

THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
High Altitude Engineering Laboratory
Departments of
Aerospace Engineering
Meteorology and Oceanography

Technical Report

THEORETICAL MODEL FOR CONVERSION
OF OBSERVED NEUTRAL AND ION DENSITIES TO
AMBIENT DENSITIES FOR ORBITING GEOPHYSICAL OBSERVATORIES

B. B. Hinton, R. J. Leite, and C. J. Mason

ORA Project 03346

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GRANT NO. NGR 23-005-383

WASHINGTON, D. C.

administered through

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

January 1970

Abstract

A spacecraft perturbs the density of ions and neutral molecules and atoms as it moves through the tenuous upper atmosphere of the earth. In this report expressions are derived for the resultant perturbations on measured values of the particle densities for mass spectrometer experiments carried on the OGO spacecraft. Using these equations the ambient values can be recovered from the measured densities for neutral particles, and from the measured fluxes for positive ions.

Table of Contents

	Page
Abstract	ii
List of Figures	iv
TEXT	
I Introduction and Purpose	1
II Assumptions and Elementary Considerations	2
III Expansion to Include Multiple Reflection Processes	7
IV Positive Ion Measurements	13
V Conclusions	14
References	14

LIST OF FIGURES

		Page
Figure 1	Incident Particles	3
Figure 2	The Solid Angle Element	4
Figure 3	Number Densities	5
Figure 4	The OGO Spacecraft	6

I. INTRODUCTION AND PURPOSE

The neutral and positive ion concentrations obtained by in situ mass spectrometer measurements on an earth-orbiting vehicle must be adjusted to account for the influence of the vehicle.* The following work was initiated to develop the proper transfer functions for converting the OGO-4 Mass Spectrometer measurements of neutral-particle concentrations and positive-ion fluxes into ambient neutral and ionic number densities. The purpose of this report is to present a set of theoretical expressions from which the detailed calculations can proceed. It is planned to present the actual program for carrying out these calculations as a separate report.

The near polar orbit of OGO-4 has a perigee of about 400 km and an apogee of 900 km. In this altitude range inter-particle collisions may be neglected since the spacecraft dimensions are orders of magnitude smaller than the mean free path. Except for electrons and the lightest neutrals and ions the spacecraft speed is much greater than the thermal speeds. Therefore, at least for the intermediate and heavier masses, one would expect the densities and fluxes to be significantly greater on surfaces looking into the velocity vector, and significantly less on surfaces looking away from the velocity. However, the density and flux are determined not only by exposure to the incident particles, but also by reflections of particles from one surface to another. The calculation of this transfer from one surface to another as well as the directly incident particle densities and fluxes is considered in this report.

*See reference (1)

II. ASSUMPTIONS AND ELEMENTARY CONSIDERATIONS

The problem is not solvable without certain simplifying assumptions because the interaction of atoms and molecules with surfaces is a poorly understood phenomenon, the surfaces themselves are not well defined, and, in addition, the surface structure may even change with time. Consequently, in order to approximate the real situation by a solvable problem we shall assume the following:

- (ia) The problem can be approximated for atomic oxygen by a steady state in which the flux of atoms arriving at a surface is equal to the flux of the same species of atoms plus the flux of the recombined diatomic molecules (of the same species of atoms) leaving the surface, the incident molecular oxygen being neglected;
- (ib) For other species there is assumed to be no chemical change; hence the incoming and outgoing fluxes are assumed to be equal.
- (ii) The molecules leaving the surface have interacted strongly enough with the surface that they have "forgotten" their direction of arrival and are directed isotropically; or alternatively, we could assume that on the molecular, or atomic scale the surfaces are "rough" and arrive at the same isotropic distribution.
- (iii) Intermolecular collisions are neglected.

In order to arrive at the solution, use will be made of the solutions of two "elementary" problems. The first of these is the calculation of the incident flux of ambient particles on a moving surface and the other is the transfer of particles from one surface to another.

