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A Regime Shift Model of the Recent Housing Bubble in the United States

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A REGIME SHIFT MODEL OF THE RECENT HOUSING BUBBLE IN THE UNITED STATES

by

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Abstract

of

A REGIME SHIFT MODEL OF THE RECENT HOUSING BUBBLE IN THE UNITED STATES

It has been widely assumed that there was a bubble in the U.S. housing market after1999. This paper analyzes the extent to which that was true. We define a bubble as: (1) a regime shift that is characterized by a change in the properties of deviations from the fundamentals of house price growth, and (2) where a shock to the fundamental equation is more self sustaining and volatile than in other periods. We model the fundamentals of price growth as a lagged adjustment of prices to the expected present value of future rent. We then study the autoregressive behavior of the residuals thus generated. We look at changes in momentum (the extent to which a shock to house price growth leads to further increases in house price growth) of the residuals. Our results from 44 Metropolitan Statistical Areas for the period of 1980-2005 (quarterly data) are mixed. There is evidence of momentum in house price growth throughout the period, and momentum did increase after 1999, indicating a regime shift; but by a modest amount, and while momentum was sometimes strong it was not explosive. The regime shift was less apparent in the likely bubble candidate cities along the coasts, which had shown high growth in the past. The evidence on volatility is strong. In general, volatility did not increase in the nonbubble MSAs, and it decreased in the faster-growing bubble MSAs.

I. Introduction

The recent property market in the United States has been widely perceived as having a bubble. Figure one shows the rate of growth of house prices across nine census regions (not labeled in the figure) from 1975 through 2005. The figure clearly shows that house prices tend to move together, although there are periods of large dispersion across regions. The proposed bubble period is post 1999. What makes this period different is the sharp acceleration in prices, especially in some regions and especially relative to inflation (not shown in the figure), during a period of relatively stable growth and little change in economic conditions. In earlier periods, the rapid house price change could plausibly be explained by changes in interest rates (for example, the early 1980s) or on regional recessions or expansions (such as the ups and downs of oil prices). These do not appear to be especially strong candidates for explanation in the late 1990s and after.

A few factors might show explanatory power on the property price movement. Figure two shows national rates of growth of house prices, the ten-year Treasuries, an index of imputed homeowner rents, and the Consumer Price Index (all of which are we use to explain housing fundamentals in the later part of the paper). In general, property prices move in step with the other series in Figure two. However, the acceleration in house prices after 1999 does not appear to be consistent with the other data. Although interest rate declines could be a factor (Long-term Treasuries dropped by 300 basis points, and real rates fell by close to the same amount), they cannot explain the regional variation in Figure One. Hence, a first glance at the data suggests that something unusual occurred in U.S. markets after 1999, and especially after 2003

This paper analyzes the post 1999 behavior of house price growth in the U.S. In particular, we look at the extent to which we can characterize the period as having a “bubble” relative to the “fundamentals” of price growth. The definition of a bubble is often vague and not widely agreed on. Our notion of a bubble is that it is: (1) a regime shift that is characterized by a change in the properties of deviations from the fundamentals of house price growth, and (2) where a shock to the fundamental equation is more self sustaining (increased momentum) and volatile than in other periods. The fundamentals of price growth come from lagged responses to the present value of expected future rent.

Various methods have been proposed for testing bubbles in financial markets. Early work relied on econometric models such as variance-bound tests. However, these methods, which compare the actual data with fundamentals, have been criticized because of the specification errors of the fundamentals. Since then, tests for stationarity and cointegration as tests for absence of speculative bubbles have been proposed (see, for example, Diba and Grossman (1988) and Hamilton and Whiteman (1985)). Evans (1991), however, shows that these methods tend to reject the presence of the bubbles too often even if they are artificially induced in the Monte Carlo simulations. The literature of testing bubbles then moved on to the introduction of the more effective regime switching models first presented by Blanchard and Watson (1982). These models look at bubbles as changes in regime, and then analyze properties of price processes in out of the bubble regimes.[1] Our model is a variant of regime shift models.

Apart from Roche (2001), who studies the Dublin market from 1976 to 1999, regime switching models have not been widely applied in real estate research in explaining house

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[1] In a recent study, Baddeley (2005) incorporates destabilizing effects from bubbles, herding, and frenzies in the study of regime shifts conditional on institutional and political changes. She argues that in a less informed market such as real estate, thence where herding can be serious, and where financing and uncertainty are crucial factors in determining the time to invest, market booms and busts tend to be more pronounced.

price bubbles. We postulate two types of regimes: the first is “pre bubble,” which we assume takes up most of our sample period (1980-1999) and which is characterized by a process for house prices that we describe as coming from the “fundamentals” of the market in a manner loosely consistent with price being determined by expected present value of rents, and the second is the “bubble candidate” period of 2000-2005. The structure of our model is similar to papers on housing bubbles by Black *et al.* (2006), Chan *et al.* (2001) and Hwang *et al*. (2006).

We first develop a model of the fundamentals of house price growth from panel data on rents (rental equivalent for owner-occupied housing), interest rates and house prices across 44 Metropolitan Statistical Areas (MSAs) in the U.S. We then use the model, and variations, as a benchmark from which to generate residuals. Assuming the residuals follow an autoregressive process, we test whether a bubble exists, and if so, its magnitude, by studying how the residual processes change after 1999. In particular, we look at the extent to which momentum in the process (measured by the sum of the coefficients of the process) increases after 1999, and whether the volatility of the error terms in the residual process increases. We do this for two sets of panel data: one for slow growth or “nonbubble” MSAs, largely cities in the center of the country, which are defined as MSAs whose house prices grew on average less than 2% per year faster than rent; and the other for a set of “bubble candidate” MSAs, largely coastal cities, whose prices grew more than 2% faster than rent (see Appendix 1).[2]

We find evidence of momentum throughout the period and some evidence that momentum increased after 1999, but not by a lot. We find no evidence of an increase in volatility. We also do not find evidence of explosive momentum (sum of coefficients

[2] The long run trend in our model is for prices to grow at about a 2% per year faster rate than rents.

greater than one) after 1999, nor do we find much difference in price growth behavior between the bubble and non bubble candidate cities. We do find that momentum operates with a long lag. There were always bubbles, but not a large regime shift, at least not in our sample period.

The paper is organized as follows. The next section provides a discussion on bubbles and regime switching models that have been widely applied in financial markets. Section III discusses how the housing price growth can be modeled. In particular, we suggest the fundamental equation from which bubbles in the market can be tested. Section IV describes the data employed, while Section V presents the results. Section VI discusses the robustness of our tests, and Section VII concludes the study.

II. Bubbles and Regime Switching

There has been considerable research on modeling the price movements of stock markets in the desire of capturing the deviations from the fundamental values. [3] Two versions of these models are the fads model proposed by Summers (1986) and the stochastic bubbles model suggested by Blanchard and Watson (1982). The latter type was subsequently extended by Van Norden and Schaller (1993, 1996), and Van Norden, (1996), who use switching regressions to describe the time-varying relationship between returns and deviations from the fundamentals.

The Fads model

Borrowing from Fama and French (1988) and Cutler, Poterba and Summers (1991), we can describe fads models as follows. The logarithm of market price of an asset is assumed

[3] Other proposed sources of bubbles are, for example, overconfidence of speculators coming from two different groups such that the deviations in price expectations create trading (Scheinkman and Xiong (2003)), and money illusion as a result of reduction in inflation, and hence nominal mortgage costs (Brunnermeier and Julliard (2006)).

to be divided into (1) a non-stationary part that describes the fundamental price and (2) a stationary component that implies the returns are predictable (from previous returns). Both components are autoregressive and subject to different white noises with their own distributions. Given a proxy of the fundamental price because of measurement error, all these imply

(1) $p_{t+1} - p_t = \beta_0 + \beta_1 \left(p_t - p^x{}_t\right) + e_t$

where $p^x{}_t$ is the available proxy of the fundamental price, and $e_t \sim iid\left(0, \sigma_\omega{}^2\right)$. This regression equation gives the excess returns as a function of differences between the log of the proxy for the fundamental and the log the observed price. In financial markets, one commonly used proxy is the dividend, and the explanatory variable in the equation is the lagged log dividend/price ratio. Hence, price growth is a function of current price and lagged fundamentals. Furthermore, because current price (via equation (1)) depends on the dividend/price ratio lagged again, iterating equation (1) implies that price appreciation depends on a long lagged function of the proxy for fundamentals.

Applying this model to house prices requires some modification. First, the assumption that the fundamentals follow a random walk and that the fads part is stationary is not likely to hold. We expect that to be the case because of obvious inefficiencies in real estate markets: (1) transaction costs in real estate are high, (2) owner-occupiers are only in the market occasionally, and (3) the tax benefits accrued to homeowners reduce their costs but not costs for speculators, thus making arbitrage difficult. As a result, there is likely to be momentum even of fundamental prices all the time. Beyond that, we want to pose expectations as about *changes* in prices rather than *levels*, and model fundamentals applied to growth rates to see if residuals from this have different properties in the post 1999 period. In other words, we do not impose the assumption that residuals from equations like (1) are independently and identically distributed (*iid*).

The Regime Switching Model

When the regression error term, e_t, is heteroscedastic, the fads model can lead to regime-switching for stochastic bubbles (which are stochastic because they either survive or collapse, subject to some probabilities). The existence of two possible outcomes of the bubbles means that there are two regimes generating market returns, the bubble being the more volatile of the two. Tests are conducted on whether the two volatilities are significantly different.

We extend the regime switching model by relaxing the assumption that the error term in the autoregressive fundamental price process is white noise. We can then arrive at an equation similar to equation (1), with longer lags, and we assume (and test) that the e_t follows an autoregressive process of the form

(2) $e_t = \sum_t^T \omega_{t-j} e_{t-j} + \upsilon_t$.

A regime shift to a bubble regime is characterized by an increase in the volatility of υ_t and in the size of the coefficients of the process for e_t, which is measured by $\sum_t^T \omega_{t-j}$.

III. Modeling House Price Growth

In this section, we develop a model similar to that in equation (1) for the housing market, and adding different stochastic properties. The basis of the model is the intertemporal behavior of households that choose between housing, h_t, whose purchase price is P_t and a representative consumer good, c_t whose price is P_t^c.

The Basic Model

Consider a household that, given information set, Ω_t, about the uncertain future house prices and interest rates maximizes an intertemporal utility function of the form

(3) $E(\Sigma_t^T U(c_t, h_t)\beta^t$

over some time horizon *T*, subject to the constraint that the present value of expenditure (cash flows) equal the present value of income plus initial wealth.

It is straightforward to show that (see Dougherty and Van Order (1982) for a derivation of a nonstochastic version) a first order condition can be expressed as

(4) $$\frac{U_{ht}}{U_{ct}} = \left(r_t - \frac{E(P_{t+1}|\Omega_t) - P_t}{P_t} \right) \frac{P_t}{P_t^c}.$$

where *r* is approximately given by $(1-\theta)i - \pi + \alpha$, with θ being the tax rate, i the risk-free interest rate, π rental growth rate, and α is a constant term. We can think of *r* as the "cap rate" for our representative property. It captures other costs like depreciation and property taxes which might be assumed proportional to property value and, if we allow risk, a risk premium. A broader specification would take account of possible cash flow effects. That is, high nominal rates can have a cash-flow effect beyond the real rate effect in (4) because of limitations on the ability of borrowers against future income, especially during periods of inflation. In that case the implied coefficient on i would be greater than 1-θ.

