

MODELING, SIMULATION AND  
VERIFICATION OF IMPACT DYNAMICS

Vol. 2 State-of-the-Art:  
Computer Simulation of  
Vehicle Impact

By:

I. K. McIvor  
A. S. Wineman  
W. H. Yang  
B. Bowman

Date:

October 31, 1973

FINAL REPORT

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Modeling, Simulation and Verification of Impact Dynamics - Vol. 2; State-of-the art, Computer Simulation of Vehicle Impact		5. Report Date 31 October 1973	
		6. Performing Organization Code	
7. Author(s) I.K. McIvor, A.S. Wineman, W.H. Yang, B. Bowman		8. Performing Organization Report No. UM-HSRI-BI-73-4-2	
9. Performing Organization Name and Address Highway Safety Research Institute and Department of Applied Mechanics and Engineering Science The University of Michigan Ann Arbor, Michigan 48105		10. Work Unit No.	
		11. Contract or Grant No. DOT-HS-031-2-481	
		13. Type of Report and Period Covered Final Report June 28, 1972- October 31, 1973	
12. Sponsoring Agency Name and Address National Highway Traffic Safety Administration U.S. Department of Transportation Nassif Building, 7th and E Streets, S.W. Washington, D.C. 20590		14. Sponsoring Agency Code	
		15. Supplementary Notes	
16. Abstract <p>In recent years a number of investigators have developed computer simulation programs to model the structural response of automobiles during vehicle impact. A variety of modeling concepts and degrees of sophistication have been employed. In this report a critical assessment of these programs is given relative to the simulation needs of NHTSA.</p> <p>To provide a basis for this assessment a simulation spectrum spanning the needs of NHTSA is defined. Relative to this spectrum it is concluded that lumped mass-generalized spring models using experimental crush data to define component behavior adequately provide Level 3 capability (relative deformation of components and gross rigid body acceleration of passenger compartment in collinear impact). Advanced simulation capability (Levels 4 and 5) is not currently available. More sophisticated frame models have not been qualified as overall vehicle simulations, but have potential as components or modules in future advanced simulations. Problem areas which must be solved to advance the state-of-the-art are delineated.</p>			
17. Key Words		18. Distribution Statement	
19. Security Classif.(of this report)	20. Security Classif.(of this page)	21. No. of Pages	22. Price

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the National Highway Traffic Safety Administration.

## TABLE OF CONTENTS

	Page
Chapter 1: Introduction	
1.1 Scope of the Study	1
1.2 Simulation Needs of NHTSA	3
1.3 Definition of Simulation Spectrum	4
1.4 Simulation Levels Required for NHTSA Uses of Computer Simulation	8
Chapter 2: Major Modeling Concepts	
2.1 Introduction	14
2.2 Simplified Spring-Mass Models	14
2.3 Hybrid Simulations	17
2.4 Generalized Resistance Models (BCL Simulation Program)	20
2.5 Frame Models	23
2.5.1 Calspan (Shieh) Plastic Hinge Program	24
2.5.2 Simulation Program "KRASH"	26
2.5.3 Simulation Program "CRASH"	27
2.5.4 Thompson Program	30
2.6 General Purpose Structures Programs	31
Chapter 3: Assessment of the Current State-of-the-Art	
3.1 Simulation Levels of Current Models	35
3.2 General Discussion	36
Chapter 4: Numerical Methods	
4.1 Current Methods of Numerical Integration	39
4.2 General Discussion of Numerical Integration	47
4.3 Conclusions	51
Chapter 5: Developments in Computer Hardware	
5.1 Man-Machine Interface Devices	52
5.2 Hardware Developments	53
5.3 Current Research in Computer Technology	64

TABLE OF CONTENTS  
(Continued)

	Page
Chapter 6: Conclusions	
6.1 Current State-of-the-Art of Computer Simulation of Vehicle Impact	68
6.2 Conclusions with Respect to Specific Programs	68
6.3 Needed Areas of Development	71
References	73
Appendix: General Bibliography	76
A.1 General Studies in Crashworthiness	76
A.2 Computer Simulation of Vehicle Structures	78
A.3 General Studies in Structural Analysis	81
A.4 Impact Testing	83

## LIST OF TABLES

	Page
Table 1: Correlation of Function With Computer Simulation Use	4
Table 2: General Purpose Structures Programs	34
Table 3: Input and Output Methods for Basic Terminal Types	54

## LIST OF FIGURES

Figure 1: Level of Simulation Required For Various Simulation Uses	12
Figure 2: Benefit of Simulation Levels	12
Figure 3: Error Vs. Time Step	41

## CHAPTER 1

### INTRODUCTION

#### 1.1 SCOPE OF THE STUDY

The major mission of the National Highway Traffic Safety Administration (NHTSA) is to set reasonable and cost effective standards with respect to vehicle safety. Structural crashworthiness obviously plays a major role in this mission. The increasing concern with crashworthiness of automobiles has imposed the need for much greater understanding of vehicle structures in the crash environment. For this purpose the ability to model vehicle impact using computer simulation is attractive. The problem, however, is exceptionally complex and, in any general sense, beyond the scope of current technology. One of the major goals of this study was to determine the state-of-the-art of computer simulation to assess its relevance to the mission of NHTSA. This volume (Volume 2 of the four volume Final Report) reports the results of the state-of-the-art study.

There are two major factors which served to focus the study. First is the importance of assessing the state-of-the-art relative to the needs of NHTSA. For this purpose, a familiarization study was conducted to establish what functions in the crashworthiness effort requires the use of computer simulation. The results of this study are given in the next section. It was clear that the required level of sophistication varies widely for the various simulation uses identified.

In section 1.3 we define various levels of simulation that serve to define a spectrum covering the range of required sophistication. The correlation of simulation uses with this spectrum is discussed in section 1.4.

The second major factor restricting the scope of the study is that crashworthiness studies require the capability of modeling the large, dynamic plastic response of structures. Several modeling concepts applied to vehicle structures are not relevant. For example, linear vibration studies and static stress analysis are not particularly germane to the study. Thus we may restrict attention here to attempts to simulate the behavior of vehicle type structures in a crash environment.

In recent years a number of investigators have developed computer simulation programs for this purpose. A variety of modeling concepts and degrees of sophistication have been employed. The introduction of a simulation spectrum provides a convenient framework for assessing their capabilities relative to the needs of NHTSA. The various concepts are discussed in detail in Chapter 2. Based on this evaluation the current state-of-the-art is summarized in Chapter 3.

In addition to current simulation programs, there are two other areas which are relevant to assessing the potential of computer simulation to meet NHTSA needs. These areas are methods of numerical analysis and computer hardware capability, which are discussed in Chapters 4 and 5 respectively. Finally the major conclusions of the study are given in Chapter 6.



## 1.2 SIMULATION NEEDS OF NHTSA

In this study the state-of-the-art of computer simulation of vehicle impact is to be assessed relative to the needs of NHTSA. In carrying out their basic mission, NHTSA performs a number of functions necessary for the planning and development of appropriate standards in the area of vehicle crash-worthiness. To identify these functions, a familiarization study was conducted through interviews with various NHTSA personnel. The major functions which were identified are:

1. Predict level of protection in various crash environments.
2. Determine vehicle compatibility in traffic mix.
3. Development and evaluation of new structural concepts.
4. Determine feasibility of design.
5. Establish cost-effectiveness.
6. Development of compliance procedures.

Computer simulation of vehicle impact provides a necessary research tool in support of these functions. Specific uses of computer simulation are:

1. Predict occupant compartment behavior under various crash conditions.
2. Identification of basic phenomenon in impact.
3. Parameter studies to identify behavior trends.
4. Sensitivity of control variables to parameters.
5. Judge component compatibility.
6. Design experiments and interpret experimental data.
7. Compliance verification.

The correlation of these specific simulation uses with the various functions is shown in the following table.

Function	Computer Simulation Use						
	1	2	3	4	5	6	7
1	X					X	
2	X	X	X	X		X	
3		X	X	X	X	X	
4			X	X	X		
5			X	X	X		
6	X					X	X

TABLE 1: CORRELATION OF FUNCTION WITH COMPUTER SIMULATION USE

### 1.3 DEFINITION OF SIMULATION SPECTRUM

Vehicle impact and crashworthiness studies primarily require the capability of modeling nonlinear dynamic response. Linear vibration studies and static stress analysis are not relevant. Thus we restrict attention here to vehicle simulation which includes both material and geometric nonlinearities. We define in general terms five levels of simulation, each higher level representing increasing sophistication. The divisions are somewhat arbitrary, but viewed as a group form the spectrum of potential simulations. Such a spectrum is clearly important. Even without detailed cost studies, it is evident that an attempt to develop a single simulation program for all NHTSA functions is not only economically unwise but also could inhibit focusing on specific issues. By relating NHTSA functions to a simulation spectrum we will provide a measure for evaluating present modeling efforts and required future developments.

#### Level 1 Simulation:

Level 1 simulations are models with up to five or six degrees of freedom, the variables representing displacements and possibly rotations of lumped masses. Typically the model involves 2-3 lumped masses and a few (less than ten) generalized resistances. Detailed geometry and material behavior is not modeled. Geometry and the generalized resistances are defined by a small set of parameters. There is no attempt to relate the resistances to specific vehicle components, but rather they represent overall vehicle characteristics.

The limited variables restrict results to overall gross displacements and average rigid body accelerations. The modeling is restricted to a specific loading situation. Level 1 simulation is expected to give qualitatively meaningful results.

#### Level 2 Simulation:

Level 2 simulations are models with up to twenty degrees of freedom, the variables again representing displacements and rotations of lumped masses. The number of masses and generalized resistances may be greater than Level 1 simulation, but geometry and resistances are still defined by relatively few parameters. At this level, however, the generalized resistances represent specific vehicle components.

The greater number of variables permit obtaining relative displacements between components. Generalized resistances are now related directly to force deformation

characteristics of components, but limited parameters permit modeling only the gross features. The modeling is restricted to a specific loading situation. Level 2 simulation is again expected to primarily give qualitative results but for a wider range of variables including the effect of specific components.

#### Level 3 Simulation:

Level 3 simulation will have essentially the same order of degrees of freedom as Level 2 simulation, i.e. the inertia modeling is the same level of sophistication. The essential difference is the increase in sophistication in modeling component behavior. The force deformation behavior of the generalized resistances are obtained either from experimental tests or detailed static modeling of specific components.

At this level the component tests or modeling will be for specific load conditions which restricts the simulation to similar loading situations. With this level of simulation it is expected to obtain engineering accuracy for relative displacements. Thus good estimates of the deformed structure should be obtained within the limitations of the number of variables employed. The rigid body accelerations of the lumped masses should quantitatively correlate with averaged physical accelerations, but magnitude and phasing of acceleration peaks cannot be expected.

#### Level 4 Simulation:

Level 4 simulation will require on the order of three-four hundred degrees of freedom. This level should permit the dynamic modeling of major components including inertia and

strain rate effects under reasonably general loading conditions. Other vehicle components will be modeled with less sophistication.

The number of variables employed should permit sufficient detail to obtain displacement and acceleration time histories at a number of significant points in the vehicle. Thus it should be possible to obtain the acceleration environment of the occupant compartment in some detail. The model should be sufficiently general to handle a variety of loading situations. It is expected that displacement histories should be accurate within a few percent. The time average of acceleration histories at the computed points should also be accurate, but some discrepancies in peak values is expected.

#### Level 5 Simulation:

Level 5 simulation is a modeling of the vehicle structure in sufficient detail to give pointwise results for the displacement and acceleration histories throughout the vehicle. Probably on the order of one-two thousand degrees of freedom will be required. Modeling will be based on material stress-strain behavior and detailed geometry of components. The modeling should include joint eccentricities, joint efficiency and local deformation effects.

This level of simulation will give the displacement and acceleration environment of the occupant compartment in complete detail with accuracy of all variables within the confidence level of the input data.

1.4 SIMULATION LEVELS REQUIRED FOR NHTSA USES OF COMPUTER  
SIMULATION

In this section we briefly discuss our perceptions of the level of simulation required by the various uses of simulation outlined in Section 1.2.

1. Predict occupant compartment behavior under various crash conditions.

To judge occupant behavior (or to exercise occupant simulation models) requires determining the dynamic environment of the passenger compartment. The lowest level of simulation providing quantitative prediction capability is Level 3. Level 3 simulation will provide an average acceleration and displacement associated with the occupant compartment. This will suffice to exercise simple occupant models and to make gross predictions on occupant behavior. As knowledge in biomechanics increases, however, there will be a need to obtain more detailed occupant behavior with a consequent need for increased sophistication in predicting the behavior of the compartment. Thus Level 4 and 5 simulations will play an increasingly important role.

2. Identification of basic phenomenon in impact.

A fairly wide range of simulation is useful in identifying characteristic features in a variety of crash situations. In general, however, one would want to examine specific component behavior. Thus Level 2 is the minimum simulation level. On the

other hand characteristic behavior is likely to be obscured by the complexities of Level 5 simulation. In general Levels 2-4 should suffice for this need.

3. Parameter studies to identify behavior trends.

Detailed parameter studies become prohibitive if the model is too complex. Even Level 3 which requires detailed component modeling is of marginal use. Fortunately in evaluating trends it is unnecessary to have a high level of confidence in the quantitative value of the output, but only in the magnitude of change with changing parameters. Thus in evaluating the general effect on crashworthiness of basic parameters, Level 1 and 2 simulations will suffice.

4. Sensitivity of control variables to parameters.

Similar remarks hold for identifying the most significant parameters that affect general crashworthiness. In addition, however, there is a need to judge the sensitivity to details of the component behavior requiring Level 3 simulation. Thus the first three levels of simulation are useful in this area.

5. Judge component compatability.

In judging component compatability it is again not necessary to have detailed and highly accurate results. It is necessary to be able to model the relative effect of component behavior. For

examining general concepts, a mix of fixed and variable force elements for example, Level 2 simulation will suffice. To examine actual components, however, Level 3 simulation is required.

6. Design experiments and interpret experimental data.

In the research effort in support of NHTSA's crashworthiness program, computer simulation and experimental tests are mutually dependent partners. Experiments are necessary for model verification and to identify physically important events to aid the modeling effort. Conversely simulation results may identify new situations and suggest the nature of the experiment required for physical verification. Further a simulation model often provides the framework for the evaluation of the new experimental data. These purposes require simulations with a quantitative predictive capability. Thus in general simulation levels 3-5 may be useful. The degree of sophistication is basically determined by whether the model includes the particular variables on which attention is focused. In this respect Level 3 simulation is likely to be marginal.

7. Compliance verification.

To judge compliance with standards solely from computer simulation requires both great detail and a high level of confidence. For example, to determine windshield integrity would require the



detailed deformation of the supporting frame. Only Level 5 simulation can provide this type of information.