In Fig. 1. consider the surface S , which is non-concave; that is, the solid angle subtended by S at any point on the surface is zero. Other surfaces S' may subtend non-zero solid angles when viewed from points on S however. The velocity of the surface is \vec{V} . The Maxwellian velocity distribution of the ambient particles relative to S is $f_3(\vec{v}_m) = f(\vec{V} - \hat{r}v)$ where \vec{v}_m is the Maxwellian velocity of a particle, f_3 the three dimensional distribution, and \vec{v} the relative velocity of the spacecraft with respect to the particles

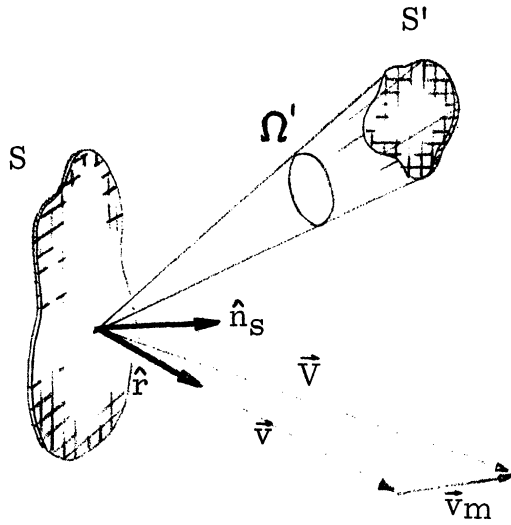


Fig. 1. Incident particle.

($\vec{v} + \vec{v}_m = \vec{V}$). The particles approaching a point on S from the direction r are those for which $v(\hat{r} \cdot \hat{n}_s) \geq 0$ where n_s is the unit normal at the point, thus Φ_{in} , the flux per unit area at the point on S, is given by (1) where f_1 is the one-dimensional Maxwellian distribution and N_{in} , the number density of incoming particles, is defined below.

$$\Phi_{in} = N_{in} \int_{\Omega_0} \int_{\vec{v} \cdot \hat{n}_s \geq 0}^{\infty} f_1(\hat{r} \cdot (\vec{V} - \hat{r}v)) v \hat{r} \cdot \hat{n}_s dv d\Omega. \quad (1)$$

The integration in solid angle is to be carried out over the entire hemisphere less Ω' the solid angle shielded from the in-coming flux by other surfaces. The vector \hat{r} in (1) is a unit vector in the direction of \vec{V} . Elsewhere in this discussion use will be made of an abbreviated form of (1), as in (4) using

$$\langle v_n \rangle_0 = \int_{\Omega_0} \int_{\vec{v} \cdot \hat{n}_s \geq 0}^{\infty} f_1(\hat{r} \cdot (\vec{V} - \hat{r}v)) v \hat{r} \cdot \hat{n}_s dv d\Omega. \quad (2)$$

and the number density N_{in} of incoming particles just outside S,

$$N_{in} = N_a \int_{\Omega_0} \int_{\vec{v} \cdot \hat{n}_s \geq 0}^{\infty} f_1(\hat{r} \cdot (\vec{V} - \hat{r}v)) dv d\Omega, \quad (3)$$

where N_a is the undisturbed ambient density.

Thus we have

$$\Phi_{in} = N_{in} \langle \hat{v} \cdot \hat{n}_o \rangle. \quad (4)$$

To develop a notation parallel with that to be used for the particle transfer between two surfaces, it is desirable to have a differential form of

(1):

$$\frac{d\Phi_{in}}{d\Omega} = \varphi_{in} = N_{in} \int_{\hat{r} \cdot \hat{n}_s \geq 0}^{\infty} f_1(\hat{r} \cdot (\bar{V} - \hat{r}v)) v \hat{r} \cdot \hat{n}_s dv. \quad (5)$$

In (5) φ_{in} is the flux per unit area onto S per unit solid angle in a direction \hat{r} .

The second elementary problem is the transfer of particles to a surface S_j from a second surface S_i , at rest with respect to S_j . From assumption (1) the element of flux emitted in the direction \hat{r}_{ij} (unit vector from a point p_i on S_i to a point p_j on S_j) by an element of area dS_i into an element of solid angle $d\Omega_j$ is,