Equation (4) says that the marginal rate of substitution between housing and the other good equals the ratio of the implicit rent on housing, $\left(r_t - \frac{E(P_{t+1}|\Omega_t) - P_t}{P_t} \right) P_t$, divided by the price of the other good. $(U_{ht}/U_{ct})P_t^c$ can be defined as the household's imputed rent. Equation (4) along with the other constraints and parameters can be used to generate the demand for housing. This can then be attached to a model of housing production to generate a model of house prices. Building in allowances for transactions and moving costs in *r* would imply adjustment to the marginal condition in equation (4) with a lag, and the model would be very complicated and probably sensitive to particular specifications..

An alternative to complicated model building is to take advantage of the fact that housing is rented as well as owned, and that we can assume that the owner acts as a landlord renting to him or her self. This implies a useful separation for modeling. In effect, rents summarize all of the local market conditions that are determined by income and wealth and supply elasticities, and we can take rent as given and express price as the present value of rent. This allows us to employ models like the price-dividend models used to analyze stock prices (the same rationale, and hence modeling approach, is also adopted by Brunnermeier and Julliard (2006)).

Consider a household that is identical to the one that we have analyzed except that it rents at price R_t rather than owns. Because prices in this period are known, its first order condition corresponding to equation (4) is nonstochastic and is given by

(5) $U_{ht} / U_{ct} = R_t / P_t^c$

For a household that is just indifferent between owning and renting[4] we can equate expressions (4) and (5) to obtain a solution that

(6) $P_t = \dfrac{R_t + E\left(P_{t+1} \middle| \Omega_t\right)}{D_t}$

where $D_t = 1 + r_t$. The variable r_t incorporates a premium for risk for investing in real estate. Equation (6) says that price equals the current rent plus the sales price in the next period. This is a rational expectations (perfect foresight) model, and like most such models it is indeterminate. That is, there are many current levels of price that are consistent with equation (6). For example, consider the simplest case where rents and r_t are constant and is equal to R and r. Then a solution to equation (6) is

(7) $P_t = \dfrac{R}{r}$.

However, an infinity of initial levels of P_t can be chosen for which (solving equation (6) for P_{t+1}) the pricing equation

(8) $P_{t+1} = P_t D - R$

[4] Households that are approximately indifferent between owning and renting are likely to be in the lower tax brackets. Moreover, Cauley and Pavlov (2002) mention that models using rental costs provide a lower bound of the price because "pride of ownership" has not been priced.

still holds. While equation (7) reflects the Gordon model and provides a stable equilibrium price, equation (8) leads to explosive price moves because *D* is greater than one. In other words, equation (7) corresponds to what we think of as a fundamental solution, while equation (8) corresponds to an explosive bubble.

More generally, equation (8) can be solved recursively to obtain

(9) $P_t = \sum_{i=0}^{\infty} E(R_{t+i} / D^i_{t+i} | \Omega_t) + \lim E(P_{t+1+i} / D^i_{t+1+i} | \Omega_t)$.

The transversality condition is that the second term approaches zero, so that the fundamental equation becomes

(10) $P_t = \sum_{i=0}^{\infty} E\left(R_{t+i} / D^i_{t+i} | \Omega_t\right)$.

However, as before, this is not the only solution. A bubble process that satisfies

(11) $B_{t+1} = B_t D_t + e_t$

will also satisfy equation (6), and it will tend to be explosive.

Special Cases

Equation (15) is quite complicated because of covariances, such as those coming from stock-flow adjustments of rents and prices over time, among the variables in it. For instance, we should expect interest rates and future rents to be correlated on the grounds that a rise in interest rates will, given rents, lower property values, but on the other hand induce less production in the future, and thus higher rents. Indeed, if supply is perfectly elastic in the long run, a rise in interest rate will produce a gradual decline in rents with no long run price change.

We consider first a very simple model with constant interest rates and a steady growth rate of expected rents. Then we can adapt the Gordon model

(12) $P_t = R_t / (\phi i - \pi^* + \alpha)$.

That is, r is extended to $(\phi i - \pi^* + \alpha)$, where (ϕ) is a coefficient that incorporates both the tax and cash flow effects on the effect of i, the interest rate, and π^* is the expected rates of growth of rent. The model does not allow us to predict whether changes in i or π^* will have a larger effect on price. Notice that the model will always work better by including more exogenous variables such as level of supply or average personal income. However, if our model is correct, rent should be a summary statistic that has already accounted for supply and demand.

Taking first differences and logarithms of expression (12), we have

(13) $GP_t = \pi_t - \Delta \ln(\gamma i - \pi^*)$

where GP_t is the growth rate of house prices and, π_t is the current rate growth of rents. Equation (13) can be approximated by

(14) $\rho_t = GP_t - \pi_t = \alpha - \beta_i \Delta i + \beta_\pi \Delta \pi^*$

where ρ_t is the rate of growth of house prices minus the rate of growth of rent and the βs are positive. This can for instance be estimated by assuming that $\Delta \pi^*$ is a function of past levels of $\Delta \pi$. However, preliminary estimates of (14) do not work well; longer lags are necessary for the model to fit well and/or make sense. So we extend the lag structure.

Adjustment to Equilibrium

Equation (14) only holds in the simple Gordon Model, which involves two key assumptions: (1) the current price is the equilibrium price, and (2) rent growth is expected to be constant.

The high transaction costs in housing markets make the first assumption difficult to take seriously. Moreover, ownership of single family housing in the U.S. is driven in many ways by tax advantages[5] that are received by property owners only on their first or second house, which precludes serious arbitrage. Home buyers tend to enter the market and obtain information about property only at times when they are seriously interested in buying. Hence, the information needed for equation (9) to hold is dispersed only gradually among different households. For these reasons, we expect prices to adjust with a lag to the equilibrium price. Furthermore, the second assumption about steady price growth is not likely to hold in the short run. Expected future rent growth is probably not constant and is probably correlated with interest rates and past growth in rents and prices. Theory does not tell us much about how to model these.

To solve the problems discussed above, we impose the following structures on the fundamentals model. First, we model the formulation of expectations as composed of two parts: a short run part that reflects current information for the next few years, and a second longer run, "stabilized",[6] part that looks like the Gordon Model, and to which expectations adjust after a period of time. That is, we assume that after some period the best that traders can do is to project steady growth. Before that, we allow rents to vary from the trend. Second, we assume that the present value formulation provides the equilibrium price, but that the market price only adjusts gradually to it by following a generalized geometrically distributed lag. The combination of these two structures implies that prices or the growth in prices adjust gradually to a long run Gordon model.

Determining the Equilibrium Price

[5] In particular, homeowners get to deduct much of the cost (e.g., mortgage interest and opportunity cost on equity) of operating the house (renting to themselves) and pay virtually no capital gains taxes without paying tax on the imputed rent (see Gyourko and Sinai, 2003, for the study of the impact of tax subsidies on home owners in various MSAs).

[6] This corresponds to the notion of a stabilized Cap Rate.

First we write the equilibrium price as the present value of an irregular rent stream for τ periods, followed by steady stream, or

$$(15) \qquad P_t^e = \sum_{j=t}^{t+\tau} \frac{R_j}{(1+\gamma i+\alpha)^j} + \frac{R_t / r}{(1+\gamma i+\alpha-\pi^*)^{t+\tau}} .$$

Alternatively, adding and subtracting $G_t = \sum_{j=t}^{t+\tau} \frac{R_t}{\left(1+\gamma i+\alpha-\pi^*\right)^j}$, and dividing by R_t, we have

$$(16) \quad \frac{P_t^e}{R_t} = \sum_{j=t}^{t+\tau} \frac{\delta R_t}{R_t(1+\gamma i+\alpha)^j} + \frac{1}{r} = G_t + \frac{1}{r} .$$

We assume that actual price adjusts to the difference between current and equilibrium price, in logarithms. Let $p_t = \log(P_t / R_t)$ and $p_t^* = \log(P_t^* / R_t)$. We extend the adjustment model in equation (1) to include possibly longer lags, that is,

$$(17) \quad p_{t+1} - p_t = \sum_{j=0}^{T} \lambda_j (p_{t-j}^e - p_{t-j}) .$$

Taking first differences and rearranging, we have

$$(18) \quad \rho_t = p_{t+1} - p_t = \sum_{j=0}^{T} \beta_j (p_{t-j} - p_{t-j-1}) + \sum_{j=0}^{T} \theta_j (G_{t-j} - G_{t-j-1} + 1/r_{t-j} - 1/r_{t-j-1}) .$$

A linear approximation to this can be written as

$$(19) \quad \rho_t = \alpha - \sum_{j=0}^{T} (\beta_{t-j}^i \Delta i_{t-j} + \beta_{t-j}^{\pi^*} \Delta \pi_{t-j}^* + \beta_{t-j}^G \Delta G + \beta_{t-j}^\rho \rho_{t-j})$$

Both $\Delta\pi^*$ and ΔG are expectations variables. We assume that they are taken from the current information set, which contains recent and past levels of interest rates, rents and prices. We can then rewrite equation (19) as

$$(20) \quad \rho_t = \alpha - \sum_{j=0}^{T} (\gamma_{t-j}^i \Delta i_{t-j} + \gamma_{t-j}^\pi \Delta \pi_{t-j} + \gamma_{t-j}^\rho \rho_{t-j}),$$

which says that the current rate of growth of house prices relative to rents is a linear function of past change in interest rates, rent growth, and lagged changes in price growth net of rent growth.

A less structured version of this is

(21) $\rho_t = \alpha - \sum_{0}^{T+T'} (\gamma^{i}_{t-j} \Delta i_{t-j} + \gamma^{\pi}_{t-j} \ \Delta\pi_{t-j})$

in which lagged ρ_t is dropped, while the lag is lengthened by *T*'.

Both versions impose the constraint that in the long run an increase in rent of 1% will increase house price growth by 1%. Hence, after *T* or *T*' periods the model reverts to the Gordon model if α is zero. The presence of α allows rents and prices to have different trends. Reasons why this might be the case, primarily measurement error, are discussed below.

Estimates of equations (20) and (21) will generate residuals, e_t. And instead of imposing e_t as *iid*, we assume that it follows the autoregressive process

(22) $e_t = \sum_{j=t}^{T} \omega_{t-j} e_{t-j} + \upsilon_t$

where υ_t is *iid.* The process for e_t is a variant of the *B* process in expression (11). For a rational bubble, the sum of coefficients must be greater than one. Our tests are (1) of the amount of, and changes in, momentum as measured by whether $\sum_t^T \omega_{t-j}$ is greater than one and/or increased during the post 1999 period, and (2) whether the variance of υ_t was higher during the post 1999 period.

IV. Data and Estimation

Our measure of house price is the quarterly house price index released by the Office of Federal Housing Enterprise Oversight (OFHEO), which provides the widely quoted residential (single-family) house price index for over 100 individual Metropolitan

Statistical Areas (MSAs) since 1980. The rent series is the "owner's equivalent rent of primary residence" obtained from the Bureau of Labor Statistics, from which we also acquire the local Consumer Price Indices. After matching these three series, data for a total of 44 MSAs can be used.[7] We use the 10-year Treasury as a measure of nominal risk-free rate.[8]

Three data concerns are in order. First, the price index may not hold quality constant. The OFHEO index looks at the same house twice but does not adjust for home improvement between observations, so it may over estimate growth in house prices. Second, measured rent may grow too slowly because of the agency cost of renting and measurement errors in the rental index. That is, even if we have matched prices and rents for owners indifferent between owning and renting, there is reason to believe that renters take less good care of property than do owners. Crone *et al* (2006) and Gordon and van Goethem, (2004) both discuss the extent to which the CPI rental index has underestimated rent growth over time (especially before 1985). If any of the above is the case, then there will be a tendency for our measure of P to grow faster than our measure of R (that is, for α to be positive), which is indeed what we find. Third, the price and rent series do not necessarily match up in the sense of the price series representing price growth for a household that is indifferent between owning and renting, probably a household in a relatively low tax bracket. We note here that the OFHEO index only covers prices of houses whose mortgages can be purchases by Fannie Mae or Freddie Mac. This imposes a limit, which is indexed to house prices over time and excludes approximately the top 10% of the market (by number of loans). Hence, the price data do at least exclude those

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[7] In order to maximize the length of the time-series, we eliminate those MSAs that have short rent indices.

