of the ejector is to produce a large induced mass flow

with a small increase in pressure as with over-fire steam-air ejectors for boilers. Conditions are such that thermal choking is not likely to occur even in a constant area duct, but the principle of heat addition at the highest attainable static pressure still applies so that in general the process demands an increasing area as the streams mix.

Thus, the mixing tube and the diffuser are indistinguishable. The theory presented here does not consider the two-dimensional flow process in sufficient detail to predict what the optimum mixing length should be, and to this extent reliance must be placed on experience. An angle of divergence of about 7° has been found to give nearly optimum performance in many diffusers. The subsonic case is, therefore, relatively simple, and any restrictions that may apply to the process can also be handled through charts such as Figure 5, and Equations (13) and (14).

4. Resume of Ejector Design for Identical Fluids

There are two cases to be considered. One case is where the stagnation temperature of the induced stream is higher than that of the driving stream, and in the other case, the temperatures are reversed.

Analysis of the first case by Equation (3), yields a common pressure, p_c, which also equalizes the velocities. Only the temperature irreversibility remains, and this is fixed. The temperature equalization process does not change the pressure and velocity of the stream and is done in a constant area duct. A mean temperature is computed, which fixes the state of the mixed stream. Reduction to stagnation condition is simple and straightforward by Equation (4). The final pressure, is computed easily with Equations (4) and (5).

Analysis of the second case reveals that there is no common pressure to which the streams may be brought that will equalize the velocities. The best that can be done is to bring the streams together at zero pressure under which condition the velocity differences are minimized. Actually this is a limiting process which cannot be attained and which, if it could be attained, would for a practical design result in excessive flow loss. The best pressure to choose cannot be predicted without considering the wall friction in the ejector duct and this is being omitted in this study. If the induced stream is allowed to expand over even a small pressure ratio, it gains considerable speed and reduces the velocity losses.

Once p_c has been selected, a flow ratio is selected. This determines the geometry of the entrance nozzles for both streams and determines their respective exit areas. Converging-diverging nozzles for both streams is the likely case. The two streams are allowed to reach temperature equalization without mixing and by a constant pressure process. The result is unchanged pressure and velocity, but different areas for each stream. The two streams are now at the same pressure and temperature, but at different velocities. Next, the velocities are equalized on a total momentum basis which reduces the streams to identical conditions, but leaves over a quantity of heat Q. Area changes also accompany the velocity equalization.

The combined stream is now brought to choking (if supersonic) by simultaneous heat addition and area change. The area may either increase or decrease, but will most likely decrease. The overall change

in area is the sum of the changes made in each step. Thus, the area change from the nozzle exits to the choked throat is known. The exact way in which this area changes with respect to distance along the duct cannot be predicted from the analysis, but since the velocity differences are greatest at the exit to the primary nozzles, the rate of heat generation is also greatest at this point. This suggests that the area reduction should be rapid at first, and diminishing toward the choked throat. According to this, a parabolic relation between area and distance would be better than a straight line relation. The usual conical bore has a parabolic area relation, but curves the wrong way so that the area change is most rapid near the choking section. It will be later seen that the gas dynamics may dictate the shape more strongly than these considerations.

5. The Constant Volume Process with Heat Addition

The general problem of solving the conservation equations for one-dimensional flow rests largely on the evaluation of the external boundary force in the momentum equation. Certain assumptions concerning the flow process implicitly involve this force. Thus, the assumption of constant entropy, constant area, constant volume or constant pressure processes leads to solvable sets of the conservation equations.

The method of handling the heat additions used in the previous section is based on the principle of adding heat at the highest pressure allowable by the dynamic properties of the stream. If, however, a constant volume process is assumed for the heat addition, there will also be a pressure increase, which will be less than that for the constant area process. The computation of the constant volume process is much simpler, however.

The flow relations governing heat addition at constant volume are stated below, and the process must satisfy these equations (see Appendix VI).

$$pv + \frac{V^2}{2g} = K_1 \quad \text{Bernoulli's equation for incompressible fluid}$$

$$\frac{\gamma}{\gamma-1} pv + \frac{V^2}{2g} - Q = K_2 \quad \text{Energy equation.}$$

$$pv = RT \quad \text{Equation of state.}$$

$$T + \frac{V^2}{2g} = K_3$$

$$Q + \frac{C_v V^2}{2gR} = K_4$$

$$Q - pv\frac{C_v}{R} = K_5$$

These equations will be used with the data of the constant area example to compute the stagnation pressure of that example.

The conditions of the example are re-stated here.

Area	-	1 ft^2	γ	-	1.40
T	-	600°R	R	-	53 ft lb/lb°F
p	-	1000 psfa	M	=	1.07
V	-	2000 ft/sec	v	=	31.8 ft^3/lb
Q	=	20 Btu/lb	w	=	62.8 lb/sec

From the Q-V relation

$$V_2^2 = V_1^2 - \frac{2gRQ}{C_v} = (2000)^2 - \frac{(64)(53)(20)(778)}{133} = 2000^2 - 397000$$

$$V_2 = 1894 \text{ ft/sec}$$

From the T-V relation

$$T_2 = T_1 + \left(\frac{V_1^2 - V_2^2}{2Rg}\right) = 600 + \frac{397000}{64 \times 53} = 600 + 117 = 717°R$$

From the p-Q relation

$$p_2 = p_1 + \frac{QR}{vC_V} = 1000 + \frac{(20)(778)(53)}{(31.8)(133)} = 1000 + 195.0 = 1195 \text{ psfa}$$

From which

$$A = \frac{wv}{V} = \frac{(62.8)(31.8)}{1894} = 1.05 \text{ ft}^2$$

The stagnation pressure for this flow is

$$p_0 = p\left[\frac{V^2}{2gc_pT} + 1\right]^{\gamma/\gamma-1}$$

$$p_0 = 1195\left[\frac{(1894)^2}{(64)(188)(717)} + 1\right]^{3.5} = 1195(3.38) = 3700 \text{ psfa}$$

It will be recalled that the stagnation pressure for the choking, heat addition process was 4250 psfa, which shows the advantage of adding the heat at higher pressure.

C. Analysis of Ejector Operating with Different Fluids

This is the most general case, and it is seen at once that there is a composition irreversibility involved that is basic and cannot be avoided. The position has been taken earlier, however, that an ejector is characteristically incapable of a diffusion process, and, therefore, should not be held accountable for this failing. The limiting process for ejectors was defined without the diffusion work for this reason.

The general aspects of the problem for two different fluids are otherwise much the same as for identical fluids. There are, however, certain differences, and these lie mainly in the greater flexibility afforded by operating with different fluids.

It will be recalled that in the discussion of identical fluids, it was found that under the special condition that the induced stream has the higher stagnation temperature, it was possible to arrange matters so that the velocity irreversiblility was suppressed. This was possible because with higher stagnation temperature the induced stream has the greater limiting velocity and it was thus possible to find a common pressure for both fluids at which the velocities would be equal. In order to suppress the velocity difference in the two streams, it is only necessary that $(C_pT_o)_i > (C_pT_o)d$. This is because $V_\ell = \sqrt{C_pT_o}$. Since for identical fluids, $C_{pi} = C_{pd}$, then it is necessary for $T_{oi} > T_{od}$.

In the case of different fluids, the conditions are not so binding. In this case, it is only necessary that the product $(C_pT_o)_i > (C_pT_o)d$.

The possibility of having C_{pd} different than C_{pi} brings about a very important condition wherein it is possible to eliminate both velocity and temperature differences in the two streams and arrive at essentially reversible operation.

It is noted that for isentropic expansion,

$$\left(\frac{V^2}{2g}\right)_d = C_{pd}\left(T_{od} - T_d\right)$$

and

$$\left(\frac{V^2}{2g}\right)_i = C_{pi}\left(T_{oi} - T_i\right)$$

For the two streams to have the same velocities, it is only necessary that

$$C_{pd}\left(T_{od} - T_d\right) = C_{pi}\left(T_{oi} - T_i\right)$$

and the condition that this occurs at a common temperature, T_c, is that $T_c = T_d = T_i$

$$\therefore \quad T_c = \frac{C_{pi}T_{oi} - C_{pd}T_{od}}{C_{pi} - C_{pd}} \tag{8}$$

It is noted that this condition is independent of the stagnation pressures of the streams. It is desired that the pressures also be equal, so that by writing

$$p_i = p_{oi}\left(\frac{T_i}{T_{oi}}\right)^{k/k-1}$$

and

$$p_d = p_{od}\left(\frac{T_d}{T_{od}}\right)^{\gamma/\gamma-1}$$

and letting $p_c = p_d = p_i$ and $T_c = T_i = T_d$

The result is

$$\frac{p_{od}}{p_{oi}} = \frac{\left(\frac{T_c}{T_{oi}}\right)^{k/k-1}}{\left(\frac{T_c}{T_{od}}\right)^{\gamma/\gamma-1}} \tag{9}$$

This means that for selected stagnation temperatures of the two streams, there is a particular ratio of the stagnation pressures that at some common pressure, p_c, will eliminate both pressure and velocity differences. Perhaps an example will help to bring out the significance of these relations.

Suppose the driver is assumed to be air, $C_p = 0.24$, $\gamma = 1.40$ and the induced fluid is helium, $C_p = 1.25$, $k = 1.67$.

In order to eliminate velocity differences, it is necessary that

$$C_{pi} T_{oi} > C_{pd} T_{od} \qquad \text{or} \qquad T_{od} < T_{oi} \frac{C_{pi}}{C_{pd}}$$

In this case

$$T_{od} < T_{oi} \frac{1.25}{0.24} = 5.20\, T_{oi}$$

If the stagnation temperature of the helium is 500°R, then $T_{o(air)} < 5.2(500) = 2600°R$ in order to eliminate velocity differences.

Select $T_{o(air)} = 1000°R$

Then by Formula (8)

$$T_c = \frac{(1.25)(500) - (0.24)(1000)}{1.25 - 0.24} = 381\,°R$$

and by Formula (9)

$$\frac{p_{od}}{p_{oi}} = \frac{(381/500)^{2.5}}{(381/1000)^{2.5}} = \frac{0.506}{0.034} = 14.90$$

Thus, if $p_{oi} = 10$ psia, $p_{od} = 10(14.9) = 149$ psia.

Thus, the stagnation condition for elimination of velocity and temperature irreversibilities are

$$p_{o(air)} = 149 \text{ psia} \qquad p_{o(He)} = 10 \text{ psia}$$

$$T_{o(air)} = 1000°R \qquad T_{o(He)} = 500°R$$

The mixing pressure p_c is given by

$$p_c = p_{o(air)}\left[\frac{T_c}{T_{o\,(Air)}}\right]^{\gamma/\gamma-1} = 149\,(0.381)^{3.50} = 5.07 \text{ psia}$$

$$p_c = p_{o\,(He)}\left[\frac{T_c}{T_{o(He)}}\right]^{k/k-1} = 10\,(0.762)^{3.5} = 5.07 \text{ psia}$$

The mixing velocity is $V_c^2 = (2g)C_p(1000-381) = (64)(188)(619) = 7.9 \times 10^6$ and $V_c = 2800$ ft/sec .

It is to be noted that the fixed pressure ratio also fixes the driver pressure that will give reversible operation for any constant induced stagnation pressure. The flow ratio will, of course, determine the final stagnation pressure of the combined stream and this may vary between p_{od} for zero flow ratio to p_{oi} for infinite flow ratio. If it is desired to increase the pressure ratio for a given flow ratio, this may be done by increasing the stagnation temperature of the driver. Suppose the driver temperature in the example above were raised to 2000°R. (It must be kept below 2600°R for reversible operation.) Then p_{od}/p_{1i} = 440, and p_{od} = 4440 psia for p_{oi} = 10 psia.

The remainder of the computation is relatively simple. It is noted that the mixing of two perfect gases at the same pressure and temperature causes no change in the pressure and temperature (Appendix III). Therefore, the mixing is at constant area, constant pressure, constant velocity and constant temperature. The reduction to isentropic conditions depends on the computation of mean values for the heat ratio and the specific heat and these will depend on the assumed flow ratio. Suppose in the example just given that the flow ratio is taken to be unity. Then

$$C_{pm} = 1/2(1 \cdot C_{p(He)} + 1 \cdot C_{p(air)}) = 1/2(1.25 + .24) = 0.75$$

$C_{Vm} = 1/2(1 \cdot C_{V(He)} + 1 \cdot C_{V(air)}) = 1/2(0.753 + 0.171) = 0.462$

Specific heat ratio = 0.75/0.462 = 1.62 = β.

So that

$$p_{om} = p_c \left[\frac{V_c^2}{2g C_p T_c} + 1\right]^{\beta/\beta-1} = 5.07\,(1.52)^{2.61} = 15.10 \text{ psia}$$

If the operating conditions for the two fluids are such that a temperature difference remains, or that there remains both temperature and velocity difference, then the procedure is the same as for the identical fluid case where both temperature and velocity differences exist. In this case, however, when computing the effect of the addition of the irreversible heat, the average stream properties, based on flow ratio, are used.

It is to be stressed that the entire means of obtaining reversible operation depends on the proposition that $C_{pi} > C_{pd}$. If $C_{pi} < C_{pd}$, then either temperature or velocity difference may be eliminated, but not both.

Since $C_p = (\gamma/\gamma-1)\ \overline{R}/\overline{M}$ it is evident that the approach to reversible operation and, hence the performance of the ejector, depends not only on molecular weight, but also depends on the value of γ. Now γ is related to the molecular structure of the gas, so that the general statement is that ejector performance depends on the molecular weight and the molecular structure of the working fluids. In view of this, it is evident why the attempts(3) to analyze ejector performance on the basis of molecular weight alone have been only partially successful.

The rather poor performance of steam driven air ejectors is also understandable on this basis. Computation on the data of several examples gave efficiencies of approximately 20-30 per cent of reversible values. Since $C_{p(air)}$ can be taken as 0.24 and that of steam as approximately 0.45 and since $T_{o(air)}$ is usually less than $T_{o(steam)}$ it is evident that $(C_pT_o)_i < (C_pT_o)_d$. This is in the wrong direction for reversible operation. Heating of the air should be very beneficial for this operation, but only if the ejector is properly designed with respect to nozzle areas and mixing tube geometry so as to take advantage of the more favorable conditions.

The use of steam jet ejectors in water refrigeration systems is well known. This is a case of identical fluids with the driving stream warmer. It appears that considerable improvement in performance could be obtained if a refrigerant of greater heat capacity were substituted for the water.

D. Gas Dynamics and Experimental Investigation

The preceding sections have dealt with the thermodynamic relations and limitations which are believed basic to the ejector operation. That there are inherent irreversibilities involved in the mixing process is of course easily recongized and an attempt has been made to provide a realistic and workable model for the evaluation of those irreversibilities. It should be emphasized that the proposed model accounts for the irreversibility due to an assumed form of viscous fluid shear only and says nothing concerning the possible losses due to turbulence, acoustical energy, etc. that may arise from the interaction of the two streams. It is merely proposed that fluid shear is the predominant loss factor and is the basic factor which accounts for the operation of the ejector and cannot therefore be minimized by design technique. In addition to fluid shear, turbulence, acoustical energy, etc., there are other sources of irreversibility in the operation. One of these is wall friction, which was neglected in the preceding analysis. Some account will be made of this factor in this section. Another source of loss that applies to the case involving supersonic flow is the hydrodynamic shock. It will be seen that this factor occupies a position of major importance in the ejector analysis, and since the operation of ejectors frequently involves supersonic flow, particular attention will be given to it. It should be pointed out that not all the manifestations of shock are as yet clearly understood, and in particular the case of shock-boundary layer interaction

is not yet amenable to computation. This case occurs when the mixing involves a supersonic and a subsonic stream as is frequently the case in operating ejectors. Later, additional attention will be given to this and related subjects.

An experimental investigation has been conducted that has the objective of evaluating the performance based on basic thermodynamic effects, and of providing some insight into the gas dynamics involved. The results of the experiments indicate that the understanding of the gas dynamics is quite incomplete and that significant improvements in design may be found in this field.

The experiments have been done using air as both the driving fluid and the induced fluid. Aside from its obvious advantages of availability, safety, etc., air has the advantage of being a nearly perfect gas whose properties are available in tabular form. This is a real convenience in a study of this kind.

The research equipment used is a two-dimensional flow test ejector designed according to the recommendations of Elrod(1) to attain a maximum suction of 1 psia at a driving pressure of 70 psig against 1 atm back pressure. The test ejector is filled with glass walls optical studies, is equipped with wall static pressure taps along the flow channel and has provisions for total head measurements across the mixing tube. The plan and location of pressure taps for the test ejector are shown in Figure 6 along with a photograph in Figure 7.

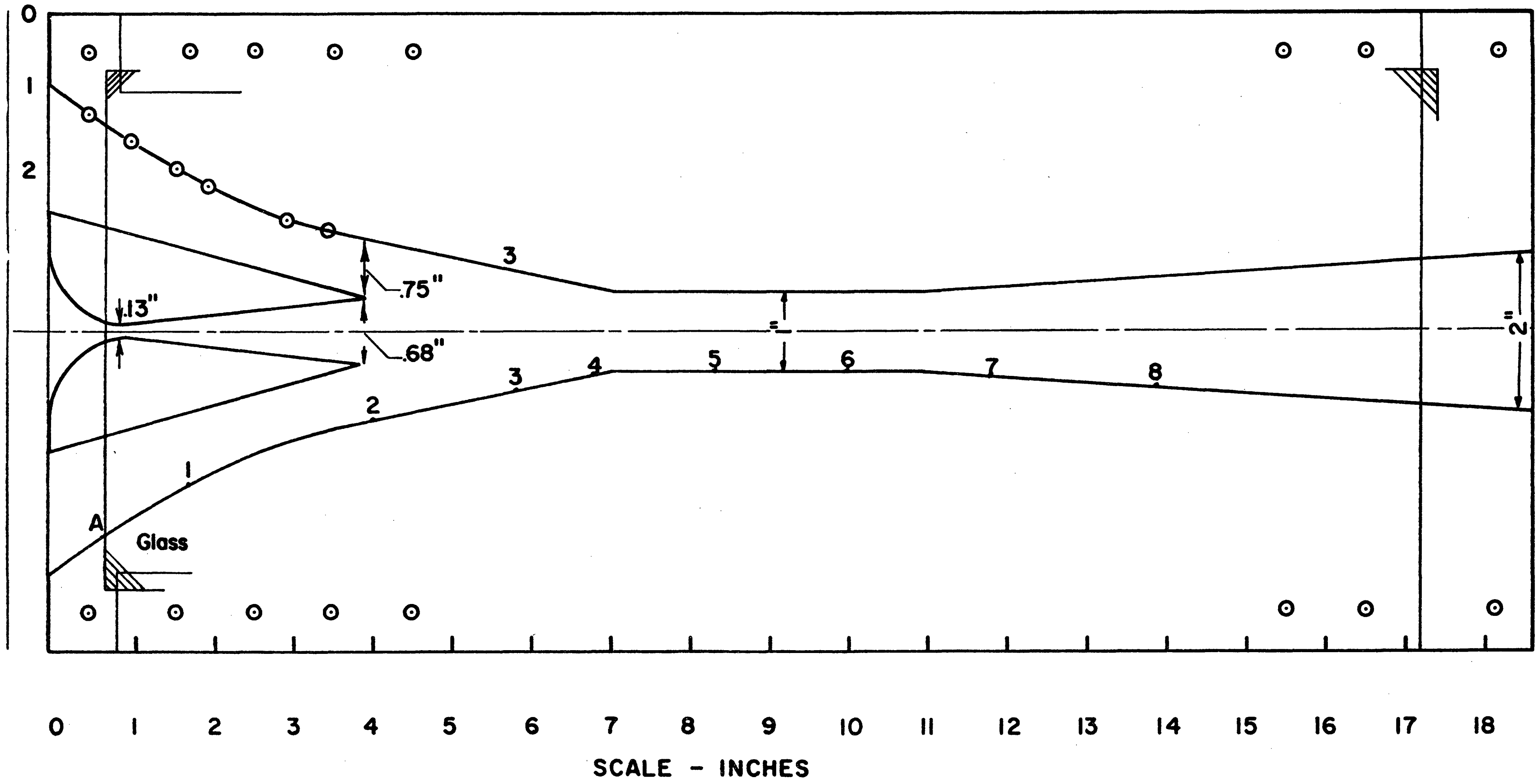


Figure 6. 1/2 Scale Layout of Test Ejector Showing Location of Wall Pressure Taps.