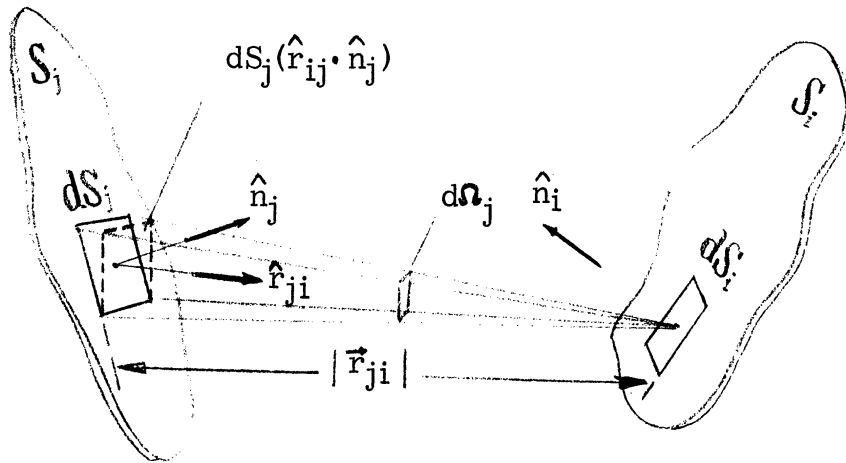


Figure 2. The Solid Angle Element

where Φ_i is the efflux per unit area of dS_i into the entire hemisphere. The portion of this flux arriving at a unit area of dS_j , φ_{ij} , may be found by observing that $d\Omega_j = dS_j (\hat{n}_j \cdot \hat{r}_{ji}) / r_{ji}^2$ in Figure 2 and substituting for $d\Omega_j$ in the above equation to obtain,

$$\frac{d}{ds_j} \frac{d\Phi}{d\Omega_j} = \varphi_{ij} = \Phi_i (\hat{n}_i \cdot \hat{r}_{ij}) (\hat{n}_j \cdot \hat{r}_{ji}) / 2\pi r_{ji}^2 . \quad (6)$$

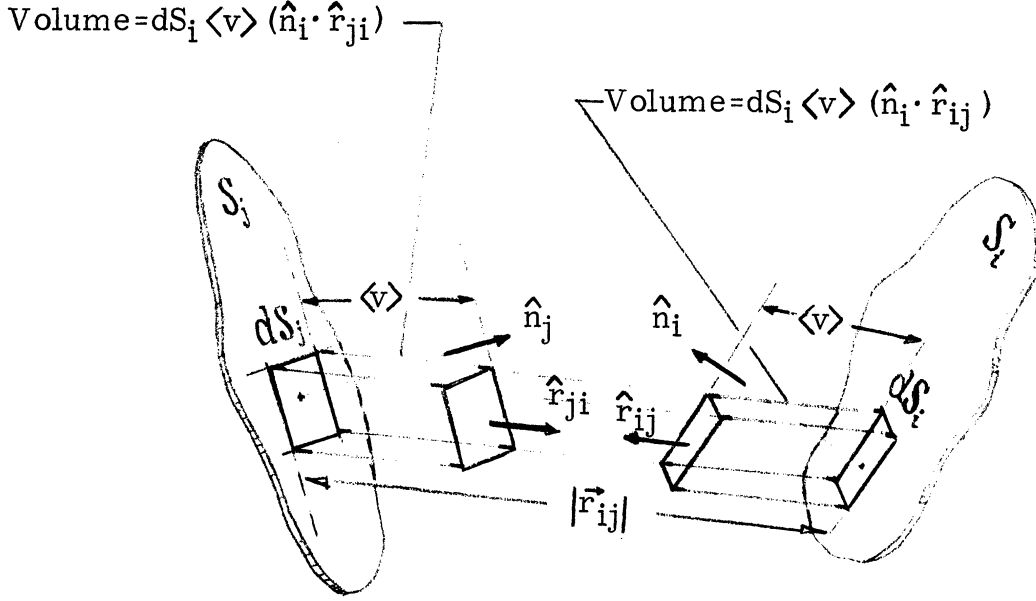


Figure 3. Number densities.

From Fig. 3 it can be seen that the flux per unit area leaving dS_i makes a contribution to the total number density just outside dS_i and its magnitude is determined by considering the number of particles contained in an elemental volume $\langle v \rangle dS_i \hat{n}_i \cdot \hat{r}_{ij}$:

$$(dN_i)_{out} = \Phi_i / 2\pi (n_i \cdot r_{ij}) \langle v \rangle dS_i . \quad (7)$$

