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SPACE-CHARGE EQUILIBRIUM IN A MAGNETRON

A STATISTICAL APPROACH

Technical Report No. 10
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ABSTRACT

This report discusses the steady-state space-charge distribution in a d-c magnetron when its anode voltage is lower than its cut-off voltage. It is shown that the discrete electron-to-electron interaction has a cumulative effect on the space-charge distribution that is not negligible. The distributions derived by Brillouin, Slater, and others are not steady-state distributions, since they are obtained without regard to the discrete interaction. The actual distribution and its dependence on the ratio of anode voltage to cut-off voltage are discussed qualitatively, but no attempt is made to calculate the space-charge distribution explicitly.

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I. INTRODUCTION

As an initial state for the oscillating magnetron the non-conducting (or approximately non-conducting) condition of a d-c magnetron has a considerable interest and has received appreciable attention, both of a theoretical and an experimental nature. Nontheless, a satisfactory agreement has not been reached about the shape of the electron orbits and the detailed distribution of potential and electron density in such a space charge.

The solutions presented necessarily rest on postulates and assumptions that constitute idealizations of the real conditions. So does, of course, all physical theory, but in this case well-known factors have been neglected with the justification that their effect is judged to be so small that the approximate solution resulting will be close enough to be of value. Furthermore, the mathematical difficulties of a more rigorous approach appeared prohibitive.

It is the purpose of this report to reconsider the conventional simplifications on which these solutions are based, to discuss whether or not they introduce appreciable errors, and to investigate the feasibility of a solution from a more realistic set of assumptions. Since the space charge is essentially a gas formed by discrete electrons, classical statistical mechanics offers a logical approach to a fresh study of the problem.

We shall first place the problem into the framework of statistical mechanics. The next step is to consider the solutions proposed so far in this new light and to show that they can not possibly represent steady states. This discussion does not directly indicate whether the errors in the distribution of space charge and potential are large or small. Finally, the problem of finding a solution consistent with the laws of statistical mechanics will be investigated and the result compared with the previously suggested solutions.

II. EQUATIONS FOR ELECTRON MOTION IN A LINEAR MAGNETRON

Since we are going to apply the methods of statistical mechanics to the problem of electron motion in a magnetron, it is convenient to write the equations of motion in Hamiltonian form. The total energy of an electron is:

$$W = \frac{1}{2m}\left[\vec{p} - e\vec{A}\right]^2 - eE ,\tag{1}$$

where $\vec{p}$ is the momentum vector, A the vector potential of the constant magnetic field, and E the scalar electric potential. The momentum $\vec{p}$ is given by

$$\vec{p} = m\vec{v} + e\vec{A} ,\tag{2}$$

where $\vec{v}$ is the velocity vector of the electron.

To simplify the mathematical processes, we choose a plane magnetron and a Cartesian coordinate system (Fig. 1). The cathode is represented by
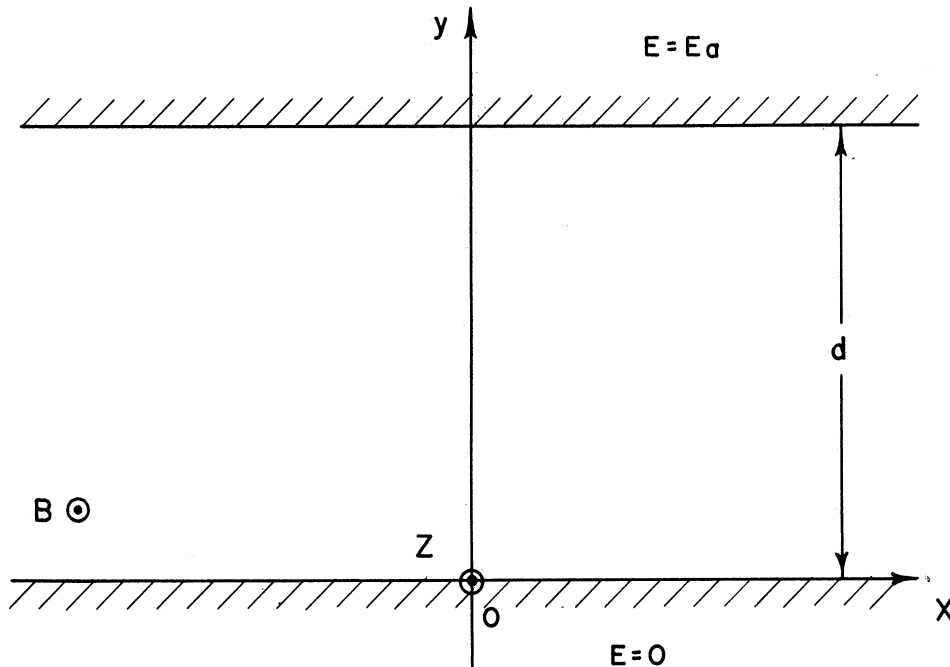

FIG. I

COORDINATE SYSTEM FOR PLANE MAGNETRON

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the plane $y = o$, the anode by the plane $y = d$. The uniform magnetic flux

vector $\vec{B}$ is parallel to the positive z-axis. The potential of the cathode

is taken to be zero and the potential of the anode positive is equal to E_a.