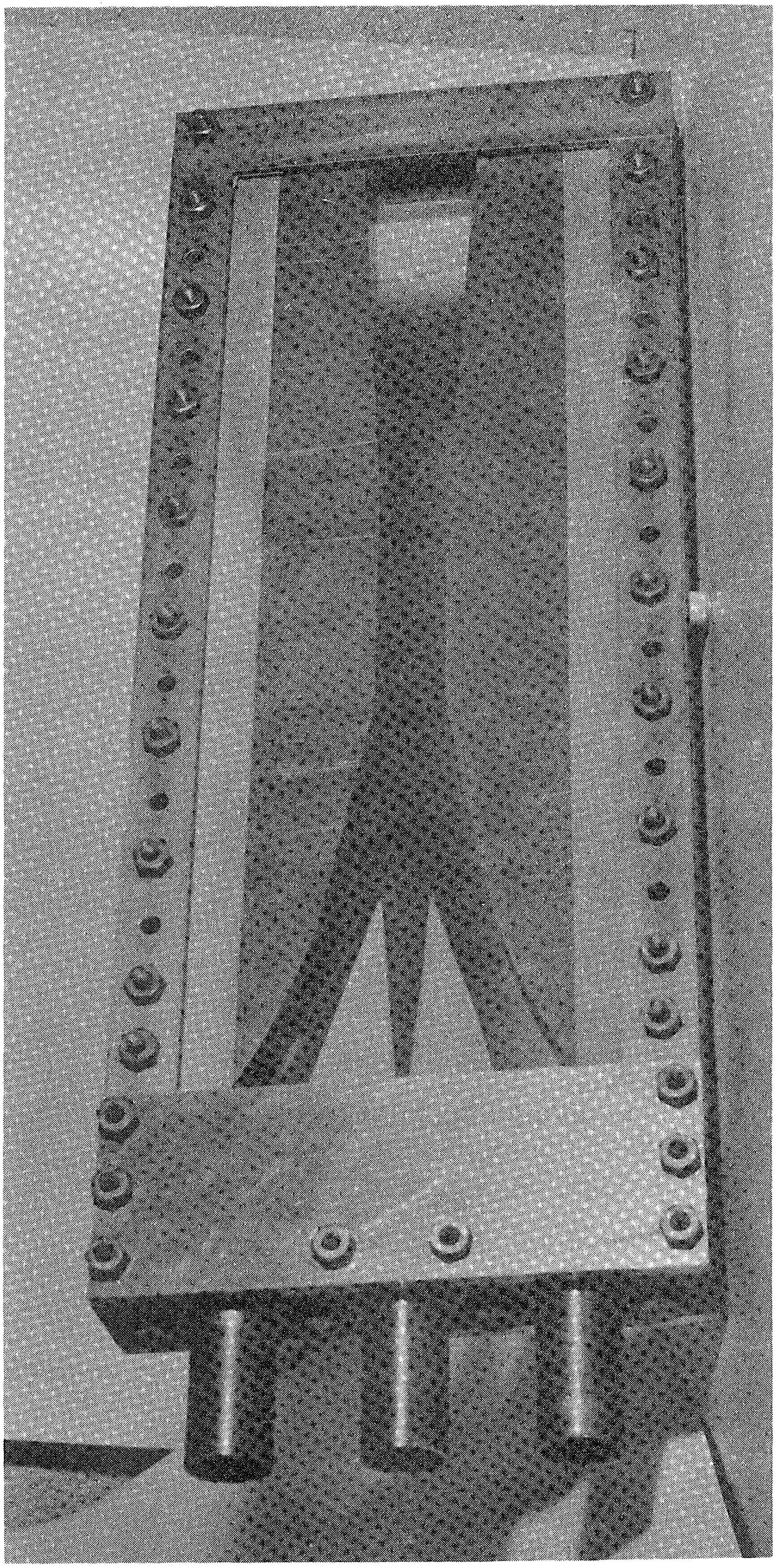


Figure 7. Photograph of Test Ejector.

Pressure measurements in the higher ranges were made with calibrated Bourdon-tube gauges and in the lower ranges with mercury manometers. The average barometric pressure (14.35 psia) is used for 1 atm. Temperatures of the flow were measured near stagnation conditions with open stem thermometers. Optical measurements were made by photographic means using a spark schlieren system that was adaptable to shadow studies. The schlieren optical system is a well known tool used in gas flow studies and will not be described here. The system is described in many references, a particularly good one being reference 13.

In all tests, the outlet from the ejector was open to the atmosphere. Suction air was drawn from the atmosphere with various degrees of throttling and the driving air was obtained at various pressures by throttling a 100 psig supply.

The tests were made about three different operating conditions corresponding to closed suction, partially throttled suction, and open suction. At each condition, the effects of driver stagnation pressures from 20-90 psig were studied.

1. Closed Suction or Totally Throttled Ejector

This case forms the basis for evaluation of gas dynamic losses, and will be considered first. Inasmuch as there is no induced flow, the thermodynamics of this stream do not enter, and the particular concern is with the gas dynamics of the driving stream. The subject of primary interest is the maximum suction attained.

In order to compute the maximum suction, it is necessary to analyze the starting condition for the ejector. The idea of starting the ejector rests on the possibility of attaining supersonic flow in the mixing tube. Since the driving nozzle is a converging-diverging duct, the maximum suction will be attained only when supersonic flow is attained at the mouth of this nozzle, which condition corresponds to minimum pressure in the suction chamber. It will be shown that the starting condition involves the driving nozzle and also must be satisfied by the flow in the mixing tube and in the subsonic diffusor at the end of the ejector.

Consider the ejector with closed suction diagrammed in Figure 8. In the beginning before driver pressure is applied, the ambient pressure p_m exists in all parts of the ejector and before flow is possible the stagnation pressure in the ejector must become greater than p_m. As p_{od} is gradually increased it becomes greater than p_m so that flow is possible. At first, the flow is altogether subsonic, and there will be a pressure reduction at d_1, an increase at d_2, a decrease at d_3, and an increase to p_m at d_4, all in accord with the principles of subsonic compressible flow. As the pressure p_{od} continues to increase however, the flow at d_1 will become critical. At this point p_{od} is considerably in excess of p_m, and the flow must adjust itself in some way to account for the excess of stagnation pressure. The way it does this is by means of a hydrodynamic shock in the expanding portion of the driving nozzle. It is assumed for the present, that the flow always "wets" the entire driving nozzle so that flow separation from the

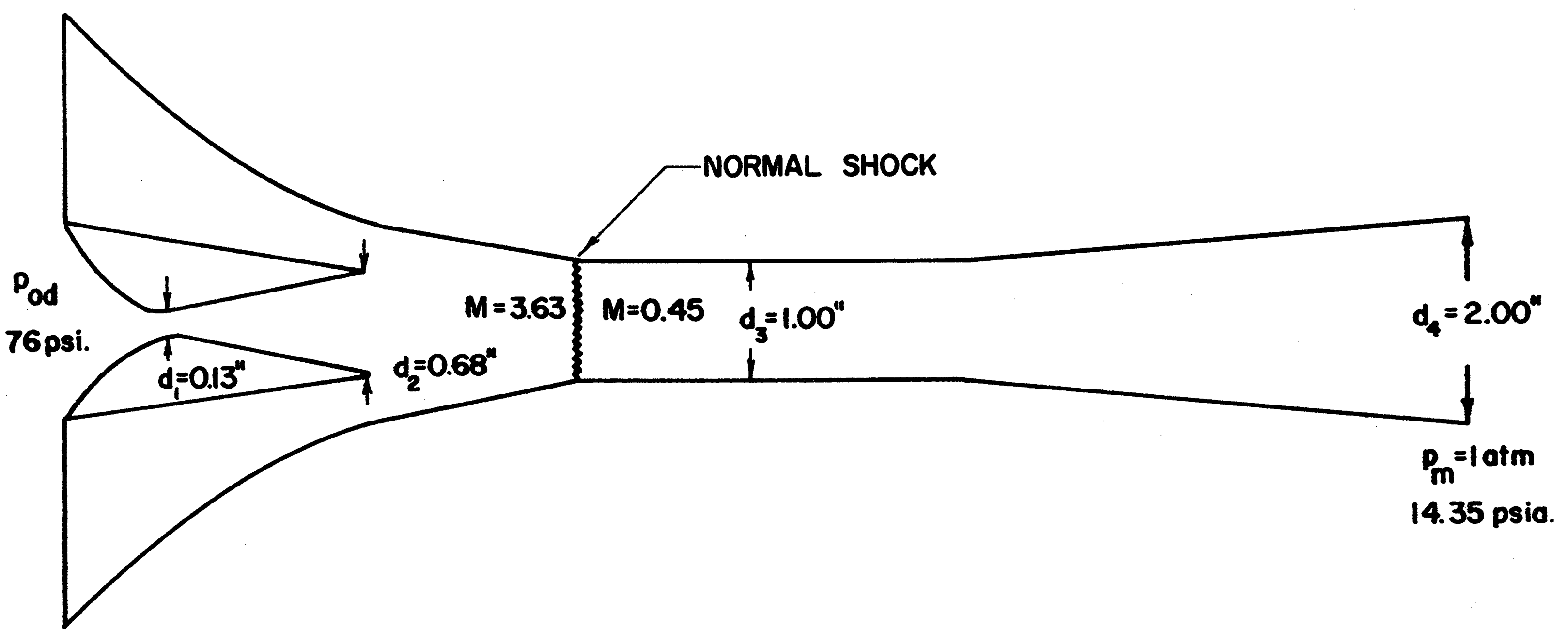


Figure 8. Starting Condition for Completely Throttled Suction Ejector.

wall of the nozzle does not occur. Uner this assumption, the shock will be normal to the flow.

Between d_1 and the position of the shock, supersonic flow will exist, but behind the shock, subsonic flow exists and the relations for subsonic flow govern the process.

If it is assumed that $d_4 >> d_3$ then for a perfect subsonic diffusor the static pressure of the stream at d_4 is also the stagnation pressure of the stream, and $p_4 = p_m$. Therefore the condition of equilibrium for the flow is that behind the shock $p_o > p_m$.

The relations governing the pressure changes across normal shocks are developed from the theory of shock waves.* The change in static pressure across a normal shock is given in terms of the approaching Mach number as

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}$$

where p_2 is the static pressure behind the normal shock and p_1 the static pressure in front. Other applicable relations are,

$$\frac{\rho_2}{\rho_1} = \frac{2}{(\gamma+1) M_1^2} + \frac{\gamma-1}{\gamma+1}$$

and

$$M_2^2 = \left(\frac{\gamma+1}{2\gamma}\right)^2 \frac{1}{M_1^2 - \frac{\gamma-1}{2\gamma}} + \frac{\gamma-1}{2\gamma}$$

where M_2 is the Mach number behind the shock, which for normal shock is always subsonic.

The relation between static pressure and stagnation pressure is given in terms of the local Mach number as

$$\frac{p_0}{p} = \left(\frac{\gamma-1}{2} M^2 + 1\right)^{\gamma/\gamma-1}$$

These relations allow computation of all changes of state occurring across

* See for example Ref. 9, Chap. 3.

the normal shock. These relations are available in tabular form for air (Ref. 9), and are convenient to use.

The starting condition for the ejector can now be analyzed. This will be done in terms of the design of the test ejector (Figure 6) for which the following dimensions apply.

$d_1 = 0.13$ in.

$d_2 = 0.68$ in.

$d_3 = 1.00$ in.

$d_4 = 2.00$ in.

and since the channel is one inch wide the above dimensions are also the equivalent channel area in square inches.

As long as the normal shock is in the expanding part of the driving nozzle, the area of the shock will range from 0.13 sq in. (throat) to 0.68 sq in. (mouth). Therefore an area is assumed for the shock and computations are made about that point. For example assume the area of the shock to be 0.40 sq in. The area ratio from throat to shock is $0.13/0.40 = 0.325$. From the air tables of compressible flow the Mach number approaching the shock is found to be $M_1 = 2.66$. The changes across the shock are found from the tables to be as follows:

$$M_2 = 0.498 \quad , \qquad p_{02}/p_{01} = 0.438$$

but $p_{02} \geq p_m = 1$ atm (14.35 psia), therefore $p_{od} \geq 14.35/.438 = 33$ psia.

As mentioned earlier, the idea in starting the ejector is to develop supersonic flow in the part of the ejector between the mouth of the driving nozzle and the entrance to the mixing tube. This part is

called the suction chamber and minimum pressure will be attained here when the flow is supersonic through the suction chamber as this corresponds to maximum expansion of the driving fluid. With the shock standing at 0.40 sq in. in the driving nozzle, only subsonic flow is possible beyond this point, for if the shock were to move downstream its area would have to increase, corresponding to $M > 2.66$. The loss in p_o across the shock will increase with the result that behind the shock $p_o < p_m$. Therefore the flow behind the shock will diminish, mass will accumulate resulting in higher pressure behind the shock which increases its celerity causing it to move back upstream against the oncoming flow to the position of equilibrium. On the other hand, if the shock were to move upstream, the shock losses would decrease. Behind the shock $p_o > p_m$ so that the flow is accelerated. The pressure ratio across the shock is decreased, reducing its celerity, causing it to move back downstream to the position of equilibrium at the point 0.40 sq in.

The same considerations as above show that the pressure p_{od} necessary to sustain a shock at the mouth of the driving nozzle is 53 psia. When located at the mouth however, the shock is in an unstable configuration so that a slight additional increase in p_{od} will produce oblique shock at the mouth of the driving nozzle (see Figure 9).* The oblique shock causes a smaller reduction in p_o than the normal shock however, so that even greater subsonic flow is permissible and the tendency of the flow is to become supersonic. This tendency is resisted by subsequent events, so that the flow in the suction chamber cannot yet become supersonic, for if it is assumed that supersonic flow is developed

* Figure 9, represents a simplification of the actual configuration, sufficiently accurate for the purpose here, see Chap. VIII, Ref. 9 for a more precise analysis.

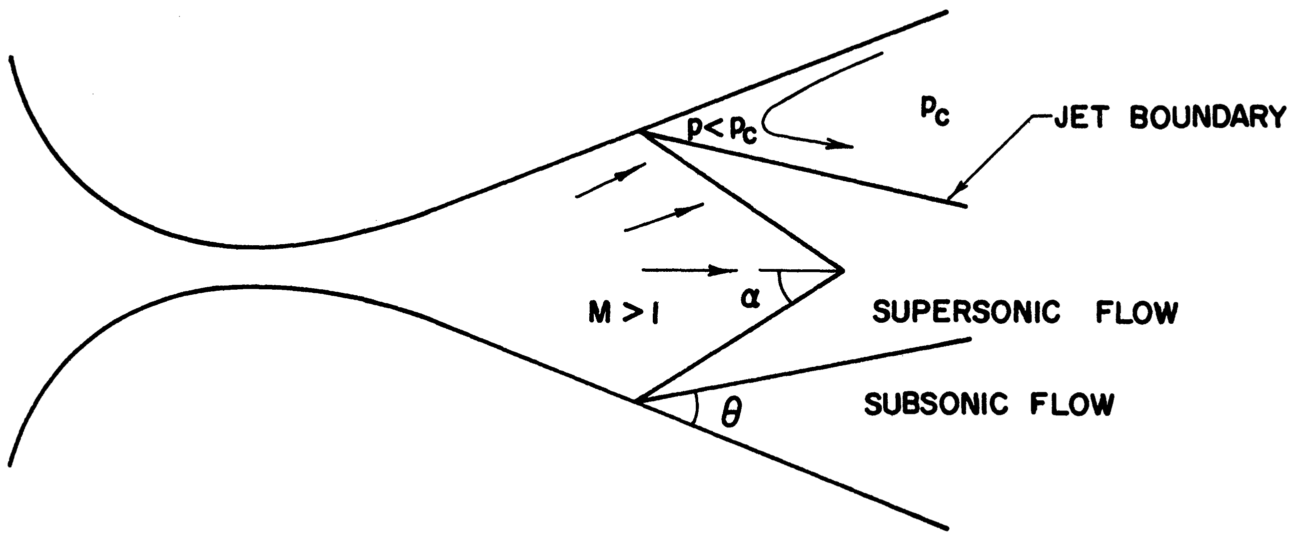


Figure 9. Flow Separation in Driving Nozzle
with Oblique Shock.

at the mouth of the driving nozzle then it will continue on as an approximately parallel or contracting flow having the area of the nozzle mouth which is 0.68 sq in., and will not fill the mixing tube, the area of which is 1.00 sq in. The flow in the mixing tube will then consist of a core of supersonic flow surrounded by a more or less stagnant region in which pressure may be communicated from the region p_m back to region p_c thus voiding the assumed condition which made supersonic flow at the nozzle mouth possible. On the other hand, if it is assumed that the flow expands so that it fills d_3, the shock will have the area 1.00 sq in. and the correspondingly higher Mach number entering the shock will cause losses in p_o that are too great to sustain the subsonic flow. It must therefore be assumed that p_{od} is increased to a value that will support a normal shock at d_3 before the ejector will start. The value of p_{od} that will support a shock at this location is 70 psia. A correction factor of 1.04 may be applied to this pressure to account for the loss in total head at the discharge d_4, and if according to Stodola[8] we assume the nozzle efficiency to be 95% the required p_{od} becomes 83 psia = 68 psig. This value is satisfactorily close to the observed value of 84.35 psia taken from the experimental data (Figure 10). The theoretical suction pressure for p_{od} = 84.35 psia corresponding to full expansion in the driving nozzle is 1.67 psia. This is compared to the pressure 1.00 psia for which the ejector was designed. The observed pressure at station 2 (Figure 10) of the suction chamber is 2.95 psia, so that the theoretical pressure does not check with the observed pressure and neither checks with the design pressure. It is immediately apparent