The cross sectional area of the volume is $dS_i \hat{n}_i \cdot \hat{r}_{ij}$, $\langle v \rangle$ is the average particle speed in the \hat{r}_{ij} direction and represents the length of the volume, and $\Phi_i / 2\pi$ is the number of particles leaving per unit time. Referring to the form of (2), this can be expressed as

$$(dN_i)_{out} = \Phi_i / 2\pi \langle v_n \rangle \quad (8)$$

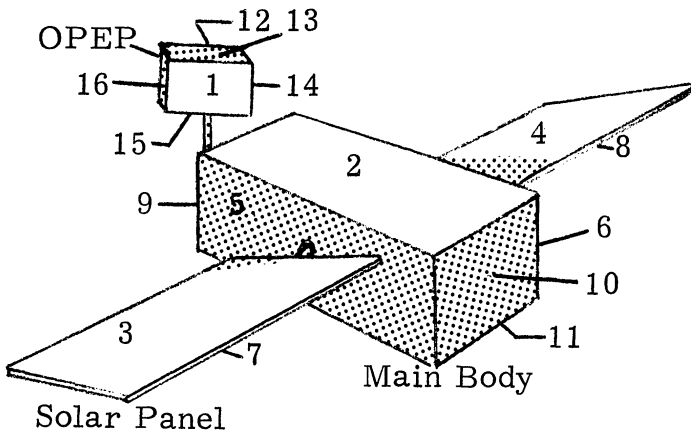
Similarly, the influx per unit area of dS_j from dS_i contributes

$$(dN_j)_{in} = \phi_{ij} / ((\hat{n}_j \cdot \hat{r}_{ji}) \langle v \rangle) dS_i = \phi_{ij} \frac{1}{\langle v_n \rangle} \quad (9)$$

to the number density just outside dS_j .

The use of $\langle v \rangle$ in (8) and (9) is justified by assumption (ii) as well as the fact that ϕ_i in (8) refers only to the flux leaving S_i and ϕ_{ij} refers only to flux incident upon S_j .

In section III the results obtained above will be applied to the OGO-IV spacecraft pictured schematically in Fig. 4. The numbers refer



to various surfaces. Quantities related to these surfaces will be labeled with subscripts corresponding to these numbers. The mass spectrometer experiments are mounted in the OPEP* at the surface labeled "1" in the figure.

Figure 4. The OGO Spacecraft.

Viewed from this surface the shaded areas are shielded and play a secondary role in the problem. Surface 10 cannot be seen from any other surface to be considered; hence, in the absence of intermolecular collisions (assumption (iii)) this surface plays no role in the problem. Surfaces 11, 12, 13, 14 and 15 are ignored also. Surface 11 can only exchange flux with the portions of 7 and 8 which cannot exchange this flux with 1, 2, 3, 4, or 9. This flux from 11 can contribute only in a multiple-bounce process, which can be neglected since the incident flux on 11 should itself be negligibly small as the surface is always parallel to the velocity \vec{V} when the spacecraft is functioning properly.

* Orbital Plane Experiment Package

Similar reasoning leads to the elimination of 12, 13, 14, 15, and 16.

Consequently the only surfaces which must be considered are 1, 2, 3, 4, 5, 6, 7, 8, and 9. Furthermore, no flux transfer between certain pairs of these is permitted by the geometry.

III. EXPANSION TO INCLUDE MULTIPLE REFLECTION CONTRIBUTIONS

The notation to be used in this section is similar to that already introduced, but in the interest of clarity it is explicitly defined below, and useful elementary results are restated.

Φ_i is the total influx per unit area at a point on the surface S_i from all directions.

Φ_{ij} is the total influx per unit area at a point on a surface S_i from the entire surface S_j , except that when $i=j$, Φ_{ij} denotes the influx per unit area at a point onto S_j from all "free" space.

φ_{ij} is that portion of the flux from a unit area of dS_i falling upon a unit area of dS_j . For $i=j$, φ_{ij} is the flux per unit area falling upon dS_j from "free" space having trajectories within the solid angle $d\Omega$.