No variations are taking place with x and z.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0 \ . \tag{3}$$

The vector potential $\vec{A}$ then has only one component, A_x, such that

$$- \frac{\partial A_x}{\partial y} = B_z = B \ , \tag{4}$$

$$A_x = - B \, y \ , \tag{5}$$

since we can choose A as well as E to be zero at the cathode.

The Hamiltonian (Eq 1) then is

$$W = \frac{1}{2m}\left\{(p_x + eB \, y)^2 + p_y{}^2 + p_z{}^2\right\} - eE \ , \tag{6}$$

and the canonical equations of motion

$$- \frac{dp_x}{dt} = \frac{\partial W}{\partial x} = 0 = m \frac{dv_x}{dt} + eBv_y \ , \tag{7}$$

$$- \frac{dp_y}{dt} = \frac{\partial W}{\partial y} = \frac{1}{2m} \, (p_x + eB \, y)\cdot eB - e \frac{\partial E}{\partial y} = eBv_x - e \frac{\partial E}{\partial y} \ , \tag{8}$$

$$- \frac{dp_z}{dt} = \frac{\partial W}{\partial z} = 0 \ , \tag{9}$$

$$\frac{dx}{dt} = \frac{\partial W}{\partial p_x} = \frac{1}{m} \, (p_x + eB \, y) = v_x \ , \tag{10}$$

$$\frac{dy}{dt} = \frac{\partial W}{\partial p_y} = v_y \ , \tag{11}$$

$$\frac{dz}{dt} = \frac{\partial W}{\partial p_z} = v_z \ . \tag{12}$$

The $\underline{\text{constants of the motion}}$, or the parameters that determine the energy state of each electron, are W, p_x and p_z. Optionally, we can specify the state of an electron by p_x, p_z and p_{yo}, the last quantity being the value of p_y at the cathode $(y = 0)$. It should be noted that p_{yo} may be imaginary, since the orbit of an electron may not necessarily reach the cathode.

In the study of the space charge in the magnetron we shall be interested in the distribution of the electrons in $\underline{\text{phase space}}$ or μ-space, i.e., a six-dimensional space with the coordinates x, y, z, p_x, p_y, p_z. Actually, a subspace y, p_x, p_y will contain all the boundary surfaces and orbit projections necessary for the study of the problem.

III. EQUILIBRIUM STATES

If the cathode of the magnetron is at constant temperature and
no current flows to the anode, no energy is received or lost by the swarm
of electrons in the tube. It should, under these hypothetic conditions, be
possible to consider the magnetron as a closed system in thermal equilibrium.
In this section we shall consider the conditions to be satisfied for thermal
equilibrium in a magnetron.

When a minute current flows to the anode the problem becomes a
transport problem, and the solution is obtained from the thermal equilibrium
by applying a small perturbation to the distribution function for the elec-
trons. As the current is increased, the diffusion of electrons through the
space charge from the cathode gradually changes to a steady flow of the whole
space charge towards the anode. The intermediate conditions, with severely
distorted distribution function but no coherent flow pattern, offer the greatest
mathematical difficulties; unfortunately, there are reasons to believe that
this is the actual state of affairs in a cut-off magnetron.

Since the space charge is assumed to be in equilibrium with the
thermionic cathode emitter, the distribution density function at $y = 0$ is
necessarily of the Maxwell-Boltzmann type.

$$P_0 = A \cdot \exp \left[- \frac{\alpha}{2m}(p_x^2 + p_{yo}^2 + p_z^2) \right] \qquad (13)$$

The distribution density function P at an arbitary point
approaches its equilibrium value under the influence of two simultaneous
processes, convection by electron motion, and exchange of energy and momentum
between the electrons because of Coulomb interaction.

We shall not include here the complete classical derivation of the distribution density function. It is only necessary to investigate the consequences of the magnetic field and the curvilinear orbits of the electrons.

