THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Mechanical Engineering

Progress Report

A SIMILITUDE STUDY OF THE DRAG AND SINKAGE OF WHEELS USING THE SINKAGE-PARAMETER SYSTEM OF SOIL VALUES

H. H. Hicks, Jr.
D. K. Kapur

E. T. Vincent
Supervisor

ORA Project 2860

under contract with:

DEPARTMENT OF THE ARMY
DETROIT ORDNANCE DISTRICT
CONTRACT NO. DA-20-018-ORD-18955
DETROIT, MICHIGAN

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

April 1961

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

b - Wheel width or characteristic dimension - L . . . . in.

c - Proportionality constant - n.d. . . . .

d - Wheel diameter - L . . . . in.

D - Soil depth - L . . . . in.

g - Acceleration due to gravity - L/T^2 . . . . ft/sec^2

k_c - Sinkage modulus (cohesive) - F/L^{n+1} . . . . lb/in.$^{n+1}$

k_ϕ - Sinkage modulus (frictional) - F/L^{n+2} . . . . lb/in.$^{n+2}$

n - Sinkage index - n.d. . . . .

R - Wheel rolling resistance - F . . . . lb

W - Wheel load - F . . . . lb

z - Wheel sinkage - L . . . . in.

α - Wheel aspect ratio - n.d. . . . .

μ - Coeff. of friction (wheel to soil) - n.d. . . . .

ρ - Soil density - M/L^3 . . . . slugs/ft^3

ABSTRACT

Analytical and experimental studies of wheel performance in soil have been hampered by the lack of a generally valid or broadly applicable system of soil values. The Ordnance Tank and Automotive Command—Land Locomotion Laboratory has adopted a system of soil values by which theoretical predictions of wheel performance have been made. These predictions check well with experiment in some cases but not in others, and owing to the approximations made in the theory, it is not possible to distinguish between the discrepancies that are chargeable to the analysis and those that may be due to an inadequacy of the soil-value system used.

In the work presented here a successful correlation was obtained by the similitude method of experiment, for the drag and sinkage of wheels using the sinkage parameters of the Land Locomotive Laboratory's soil-value system. This appears to confirm the validity of these parameters for sand, and defines the method for extending the work to cohesive soils. The tests were made in the laboratory by the application of new concepts of experimentation that offer a means of relating laboratory results to field conditions, and include an investigation of wheel sinkage and drag as related to wheel shape, diameter, load, and soil depth.

INTRODUCTION

Theoretical and experimental investigations of the drag and sinkage of wheels in soil were initiated long ago.[1] Early experimental investigations consisted of isolated tests conducted without reference to quantitatively measured soil properties and the results obtained are therefore unrelated to each other or to any particular system of soil-wheel mechanics.

Theoretical investigations have been hampered fundamentally by the lack of a generally valid system of soil parameters and practically by the complexity of the analytical methods. These efforts have nevertheless produced some working formulas for the prediction of wheel performance that are fairly accurate over limited ranges of conditions, but precise predictions of broad applicability have yet to be achieved.

About a decade ago, Nuttall[2] pioneered the use of scale-model wheels in laboratory testing and obtained good correlations in homogeneous soils by using the Coulomb soil properties as part of the system variables. In a subsequent study[3] he applied the dimensional analysis to Bernstein's[4] theoretical work and by experiment was able further to demonstrate the basic applicability of Bernstein's method. In this study Nuttall redefined two of Bernstein's sinkage parameters in terms of the Coulomb soil properties, leaving the third unchanged as well as the basic form of Bernstein's prediction equations.

Thus the Bernstein equation for drag as modified by Goríatkín[3] is

$$R/W = \frac{\left(\frac{3}{3-\mu}\right)^{\frac{2\mu+2}{2\mu+1}}}{1+\mu}\left[\frac{W}{b(a_1 + ba_2)d^{\mu+1}}\right]^{\frac{1}{2\mu+1}} \qquad (1)$$

where μ, a_1, a_2 are Bernstein's sinkage parameters that, broadly speaking, are functions of the system boundaries as well as the mechanical properties of the soil. Nuttall redefined

$$\left.\begin{aligned} a_1 &\equiv \frac{\lambda}{d^\mu}\, c \\ a_2 &\equiv \frac{\sigma}{d^\mu}\, g\rho \end{aligned}\right\} \quad \lambda \text{ and } \sigma \text{ are nondimensional proportionality factors} \qquad (2)$$

and leaving μ unchanged, obtained for drag

$$R/W = \frac{(1/\lambda)^{\frac{1}{2\mu+1}} \left(\frac{3}{3-\mu}\right)^{\frac{2\mu+2}{2\mu+1}}}{1+\mu} \left(\frac{W}{bd\lambda c + b^2 d\sigma\gamma}\right)^{\frac{1}{2\mu+1}} \quad . \qquad (3)$$

With minor additional modifications Nuttall was able to use Eq. (3) as the basis for successful correlations of experimental data taken in both friction and cohesive soils. In spite of this success, however, certain features of the approach seem to preclude its development into a general soil-wheel mechanics. It will be noted that, in Eq. (2), the Bernstein constants a_1 and a_2 are redefined in terms of the Coulomb soil properties c and ρ, but the Coulomb properties are essentially point properties and apply to soil in the mass, only if the mass is homogeneous. Therefore whatever of soil inhomogeneities are represented by the Bernstein parameters a_1 and a_2 are lost when they are redefined in terms of Coulomb properties, and any accounting for soil inhomogeneity must be borne by the remaining Bernstein parameter μ. Nuttall has offered nothing that would allay skepticism about this latter point and it appears safe to assume that Eq. (3) is applicable only to well-compacted homogeneous soils.