[8] We have tried to proxy the real interest rate by the ten year Treasury Inflation-Protected Securities (TIPS). However, since the earliest available TIPs begins listing in 1997, we are not able to obtain a reliable real interest rate series.

owners who, say, for tax reasons, are the furthest from being indifferent between owning and renting.

With these data, we first estimate variants of the fundamentals of price growth from the specifications of ρ in equations (20) or (21). We estimate fundamentals over the entire period. We vary these models by changing lag length. From these are generated residuals, which we model as given by equation (22) for various lag lengths. For each fundamental equation, we estimate four residual equations for a given lag length. These groups of regressions come from dividing the MSA sample into two groups: fast growing (bubble candidate) MSAs (those that are widely perceived as overheated markets, and mostly whose house prices over the period grew on average at a 2% per year faster rate than rents grew), and the rest as non bubble states. These bubble states are depicted with asterisks in Appendix 1. This grouping is meant to capture the possibility that the bubble candidates are more susceptible to bubbles. We also divide the sample into (1) the 1999 and earlier, pre bubble period, and (2) the post 1999 regime shift candidate period.

Our tests are of the extent to which there was a regime shift. If there was a regime shift, we should expect the sum of the coefficients in estimates of equation (22) to be larger in the post 1999 period, perhaps larger in the bubble MSAs, and the variance of the residuals in the error regression to be higher post 1999, and perhaps higher in the bubble MSAs.

V. Results

Estimates of the Fundamentals

Tables 1 and 2 summarize the estimates of our two fundamental equations (20) and (21) respectively using the entire panel of data across MSAs for the entire sample period. The variables in general have the right signs. The signs within groups are also consistent, generally negative for interest rate and positive for past rent growth and lagged ρ.

Consider the 8-lag model in Table 1 (the third column). Long run effects are given by the sum of coefficients for the three variables. The sum of the interest rate coefficients is around -2.5, that of the rent growth is 2.3, while the sum of lagged ρ coefficients is 0.6 (see Table 3). That the sum of coefficients of interest rate changes is close to, but slightly bigger than, the sum of the rent growth change coefficients is consistent with the notion that price is driven by real interest rates with the tax effect being more than offset by the cash flow effect. However, that result does hold for our other specification without lagged ρ.[9] All versions of our models have constant terms of around 0.0025, reflecting a quarterly difference in growth rates of about 0.25%, or 1% per year. In the models without lagged ρ the constant term was around .5. The results suggest that in the long run prices grow at close to a 2% faster annual rate than rents.

The long run effect of a change has to take into account feedback through the gradual adjustment of ρ. Long run equilibrium in growth rates takes place when past and current levels of ρ are the same. The cumulative effect of a one-time shock to i on ρ after T periods is given (rounded) by

$$(23) \qquad \rho = \sum_{0}^{T} (\gamma_{t-j}^{i})\Delta i + \sum_{0}^{T} (\gamma_{t-j}^{\rho})\rho) = \text{-}2.5\Delta i + 0.6\rho .$$

The data used in the model are all at quarterly rates, including the 10-year Treasuries. Hence a 100 basis points increase in 10-year Treasury rates is a 25 basis points increase in the quarterly rate. As a result, the long run effect of the 10-year Treasuries must be divided by four. Thus, a 100 basis points increase in interest rates will have a long run

[9] For instance we might expect the sum of coefficients of lagged rent growth changes to be low because as proxies for future expectations they are subject to measurement error.

impact of -0.25 × 2.5 / (1 - 0.6), or about -1.6% on price relative to rent. The long run impact of a change in rates in the model is like duration. One might expect a somewhat bigger number for a long term asset. However, as was discussed above, it is likely that when interest rates change, expectations about future rents also change; in the usual stock flow model of housing adjustment, a decrease in interest rates will cause construction to increase, which will decrease future rents, lowering the numerator in the present value formula.

The model with 4 lags does not fit well or make much sense. With 12 lags, the results are similar, although the fit is somewhat better. The sum of interest rate coefficients is about -1.7, the sum of rent growth 1.6, and the long run effect of an interest rate change is -1.3 (see Table 3). In the 16 lag case, the model rejects the fixed effects. Nevertheless, we still obtain similar results.

This appears to be a respectable model of fundamentals in the sense of having sensible coefficients. It also suggests that, because of the significantly positive coefficients for lagged ρ, there is momentum in house price growth over the entire period. A onetime shock to ρ feeds back into the model gradually and fades gradually. The strength of the momentum will also depend on the autoregressive properties of the errors in equation (22), which are analyzed below. The model implies a significant lag in the effect of an interest rate change on house prices.

Table 2 has estimates of the fundamental equation without lagged ρ for 8, 12, and 16 quarter lags. The lag lengths are longer than those in Table 1 because, by not including the lagged ρ, the effects captured by other explanatory variables should tend to be longer. It is obvious that not including the lagged dependent variable significantly reduces the explanatory power of the fundamental equation. Stated differently, the "memory effect" previously shown by the lagged dependent variables to capture the momentum has to be shifted to other exogenous variables, thus requiring even longer history from these variables. Results are however in some ways similar. The sum of coefficients of interest rate change is -6.2, which gives about the same long run effect of interest rates on price. On the other hand, the effect of rent inflation is much less at 3.5. Shorter lag specifications produce worse fits, and the coefficients make less sense.

Table 3 presents a summary of the results of the coefficients for the various specifications of the fundamentals, as well the effects of a one time increase in the 10 Treasury rate, and adjusted R-squared. An obvious result is that longer lag specifications fit better, and their coefficients make more economic sense.[10]

Error Equations

We use the fundamental equation(s) to generate errors equations. In particular, we employ the 8-lag and 12-lag fundamental regression equations as depicted in Table 1 to generate residuals, which are then used to estimate variants of the autoregressive model as in equation (22). As described above, we divide the available data into bubble and nonbubble MSAs, and we produce separate estimates of the error model by these MSA divisions in the pre- and post-1999 period. The results using residuals from the fundamental equation with 8 lags and 12 lags are depicted respectively in Appendices 2 and 3. Results for the same error equations for residuals from the 8-lag fundamental equation without lagged ρ (that is, regression results from Table 2) are shown in Appendix 4, while Appendix 5 depicts the corresponding findings for the 12-lag model.

[10] We initially tried to establish the fundamental model with local CPI to capture the MSA specific inflation, and in a way, deduce the real interest rate. However, adding the variable does not increase the explanatory power and intuition of the model significantly. We therefore maintain the current model for parsimony.

Our concern is with the sum of the coefficients and the volatility of the disturbance. Table 4 presents summary results for the sum of the coefficients by model and lag structure. Consider Panel B, which presents results for the model without lagged dependent variable, ρ. In all of the specifications, the sum of coefficients is positive before 1999; and in all cases the sum increased after 1999. On average, the increase in sums was around 0.2 or 0.3. While the bubble MSAs had higher sums, the increase in sum was, if anything, lower in the bubble MSAs. Running across the table, it is easy to see that while the sums of the coefficients for the non-bubble MSAs show mixed patterns in both types of fundamental equations, those for the bubble MSAs almost ubiquitously (except for 12-lagged error terms in the 12-lagged fundamental) decrease.

A closer look at Appendices 2 to 5 reveals that the lagged errors of the bubble MSAs are significant mostly for only the first few lags; and this is especially true when more lags are included. The MSAs had faster, more front-loaded, adjustments in the post-bubble period. Something did happen post 1999. There was a regime shift, but not a large one. In fact, the regime shift tends to be smaller when longer lags are considered in the case of bubble MSAs. Furthermore, in none of the cases was the sum of coefficients close to one; tests on this were rejected in all cases. Momentum increased, but it was not explosive.

Panel A produces results for cases with lagged ρ. It is more complicated because the presence of lagged ρ adds momentum to the system along with momentum added by the errors. In the pre-1999 period, the coefficient sums were generally negative, thus offsetting some of the positive momentum from the positive coefficients of lagged ρ. The results for before and after 1999 were similar; the sums tended to increase by around 0.3, though with more variability.

Summing up the basic results for the coefficients sum, we are able to observe momentum throughout the period, and the adjustment lags were long. There is also some evidence of a regime shift in the post-1999 period. However, unlike financial markets, there is no evidence of an explosive bubble associated with the regime shift. Finally, there is no evidence that the "bubble" MSAs were worse. Indeed, our model is able to explain the price movements in the bubble MSAs relatively better than the non-bubble ones.

Volatility

Table 5 presents results for testing the changes in the volatility of the errors in equation (22). Panel A corresponds to results generated from the fundamental with lagged regressand, ρ in Appendices 2 and 3; while Panel B presents results without lagged ρ from Appendices 4 and 5. We apply the Goldfield-Quandt test for the differences in variances. The results of the tests can be read from the "Pre/Post-1999 Test" rows. Bold face numbers show cases where the hypothesis that the variances are different is accepted. In the nonbubble MSAs the hypothesis is always rejected. In the bubble MSAs the hypothesis is almost always accepted. However, in all those case the variance fell after 1999. Hence, once again, there is some evidence of a regime shift in the bubble MSAs. However, the shift is toward a more stable regime after 1999.[11]

Reviewing the actual market movement may prompt a query on the choice of the cutoff period. That is, it is possible that the bubble became bigger in the later part of the post-bubble period. We therefore run error equations from the fundamentals in Appendix 2 with 8 lags (results omitted here). In both periods of 2002 through 2005 and 2003

[11] We have also tested the variances of the 44 individual MSAs. There is only an average of one or two MSAs that have statistically significant change in variance between to pre- and post-bubble periods in any of the cases. We therefore omit the results here for purpose of simplicity.

through 2005, the coefficient sums are either the same as those in the post-1999 period as a whole or, in the case of the bubble MSAs, lower. Table 6 depicts the variances in the various sub-periods. The rows noted as "GQ Test: 99 vs 02" and "GQ Test: 99 vs 03" exhibit tests for increases in variance. Variances did go up in both the post-1999 versus 2002-2005 and post-1999 versus the 2003-2005 periods, but differences are not statistically significant. We can thus conclude that our results are not sensitive to the cutoff point at 1999.

Fundamentals without Lagged Regressands

As mentioned earlier, the fundamental equation without lagged dependent variable, ρ, requires longer lags because the effects captured by other explanatory variables should tend to be longer. We run the tests again with 20 lags and 24 lags on the explanatory variables. We then adopt the 12-lag error equation model. Results are shown in Appendix 6. Panel A of the Table presents the panel regression results, while Panels B and C depict the regression results of the 12-lag error equation and the test of difference in variances between the pre and post bubble periods respectively.

As expected, the explanatory power increases with increase in the number of lags included, albeit marginal. The error equation results also show that the MSAs price versus rent growth rates adjust relatively faster in the post-bubble period. As more lags are included in the fundamental, the errors tend to carry more momentum (sum of coefficients is bigger). Nevertheless, they are still non-explosive. Finally, only the bubble MSA group shows a change in the volatility from the pre versus post bubble period in the 24-lag fundamental equation case; and is only barely statistically different. Once again, volatility in general decreases in the post-bubble period.

VI. Robustness of the Fundamentals

A major complication in our study is that the results of the error equations might be sensitive to the fundamental equations employed. We therefore estimate some variations of the fundamentals to see if the error equations still lead to findings that are similar to the ones we obtained in the previous section.