The results of the above discussion are summarized by the bar graph shown in Figure 1. The ordinate represents the various simulation uses according to the number assigned above. The abscissa is the level of simulation as defined in section 1.3. It is clear that the range of simulation uses cover the entire simulation spectrum.

Finally we attempt to quantify the benefit to NHTSA of the various levels of simulation. For each level of simulation, we assign a benefit unit of unity for each simulation use it serves. The result normalized to a scale of ten is shown in Figure 2 as the solid line.

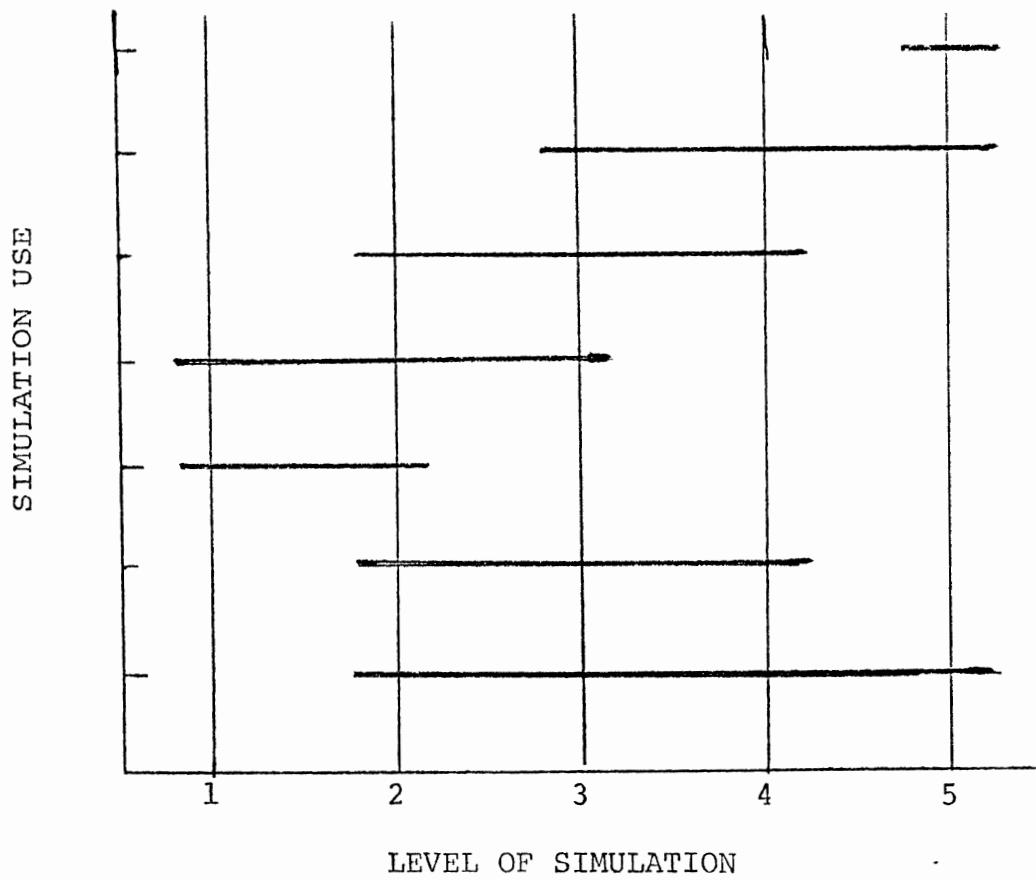


Fig. 1 Level of Simulation Required for Various Simulation Uses

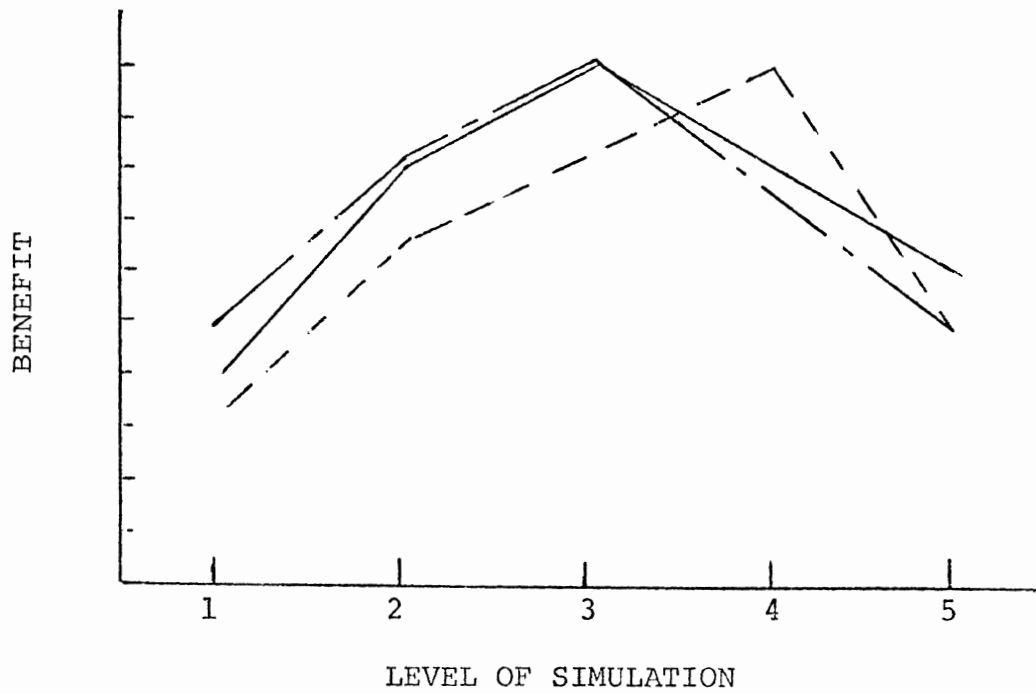


Fig. 2 Benefit of Simulation Levels

Two additional curves are shown to demonstrate possible weighting of the base curve determined by priority decisions within NHTSA. For example, the dash-dot curve is obtained if each simulation use is weighted by the number of functions it serves according to Table 1. A more discriminating weighting is illustrated by the dash curve. It represents doubling the benefit associated with predicting occupant compartment behavior and interpreting experimental occupant compartment data with a confidence associated with Level 4 simulation. In our view this is the lowest level of vehicle simulation that will provide sufficient detail to realistically exercise occupant simulation models to determine injury. Thus this particular weighting is biased toward using vehicle simulation as a tool to determine occupant injury. Of course, any number of examples could be cited. It is unlikely, however, that any reasonable weighting will change the basic conclusion that substantial benefit is obtained for a range of simulation levels.

## CHAPTER 2

### MAJOR MODELING CONCEPTS

#### 2.1 INTRODUCTION

In recent years a number of investigators have attempted to model vehicle response in crash environments in a manner appropriate for crash-worthiness studies. The approach and level of sophistication varies widely. In this chapter the major modeling concepts that have appeared in the technical literature are discussed, and their capabilities and limitations are identified. Although all the programs discussed have merit either as direct vehicle simulations or as important contributions in advancing the state-of-the-art, no current simulation meets the requirements of Level 4 or Level 5 simulation.

The programs reviewed can be classified in four main categories. They are (1) simplified spring-mass models, (2) hybrid models, i. e. models requiring experimental crush data as input for exercising the program, (3) models employing a variety of input specified generalized resistances, and (4) models based on treating the vehicle as a frame structure. Within the latter category, a number of programs have been developed using different structural concepts. These categories are discussed in the following four sections. Finally, in section 2.6 the applicability of available general purpose structural computer codes to the vehicle impact problem is assessed.

#### 2.2 SIMPLIFIED SPRING-MASS MODELS

The simplest approach to vehicle impact is to model the structure as a mechanical system consisting of masses connected by springs. Although there is a variety of such models in the published literature (and undoubtedly many more in the unpublished area), we feel some general statements about the nature and appropriate use of these models can be stated.

We have reviewed in detail the vehicle models proposed by Emori (1), Tani and Emori (2), and Muria and Kawamra (3). These models represent a typical sample of the simplified models available in the literature. They range from a single mass and spring to a three mass model with eight generalized resistances. In general, we will define simplified models as those having a few masses and less than ten degrees of freedom. The second characteristic is that the generalized resistances represent gross structural properties and are not necessarily identified with specific vehicle components.

All three authors claim reasonable agreement between calculated results using the model and experimental results. There are, however, a number of observations that may be made. First, agreement of displacement variables is considerably better than decelerations. Although peak deceleration may be quite close in the examples cited, the deceleration time curve is matched only in its gross features. As discussed below, this is more a function of judicious choice of parameters than a measure of model confidence.

This leads to the major observation - there is a high degree of arbitrariness in the definition of the generalized resistances employed in the model. All the authors employ piecewise linear force - deformation curves representing a plastic yielding structure. Each resistance represents a gross structural characteristic. For example, in reference (3) two of the resistances are defined as "front end upper member" and "front end lower member". The determination of the parameters characterizing the resistance is even more vague, as illustrated by a typical quote from reference (2). "The load-deformation characteristics of each nonlinear spring were determined by both presumptive calculations and experiments".

Thus, we conclude that agreement between model predictions and experiment represent a high degree of intuitive judgment by the investigator with a strong element of empirical curve fitting. This, of course, is not without merit. It demonstrates that simple models can describe qualitatively those features of vehicle impact which are compatible with the

limited variables of the model. On the other hand a high level of confidence cannot be ascribed to quantitative results except for the experimental conditions (and possibly even more significant, the exact experimental procedure) to which the model was "tuned". Thus simplified models are not useful as a predictive tool in a quantitative sense, but rather as a qualitative measure of general behavior.

Today this limitation of simplified models is generally understood. Within the National Highway Traffic Safety Administration, for example, the report by Carter (4) and the work of Spencer (5) are example of appropriate uses of simplified models for parameter and sensitivity studies. Nevertheless recognition of the nature of simplified models has implications for their use in crashworthiness studies. The first consideration is the futility of comparative studies to determine the "best" model. This basically follows from the nature of the generalized resistances which represent overall structural characteristics but are not readily identified with specific vehicle components. In general, there is no systematic way in which given a set of resistances for one model an equivalent set of resistances can be computed for a different model. An experienced investigator, for example, could tune all three of the models studied to get good agreement with measured overall displacement between engine and passenger compartment. In this case, the first model is not appropriate because it does not include the appropriate variable.

Thus the choice of a simplified model for a particular study is based not on its accuracy beyond initial model verification. Rather the model is selected on the basis of having variables appropriate for the particular study. In the side impact study reported in (5), for example, a single mass model was used. To consider non-centered impact, however, the mass was allowed to have a rotational degree of freedom and the moment of inertia of the vehicle was a significant parameter. Such a model would not be appropriate for a frontal barrier study in which relative engine displacement was of interest. Here two masses each with a single translational degree of freedom would be required.

These considerations lead to the conclusion that it is not desirable to choose one or two "best" simplified models. Rather NHTSA requires the capability to create and exercise a wide variety of models appropriate for particular qualitative studies. What is needed is a general simulation program for mechanical systems which automatically generates equations of motion from input describing the model variables. At the present time, the IBM CSMP package is employed by NHTSA to meet this need. Since for programs of this size, computer efficiency is not a crucial issue, this provides a satisfactory capability. In general, we conclude that the Level 1 simulation needs of NHTSA are adequately met within this context. More general mechanical simulations are discussed in section 2.4 below.

### 2.3 HYBRID SIMULATIONS

At the present time, hybrid simulation based on the work of Kamal (6) has been the most successful approach to predictive capability for vehicle impact. Its use has wide acceptance within the automotive industry. To our knowledge there are two operating programs in use, the Kamal program at General Motors and the CSS program developed by Autosafety Engineering Corporation (7). Both are considered proprietary in detail, but their general features and application to specific problems are available.

The present programs are basically three lumped masses with eight resistances. The resistances are identified with specific vehicle components or subassemblies. The force deformation curve for each resistance is determined experimentally from static crush tests and supplied to the program in digitized tabular form. Dynamic resistances are accounted for by an empirical "strain rate factor". The programs are limited to symmetric frontal or rear impact. The cost of exercising the program is economical (on the order of tens of dollars). The static crush

tests, however, are expensive (on the order of thousands of dollars). In many situations this is alleviated as a library of component force deformation curves is compiled.

The demonstrated results for frontal impact are good. Accurate values for the relative deformation of components and overall vehicle crush are obtained. The energy dissipated in each component is also obtained and the total energy accounted for within a few percent. The computed rigid body accelerations are less satisfactory but sufficient to make engineering judgment on design. Typically experimental results for accelerations show high frequency oscillations about an average value. The high frequency peaks are not obtained in simulation, but the average value is predicted within engineering accuracy. The gross nature of the inertial modeling (three lumped masses) is the reason for the discrepancy.

In evaluating the present programs there are two major problems that limit their general use. The first is the dynamic correction factor. Although there is considerable information on dynamic stress-strain curves for common metallic materials, equivalent information for structural force deformation curves is not known. The basic difficulty is that the strain rate may vary spatially over the structure with local strain rates differing by order of magnitude from the average rate. Thus, at the present time the dynamic factor is set empirically. This requires considerable judgment and experience. There is evidence, for example, (reference 8) that different factors may be required for different structural configurations.

The second problem is the care that must be exercised in conducting the static crush tests. Correct simulation depends upon the static deformation mode coinciding with the dynamic mode. The crush test must be carried out to insure this similarity. This may require special constraints and/or loading procedures. Again considerable judgment and experience must be exercised in the design of the tests. These problems in general reduce the confidence level of the simulation in the absence of experimental confirmation for a particular run. This is due to the



difficulty of objectively measuring the judgment factors involved and reliance must be placed on subjective evaluation of the experience of the investigator.

Nevertheless the concept of hybrid simulation in future developments is appealing. The complexity of some vehicle components may preclude analytical modeling in the foreseeable future. There are, however, some difficult problems in generalizing the present simulations to other crash environments. Even a relatively simple situation as an unsymmetric pole test presents major difficulties.

The crucial problem is to define the experimental information required which is consistent for a given model. When the only degrees of freedom are uni-directional translational displacements, the required force-deformation curve is relatively easy to define. When other displacement and rotational degrees of freedom are introduced, which is necessary for any type of unsymmetric loading, the problem is much more difficult. The problem can be simply illustrated. In writing the equation of motion of a given mass in the x-direction, for example, we need the force in that direction exerted by the component on the mass. For the large plastic deformations of interest, this force will depend upon all the degree of freedom variables and probably their time in history. How to define a series of tests to experimentally determine this function of several variables is not obvious. Further the empirical incorporation of dynamics factors in this situation is probably not feasible.