It seems evident, however, that natural soils in the field are neither homogeneous nor always compacted, and laboratory work must somehow take this fact into account if it is to provide results that are of more than academic interest.

The work presented here has been done with these problems in mind; the Bernstein and Coulomb soil parameters have been abandoned in favor of those used by the Land Locomotion Laboratory, and attention will now be given to the theory connected with these circumstances.

THEORY

Rational analyses[1,4] indicate there is an intimate connection between wheel sinkage and drag. Thus it is reasonable to assume that they are both functions of the same independent variables. Among these variables are those termed the sinkage parameters. The sinkage parameters considered here are those advocated by Bekker[5] and are obtained from the stress-strain relationship set up by forcing a rectangular flat plate into the soil with an instrument called a bevameter. The parameters are as follows:

k_ϕ - Sinkage modulus (frictional) lb/in.^{n+2}

k_c - Sinkage modulus (cohesive) lb/in.^{n+1}

n - Sinkage index (nondim.)

For low-speed tests on solid, rectangular cross-sectional wheels, the remaining independent variables are taken to be:

d - Wheel diameter (in.)

α - Wheel aspect ratio (nondim.)

D - Soil depth (in.)

μ - Coeff. of friction (soil-wheel)

W - Wheel load (lb)

The dependent variables are:

R - Wheel rolling resistance (lb)

z - Wheel sinkage (in.)

In evaluating the sinkage parameters it is assumed that the stress-strain relationship is expressible by an equation of the form

$$p = (k_\phi + k_c/b)\, z^n \tag{4}$$

and the sinkage parameters are the constants of this equation.

A dimensional analysis of the above listed variables yields the following equations:

$$R/W = (d/D)^{a_1} (W/d^{n+2}k_\phi)^{a_2} (k_c/dk_\phi)^{a_3} (n)^{a_4} (\alpha)^{a_5} (\mu)^{a_6} \tag{5}$$

$$z/d = (d/D)^{b_1} (W/d^{n+2}k_\phi)^{b_2} (k_c/dk_\phi)^{b_3} (n)^{b_4} (\alpha)^{b_5} (\mu)^{b_6} \tag{6}$$

Since it was decided to study the effect of the independent variables W,D,d, and α on the dependent variables z and R, the model rules require the following conditions to be met:

$$k_c = c_1 \, d \, k_\phi \tag{7}$$

$$W = c_2 \, k_\phi \, d^{n+2} \tag{8}$$

$$D = c_3 \, d \tag{9}$$

These equations require that, to obtain complete similitude with geometrically scaled models, the tests must be conducted in soils whose qualities change as the scale of the model changes. More specifically, laboratory test soils must be available in which k_c may be varied independently of k_ϕ, n, and μ.

We do not yet have the ability to compound laboratory test soils that will satisfy these particular requirements, but progress is being made on the problem. For the present, valuable work may be done in sand, for which k_c is essentially zero. For this soil the model rules require only that

$$W = c_2 \, k_\phi \, d^{n+2} \tag{8}$$

$$D = c_3 \, d \; . \tag{9}$$

These conditions can be met in a single soil.

The particular forms of Eqs. (5) and (6) permit a considerable simplification in testing, for by taking the soil parameters only in semi-infinite homogeneous beds with a single bevameter plate, the soil parameters are constant for all conditions of test. Changes in the boundary conditions due to finite test-bed depth are accounted for by the depth coefficient (d/D). Those effects due to wheel shape are accounted for by the aspect ratio α. These are the conditions that have been applied in our laboratory tests which are believed to be of the utmost value in relating these tests to practical field conditions.

The d/D ratio represents a type of inhomogeneity easily produced in the laboratory and simulates what is probably the most frequently occurring inhomogeneity of natural soils, namely, a more or less homogeneous soil bed overlying a hard pan at some depth.

EXPERIMENTAL WORK

In accordance with the preceding theory, low-speed drag and sinkage tests were conducted on two geometrically similar, solid, aluminum wheels of rectangular cross section, the diameters of the wheels being 6.625 in. and 12.50 in. The aspect ratios of the wheels tested were 0.27, 0.52, and 0.84. The soil used was a sand for which $k_\phi = 3.40$ and $n = 1.05$ are most probable values. The tests with variable aspect ratio were conducted in deep beds of soil corresponding to practically semi-infinite conditions, while the tests conducted in various depths of soil were made at the single aspect ratio $\alpha = 0.27$. All tests were conducted at the relatively low speed of 0.25 ft/sec. This was done because experimental difficulties are minimized at low speed, and is justified by the results of other tests[6] which showed the effect of speed to be negligible below a speed of 5.0 ft/sec with a 12.5-in.-diameter wheel.

It should be noted that unexpected experimental difficulties are encountered in work of this type. The principal difficulty is the preparation of the soil to a uniform and repeatable consistency. This problem was never completely solved and consequently the data presented here represent the mean of many test runs, in some cases numbering twelve. Another problem is associated with the large span of forces to be measured, covering a range of approximately a hundred to one. This demands considerable precision and ruggedness in the test apparatus, qualities that are somewhat mutually exclusive.