We first separate the data set for the fundamentals those for bubble MSAs and non-bubble MSAs and estimate separate panel regressions (regression results depicted in Appendix 7). The rationale is that, assuming bubble and non-bubble markets are separate groups, intra-group markets might share identical effects from the factors in the fundamental equation, but not inter-group markets. As expected, the error equations (with 8, 12, and 16 lags) shown in Appendix 8 are different between the two groups of MSAs, as well as different from the previous test without the separation, because the panel regressions should be able to better capture the common characteristics of the two MSA groups. The sensitivity to a change in the regression does not however alter our previous conclusion. First, the sums of the coefficients of the lagged errors are far less than one. Second, the volatilities in the pre- and post-bubble period are very similar (see Table 7). Even if the hypothesis that variances in the two periods are statistically the same is occasionally rejected, the difference is minimal. This is similar to the findings in the previous section.

Our second variation is to include the inflation rate into the fundamental equation. This allows the discount rate to be thought of composed of a real rate plus real rent growth, and these might not have the same coefficients (e.g., because of different measurement errors). Furthermore, local inflation may contain information about rent, or its determinants that is not found in the rental equivalent index (e.g., the rent numbers might be too smooth or grow too slowly relative to the true numbers. We therefore obtained changes in inflation rates for each individual MSA from the local CPI series available from the Bureau of Labor Statistics, and included the series in the fundamental equation, again with 8 lags, from which we obtain the variance tests from the error equations.[12] The error equation results are tabulated in Appendix 9, while the comparison of variances in the pre- and post-1999 period is exhibited Table 8. It is clear from the tables that, again, albeit the high sensitivity of the results to the fundamental equation, the basic result is still that there are only small traces of bubbles/regime shifts in the property market in the U.S. in the period of study.

Another robustness check is to test if the behavior of the models differs when the pre-bubble period is separated from the post-bubble period in estimating the fundamental equation. However, the very short post-bubble period data does not have enough degrees of freedom for testing on the two periods separately.

VII. Conclusions

Perhaps the best way to characterize housing markets during our sample period is that there were always small bubbles, but not a large regime shift. There does appear to have been a small regime shift after 1999, but it was weaker in the likely bubble candidate cities along the coasts, cities which had shown high growth throughout the period. There is evidence of momentum in house price growth throughout the period, and the momentum did increase after 1999, but not by a lot. These results appear to hold if we consider post-2002 as the bubble period. The evidence for volatility is strong. In general, volatility not only did not increase in the nonbubble MSAs, but actually decreased in the faster-growing bubble MSAs. Hence, evidence for a bubble across regions is modest, and

[12] We do not present the panel regression results with local inflation because our focus is on the behavior of the residuals from the error equations thus generated.

somewhat mitigated by the long lags suggested by the model. Results are not very sensitive to the variations in lag length and lag structure that we tried.

We have not tested for local results, so we cannot exclude strong local bubbles. We have also not ruled out that income (see Black *et al*.) might be a better proxy for rent than our current rent series extracted from the CPI. Asian immigrants mostly to coastal cities are another possible explanation for overheating the real estate market, but are unlikely to be a source of bubbles. It is for future extension and more complicated modeling to test effects of such immigrants, or other demographic patterns, as an explanation for why growth rates in bubble cities tend to be more sustainable.

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Figure one
Annual Growth Rates of House Prices by 9 Census Regions

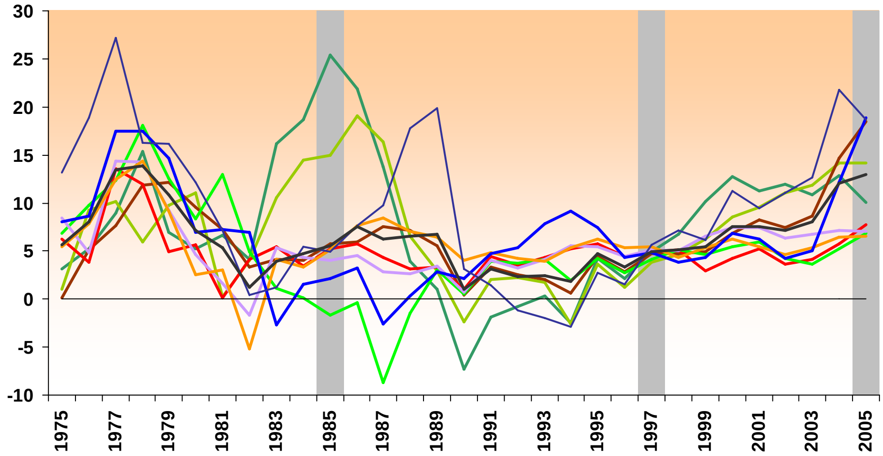

Figure Courtesy of Freddie Mac

Figure Two Growth Rates of House Price Index, Ten-year Treasury Bonds, Consumer Price Index, and Rent Index, at the National Level

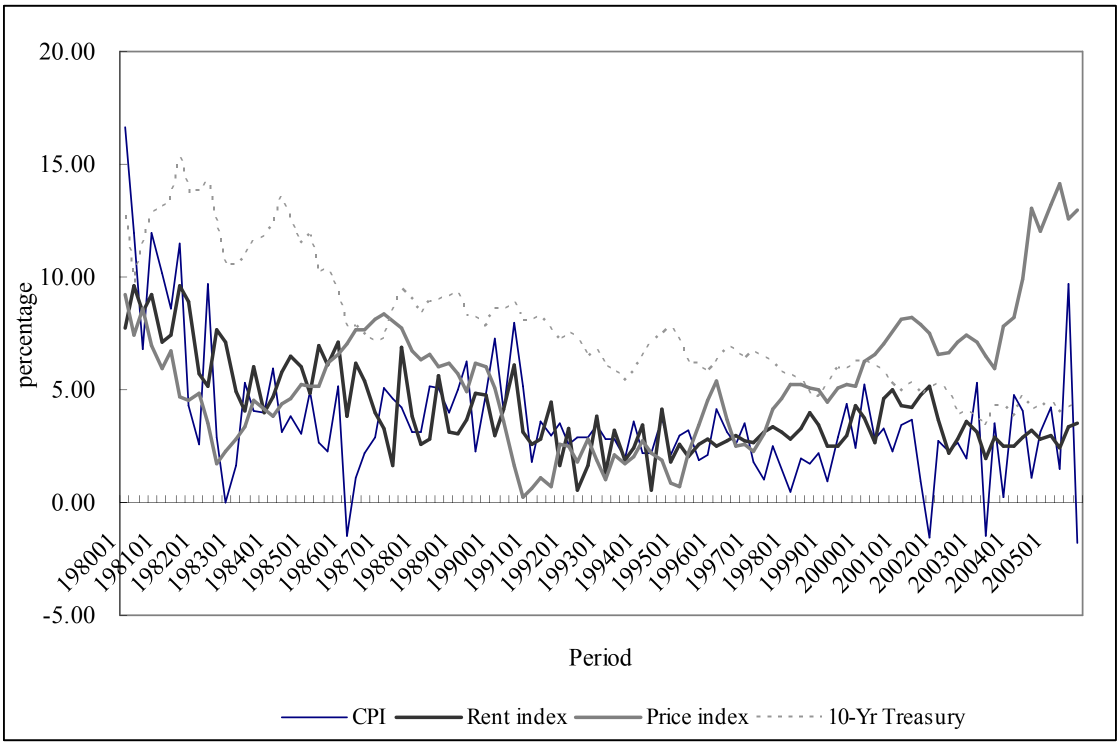

Table 1 Basic Regression Results for the Fundamental Equation with Lagged Regressands (Various Lags)

Equation being Difference between Growth Rates of House Prices versus Rent (Regressand) on Lagged Changes in 10-year Treasury, Local Rent Growth, and Regressands (MSA Fixed Effects Omitted)

Variables	4 Lags	8 Lags	12 Lags
Nominal Interest			
Lag 1	0.2436	-0.1920	-0.0360
2	-0.5360 **	-0.5320 **	-0.3400
3	-0.1040	-0.3720 *	-0.1120
4	-0.2120	-0.0160	0.1752
5		-0.0004	-0.0360
6		-0.6960 ***	-0.4360 **
7		0.1600	-0.0600
8		-0.8240 ***	-0.7960 ***
9			-0.0360
10			-0.0600
11			0.0908
12			-0.0360
Rent Growth			
Lag 1	-0.0323	0.1352 ***	0.2117 ***
2	-0.1234 **	0.1661 ***	0.2849 ***
3	-0.0265	0.2388 ***	0.3677 ***
4	0.0935 ***	0.4309 ***	0.3734 ***
5		0.3892 ***	0.3161 ***
6		0.4337 ***	0.2982 ***
7		0.4076 ***	0.2403 ***
8		0.0705 **	-0.1196 **
9			-0.1500 ***
10			-0.1495 ***
11			-0.0921 **
12			0.0376
Regressand			
Lag 1	-0.1255 ***	0.0257 *	0.1537 ***
2	0.1081 ***	0.1319 ***	0.1653 ***
3	0.0884 ***	0.0756 ***	0.1476 ***
4	0.2012 ***	0.2248 ***	0.0964 ***
5		0.0394 ***	0.0019
6		0.0331 **	-0.0026
7		0.0382 ***	0.0165
8		0.04576 ***	0.0550 ***
9			0.0343 ***
10			0.0285 **
11			-0.0144
12			0.01910 ***
Adjusted *R*-square	0.083678	0.203536	0.230679

*"***", "**", "*" represent significance at 1%, 5% and 10% respectively.*

Table 2 Basic Regression Results for the Fundamental Equation Without Lagged Regressands (Various Lags)

Equation being Difference between Growth Rates of House Prices versus Rent (Regressand) on Lagged Changes in 10-year Treasury and Local Rent Growth (MSA Fixed Effects Omitted)

Variables	8 Lags	12 Lags	16 Lags
Nominal Interest			
Lag 1	0.1160	0.3084	0.6500 ***
2	-0.0280	0.1308	0.1152
3	-0.1680	0.1220	-0.0240
4	-0.1560	0.4172 *	-0.0160
5	0.0460	0.3088	0.3340
6	-0.6760 ***	0.0080	-0.2880
7	0.2828	0.2140	-0.1800
8	-0.9160 ***	-0.4960 **	-1.2040 ***
9		-0.0440	-0.8600 ***
10		-0.1840	-0.5680 ***
11		0.1128	-0.0680
12		-0.1480	-0.3280
13			-0.8880 ***
14			-1.6320 ***
15			-0.3880 **
16			-0.8040 ***
Rent Growth			
Lag 1	0.2185 ***	0.1618 ***	0.1778 ***
2	0.2394 ***	0.1869 ***	0.2254 ***
3	0.3369 ***	0.2290 ***	0.2588 ***
4	0.3846 ***	0.2449 ***	0.1969 ***
5	0.3836 ***	0.2773 ***	0.1888 ***
6	0.4420 ***	0.3448 ***	0.2311 ***
7	0.4378 ***	0.3439 ***	0.2623 ***
8	0.0846 ***	-0.0071	0.0860
9		-0.0681	0.0678
10		-0.1065 **	0.0731
11		-0.0543	0.1531 ***
12		0.0351	0.3122 ***
13			0.3224 ***
14			0.3574 ***
15			0.3367 ***
16			0.1046 ***
Adjusted *R*-square	0.07351	0.079149	0.101218

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Table 3 Comparison of Sum of Coefficients From Fundamental Equations, Equations (40) and (41).

Coefficients are from Tables 1 and 2.