Under equilibrium conditions the convection-current element formed by a certain group of electrons in phase space must be continuous throughout its path through x-y-z space. The expected value of a current element formed by electrons of y-directed momentum between p_{yo} and $p_{yo} + dp_{yo}$ at the cathode is, therefore, equal to the corresponding element at any other value of y.

$$eP_o \; p_{yo} \; dp_{yo} \quad = \quad eP \; p_y \; dp_y \; . \tag{14}$$

However, conservation of energy requires

$$W \quad = \quad \frac{1}{2m}\left\{p_x^2 + p_{yo}^2 + p_z^2\right\} \quad = \quad \frac{1}{2m}\left\{(p_x + eB \; y)^2 + p_y^2 + p_z^2\right\} - eE, \tag{15}$$

and consequently

$$p_{yo} \; dp_{yo} \quad = \quad p_y \; dp_y \; . \tag{16}$$

Continuity therefore requires that the distribution density at y is

$$P \; dp_y \quad = \quad P_o \; dp_y \; . \tag{17}$$

It should be noted that the distribution density function P is given by (17) only for the regions in phase space accessible to electrons emitted from the cathode. We shall later map these regions.

The interaction between the electrons during thermal equilibrium must be such that the same number of electrons are removed from a certain cell in phase space as are entering into the cell during the same time interval.

Suppose that a certain encounter involves n electrons with the initial energy states $W_1 \ldots W_n$, and after the encounter, the energy states $W_1' \ldots W_n'$. Conservation of energy requires that

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$$W_1 + W_2 + \ldots W_n = W_1' + W_2' + \ldots W_n' \quad . \tag{18}$$

The rates at which such encounters and the inverse encounters take place should be equal and are proportional to

$$P_1 \cdot P_2 \cdot P_3 \ldots P_n = P_1' \cdot P_2' \ldots P_n' \tag{19}$$

Since the exponential function transforms a sum into a product, it is obvious that the Maxwell-Boltzmann distribution function satisfies the requirements (18) and (19), as well as (13) and (17). Thus we can write:

$$P = A \cdot e^{-\alpha W} \quad , \tag{20}$$

where A is constant related to the total number of electrons in the space charge and $\alpha = 1/kT$.

It is interesting to note that the presence of the magnetic field and the consequent curvature of the electron orbits does not affect the distribution function. The factors that determine the distribution arise from conservation of energy and isotropy of interaction.

In order to find a space-charge distribution compatible with thermal equilibrium, we integrate Eq 20 with respect to p_x, p_y, and p_z from $-\infty$ or $+\infty$. The result is

$$\rho = \rho_0 e^{\alpha e E} \tag{21}$$

This relation is combined with Poisson's equation to

$$\frac{\partial^2 E}{\partial y^2} = -\frac{\rho}{\epsilon_0} = -\frac{\rho_0 e^{\alpha e E}}{\epsilon_0} \quad , \tag{22}$$

$$\frac{1}{2}\left(\frac{\partial E}{\partial y}\right)^2 = -\frac{\rho_0}{\alpha e \epsilon_0} e^{\alpha e E} + C_1 \quad . \tag{23}$$

In order to perform the second integration, we introduce

$$Q = e^{\alpha eE},\qquad(24)$$

$$\frac{dQ}{Q\sqrt{a+Q}} = \frac{-2\,\rho_0\,\alpha e}{\epsilon_0}\,dy\;.\qquad(25)$$

Since the first term in Eq 23 is always positive, C_1 and a must be negative if a potential minimum exists.

Changing the sign of a and integrating, we get

$$\frac{2}{\sqrt{a}}\tan^{-1}\sqrt{\frac{Q-a}{a}} = by + C_2\qquad(26)$$

$$\rho = \frac{\rho_0\cdot a}{\cos^2 b(y-y_m)} = \rho_0\left[\frac{\cos by_m}{\cos b(y-y_m)}\right]^2\qquad(27)$$

$$E = \frac{2}{\alpha e}\log\frac{\cos by_m}{\cos b(y-y_m)}\qquad(28)$$

where

$$b = \sqrt{\frac{-2\,\rho_0\,\alpha e}{\epsilon_0}}\qquad(29)$$

The boundary conditions are represented by the constants ρ_0, a, and y_m. For negative values of y_m the solution represents temperature-limited conditions.

Fig. 2 indicates the space-charge distribution required for this thermal equilibrium. It obviously does not resemble the distribution in a cut-off magnetron, since it demands emission of electrons from the anode with the same temperature as those emitted from the cathode but with considerably higher density,