This problem has been considered in the purely elastic case. Klosterman (9) gives methods of experimentally determining stiffnesses and compliances for elastic vibrations of automobiles. The model approach used, however, cannot be extended to the large deformation plastic case. To our knowledge the problem in the plastic range has not been considered.

It is possible, of course, that much of the difficulty can be avoided in specific situations as in the case with frontal barrier impact. In a nearly centered side impact, for example, it is probably possible to express the necessary force deformation curve in terms of a single degree

of freedom variable which is feasible experimentally. Thus, future hybrid simulations will be limited to restricted loading conditions, a different simulation and test procedure being required for each condition.

We conclude that currently used hybrid models provide Level 3 simulation capability within the restriction of collinear impact. Their use, however, requires experience and judgment in obtaining experimental crush data. Finally, the potential for generalizing hybrid models to higher levels of simulation is small.

#### 2.4 GENERALIZED RESISTANCE MODELS (BCL SIMULATION PROGRAM)

Battelle Columbus Laboratories (BCL) has developed a computer simulation program for collinear car/car and car/barrier collisions (10). This program is based on a mathematical model with 4 masses and up to 35 individual nonlinear resistances. The masses are restricted to unidirectional motion.

Since the focus of BCL's study was to develop a flexible computer program, each mass or nonlinear resistance of the mathematical model does not represent any specific part or member of the vehicle. The determination of the candidate mass and resistance assignments are left to the user. He can leave these as blank, i. e. simplify the model, but can not change the basic configuration of the model. For a proper choice of masses and resistance, however, BCL's program can be applied to front, side and rear collinear impact.

In the program, the characteristics of the resistance members can be classified into six different types, each being represented by a program subroutine. They are:

1. A model of elastic-plastic "spring" capable of transmitting compression force only.
2. A model of a fixed-stroke variable-orifice hydraulic cylinder.
3. A model of an elastic-plastic "spring" which has both tension and compression capability.

4. A generalized model for elastic-plastic springs with tension and/or compression capability which may be described by a set of force versus deflection points and a representative unloading spring rate.
5. A model of variable-stroke, variable-orifice hydraulic cylinder.
6. A model of damping element which produce force proportional to velocity.

Considerable effort has been made to make the program user oriented. Preparation of input data is simplified and thoroughly explained. The program provides output in the form of graphical plots as well as printed output. A particularly nice feature is the ability to cross plot a variety of variables. Further software development to effect an interactive capability could be developed if desired. Thus the BCL simulation program is a flexible system designed with NHTSA needs in mind. Within the restriction of collinear impact, it meets the requirements of Level 2 simulation.

Although to date only limited use of the BCL program as a predictive tool has been reported (11), it undoubtedly has potential. The various options for the resistance members outlined above give the BCL model the capability of modeling a wide variety of hypothetical force deformation relations. The problem, of course, is that there is no systematic way to determine the parameters defining these relations. Presumably they could be fit to experimental component crush data. In fact using option 4 experimentally determined curves can be used directly.

It is not yet known, however, how detailed a fit is required to obtain quantitative results. Further until the model parameters can be related to geometric and material properties of the components, the use of the BCL model as a Level 3 simulation is in effect a hybrid model. Its use as a predictive tool requires experimental crush data for each component and is thus subject to all the limitations of hybrid models discussed above.

Thus from the viewpoint of Level 3 simulation, the BCL model is a generalization of the Kamal model (6). With proper choice of the lumped masses, the configuration of the two models can be made identical. If a current version of the Kamal program was available, it would be beneficial for a number of reasons to make comparative studies between the two models.

Some reasons are:

1. The Kamal model has undergone verification against full scale barrier tests. A comparative study would give at least a preliminary verification of the BCL model without actual physical testing.
2. The study would demonstrate the relative efficiency and accuracy of the numerical integration method and the algorithms for interpolation and look up of the tabular input data.
3. Such a study could be very useful in relating the hypothetical resistance members of the BCL model to actual component data. The essential characteristics of component behavior could be delineated by comparing the output for a series of hypothetical resistance members which match actual component data with increasing detail.

It should be noted that the BCL model has an option to incorporate a dynamic correction factor.

It has the form

$$F_{\text{dynamic}} = F_{\text{static}} \cdot C_v$$

where

$$C_v = A + B \log_{10} V_0$$

in which  $V_0$  is the impact velocity. A and B are chosen to give  $C_v = 1.3$  at an impact velocity of 30 mph. As in the Kamal model this overall magnification factor must be considered empirical. In this respect the BCL model has the same limitations.

Finally it is perhaps appropriate to comment here on some of the recommendations for future work made in the BCL report. A subroutine

to monitor energy absorption should definitely be included as it is essential to the use of the model for many of the Level 2 simulation needs of NHTSA. A number of recommendations deal with the desirability of incorporating accelerometer simulation and/or filtering capability. This is a complex question which is primarily related to the comparison of model and experimental data and will receive more attention in our impact testing study. At the present time, however, we doubt that the increased cost of adding degrees of freedom for accelerometer simulation is justified. As the report points out there is a need to study the integration procedure to obtain optimal accuracy relative to the model's capability to predict the actual vehicle response. If this can be done successfully, it should be unnecessary to further process the model output.

It is also premature to consider extending the BCL model to two-or-three dimensional motion capability. It is certainly desirable, of course, to remove the restriction to collinear motion. The difficulty is that a generalized spring representation of a component is useful only for collinear motion. To attempt a two or three dimensional arrangement of springs to represent components like a frame or sheet metal is futile until basic and detailed modeling studies for specific components are available.

We conclude that the BCL program provides NHTSA with a general and flexible Level 2 simulation capability. Moreover its use as a Level 3 simulation is feasible provided experimental crush data is available for defining the generalized resistances. It has, however, limited potential for use as an advanced simulation program.

## 2.5 FRAME MODELS

Recently a number of investigators have independently developed more general programs directed towards Level 4 and Level 5 simulations. Although a variety of structural techniques have been employed, they all model the vehicle as an assemblage of frame members interconnected at discrete nodes. The frame members are taken as straight beams with uniform cross section between nodes. Inertial modeling consists of

lumped point or rigid body masses at the nodes. With one exception, the simulations are three dimensional and allow for general loading conditions. These programs are reviewed in the following subsections.

### 2.5.1 CALSPAN (SHIEH) PLASTIC HINGE PROGRAM

A dynamic elastic-plastic program for computing the response of planer frames has been developed by Shieh (12). The major simplifying assumption is the structural concept of a plastic hinge. The structure is idealized as a two-dimensional frame consisting of beam members connected together at joints. The beam masses, and the mass of any object attached to the structure, such as an engine, are lumped. The structure has a number of points called joints where

1. two or more non-collinear beam elements are joined
2. concentrated masses or forces are located
3. the frame is supported
4. special information is required.

The beams undergo only small elastic deformation between their ends. All plastic deformation is confined to the joint locations where plastic hinges may form. This permits large joint rotations of the beams which account for the large change in structural geometry.

Only two-dimensional motion of the frame is considered. Beam ends translate in a plane, while the beam rotates about an axis perpendicular to the plane. In order to describe the behavior of a typical beam, a local rotating and translating coordinate frame is defined so that one axis always passes through the ends of the beam. With respect to this rotating frame, beam deformation is restricted to be small and is caused by bending and axial extension or compression. Motion of the beam with respect to its local coordinate system is transformed to a description in a global fixed system. This is the main source of non-linearity in the equation of motion.

A number of approximations and assumptions are inherent in introducing the concept of a plastic hinge. In addition to assuming the extent of the plastic zone is small, it also neglects any elastic-plastic

bending at the cross section. Thus the cross section is considered either fully elastic or fully plastic as determined by the yield condition. In the present study the effect of axial force on the yield condition is neglected. Thus a hinge is introduced whenever the bending moment at the node reaches a critical specified value. The moment is then specified to be constant until the rate of plastic work becomes negative at which time the section is again considered to be elastic.

The formulation seems reasonable within the framework of the basic assumptions, and appears to have good potential for application to crashworthiness studies. There are, however, some deficiencies:

- 1) Potential plastic bending locations must be a specified a priori.
- 2) The method of assigning lumped masses appears to be left to the ingenuity of the user; no study of different assignments of lumped masses has been made.
- 3) It is not clear how to apply this two-dimensional analysis to existing frame type cars. The dominant example type in the report involved a specially designed frontal frame.
- 4) Existing car frames may be subject to bending about two axes even in frontal pole collisions. The present model cannot handle this situation.

Correlation with experiments has been demonstrated for specialized frame structures with respect to overall deformation and average accelerations. Detailed correlation has not been demonstrated. The current restriction to planer frames, of course, limits its use as an overall vehicle simulation. Even for symmetric loadings, biaxial bending and torsion will be induced in typical automotive frame structures. It should also be noted that the assumptions inherent in the concept are too restrictive for predicting the detailed response associated with Level 5 simulation. This follows from the fact that realistic relationships between the stress-resultants and the deformation cannot be established without detailed consideration of the stress distribution on the cross section. Nevertheless, the Shieh program has considerable potential for Level 4 simulation if it can be generalized to a three dimensional capability.

### 2.5.2 SIMULATION PROGRAM "KRASH"

Recently a different approach has been employed by Wittlin and Gamon (13) in their simulation program "KRASH." This program was developed for aircraft type structures. In principle, however, it is applicable to vehicle impact. In concept it is a three-dimensional extension of the BCL model consisting of masses connected by straight line one-dimensional "beam" elements. Each mass now has six degrees of freedom, three translational and three rotational. The model equations are obtained by writing the equations of motion for each mass by summing the forces and moments acting on the mass from the generalized beam resistances. The program includes occupant masses that may be coupled to the structure.

In treating the generalized resistances, however, the program is essentially a frame model. Each "beam" element transfers a general force (three components) and general moment (three components). Thus the structure is replaced by an equivalent three-dimensional frame. The large deformation is treated by piecewise linearization. In each time step the forces and moments are determined from a linear stiffness matrix (the elastic stiffness matrix) which is adjusted for plasticity by multiplying by a stiffness reduction factor. The stiffness reduction factor is experimentally determined from overall force (moment) - displacement (rotation) curves obtained from static crush data. In this respect it is a generalization of the "Kamal" model.

Although the KRASH program appears to have potential as a general three-dimensional Level 3 simulation, there are serious questions about the feasibility of the procedure. The stiffness reduction factor concept employed in the program is theoretically incorrect in three-dimensional problems. The procedure employed implies that each element of the plastic stiffness matrix depends upon the current value of only a single deformation variable, whereas in general they depend upon the entire deformation history. Thus it is impossible to define a unique "load-stroke" curve for the experimental determination of the reduction factor as postulated by the KRASH formulation.



We conclude that experimentally determined stiffness reduction factors are meaningful only if the component test closely duplicates the dynamic deformation experienced in the actual vehicle impact. It is questionable whether this is experimentally feasible for general three-dimensional response except possibly under very special loading conditions. In addition the experimental difficulties discussed above in connection with extending the Kamal model are relevant here.

Thus it is likely that KRASH can be used as a Level 3 simulation only under restricted circumstances. It may prove useful as a three-dimensional Level 2 simulation where hypothetical reduction factors can be chosen based on experience and judgment for a particular qualitative study.

### 2.5.3 SIMULATION PROGRAM "CRASH"

A more general finite element frame model has been developed by Young (14) in the simulation program CRASH. The program is three-dimensional and considers both geometric and material nonlinearities. Material behavior is limited to plasticity theory. The basic beam element has uniform properties, but nodes may be specified arbitrarily. No prior assumption on location of plastic zones is required. Inertial modeling is accomplished by lumped masses at the nodes, the assignment of masses being left to the judgment of the user. Moments and forces at the nodes are computed by numerical integration of the stress distribution over the cross section. Thus the actual stress-strain behavior of the material may be used directly at the expense of monitoring the stress state at locations across the cross section.

The motion of the structure is governed by a system of nonlinear second order differential equations for the displacements of the end points and orientations of the structural bars. This system is not solved by a standard numerical integration procedure for differential equations. Instead, these equations are interpreted as the conditions which express that a certain functional  $S$  be a minimum at the desired displacement solution. The problem is thus reduced to finding the minimum of a

specific functional  $S$ . This functional  $S$  is expressed in terms of an energy function whose form depends on whether the structure is a truss or a frame. If the bar is entirely elastic, the energy function represents elastic energy. If plastic deformations occur, the energy function also includes dissipative energy. The central point of the formulation reduces to obtaining an appropriate energy expression.

Inclusion of plastic effects produces several problems in computing the energy expression. A general analytic formulation of a bending moment-curvature relation for bending about two axes is too complicated. The use of an unsymmetric cross-section makes the derivation of such a relation hopeless. A general analytical formulation for shear stress or deformation modeshape is also extremely difficult to obtain. Young overcomes this difficulty by introducing a number of approximations:

Buckling deformation is neglected.

Elastic modeshapes for bending and torsional deflections are used.

Material shear deformation is assumed elastic.

Plastic response is entirely one dimensional and tensile.

The elastic shear center coincides with the plastic shear center.

Axial strain varies linearly over the cross section.

Frame members are restricted to idealized thin-walled sections.

In general these assumptions are restrictive, but appear reasonable. They do permit consideration of elastic-plastic behavior of the cross section. The program was exercised on a number of example problems and correlated with known results and/or experiments. These problems, however, were not comparable to actual vehicle structures in degree of complexity.

Melosh (15) has applied Young's CRASH simulator to the analysis of a Mustang. The Mustang has a unitized structure consisting of stiffeners connected by sheet metal. There are both straight and curved members of complicated cross-section. Melosh developed both a truss and a frame model to account for complexity. No formal procedure for developing these models is presented. However, the essential ideas can be

deduced by a careful reading of Melosh's discussion. The easily identified stiffeners are given a literal representation, meaning that almost the actual cross-section and material properties are used in the model. The sheet metal connecting the stiffener appears to be neglected. Sections such as front fender supports, firewall and front door do not have obvious models. These are approximated by bars which apparently include the same volume of material as the original sections and attempt to follow primary load path through them.

Melosh's discussion of these results do not give a very adequate description of the behavior of truss and frame elements after impact. It is difficult to assess the relative importance of buckling and yielding and how well these are modeled. Melosh suggests that the failure of the truss simulation should be attributed to the inability of only 14 elements to adequately model the side of the car. There are a number of reasons why the frame simulation could be too stiff:

The above mentioned kinematic constraints undoubtedly increased the stiffness significantly.

Sections such as the front fender supports, firewall and front door were approximated by bars which included the same amount of material as the section. Since not all the material on the actual car provides resistance in impact, an equal volume approximation probably provides overly stiff bars.