	With Regressand on RHS			Without Regressand on RHS		
Variables	**Lag = 4**	**Lag = 8**	**Lag = 12**	**Lag = 8**	**Lag = 12**	**Lag = 16**
Change in Interest Rate	-0.6084	-2.4724	-1.6820	-1.4992	0.7500	-6.1488
Change in Rent Growth	-0.0886	2.2720	1.6186	2.5272	1.5876	3.3543
Change in lagged LHS	0.2723	0.6145	0.7016	N/A	N/A	N/A
LR effect of Change in Interest rate 100bp	-0.209	-1.606	-1.409	-0.373	0.1975	-1.5372
Adjusted *R*-squared	0.0837	0.2035	0.2307	0.0735	0.0791	0.1012

TTable 4 Comparison of Sums of Coefficients of the Error Equation from the Fundamental Equation

Panel A: Fundamental Equation with Lagged Regressands						
	Fundamental Lag = 8			**Fundamental Lag = 12**		
Category	**Lag = 8**	**Lag = 12**	**Lag = 16**	**Lag = 8**	**Lag = 12**	**Lag = 16**
Non-Bubble MSA Pre-1999	-0.3912 (884.72)	-0.3751 (902.73)	-0.2762 (744.39)	-0.4537 (638.92)	-0.4346 (884.99)	-0.5203 (698.89)
Non-Bubble MSA Post-1999	0.1813 (283.16)	-0.2747 (101.62)	-0.0371 (97.52)	-0.1182 (283.75)	-0.2932 (200.89)	-0.0422 (64.00)
Differences	**0.5725**	**0.1005**	**0.2391**	**0.3355**	**0.1414**	**0.4781**
Bubble MSA Pre-1999	0.1140 (348.46)	0.0593 (356.49)	0.0677 (298.91)	0.0054 (345.31)	0.0313 (379.78)	-0.0221 (431.35)
Bubble MSA Post-1999	0.4775 (98.58)	0.1356 (65.21)	0.0550 (64.97)	0.3376 (140.37)	0.0411 (87.17)	-0.1154 (85.11)
Difference	**0.3661**	**0.0763**	**-0.0127**	**0.3322**	**0.0097**	**-0.0933**
Panel B: Fundamental Equation without Lagged Regressands						
	Fundamental Lag = 8			**Fundamental Lag = 12**		
Category	**Lag =8**	**Lag=12**	**Lag=16**	**Lag =8**	**Lag=12**	**Lag=16**
Non-Bubble MSA Pre-1999	0.3573 (796.63)	0.3372 (816.20)	0.3272 (651.50)	0.2615 (499.95)	0.3384 (681.97)	0.3120 (549.87)
Non-Bubble MSA Post-1999	0.6710 (245.08)	0.4270 (79.96)	0.7063 (59.92)	0.5976 (243.33)	0.6663 (162.79)	0.7259 (35.87)
Differences	**0.3138**	**0.0898**	**0.3791**	**0.3360**	**0.3279**	**0.4139**
Bubble MSA Pre-1999	0.5647 (375.36)	0.5157 (382.91)	0.4827 (295.68)	0.5498 (256.06)	0.5386 (275.45)	0.4722 (317.88)
Bubble MSA Post-1999	0.8393 (111.46)	0.7453 (72.78)	0.6933 (69.39)	0.8399 (112.74)	0.7335 (70.17)	0.6935 (67.00)
Difference	**0.2747**	**0.2296**	**0.2105**	**0.2901**	**0.1949**	**0.2213**

Note: Numbers within parentheses are the F-values for testing the hypothesis that the sum of the coefficients equals 1. All the above results indicate the null hypothesis is rejected, or the coefficients do not sum to unity.

Table 5 Test of Differences in Variance between the Pre- and Post-Bubble Period in the Bubble and Non-Bubble MSAs in Various Fundamental Equations

Panel A: Comparing 8-lag versus 12-lag Fundamental Equations with Lagged Regressands						
	Non-Bubble MSAs			**Bubble MSAs**		
	8-lag Fundamental	**12-lag Fundamental**	**GQ Test [2]**	**8-lag Fundamental**	**12-lag Fundamental**	**GQ Test [2]**
8-lag residuals						
Pre-1999	1.64×10^{-4}	1.51×10^{-4}	1.1285	3.98×10^{-4}	3.65×10^{-4}	1.0262
Post-1999	1.86×10^{-4}	1.65×10^{-4}	1.0802	2.72×10^{-4}	2.29×10^{-4}	1.0261
Pre/Post-1999 Test [1]	0.88406	0.91429		**1.45973 ***	**1.59046 ***	
12-lag residuals						
Pre-1999	1.48×10^{-4}	1.44×10^{-4}	1.0412	3.9479×10^{-4}	3.2029×10^{-4}	1.1282
Post-1999	1.96×10^{-4}	1.26×10^{-4}	**1.4083 ***	$2.366 \times 10^{-4}4$	2.07×10^{-4}	1.0300
Pre/Post-1999 Test [1]	0.75616	1.14740		**1.67166 ***	**1.54797 ***	
16-lag residuals						
Pre-1999	1.43×10^{-4}	1.41×10^{-4}	1.0243	3.59×10^{-4}	2.70×10^{-4}	1.1536
Post-1999	1.34×10^{-4}	1.30×10^{-4}	0.9810	2.67×10^{-4}	2.41×10^{-4}	0.9946
Pre/Post-1999 Test [1]	1.06339	1.09061		**1.34328 ***	1.12256	
Panel B: Comparing 8-lag versus 12-lag Fundamental Equations without Lagged Regressands						
	Non-Bubble MSAs			**Bubble MSAs**		
	8-lag Fundamental	**12-lag Fundamental**	**GQ Test [2]**	**8-lag Fundamental**	**12-lag Fundamental**	**GQ Test [2]**
8-lag residuals						
Pre-1999	1.76×10^{-4}	1.56×10^{-4}	1.1285	4.37×10^{-4}	4.26×10^{-4}	1.0262
Post-1999	1.88×10^{-4}	1.74×10^{-4}	1.0802	3.30×10^{-4}	3.22×10^{-4}	1.0261
Pre/Post-1999 Test [1]	0.93295	0.89302		**1.32176 ***	**1.32163 ***	
12-lag residuals						
Pre-1999	1.54×10^{-4}	1.48×10^{-4}	1.0412	4.31×10^{-4}	3.82×10^{-4}	1.1282
Post-1999	1.97×10^{-4}	1.40×10^{-4}	**1.4083 ***	$2.89 \times 10^{-4}4$	2.81×10^{-4}	1.0300
Pre/Post-1999 Test [1]	0.78288	1.05897		**1.49222 ***	**1.36237 ***	
16-lag residuals						
Pre-1999	1.48×10^{-4}	1.44×10^{-4}	1.0243	3.88×10^{-4}	3.36×10^{-4}	1.1536
Post-1999	1.46×10^{-4}	1.49×10^{-4}	0.9810	2.96×10^{-4}	2.97×10^{-4}	0.9946
Pre/Post-1999 Test [1]	1.01278	0.96997		**1.31219 ***	1.13133	

Continue…

(Table 5 Continued)

Panel C: Comparing 8-lag Fundamental Equations with and without Lagged Regressands						
	Non-Bubble MSAs			**Bubble MSAs**		
	With Regressands	**Without Regressands**	**GQ Test** [2]	**With Regressands**	**Without Regressands**	**GQ Test** [2]
8-lag residuals						
Pre-1999	1.64×10^{-4}	1.76×10^{-4}	0.9345	3.98×10^{-4}	4.37×10^{-4}	0.9109
Post-1999	1.86×10^{-4}	1.88×10^{-4}	0.9871	2.72×10^{-4}	3.30×10^{-4}	0.8234
12-lag residuals						
Pre-1999	1.48×10^{-4}	1.54×10^{-4}	0.9595	3.95×10^{-4}	4.31×10^{-4}	0.9153
Post-1999	1.96×10^{-4}	1.97×10^{-4}	0.9935	2.36×10^{-4}	2.89×10^{-4}	0.8164
16-lag residuals						
Pre-1999	1.43×10^{-4}	1.48×10^{-4}	0.9667	3.59×10^{-4}	3.88×10^{-4}	0.9248
Post-1999	1.34×10^{-4}	1.46×10^{-4}	0.9181	2.68×10^{-4}	2.96×10^{-4}	0.9042

Panel D: Comparing 12-lag Fundamental Equations with and without Lagged Regressands						
	Non-Bubble MSAs			**Bubble MSAs**		
	With Regressands	**Without Regressands**	**GQ Test**	**With Regressands**	**Without Regressands**	**GQ Test**
8-lag residuals						
Pre-1999	1.51×10^{-4}	1.56×10^{-4}	0.9706	3.65×10^{-4}	4.26×10^{-4}	0.8573
Post-1999	1.65×10^{-4}	1.74×10^{-4}	0.9476	2.29×10^{-4}	3.22×10^{-4}	0.7125
12-lag residuals						
Pre-1999	1.44×10^{-4}	1.48×10^{-4}	0.9734	3.20×10^{-4}	3.82×10^{-4}	0.8378
Post-1999	1.26×10^{-4}	1.40×10^{-4}	0.8995	2.07×10^{-4}	2.81×10^{-4}	0.7376
16-lag residuals						
Pre-1999	1.41×10^{-4}	1.44×10^{-4}	0.9763	2.70×10^{-4}	3.36×10^{-4}	0.8024
Post-1999	1.30×10^{-4}	1.49×10^{-4}	0.8704	2.41×10^{-4}	2.97×10^{-4}	0.8099

1. The "Pre/Post- 1999 Test" is test for statistical difference between the pre- and post-bubble periods (Goldfeld-Quandt Test is used).
2. The "GQ Test" is test for statistical difference between two fundamental equations.
* implies that the Goldfeld-Quandt Test rejects the null hypothesis that the variances between the 8-lag and 12-lag fundamental equations are statistically the same at 5% significance level (compared to an *F*-value of 1.3)

Table 6 Test of Differences in Variance between Various Post-Bubble Periods in the Bubble and Non-Bubble MSAs in 8-Lag Fundamental Equations with 4-Lag Error Equation

	Non-Bubble MSAs	Bubble MSAs
Post-1999 variance	2.0000×10^{-4}	2.5793×10^{-4}
Post-2002 variance	2.3126×10^{-4}	2.6954×10^{-4}
Post-2003 variance	2.6627×10^{-4}	3.0509×10^{-4}
GQ Test: 99 vs 02	0.8648	0.9569
GQ Test: 99 vs 03	0.7511	0.8454
GQ Test: 02 vs 03	0.86853	0.88346

Table 7 Test of Differences in Variance between the Pre- and Post-Bubble Period in the Bubble and Non-Bubble MSAs in Fundamental Equations with Separation of Non-Bubble and Bubble MSAs

Error Equation	Non-Bubble MSAs			Bubble MSAs		
	Pre-1999	Post-1999	GQ Test [1]	Pre-1999	Post-1999	GQ Test [1]
8-lag	1.72×10^{-4}	1.38×10^{-4}	1.2396	3.94×10^{-4}	2.94×10^{-4}	**1.3395 ***
12-lag	1.55×10^{-4}	1.44×10^{-4}	1.0791	3.90×10^{-4}	2.74×10^{-4}	**1.4236 ***
16-lag	1.51×10^{-4}	1.48×10^{-4}	1.0254	3.49×10^{-4}	2.53×10^{-4}	**1.3755 ***

1. GQ Test compares the variances between the pre- and post-1999 period.

* implies that the Goldfeld-Quandt Test rejects the null hypothesis that the variances between the 8-lag and 12-lag fundamental equations are statistically the same at 5% significance level (compared to an *F*-value of 1.3)

Table 8 Test of Differences in Variance between the Pre- and Post-Bubble Period in the Bubble and Non-Bubble MSAs in Fundamental Equations with Local Inflation in the Fundamental