$X_{ij} = \frac{1}{2\pi} [(\hat{n}_i \cdot \hat{r}_{ij}) (\hat{n}_j \cdot \hat{r}_{ji}) / r_{ij}^2]$, from (7) where n_i and n_j are the unit surface normals at points on S_i and S_j and \vec{r}_{ij} is the vector from the point on S_i to the point on S_j . Note also that $X_{ij} = X_{ji}$.

$$\varphi_{ij} = X_{ij} \Phi_i$$

$\langle v_n \rangle_{ij} = (\hat{n}_j \cdot \hat{r}_{ji}) \langle v \rangle_j$ where $\langle v \rangle_j$ is the average speed of particles emitted from S_j .

$$\varphi_{oj} = (\langle v_n \rangle_o N_{in})_j \text{ (from (2) and (3))}$$

$$\Phi_j = \sum \Phi_{ij}$$

$$\Phi_{ij} = \int \varphi_{ij} dS_i$$

$$\Phi_{oj} = \int_{\Omega_0} \varphi_{oj} d\Omega$$

The remainder of this section is devoted to finding symbolic expressions for Φ_{21} , Φ_{31} , Φ_{41} , Φ_{51} , and Φ_{91} which are to be compared with Φ_{01} ; all of which are evaluated near the location of the mass spectrometer sensors. The procedure consists in expanding the Φ 's, as sums of other Φ 's, and expanding these in turn with the aim of obtaining the ultimate expansion in terms of Φ_{0j} 's since these can be related to the ambient density. Each re-expansion generates terms of higher orders in the X's. For the geometry considered one can assume that terms beyond some order may be neglected. Considering the severity of assumption (ii) and the fact that the integrated X's are of the order of 0.2 or less, it is reasonable to limit consideration to order two or three in X. Consider:

$$\begin{aligned} \Phi_{21} = & \int \varphi_{21} dS_2 = \int \Phi_{20} X_{12} dS_2 = \int \Phi_{02} X_{12} dS_2 + \int \Phi_{12} X_{12} dS_2 + \int \Phi_{32} X_{12} dS_2 \\ & + \int \Phi_{42} X_{12} dS_2 + \int \Phi_{72} X_{12} dS_2 + \int \Phi_{82} X_{12} dS_2. \end{aligned}$$

It is necessary to continue by expanding Φ_{12} , Φ_{32} , Φ_{42} , Φ_{72} , and Φ_{82} . For example,

$$\Phi_{12} = \int \Phi_{01} X_{12} dS_1 + \int \Phi_{21} X_{12} dS_1 + \int \Phi_{31} X_{12} dS_1 + \int \Phi_{41} X_{12} dS_1 + \int \Phi_{71} X_{12} dS_1 + \int \Phi_{81} X_{12} dS_1$$

Substitution of these expansions into Φ_{21} and subsequent re-expansion of the terms they contain leads to a sum for Φ_{21} which contains terms of increasing orders in the X's. The same is of course true for Φ_{31} and Φ_{41} . If we omit all terms of order three and higher (10), (11), (12), and (13) result:

$$\begin{aligned} \Phi_{21} = & \int \Phi_{02} X_{12} dS_2 + \int X_{12} \int \Phi_{01} X_{12} dS_1 dS_2 + \int X_{12} \int \Phi_{03} X_{23} dS_3 dS_2 + \int X_{12} \int \Phi_{04} X_{24} dS_4 dS_2 \\ & + \int X_{12} \int \Phi_{07} X_{27} dS_7 dS_2 + \int X_{12} \int \Phi_{08} X_{28} dS_8 dS_2; \end{aligned} \quad (10)$$

$$\Phi_{31} = \int \Phi_{03} X_{13} dS_3 + \int X_{13} \int \Phi_{01} X_{13} dS_1 dS_3 + \int X_{13} \int \Phi_{05} X_{35} dS_5 dS_3 + \int X_{13} \int \Phi_{02} X_{23} dS_2 dS_3; \quad (11)$$

$$\Phi_{41} = \int \Phi_{04} X_{14} dS_4 + \int X_{14} \int \Phi_{01} X_{14} dS_1 dS_4 + \int X_{14} \int \Phi_{06} X_{46} dS_6 dS_4 + \int X_{14} \int \Phi_{02} X_{24} dS_2 dS_4; \quad (12)$$

$$\Phi_{91} = \int \Phi_{09} X_{19} dS_9 + \int X_{19} \int \Phi_{0,16} X_{1,16} dS_{16} dS_9. \quad (13)$$

For a spectrometer with an "enclosed" source or a "closed" pressure gauge, the flux through the entrance aperture due to Φ_{21} , Φ_{31} , Φ_{41} , and Φ_{91} determines the contribution of the spacecraft surfaces through reflections, to the measured densities. One might consider $(\Phi_{01} - \Phi_{01}' + \Phi_{21} + \Phi_{31} + \Phi_{41} + \Phi_{91}) / \Phi_{01}$ to be the relative "error" in ignoring the influence of neighboring surfaces, where Φ_{01} is the directly incident flux calculated by (1) and Φ_{01}' is the flux one would obtain if the entire hemisphere were free of obstacles to the incoming flux.