Yielding of the door and its frame is not modeled. These are treated as a single elastic member.

Lateral beam deflections during plastic response is usually much greater than elastic deformation. Thus, use of elastic modeshapes in the plastic range increases structural stiffness.

Superposition of buckling modeshapes on the plastic deformation modeshapes may not adequately account for loss of stiffness due to buckling. A better approach would be to include more joint locations.

The computer simulations for a truss and a frame structure impacting a barrier at 31.2 mph predicts B post decelerations. These decelerations are compared to measured B post deceleration in a 30 mph crash. For the frame simulation acceleration grows too rapidly and

occurs in too short a time compared to the measured deceleration. This indicates that the frame model is too stiff. The truss simulation gives a much closer approximation to car response in terms of stiffness but does not correlate in detail.

It is unlikely that these results could be improved significantly by using more elements. The basic difficulty is the inadequacy of the frame concept to model the entire vehicle. At the present time there is no rational way to choose cross sectional properties so that a beam is equivalent to many actual structural components. Another source of modeling error is the structural joints. In the Melosh simulation the joints are treated either as frame nodes or pins which essentially neglects any effect of joint inefficiency. Also local deformation of the cross section is not considered.

#### 2.5.4 THOMPSON PROGRAM

The final frame program to be discussed has recently been developed by Thompson (16). The program is proprietary, but a general description is given in the reference cited. Basically the program is a finite element frame program with nonlinear geometry and plastic deformation capability. Although differing in some key respects, it is similar in size and concept to CRASH. It is considerably more flexible in treating cross sectional properties and is thus more adaptable to vehicle modeling. (As with all frame models, of course, the basic modeling problem of replacing actual components with equivalent beams remains.) It is also more general in material properties including strain rate sensitivity.

It also differs in another important respect. Rather than derive a plastic stiffness matrix which must be recomputed at each time step, the program employs an elastic stiffness matrix and a stiffness reduction factor. Unlike KRASH, where the reduction factor is postulated as being known from experiment, the present program computes this factor at each time step by taking the ratio of the actual moment about the neutral axis to the fully elastic moment. This requires pointwise integration

across the cross section and an iterative procedure for converging to the plastic stress-strain curve at each point. This is computationally a major task. Relative efficiency between this and the CRASH formulation is not known, but they are probably computationally of the same order of magnitude.

Although the Thompson reduction factor accounts for deformation history, it still may be criticized on theoretical grounds. The procedure is valid for symmetric bending, but in general is not correct. The range of loading conditions for which the procedure will give reasonable results is speculative. We believe, however, that reasonable results can be expected provided the resultant moment vector has small deviation from the neutral axis and torsion and axial effects are not significant.

In reference (16) correlation between results of simulation and tests was demonstrated for two experiments. The first was a dynamically loaded beam, and the second was a side impact study. In both cases the program was used to predict the time-varying nodal forces when the experimental nodal displacements were used as input at each time step. This is quite different, of course, than predicting the dynamic response from initial conditions. Thus on the basis of published results, the Thompson model cannot be considered as fully validated. Its use for overall vehicle simulation has not been reported.

## 2.6 GENERAL PURPOSE STRUCTURES PROGRAMS

The term "general-purpose" as applied to structural computer programs means that the program is intended to solve more than one problem. This generality can be due to inclusion of a library of different finite elements, or due to an analytical sophistication allowing solution of dynamic, stability or nonlinear problems in addition to the common case of the static elastic problem. The proceedings of two recent symposia (29) (30) provide a good background on organization and management of large general purpose programs.

Perhaps the most famous and widely available general purpose program is NASTRAN, created by a government-industry team under

sponsorship of NASA. Nastran was first operational for solution of static, elastic problems but was soon expanded to include stability and dynamics problems. At least 23 elements are available in the NASTRAN library at present, making it the most general of the general purpose programs.

From the standpoint of a crashworthiness model, it is unlikely that a general purpose program will provide the answer. Such a program would need to include nonlinear material properties, transient response, instabilities and possibly a rather adaptable element which would accept empirical stiffness data as generated by static crush tests. Although in many respects NASTRAN is close to this goal, with piecewise linear static analysis, elastic buckling analysis and a direct transient response analysis, a crashworthiness model of an automobile could not be directly modelled with NASTRAN. Some modification of the NASTRAN executive system would be needed to combine all the features above which are currently separately available. Because of the complexity of NASTRAN, it would probably be more difficult to modify it than to develop a new, special purpose program for the crashworthiness model.

A number of inelastic general purpose programs have been written typified by "MARC 2," developed by Marcal (17). The program is static but is quite general with respect to material and geometric nonlinearities. Since the program is incremental, the inclusion of dynamic inertia effects would not be difficult.

On the basis of our study of reference (17) the following comments are made:

- 1) Major attention is given to versatility and generality of the program, but efficiency of the numerical algorithms is not emphasized.
- 2) Matrix generation and handling are organized with a view to minimize the coding required to implement new elements.
- 3) The program was tested on five different computers for various problem sizes. For large problems, large discrepancies of total system time were found for the various computers. Clearly the program is not optimal for all computers.

4) A few case studies show slower running time and therefore higher cost compared to some special purpose programs.

5) Because of its general nature, it is difficult for a new user to employ.

There are a number of other programs capable of treating structural problems related to those arising in vehicle impact. Two excellent reviews of general purpose programs are available (31) (32). In Table 2 we have identified five programs which could be considered for use in vehicle simulation.

From the above discussion of NASTRAN and MARC 2 it appears that the general nature of such programs and their need to employ algorithms which are suitable for a wide range of elements restricts their efficiency in specialized applications.

Finally it is clear that advanced simulations will require the development of specialized modeling concepts. Although general purpose programs may have a variety of elements, they are all derived from consideration of continuum structure. Thus their use in vehicle simulation would require substantial modifications. We conclude that the direct application of such programs to vehicle structures is both inefficient and incomplete, and specializing them for this application represents a programming effort comparable to the development of a new program.

Name	Originator	Features	Proprietary
MARC 2	P. V. Marcal	Incremental, nonlinear material and geometric effects, time dependent material behavior	Yes
NOSAP	Lockheed Missile and Space Company	Incremental, large strain, varying material properties, geometric nonlinearities	Yes
NASTRAN	NASA	Very large element library, piecewise linear elastic, stability, direct transient response	No
BERSAFE Phase III	Berkeley Nucl. Lab., England	Material and geometric nonlinearities, Large element library	Yes
MAGIC III	Air Force Flight Dynamics Laboratory	Large element library, dynamic loading, stability	No
EPACA	Franklin Institute Research Lab.	Will have incremental, elastic-plastic material, elastic buckling, plastic buckling, transient response, large element library including pipes and curved beams. Not fully operational until end of 1973.	?

TABLE 2: GENERAL PURPOSE STRUCTURES PROGRAMS



## CHAPTER 3

### ASSESSMENT OF THE CURRENT STATE-OF-THE-ART

#### 3.1 SIMULATION LEVELS OF CURRENT MODELS

Based on the discussion in the previous chapter, the capability of present vehicle simulations relative to the simulation spectrum may be summarized as follows:

##### 1. Simplified Mass-Spring Models

In general they are suitable for Level 1 simulations when exercised by an experienced investigator. They must be considered as qualitative and have no potential for extension to higher levels of simulation.

##### 2. Hybrid Models

Hybrid models have been qualified as Level 3 simulations for collinear impact. They require judgment and experience in obtaining experimental crush test data for input. Their potential for generalizing to higher simulations is poor.

##### 3. Generalized resistance (BCL) Model

This model has been qualified as a Level 2 simulation which adequately meets NHTSA needs at this level. Provided sufficient experimental data is available for defining the generalized resistances it may be used as a Level 3 simulation comparable to hybrid models. Its restricted to collinear impact and has little potential for extension to higher simulation levels.

##### 4. Plastic Hinge Frame (Shieh) Model

This program has not been qualified as an overall

vehicle simulation. If it can be extended to three dimensions, it has considerable potential as a component module in Level 4 simulation.

5. Frame Program "KRASH"

This program may be used as a three dimensional Level 2 simulation. Under limited circumstances it has potential for a three dimensional Level 3 simulation. Its basic hybrid nature and theoretical limitations precludes its use for higher simulations.

6. Frame Program "CRASH"

This program has not been qualified as an overall vehicle simulation. Its complexity precludes its use except for Level 5 simulation, where it has potential as a component module. The validity of some of its assumptions remain to be established.

7. Thompson Frame Program

This program has not been qualified as an overall vehicle simulation. In complexity it is similar to "CRASH" and thus has potential only for Level 5 simulation. There are some theoretical limitations to its basic approach and its validity in general impact.

3.2 GENERAL DISCUSSION

The most striking feature of the current state-of-the-art is the success of hybrid models for quantitative prediction when to date there are no published reports qualified vehicle simulations using the more analytically sophisticated frame models. There are two major factors that account for

this situation. Despite their apparent greater modeling detail, no current frame simulation accounts for local deformation of the cross section. Further joint efficiencies<sup>1</sup> and eccentricities are not taken into account. Both effects play a significant role in the energy dissipated by the structure and are inherently accounted for in experimental crush data. The second factor is that the single force deformation curve required for collinear impact can be obtained experimentally for non-frame components like exterior sheet metal, fire wall, unitized forestructure, motor mounts, etc. In contrast there is no rational way to choose the cross section properties of an equivalent beam element to use in a frame model. Thus the evidence strongly suggests that a purely frame model is inadequate for a complete vehicle simulation. In addition advanced simulations cannot be realized without including effects of local deformation and joint behavior.

The current computational success of hybrid models have, however, about reached their maximum potential as an overall vehicle simulation. It is unlikely that they can be developed beyond their present Level 3 simulation capability. The major technical difficulty is the problem of obtaining the required experimental relationships between the generalized displacements for three dimensional deformations. For collinear impact only a single force and displacement variable are involved. In the general case, however, not only must a

---

<sup>1</sup>

The Thompson model incorporates an empirical joint efficiency factor but the choice and use of this factor was not discussed.

matrix relation be determined, but also this relationship is not unique and depends upon the loading history. Thus a definitive experiment cannot be performed. This greatly limits the hybrid concept since three dimensional crush data must be obtained, which in itself is a major task, for every loading configuration.

In contrast the frame simulation programs have demonstrated considerable potential for advancing the state-of-the-art. As discussed above frame models are also inadequate for overall vehicle simulation. They can, however, serve as accurate modeling techniques for major vehicle components and thus serve as the basis for advanced simulations.

With respect to the potential of specific simulations, the frame program KRASH has major deficiencies. The empirical stiffness reduction factor makes KRASH a three dimensional version of the hybrid concept. The major experimental difficulties probably precludes its use except for qualitative studies. The current Shieh program is also limited due to its restriction to planer frames. It does, however, have merit for use as a module in Level 4 simulation if it is generalized to three dimensional deformation. The program CRASH and the finite element program of Thompson both consider the detailed elastic-plastic stress distribution over the cross section. This computational complexity precludes their use for Level 4 simulation, but probably will be required in a Level 5 frame module.

CHAPTER 4  
 NUMERICAL METHODS

4.1 CURRENT METHODS OF NUMERICAL INTEGRATION

In this section we review in some detail the numerical methods currently employed in the simulation programs discussed in Chapter 2. Not all of the programs discussed are sufficiently well documented in the literature to assess the details of their numerical procedure. Thus the reports of Young [12] and Herridge and Mitchell [10] will serve as the basis for discussion. All three reports are basically concerned with a system of nonlinear ordinary differential equations which are integrated by an incremental approach. In Young's report [14], the equations take the form

$$M_i \ddot{x}_i + \frac{\delta U}{\delta X_i} = F_i(t) \quad (1)$$

for each degree of freedom where  $M_i$  is the mass,  $F_i$  is the forcing function and  $U$  is the total strain energy which can be highly nonlinear. The first variation of  $U$  may not have a simple analytic representation.

The time integration is based on a method proposed by Newmark [17]. In an interval  $\Delta t$ ,  $X_i(t)$  is approximated by a third degree polynomial in  $t$ . Assume the displacement, velocity and acceleration are known at the beginning of the time interval, denoted by  $X_{oi}$ ,  $\dot{X}_{oi}$  and  $\ddot{X}_{oi}$ . The

polynomial has the form

$$X_i(t) = \dot{X}_{oi} t + \frac{\ddot{X}_{oi}}{2} t^2 + \frac{1}{(\Delta t)^3} [X_{ei} + \dot{X}_{oi} \Delta t + \ddot{X}_{oi} \frac{\Delta t^2}{2}] t^3 \quad (2)$$

The solution  $X_i(t)$  in the interval is determined by equation

(2) if  $X_{ei}$  is known at the end of the interval  $\Delta t$  is chosen,

we may substitute (2) into (1) and obtain the governing equation for  $X_{ei}$ . It is

$$M_i X_{ei} + \frac{(\Delta t)^2}{6} \frac{\delta U}{\delta X_{ei}} = M_i [X_{oi} + \dot{X}_{oi} \Delta t + \frac{\ddot{X}_{oi}}{2} \Delta t^2] + \frac{(\Delta t)^2}{6} F_i(t_e) \quad (3)$$

This is a nonlinear algebraic system for the vector unknown  $X_{ei}$ . If a sufficiently simple analytic representative for the variations of  $U$  is available, equation (3) may be solved directly or iteratively. This, however, is not the present case. Young proposes that instead of solving (3), a functional

$$S = \sum_{i=1}^N M_i \left[ \frac{3}{(\Delta t)^2} X_{ei}^2 - \left( \frac{6}{(\Delta t)^2} X_{oi} + \frac{6}{\Delta t} \dot{X}_{oi} + 2 \ddot{X}_{oi} \right) X_{ei} \right] - F_i X_{ei} + U + C \quad (4)$$

is introduced such that its minimum is associated with the solution of (3). The minimization procedure used is the Fletcher-Powell method [18] [19] which does not require the derivatives of  $U$ . The author contends that this minimization procedure is more economical than direct methods. This is not an obvious assertion. Although the Fletcher-Powell algorithm does not require the derivatives, it does require the function  $S$  to be evaluated a number of times. Even for a complicated function  $U$  it may be more efficient to consider (3) directly using numerical approximations for the derivatives of  $U$ . At the present time, this should be considered an open question.

In any case, we still need to choose the time step size  $\Delta t$ . If  $\Delta t$  is too large, there will be significant error due to truncating (2) at cubic terms. If  $\Delta t$  is too small, computer round off errors will lead to instability. This behavior is shown schematically in Figure 2.