Error Equation	Non-Bubble MSAs			Bubble MSAs		
	Pre-1999	Post-1999	GQ Test [1]	Pre-1999	Post-1999	GQ Test [1]
8-lag	1.57×10^{-4}	1.18×10^{-4}	**1.33123 ***	3.90×10^{-4}	3.30×10^{-4}	1.18236
12-lag	1.50×10^{-4}	1.25×10^{-4}	1.20338	3.88×10^{-4}	2.94×10^{-4}	**1.31859 ***
16-lag	1.46×10^{-4}	1.30×10^{-4}	1.11997	3.48×10^{-4}	2.72×10^{-4}	1.27802

1. GQ Test compares the variances between the pre- and post-1999 period.

* implies that the Goldfeld-Quandt Test rejects the null hypothesis that the variances between the 8-lag and 12-lag fundamental equations are statistically the same at 5% significance level (compared to an *F*-value of 1.3)

Appendices

Appendix 1 Annualized Average Growth Rates (in percentage) of Price, Rent, Difference between Price and Rent, and Local CPI of Individual MSAs (Asterisk indicates MSAs separated as bubble candidates)

MSAs	No. of Obs.	Price Growth	Rent Growth	Price-Rent Growth	Local CPI
Akron, OH	103	3.9768	3.5016	0.4752	3.7448
Anchorage, AK*	96	3.1673	2.5849	0.5824	2.4593
Ann Arbor, MI	102	5.1890	3.4639	1.7251	3.5542
Atlanta-Sandy Springs-Marietta, GA	104	4.6340	3.8777	0.7563	7.1204
Atlantic City, NJ	87	6.5720	3.7304	2.8416	3.2089
Baltimore-Towson, MD	36	9.2178	4.1863	5.0315	2.5386
Boston-Quincy, MA *	104	8.2892	4.9242	3.3650	4.0825
Boulder, CO	104	5.8883	3.7984	2.0899	3.7073
Bremerton-Silverdale, WA	103	5.4960	3.4012	2.0948	3.6060
Chicago-Naperville-Joliet, IL	104	5.4657	4.3685	1.0972	3.6755
Cincinnati-Middletown, OH-KY-IN	104	3.8449	3.3949	0.4501	3.4088
Cleveland-Elyria-Mentor, OH	104	3.9130	3.5016	0.4114	3.7448
Dallas-Plano-Irving, TX	104	3.0029	3.3065	-0.3036	3.5913
Denver-Aurora, CO	104	4.9855	3.7984	1.1872	3.7073
Detroit-Livonia-Dearborn, MI	104	4.7089	3.3968	1.3120	3.4888
Flint, MI	104	4.6088	3.3968	1.2120	3.4888
Fort Lauderdale-Pompano Beach-Deerfield Beach, FL *	104	6.4542	3.9566	2.4976	3.8114
Fort Worth-Arlington, TX	104	2.7439	3.3065	-0.5626	3.5913
Gary, IN	104	3.7292	4.3685	-0.6393	3.6755
Greeley, CO	81	4.6503	2.9757	1.6746	2.8996
Honolulu, HI *	104	8.4394	4.0158	4.4236	3.6627
Houston-Sugar Land-Baytown, TX	104	2.4987	3.1129	-0.6142	3.2148
Kansas City, MO-KS	104	3.6772	3.6694	0.0078	3.3558
Lake County-Kenosha County, IL-WI	104	5.1801	4.3685	0.8116	3.6755
Los Angeles-Long *Beach-Glendale, CA	104	7.2382	4.7031	2.5351	3.7761
Miami-Miami Beach-Kendall, FL*	104	6.5227	3.9566	2.5661	3.8114

(Appendix 1 continued…)

MSAs	No. of Obs.	Price Growth	Rent Growth	Price-Rent Growth	Local CPI
Milwaukee-Waukesha-West Allis, WI	104	4.6754	3.6459	1.0295	3.5204
Minneapolis-St. Paul-Bloomington, MN-WI	104	5.2949	3.6892	1.6058	3.7092
New York-White Plains-Wayne, NY-NJ *	104	7.9851	4.6493	3.3358	3.9667
Philadelphia, PA	104	6.3239	4.3361	1.9878	3.7826
Pittsburgh, PA	104	3.8733	3.2084	0.6649	3.6355
Portland-Vancouver-Beaverton, OR-WA *	104	5.5337	3.3116	2.2221	3.4013
Racine, WI	90	5.0893	3.0796	2.0097	2.7917
Riverside-San Bernardino-Ontario, CA *	104	6.3798	4.7031	1.6767	3.7761
Salem, OR	98	5.1685	3.5155	1.6530	3.6083
San Diego-Carlsbad-San Marcos, CA *	104	7.0325	4.8219	2.2106	4.2708
San Francisco - San Mateo - Redwood City, CA *	104	7.7118	5.0223	2.6895	3.8847
San Jose-Sunnyvale-Santa Clara, CA *	104	7.9135	5.0223	2.8912	3.8847
Seattle-Bellevue-Everett, WA *	104	6.2482	3.7498	2.4983	3.8443
Tacoma, WA *	104	5.9906	3.7498	2.2407	3.8443
Tampa-St. Petersburg-Clearwater, FL *	32	11.0700	4.3738	6.6962	3.7665
Washington-Arlington-Alexandria, DC-VA-MD-WV*	36	10.8923	4.1863	6.7061	2.5386
Wilmington, DE-MD-NJ	103	6.1454	4.3271	1.8182	3.6888

Appendix 2 Results of Error Equations from Fundamental Equation *with* 8-Lag Regressands for Non-Bubble versus Bubble MSAs in Pre- and Post-Bubble Period (various lags)

Panel A: 8-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0012 ***	0.0024 ***	-0.0024 ***	0.0043 ***
1	-0.0689 **	0.1654 ***	0.1722 ***	0.3480 ***
2	-0.1032 ***	-0.1405 ***	0.0614 **	-0.0176
3	0.0918 ***	0.0553	0.1181 ***	0.4030 ***
4	-0.0144	-0.1233 ***	-0.1325 ***	-0.2591 ***
5	-0.0881***	-0.0114	-0.1060 ***	-0.0831
6	-0.0535 **	0.0328	0.0107	0.0343
7	-0.0705 ***	-0.0798 *	0.0796 ***	-0.0786
8	-0.0844 ***	0.2829 ***	-0.0920 ***	0.1306 **
Adjusted *R*-Square	0.0526	0.1297	0.0913	0.2537
Durbin-Watson	1.938	1.801	1.979	1.998

Panel B: 12-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0009 **	0.0034 ***	-0.0023 ***	0.0078 ***
1	-0.0429	0.3202 ***	0.2001 ***	0.2555 ***
2	-0.0910 ***	-0.2622 ***	0.0872 ***	-0.1513 ***
3	0.0798 ***	0.1869 ***	0.0680 **	0.4616 ***
4	-0.0161	-0.1998 ***	-0.1141 ***	-0.3444 ***
5	-0.0651 **	-0.0608	-0.1500 ***	-0.0200
6	-0.0148	-0.0118	0.0539 *	0.0850
7	-0.0427	-0.1642 ***	0.0745 ***	0.0279
8	-0.1360 ***	0.3332 ***	-0.1222 ***	0.0204
9	0.0142	-0.3454 ***	0.0540 **	0.0459
10	-0.0146	0.0991	0.0221	-0.0056
11	-0.0234	-0.2324 ***	-0.0441 *	-0.1428 **
12	-0.0226	0.0624	-0.0701 ***	-0.0965
Adjusted *R*-Square	0.0485	0.2119	0.1234	0.271
Durbin-Watson	1.960	1.931	1.989	2.05

Continue…

(Appendix 2 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0008 **	0.0037 ***	-0.0024 ***	0.0098 ***
1	0.0085	0.2554 ***	0.2983 ***	0.1807 ***
2	-0.0940 ***	-0.2357 ***	0.0586 *	-0.1659 **
3	0.0977 ***	0.1932 ***	0.0728 **	0.5416 ***
4	-0.0287	-0.2203 ***	-0.1408 ***	-0.4077 ***
5	-0.0435	0.1398 **	-0.1351 ***	0.0075
6	-0.0249	0.0216	0.0642 **	-0.0191
7	-0.0395	-0.0684	0.0575 *	0.2096 **
8	-0.1643 ***	-0.1333 **	-0.1717 ***	-0.0494
9	0.0323	-0.1209 *	0.1211 ***	0.0100
10	-0.0193	0.0560	0.0004	-0.0143
11	-0.0097	-0.1708 ***	-0.0536 **	-0.1258
12	0.0252	0.1336 **	-0.0704 ***	-0.1899 **
13	0.0155	-0.0772	0.0180	0.0239
14	-0.0060	0.0502	-0.0037	0.0823
15	-0.0108	-0.0138	-0.0132	-0.0349
16	-0.0149	0.1534 **	-0.0348	0.0063
Adjusted *R*-Square	0.0565	0.1766	0.1959	0.3072
Durbin-Watson	1.965	1.892	1.927	2.086

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Appendix 3 Results of Error Equations from Fundamental Equation with 12-Lag Regressands for Non-Bubble versus Bubble MSAs in Pre- and Post-Bubble Period (various lags)

Panel A: 8-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0012 ***	-0.0021 ***	0.0024 ***	0.0044 ***
1	-0.0689 **	0.0809 **	0.1077 ***	0.2113 ***
2	-0.1032 ***	0.0700 ***	-0.2137 **	-0.0267
3	0.0918 ***	0.0000	0.0151	0.3144 ***
4	-0.0144	-0.0197	-0.0748	-0.1263 **
5	-0.0881***	-0.1535	-0.0731 ***	-0.0999 *
6	-0.0535 **	0.0590	0.0141 **	0.0527
7	-0.0705 ***	0.0816 ***	-0.1310 ***	-0.1013
8	-0.0844 ***	-0.1127 ***	0.2376 ***	0.1135 *
Adjusted *R*-Square	0.0543	0.1002	0.0542	0.1304
Durbin-Watson	1.968	1.954	1.989	2.001

Panel B: 12-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0009 **	0.0026 ***	-0.0021 ***	0.0081 ***
1	-0.0767 **	0.1707 ***	0.1782 ***	0.1124 ***
2	-0.1100 ***	-0.2418 ***	0.0549 *	-0.1733 ***
3	0.0187	0.0938 *	0.0016	0.3327 ***
4	0.0198	-0.1221 **	-0.0339	-0.2205 ***
5	-0.0223	0.0261	-0.1384 ***	-0.0687
6	-0.0317	0.0830 *	0.0648 **	0.1342 **
7	-0.0337	-0.0999 **	0.0590 **	0.0148
8	-0.1948 ***	-0.1366 **	-0.1538 ***	0.0376
9	-0.0024	-0.1132 *	0.0901 ***	0.0593
10	-0.0667 **	-0.0220	0.0000	-0.0090
11	-0.0040	-0.0601	-0.0473 *	-0.0705
12	0.0693 ***	0.1289 **	-0.0439 *	-0.1080 *
Adjusted *R*-Square	0.0648	0.1406	0.0983	0.1078
Durbin-Watson	1.9650	1.8040	1.9380	2.0660