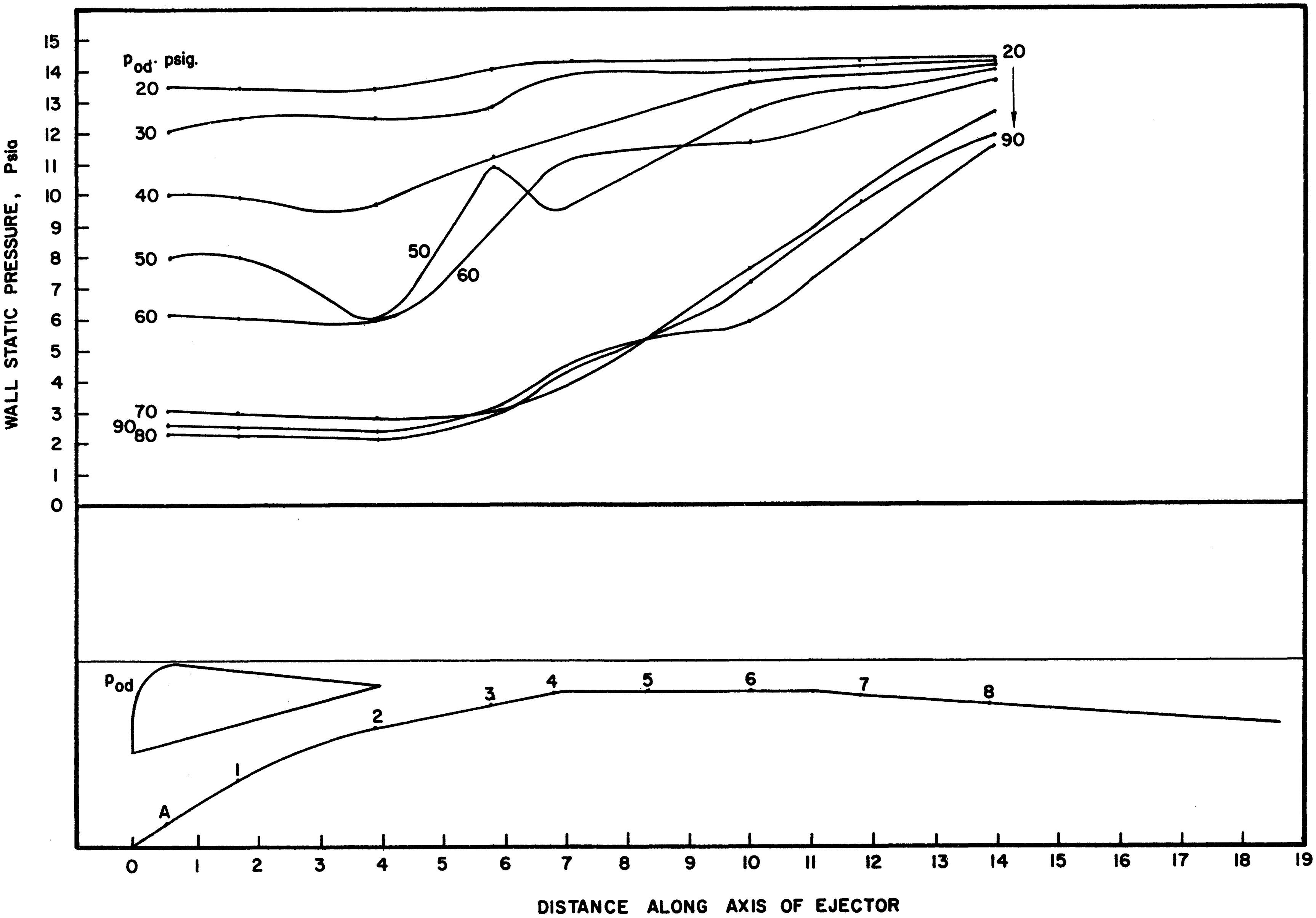
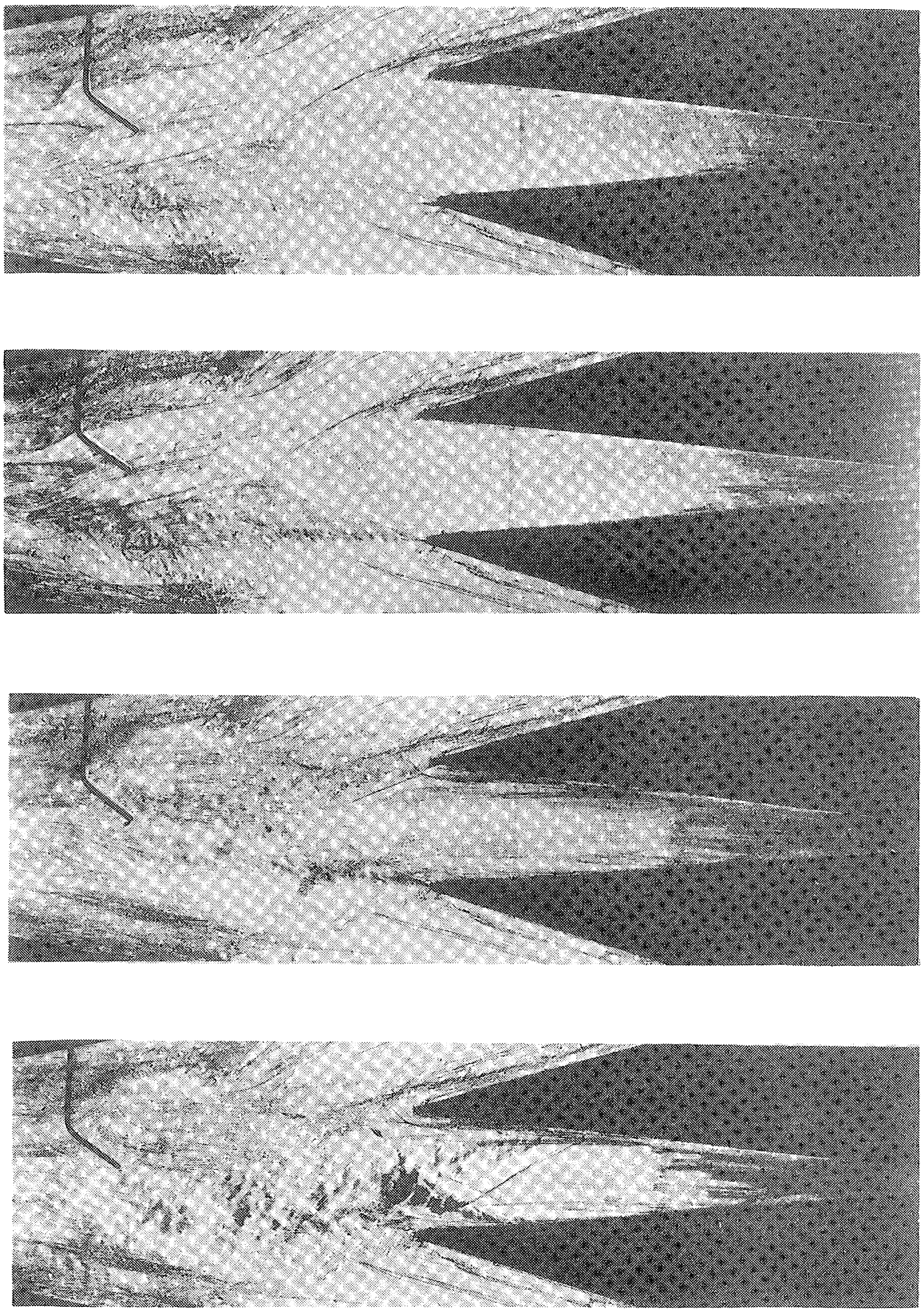


Figure 10. Experimental Pressure Data for Totally Throttled Ejector.

that the maximum suction is better predicted from the theoretical expansion ratio of the driving nozzle than from the recommended design procedure. The discrepancy between the theoretical suction pressure 1.67 psia and the observed suction pressure of 2.95 psia for p_{od} = 84.35 psia can be accounted for on the basis of the dimensions of the flow leaving the driving nozzle, for if this flow corresponds to the theoretical expansion ratio of the nozzle, then it will have the area 0.68 sq in. corresponding to the mouth of the driving nozzle. In order to expand to the area of the mixing tube where a shock of 1.00 sq in. area can be sustained and which was the basis of the computation, the flow must pass through a length of approximately stagnant air, which causes frictional losses over and above those allowed for in the computation. Therefore the maximum suction will not correspond to p_{od} = 84.35 psia, but to a somewhat higher driver pressure. The means is not at hand to compute this precisely, but the experimental data (Figure 10) show the maximum suction to correspond to p_{od} = 94.35 psia, and is p_c = 2.30 psia.

An examination of the schlieren photos of the driving nozzle (Figure 11) shows that for p_{od} = 84.35 psia, an oblique shock stands at the mouth of the nozzle at an angle of 23° to the flow. The tables for oblique shock (Ref. 9) show that the pressure behind this shock should be 2.88 psia as compared to the observed value of 2.95 psia. The flow actually contracts on leaving the nozzle before it expands to fill the mixing tube. When p_{od} = 94.35 psia, the schlieren data (Figure 11) show the shock standing at the mouth of the driving nozzle to be at

← Flow



90 psig (104.35 psia)

80 psig (94.35 psia)

p_{od}

70 psig (84.35 psia)

60 psig (74.35 psia)

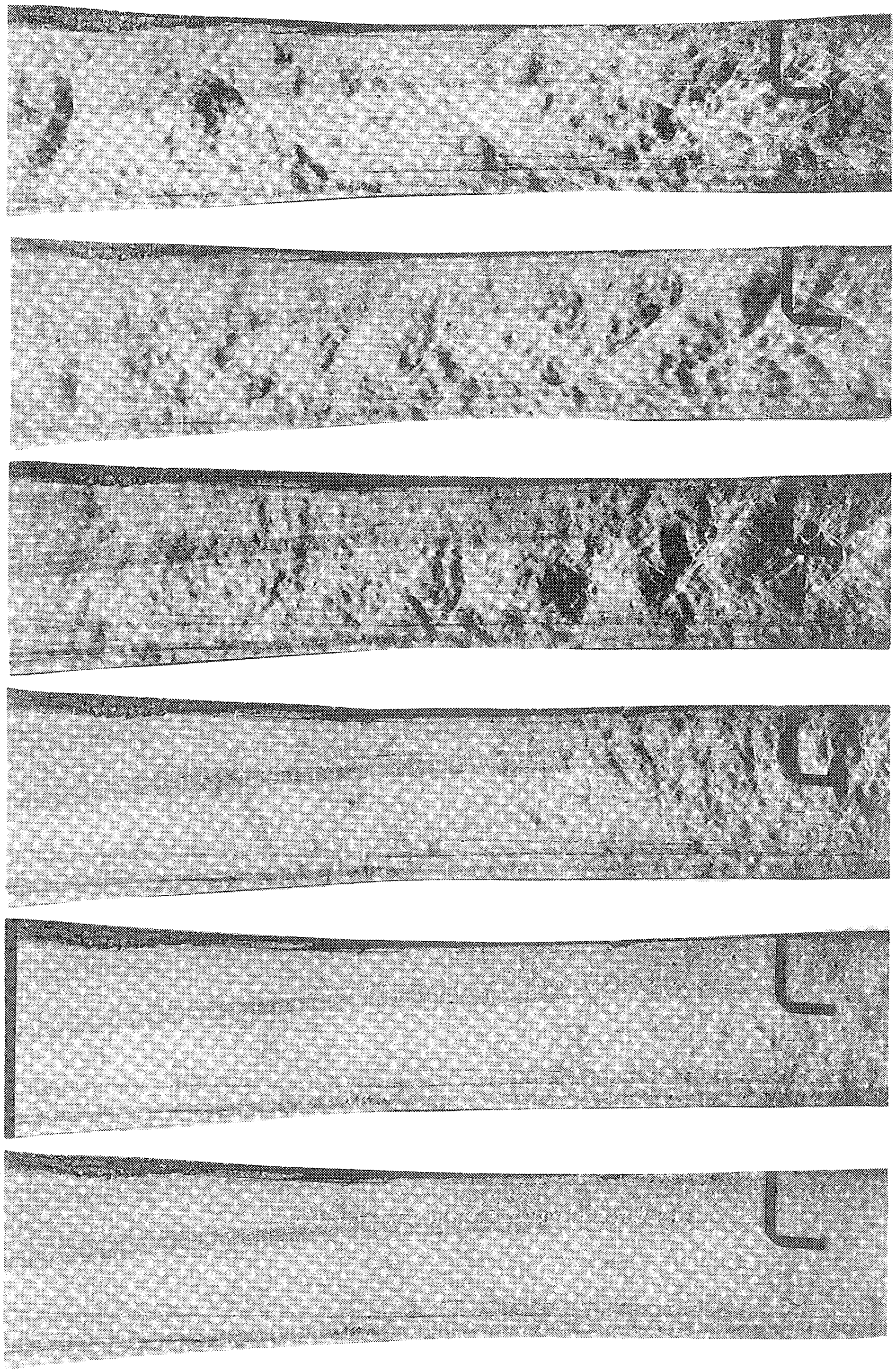
Figure 11. Schlieren Photographs of Driving Nozzle Flow Separation of Various Values of p_{od} with Closed Suction.
KNIFE EDGE VERTICAL

the angle 18° with respect to the flow. This is a very weak shock which amounts to little more than a Mach line of compression. The tables give the pressure ratio across this shock to be 1.20 and the computed value of p_o is 2.24 psia as compared to the observed maximum suction value of 2.30 psia. At higher pressure for p_{od}, the expansion ratio of the driving nozzle limits the suction obtained and the suction pressure increases accordingly. For p_{od} = 104.35 psia, the compression lines at the mouth of the nozzle are very faint and the stream can be seen to leave the nozzle with parallel flow (Figure 11). In this case the observed p_c has increased to 2.40 psia.

As a further corroboration of the interpretation, the schlieren data for the flow in the mixing tube (Figure 12) show approximately normal shock for p_{od} = 84.35 psia at the point where the flow fills the tube. Comparing this to the data for p_{od} = 94.35 psia shows that for the latter case, the flow fills the tube earlier thereby experiencing less friction loss, which accounts for the better correlation between computed and observed values for p_c at p_{od} = 94.35 psia. In the cases of p_{od} = 94.35, 104.35 psia, the ejector is overdriven with respect to energy of the flow, and approximately normal shocks do not appear until the flow has passed the mixing tube and entered the diffusor.

Up to this point, it has been assumed that the starting condition is characterized by the regime of nozzle flow that has no separation of the flow from the nozzle walls. Actually separation does occur in most if not all cases where the pressure ratio across the nozzle is less than that for complete expansion and this modifies

Flow



90 psig (104.35 psia)

80 psig (94.35 psia)

70 psig (84.35 psia)

60 psig (74.35 psia)

50 psig (64.35 psia)

40 psig (54.35 psia)

p_{od}

Figure 12a. Schlieren Photographs of Mixing Tube Flow at Various Values of p_{od} with Suction Closed. KNIFE EDGE VERTICAL.

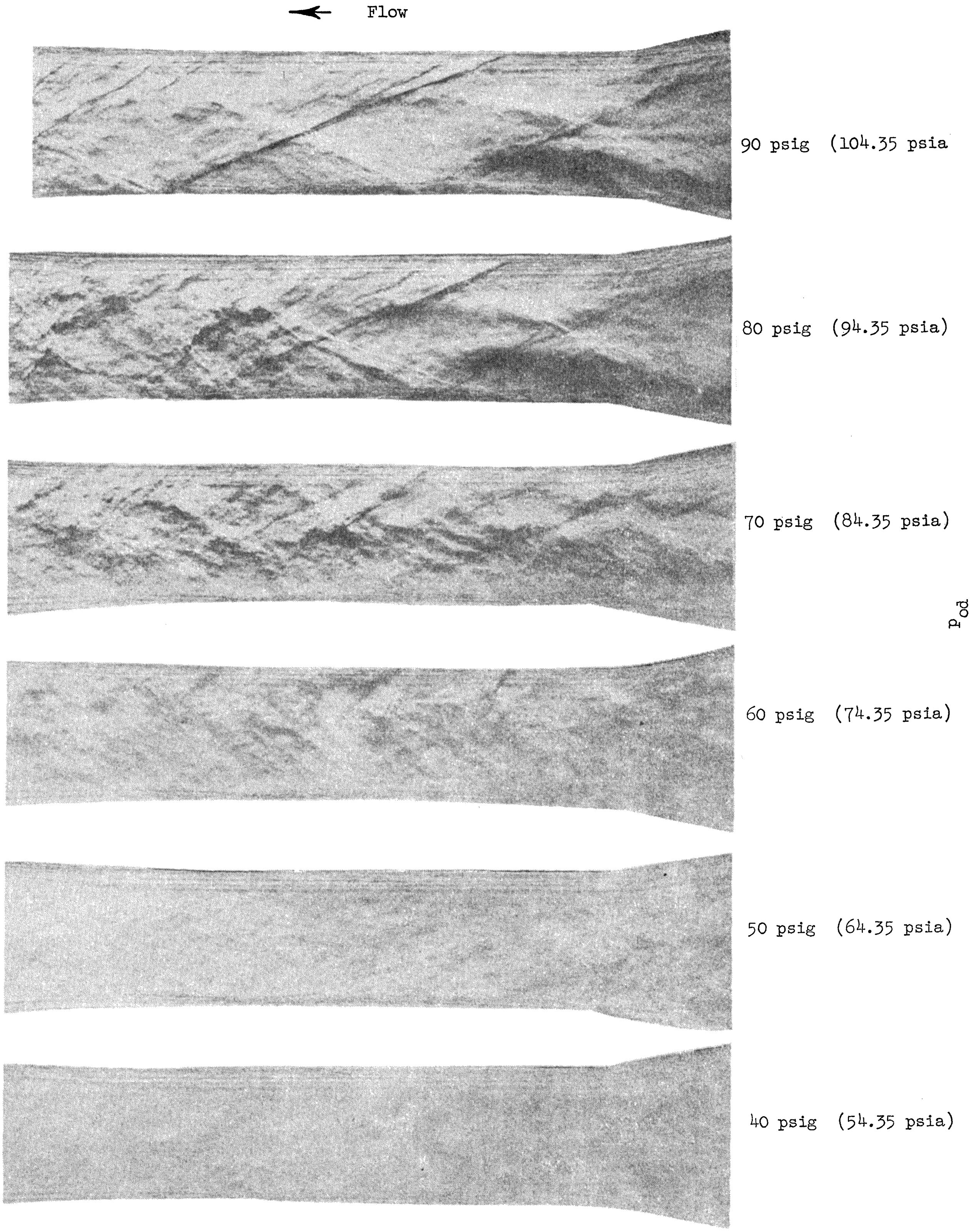


Figure 12b. Schlieren Photographs of Mixing Tube Flow at Various Values of p_{od} with Suction Closed. KNIFE EDGE HORIZONTAL.

the viewpoint on starting requirements. As to the condition which will cause flow separation to occur, it must be concluded that this is an unsolved problem in gas dynamics. A moderate amount of work has been done on this subject in the past 50 years, and some insight has been gained into the problem.