The test results are given in Figs. 1-4. In Figs. 1 and 2, the sinkage and drag coefficients are given vs. the load coefficient with the depth ratio as a parameter. In Figs. 3 and 4, the sinkage and drag coefficients are given vs. the load coefficient with the aspect ratio as a parameter.

A general view of the tow tank is shown in Fig. 5, and a detailed view of the wheel carriage is shown in Fig. 6.

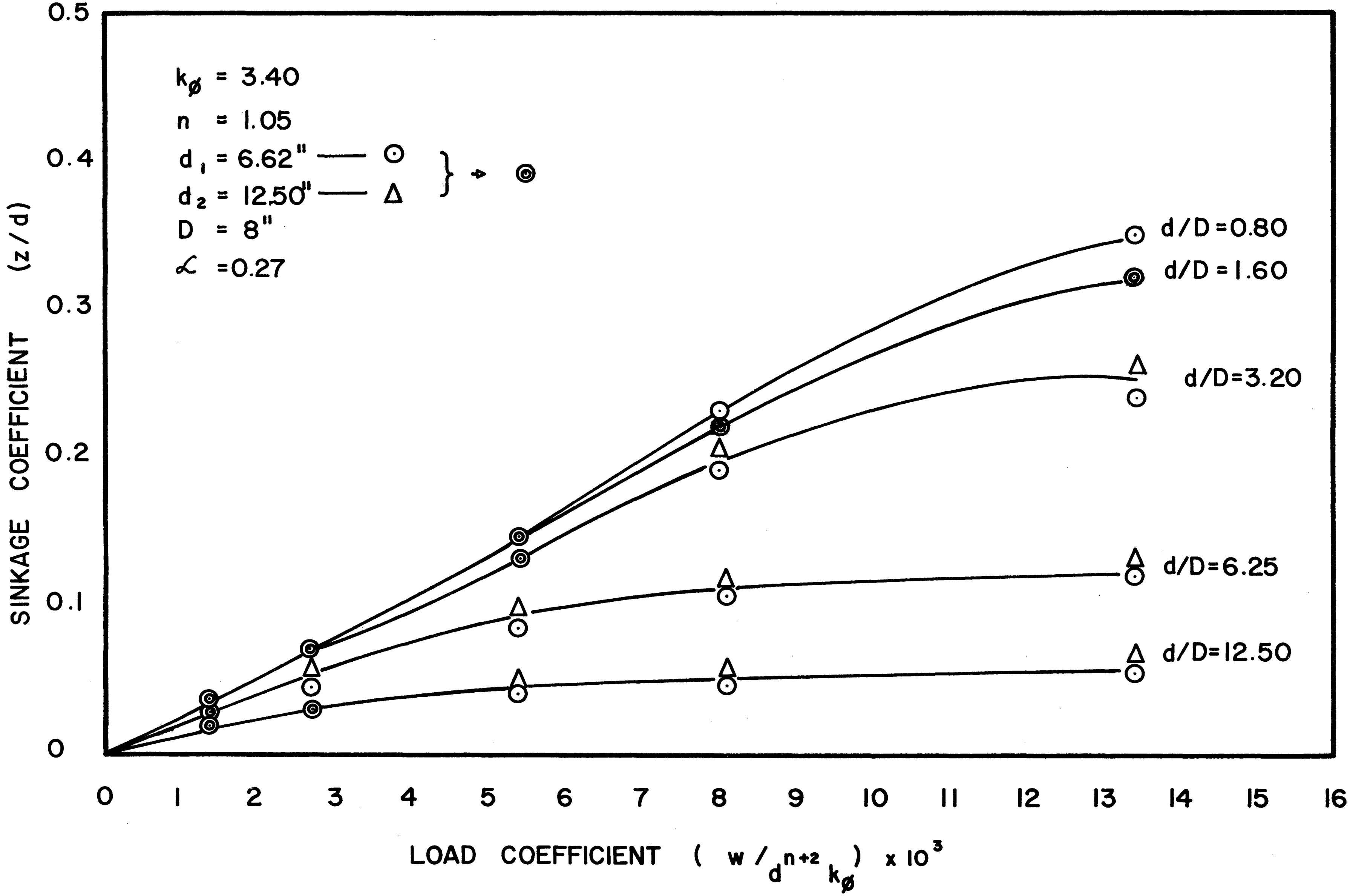

Fig. 1. Plot of non-dimensional sinkage coefficient (z/d) vs. non-dimensional load coefficient ($w/d^{n+2}k_\phi$) for rectangular cross-sectional wheels.