$$\rho_a = \rho_0\, e^{\alpha eE_a}\qquad(30)$$
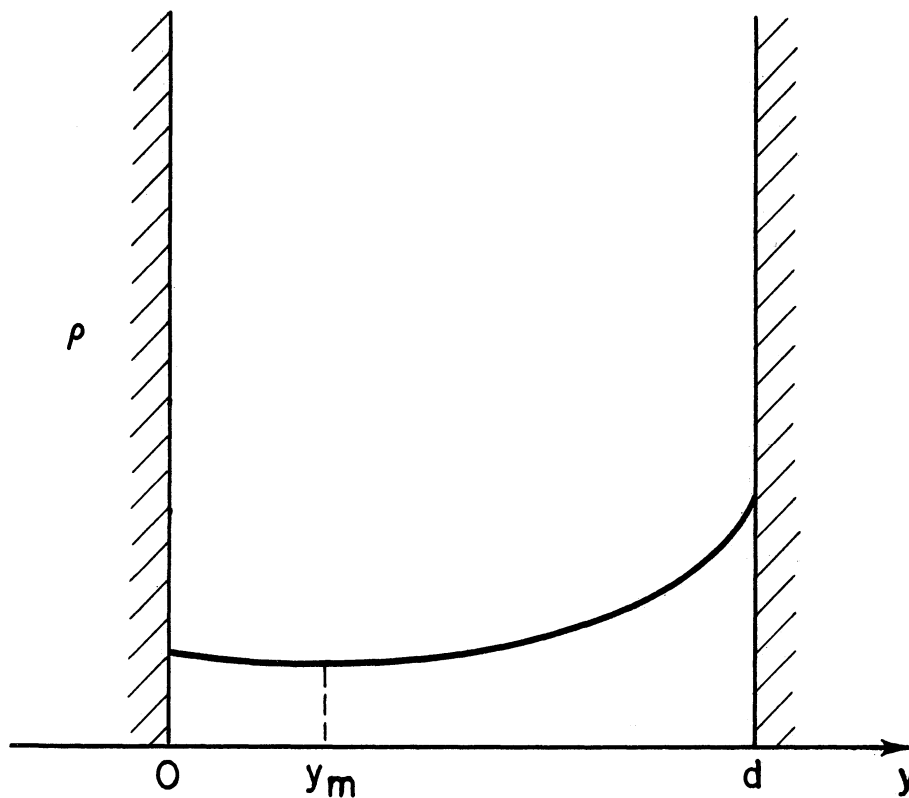
FIG. 2
SPACE-CHARGE DISTRIBUTION
AT THERMAL EQUILIBRIUM

Nontheless this equilibrium has a certain interest to us, since the difference between this distribution and the actual distribution in any particular volume element in real space dictates the amount of diffusion that takes place in that element. A steady state is reached when the diffusion into every energy state is equal to the diffusion out of the same energy state.

Before we investigate more closely this diffusion process, we shall in the next section discuss the representation of electron energy states as points and orbits in phase space.

IV. INITIAL STATES AND PHASE-SPACE REPRESENTATION OF ELECTRON ENERGY STATES

When a magnetron is switched to a source with a voltage E smaller than the cut-off voltage of the magnetron, a charge Q will flow to the magnetron. The source supplies the energy EQ, but the stored energy in the magnetron is only 1/2 EQ. The difference is lost in the circuit resistance and, possibly, in a temporary increase in the electron temperature of the magnetron space charge above the cathode temperature. These transient effects we shall disregard and assume that the initial conditions are consistent with conservation of energy and momentum for each electron emitted from the cathode and with Poisson's law.

As stated earlier in this report, the energy state of an electron is characterized by its energy W and two of the components of its momentum, p_x and p_z, these three quantities being constants of the motion, as long as energy and momentum are conserved. On the other hand, the component p_y and the potential energy vary during the motion.

In order to map the regions in phase space accessible to the electrons emitted from the cathode it is sufficient to consider the space p_x, p_y, y.

The accessibility criterion is obtained from

$$2m\,W = p_x^2 + p_{yo}^2 + p_z^2 = (p_x + eBy)^2 + p_y^2 + p_z^2 - 2m\,eE$$

or

$$p_y^2 = p_{yo}^2 + 2m\,eE - e^2B^2y^2 - 2p_x\,eBy \tag{31}$$

where p_y and p_{yo} have to be real quantities.

Fig. 3 shows a sketch of the boundary surface between the accessible and the inaccessible part of p_x-p_y-y-space. It is, of course, qualitative only, since E (y) is not known but related to the space-charge distribution by Poisson's