Stodola* experimented with the phenomena around the year 1920, but did not offer a theory for it. Other observations have been made by Stanton,[10] Frazer[11] and by Summerfield. Summerfield has made an attempt at an analysis of the problem, and he assumes (Figure 9) that the boundary streamline of the supersonic flow is nearly parallel to the axis of the flow and that the region of subsonic flow is essentially at rest and at the ambient pressure at the mouth of the nozzle. Using this assumption it was stated that a relation had been derived, and this relation presented in graphical form gives fairly good correlations with experimental data taken from an acid-aniline rocket motor operating in the regime of separated flow. The graphical correlation does not check the data of Figure 11 very well however. J. D. McKenney[12] has performed experiments on flow separation in two-dimensional flow nozzles, and reports that the separation is unsymmetrical and unstable at pressure ratios less than 12. This agrees with the data of Figure 11, but at low pressure ratios (4 and under) the flow separation occurs in an oblique shock, and the evidence of instability is not conclusive. At very low pressure ratios (less than 2) Figure 11 shows the shock to become approximately normal as in the classical viewpoint. Another interesting phenomenon observed in Figure 11, is thatacross the flow in the direction of the optical axis,

* Ref. 8, pages 88-93.

the flow does not separate symmetrically, but one side leads the other. This is especially true for p_{od} in the range 20-40 psig (34.35-44.35 psia).

Some experiments were done with string tufts attached to the mouth of the driving nozzle and these tests indicate a strong reverse flow in the nozzle in the regime of separated flow. This is in contradiction to Summerfield's assumption of a stagnation state.

A theoretical examination of nozzle flow separation is a study in itself and will not be gone into further here. It appears however that a successful theory for flow separation must consider the subsonic pressure communication in the zone of separation. Taking this assumption, it can be seen that the pressure communication is more difficult with slowly diverging nozzles than with rapidly divergent ones. Accordingly the slowly divergent nozzle should tend to operate in the regime of normal shock, while the rapidly divergent nozzle should have flow separation with oblique shocks. The appearance of the normal shock near the throat where divergence is small (Figure 11) tends to substantiate this hypothesis.

Flow separation in the driving nozzle may influence the ejector operation in several ways, but appears most important in relation to the starting condition. It is possible that flow separation may actually make the ejector easier to start as will be seen presently.

The starting condition has thus far been examined with respect to the shock in the driving nozzle and the subsonic flow beyond the shock. It will now be shown that the mixing tube area must also satisfy certain requirements in order that the ejector will start. It may be

readily shown that for any critical gas flow ($M = 1$), the area of the throat is related to the stagnation pressure of the flow by the simple relation $Ap_o = K$, where A is the area of the throat, and K is a constant for the particular flow. When the flow in the driving nozzle enters a shock, it is evident that on emerging from the shock, the flow can never again be reduced to area d_1 (Figure 8) but must always thereafter occupy some larger area. This is because of the reduction in p_o caused by the shock. The largest reduction in p_o occurs when a normal shock is located in the mouth of the driving nozzle because this corresponds to the maximum Mach number attainable in the nozzle. At this point in the test ejector, $M = 3.22$ in front of the shock and $M = 0.463$ behind the shock. The shock tables show p_o to be reduced by the factor 0.272 and therefore the minimum area through which the flow can subsequently pass is $0.13/0.277 = 0.48$ sq in., which corresponds to $M = 1$, and for the design of the test ejector corresponds to the minimum permissible area of the mixing tube. If the test ejector had been designed with this modification instead of the 1.00 sq in. mixing tube then it would possess a real advantage as a device for producing maximum suction (see Figure 13).

The reason for this is as follows. Assume the test ejector is modified as above and the only change made is to make $d_3 = 0.48$ sq in. instead of 1.00 sq in. Under these conditions, since $d_4/d_3 = 4$ and since d_3 is a throat, it is only necessary that at d_3, $p_o > p_m$.

Also at d_3, $p \cong 0.528$ atm = 7.60 psia. Now after leaving the normal shock at the nozzle the flow does not have to expand to 1.00 sq in. as with the test ejector, but is soon reduced, so that excluding other

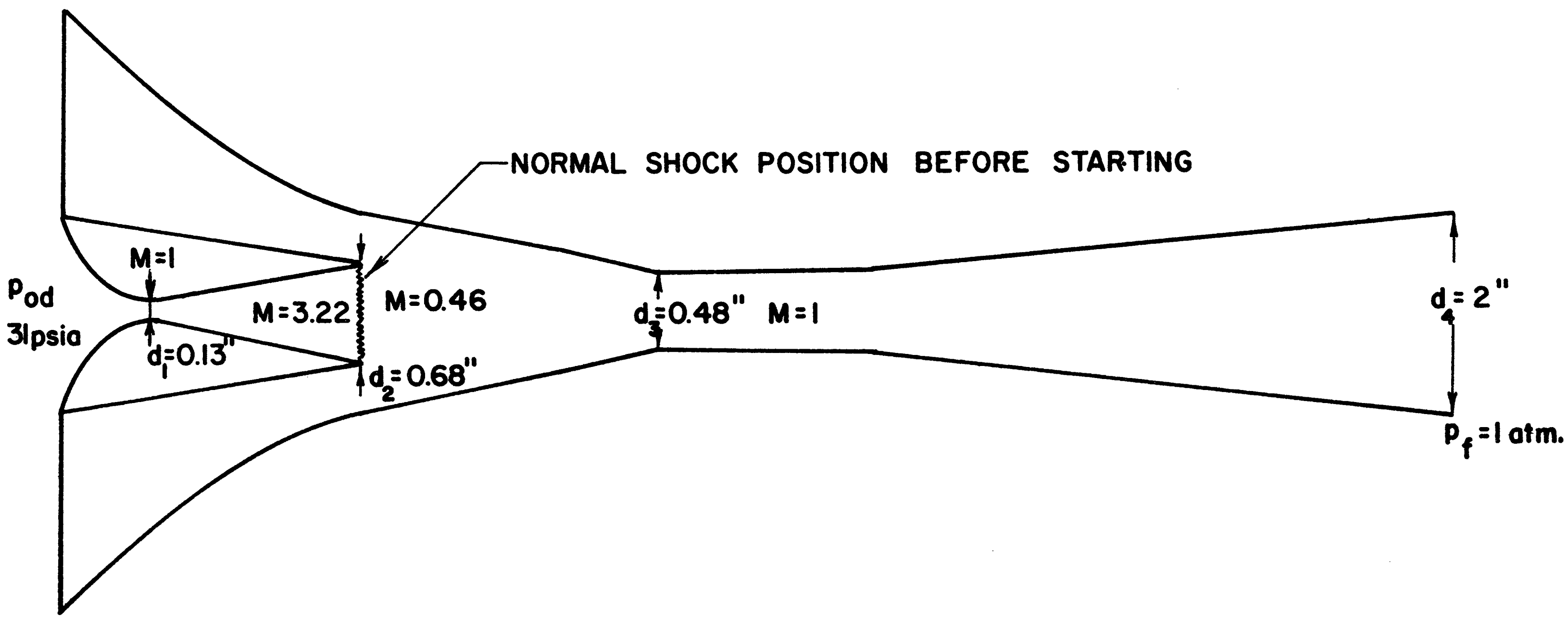


Figure 13. Modification of Completely Throttled Ejector to Obtain Maximum Suction That Allows Positive Starting.

losses but shock the pressure of the subsonic flow leaving the shock and having area 0.68 sq in. is 0.86 atm = 12.30 psia. At this point $p_o \cong 1$ atm = 14.35 psia. The shock tables show that the loss in p_o across the nozzleshock is the factor 0.277, so that the necessary requirement for p_{od} is $p_{od} = 14.35/0.272 = 53$ psia. The normal shock at the mouth of the driving nozzle is in an unstable configuration and a perturbation of pressure will send it toward d_3 where the duct is contracting. This means that the supersonic part of the flow is taking shock at lower Mach numbers which reduces the pressure ratio across the shock, which also reduces its celerity and it is thus swept on through d_3 into the region between d_3 and d_4 and supersonic flow exists at d_3, and the ejector is started. After the ejector is started, the back pressure p_m may be increased until the shock is forced back toward d_3 and is reduced in size to 0.48 sq in.

This is the minimum loss condition for the flow system. On the other hand, once the ejector is started against $p_m = 1$ atm, it will continue to run if p_{od} is reduced, as an alternative to increasing p_m. The reduced value p_{od} may be obtained by considering that supersonic flow exists at d_3 and the shock area is 0.48 sq in. According to this the Mach number at d_3 corresponds to the area ratio $d_1/d_3 = 0.13/0.48 = 0.27$ and is $M_3 = 2.86$. At M = 2.86, for normal shock, the loss in p_o is the factor 0.370 and for $p_m = 1$ atm this gives $p_{od} = 39$ psia. Therefore the modified ejector will start at $p_{od} = 53$ psia, and will run at $p_{od} = 39$ psia against $p_m = 1$ atm. The suction pressure corresponds to $p_{od} = 39$ psia and $M_2 = 3.22$ and is $p_c = 0.76$ psia.

There is a step beyond this modification however, which will produce still improved performance. It was shown above, for the otherwise fixed design of the test ejector, that d_3 could be reduced to 0.48 sq in. Areas for d_3 less than this would not allow the ejector to start and supersonic flow could not be developed in the suction chamber. However, once the ejector is started, d_3 may be further reduced to d_1 and the ejector will continue to run because the shocks are beyond d_3. In this case and as usual assuming no flow losses except those due to shock p_{od} may be reduced to p_{od} = 1 atm and p_c = 0.282 psia.

The characteristics of the closed suction test ejector and the modifications, are summarized in the following table. It is assumed in all cases that p_m = 1 atm.

Table I shows for the test ejector design, the performance sacrificed by not designing the ejector for maximum suction. Of course, the purpose of an ejector is not necessarily to produce maximum suction with minimal flow. Indeed the more usual case of application is as a pump for induced flow. The case of closed suction serves however as a guide to the design of maximum suction ejectors, and as will be seen in the next section the experimental data serves as a basis for distinguishing the flow losses due to gas dynamics from those proposed in the basic thermodynamic analysis.

Before ending the section, it is now appropriate to consider further, the effect of flow separation in the driving nozzle on the problem of starting the ejector. The occurrence of oblique shock due to flow separation in the driving nozzle instead of the classically assumed normal

TABLE I

CHARACTERISTICS OF FAMILY OF RELATED EJECTORS

Design	Test Ejector	Positive Start Mod. Normal Shock	Positive Start Mod. Oblique Shock	Max. Suct. Mod.
d_3	100 sq. in.	0.48	.28	0.48-0.13 Variable
p_{od} (start)[1]	76 psia	53 psia	31 psia	53 psia
p_{od} (run)[1]	76 psia	39 psia	24.5 psia	14.35 psia
Suction[1]	1.49 psia	0.76 psia	0.48 psia	0.282 psia
(p_{od})(start)[2]	83 psia	58 psia	34 psia	58 psia
(p_{od})(run)[2]	83 psia	43 psia	27 psia	15.90 psia
Suction[2]	1.07 psia	0.84 psia	0.53 psia	0.31 psia
p_{od} (start)	95 psia[3]	66[4]	39[4]	66[4]
p_{od} (run)	95 psia[3]	49[4]	30.7[4]	18[4]
Suction	2.95 psia[3]	1.52[4]	0.95[4]	0.56[4]

[1] Considering only shock losses.
[2] Considering shock losses and accountable flow losses.
[3] Experimental
[4] Estimation based on experimental data.

- - - - - - - - -

shock means that smaller than expected losses in p_o of the stream are incurred. This in turn leads to the possibility of starting the ejector at smaller areas of the throat d_3 than is predicted on the basis of normal shock in the driving nozzle. The schlieren data (Figure 11) indicate that a shock angle at separation of approximately 30° applies to the experimental

driving nozzle near the starting condition. When the shock angle is applied to the flow approaching the shock, it is found that the flow behind the shock is still supersonic instead of subsonic. Computing conditions at the mouth of the nozzle gives $M_2' = 3.22$ and $M_2'' = 2.40$ where M' applies to the flow ahead of the shock and M" applies to the flow behind the shock. The choking area ratio for the flow at $M = 2.40$ is 0.42 and since the area of the $M = 2.40$ flow is 0.68 sq in. corresponding to d_2, the area d_3 is $0.42 \times 0.68 = 0.28$ sq in. Thus d_3 may be reduced from 0.48 sq in. for positive start with normal shock to 0.28 sq. in for positive start with oblique shock. The corresponding values for p_{od}, p_c, d_3 are entered in Table I under oblique shock.

In summation, it is seen that the principles of gas dynamics can be successfully applied to predicting the maximum suction of the ejector, and by using conventional factors to account for flow losses, the result may be predicted to an error of about 50%. It is quite probable that if based on further experimental work, and additional account is taken of supersonic flow losses in free stream boundaries, this error can be greatly reduced.

2. Partially Throttled Suction Ejector

In the section on thermodynamics, an ejector model was proposed which was to account for the unavoidable losses in an ejector due to the fluid shear between the two streams. In a practical ejector, there will naturally be losses due to wall friction, pressure shock, etc., that are in addition to shear losses, but these losses are of a kind that might be minimized if not eliminated by proper design. The losses due to fluid shear are supposed to be an inherent part of the process and cannot be changed. The purpose of the research devoted to this section is to attempt to evaluate the proposed theory as regards shear losses.

The experimental work consists of optical and pressure measurements on the test ejector as was the case with closed suction, but in this case, the induction nozzle is not closed, but is throttled through a pair of 0.280" diameter sharp edge orifices.

The inlet air to the orifices is drawn from the atmosphere and the flow conditions at the mouth of the inductor nozzle are computed according to the standard methods for orifices, as functions, of the pressure ratios across the orifices reduced to the conditions at the nozzle mouth. These data are presented in Figure 14.

As to the possibility of testing, there are numberless different tests which could be made on the test ejector, but fortunately only relatively few are significant to the purpose at hand. If, for example, it is desired to evaluate the shear losses, the conditions for doing this are at an operating point where flow losses are minimized and where shear effects are prominent. Now according to the theory of shear losses evaluated by velocity and temperature equilization, the best operating point

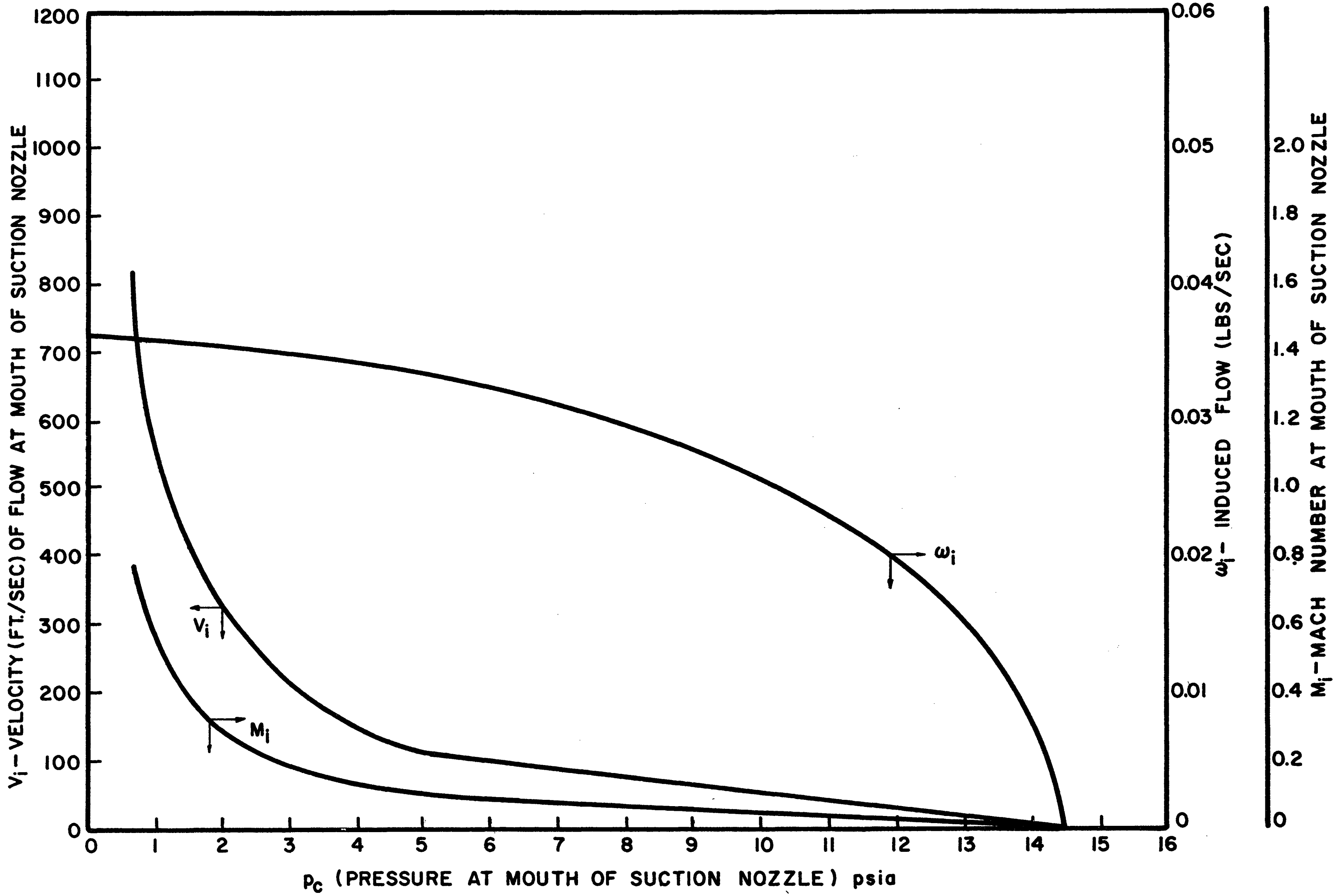


Figure 14. Relations Between State of Induced Flow and Static Pressure at Mouth of Suction Nozzle with Partially Throttled Flow. (0.280" Orfices)

for the ejector is when V_i and V_d at the beginning of mixing are as nearly equal as possible. Assuming also that the driving nozzle works most efficiently at fully developed flow, there obtains for the test ejector design $M_d = 3.22$, $M_i = 1.00$, for the best possible conditions.