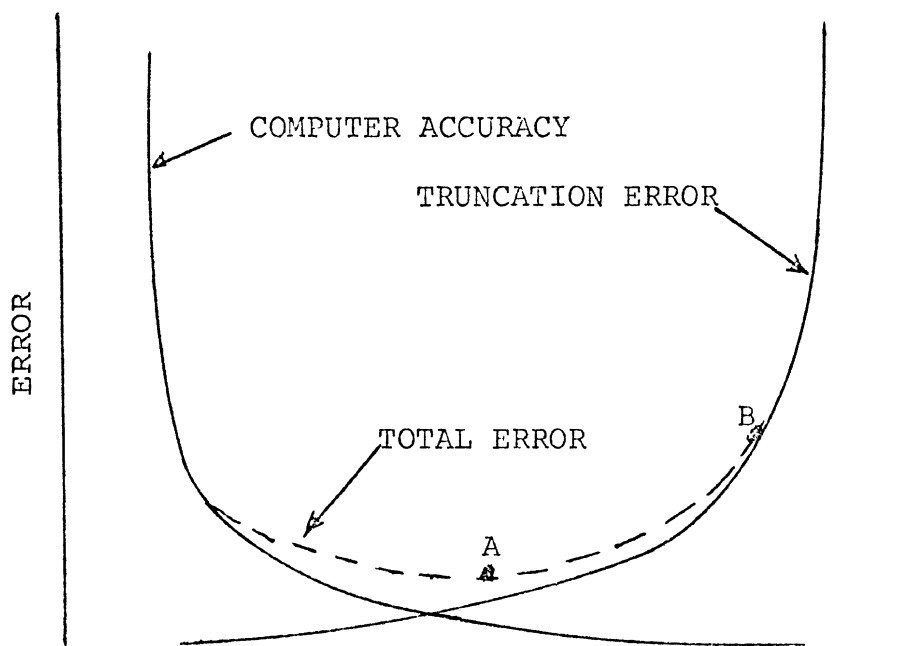


Figure 3 Error Vs Time Step

The point A denotes the time step giving minimum error, but this is not necessarily the optimal time step from the viewpoint of efficiency. If we can establish a bound on permissible error, say the error level associated with Point B in Figure 2, then the corresponding  $\Delta t$  will be the most efficient for the given accuracy. (If the permissible error bound is less than point A, we have no recourse but to increase the number of significant digits by using double precision.)

Based on the above reasoning the CRASH program uses a measure of truncation error as the criterion for choosing  $\Delta t$ . For each degree of freedom the residual equilibrium error in (1) is approximated by

$$\Delta F_i = [ S(t_i, x_i + \Delta x_i) - S(t_i, x_i - \Delta x_i) ] / 2\Delta x_i \quad (5)$$

since equations (1) are satisfied if  $\partial S / \partial x_i = 0$ . This error is weighted to emphasize the error in large displacements by defining the weighted error as

$$E_i = \Delta F_i x_i / S_i \quad (6)$$

The procedure to determine  $\Delta t$  is as follows. A trial  $\Delta t$  is selected and the minimization procedure carried out. The error  $E_i$  is then evaluated at the beginning, middle, and end of the time step. It should be noted that if the minimization is done exactly the error at the beginning and end of the time step will be zero. The actual minimization error is controlled by the error control (essentially number of iterations) in the Fletcher-Powell algorithm. The report



asserts that very tight error control can be imposed economically (essentially to number of significant figures available) since the major cost of the algorithm is in determining the neighborhood of the minimum.

In any case, the minimization error should not affect the choice of  $\Delta t$  for the forward integration. Thus the report defines truncation error as

$$TE = EH - (EZ + EF)/2 \quad (7)$$

where  $EZ$ ,  $EH$ , and  $EF$  are the maximum values of all  $E_i$  at the beginning, middle and end of the trial time step. Thus truncation error is the difference between the total residual error at the time interval midpoint and the average minimization error. If  $TE$  is greater than the error bound the procedure is repeated with a time step  $0.6\Delta t$ ; if  $TE$  is less than a lower bound the procedure is repeated with a time step  $1.85\Delta t$ .

We have gone into some detail in the above discussion to demonstrate a number of points. First the error definition used is somewhat arbitrary. It does, however, insure that the error introduced by the use of (2) is bounded and hence the solution will be stable. It also insures that reasonable if not optimal time steps will be used consistent with the bound imposed. Nevertheless a number of questions remain. No information is given to the choice of the upper and lower error bounds, nor to the effect of this choice on overall accuracy and efficiency. How are these factors affected by the error definition itself? At the present time such questions

can be resolved only through the experience of the user.

There is a definite need for more study in this area.

Shieh [12] uses a direct stiffness matrix formulation.

In a given time interval  $t_n \leq t \leq t_{n+1}$ . The governing equations

are:

$$[M] \ddot{\underline{u}} + [K(\underline{u})] \underline{\bar{u}} = \underline{P}(t) + \underline{Q}(\underline{u}(t)) \quad (8)$$

where M is the mass matrix, K the stiffness matrix  $\underline{P}$ ,  $\underline{Q}$  the forcing function vectors;  $\underline{u}$  is the displacement vector and  $\underline{\bar{u}}$  is the incremental displacement vector in the time interval. Large displacement and nonlinear material behavior require modification of K and  $\underline{Q}$  as the deformation proceeds.

Integration of equation (6) is done by a standard Runge-Kutta subroutine regarding K and  $\underline{Q}$  constant. The integration terminates at  $t = t_{n+1}$  and thus obtains  $\underline{\bar{u}}(t_{n+1})$ . A series of tests are performed to check plastic loading or unloading conditions, in terms of forming and disappearing of plastic hinges. K and  $\underline{Q}$  are then updated according to these tests. With these updated values the computation proceeds to the next time interval.

There are a number of points to consider in evaluating the numerical methods used in Shieh's report. The Runge-Kutta routine (see [20], for example) used as the basic integration method is a fixed time step prediction method. To increment the solution it uses a fourth order prediction formula to compute the solution at the next time point. There is no iterative procedure at each time step to control local error. The choice of  $\Delta t$  must be fixed by the user.

The basis for the choice can be established only by numerical convergence studies over the time range of interest.

In many problems Runge-Kutta is likely to be inefficient for large systems. The size of the time step required for a given local error is likely to vary widely over the time interval of interest. Since there is no local error control, the fixed time step size is controlled by the minimum value. But this may be considerably smaller than required for most of the interval.

Shieh does not discuss his choice of  $\Delta t$ . Presumably it is based on his experience with convergence for the specific examples presented. A new user will need to explore carefully the choice of  $\Delta t$  in applications to new situations. Moreover the use of a fixed time step will probably be inefficient.

There is another source of error in the Shieh procedure arising from linearizing the equations in the incremental displacement vector  $\bar{u}$ . This is not necessary since the Runge-Kutta method could be used to integrate the nonlinear equations directly, but at the expense of having to evaluate  $K(\underline{u})$  at each time step. Thus, the linearization undoubtedly increases the efficiency of the numerical procedure substantially.

Unfortunately there is no provision for estimating the error introduced. Since the error is accumulative, it in fact could lead to instability. In the report there is no discussion of choosing the interval  $(t_n \leq t \leq t_{n+1})$  to incur accurate results. Again, of course, numerical convergence studies for a particular example can establish credence of the results. But in new

applications, the necessity may offset any economic advantage gained by the linearization procedure.

Finally we consider the numerical procedure employed in BCL program [10]. The BCL program uses a standard subroutine which provides four integration modes. In the discussion and examples, however, it is implied that the mode providing variable step integration should be used. This mode is similar to the integration method employed in CRASH in that it provides local error control and automatic time step selection. The details of error definition and prediction formula are somewhat different.

To employ the subroutine the dynamic equations of the system must be expressed as a system of first order differential equations in the form

$$\dot{\underline{y}} = \underline{F}(\underline{y}, t) \quad (9)$$

where  $\underline{y}$  is the state vector and  $\underline{F}$  a vector function. The form of equation (9) is not restrictive, although to express (1) in this form, for example, requires inversion of the mass matrix as well as introducing the velocities as state variables.

The integration procedure uses a fourth order Adams-Moulton predictor-corrector method [20] which essentially plays the role of (2) in CRASH. Based on values of  $\underline{y}$  at previous time steps the value of  $\underline{y}$  at a time point increased by a trial  $\Delta t$  is predicted. With this predicted point an iterative formula is used to compute a corrected value. If we denote  $P_i$  as the predicted value of the  $i$ th element

and  $C_i$  as the corresponding corrected value, the local error is defined as

$$E = W_i \text{ Min. } \left( \left| \frac{P_i - C_i}{C_i} \right|, \left| \frac{P_i - C_i}{C_i} \right| \right) \quad (10)$$

where  $W_i$  is an arbitrary weighting function specified by the user. The value of  $E$  is then compared to specified upper and lower error bounds. If the upper bound is exceeded, the time step is decreased, whereas if  $E$  is less than the lower bound, the time step is increased.

Thus, the BCL integration procedure is equivalent to CRASH in its use of error control and automatic time step selection to increase efficiency. The relative efficiency of the two methods for a given accuracy is unknown. Further the questions raised concerning the CRASH method are also applicable here. The effect of the choice of the upper and lower error bounds and the weighting function  $W_i$  on accuracy and efficiency can be established only through the experience of the user.

#### 4.2 GENERAL DISCUSSION OF NUMERICAL INTEGRATION

A discrete model of an elasto-plastic structure dynamics problem invariably leads to a system of ordinary nonlinear differential equations as discussed above. Such systems are usually large in dimension. The functions involved in the systems are often very complicated. The economic factor

thus plays an important role in deciding the feasibility of computer simulation of dynamic response of a nonlinear structure.

The methods used in the programs discussed above fall into two categories of classical methods, namely polynomial extrapolation method and the classical Runge-Kutta method. Although, these methods are time tested and are relatively easy to program, our goal is to find or construct a highly efficient method for large scale structure dynamics problems.

The subject of numerical methods for initial value problems in ordinary differential equations has received a great deal of attention in recent years. Progress has been made together with computer evolution. Both the implementation of classical methods has been improved and new methods have been developed based on better understanding of convergence and numerical stability. Efficiency of a method is gained through algorithmic improvements. The current state of numerical analysis of initial value problems in ordinary differential equations is thoroughly reviewed in two recent books by Gear [21] and Lapidus and Seinfeld [22].

Numerical analysts have put their emphasis on the multivalue-multistep methods, which compute the solution at a new mesh point from the solution and its derivatives at several previous mesh points. The computation consists of two processes called prediction and correction. Hence, the method is some times called predictor-corrector method (PC).

A predictor is usually an explicit formula which gives a predicted value of the solution at the new mesh point from the known information at a subset of previous mesh points. A

corrector can be explicit or implicit. An implicit corrector require an iterative procedure to obtain the corrected value of the solution at the new mesh point. An implicit formula is always more time consuming than an explicit one. On the other hand it has been proven that an implicit corrector has a larger range of tolerance for prediction without divergence [21]. For example, the region of stability for the implicit Adam-Moulton method is larger by a factor of ten than that of the explicit Adams-Bashforth method. It pays a price in computational time since the iteration procedure is required to evaluate the functions and compute the corrected values a number of times. If we denote prediction by  $P$ , evaluation by  $E$  and correction by  $C$ , many methods in this category can be denoted by  $P(EC)^k$  or  $P(EC)^kE$ , depending on whether the method ends with an evaluation or a correction and where  $k$  is the number of iterations specified. The value of  $k$  may be automatically determined by the computer for each step if an error estimate is available.

The Runge-Kutta method has no corrector. It requires a fixed number of intermediate evaluations. A trade-off between implicit and explicit depends on how time consuming it is to evaluate the functions involved in the system. Thus the choice of method is problem dependent. The classical Runge-Kutta method has been generalized to include an implicit procedure [23].

The choice of step size for incrementing the independent variable is an important factor to achieve efficiency and economy. This is related to the practical control of errors.

There are two basic criteria for choosing an optimal step size. One is to obtain an expression for the work (Computation) per step with arbitrary size. The optimal step is obtained by minimizing the work subject to the constraint of an error bound. (Page 76, Gear [21]). A second procedure is to establish an error expression in terms of step size and then minimize the error. This procedure may be programmed providing automatic control of step size.

The effort to develop automatic control of step size may in general be quite complicated. It is, however, an absolute necessity for treating the class of problems denoted as "stiff equations". This class has a wide range of time constants. For example consider the vector  $y(t)$  with elements  $y_i(t)$  ( $i=1,2,\dots,n$ ) which is the solution to

$$\ddot{\underline{y}} = \underline{f}(\dot{\underline{y}}, \underline{y}, t)$$

Depending upon the nature of  $f$  the elements  $y_i(t)$  may behave

quite differently from each other. For example  $\left| \dot{y}_p(t) \right| / \left| \dot{y}_q(t) \right|$  may be on the order of  $10^4$  for particular values of  $p$  and  $q$ .

Unfortunately most physical problems involving the dynamic response of structures fall into this category. (Basically this arises from the broad frequency spectrum of structures). Thus in general it can be anticipated that control of step size is required if accurate solutions are to be obtained efficiently. Methods for treating "stiff



equations" are a major development area in numerical analysis. Current capability is discussed in detail in [22].

#### 4.3 CONCLUSIONS

With respect to numerical integration methods, our conclusions are:

1. A satisfactory simulation program must provide local error control and automatic time step selection. This capability is required to effect necessary compromise between accuracy and efficiency.
2. Within the present state-of-the-art the BCL and CRASH simulations are satisfactory with respect to numerical integration methods. The Shieh program is deficient in this area.
3. Although there is considerable intuitive understanding based on experience for choosing error bounds, there is a strong need for systematic study of the effect of local error bounds on accuracy and efficiency. Related questions are the appropriate definition for the error measure and the choice of error weight functions.
4. Recent developments in numerical analysis show considerable promise for improving the efficiency of integration for the class of equations relevant to the vehicle impact problem.

## CHAPTER 5

### DEVELOPMENTS IN COMPUTER HARDWARE

#### 5.1 MAN-MACHINE INTERFACE DEVICES

An exhaustive survey of mass-machine interface devices, or "terminals," would be voluminous. Presented here is a compendium of interface devices. Terminal devices which have no scientific, or engineering, application are not discussed. (An example would be a terminal which would accept punched card input and give, for example, filled-out W-2 forms as output.)

Device referred to here are in general "terminals" as opposed to "peripherals". The distinction has to do with the manner of data transmission. If transmission line length is short, then it is economical to transfer a word at a time. Thus, computer peripherals are often in the same room with the computer and eight lines would be used for transferring an eight-bit word. Long distances make this "parallel transmission" impractical and costly. Terminals, therefore, make use of data communications equipment which breaks down the character of a word into bits to arrange them into a standard code, and to send the characters out a bit at a time over one line (serial transmission).