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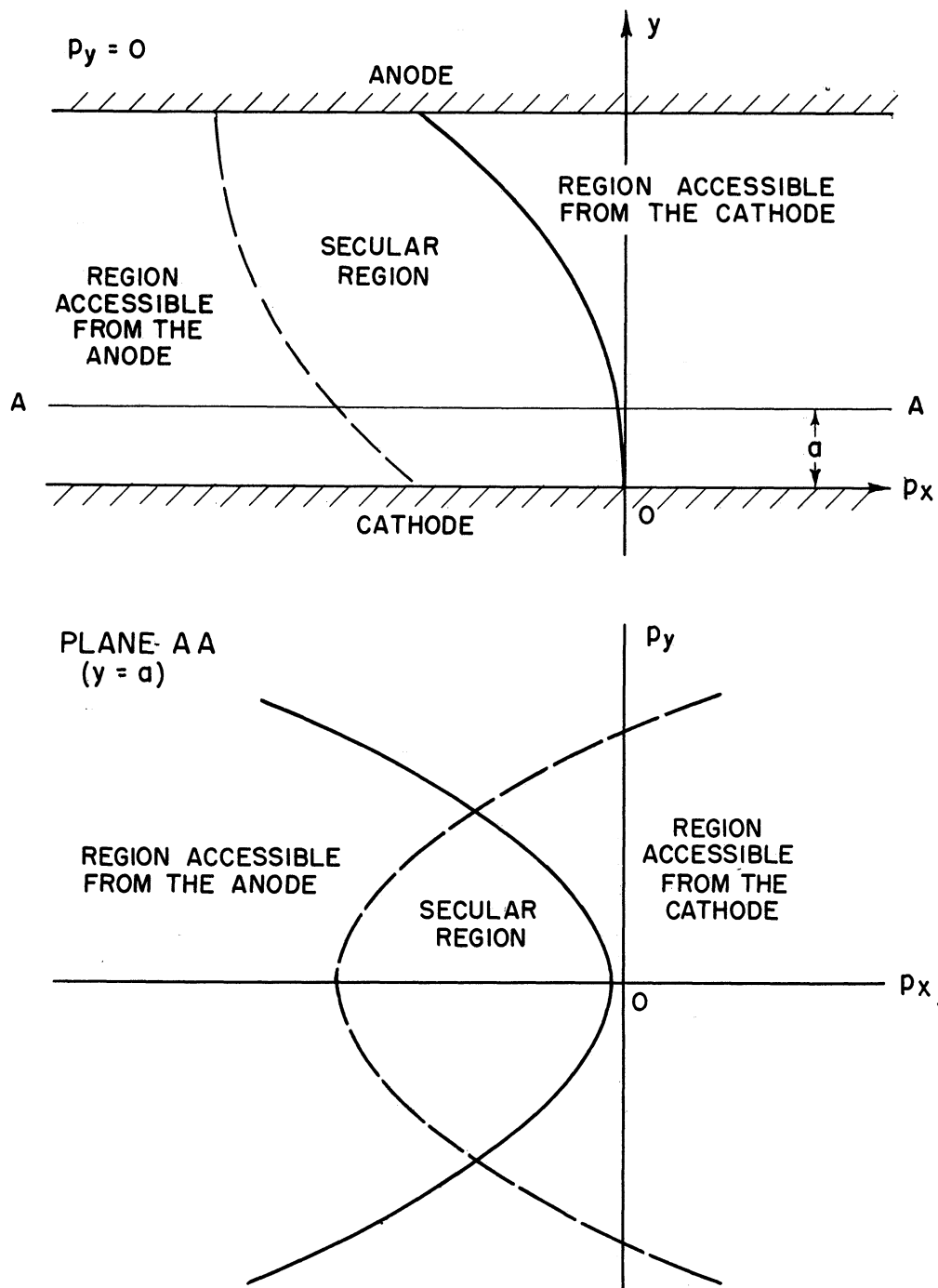
FIG. 3

REGIONS IN PHASE SPACE ACCESSIBLE TO ELECTRONS
WHOSE ORBITS INTERSECT THE ANODE OR CATHODE
PLANE.

law. Integration of the distribution function (20) over the accessible space with respect to p_x and p_y would give the relation between p, E, and y that, combined with Poisson's equation, determines the initial conditions.

The inaccessible region has the shape of a distorted cone with its apex on the axis $p_y = 0$, but in general not at $p_x = 0$. The axis $p_y = 0$ to the left of the apex is a generatrix of the surface. It should be pointed out that the fact that the inaccessible region reaches the axis does not mean that electrons with $p_y = 0$ and p_x in this range can not escape from the cathode. They do, but only tangentially to the p_x-p_y-plane, describe a broad elliptic orbit in the y-p_y-plane and reach their turning point on the upper part of the boundary line of the accessible region or hit the anode.

In Fig. 3 the boundary for interception of the electrons by the anode is also indicated. In other words, any electron to the left of this surface will be removed by impact on the anode. The space-charge density can therefore be considered zero to the left of this boundary except where this region overlaps the region accessible from the cathode.