Continue…

(Appendix 3 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0013 ***	0.0029 ***	-0.0022 ***	0.0097 ***
1	-0.0581 *	0.2730 ***	0.1449 ***	0.0532
2	-0.1234 ***	-0.3271 ***	0.0316	-0.2007 ***
3	0.0201	0.2929 ***	0.0495	0.4080 ***
4	0.0037	-0.1658 **	-0.0466	-0.2747 ***
5	-0.0340	0.1065 *	-0.0558 *	-0.0334
6	-0.0275	-0.0039	0.0307	0.0318
7	-0.0416	-0.0945 *	0.0585 **	0.1630*
8	-0.1852 ***	-0.2334 ***	-0.1732 ***	-0.0077
9	0.0182	0.0028	0.0691 *	0.0034
10	-0.0680 **	-0.2459 ***	-0.0057	-0.0235
11	-0.0156	0.2023 **	-0.0085	-0.0818
12	0.0652 **	0.1475 *	-0.0378	-0.1742 **
13	-0.0206	-0.0242	-0.0170	0.0311
14	-0.0355	0.0682	-0.0182	0.0299
15	-0.0263	-0.0734	-0.0251	-0.0579
16	0.0080	0.0327	-0.0185	0.0182
Adjusted *R*-Square	0.0618	0.2563	0.0765	0.2181
Durbin-Watson	2.0280	2.0210	1.8920	2.0640

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Appendix 4 Results of Error Equations from 8-Lag Fundamental Equation *without* Lagged Regressands for Non-Bubble versus Bubble MSAs in Pre- and Post-Bubble Period (various lags)

Panel A: 8-lags

Number of Lags	Non-Bubble MSAs		Bubble MSAs	
	Pre-bubble Period	Post-bubble Period	Pre-bubble Period	Post-bubble Period
Intercept	-0.0007 *	0.0031 ***	-0.0019 ***	0.0049 ***
1	0.0343	0.2425 ***	0.2074 ***	0.3658 ***
2	0.0450 *	-0.0481	0.1810 ***	0.1047 **
3	0.2028 ***	0.1219 ***	0.1593 ***	0.4155 ***
4	0.1585 ***	-0.0309	0.0431	-0.0792
5	-0.0355	0.0365	-0.1169 ***	-0.1608 ***
6	-0.0017	0.0798 *	0.0395	0.0956
7	-0.0311	-0.0355	0.0783 ***	-0.1040
8	-0.0151	0.3049 ***	-0.0270	0.2016 ***
Adjusted *R*-Square	0.0755	0.1568	0.1769	0.8442
Durbin-Watson	1.9480	1.8170	1.9880	2.0160

Panel B: 12-lags

Number of Lags	Non-Bubble MSAs		Bubble MSAs	
	Pre-bubble Period	Post-bubble Period	Pre-bubble Period	Post-bubble Period
Intercept	0.0002	0.0042 ***	-0.0018 ***	0.0083 ***
1	0.0527 *	0.3902 ***	0.2325 ***	0.2773 ***
2	0.0167	-0.1589 ***	0.2010 ***	-0.0300 ***
3	0.1871 ***	0.2422 ***	0.1061 ***	0.4919
4	0.1348 ***	-0.0936	0.0572 *	-0.1328 ***
5	-0.0125	0.0020	-0.1670 ***	-0.0723 **
6	0.0141	0.0470	0.0830 ***	0.1894
7	0.0036	-0.0936 **	0.1056 ***	-0.0020 ***
8	-0.0992 ***	0.3611 ***	-0.0711 **	0.0905
9	0.0372	-0.2805 ***	0.0421	0.0958
10	-0.0118	0.0905	0.0254	-0.0035
11	-0.0070	-0.1453 **	-0.0651 ***	-0.1391 *
12	0.0214	0.0658	-0.0341	-0.0200
Adjusted *R*-Square	0.0773	0.2056	0.2068	0.4098
Durbin-Watson	1.9730	1.9500	1.9950	2.0370

Continue…

(Appendix 4 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0002	0.0039 ***	-0.0020 ***	0.0104 ***
1	0.0941 ***	0.3514 ***	0.3270 ***	0.2151 ***
2	0.0181	-0.1253 **	0.1636 ***	-0.0299
3	0.1925 ***	0.2614 ***	0.0974 ***	0.5820 ***
4	0.1095 ***	-0.1086 *	0.0271	-0.2049 **
5	-0.0011	0.1652 ***	-0.1621 ***	-0.0420
6	-0.0039	0.0924	0.0931 ***	0.0833
7	0.0056	-0.0115	0.0893 ***	0.1529
8	-0.1316 ***	-0.1397 **	-0.1179 ***	0.0262
9	0.0455 *	-0.0328	0.1217 ***	0.0486
10	-0.0113	0.0772	0.0153	-0.0120
11	0.0177	-0.0739	-0.0678 **	-0.1618 *
12	0.0671 ***	0.1520 **	-0.0292	-0.1217
13	0.0123	0.0244	-0.0108	0.0668
14	-0.0392 *	0.0358	-0.0219	0.1100
15	-0.0334 *	-0.0137	-0.0187	-0.0598
16	-0.0147	0.0520	-0.0234	0.0407
Adjusted *R*-Square	0.0835	0.2242	0.2682	0.3758
Durbin-Watson	1.9670	1.8140	1.9270	2.0660

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Appendix 5 Results of Error Equations from 12-Lag Fundamental Equation *without* Lagged Regressands for Non-Bubble versus Bubble MSAs in Pre- and Post-Bubble Period (various lags)

Panel A: 8-lags

Number of Lags	Non-Bubble MSAs		Bubble MSAs	
	Pre-bubble Period	Post-bubble Period	Pre-bubble Period	Post-bubble Period
Intercept	-0.0007 *	0.0032 ***	-0.0018 ***	0.0050 ***
1	0.0318	0.2374 ***	0.2191 ***	0.3548 ***
2	0.0428	-0.1029 **	0.2125 ***	0.0895 *
3	0.1656 ***	0.1405 ***	0.1023 ***	0.4213 ***
4	0.1322 ***	0.0174	0.0478	-0.1036 *
5	0.0102	0.0009	-0.1531 ***	-0.1253 **
6	0.0054	0.0827 *	0.0879 ***	0.0891
7	-0.0111	-0.0844 *	0.1102 ***	-0.0861
8	-0.1154 ***	0.3060 ***	-0.0770 ***	0.2002 ***
Adjusted *R*-Square	0.0681	0.1398	0.1917	0.3818
Durbin-Watson	1.9610	1.9360	1.9890	2.0230

Panel B: 12-lags

Number of Lags	Non-Bubble MSAs		Bubble MSAs	
	Pre-bubble Period	Post-bubble Period	Pre-bubble Period	Post-bubble Period
Intercept	0.0004	0.0029 ***	-0.0019 ***	0.0087 ***
1	0.0730 **	0.3120 ***	0.3131 ***	0.2458 ***
2	0.0374	-0.1369 ***	0.1779 ***	-0.0325
3	0.1546 ***	0.2396 ***	0.0942 ***	0.4762 ***
4	0.1128 ***	-0.0310	0.0285	-0.1487 **
5	0.0226	0.1211 ***	-0.1387 ***	-0.0573
6	-0.0115	0.1693 ***	0.0830 ***	0.1941 ***
7	0.0025	-0.0441	0.0817 ***	0.0351
8	-0.1518 ***	-0.0939	-0.1144 ***	0.0934
9	0.0365	-0.0536	0.1149 ***	0.1071
10	-0.0425 *	0.0464	0.0116	0.0004
11	0.0117	-0.0391	-0.0745 ***	-0.1257 *
12	0.0933 ***	0.1765 ***	-0.0388	-0.0544
Adjusted *R*-Square	0.0763	0.2080	0.2500	0.3873
Durbin-Watson	1.9600	1.7940	1.9440	2.0510

Continue…

(Appendix 5 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.0010 ***	0.0030 ***	-0.0023 ***	0.0101 ***
1	0.0811 **	0.4261 ***	0.2726 ***	0.2042 ***
2	0.0196	-0.2487 ***	0.1528 ***	-0.0340
3	0.1577 ***	0.4521 ***	0.1462 ***	0.5749 ***
4	0.1045 ***	-0.1279 *	0.0153	-0.2185 **
5	0.0220	0.1936 ***	-0.0531 *	-0.0328
6	0.0124	0.0460	0.0415	0.0748
7	0.0081	-0.0423	0.0747 **	0.1904 *
8	-0.1315 ***	-0.2062 ***	-0.1405 ***	0.0434
9	0.0655 **	0.0915	0.0997 ***	0.0617
10	-0.0402	-0.1917 **	0.0176	-0.0123
11	0.0149	0.2548 ***	-0.0102	-0.1401
12	0.1014 ***	0.1881 **	-0.0190	-0.1173
13	-0.0257	-0.0472	-0.0388	0.0429
14	-0.0477 *	0.0687	-0.0295	0.0707
15	-0.0410	-0.1115 *	-0.0397	-0.0524
16	0.0109	-0.0195	-0.0175	0.0379
Adjusted *R*-Square	0.0815	0.3360	0.2703	0.3564
Durbin-Watson	2.0330	1.9890	1.9020	2.0510

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Appendix 6 Results of the Fundamental Equation *without* Lagged Regressands with 20 and 24 lags

Equation being Difference between Growth Rates of House Prices versus Rent (Regressand) on Lagged Changes in 10-year Treasury, Local Rent Growth, and Regressands (MSA Fixed Effects Omitted)

Panel A: Regression Results of Various Lags in Fundamental Equation				
	Fundamental with 20 Lags		**Fundamental with 24 Lags**	
Lag	**Δ Interest Rate**	**Δ Rent Growth**	**Δ Interest Rate**	**Δ Rent Growth**
1	0.0021***	0.1924***	0.0024 ***	0.1965***
2	0.0003	0.2314***	0.0004	0.2440***
3	-0.0003	0.2508***	0.0006	0.2659***
4	0.0013**	0.1881***	0.0019***	0.2101***
5	0.0015**	0.1832***	0.0025***	0.2024***
6	-0.0007	0.2225***	0.0001	0.2340***
7	-0.0022***	0.2393***	-0.0018***	0.2459***
8	-0.0031***	0.0446	-0.0029***	0.0438
9	-0.0021***	0.0215	-0.0017***	0.0102
10	-0.0016***	0.0240	-0.0017***	0.0015
11	-0.0005	0.1021*	-0.0006	0.0697
12	-0.0014**	0.2595***	-0.0005	0.2174***
13	-0.0025***	0.2697***	-0.0020***	0.2150***
14	-0.0040***	0.3005***	-0.0028***	0.2327***
15	0.0002	0.2826***	0.0012**	0.2009***
16	-0.0017***	0.0597*	-0.0002	-0.0362
17	-0.0020***	-0.0452***	-0.0004	-0.1477***
18	-0.0015***	-0.0545***	-0.0003	-0.1502***
19	0.0001	-0.0301***	0.0010*	-0.1174***
20	0.0012**	-0.0086	0.0014***	-0.0871***
21			0.0025***	-0.0644***
22			0.0017***	-0.0419***
23			0.0012**	-0.0122
24			0.0018***	-0.0044
Sum of Coefficients	-0.0168	2.7333	0.0037	1.9287
Adjusted R^2	0.1375		0.1562	

Continue…

(Appendix 6 continued…)