For certain orientations of the spacecraft, terms of order three in the X's or perhaps even higher, which were discarded in (10), (11), (12), and (13) may actually be larger than some terms retained in these equations. For these cases, certain of the retained terms may be negligible. The four equations will, however, always contain the dominant terms. It will frequently happen that the last two terms in (11) are small. The last two terms in (12) and (13) will be very small in noon-midnight orbits. In sunrise-sunset orbits the second term of (11) is zero, since the OPEP cannot "see" S_2 . In these orbits either Φ_{31} or Φ_{41} is zero since the OPEP is exposed to only one solar panel. The entire contribution of (13) is rather small, though perhaps not always negligible.

For future reference each of the additive terms in (10) through (13) will be given a symbolic name by extending the Φ notation already introduced:

$$\begin{aligned}
\int X_{12} \Phi_{02} dS_2 &= \Phi_{021}; & \int X_{12} \int X_{21} \Phi_{01} dS_1 dS_2 &= \Phi_{0121}; \\
\int X_{12} \int X_{23} \Phi_{03} dS_3 dS_2 &= \Phi_{0321}; & \int X_{12} \int X_{24} \Phi_{04} dS_4 dS_2 &= \Phi_{0421}; \\
\int X_{12} \int X_{27} \Phi_{07} dS_7 dS_2 &= \Phi_{0721}; & \int X_{12} \int X_{28} \Phi_{08} dS_8 dS_2 &= \Phi_{0821}; \\
\int X_{13} \Phi_{03} dS_3 &= \Phi_{031}; & \int X_{13} \int X_{31} \Phi_{01} dS_1 dS_3 &= \Phi_{0131}; \\
\int X_{13} \int X_{35} \Phi_{05} dS_5 dS_3 &= \Phi_{0531}; & \int X_{13} \int X_{23} \Phi_{02} dS_2 dS_3 &= \Phi_{0231}; \\
\int X_{14} \Phi_{04} dS_4 &= \Phi_{041}; & \int X_{14} \int X_{41} \Phi_{01} dS_1 dS_4 &= \Phi_{0141}; \\
\int X_{14} \int X_{46} \Phi_{06} dS_6 dS_4 &= \Phi_{0641}; & \int X_{14} \int X_{24} \Phi_{02} dS_2 dS_4 &= \Phi_{0241}; \\
\int X_{19} \Phi_{09} dS_9 &= \Phi_{091}; & \int X_{19} \int X_{1,16} \Phi_{0,16} dS_{16} dS_9 &= \Phi_{0,16,9}.
\end{aligned} \tag{14}$$

For "open source" instruments the number density contributions of the fluxes must be determined. In order to do this by means of relations similar to (8) and (9), it is necessary to make an assumption about the relation of $\langle v_n \rangle$ before a surface collision with $\langle v_n \rangle$ after this collision. The simplest assumption to make is that the particles are fully accommodated. That is, $\langle v_n \rangle$ may be calculated from the Maxwellian distribution characteristic of the temperature of the last surface touched by the particles, or in the case of those incident from "free space", $\langle v_n \rangle_o$ is to be calculated from $f(\vec{V}-\vec{v})$ with a temperature corresponding to the ambient atmospheric temperature. Unfortunately, full accommodation is probably a poor assumption. For this reason, formulae will be derived for the number densities which allow the assumption about $\langle v_n \rangle$ to be made later. To take care of this, the notation must be expanded. Let $\langle v_n \rangle_{o, i_1 \dots i_k}$ be the average speed of particles incident from space onto S_{i_1} , and subsequently transferred to S_{i_2}, \dots to S_{i_k} . Note $\langle v_n \rangle_{o, i_1 \dots i_k}$ is the mean normal speed after interaction with S_{i_k} not before interaction with S_{i_k} .