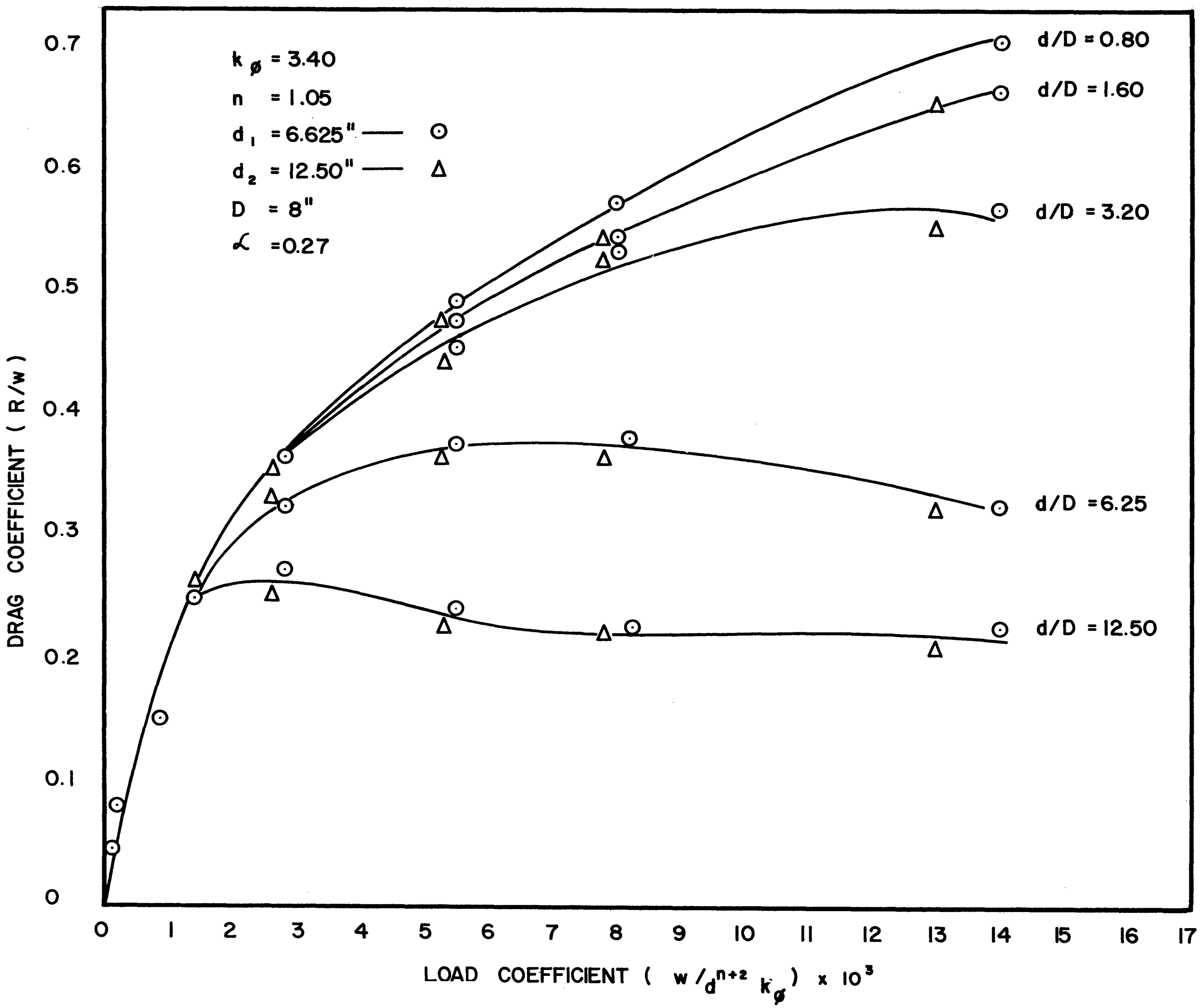

Fig. 2. Plot of non-dimensional drag coefficient (R/w) vs. non-dimensional load coefficient ($w/d^{n+2}k_\phi$) for rectangular cross-sectional wheels.