Table 2 shows some of the variety of input and output methods by which data can enter and exit a transfer medium for the basic terminal types [24].

There are several parallel trends to be seen in the rapidly developing function of the terminal as a man-machine interface.

A primary trend can be characterized as "user orientation." One way to bring computers closer to non-computer-oriented people in the performance of their jobs is to make a terminal so smart that it can tell a user how to run it. One such example is a terminal offered by Friden Div., Singer Co., San Leandro, California. When an operator begins, the terminal tells him what to do next from a display of 40 message instructions. When an entry is correct, the computer acknowledges it. If the entry is wrong, the terminal informs the operator what to do to correct it. Terminals are also made more user oriented by increasing the number of function switches and keys. At present, interactive display terminals require skilled operators. But progress is being made here. One significant development predicted for the 1970's by top level experts in the field is software (and hardware) necessary to permit close machine interaction and to facilitate the use of display terminals by casual users. [25]

## 5.2 HARDWARE DEVELOPMENTS

The general trend is in the direction of increased function sophistication, i.e., man-machine interface devices with greater flexibility range of applications. A brief survey of such devices include:

Input-Output Interfaces -- An example of function sophistication is a graphic input system which automatically scans documents of graphic data. (Alden Electronic and Impulse Recording Equipment Company, Westboro, Mass.) Data is inputted by this digital facsimile scanner without going through time-

		INPUT METHODS															
TERMINAL TYPE	Alphanumeric	Numeric	Special-purpose	Punched cards	Paper tape	Standard	Cassette	Cartridge	Disk	Drum	Direct from computer	Light pen	Tablet	Joystick	Voltage probe	Cursor	Microfilm
Teletypewriter	•			•	•						•						
Alphanumeric crt	•	•	•				•	•			•					•	
Graphic crt	•	•	•						•	•	•	•	•	•	•	•	
Remote batch	•	•	•	•	•	•			•	•	•						•
		OUTPUT METHODS															
TERMINAL TYPE	Serial printer	Line printer	Paper tape	Punched cards	Standard mag. tape	Cassette	Cartridge	CRT display	Alphanumeric display	Microfilm	Direct to computer	Disk	Drum	Plotter	Facsimile		
Teletypewriter	•		•								•						
Alphanumeric crt						•	•	•			•						
Graphic crt								•			•				•		
Remote batch	•	•	•	•	•			•		•	•	•	•	•			

TABLE 3. Input and Output Methods for Basic Terminal Types

consuming XY point location and identification. Documents accommodated can measure 54 inches in width and be of any length. In contrast, graphical input systems are available which are simpler and less expensive than scanners or the conventional light pen systems. One system, for example, makes use of a ballpoint pen which sends sound signals to strip microphones that record X and Y coordinates. (Science Accessories Corporation, Southport, Conn.) High resolution is not sacrificed, yet software and simplified hardware result in a low cost system. General-purpose digitizing and graphical data acquisition systems will certainly undergo further development in the years ahead.

Terminals that can read information directly and transmit it to a central computer are an important entry in the field of computer terminals and communications. Such terminals are called OCR interfaces - for "optical character recognition." Technology is now progressing to the point that this may become a practical, useful way of communicating with a computer. A well-conceived OCR terminal system, making maximum use of preprinted and handprinted information, will require fewer operators than a system using non-OCR terminals. At the same time, the accuracy will be improved and time lags will be shortened. At present, however, OCR devices are not marketed with appropriate sizes, speeds, and interfaces for moderate-volume terminal work. Worst of all, prices are very high. But with the development of improved OCR techniques this type of system should find greater and greater use.

OCR systems may find most immediate extensive use in processing microfilm input. Present microfilm readers are still very expensive, but output devices are now available at more moderate cost which will produce graphical and alphanumeric output on microfilm.

Microfilm as computer-generated hardcopy is relatively new. Another type of hardcopy output which is still undergoing improvement is electro-optical printout of CRT displays. Continual improvements are being made in softcopy output as well as CRT display. Software and hardware development and system studies aimed at making interactive dynamic display terminals more practical are currently underway. Most current developmental work is still in increasing the scope of static display capabilities. (Interactive display terminals, static and dynamic, are discussed in a later section.) New non-interactive CRT terminals, for example, use a larger memory and a paging scheme which allows lines to be scrolled up or down across the screen face. Low cost CRT display terminals are also now available. The lowest priced unit may be the TeleComputer display terminal at about \$960. (Digi-Log Systems, Inc., Conshohocken, Penn.) A simple dip lead is attached to the antenna of any television, which provides the CRT. The keyboard is portable in a briefcase.

Programmable Terminals -- A wide range of devices is available which cannot only serve as interfaces to a central computer but also act as small stand-alone computers. These are called

programmable terminals, intelligent terminals, smart terminals, super terminals, or terminal computers. They are general-purpose stations that emphasize manual keyboard entry and user programming.

The presence of a minicomputer inside the terminal means that the user can acquire, sort, edit, update, file, calculate, format, and manipulate source data and local records offline. Timesharers can solve parts of their problem locally, letting the central computer take over when large amounts of data must be manipulated and updated or when other factors dictate use of the more powerful machine. Programmable terminals are typically priced between \$3000.00 and \$15000.00.

Programmable terminal functions break down into three broad areas: input/output, data communication, and processing. Manual input is normally by a typewriter-like keyboard for alphanumeric data. In some cases a Selectric typewriter serves for both keyboard entry and printout. Additional function keys on some keyboards can be tied to internal software or firmware routines to ease terminal operation.

A CRT or serial printer serves for terminal output. Where one is standard, the other is often an option. With a CRT, a larger portion of the terminal's memory will probably be devoted to display functions, and a character generator will be present. Data received from the central computer can often be placed in auxiliary storage (magnetic cassettes or cartridges) and printed later at a convenient time under program control.

Transmission speeds depend somewhat on the application since both sender and receiver must be compatible. If the minicomputer has enough processing power, it can be put to work to interface the terminal with nearly any central computer or other terminal. Through programming, it can convert transmission codes, change speeds, and switch from asynchronous to synchronous mode and vice versa. In some cases the manufacturer will microprogram a read-only memory to meet the customer's transmission needs. Some makers offer interchangeable hard-wired communications controllers on plug-in cards that fit inside the terminal.

Processing capabilities vary over a wide range. In some cases the manufacturers have slightly beefed up a more conventional terminal, while others have started with a small computer and added data communications. Here, the user must balance his remote processing needs against the cost of doing the same jobs at the central computer. A terminal processor with only limited ability may be enough to handle nuisance jobs that unnecessarily tie up the central computer. Some terminals have a read-only memory which holds programs that define functions such as data capture, formats, file generators, file maintenance, search, edit, and display. The operator simply pushes buttons to access these programs. (Such a terminal would be ideal for the user who does not want to concern himself with programming because his needs can be met by standard routines.) Programmable terminals typically have a 2K to 8K byte memory and a read-only memory that defines 40 to 50 macro-instructions.



Interactive Display Terminals -- A display console is termed "interactive" if it has the ability to serve not only as an output device, but as a real time, on-line input medium as well. A variety of devices under operator control are used for this purpose.

The principal means for human input at the display terminal are normally two keyboards. An alphanumeric keyboard serves generally the same purpose as in standard non-display terminals. A control keyboard with switches and buttons allows extensive manipulation and user control of the display information.

Means are provided for inputting information in graphic form. Typically, this information enters through either a "tablet" or a CRT. A tablet is a horizontal writing surface which digitizes the position of a hand-held stylus, i.e., stores its XY location in memory. A cursor is a cross which indicates and possibly digitizes a position. The cursor may be moved on the screen by several methods, which include: (1) a "joystick," an electromagnetic control controlling combined X and Y motion and (2) a light pen, which optically draws the cursor along to a new position.

Forthcoming advancements in the state-of-the-art of interactive display graphics are discussed later in this section. The following represents an approximate summary of the current state-of-the-art. A computer-aided design process is used for illustration [26]. The CRT is used under cursor/light pen/keyboard control in roughly the following manner:

- (1) The screen is cleared
- (2) Points and lines and arcs defined between them are drawn either by light pen or keys
- (3) Geometric constraints such as parallelism, perpendicularity, equal angles or lengths, etc., are imposed
- (4) Smoothing and linearizing take place
- (5) Previously defined macros or drawn pictures may be retrieved from memory and connected into the assembly
- (6) Selective erasure is used to alter the configuration
- (7) Entire subpictures or portions of the screen may be translated, rotated about any arbitrary axis, or non-uniformly scaled about an arbitrary axis
- (8) The contents of the entire screen may be rotated through a perspective 3-space
- (9) The display may "zoom" in or out on an arbitrary region or component
- (10) On the basis of function keys standard computations may be performed on the picture in the screen. Values of parameters can be varied by use of the light pen.

Continual improvements are being made in interactive display systems. An example is the manner of rotation of a display. Consoles now have built into the hardware rotation matrix logic that permits rotation (in 3-D) of a display about any axis without the need to access the computer. Three-dimensional joysticks are available. It is possible to zoom in and magnify dense sections of a three-dimensional layout and make modifications to the blowup. It is possible to gain

an apparent position inside a multilayered display and rotate it around you. Hardware is doing more and more of the work, lessening software requirements. Higher speed function generators are available. Analog input can be used with some systems as well as direct operator input.

The primary reason that interactive display terminals are not used more widely than they are is that of cost. But there are other reasons. There is a need to expand existing machine dependent display languages to machine independent "display compilers." Users still need to know too much about the pathologies of the system and cannot program for the display in a strictly program-oriented language. Also, further advances in the state-of-the-art in pattern recognition are needed in order to lessen the keyboard input required to specify in detail what is on the screen.

The most ambitious known state-of-the-art advancement is still under development. It is IBM's DESIGNPAD [27]. DESIGNPAD provides a solution to at least one of the problems mentioned in the preceding paragraph. Complete reprogramming often necessitated by small changes in functional specifications for different applications has been obviated by generalizing DESIGNPAD to accept all problems that can be represented by labelled block diagrams.

The experimental DESIGNPAD system falls within the definition of an "interactive display terminal," but it is far more powerful than conventional programmable, or intelligent, terminals. The "terminal" consists of an IBM 1130 computer combined with an IBM 2250 Model 4 display unit. These

are connected to a time-shared IBM 360/67.

All graphic or textual input or output information is carried by conceptual "modelling sheets." The sheets are a two-dimensional medium equivalent to a sheet of paper sixty feet square. The user can work on whatever part of the sheet is visible on the display unit and can easily access any portion of the sheet. "Viewports" and "windows" are used to examine graphic input and output. The display screen can be divided into up to four rectangular areas of adjustable size. These are the "viewports." With them the user can view parts of different sheets simultaneously. A "window" is the boundary of a displayed portion of a sheet. "Windowing" is a procedure allowing continuous change of the window position, i.e., controlled scanning over a sheet. DESIGNPAD provides a facility for filing and retrieval of sheets.

DESIGNPAD does not include dynamic scaling, i.e., the "zoom" feature of previously mentioned systems. It incorporates only windowing. Dynamic windowing in DESIGNPAD is carried out as one function of a "display file" rather than by special hardware so that the system will be more transferable to different locations. The display file is a set of commands (graphic orders) to the display unit to generate an image. In order to accomplish windowing, use is made of a "cellular structure" for the display file. Each sheet is partitioned, conceptually, into cells one and a half inches square. (Actual use of the DESIGNPAD system might suggest another size to be more appropriate.) The sheet's display file is divided into corresponding groups of graphical data, incremental

graphic orders. An advantage of cellular sheets is greater efficiency in searching display files on the XY position of the input device. DESIGNPAD uses the light pen (or tablet stylus) only to obtain XY positional information, so all graphical entities at that position must be searched for; but the search need not go beyond a single cell of the complete display file.

Block diagrams are constructed through use of a set of facilities called the "drawing package." It may be used, for example, to copy a block from one place to another. It is used to draw lines or a series of lines and to delete blocks or lines from a sheet. Textual entities are block components which can be created, modified, or deleted by a text-editing facility in the drawing package. Endpoints and attachment points for a block diagram are defined by the drawing package and "hooks," which associate textual entities with particular blocks, can be created or deleted.

Analysis of the completed block diagram is performed by a user-supplied program. This program can call an output package provided as a part of DESIGNPAD. This package provides subroutines for plotting various kinds of graphs. Projections of three-dimensional surfaces for functions of two variables may be obtained. All output is placed on a sheet so it can be filed for later study. The windowing facility is used to examine all output on the sheet.

A display terminal is considered to be "dynamic," as opposed to "static," if it can provide dynamic (animated) output. DESIGNPAD has this capability since it has an action

file that can describe a series of changes to a sheet and thus direct dynamic output "frame by frame." The action file is stored in the 1130 and can thus be called upon to direct an animated sequence any number of times.

### 5.3 CURRENT RESEARCH IN COMPUTER TECHNOLOGY

Research in computer technology continues, of course, on many fronts. It has previously been mentioned that OCR interfaces are still undergoing development. While it seems unlikely that OCR systems will ever be a tool that everyone can afford, it may be hoped that improved OCR techniques will result in less expensive systems. Advances might also be expected in the areas of improved software for virtual storage paging algorithms and added levels in storage hierarchies, both of which would result in lower execution costs because of reduction in CPU times. Variable micrologic may come to pass. Hardware in general will become more densely packed and miniaturized, thus allowing systems of lesser physical dimensions and resulting in overall reductions of transmission times. Low-cost intelligent terminal devices may become available. Display terminals will appear that can be used by non-computer-oriented operators. Multilevel computer and terminal networks are predicted; some processing will be done at the terminal, some at the first level computer (geographically close), some at the second level computer (farther away and more power), and so forth, with all processing done at the lowest possible level in the network.

Many developments are expected in the area of computer output. The SEER technique (System for Event Evaluation and Review) was recently used in order to develop a list of potential events in this area and to assess the probability that those events will transpire before certain dates. The technique involves two phases of interaction with top level experts in the field and makes use of intuitive and normative characteristics. A summary of the results of this investigation follows [25]. Numbers in parentheses are predicted 50% and 90% probability dates.

- (1) High resolution TV viewers will come into being, providing the flexibility of electronic magnification variation and aspect ratio control to give a user a "universal" viewer for a wide variety of optical format microfilms (1972, 1974)
- (2) High quality micro-medium for storing information of permanent value (but low usage rate) in a manner capable of direct input to a computer (1974, 1976)
- (3) There will be a radical change in the policy and methods of publication. Copyright laws are a chief obstacle to wider publication in microforms, and publishing houses are struggling with the problem with an eye on the possibility of microform publications (1974, 1978)
- (4) Marriage of microform with other information processing equipment will continue to increase the utility of microform from only a storage medium to a dynamic and important element in active systems (1976, 1981).