The volume between the two boundaries contains all the possible energy states whose electron orbits reach neither the anode nor the cathode. We shall refer to this volume as the <u>secular</u> region since the life of an electron energy state in this region is very long compared with the period of the cyclic motion of the electron. It should be noted that:

1. <u>Only an infinitesimal change in momentum is required for an electron to cross the boundary into this volume.</u>

2. <u>The electron population of this volume will continue to increase until the current flowing to the anode through the opposite boundary exceeds the current entering the volume from the cathode-accessible region.</u>

3. <u>Whether the discrete electron-electron interaction is weak or strong determines primarily the time required to reach a steady state but not necessarily the final space-charge distribution.</u>
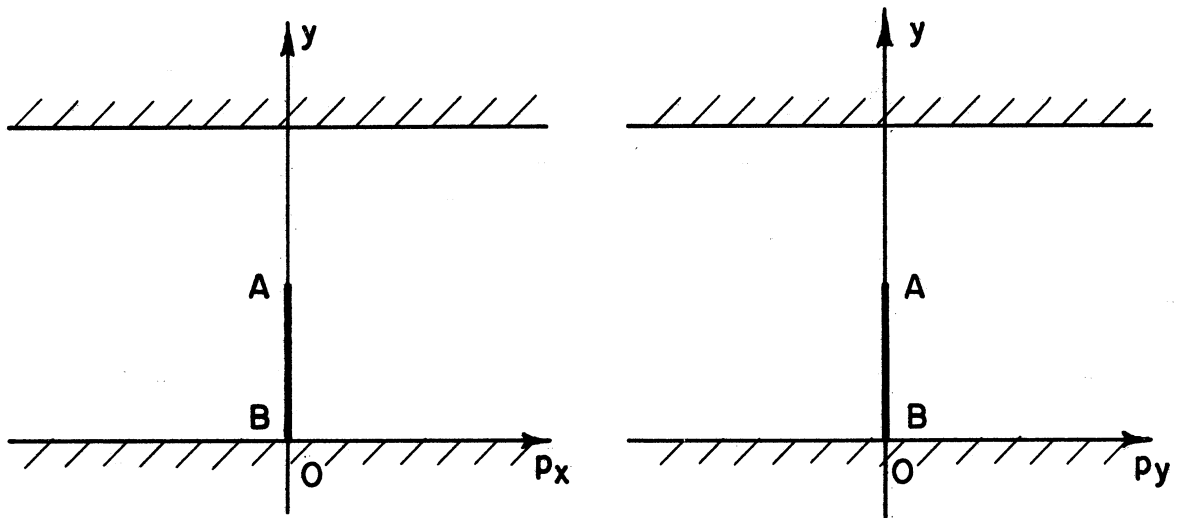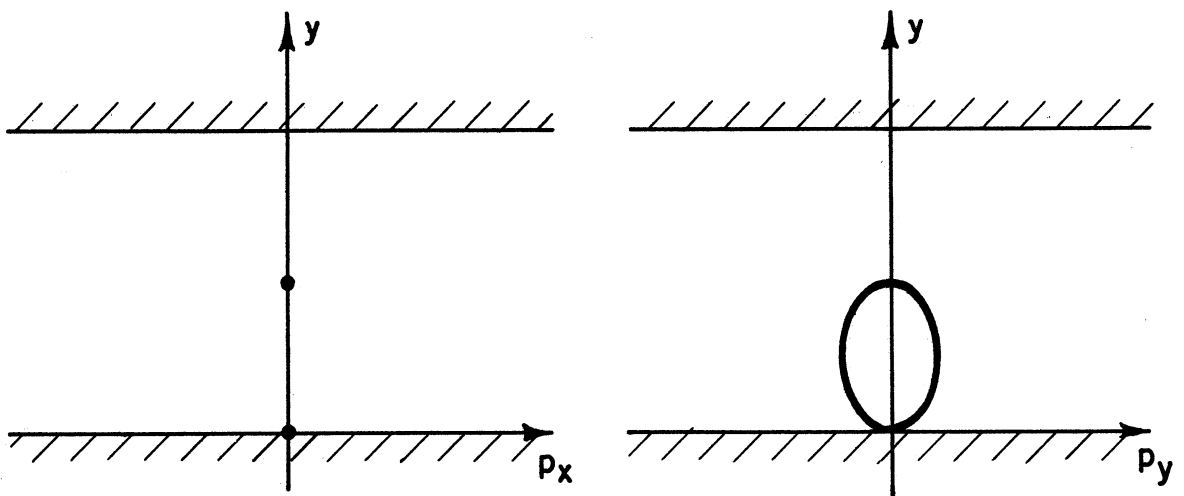
FIG. 4
BRILLOUIN DISTRIBUTION IN PHASE SPACE

FIG. 5
DOUBLE-STREAM DISTRIBUTION IN PHASE SPACE

Before discussing the possibility of determining at least roughly

the final space-charge distribution, we shall give some brief comments on

previously proposed solutions to the space-charge distribution in the cut-off

magnetron. Common to them all is that the solution is identified with some

form of what we have called initial conditions; sometimes the justification

is given that the interaction between discrete electrons is small enough to

be neglected. The three underlined conditions above show that this reasoning

is not correct because of the effect of the interaction, however weak, is cumu-

lative.