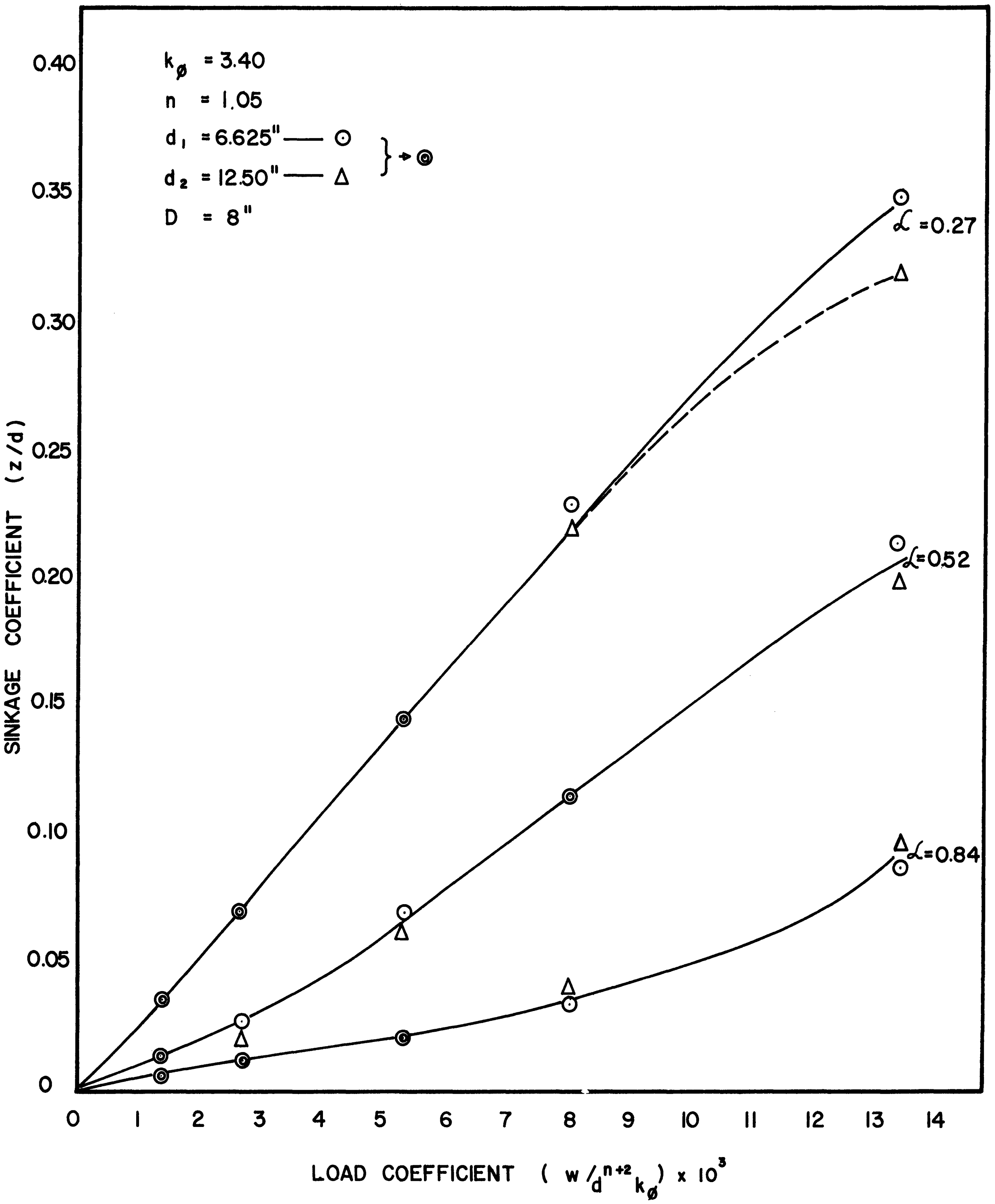

Fig. 3. Plot of non-dimensional sinkage coefficient (z/d) vs. non-dimensional load coefficient ($w/d^{n+2}k_\phi$) at various aspect ratios for rectangular cross-sectional wheels.

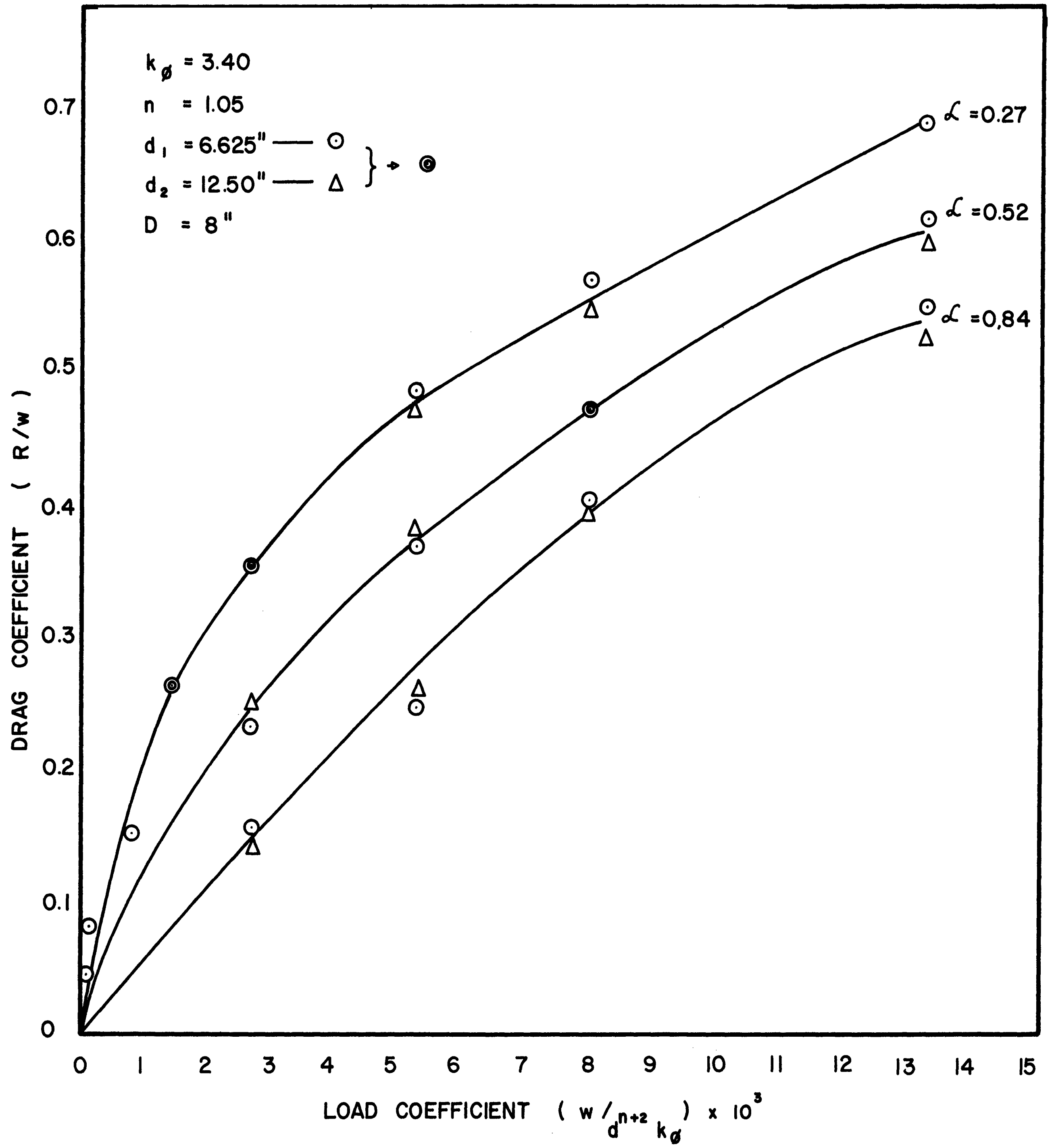

Fig. 4. Plot of non-dimensional drag coefficient (R/w) vs. non-dimensional load coefficient ($w/d^{n+2}k_\phi$) at various aspect ratios for rectangular cross-sectional wheels.