- (5) Professional literature dissemination in microform (1977, 1981)
- (6) Use of conventional printed materials will decline and be replaced by high density media and soft display (1982, 1988)
- (7) Hard copy devices such as teletypewriters, electric typewriters, and high speed printers will no longer have a cost advantage over microform and soft display (1973-50%, 1975-80%).
- (8) Peripherals capable of accepting technical data recorded on standard input/output media and producing graphics-quality technical reports will come into widespread use (1975, 1978)
- (9) Standard television sets will come into substantial use as input/output terminals (1973, 1975)
- (10) Solid state, low cost, direct view display devices with selective erasure (1974, 1977)
- (11) Holographic techniques may compete with and/or supercede the use of TV consoles for man/machine interface (1980, 1985).

A general observation from the foregoing list is that microform technology is expected to have a very significant effect on computer presentation in the 1970's. Computer-output-microfilm (COM) will have great effect on the future direction of the microfilm industry. Computer-input-microfilm (CIM) does not enter the picture as strongly, but it will develop to a degree that microform will become a dynamic medium.



Development of computers with greater efficiency, greater "computing power," and increased throughput capability obviously goes hand in hand with defining as clearly as possible various meaningful measures of system performance. There is a basic need for a "Theory of Computer Performance." Theoretical work is being carried out in this area and hopefully may result in advances in system configuration and hardware/software structures that will achieve denser, more compact code.

Finally, latest indications are that IBM's fourth generation of computers might be announced in about 1977 or 1978. IBM could possibly scrap the traditional stored-program approach for a totally new technology. An example of such an approach would be an iterative circuit computer.

CHAPTER 6  
CONCLUSIONS

6.1. CURRENT STATE-OF-THE-ART OF COMPUTER SIMULATION OF VEHICLE IMPACT

Our conclusions concerning the state-of-the-art of computer simulation are:

- 1) Level 1 and Level 2 simulation needs of NHTSA are adequately met by available simulation programs. In particular, the BCL program is well designed to meet Level 2 simulation needs.
- 2) Within the restriction of collinear impact, Level 3 simulation may be obtained with hybrid models, i. e. models requiring experimental crush data for components as input data. Although only limited application of the BCL model have been reported in this mode, it appears to serve as an adequate Level 3 simulation. Considerable care must be exercised in obtaining crush data in the appropriate dynamic deformation mode.
- 3) No currently available simulation based on a frame model has been qualified as a vehicle simulation. Moreover, it is unlikely that advanced simulations can be developed based solely on the frame concept. Nevertheless, both the frame program developed by Shieh based on the plastic hinge concept and finite element frame programs currently available have potential as "modules" for advanced simulations.
- 4) Although hybrid models adequately serve as Level 3 simulations, their potential for advanced simulations is small.

6.2. CONCLUSIONS WITH RESPECT TO SPECIFIC PROGRAMS

Our review of the simulation programs currently available have led to the following conclusions:

- 1) There are available a number of simplified models that meet Level 1 simulation requirements. These are adequate for simple qualitative studies. Since execution time is not critical for

such programs, a standard mechanical simulation program like the IBM-CSMP package is adequate for exercising such models.

- 2) Hybrid models typified by the Kamal Program have to date been the most successful in obtaining quantitative results for actual crash events. Within the restriction of collinear impact, the results obtained meet Level 3 simulation requirements. There are a number of disadvantages. Considerable experience and engineering judgment is required to obtain satisfactory component test data. The dynamic correction factor is empirical and based entirely on experience. There is little hope of generalizing the program to meet higher level simulation requirements.
- 3) The BCL Simulation Program meets all the requirements of Level 2 simulation within the restriction of collinear impact (front, rear, side). It is user oriented and has considerable flexibility. It adequately meets NHTSA needs for Level 2 simulation. In concept the BCL program also gives NHTSA the capability of Level 3 simulation provided sufficient experimental crush data is available to define the generalized resistances. Use in this mode is subject to the same restrictions as hybrid models.
- 4) The plastic hinge frame model developed by Shieh has not been qualified as a vehicle simulation. The modeling concept is inadequate for an overall vehicle simulation. The program has demonstrated, however, the potential of the plastic hinge concept for the development of a frame component simulation. Inherent limitations of the plastic hinge idealization probably preclude its use as the major frame component for Level 5 simulation. It does, however, have merit for Level 4 simulation if it can be generalized to three dimensional frames.

- 5) The simulation program "KRASH" is a three dimensional frame program which relies on experimental crush data for specifying frame component behavior. The theoretical framework, however, is not valid. Thus the program will be useful for general Level 2 simulations where the force-deformative characteristics may be postulated based on judgment and experience. Under restricted loading conditions it might be possible to obtain the necessary experimental crush data for use on a Level 3 simulation. In general, however, size restrictions may prove prohibitive for general Level 2 and Level 3 simulations. Within its present modeling concept, it does not have potential for use on a higher level simulation.
- 6) The simulation program "CRASH" is a general elastic-plastic frame program based on finite element technology. A number of assumptions are made in formulating the basic beam element, but it does account for elastic-plastic behavior of the cross section. It has not been qualified on an overall vehicle frame simulation due to the inadequacy of the frame concept and neglecting realistic joint behavior. Its size prohibits its use in Level 4 simulation, but it has potential for providing the basis of a frame module for Level 5 simulation.
- 7) The simulation program developed by Thompson is in size and complexity comparable to "CRASH." In detail it has features that are somewhat more convenient for vehicle applications. Although it is also based on finite element technology, a different concept is used to account for elastic-plastic behavior of the cross section. The procedure employed is not generally valid, and its limitations are not currently known. Qualifications of the program have not been demonstrated in the literature. As is the situation with "CRASH," its application to overall simulation is limited by the frame concept, but it would be a candidate for use as a frame module for Level 5 simulation.

### 6.3. NEEDED AREAS OF DEVELOPMENT

The assessment of the current state-of-the-art has identified a number of areas that must be developed if the state-of-the-art is to be extended to advanced (Level 4 and Level 5) simulations. NHTSA need, technical and economic feasibility, and a recommended methodology for the development of advanced simulations is discussed in detail in Volume 1 of the final report (28). Here we briefly summarize the development areas identified.

They are:

#### 1) Basic Modeling Concepts

Although there is some need for further development of numerical integration techniques, numerical analysis and computer hardware are not the limiting conditions on advanced simulations. To date vehicle components have been modeled as generalized resistances or frame members. Although both concepts are useful, they are not sufficient for overall vehicle simulation at advanced levels. There is a great need for new modeling concepts appropriate to specific components but which can be integrated into a total vehicle simulation. Of critical importance are methods for treating two-dimensional components like body sheet metal and unitized construction.

#### 2) Joint Behavior

Vehicle joints play a dominant role in the deformation mode and energy dissipation during impact. There are at least three factors that need to be considered in modeling efforts. They are joint eccentricities, joint efficiency of spot welded connections, and local deformation of the cross section. Presently, these are inherently accounted for in experimental crush data, but are neglected in the frame simulations. Since hybrid models are essentially limited to collinear impact, advanced simulations will require the rational but efficient modeling of joint behavior. Presently our basic knowledge in this area is limited.

3) Dynamic Effects

Dynamic strain rate effects are for the most part currently modeled empirically by an overall correction factor. As specific components are modeled in detail by different concepts, this procedure becomes increasingly invalid. The basic difficulty is that correction factors are based on average strain rates, whereas actual strain rates may vary widely at different points in the structure. Although considerable is known about material strain rate effects, its influence on structural behavior has not been assessed in detail.

## REFERENCES

1. Emori, R.I. "Analytical Approach to Automobile Collision." SAE Paper 680016, January, 1968.
2. Tani, M. and Emori, R.I. "A Study on Automobile Crashworthiness." SAE Paper 700175, January, 1970.
3. Miura, N. and Kawamura, K. "An Analysis of Deformation Mechanisms in Head-On Collision." SAE Paper 680484, May, 1968.
4. Carter, R.L. "Passive Protection at 50 Miles per Hour." U.S. Dept. of Transportation, Report No. DOT-HS-810-197, 1972.
5. Spencer, P.R. "Parametric Study of Side Impact Vehicle Aggressiveness." 18th Annual Meeting of Institute of Environmental Sciences, May 2, 1972.
6. Kamal, M.M. "Analysis and Simulation of Vehicle to Barrier Impact." SAE Paper 70044, 1970.
7. Autosafety Engineering Corporation, "Crash Environment Systems Studies," undated.
8. McLean, R., Private Communication.
9. Klosterman, A.L. "A Combined Experimental and Analytical Procedure for Improving Automotive System Dynamics," SAE Paper 720093, January, 1972.
10. Herridge, J.T. and Mitchell, R.R., "Development of a Computer Simulation Program for Collinear Car/Car and Car/Barrier Collisions," BCL Report DOT-HS-800-645, January, 1972.
11. Herridge, J.T. and Mitchell, R.K., "Initial Crash Test Screening Studies Utilizing the NHTSA/BCL Computer Simulation Program..." BCL Report, POT-HS-800-730, August 1972.
12. Shieh, R.C. "Basic Research in Crashworthiness II - Large Deflection Dynamic Analysis of Plane Elasto-Plastic Frame Structures..." CAL Report YB-2987-V-7, August 1972.

13. Witlin, G. and Gamon, M., "Experimental Program For Developing Rotary Wing Aircraft Structural Crashworthiness Criteria and Concepts," Lockheed-California Co., Final Report, Contract No. DAAJ02-71-C-0066.
14. Young, J. W. "CRASH: A Computer Simulator of Nonlinear Transient Response of Structures," Philco-Ford Corp., Report No. DOT-HS-091-1-125-B, 1972.
15. Melosh, R. J. "Car-Barrier Impact Response of a Computer Simulated Mustang," Philco-Ford Corp., Report No. DOT-HS-091-1-125A, 1972.
16. Thompson, J. E. "Vehicle Crush Predictions Using Finite Element Techniques," SAE Paper No. 730157, January, 1973.
17. Marcal, P. V. "On General Purpose Programs for Finite Element Analysis, with Special Reference to Geometric and Material Nonlinearities," Division of Engineering, Brown University, Report No. ARPAE-73, May 1970.
18. Newmark, N. M. "A Method of Computation for Structural Dynamics," Journal EMD, Trans. ASCE, Vol. 85, No. EM3, 1959.
19. Fletcher, R. and M. J. Powell, "A Rapidly Convergent Descent Method for Minimization," Computer Journal, Vol. 6, 1963.
20. Stewart, G. W. "A Modification of Davidon's Minimization Method to Accept Difference Approximations of Derivatives," Journal Association for Computing Machinery, Vol. 14, 1967.
21. Hamming, R. W. Numerical Methods for Scientists and Engineers, McGraw-Hill, 1962.
22. Gear, C. W. "Numerical Initial Value Problems in Ordinary Differential Equation," 1971 Prentice Hall.
23. Lapidus, L. and C. Seinfeld "Numerical Solution of Ordinary Differential Equations," 1971 Academic Press.
24. Butcher, J. C. "Implicit Runge-Kutta Processes," Math. Comp. 18, pp. 50-64, 1964..
25. Data Processing Magazine, February, 1971.



26. Business Automation, September, 1971, p. 33.
27. "A Survey of Pictorial Data Processing Techniques and Equipments,"  
Defense Documentation Center, AD-626-155, p. 42.
28. IBM Systems Journal, No. 2, 1971, pp. 143-163.
29. McIvor, I. K., "Modeling, Simulation and Verification of Impact  
Dynamics, Vol. 1, Executive Report," Report No. UM-HSRI-  
BI-73-4-1, The University of Michigan, Oct. 1973.
30. "Computer Software in Structural Analysis," Proc. 2nd ASME  
Pressure Vessels and Piping Conference, Denver, Colo.,  
Sept. 1970.
31. "Engineering Computer Software," Proc. ASME 1st National Congress  
on Pressure Vessels and Piping, May 1971.
32. "On General Purpose Finite Element Computer Programs," ASME  
Winter Annual Meeting, New York, Nov. 1970.
33. "Three Dimensional Continuum Computer Programs for Structural  
Analysis," ASME Annual Winter Meeting, New York, Nov. 1972.

APPENDIX  
GENERAL BIBLIOGRAPHY

A. 1. GENERAL STUDIES IN CRASHWORTHINESS

- Bozich, D. J. "Research in Crashworthiness of Vehicle Structures,"  
Wyle Lab, Huntsville, Al., Rept. No. WR-68-3, V. 1., 1968.
- Carter, R. L. "Passive Protection at 50 Miles Per Hour," U. S.  
Dept. of Trans., Rept. No. DOT HS 810 197, 1972.
- "Investigation of Motor Vehicle Performance Standards for Crash-  
worthiness of Vehicle Structure - Phase 1." Fairchild-Hiller  
Corp., Republic Aviation Division, DOT HS-800 531, Jan. 1968.
- Fenton, J. "Vehicle Secondary Safety and Structural Design."  
Automotive Design Engineering, Vol. 11, April 1972, pp. 19-21.
- Glancy, J. J. and Desjardins, S. P. "A Survey of Naval Aircraft  
Crash Environments with Emphasis on Structural Response."  
Dynamic Science, Phoenix, Arizona, Rept. 1500-71-43, Dec. 1971
- Haines, J. A. and Armstrong, R. W. "Space Zone Comfort Concept  
for Upper Torso Restraint." National Bureau of Standards  
Rept. 10384, Nov. 1970.
- Haines, J. A. and Water, H. P. "Restraint Requirements for Side  
and Rear Facing Seats." National Bureau of Standards Rept.  
10386, Jan. 1971.
- Haynes, A. L. "Studies of Barrier and Car-to-Car Impacts."  
Presented at the meeting of the Harvey Cushing Society, Detroit,  
Michigan, April 1957.
- Johnson, P. R. and Wiltse, W. E. "Front Structural As It Affects  
Occupant Injury-Reduction and Survival." Proceedings of  
Automotive Safety Seminar, General Motors Corporation,  
July, 1968.
- Lundstron, I. C. "Vehicle Crash Research and Manufacturing  
Experience," SAE-680543, 1968.