On the other hand, according to the theory, improved performance would result if $M_i > 1.00$ (for $T_{oi} = T_{od}$). This could be achieved however, only by modification of the induction nozzle of the test ejector and was not done since it would defeat the purpose of the test which is not directly aimed at improved performance, but is intended to evaluate the theory of shear losses.

The effect of the significant test variables is perhaps most easily found through the development of a tabular or graphical relation between these variables. What is needed is a way to determine the possible operating conditions for the test ejector. The experimental conditions themselves limit the possible range of operating conditions and thus reduce the number of choices. In general, the experimental conditions amount to the back pressure being 1 atm., and the induced fluid having stagnation pressure of 1 atm. The stagnation temperatures are also fixed. In addition there are design factors which limit the range of possible operation. These amount to the expansion ratio of the driving nozzle, the expansion ratio of the induction nozzle, their areas, the area of the mixing tube, and the dimensions of the subsonic diffusor. Also there are certain assumptions which still further limit the range of possible operation. It will be assumed that fully developed flow exists in the driving nozzle because this corresponds to its most

efficient operation. The induction nozzle can have the maximum velocity $M = 1$ since it is a converging nozzle only. It may also operate at lower Mach numbers. With these limitations in mind, the procedure is as follows:

1. Choose a parametric design pressure p_c for the mixing chamber. This will fix p_{od} since the driving fluid is in supercritical flow through a fixed geometry nozzle. If p_c is chosen to be 1.00 psia, this gives at the mouth of the driving nozzle $M_d = 3.22$, $p_{od} = 51$ psia. From the test conditions $T_{od} = 540°R$, so that at the driving nozzle mouth $T_d = 175°R$, $w_d = 0.153$ lb/sec, $V_d = 2090$ ft/sec.

Choosing p_c to be 1.00 psia does not fix the flow in the induction nozzle because it is in sub-critical flow. For low Mach numbers at the mouth of the induction nozzle, it is safe to assume that approximately the same pressure exists at the entrance, and this gives the pressure on the downstream side of the flow metering orfices which in turn leads to the weight flow in the induction nozzle. The weight flow at the mouth of the induction nozzle together with the assumed pressure p_c and an assumed Mach number lead to T_i at the nozzle mouth. At higher Mach numbers at the mouth of the induction nozzle, account must be made of the pressure drop from the entrance to the mouth. Also a slight correction for pressure recovery behind the metering orfices can be applied. These factors were considered in arriving at the data in Figure 14 for the induction nozzle. Thus a set of values M_i are chosen, and at each M_i there are computed corresponding T_i, w_i, V_i all at $p_c = 1$ psia.

2. The conditions have now been obtained for the application of the theory of temperature and velocity equalization. In applying these relations, the rule is followed that the best possible ejector will be designed that is self starting. This is because the test ejector is of fixed geometry and cannot start with one configuration, and run with another more favorable one. In the application of the theory this implies in general that the best possible ejector will usually have a contracting mixing tube, the minimum area of which will just allow the ejector to start as was discussed in the preceding section. After starting, the minimum area will run with supersonic flow, and it is assumed that a normal shock exists at this minimum area to convert the stream to subsonic flow before entering the subsonic diffusor. From the viewpoint of the effect on the p_o of the mixed streams, this is the best place to have the shock and so it is assumed to be there in making the calculations.

3. After the mixed streams have passed through the stablizing shock at the minimum area of the mixing tube, there will be a new and lower p_o called p_{o2}. This value of p_o must be greater than 1 atm, or the flow is impossible. If it is less than 1 atm, then the parametric design pressure p_c must be increased by a corresponding amount and this leads to an increase in p_{od} by the same proportion. Since p_{oi} is fixed, the change involves some adjustments in the quality of the induced stream, but these were found to be negligible over the ranges considered.

The application of these principles leads to the data in Table II which has a set of special symbols.

TABLE II

COMPUTATIONS FOR PREDICTING TEST EJECTOR PERFORMANCE

M_i	T_i	V_i	p_{oi}	A_m	M_m	A_r^*	A_s^*	p_{om_1}
	°R	ft/sec	psia	in.^{2}		in.^{2}	in.^{2}	psia
0	530	0	1.00	0.68	3.22	0.13	0.48	51
.10	530	113	1.01	0.79	2.98	0.19	0.54	35
.25	523	280	1.045	1.04	2.62	0.35	0.74	20
.50	504	550	1.19	1.31	2.37	0.57	0.95	14
1.00	442	1030	1.89	1.78	2.20	0.89	1.37	10.7

M_i	w_i	w_i'	p_{od}'	p_{oi}'	p_c'	M_s	p_{om_2}
	lb/sec	lb/sec	psia	psia	psia		psia
0	0.00	0.00	40.5	0.80	0.80	2.86	18.9
.10	0.006	0.005	47	0.92	0.92	2.58	16.3
.25	0.015	0.019	65	1.32	1.27	2.29	11.8
.50	0.030	0.044	75	1.75	1.47	1.98	10.2
1.00	0.065	0.119	93	3.45	1.83	1.91	8.2

where

M_i, T_i, V_i, M_d, T_d, V_d, represent conditions at the mouths of the respective nozzles.

A_m, M_m represent the conditions of the flow after velocity and temperature equalization at constant pressure.

M_s = Mach number entering shock at A_s^*

A_r^* = area of the mixed flow reduced to M = 1

A_s^* = area of the mixed flow that will allow the ejector to be self starting.

p_{om_1} = stagnation pressure of the mixed streams at A_s^* upstream of shock.

p_{om_2} = stagnation pressure of the mixed stream at A_s^* downstream of shock.

p_{oi} = pressure of induced stream at entrance to induction nozzle, just behind metering orfice.

p_c = parametric design pressure at mouths of driving and induction nozzles or in the suction chamber.

p_c', p_{od}', p_{oi}', w_i' = values for the respective streams when p_{om_2} has been normalized to 1 atm., or slightly greater, taking into account subsonic diffusor geometry.

The significance of Table II can now be considered. Suppose the case in Table II is considered where M_i = 1.00. This corresponds to the most favorable condition of operation attainable in the test ejector with respect to fluid shear losses. It is seen from Table II however, that the minimum starting area for this condition is A_s^* = 1.37 sq in., so it can be concluded that operation about this point is not possible in the test ejector since the corresponding area for the test ejector is only 1.00 sq.in. On the other hand, operation about the point M_i = 0.10 is not possible since this would require the mixing tube to be constricted to 0.54 sq in. whereas it has the area 1.00 sq in.

The possible operation of the ejector whereby it will be self starting is seen to correspond closely to $M_i = 0.50$ since the required starting area is 0.95 sq in., and the corresponding area of the test ejector is 1.00 sq in.

The computed performance in the range $M_i = 0.50$ can be compared to the experimental data taken with partially throttled suction (Figure 15). It is observed experimentally that the test ejector "starts" in the range p_{od} (60-70 psig)(74.35-84.35 psia). Maximum suction (4.40 psia) is attained at p_{od} = 70 psig (84.35 psia). The starting condition is also observed in the schlieren photographs (Figure 16) where the flow in the mixing tube suddenly changes from subsonic flow (p_{od} - 60 psig) to supersonic flow (M = 1.70) for p_{od} = 70 psig (84.35 psia). Interpolating Table II for the starting condition corresponding to $A_s^* = 1.00$ sq in. gives $p_{od}' = 78.35$ psia which checks well enough with the observed value p_{od} (74.35-84.35 psia). Using Figure 14 for the induction nozzle shows that when $p_c = 4.40$ psia as experimentally observed, $M_i = 0.12$ and $w_i = 0.034$ lb/sec. On the other hand the computed values (Table II) are $M_i = 0.50$ and $w_i' = 0.044$ lb/sec. This variation in M_i will have a very minor influence on the computed performance since the difference in V_i and V_d is large regardless of whether $M_i = 0.50$ or 0.12. The difference in weight flow is of greater significance however, and must be charged against the accuracy of the theory.

Before an accounting can be made of the accuracy of the theory, some means must be devised to account for the other losses in the ejector. By other losses is meant the nozzle losses, non-uniform flow velocity, friction losses, diffusor losses and shock losses. None of these except the most ideal of shock losses were considered in developing Table II.

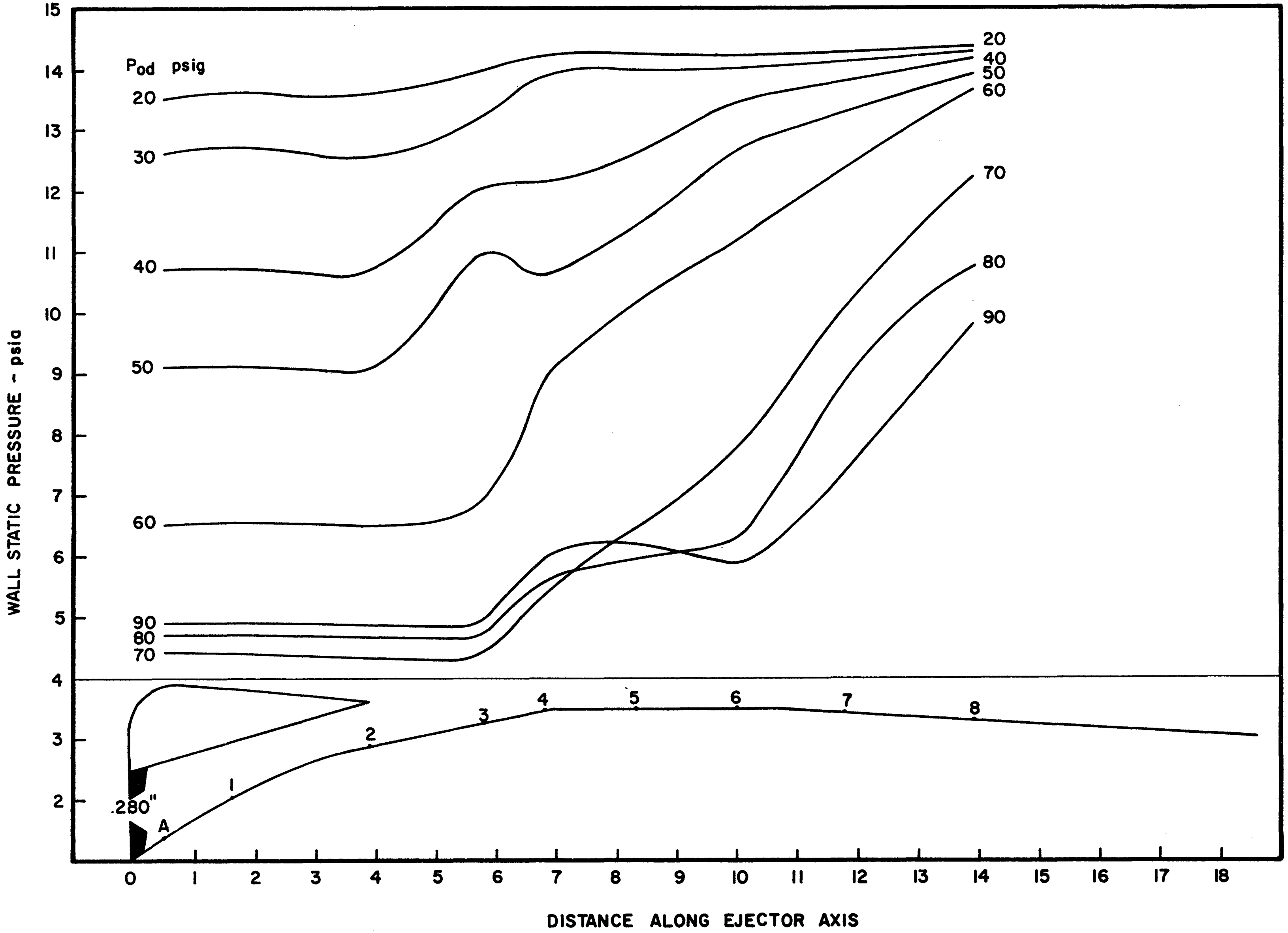
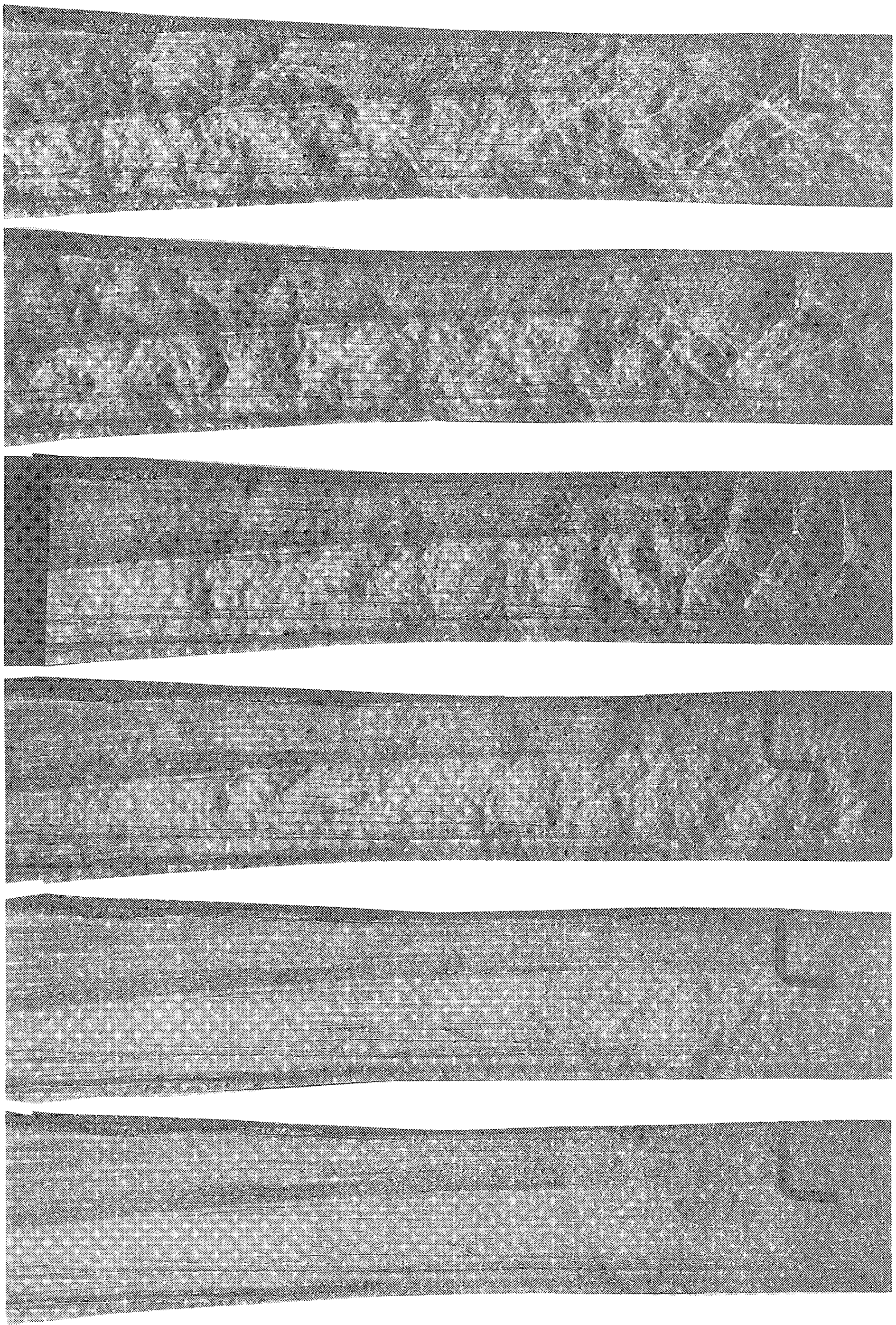


Figure 15. Experimental Pressure Data for Partially Throttled Ejector. (0.280" Suction Orfices)

It would have been possible to develop Table II by assuming certain values of nozzle and diffusor efficiencies, friction factors and the like, but there is a better way to account for these factors and that is from the data of the test ejector itself. The operation of the test ejector with closed suction will account for all loss factors except those due to the induced fluid. Thus the performance of the closed suction ejector serves as a basis with which to compare the performance of the throttled ejector and so it is possible to evaluate this performance as though these extraneous losses were removed.

The comparison to be made involves the operation in the range p_{od} = 70 psig where the test ejector gives p_c = 4.40 psia, w_i = 0.034 lb/sec and Table II for the ideal ejector gives p_c = 1.47 psia, w_i' = 0.044 lb/sec. The basis for evaluating extraneous losses is taken from the experimental data for the closed suction (Figure 10) for p_{od} = 70 psig and this gives p_c = 2.95 psia. It is noted from Figures 12 and 16, that the shock patterns for the throttled and closed suction ejectors are nearly identical at p_{od} = 70 psig, so that flow losses from this source can be expected to be equal. Since the discharge pressure in all cases is 1 atm, and since the compression of the induced flow involves a relatively narrow range of pressures and also because the compression of the induced stream is nearly isothermal due to the mixing with the driving stream, it is possible to consider the relative work done on the induced streams by compression as substantially linear with variations in p_c. On this basis, it is noted that the ideal ejector (Table II) when corrected by the base suction pressure 2.95 psia as obtained from the closed suction case (Figure 10)

← Flow



90 psig (104.35 psia)

80 psig (94.35 psia)

70 psig (84.35 psia)

60 psig (74.35 psia)

50 psig (64.35 psia)

40 psig (64.35 psia)

p_{od}

Figure 16. Schlieren Photographs of Mixing Tube Flow of Various Values of p_{od} with Suction Partially Throttled. KNIFE EDGE VERTICAL.

gives a corrected suction pressure $p_c = 1.47 + 2.95 = 4.42$ psia. This compares remarkably well with p_c observed for the partially throttled jector which is 4.40 psia. The discrepancy in the weight flow ($w_i' = 0.044$ lb/sec, Table II, $w_i = 0.034$ lb/sec (Figure 14)) is mostly attributable to the theory of temperature and velocity equalization which in this case predicts weight flows about 30% greater than observed. Some corrective factors are known, but have not all been evaluated. For example, in the experimental data, if a correction is made for the relatively low Mach number $M_i = 0.12$ observed, then the discrepancy in weight flow is reduced to 25%. The adoption of Reynolds theory of turbulent shear would also apply a correction on the total work done on the induced fluid which would be in the right direction, but its magnitude has not been evaluated.