Using this notation each of the terms in (14) will contribute twice to the number density just outside a point on S_1 , once as an incident density as in (8) and once as a reflected density as in (9). Let N_{021} be the sum of the incident and reflected contributions of Φ_{021} . Then,

$$N_{021} = \Phi_{021} \left(\frac{1}{\langle v_n \rangle_{02}} + \frac{1}{\langle v_n \rangle_{021}} \right)$$

and similar relations hold for the other contributions in (14) a complete list of which is given in (15):

$$\begin{aligned}
 N_{021} &= \Phi_{021} \left(\frac{1}{\langle v_n \rangle_{02}} + \frac{1}{\langle v_n \rangle_{021}} \right); & N_{0121} &= \Phi_{0121} \left(\frac{1}{\langle v_n \rangle_{012}} + \frac{1}{\langle v_n \rangle_{0121}} \right); \\
 N_{0321} &= \Phi_{0321} \left(\frac{1}{\langle v_n \rangle_{032}} + \frac{1}{\langle v_n \rangle_{0321}} \right); & N_{0421} &= \Phi_{0421} \left(\frac{1}{\langle v_n \rangle_{042}} + \frac{1}{\langle v_n \rangle_{0421}} \right); \\
 N_{0721} &= \Phi_{0721} \left(\frac{1}{\langle v_n \rangle_{072}} + \frac{1}{\langle v_n \rangle_{0721}} \right); & N_{0821} &= \Phi_{0821} \left(\frac{1}{\langle v_n \rangle_{082}} + \frac{1}{\langle v_n \rangle_{0821}} \right); \\
 N_{031} &= \Phi_{0531} \left(\frac{1}{\langle v_n \rangle_{032}} + \frac{1}{\langle v_n \rangle_{031}} \right); & N_{0131} &= \Phi_{0131} \left(\frac{1}{\langle v_n \rangle_{013}} + \frac{1}{\langle v_n \rangle_{0131}} \right); \\
 N_{0531} &= \Phi_{0531} \left(\frac{1}{\langle v_n \rangle_{053}} + \frac{1}{\langle v_n \rangle_{0531}} \right); & N_{0231} &= \Phi_{0231} \left(\frac{1}{\langle v_n \rangle_{023}} + \frac{1}{\langle v_n \rangle_{0231}} \right); \\
 N_{041} &= \Phi_{041} \left(\frac{1}{\langle v_n \rangle_{04}} + \frac{1}{\langle v_n \rangle_{041}} \right); & N_{0141} &= \Phi_{0141} \left(\frac{1}{\langle v_n \rangle_{014}} + \frac{1}{\langle v_n \rangle_{0141}} \right); \\
 N_{0641} &= \Phi_{0641} \left(\frac{1}{\langle v_n \rangle_{064}} + \frac{1}{\langle v_n \rangle_{0641}} \right); & N_{0241} &= \Phi_{0241} \left(\frac{1}{\langle v_n \rangle_{024}} + \frac{1}{\langle v_n \rangle_{0241}} \right); \\
 N_{091} &= \Phi_{091} \left(\frac{1}{\langle v_n \rangle_{09}} + \frac{1}{\langle v_n \rangle_{091}} \right); & N_{0,16,9,1} &= \Phi_{0,16,9,1} \left(\frac{1}{\langle v_n \rangle_{0,16,9}} + \frac{1}{\langle v_n \rangle_{0,16,9,1}} \right);
 \end{aligned}
 \tag{15}$$

The above are to be compared, along with $(\Phi_{01} - \Phi_{01}') \left(\frac{1}{\langle v_n \rangle_{01}} + \frac{1}{\langle v_n \rangle_0} \right)$ to $\Phi_{01}' \left(\frac{1}{\langle v_n \rangle_{01}} + \frac{1}{\langle v_n \rangle_0} \right)$. If N_{21} , N_{31} , N_{41} , N_{91} , N_{01} and N_{01}' are defined as in (16) the error in considering only the front plate of the OPEP instead of the entire spacecraft is given by (17). The number density contributions are:

$$\begin{aligned}
N_{01} &= \Phi_{01} \left(\frac{1}{\langle v_n \rangle_{01}} + \frac{1}{\langle v_n \rangle_0} \right); \\
N'_{01} &= \Phi'_{01} \left(\frac{1}{v_n}_{01} + \frac{1}{v_n}_0 \right); \\
N_{21} &= N_{021} + N_{0121} + N_{0321} + N_{0421} + N_{0721} + N_{0821}; \\
N_{31} &= N_{031} + N_{0131} + N_{0531} + N_{0231}; \\
N_{41} &= N_{041} + N_{0141} + N_{0641} + N_{0241}; \\
N_{91} &= N_{091} + N_{0,16,9,1};
\end{aligned} \tag{16}$$