The Brillouin or single-stream space-charge distribution[1,2] in phase

space (Fig. 4) is limited to the line AB and independent of y within this range.

At first sight this distribution may appear to be a possible equilibrium at

$0°K$ temperature. However, it would be strange indeed if all electrons occupied

the same point in momentum space, although lower energy states certainly are

possible. But we must consider the possibility that the state be metastable

because of the lack of a process whereby the unoccupied energy states could

be populated. Obviously such a process does exist. Since the space charge

is formed by discrete electrons whose velocity varies with y, even though

their orbits do not intersect, the electric field at the edge of the space-

charge cloud necessarily fluctuates in magnitude and direction both in time

and space. A diffusion will therefore take place, both out into the unoccupied

space and towards the cathode. The energy required for this random motion is,

of course, supplied by the d-c electric field.

The double-stream distribution described by Slater,[4] Page and

Adams,[5,6] and others is represented by an approximately elliptic line charge

in phase space (Fig. 5). The space-charge density in real space at the cathode

and at the edge of the swarm is infinite; a finite minimum is located at an

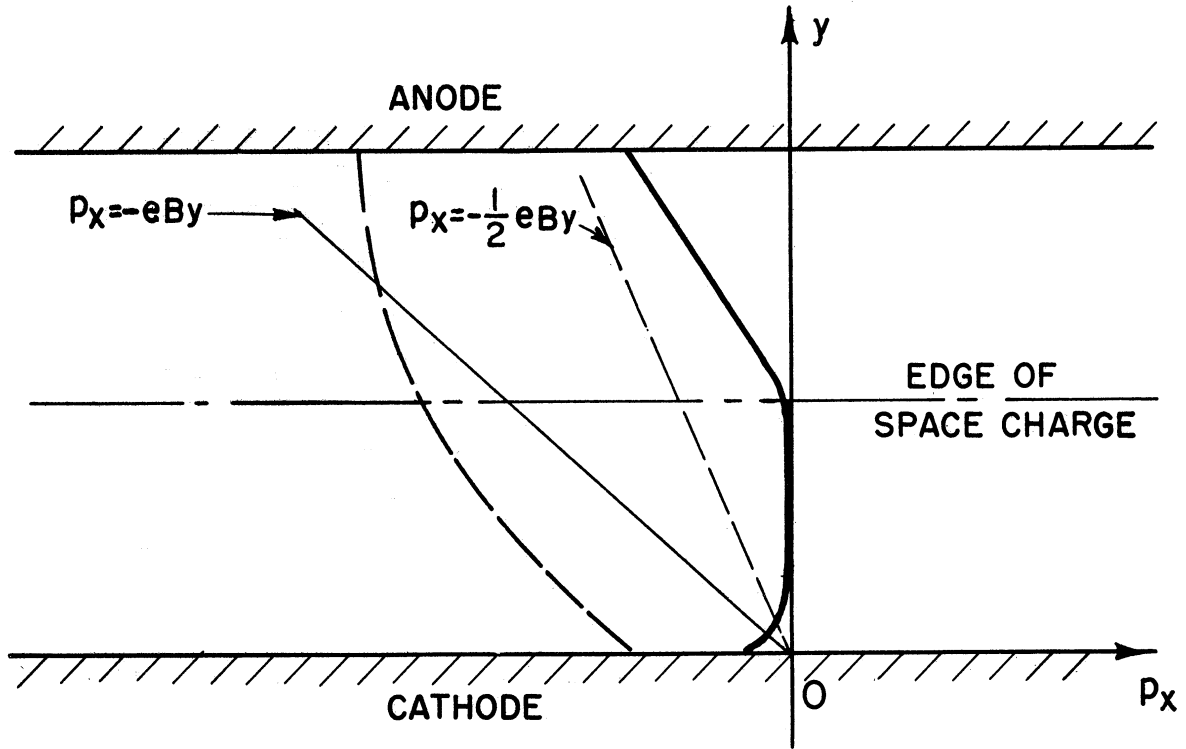intermediate plane. This is also a $0°K$ distribution, since all the electrons