The evaluation just made is the only one that allows a compari= son between the theory and experiment with the available test equipment. Basically this is because ejectors are essentially point operating devices. Additional comparisons could be made on the basis of changing the ejector design or of changing test conditions by heating the working fluids for example, or by using different working fluids, etc. Good comparisons cannot be made with the existing test apparatus outside the point con- sidered for the following reasons:

At the higher values of p_{od} (80, 90 psig) (94.35, 104.35 psia), the comparison is invalid because with the fixed expansion ratio of the driving nozzle and fully developed flow, the suction pressure is found to increase as it is seen to do in Figure 15. This reduces the flow

rate of the induced stream which further increases p_o of the stream entering the mixing tube. This increased value of p_o is dissipated by the increasingly strong shock seen in the mixing tube in Figure 16 for p_{od} = 80, 90 psig. At p_{od} below 60-70 psig, the operation enters the regime of separated flow in the driving nozzle which cannot be presently predicted by available theory.

It is thus seen from this section that the inherent performance of the ejector, excluding flow losses due to friction and shocks, can be evaluated with reasonable accuracy by means of the theoretical model proposed. The gas dynamics in the mixing tube involve the termination of shock waves in the boundary layer which consists of the induced stream. This condition represents an unexplored problem in gas dynamics and currently limits the scope of the understanding of the ejector process.

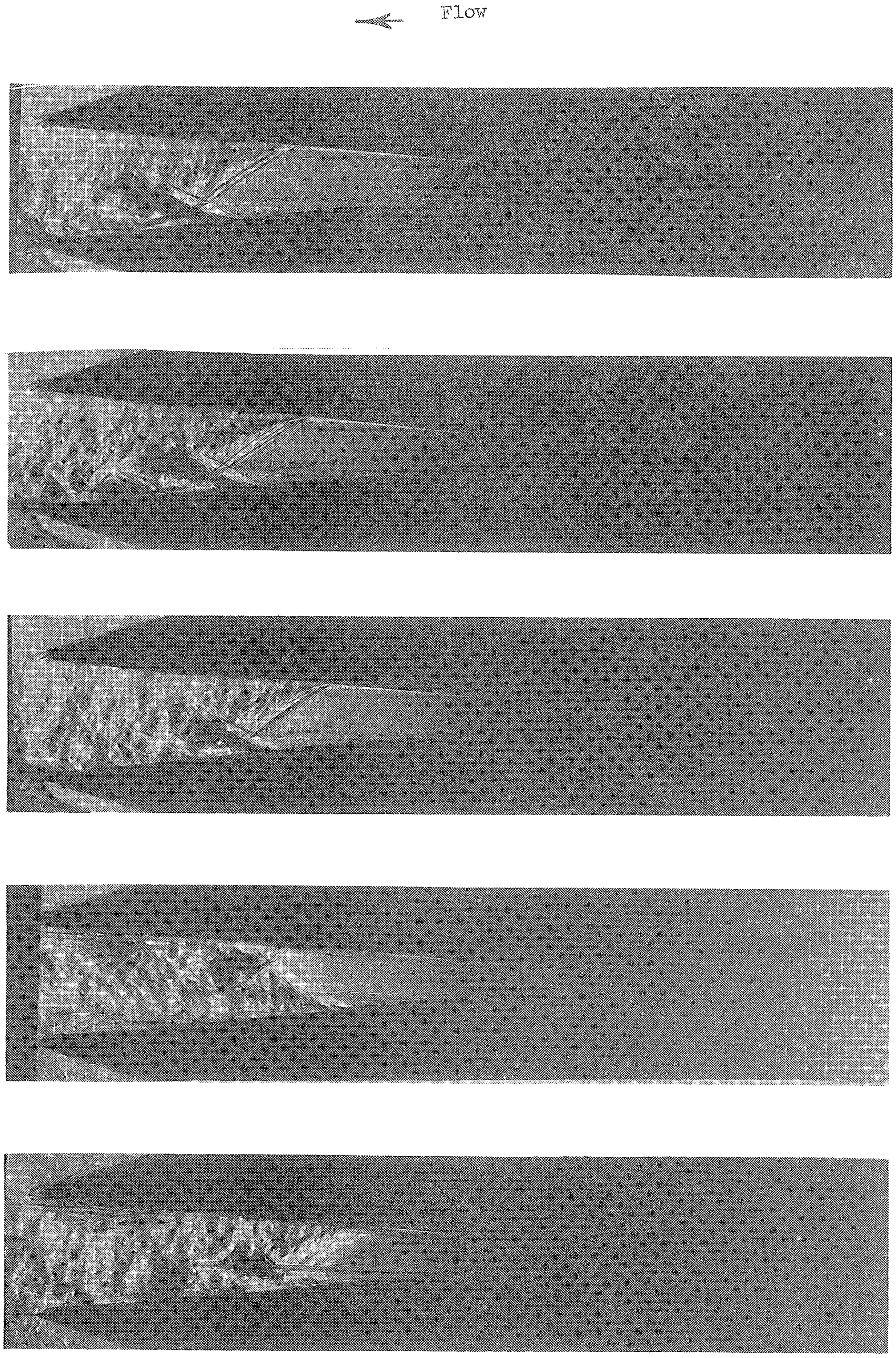
3. Unthrottled Suction Ejector

When the suction of the ejector is left unthrottled, the pressure in the suction chamber is high, so that the pressure ratio across the driving nozzle is low. This causes the driving nozzle to operate in the regime of separated flow for all available ranges of driving pressure, and as has been pointed out before, the quality of the flow emerging from the driving nozzle cannot be predicted for these conditions. There is therefore no basis for making predictions about the ejector performance.

Experiments with the test ejector have been made however with unthrottled suction, and some interesting observations have been made which bear reporting since they will become more significant with a better understanding of the driving nozzle performance with flow separation.

The fact that the driving nozzle is operating with separation at all driver pressures is clearly seen in Figure 17 which shows Schlieren photographs of the nozzle flow. At the lower driving pressure (p_{od} 20-40 psia) (34.35-54.35 psia), the separation occurs so early in the expanding part of the nozzle and with such strong and multiple shocks, that the flow emerging from the mouth of the nozzle will probably have about the same state that would be obtained by throttling the driving stream to the same flow area. This is only a hypothesis and is not supported by any other measurements made on the test ejector.

At the higher driving pressures p_{od}, (60-90 psig)(74.35-104.35 psia), the driving nozzle flow separates closer to the mouth of the nozzle and with oblique shock, which means that the flow from the mouth, of the nozzle, while not accurately predictable, is probably supersonic.



80 psig (94.35 psia)

70 psig (84.35 psia)

60 psig (74.35 psia)

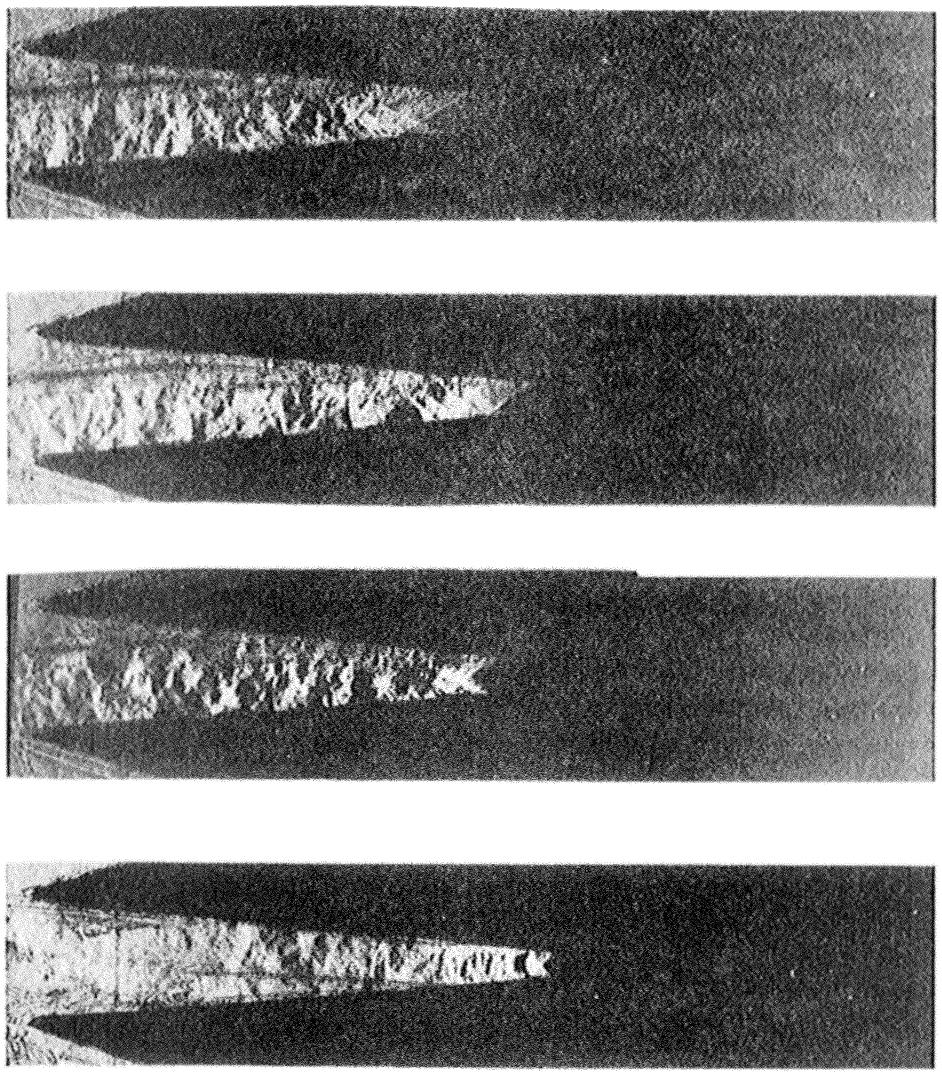
50 psig (64.35 psia)

40 psig (54.35 psia)

p_{od}

Figure 17a. Schlieren Photographs of Driving Nozzle Flow Separation at Various Values of p_{od} with Unthrottled Suction. KNIFE EDGE VERTICAL

← Flow



30 psig (44.35 psia)

20 psig (34.35 psia)

20 psig (34.35 psia)

10 psig (24.35 psia)

p_{od}

Figure 17b. Schlieren Photographs of Driving Nozzle Flow Separation at Various Values of p_{od} with Unthrottled Suction. KNIFE EDGE VERTICAL : Continuation of 17a.

Thus the performance of the ejector can be expected to improve out of proportion to the pressure increase at the higher driver pressure. This is actually apparent from Figure 18 which shows that the pressure drop across the suction nozzle is negligible for p_{od} = 20 psig, but becomes significant for p_{od} = 60, 70, 80, 90 psig. These pressure drops are of course indicative of the flow of induced fluid which is the only performance the ejector can have, as it is only handling flow without producing pressure increase.

Probably the most interesting phenomenon observed during the unthrottled ejector tests is the presence of approximately normal shocks at the entrance of the mixing tube for p_{od} 60-90 psig. These shocks are visible in Figure 20 at the right hand side of the photograph. These shocks do not show up as well on thephotographs as they do by visual examination and this is probably due to the fact that the flow is not perfectly two-dimensional, but varies across the channel so that the shocks are not planar, and are not aligned with the Schlieren optical axis and therefore do not produce a sharp photograph. The peculiar thing about these shocks is their stability, as they are also observable with steady Schlieren illumination. It is not ordinarily concluded from gas dynamic considerations that a normal shock can be stabilized in a straight duct, but on the contrary it is concluded that once a shock enters a straight section of flow (constant area) it will either propagate upstream, or will be swept downstream to a larger area depending on the boundary conditions. On the other hand, these conclusions are based on considerations of a single stream of flow, and do not necessarily apply to a mixing flow of two streams. As a matter of fact, a rational explanation of the phenomenon can be based on the mixing process and is as

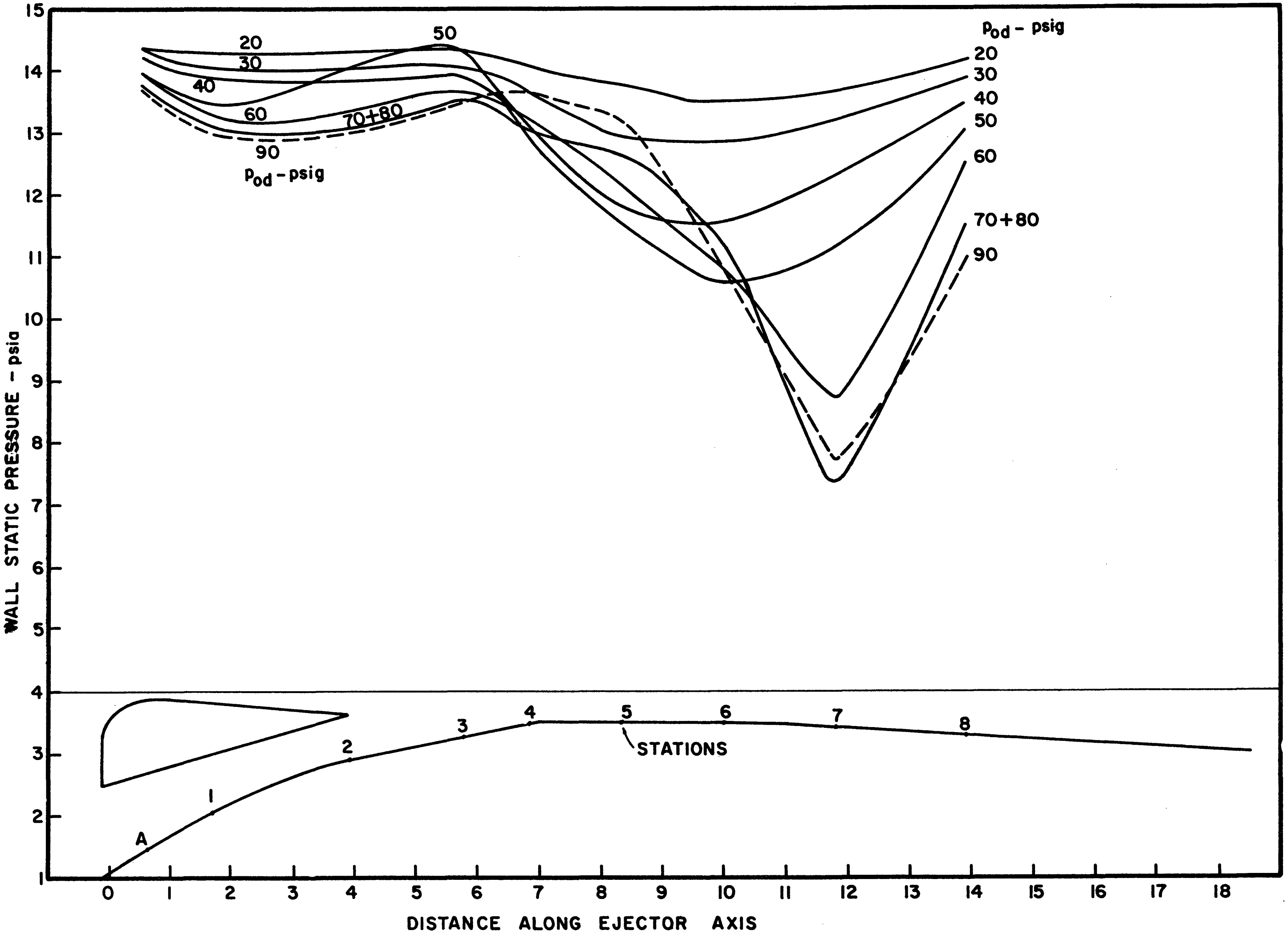


Figure 18. Experimental Pressure Data for Unthrottled Suction Ejector.