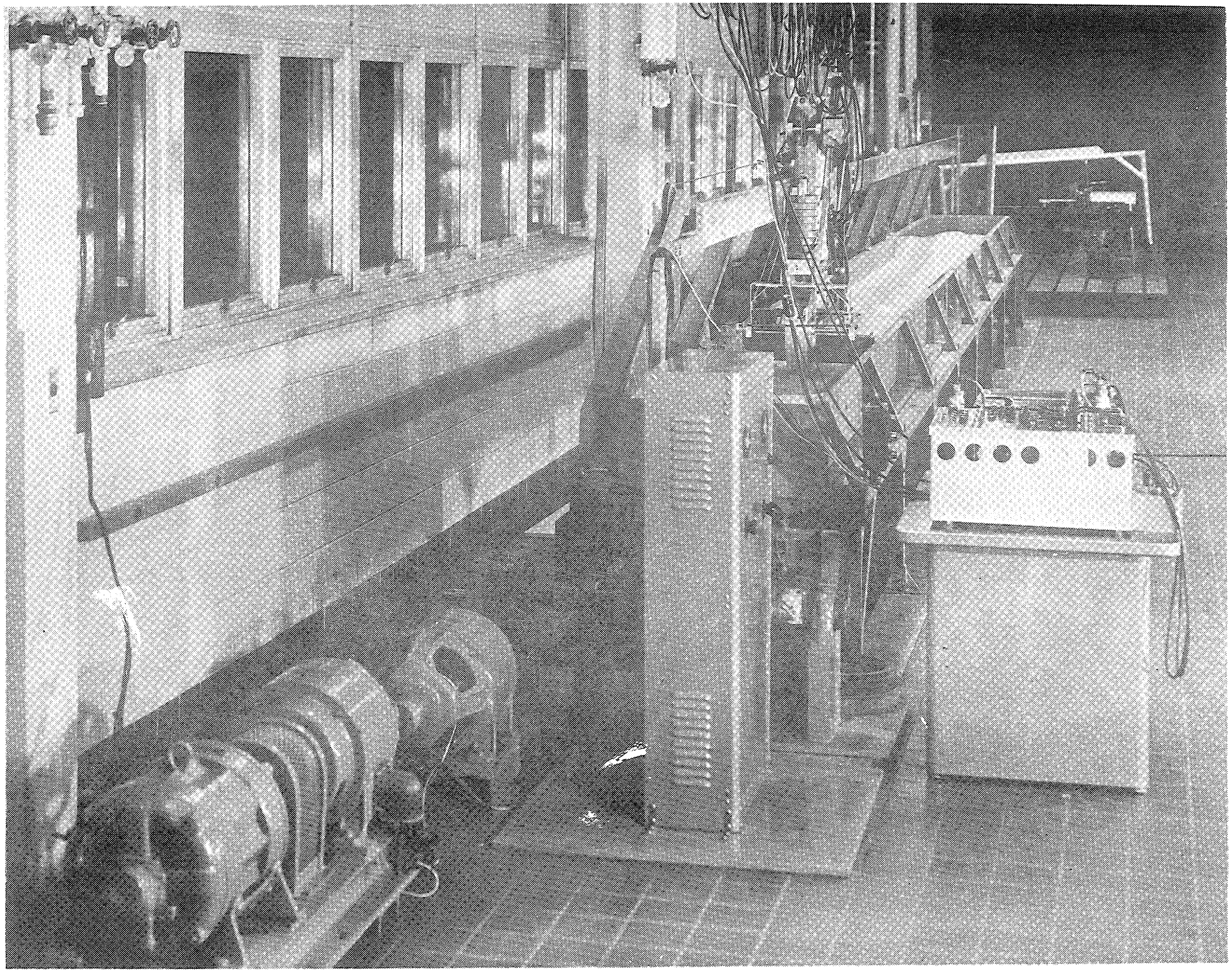

Fig. 5. The tow tank.