Panel B: Results of Error Equations				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Panel B1: Fundamental with 20 Lags				
Intercept	-0.0008**	0.0026***	-0.0022***	0.0085***
1	0.1107***	0.3265***	0.3067***	0.3105***
2	0.1309***	-0.1879***	0.1255***	-0.0940
3	0.1542***	0.3840***	0.1881***	0.5167***
4	0.0733**	-0.0061	-0.0251	-0.2220***
5	0.0043	0.1605***	-0.0428	-0.0488
6	0.0296	0.2147***	0.0048	0.1213
7	0.0368	-0.0706	0.0887***	0.1202
8	-0.0277	-0.0421	-0.1209***	0.2004**
9	0.1067***	-0.0805	0.0860***	-0.0503
10	0.0044	-0.0559	0.0287	0.0419
11	0.0369	0.0879	-0.0587*	-0.1338
12	0.0510**	0.1403**	-0.0361	0.0518
Sum of Coefficients	0.7110	0.8708	0.5450	0.8139
Adjusted *R*-Square	0.1596	0.3301	0.2785	0.3778
Panel B2: Fundamental with 24 Lags				
Intercept	-0.0007*	0.00222***	-0.0019***	0.00867***
1	0.11198***	0.33985***	0.30737***	0.31032***
2	0.12311***	-0.17152***	0.12255***	-0.09848
3	0.16115***	0.36297***	0.1947***	0.46476***
4	0.10019***	-0.02897	-0.02394	-0.226***
5	0.01296	0.12749**	-0.03269	-0.05374
6	0.0372	0.2049***	0.01237	0.13831
7	0.04401	-0.05844	0.09574***	0.14496*
8	-0.02887	-0.02371	-0.1131***	0.19723**
9	0.07893***	-0.07452	0.08044**	-0.06921
10	-0.01151	-0.03787	0.02079	0.05717
11	0.04475*	0.10242*	-0.0606*	-0.10647
12	0.06251**	0.12759**	-0.03152	0.05444
Sum of Coefficients	0.7364	0.8702	0.5722	0.8133
Adjusted *R*-Square	0.1828	0.3187	0.2916	0.3561

Continue…

(Appendix 6 continued…)

Panel C: Test pf Differences in Variance between Pre- and Post-Bubble Period		
Category	**Fundamental = 20 Lags**	**Fundamental = 24 Lags**
Non-Bubble MSA Pre-1999	1.66×10^{-4}	1.70×10^{-4}
Non-Bubble MSA Post-1999	1.68×10^{-4}	1.62×10^{-4}
Pre/Post- 1999 Test [1]	0.99123	1.05245
Bubble MSA Pre-1999	3.39×10^{-4}	3.39×10^{-4}
Bubble MSA Post-1999	2.60×10^{-4}	2.43×10^{-4}
Pre/Post- 1999 Test [1]	1.30188	**1.39361** [†]

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

1. The "Pre/Post- 1999 Test" is test for statistical difference between the pre- and post-bubble periods (Goldfeld-Quandt Test is used).

† implies that the Goldfeld-Quandt Test rejects the null hypothesis that the variances between the pre- and post-1999 periods are statistically the same at 5% significance level (compared to an *F*-value of 1.3).

Appendix 7 Regression Results for the Fundamental Equation for Bubble MSAs and Non-Bubble MSAs Separated

Equation being Difference between Growth Rates of House Prices versus Rent (Regressand) on Lagged Changes in 10-year Treasury, Local Rent Growth, and Regressands (MSA Fixed Effects Omitted)

	Variables		
Variables	**Nominal Interests**	**Rent Growth**	**Regressand**
Panel A: Non-Bubble MSAs			
[illegible]	-0.65200 **	0.08545 *	-0.21947 ***
[illegible]	-0.85200 ***	0.13290 *	0.13028 ***
[illegible]	-1.02000 ***	0.10380	0.00696
[illegible]	-0.66000 ***	0.17795 **	0.22104 ***
[illegible]	-0.25600	0.10759	0.10375 ***
[illegible]	-1.00400 ***	0.14199 *	0.07580 ***
[illegible]	0.31960	0.22916 ***	0.13573 ***
[illegible]	-0.61600 **	0.11281 **	0.10386 ***
Adjusted	0.25247		
Panel B: Bubble MSAs			
[illegible]	0.28320	0.09866 **	0.11390 ***
[illegible]	-0.34800	0.16700 ***	0.13044 ***
[illegible]	0.07520	0.28606 ***	0.11596 ***
[illegible]	0.26560	0.49461 ***	0.21936 ***
[illegible]	-0.01200	0.44867 ***	0.02737
[illegible]	-0.42000	0.48665 ***	0.01126
[illegible]	-0.12400	0.44597 ***	0.01143
[illegible]	-0.96000 ***	-0.00880	0.02318
[illegible]	0.22687		

*"***", "**", "*" represent significance at 1%, 5% and 10% respectively.*

Appendix 8 Results of Error Equations from 8-Lag Fundamental Equation with Separation of Non-Bubble and Bubble MSAs in Pre- and Post-Bubble Period (various lags)

Panel A: 8-lags				
	Non-Bubble MSAs		**Bubble MSAs**	
Number of Lags	**Pre-bubble Period**	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	0.00099 ***	-0.00254 ***	0.00184 ***	0.00447 ***
1	0.13516 ***	0.07516 **	0.53786 ***	0.18425 ***
2	-0.12286 ***	0.07666 ***	-0.39784 ***	0.02370
3	0.17815 ***	0.07047 **	0.44936 ***	0.21641 ***
4	-0.05435 **	-0.12580 ***	-0.26742 ***	-0.14515 ***
5	-0.06125 **	-0.13983 ***	0.09191	-0.09551 **
6	-0.05020 **	0.01288	-0.02054	0.03616
7	-0.08117 ***	0.08362 ***	-0.09178	-0.09629 *
8	-0.04564 **	-0.09072 ***	-0.04692	0.31113 ***
Adjusted *R*-Square	0.0793	0.0664	0.2574	0.1591
Durbin-Watson	1.938	1.991	2.016	1.873

Panel B: 12-lags				
	Non-Bubble MSAs		**Bubble MSAs**	
Number of Lags	**Pre-bubble Period**	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	0.00080 **	-0.00226 ***	0.00279 ***	0.00790 ***
1	0.16613 ***	0.09914 ***	0.56779 ***	0.12394 **
2	-0.09202 ***	0.11065 ***	-0.48747 ***	-0.10439 **
3	0.15724 ***	0.02502	0.52716 ***	0.33279 ***
4	-0.05860 **	-0.10885 ***	-0.33241 ***	-0.21416 ***
5	-0.05823 **	-0.17418 ***	0.16204 **	-0.10847 **
6	-0.01653	0.05036 *	0.01328	0.01600
7	-0.07130 ***	0.08571 ***	-0.12262 *	-0.06672
8	-0.06464 **	-0.12261 ***	-0.06870	0.30957 ***
9	0.02814	0.04245	-0.05150	-0.11414 *
10	0.00740	0.03863	-0.13741 *	0.04662
11	-0.02684	-0.04562 *	0.04492	-0.24365 ***
12	-0.01764	-0.05812 **	0.04615	-0.05493
Adjusted *R*-Square	0.0706	0.098	0.3345	0.2134
Durbin-Watson	1.953	1.986	1.974	2.070

Continue…

(Appendix 8 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	0.00065 *	-0.00236 ***	0.00279 ***	0.00984 ***
1	0.21999 ***	0.19803 ***	0.54898 ***	0.03237
2	-0.10882 ***	0.09135 ***	-0.48642 ***	-0.12456 **
3	0.17832 ***	0.02455	0.54877 ***	0.32870 ***
4	-0.08917 ***	-0.13469 ***	-0.39248 ***	-0.21646 ***
5	-0.03099	-0.15524 ***	0.27316 ***	-0.07251
6	-0.02698	0.06678 **	-0.04320	0.08423
7	-0.07308 **	0.06640 **	-0.06925	0.04278
8	-0.08411 ***	-0.18035 ***	-0.10524	0.03212
9	0.04548	0.11070 ***	-0.01847	-0.00945
10	0.00160	0.02223	-0.16214 *	0.04346
11	-0.01198	-0.06224 **	0.04352	-0.27765 ***
12	0.01884	-0.06946 ***	0.12789	-0.08889
13	0.00486	0.01393	-0.19258 **	0.05007
14	-0.01384	0.00817	0.18351 **	0.01049
15	-0.02011	-0.01089	-0.10143	-0.09360
16	0.00288	-0.03854 *	0.25325 ***	0.04688
Adjusted *R*-Square	0.0942	0.1589	0.3568	0.2110
Durbin-Watson	1.963	1.937	2.031	2.004

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.

Appendix 9 Results of Error Equations from 8-Lag Fundamental Equation with Local Inflation in the Fundamental (various lags)

Panel A: 8-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.00163 ***	0.00163 ***	-0.00254 ***	0.00450 ***
1	-0.05126 *	0.35149 ***	0.17341 ***	0.30734 ***
2	-0.11513 ***	-0.31355 ***	0.06033 ***	-0.02939
3	0.05640 **	0.32261 ***	0.08594 ***	0.22792 ***
4	-0.06002 **	-0.18798 ***	-0.13008 ***	-0.18111 ***
5	-0.07794 ***	0.08235	-0.12294 ***	-0.03666
6	-0.05090 **	0.02983	0.03482	0.01674
7	-0.04720 **	-0.04057	0.07411 ***	-0.10121 *
8	-0.09052 ***	-0.06091	-0.08169 ***	0.34916 ***
Adjusted *R*-Square	0.0446	0.0854	0.1917	0.2266
Durbin-Watson	1.9800	1.963	1.9890	1.951

Panel B: 12-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.00139 ***	0.00256 ***	-0.00242 ***	0.00841 ***
1	-0.03334	0.38500 ***	0.19820 ***	0.19789 ***
2	-0.08873 ***	-0.37852 ***	0.08625 ***	-0.15115 ***
3	0.04873 *	0.36500 ***	0.03407	0.36611 ***
4	-0.04623	-0.25366 ***	-0.11899 ***	-0.24821 ***
5	-0.06947 **	0.13173 **	-0.14020 ***	-0.03452
6	-0.01402	0.03573	0.06468 **	0.00068
7	-0.05271 **	-0.01728 **	0.05410 *	-0.06351
8	-0.11545 ***	-0.15394	-0.11180 ***	0.36649 ***
9	0.01773	0.10067	0.05457 *	-0.12022 *
10	-0.00812	-0.17881 **	0.02794	0.06897
11	-0.03055	0.08946	-0.04311	-0.27996 ***
12	-0.01969	0.00766	-0.07415 ***	-0.00499
Adjusted *R*-Square	0.0356	0.1122	0.2062	0.2496
Durbin-Watson	1.9940	1.9940	1.9520	2.0880

Continue…

(Appendix 9 continued)

Panel C: 16-lags				
Number of Lags	**Non-Bubble MSAs**		**Bubble MSAs**	
	Pre-bubble Period	**Post-bubble Period**	**Pre-bubble Period**	**Post-bubble Period**
Intercept	-0.00135 ***	0.00287 ***	-0.00272 ***	0.01004 ***
1	0.01409	0.35994 ***	0.31673 ***	0.13401 **
2	-0.09413 ***	-0.34498 ***	-0.01439	-0.15430 **
3	0.07158 **	0.35902 ***	0.07945 ***	0.35390 ***
4	-0.07171 **	-0.28205 ***	-0.14695 ***	-0.25443 ***
5	-0.05146 *	0.18385 **	-0.11819 ***	-0.00621
6	-0.02238	0.00128	0.07680 **	0.09674
7	-0.04575	0.00041	0.04431	0.01795
8	-0.14399 ***	-0.20864 ***	-0.15929 ***	0.03153
9	0.05920 **	0.14061	0.10566 ***	0.14057
10	-0.03514	-0.25528 ***	-0.00720	-0.00722
11	-0.02240	0.10516	-0.03656	-0.31179 ***
12	-0.00498	0.00303	-0.06739 **	-0.01240
13	-0.00623	-0.10628	0.00562	0.02602
14	0.00366	0.04570	0.00054	-0.03296
15	-0.00429	-0.01619	-0.02032	-0.08671
16	0.01174	0.10674	-0.03923	0.06824
Adjusted *R*-Square	0.0469	0.2018	0.1824	0.2507
Durbin-Watson	1.9910	2.0060	1.9940	2.0350

"***", "**", "*" represent significance at 1%, 5% and 10% respectively.