- McHenry, R. R. and Miller, P. M. "Automobile Structure Crashworthiness," 1970 International Automobile Safety Conference Compendium, Sponsored by SAE, SAE-700412, 1970.
- Marsh, J. C. "The 1970 International Automobile Safety Bibliography," Distributed by SAE, 1970.
- Mayor, R. P. "Basic Research into Crashworthiness of Vehicle Structures," CAL Rept. YB-2498-V-1, 1968.
- Meisler, E. and Earwood, I. "Bibliography of Research in Crashworthiness of Vehicle Structures," Wyle Lab, Huntsville, Al., Rept. No. 68-3, V. 2., 1968.
- Melosh, R. J. and Kelley, D. M. "The Potential for Predicting Flexible Car Crash Response." Automotive Safety Dynamic Modeling Symposim, Anaheim, Calif., October, 1967, Sponsored by SAE.
- Miller, P. M. and Mayor, R. P. "Basic Research in Automobile Crashworthiness - Summary Report," CAL Rept. No. YB-2684-V-6, 1968.
- Perrone, N. "Crashworthiness and Biomechanics of Vehicle Impact." Presented at the Winter Annual Meeting of the ASME, New York, New York, December, 1970.
- Rapin, M. P. "Vehicle Structural Crashworthiness," 1970 International Automobile Safety Conference Compendium, Sponsored by SAE, SAE-700413, 1970.
- Renneker, D. N. "A Basic Study of Energy-Absorbin Vehicle Structure and Occupant Restraints by Mathematical Model." Automotive Safety Dynamic Modeling Symposium, Anaheim, California, October, 1967, Sponsored by SAE.
- "Research Reports of the National Highway Traffic Safety Administration, A Bibliography." NHTSA, Washington, D. C., April 1971, PB 199 680.
- Rubenstein, M. F. "Vehicle Frame Collapse Mechanism," UCLA Motor Vehicle Safety Project Rept. No. 68-52, 1968.

Viner, J. G. "Recent Developments in Roadside Crash Cushions."  
Transportation Engineering Journal, Proc. of the ASCE, Feb.  
1972, pp. 71-87.

A. 2. COMPUTER SIMULATION OF VEHICLE STRUCTURES

Ali, R., Hedges, J. L. and Mills, B. "Static Analysis of an Auto-  
mobile Chassis Frame," Proc. Instn. Mech. Engrs., 1970-71,  
pp. 665-674.

Ali, R., Hedges, J. L. and Mills, B. "Dynamic Analysis of an Auto-  
mobile Chassis Frame," Proc. Instn. Mech. Engrs., 1970-71,  
pp. 683-690.

Culkowski, P. M. and Miller, P. M. "Application of Limit Analysis  
to the Evaluation of Automobile Structural Component Damage  
During Low Speed Collisions," CAL Rept. VJ-2968-V-1, 1972.

Deleys, N. J., Segal, D. J. and Patten, J. S. "Underride/Override  
of Automobile Front Structures in Intervehicular Collisions  
v. 2. - Car to Car Head-On Impacts," CAL Rept. No. DOT  
HS 800 646, Dec. 1971.

Dong, S. B. and Selna, L. G. "Vehicle Structural Response."  
UCLA Motor Vehicle Safety Project Report No. 68-52, Oct.  
1968.

Elliott, W. A. "Plastic Models for Dynamic Structural Analysis."  
Presented at Automotive Engineering Congress, Detroit,  
Michigan, Jan. 1971, SAE 710262.

Ellis, J. R. "A Six Degree of Freedom Model of a Car." Presented  
at the Second International Conference on Vehicle Mechanics,  
Sept. 1971, Univ. of Paris.

Emori, R. I. "Analytical Approach to Automobile Collisions."  
SAE 680016, Jan. 1968.

Emori, R. I., "Mechanics of Automobile Collisions." HSRI Library  
Accession No. 22179, Univ. of Michigan, Ann Arbor.

- Haynes, J. S. "Development and Analysis of Door Side-Impact Reinforcements." Presented at National Automobile Engineering Meeting, Detroit, Michigan, May 1972, SAE 720494.
- Hedges, J. L., Norville, C. C. and Gurdogar, O. "Stress Analysis of an Automobile Chassis Frame." Proc. Instn. Mech. Engrs. 1970-71, pp. 675-682.
- Herridge, J. T. and Mitchell, R. K. "Development of a Computer Simulation Program for Collinear Car/Car and Car/Barrier Collisions." Battelle Columbus Laboratories Rept. No. DOT HS 800 645, Jan. 1972.
- Hessel, J. J. and Lammers, S. J. "Solution of Automotive Structural Problems Using the Finite Element Method and Computer Graphics." SAE 710243, Jan. 1971.
- Kamal, M. M. "Analysis and Simulation of Vehicle to Barrier Impact." 1970 International Automobile Safety Conference Compendium, Sponsored by SAE. SAE 700414.
- Kirioka, K. "An Analysis of Body Structures." SAE 650026, 1966.
- Kirioka, K., Ohkubo, Y. and Hotta, Y. "An Analysis of Body Structures - Part II." SAE 710157, Jan. 1971.
- Leffert, R. L. "Programming Time-Sharing Digital Computers for Linear Dynamic Simulation." SAE Paper 690235.
- McHenry, R. R. and DeLeys, N. J. "Development of Analytical Aids for Minimization of Single Vehicle Accidents." Cornell Aeronautical Laboratory, Inc., CAL No. VJ-2251-V-10, July 1971.
- McHenry, R. R., Segal, D. J. and DeLeys, N. J. "Computer Simulation of Single Vehicle Accidents." Proc. 11th Stapp Carr Crash Conference. 1967.
- Maeda, T. and Uemura, H. "On Development of Vehicle Dynamics by Means of a Digital Computer." SAE Paper 690233.
- "Mathematical Simulation Models for Crashworthiness Qualifications of Motor Vehicle Structures - Chaps. 1, 4, 5, 6, 7." Phase 1 Final Report, AMF Eng. Rept. No. ASL/E153-1120-1, 1968.

- "Mathematical Simulation Models for Crashworthiness Qualification of Motor Vehicle Structures - Chaps. 3-4," Phase I Final Report AMF Eng. Rept. No. ASL/E153-1120-1, 1968.
- Mayor, R. P., Theiss, C. M. and Schuring, D. J. "Basic Research in Automobile Crashworthiness - Analytical Studies." CAL Rept. No. YB-2684-V-5, Nov. 1969.
- Melosh, R. J. "Car-Barrier Impact Response of a Computer Simulated Mustang." Philco-Ford Corp. Rept. No. DOT-HS-091-1-125A, 1972.
- Miki, K., Nagai, M. and Adachi, E. "Elastic Vibration Analysis of Passenger Car Bodies." SAE Paper 690272.
- Miura, N. and Kawamura, K. "An Analysis of Deformation Mechanism in Head-On Collision." SAE 680484, May 1968.
- Petersen, W. "Application of Finite Element Method to Predict Static Response of Automotive Body Structures." SAE 710263, Jan. 1971.
- Rup, W. J., Daly, J. M., Erten, M., et al. "Vehicle Bumper Performance Study." American Machine & Foundry Company, Advanced Systems Laboratory, Santa Barbara, Calif., Rept. ASL-E-177, Dec. 1968.
- Segel, L. and Murphy, R. W. "Dynamic Modeling in Engineering." Automotive Safety Dynamic Modeling Symposium, Anaheim, California, October, 1967, Sponsored by SAE.
- Selna, L. and Salinas, D. "Dynamic Analysis of Automotive Structural Systems." SAE-700844, Oct. 1970.
- Shieh, R. C. "Basic Research in Crashworthiness II - Large Deflection Dynamic Analysis of Plane Elasto-Plastic Frame Structures Including the Cases of Collision into a Pole or Flat Barrier." Cornell Aeronautical Laboratory, Inc., Buffalo, N. Y., CAL Rept. No. YB-2987-V-7, Aug. 1972.

- Shieh, R. C. "Basic Research in Crashworthiness II - Pre-collapse Dynamic Analysis of Plane, Ideal Elasto-Plastic Frame Structures Including the Case of Collision onto a Narrow Rigid Pole Obstacle." CAL Rept. YB-2987-V-3. July, 1972.
- Tani, M. and Emori, R.I. "A Study on Automobile Crashworthiness." SAE 700175, Jan, 1970.
- Westerwick, R. A. and Brown J. B. "A Fortran IV Program to Derive the Equations of Motion of Systems." Convair Division of General Dynamics, Technical Rept. AFFDL-TR-65-194.
- Wingenbach, W. J. and Schwarz, R. "Experimental Safety Vehicle Crashworthiness Design." SAE 720070. Jan. 1972.
- Young, J. W. "CRASH: A Computer Simulator of Nonlinear Transient Response of Structures," Philco-Ford Corp. Rept. No. DOT-HS-091-1-125-B, 1972.

A. 3. GENERAL STUDIES IN STRUCTURAL ANALYSIS

- Baronas, R. P. and Chyras, A. A. "Linear Programming Method of Displacement Analysis in Elastic-Plastic Frames." Int. J. for Numerical Methods in Engineering, Vol. 3, 1971, pp. 415-423.
- Brew, J. S. and Brotton, D. M. "Non-linear Structural Analysis by Dynamic Relaxation." Int. J. for Numerical Methods in Engineering, Vol. 3, 1971, pp. 463-483.
- Chaudbury, N. K., Brotton, D. M. and Merchant, W. "A Numerical Method for Dynamic Analysis of Structural Frameworks," Int. J. Mech. Sci. 8, pp. 149-162, 1966.
- Clough, R. W. "The Finite Element Method in Structural Mechanics," Stress Analysis, John Wiley and Sons, 1965.
- Dunham, R. S., Nickell, R. E. and Stickler, D. C. "Integration Operators for Transient Structural Response," Computers and Structures 2, pp. 1-15, 1972.

- Dupuis, G. A., Pfaffinger, D. D. and Marcal, P. V. "Effective Use of the Incremental Stiffness Matrices in Nonlinear Geometric Analysis," Div. of Eng., Brown University, Tech. Rept. N00014-0007/5, Sept. 1970.
- Fenves, S. J. and Gonzalez-Caro, A. "Network-Topological Formulation of Analysis and Design of Rigid-Plastic Framed Structures." Int. J. for Numerical Methods in Engineering, Vol. 3, 1971, pp. 425-441.
- Haliburton, T. A. and Matlock, H. "A Finite-Element Analysis of Structural Frames." Center for Highway Research, The Univ. of Texas, Austin, Research Rept. No. 56-7, July 1967.
- Hibbitt, H. D., Marcal, P. V. and Rice, J. R. "A Finite Element Formulation for Problems of Large Strain and Large Displacement," Division of Eng., Brown University, Tech. Rept. No. N00014-0007/2, June 1969.
- Hodge, P. G., Jr. "Plastic Analysis of Structure," McGraw-Hill, 1959.
- Horii, K. and Kawahara, M. "A Numerical Analysis on the Dynamic Response of Structures," Proc. 19th Japan National Congress for Applied Mechanics, pp. 17-22, 1969.
- Hurty, W. C. and Rubinstein, M. F. "Dynamics of Structures," Prentice-Hall, 1964.
- Levy, N. "Finite Element Formulation of the Stress Strain Matrix for an Elastic - Perfectly-Plastic Tresca Material," Division of Eng., Brown University, Tech. Rept. NASA NGL 40-002-080/2.
- Mallett, R. H. and Marcal, P. V. "Finite Element Analysis of Nonlinear Structures," J. Struct. Div. ASCE Vol. 94, Sept. 1968, pp. 2081-2105.
- Marcal, P. V. "Finite Element Analysis of Combined Problems of Nonlinear Materials and Geometric Behavior," Div. of Eng., Brown University, Tech. N00014-0007/1, March 1969.



- Marcal, P.V. "Finite Element Analysis with Material Nonlinearities - Theory and Practice," Div. of Eng., Brown University, Tech. Rept. N00014-0007/3, Aug. 1969.
- Nickell, R.E. "A Survey of Direct Integration Method in Structural Dynamics," Div. of Eng., Brown University, Tech. Rept. N00014-0007/9, April 1972.
- Oden, J.T. "General Theory of Finite Elements," Int. J. Numerical Method in Eng., 1969, p. 205.
- Stocker, J.E. and Clymer, A.B. "Mathematical Methods of Structural Analysis and Simulation." North American Aviation, Inc., HSRI Library Accession No. 14028, Univ. of Michigan, Ann Arbor.
- Symonds, P.S. "Limit Analysis," Handbook of Engineering Mechanics, Edited by W. Flügge, McGraw-Hill, 1962.
- Wang, C.K. "Matrix Method of Structural Analysis," International Textbook Co., 1966.
- Zienkiewicz, O.C. "The Finite Element Method in Engineering Science." McGraw-Hill, 1971.

A. 4. IMPACT TESTING

- Cornacchia, F. "Behavior of a Specific Body Structure in Crash Testing." 1970 International Automobile Safety Conference Compendium. SAE 700415.
- DeLeys, N.J. "Validation of a Computer Simulation of Single Vehicle Accidents by Full-Scale Testing." First International Conference on Vehicle Mechanics, Wayne State Univ., July, 1968.
- Emori, R.I. and Link, D. "A Model Study of Automobile Collisions." SAE 690070, Jan. 1969.
- Grant, J.W. "Laboratory Control of Dynamic Vehicle Testing." Mobility Systems Laboratory, U.S. Army Tank Automotive Command, Warren, Mich., Tech. Rept. No. 11482, Dec. 1971.

- Herridge, J. T., Mitchell, R. K., Leis, R. D. and Francis, G. A.  
"Development of a Crashworthiness Test Methodology."  
Battelle Memorial Institute, Rept. No. DOT-HS-800-647,  
(Feb. 1971).
- "Impact Capability of Safety Glazing Materials." Southwest Re-  
search Institute, San Antonio, Texas, Rept. No. FH-11-7277.
- Macaulay, M. A. and Macmillan, R. H. "Impact Testing of Vehicle  
Structure." 12th Congress International des Technique de  
l'Automobile, FISITA (1968).
- Macmillan, R. H. "Vehicle Impact Testing." 1970 International  
Automobile Safety Conference Compendium, SAE 700404.
- Mayor, R. P. and Naab, K. N. "Basic Research in Automobile  
Crashworthiness-Testing and Evaluation of Modifications for  
Side Impacts." CAL Rept. No. YB-2684-V-3, Nov. 1969.
- Miller, P. M. and Mayor, R. P. "Basic Research in Automobile  
Crashworthiness-Testing and Evaluation of Engine Deflection  
Concept." CAL Rept. No. YB-2684-V-2, Nov. 1969.
- Neilson, I. D., Kemp, R. N. and Wall, J. G. "Controlled Impact  
Investigations: Head-On Impact of Four Similar Cars from  
Different Speeds Against a Rigid Barrier." Crowthorne,  
England, RRL Rept. LR-132, 1968.
- Wilson, R. A. "A Review of Vehicle Impact Testing: How It Began  
and What is Being Done." 1970 International Automobile Safety  
Conference Compendium, SAE 700403.
- Wingenbach, W. J. and Schwarz, R. "Experimental Safety Vehicle  
Crashworthiness Design." SAE 720070, Jan. 1972.