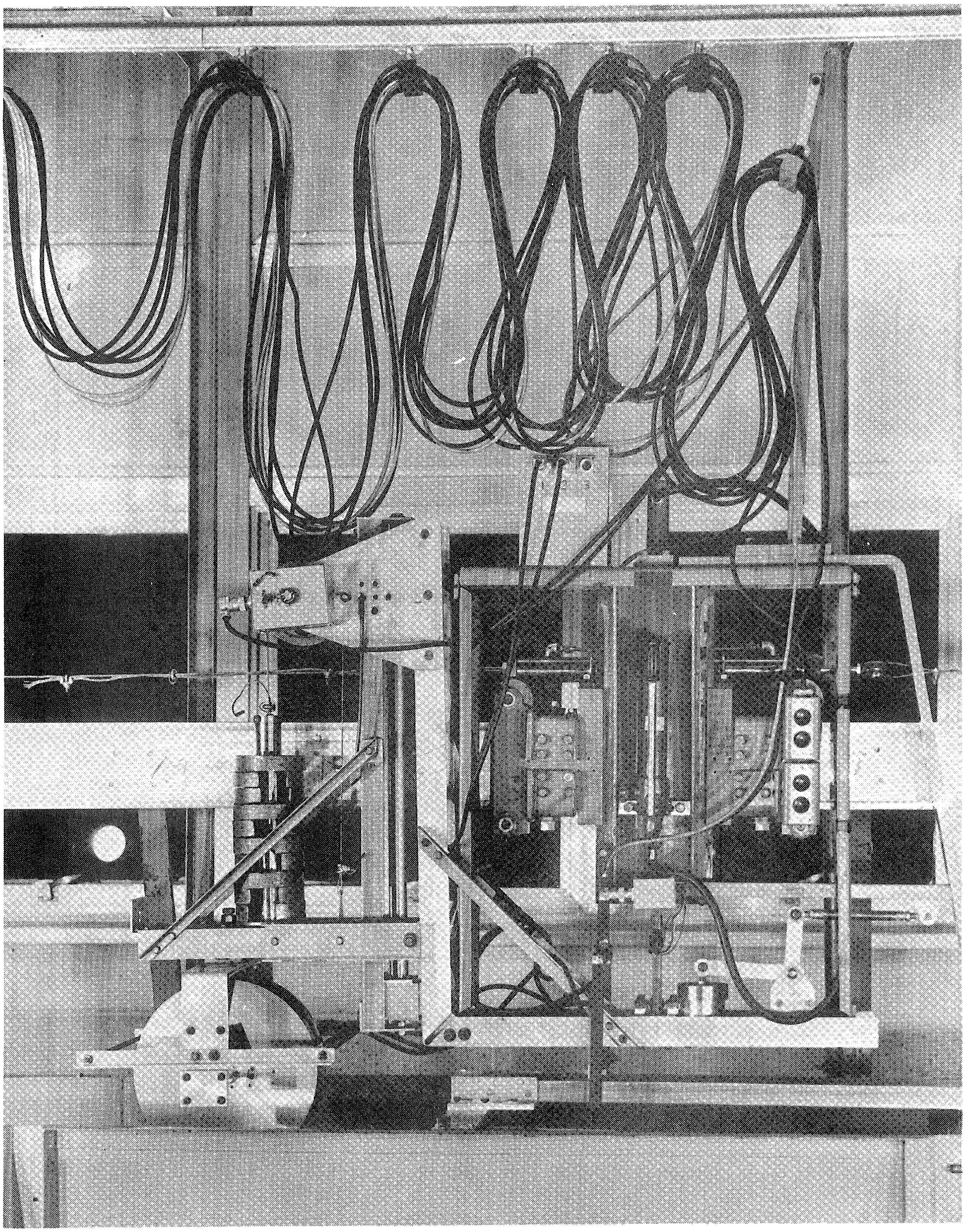

Fig. 6. The wheel carriage.

DISCUSSION

The plotted data make it apparent that a high degree of similitude has been attained in the tests, and that the independent variables selected are appropriate to the system. Accordingly, the results should be applicable to wheels of any scale as long as the assumed parameters are maintained. It is consequently important to consider carefully what this condition requires.

The sinkage parameters used are those obtained with a single (1 by 3 in.) bevameter plate, penetrating a semi-infinite bed of homogeneous soil. This means that the parameters obtained depend only on the inherent mechanical properties of the soil, and on the shape and size of the bevameter plate. If a different soil is placed under the bevameter plate, changes in the sinkage parameters will result which can be due only to changes in the mechanical properties of the soil.

Consistent with this framework of ideas, there exists a particular wheel having some diameter and aspect ratio, whose sinkage characteristics are similar to those of the bevameter plate used. In other words, an analogy exists, and because of this any variations in the basic mechanical properties of the soil will not only produce changes in the sinkage parameters as measured by the bevameter, but also cause a change in the sinkage of the wheel. Consequently the sinkage of the analogous wheel is uniquely related to the measured sinkage parameters.

If a fixed quality soil and variable geometry wheel are considered instead of the converse above, then variations in the size and shape of the wheel produce sinkages that are related to the sinkage of the analogous wheel and hence to the sinkage parameters through these geometrical variations alone. The preceding reasoning is now repeated, but the soil is considered to be of finite depth and the first effect to be observed is that a new set of sinkage parameters is obtained that is related to the original set by some factor that describes the relative "infiniteness" of the boundaries. The ratio of the characteristic dimension "b" of the bevameter plate to the soil depth "D" or specifically the b/D ratio is a factor that satisfies this requirement. It is further evident that the analogous wheel will describe a new sinkage that is uniquely related to the new sinkage parameters, but also to the original sinkage parameters through the b/D ratio. Similarly, new relations describing the effects on sinkage of changes in wheel geometry and soil quality will be obtained that are related to the original relations through the b/D ratio. Finally, since there exists an analogous wheel of characteristic dimension "d," all effects of changes in soil depth may be related to the ratio d/D instead of the ratio b/D. It is thus possible to construct a self-consistent system based on analogy, which preserves the idea of sinkage parameters that are determined by soil properties only and which accounts for variations in wheel geometry and soil depth. In this system, it is obviously necessary to take account of the d/D ratio in the correlation of test results, as this is the source of a scale effect that enters the system. Failure to do this can lead to difficulties in obtaining correlations. This was observed by Czako and Bekker[7] in an attempt to obtain sinkage parameters from the analysis of tests on wheels. Wheels of a range of diameters were tested in a bed of soil of fixed depth, and only one of these wheels having a particular diameter gave sinkage parameters comparable to those obtained with the bevameter. Apparently the analogy of wheel, soil, and bed depth to bevameter, soil, and bed depth was an optimum for this particular wheel. This observation is a manifestation of the scale effect and this difficulty has been entirely overcome in the present tests by observing the depth ratio (d/D).