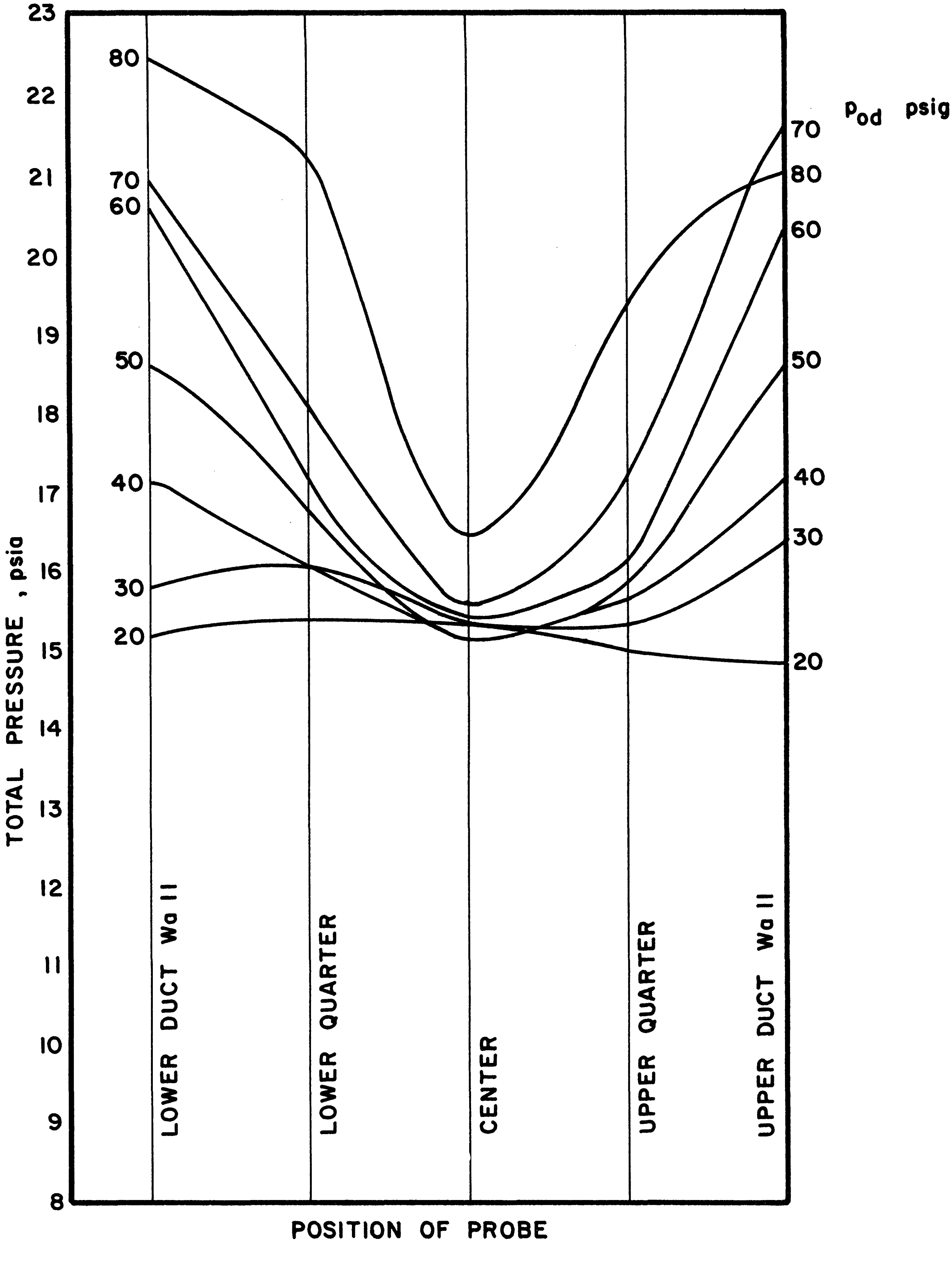
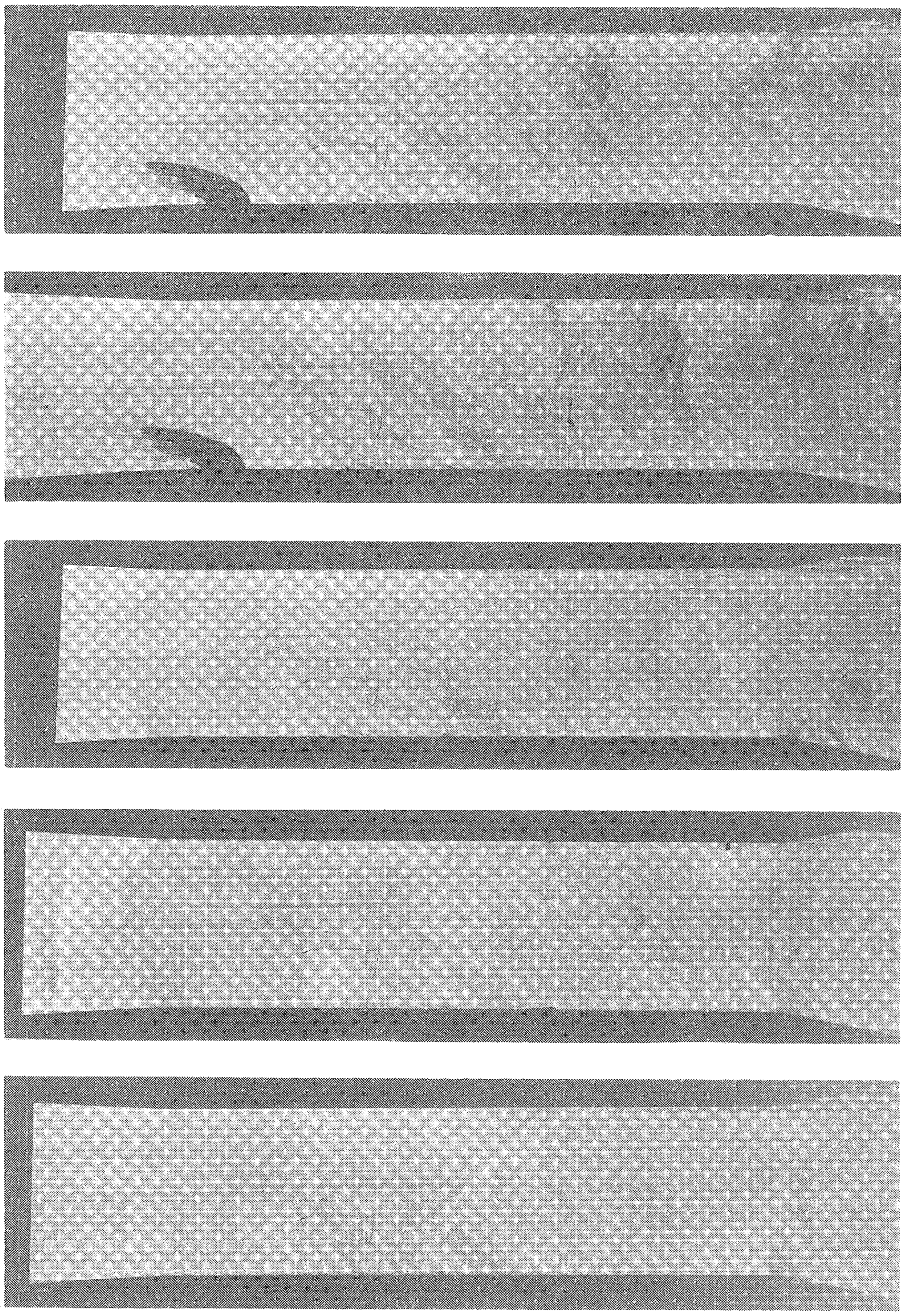


Figure 19. Experimental Pressure Data for Unthrottled Suction Ejector. Total Pressure Traverse at Station 5 Behind Normal Shock.

follows. Owing to the relatively high pressure existing at the mouth of the driving nozzle, the supersonic driving stream, which exists at high driving pressure, does not expand to fill the mouth of the driving nozzle, but is confined to a narrow channel of flow. The induced flow being subsonic may accomodate itself to the remaining channel area, and therefore occupies a wide channel in a large area. With this arrangement, the mixing presumably is accomplished after the flow has traversed some particular part of the constant area mixing tube. The compression of the supersonic driving stream accounts for the normal shocks (Figure 20) at the entrance of the mixing tube and is stabilized there by the requirements of the mixing process. For if it were assumed that the shock were swept downstream, then the high speed driving stream would continue to exist and to influence the induced stream over a greater distance which would increase the flow of the induced stream. This operation reduces p_o of the mixed stream, which becoming low enough cannot support the subsequent flow against the back pressure on the diffusor so that the flow accumulates downstream of the shock. This increases the pressure ratio across the shock and increases its celerity thus causing it to move back upstream to a position of equilibrium.

Also, if it is assumed that the shock moves upstream, the induced stream is influenced less by the driving stream, is reduced in flow and the pressure ahead of the shock increases which reduces the shock pressure ratio thus reducing its celerity. The shock will then move back downstream to a position of equilibrium. These processes depend on the fact that downstream of the shock, the velocity of the driving stream is considerably reduced because of the shock, and therefore has less influence

← Flow



p_{od}

80 psig (94.35 psia)

70 psig (84.35 psia)

60 psig (74.35 psia)

50 psig (64.35 psia)

40 psig (54.35 psia)

Figure 20. Schlieren Photographs of Mixing Tube Flow at Various Values of p_{od} with Unthrottled Suction. KNIFE EDGE VERTICAL

on the induced flow. The data of Figure 19 support this theory. These data show the results of a total head traverse made in the subsonic flow behind the shock at the entrance to the mixing tube. The traverse probe is located at station 5, Figure 18. It is seen from these data that the total head in the center of the flow which is presumably occupied by the driving stream is considerably lower than the total head at the edges of the flow which carries the induced stream. This surprising result is consistent with the process that involves a strong shock in the unmixed driving stream just ahead of the point of measurement, and hardly seems explainable on any other basis. The wall static pressure data of Figure 18 are also consistent with this point of view. It is seen here that supersonic compression takes place between station 2 and 4 (Figure 18) for all those p_{od} for which supersonic flow of the driving stream is probable (p_{od} 50-90 psig).

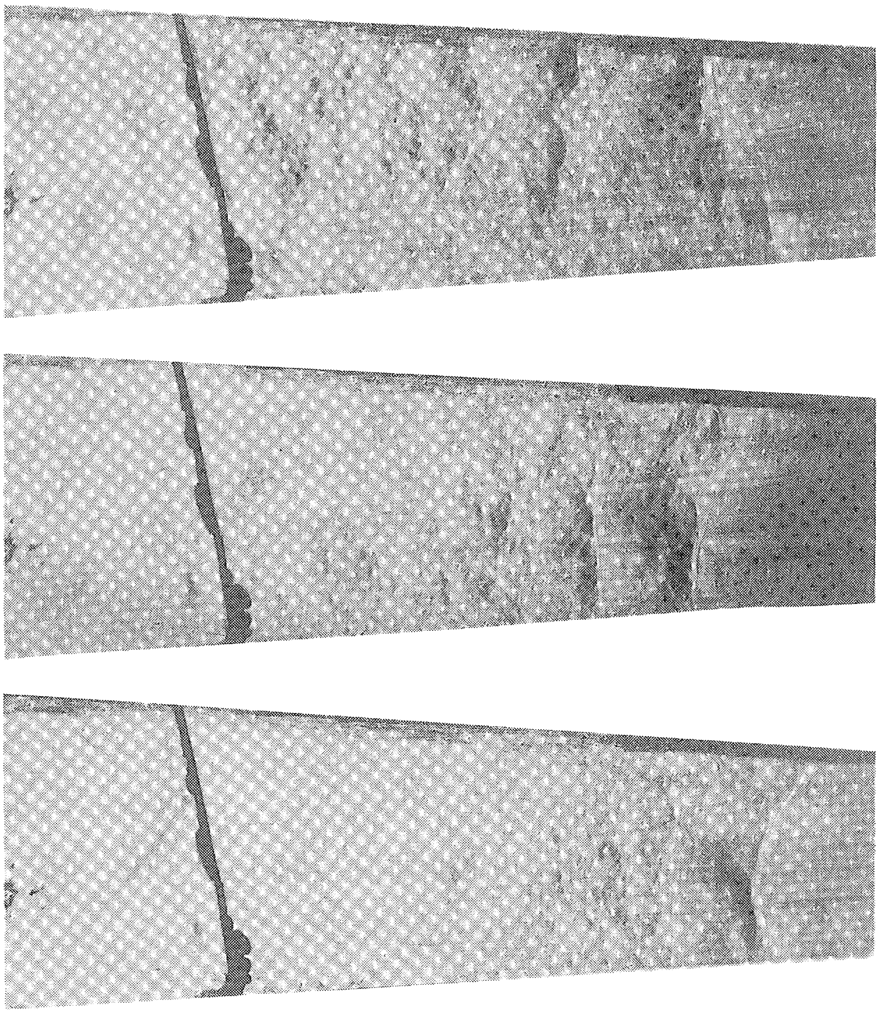
The pressure increase between stations 2-4 is not sharp as would normally be expected for a shock, but changes gradually. This, however is the pattern to be expected where a shock occurs in a supersonic stream adjacent to a subsonic stream and where the pressure measurements are made relative to the subsonic stream.

Behind the shock zone at the entrance of the mixing tube (at station 5 for example) the flow is subsonic. This is apparent from the Schlieren photos of Figure 20 and from the pressure data of Figures 18 and 19, for if the static pressure at station 5 is compared to the total pressure at station 5, and the most favorable example chosen (which is for p_{od} = 50 psig and the total pressure measured at the upper wall of the channel), then p_o for the stream is 19.0 psia and p = 11.6 psia, and

$p/p_o = 0.61$. Thus the flow is subsonic because at $M = 1$, p/p_o must be 0.53. On the other hand, the flow velocity is in the high subsonic region and taken with the value of p_o which exists (for example p_o for the stream averages about 18 psia for $p_{od} = 70$ psig) the conditions are proper for the reattainment of sonic velocity in the constant area mixing tube and for the attainment of supersonic velocity in the expanding diffusor. This the stream does by gradually reducing pressure between station 5 and 6 (Figure 18) as it attains sonic velocity. This is followed by a sharp reduction in pressure as the velocity becomes supersonic at station 7. A more or less normal shock occurs near station 7, causing a sharp increase in pressure as the flow again becomes subsonic and adjusts to the downstream boundary condition. These considerations apply particularly to the higher driving pressures (60-90 psig) where the flow is actually supersonic at the entrance to the diffusor and becomes subsonic through a normal shock. This condition is seen in the Schlieren photos of the diffusor operation (Figure 21).

This section of the study has been presented to demonstrate that a proper understanding of the ejector process depends on the application of gas dynamics as much as thermodynamics. The data and interpretations of this section are not intended to apply to an efficient operation, but are intended to be an aid in understanding the ejector process.

Flow



90 psig (104.35 psia)

80 psig (94.35 psia)

70 psig (84.35 psia)

p_{od}

Figure 21. Schlieren Photographs at Diffusor Flow at Various Values of p_{od} with Unthrottled Suction. KNIFE EDGE VERTICAL

IV. RESUME

At the beginning of this work it was felt that the application of new concepts and ideas was needed in the analysis of ejection. It appeared that the one-dimensional analysis of the problem had been almost fully developed without providing a rational basis for ejector design. The possibility of extending the two-dimensional analysis of Goff and Coogan(2) was considered, but it was decided that, considering the state of the art, it would be difficult to generalize their results, and even more difficult to use this approach as the basis for an engineering solution to the problem. Therefore what was done was to turn away from a strictly gas dynamic approach, and to try to develop a rational analysis based primarily on thermodynamic principles. This analysis has led to apparently significant results, and seems to show much more clearly than the gas dynamic analysis, the effect on performance of the various operating variables

Yet the thermodynamic analysis is limited and it is necessary to consider further the gas dynamics of the problem. In this respect an innovation has been made which combines an approximate two-dimensional analysis with the one-dimensional analysis. The result is a method of analysis which removes the ambiguity from the one-dimensional approach, and which is applicable to those conditions which cannot be idealized in a thermodynamic sense.

The gas dynamics have been further extended to consider the effects of compression and shock waves in those frequent cases which involves supersonic flow.

With respect to the experimental verification of the theory, it was soon apparent that the testing and evaluation of all the concepts developed would involve a task far too great for the modest experimental program that was possible with available funds and time. It was therefore decided to concentrate on one set of experiments with air as the experimental medium, which would serve to evaluate the theory of shear losses, since this part of the theory seemed to involve the most questionable assumptions. The results of these experiments, while limited in scope, do give evidence that the theory of shear losses is substantially correct, although the losses so predicted are somewhat lower than those actually measured. The proposed shear model itself could of course be modified, one tempting modification is the use of Reynold's turbulent shear analogy, which from the entropy viewpoint, provides an additional factor to account for the loss of stagnation pressure in a flowing stream with friction. At the present time however it appears that more experimental evaluations of the presently proposed model is needed. Another result of the experimental work is the condirmation of the Schlieren optical system as a valuable tool in ejector research.

As is often the case, this work seems to suggest more questions and areas of research than it provides answers. As to the further extension of the work, it appears that an intensive study of nozzles in over- and underexpanded flow would provide information immediately useful in ejector analysis, since the ejector theory presented here and elsewhere does not consider the events ahead of the driving nozzle mouth. It is also important that experiments be performed to test the theory of irreversible losses presented in the section on thermodynamics. Experiments

of this type with heated induction gas to equalize velocities before mixing, and also with gases of different ratio of specific heats, operated under conditions where both temperature and velocity are made equal before mixing, are needed. At the present time, there is no theory to describe the events attending the compression of a supersonic stream with mixing as occurs near the entrance to the mixing tube of the ejector. Where the induced flow is proportionately low, it is probable that the method of characteristics[14] can be used to determine the shape of the compression zone with adequate accuracy since the quality of the flow is determined primarily by the driving stream. The location of the shocks generated in the supersonic compression zone at partially and totally throttled flow is consistent with this method. On the other hand, the case involving a high rate of induced flow leads to compression shocks terminating at a subsonic flow boundary and is a case that has had only qualitative definition. The design objective in an investigation of the compression-mixing process is of course the elimination of shocks if possible.

As a final statement, the results of this work are not encouraging as related to the possibility of increasing the flexibility of the ejector, but it appears that there are real possibilities for obtaining improved performance in narrow operating ranges.

APPENDIX I

TEMPERATURE IRREVERSIBILITY

In order to analyze this problem, a process is imagined where the two fluids are expanded isentropically to a common pressure, p_c. The work obtained in this process is set aside. Then the two fluids a are allowed to reach temperature equalization irreversibily. For like fluids, this process is one of both constant pressure and constant volume, so no choice need be made. After temperature equalization is attained, the work previously set aside is applied to an isentropic process to attain a final pressure, p_{om}, for the mixture.

For the isentropic expansion,

$$W_d = w_d \, C_p \, T_{od} \left[1 - \left(p_c / p_{od} \right)^{\gamma-1/\gamma} \right]$$

$$W_i = w_i \, C_p \, T_{oi} \left[1 - \left(p_c / p_{oi} \right)^{\gamma-1/\gamma} \right]$$

$$T_d = T_{od} \left(\frac{p_c}{p_{od}} \right)^{\gamma-1/\gamma}$$

$$T_i = T_{oi} \left(\frac{p_c}{p_{oi}} \right)^{\gamma-1/\gamma}$$

$$T_m = \frac{w_d T_d + w_i T_i}{w_d + w_i} = \frac{w_d T_{od} \left(p_c / p_{od} \right)^{\frac{\gamma-1}{\gamma}} + w_i T_{oi} \left(p_c / p_{oi} \right)^{\frac{\gamma-1}{\gamma}}}{w_d + w_i}$$

$$p_m = p_c$$

For the compression of the mixture,

$$W_m = (w_d + w_i)\, C_p\, T_m \left[\left(\frac{p_{om}}{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Setting $W_m = W_d + W_i$ gives

$$(w_d + w_i)\, C_p\, T_m \left[\left(\frac{p_{om}}{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = w_d\, C_p\, T_{od}\left[1 - \left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}}\right] + w_i\, C_p\, T_{oi}\left[1 - \left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Substituting the value for Tm gives

$$w_d\, T_{od}\left[1 - \left(p_c/p_{od}\right)^{\frac{\gamma-1}{\gamma}}\right] + w_i\, T_{oi}\left[1 - \left(p_c/p_{oi}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$= \left[w_d\, T_{od}\left(\frac{p_c}{p_{od}}\right)^{\frac{\gamma-1}{\gamma}} + w_i\, T_{oi}\left(\frac{p_c}{p_{oi}}\right)^{\frac{\gamma-1}{\gamma}}\right]\left[\left(\frac{p_{om}}{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

which, solved for p_{om}, becomes

$$p_{om} = \frac{w_d\, T_{od} + w_i\, T_{oi}}{\dfrac{w_d\, T_{od}}{(p_{od})^{\frac{\gamma-1}{\gamma}}} + \dfrac{w_i\, T_{oi}}{(p_{oi})^{\frac{\gamma-1}{\gamma}}}}$$

It is seen from this analysis that where temperature irreversibility alone is considered, the final pressure is in no way dependent on the common pressure of expansion. This means that in the jet compressor where like fluids are involved, the sole necessity for gaining maximum performance is the elimination of velocity losses.

APPENDIX II

EFFECT OF HEAD ADDITION ON THE STAGNATION PRESSURE OF A FLOWING FLUID

Write the isentropic flow relation as follows.

$$\frac{V^2}{2gC_p} = T\left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}} - T$$

differentiate with respect to T and p_o and obtain

$$dT = \frac{\frac{\gamma-1}{\gamma} T p^{-1/\gamma} dp_o + p_o^{\frac{\gamma-1}{\gamma}} dT}{p^{\frac{\gamma-1}{\gamma}}}$$

$$\frac{dp_o}{dT} = \frac{p\left[1 - \left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}}\right]}{\frac{\gamma-1}{\gamma} T}$$

Since $p_o/p > 1$, the sign of the relation is negative. Therefore, if dT is positive (heat added), then dp_o is negative and the addition of heat (increase of stagnation temperature) causes a reduction of stagnation pressure.