The relative error is:

$$\frac{\Delta N}{N} = \frac{N_{01} - N'_{01} + N_{21} + N_{31} + N_{41} + N_{91}}{N_{01}} \tag{17}$$

For (15) to be correct it is necessary that the $\langle v_n \rangle$ not be a function of position on the various surfaces. The legitimacy of this should be examined. In the first place it was effectively postulated that $\langle v_n \rangle_{0, i_1, i_2, \dots, i_k}$ is a function of $\langle v_n \rangle_0, T_{i_1}, \dots, T_{i_k}$, and it is evident from (2) that $\langle v_n \rangle_0$ is a function of position on a surface since the limits of integration (schematically written Ω_0 in (2) and (3)) are functions of position on a surface. Since the particles contributing to all the N's in (15) will undergo one, two or three bounces and each interaction tends to "erase" more and more of the identity of the incident velocity $\langle v_n \rangle_0$, and since the T_i are essentially constant on S_i and T_i is approximately equal to T_j , it is reasonable to assume $\langle v_n \rangle$ to be constant across a given surface which permits the use of (15). This computation could be extended if $\langle v_n \rangle$ could be expressed as an analytic function of $\langle v_n \rangle_0$ and the T_i . Then $\langle v_n \rangle$ could be determined at each point and integrations similar to those indicated in (14) could be performed explicitly, with $\langle v_n \rangle$ included within the integral, thereby deriving new analytic expressions for the N's.

On the surfaces receiving the largest incident flux Φ_{oj} - those directed approximately into the velocity vector $\langle v_n \rangle_o$ will be nearly constant. Other surfaces contribute mostly by higher order processes, hence the statement concerning "erasing" the identity of $\langle v_n \rangle_o$ by subsequent interactions applies to these contributing surfaces.

Another aspect of equations (14) should be restated explicitly. The limits of integration are functions of the relative orientations of the OPEP and the solar paddles with respect to each other, and with respect to the main body as well. It is intended that the integrations in a given transfer process extend only over the area of the surface from which the particles are being transferred that can be "seen" from the point to which they are being transferred. Thus the limits depend upon the point to which the transfer is made. This complicated geometric analysis is the subject of a separate report in which the integral equations are set-up in more detail.

IV. POSITIVE ION MEASUREMENTS

If the incident particles are ambient ions there is no reflection from surfaces since the ions may be assumed to be neutralized on contact. In this case $\Phi_1 = \Phi_{01}$. Equations (1), (2), and (4) are still applicable on the assumption that the electric and magnetic fields around the spacecraft can be neglected when the Debye length is smaller than the characteristic spacecraft dimensions. The neighboring spacecraft surfaces still influence Φ_{01} because the spectrometer is shielded from a portion of the incoming flux, Φ'_{01} , which would reach the instrument if the surfaces were absent. For Φ_{01} the integration in (1) extends over the entire hemisphere. The relative

error if the surfaces are neglected is therefore given by $(\Phi_{01} - \Phi'_{01}) / \Phi_{01}$ for ambient ions. One can write an expression for the number density of ambient ions (18) which is of the same form as (16) and (19) gives the relative "error" in neglecting the spacecraft surfaces:

$$N_{01} = \Phi_{01} \left(\left\langle \frac{1}{v} \right\rangle_{n_{01}} \right); \quad (18)$$

$$\frac{\Delta N}{N} = (\Phi_{01} - \Phi'_{01}) \left(\left\langle \frac{1}{v} \right\rangle_{n_{01}} \right). \quad (19)$$

V CONCLUSIONS

The introduction of a few simplifying assumptions, has permitted a formal solution to be obtained for the influence of the OGO spacecraft upon atmospheric density measurements. Because of the large number of integrations over variable limits, numerical solutions are not easily obtained. Solutions for every important orientation of the solar panels, OPEP, and man body, with respect to the spacecraft velocity, can be expected to consume considerable processing time even when programmed for a digital computer.

A program for a Digital Equipment Corporation PDP-8 computer is currently being prepared to perform these calculations. This program and the characteristics of the above solutions will be the subject of a subsequent report.

REFERENCES

- (1) Hinton, B. B., R. D. Kistler, R. J. Leite, and C. J. Mason, IEEE Trans. on Geoscience Electronics GE-7 p. 107 (1969).