An infinite variety of conditions exists with natural soils in the field, but it is believed that the most frequently occurring condition is that of an approximately homogeneous soil overlying a hard pan at some depth. To the degree that this is true, the results given here should be applicable to natural soils in the field.

It was noted earlier that prior theory and experiment had produced working formulas for wheel drag that were fairly accurate over limited ranges of conditions. One of these equations due to Bernstein[1] can be expressed in the form

$$R/W = 0.86(W/\alpha\ k\ d^3)^{0.33}$$

and another due to Nuttall[3] and applicable to sand can be expressed as

$$R/W = 0.50(W/\alpha^{0.3}\ g\ \rho\ d^3)^{0.4}\ .$$

Both of these equations, especially Nuttall's, predict drag coefficients that are in reasonable agreement with selected areas of the data presented here, but neither of them follows the trend of aspect ratio very well. A better understanding of the source of this divergence can be obtained by writing these equations for a wheel of variable width which carries a constant load "c" per unit of width. For these conditions the formulas reduce to

$$R/W = 0.86(c/kd^2)^{0.33}$$

and

$$R/W = 0.50(c/g\ \rho\ d^2)^{0.4}\ \alpha^{0.2}\ .$$

In the first case it is seen that the aspect ratio drops out entirely, implying that shape has no effect on the drag coefficient and that end effects are negligible. In the second case it is apparent that

$$\lim_{\alpha \to \infty} R/W = \infty$$

and

$$\lim_{\alpha \to 0} R/W = 0 \ .$$

There is no physical basis for these conclusions, nor do the data support them. It is apparent, moreover, that additional analytical work is needed in this area.

One of the more interesting features of the drag data is the way the drag coefficient decreases with large loads in shallow soils. This possibly may have practical application, but in any event this result demonstrates the value of conducting experiments that simulate natural soil conditions.

The effect due to variation of aspect ratio also has some interesting features. The increase of the drag and sinkage coefficients of the low-aspect-ratio wheels is apparent. Some considerations in logic indicate that the drag coefficient may become infinite for zero-aspect-ratio wheels, but approaches a limiting finite value for infinite-aspect-ratio wheels.

CONCLUSIONS

1. The results of this investigation appear to confirm the validity of the Land Locomotion Laboratory's sinkage parameters as appropriate variables for predicting the drag and sinkage of wheels. The experimental work has been done in sand, but appears extendable to cohesive soils.

2. This investigation also shows that the depth of the soil is a fundamentally important variable to be considered in laboratory correlations of wheel performance.

3. Theoretical considerations introduced here indicate a basic flaw in the drag formulas given by Bernstein and Nuttall. This should stimulate a re-examination of the entire order of procedure leading to these formulas.

RECOMMENDATIONS

The work presented here appears to open up a large area in which new and valuable work may be done. The next logical goal is the extension of the work to the case of cohesive soils, but as noted before, this depends on the development of laboratory test soils with specifically controllable properties. This is therefore the immediate task, which is now progressing. Preliminary results are very encouraging, but it will require two to three months to begin to obtain significant results. Full development of the laboratory test soils will take much longer, as there is much experimental and theoretical work to be done.

The introduction of the soil depth brings a new variable into the soil-value system. It is believed, however, that this may ultimately lead to simplifications by making it possible to predict traction in terms of the sinkage parameters. This idea is based on the reasoning that the penetration test and the shear test produce the same basic action on the soil (rotation, shear, compaction, etc.) but in relative different amounts that are a consequence of the boundaries peculiar to each test. Inasmuch as the soil depth (or pseudo-soil depth for soils in the field) is an important boundary condition for the penetration test, this may be the link that relates the two sets of parameters.

The recommended program for the extension of the present work is then as follows.

1. Development of cohesive laboratory test soils.

2. Drag and sinkage of wheels in cohesive soils.

3. A broad and intensive study of the soil-value system, including natural soils, directed toward ultimate simplifications.

4. Powered wheels in cohesive soils, this program necessarily running concurrently with step 3 above.

REFERENCES

1. Bekker, M. G., *Theory of Land Locomotion*, Univ. of Mich. Press, Ann Arbor, 1956.

2. Nuttall, C. J., *Scale Model Testing in Non-Plastic Soil*, Stevens Inst. of Tech., Hoboken, N. J., Expt. Towing Tank Rpt. No. 394, 1949.

3. Nuttall, C. J., *The Rolling Resistance of Wheels in Soil*, Stevens Inst. of Tech., Hoboken, N. J., Expt. Towing Tank Rpt. No. 418, July, 1951.

4. Bernstein, R., "Problem einer experimentellen Motorflugmechanik," *Der Motorwagen*, *16*, 1913.

5. Bekker, M. G., *Off-the-Road Locomotion*, Univ. of Mich. Press, Ann Arbor, 1960.

6. Vincent, E. T., Hicks, H. H., Jr., and Kapur, D. K., *Performance Coefficients for Free Running Wheels in Sand*, UMRI Rpt. 2544-31-F, Ann Arbor, July, 1960.

7. Czako, T., and Bekker, M. G., *Determination of Vehicle Sinkage Parameters by Means of Rigid Wheels*, Part I, Dept. of The Army, O.T.A.C., Land Locomotion Laboratory, Rpt. No. 33, Center Line, Mich., May, 1958.