APPENDIX III

THE RELATIONS GOVERNING TEMPERATURE EQUALIZATION OF TWO QUANTITIES OF AN IDENTICAL GAS

Consider two quantities, w_i and w_d, of a given gas at states T_i and T_d, and at the common pressure, p. These gases are to be mixed at constant pressure. What is the resulting volume?

By the first law,

$$T_m = \frac{w_d T_d + w_i T_i}{w_d + w_i}$$

$$\bar{v}_m = \frac{(w_d + w_i) R T_m}{p}$$

where $\bar{v}_m$ is the total volume. By substituting the above value for Tm

$$\bar{v}_m = \frac{R}{p}\left(w_d T_d + w_i T_i\right)$$

however,

$$\bar{v}_d + \bar{v}_i = \frac{R}{p}\left(w_d T_d + w_i T_i\right)$$

therefore,

$$\bar{v}_m = \bar{v}_d + \bar{v}_i$$

By the constant pressure process, the final volume of the mixture is the sum of individual volumes of the components before mixing. Therefore, the total pressure and the total volume are constant for this process. These simple relations do not hold when the gases are not identical.

Where the gases are different, the following relations may be used for a constant volume process.

$$p_m = p\left[\frac{\dfrac{w_d C_{pd} + w_i' C_{pi}'}{w_d C_{vd} + w_i' C_{vi}'} - 1}{\dfrac{w_d C_{pd} T_d + w_i' C_{pi}' T_i'}{w_d C_{vd} T_d + w_i' C_{vi}' T_i'} - 1}\right]$$

$$T_m = \frac{w_d C_{vd} T_d + w_i' C_{vi}' T_i'}{w_d C_{vd} + w_i' C_{vi}'}$$

Returning to the case of identical gases, the way this process is related to the one-dimensional flow equation is as follows.

Assume first, that the two gases have been expanded isentropically to equal velocities and pressures, V_c, p_c. Assume tentatively, that a constant total area temperature equalization and mixing will result in unchanged pressure for the two streams. Then by the momentum relation

$$p_d A_d + p_i' A_i' + \frac{w_d}{g} V_c + \frac{w_i'}{g} V_c = p_m (A_d + A_i') + \left(\frac{w_d + w_i'}{g}\right) V_m$$

since $p_d = p_i' = p_c = p_m$

$$p_c (A_d + A_i') + \left(\frac{w_d + w_i'}{g}\right) V_c = p_c (A_d + A_i') + \left(\frac{w_d + w_i'}{g}\right) V_m$$

Therefore, $V_m = V_c$ and there is no velocity change. Using this result with the continuity equation establishes v_m as follows,

$$w_d + w_i' = w_m$$

or

$$\frac{A_d V_c}{v_d} + \frac{A_i' V_c}{v_i'} = \frac{A_d + A_i'}{v_m} V_c$$

so that

$$w_d + w_i' = \frac{A_d V_c + A_i' V_c}{v_m}$$

from which

$$v_m = \frac{w_d v_d + w_i v_i}{w_d + w_i}$$

However, the tentative values p_c, V_c, must also satisfy the energy equation

$$w_d\left[\left(\frac{\gamma}{\gamma-1}\right)p_c v_d + \frac{V_c^2}{2g}\right] + w_i\left[\left(\frac{\gamma}{\gamma-1}\right)p_c v_i + \frac{V_c^2}{2g}\right]$$

$$= (w_d + w_i)\left[\left(\frac{\gamma}{\gamma-1}\right)p_c v_m + \frac{V_c^2}{2g}\right]$$

from which

$$v_m = \frac{w_d v_d + w_i v_i}{w_d + w_i}$$

Therefore, the values V_c and p_c satisfy the flow equation for one-dimensional constant area flow. The pressure and velocity are unchanged after mixing. v_m is given above, and

$$T_m = \frac{w_d T_d + w_i T_i}{w_d + w_i} \quad *$$

* Also see Chapter III, B, 2 for differentiation between subsonic and supersonic cases.

APPENDIX IV

FLUID SHEAR EFFECTS

In order to examine the effect of fluid shear in a stream, consider the two-dimensional flow diagram shown in Figure 22, in which the upper face of the fluid element has a velocity relative to the lower face. The absolute velocity of the two faces does not influence the shear.

If there is no acceleration of the element (steady flow), then the summation of horizontal forces is zero.

$$-\left(\frac{dp}{dx}\,\Delta x\right)\Delta y\,\Delta z+\left(\frac{d\tau}{dy}\,\Delta y\right)\Delta x\,\Delta z=0$$

which for elemental volume dx dy dz and two-dimensional flow becomes

$$\frac{dp}{dx}=\frac{d\tau}{dy}$$

and since

$$\tau=\mu\,\frac{dV}{dy}$$

$$\frac{dp}{dx}=\mu\,\frac{d^2V}{dy^2}$$

This condition states that when shear only occurs, there is zero pressure gradient along the flow for any linear velocity profile. Irreversible work is, of course, being done on the element through fluid shear, but since the summation of horizontal forces is zero, this work cannot be interpreted as the product of force times displacement along the axis of flow. The work on the element can, however, be approached by the rotation of the element. The equivalent heat input to the element can be expressed as the product of the couple acting on the shear faces and the angular velocity of rotation of a line joining these faces.

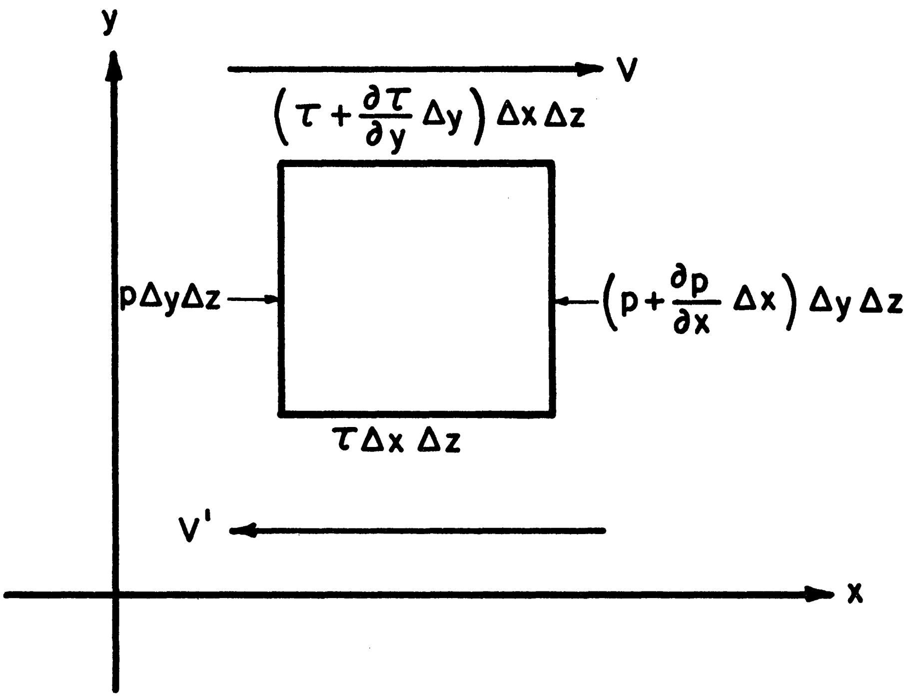


Figure 22. Fluid Shear in Two-Dimensional Flow.

Therefore,

$$\Delta q = (\tau \Delta x \Delta z) \frac{\Delta v}{\Delta y}$$

which for elemental volume becomes

$$\lim_{\Delta x \Delta y \Delta z \to 0} \frac{\Delta q}{\Delta x \Delta y \Delta z} = \frac{dq}{dv} = \tau \frac{dv}{dy} = \mu \left(\frac{dv}{dy}\right)^2$$

where dq is the heat generated in the elemental volume per unit time.

APPENDIX V

ONE DIMENSIONAL FLOW WITH HEAT ADDITION (CONSTANT PRESSURE PROCESS)

The analysis may begin with the momentum relation

$$p_1 A_1 + \frac{w}{g} V_1 + F = p_2 A_2 + \frac{w}{g} V_2$$

Since, however, $p_2 = p_1 = p$, F may be written in terms of the area differences

$$F = p(A_2 - A_1)$$

Using this value in the momentum relation gives

$$p A_1 + \frac{w}{g} V_1 + p(A_2 - A_1) = p A_2 + \frac{w}{g} V_2$$

from which

$$V_2 = V_1$$

The energy relation becomes

$$c_p T_1 + \frac{V^2}{2g} + Q = c_p T_2 + \frac{V^2}{2g}$$

from which

$$T_2 = T_1 + \frac{Q}{c_p}$$

thus,

$$v_2 = \frac{R\left(T_1 + \frac{Q}{c_p}\right)}{p}$$

The continuity equation gives

$$A_2 = A_1 \frac{v_2}{v_1} = A_1 \frac{\frac{R}{p}\left(T_1 + \frac{Q}{c_p}\right)}{\frac{R T_1}{p}} = A_1 \frac{\left(T_1 + \frac{Q}{c_p}\right)}{T_1} = A \frac{T_2}{T_1}$$

APPENDIX VI

HEAT ADDITION IN ONE-DIMENSIONAL FLOW (CONSTANT VOLUME PROCESS)

The energy equation for one-dimensional flow in differential form is as follows:

$$du + p\,dv - dQ + v\,dp + d\left(\frac{V^2}{2g}\right) = 0$$

Bernoulli's equation is

$$v\,dp + d\left(\frac{V^2}{2g}\right) = 0$$

These two equations are identical when, for the first equation,

$$du + p\,dv - dQ = 0$$

Since no shaft work is done by heat addition,

$$p\,dv = 0$$

so that

$$du = dQ \quad \text{or} \quad C_v\,dT = dQ$$

and

$$\Delta T = \frac{Q}{C_v}$$

The flow will obey Bernoulli's equation for incompressible fluids since the process is at constant volume. Bernoulli's equation is based on the laws of motion and not on the energy principle. Therefore, it immediately follows that

$$pv + \frac{V^2}{2g} = K \tag{1}$$

from which

$$dp = -\frac{V\,dV}{vg}$$

and since

$$dp = \frac{R\,dT}{v}$$

$$\frac{R\,dT}{v} = -\frac{V\,dV}{vg} \tag{2}$$

which integrates to

$$T + \frac{v^2}{2g} = K$$

also since

$$dT = \frac{dQ}{c_v}$$

which substituted into Equation (2) gives

$$dQ = -\frac{c_v}{Rg} V dV$$

which integrates to

$$Q + \frac{c_v}{2Rg} V^2 = K$$

Since the process occurs at constant volume

$$p_2 = \frac{R(T_2)}{v} = \frac{R(T_1 + \Delta T)}{v} = p_1 + \frac{RQ}{c_v v}$$

or

$$Q - pv \frac{c_v}{R} = K$$

REFERENCES

1. Elrod, H.G., Jr. "The Theory of Ejectors." Trans. A.S.M.E., 67, (Sept. 1945), 170-174.

2. Goff, J.A. and Coogan, C.H. "Some Two-Dimensional Aspects of the Ejector Problem." Trans. A.S.M.E., 64, (1942), A-153.

3. Work, L.T. and Haedrich, V.W. "Performance of Ejectors." Industrial and Engineering Chemistry, 31, (1939), 464-477.

4. Holton, W.C. "Effect of Molecular Weight of Entrained Fluid on the Performance of Steam-Jet Ejectors." Trans. A.S.M.E., 73, No. 7, (Oct. 1951), 905-910.

5. Holton, W.C. and Schulz, E.J. "Effect of Temperature of the Entrained Fluid on the Performance of Steam Jet Ejectors." Trans. A.S.M.E., 73, No. 7, (Oct. 1951), 911-913.

6. Tollmein, W. "Berechnung Turbulenter Susbreit ungsvorgänge." Zeiteschrift für angervandt Mathematik und Mechanik, 6, Heft 6, (Dec. 1926), 468-478.

7. Kuethe, A.M. "Investigations of the Turbulent Mixing Regions Formed by Jets." Trans. A.S.M.E., 57, (1935), A-87.

8. Stodola, Aurel. Steam and Gas Turbine. Translated by C. Lowenstein, New York: Peter Smith Book Co., Vol. 2, (1945), 927-931.

9. Ferri, Antonio. Elements of Aerodynamics of Compressible Flows. New York: The Macmillan Co., 1949.

10. Stanton, T.E. "On the Flow of Gases at High Speeds." Proceedings of the Royal Society, London, 111A, (1926), 306.

11. Frazer R.P. Flow Through Nozzles at Supersonic Speeds. Rep. WA-1513-1a, Ministry of Supply, DSR, EMR, London, 1941.

12. Summerfield, Martin, Foster, C.R. and Swan, W.C. "Flow Separation in Overexpanded Exhaust Nozzles." Journal of the American Rocket Society, 24, Sept.-Oct. 1954.

13. Speak, G.S. and Walters, D.J. Optical Considerations and Limitations of the Schlieren Method. Report No. 2859, Ministry of Supply, DSR, London, 1950.

14. Puckett, A.E. "Supersonic Nozzle Design" Trans. A.S.M.E. Jour. App. Mech., 68, (1946), A-265-270.

BIBLIOGRAPHY

1. Flugel, Gustav, "The Design of Jet Pumps." "Berechnung von Strahlapparaten." Forschungsheft 395, (March-April, 1939), -121. Translation NACA, Tech. Memo. do 982.

2. Courant, R. and Friedrichs K.O. Supersonic Flow and Shock Waves. New York: Interscience Publishers, 1948.

3. Shapiro, A.H. and Hawthorne, W.R. "The Mechanics and Thermodynamics of Steady One-Dimensional Flow." A.S.M.E. Jour. of App. Mech., (Dec. 1947), A317-A336.

4. Pai, S.I. "Two-Dimensional Jet Mixing of a Compressible Fluid." I.A.S. Jour. Aero. Science, (Aug. 1949), 403-409.

5. Pai, S.I. "On Turbulent Jet Mixing of Two Gases at Constant Temperature." A.S.M.E. Jour. App. Mech., (March 1955), 41-47.

6. Puckett, A.E. "Supersonic Nozzle Design." A.S.M.E. Jour. App. Mech. (Dec. 1946), A205-A270.

7. Forstall, W. and Gaylord, E.W. "Momentum and Mass Transfer in a Submerged Water Jet." A.S.M.E. Jour. App. Mech., (June 1955), 161-164.

8. Abramovich, G.N. "The Theory of a Free Jet of a Compressible Gas." N.A.C.A. TM 1058, Abs. Aero. Eng. Rev., (May 1944), 125, Translation, Report No. 377, Central Aero-Hydronamical Institute, Moscow, 1939.

9. Rouse, Hunter. "Diffusion of Submerged Jets." Trans. A.S.C.E., 115, (1950), 639-97.

10. Alexander, L.G. and Kivnick, A. Simultaneous Transport of Heat and Momentum in a Ducted Jet. Univ. of Illinois, Eng. Expt. Station, T.R.'s No. 8, 11, 12.

11. Bailey, N.P. "The Thermodynamics of Air at High Velocity." Jour. Aero. Sci., 11, (1944), 227-238.

12. Tests of a Jet Pump. Univ. of Wisconsin, Bull. No. 596, Eng. Series, 7, No. 4, 1913.

13. Bean, D.R. "A Note on the Theory of the Steam Ejector." The Engineer, 180, (1945), 131-32.

14. Bernhader, D. and Pai, S.I. "Air Turbulent Mixing in Two-Dimensional Supersonic Flow." Jour. App. Phys., 21, No. 6, (June, 1950), 616.

15. Bimmie, A.M. and Woods, M.W. "The Pressure Distribution in a Convergent-Divergent Stream Nozzle." Prov. I.M.E., 138, (1938), 229-266, 284-313.

16. Chapman, Dean R. "Laminar Mixing of a Compressible Fluid." NACA Tech. Note No. 1800, Feb. 1949.

17. Cleever, V. and Boelter, L.M.K. "Isothermal and Non-Isothermal Air Jet Investigation." Chem. Eng. Progress, 43, (March 1947) 123-124.

18. Forstall, W. and Shapiro, A.H. "Momentum and Mass Transfer in Coaxial Gas Jets." Jour. of App. Mech., 17, No. 4, (Dec. 1950), 399-408.

19. Goodman, P.B., Wood, G.P. and Brevoort, M.J. "Investigation with an Interferometer of the Turbulent Mixing of a Free Supersonic Jet." NACA TR963, 1950.

20. Hooker, S.G. "Oscillations in High Speed Jets of Compressible Fluid." Philosophical Mag., 17, (1941), 600-651.

21. Hughes, N.J.S. "Steam Expansion with Discontinuity in Velocity on the Boundary." Ministry of Aircraft Production, Aero. Research Comm. R. and M. No. 1928, British Info. Services, N.Y., also Aero. Eng. Review, 5, No. 1, Jan. 1946.

22. Kantrowicz, A. "The Formation and Stability of Normal Shock Waves in Channel Flow." NACA TN1225, 1948.

23. Keenan, J.H., Neumann, E.P. and Lustwerk , F. "An Investigation of Ejector Design by Analysis and Experiment." A.S.M.E. Jour. App. Mech., (Sept. 1950), 299-309.

24. Ladenburg, R., Van Voorhis, C.C. and Winchler, J. "Interferometer Studies of Supersonic Phenomena." Pt. I, Nov. Ord. Report 69-46 (1946).

Pt. II Nov. Ord. Report 93-46 (1946)
("The Gas Flow Around Various Objects in a Free Homogeneous Supersonic Air Stream")
Pt. III Nov. Ord. Report 9-47 (1947)
"Boundary Layer and Shock Wave Interaction Observed along Probes and Wires in Supersonic Air Stream"

25. "Wind Tunnel Ejector Theory." Aero. Eng. Review, 10, No. 8, Aug. 1951.

26. "Fluid Jets and Their Application." Trans. of the Institute of Chemical Engrs., London, 6, (1928), 66-84.

27. Miller, K.D., Jr. Flow in an Ejector Driven by Supersonic Jets. Project Squid TM No. Pr-3, Princeton Univ., May 15, 1948.

28. Neumann, E.P. Air Jet Ejectors. M. Sc. Thesis, M.I.T., 1941.

29. Royds, R. and Johnson, E. "The Fundamental Principles of the Steam Ejector." Proc. I.M.E., 145, (1941), 193-204, 146, (1941), 223-235.

30. Tsien, H.S. and Funstean, M. "Interaction Between Parallel Streams of Sub-Sonic and Supersonic Velocities." Jour. Aero. Sci., 16, (Sept. 1949), 515-528.