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FIG. 6
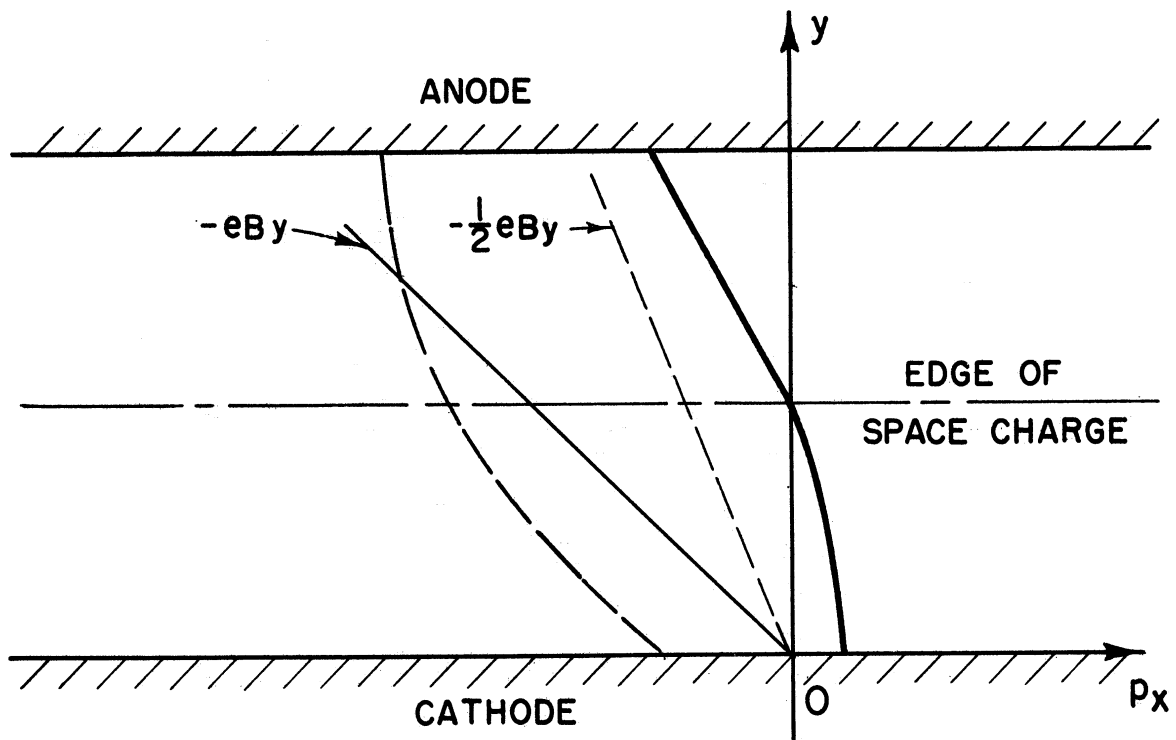SPACE-CHARGE-LIMITED CONDITIONS

FIG. 7
TEMPERATURE-LIMITED CONDITIONS

have the same energy although occupying a line rather than a point in momentum space. There can be no question about a metastable state in this case, since the electron orbits intersect with considerable relative velocity so that exchange of energy and momentum is inevitable.

Twiss has considered the modification in the initial space-charge distribution produced by the initial velocities of the electrons. The result is essentially the initial state mentioned earlier in this section as obtainable by integration of the distribution of function (20) over the accessible region of momentum space. It avoids the discontinuities and singularities of the two distributions mentioned above, but is otherwise subject to the same criticism; it is an initial state but not a steady state.

V.   DISCUSSION OF THE MAGNETRON SPACE-CHARGE DISTRIBUTION AS A DIFFUSION PROBLEM

A calculation of the steady state in the cut-off magnetron from the initial state previously indicated is complicated by the following circumstances:

1. The relationships governing the transfer of momentum between the electrons, including close encounters as well as distant encounters, are rather involved even when the state of the space charge differs very little from thermal equilibrium.

2. The actual distribution density is very irregular because of the initially abrupt variation at the accessibility boundaries.

3. The distribution density function is furthermore distorted by the increased electron temperature produced by the diffusion of electrons from the low-potential parts of the space charge to the high-potential parts, so that the energies of electrons returning to the cathode are distributed according to a higher temperature than those leaving the cathode.   (We use the term "temperature" for convenience although the distribution is not regular Maxwell-Boltzmann.)

We shall here try to arrive at a qualitative understanding of the way in which a steady state is reached without resolving quantitavely the complications enumerated above.

The drift of electrons through phase space is such that if the system were temporarily closed its state would gradually approach thermal equilibrium.

During thermal equilibrium the number of electrons that move from one particular volume element in phase space to another such element is equal to the number that moves in the opposite direction. We should expect it to be possible to calculate the instantaneous value of the interaction current from

one volume element in the phase space to another by comparing the instantaneous

distribution with the equilibrium distribution. Here, however, we should not

consider thermal equilibrium, which is determined by the boundary conditions

of the whole space charge, but the standard normal distriubtion of $(p_x + eBy)$,

p_y and p_z that has the same total population, the same energy, and the same

center of gravity as the initial distribution. The total interaction current

across a certain boundary surface in momentum space would be obtained by a

double integration over these two distribution density functions. In addition

to a function of the densities of the two volume elements in both distributions

the integrand must necessarily contain a weighting function expressing the

probability of the required change of momentum between the two volume elements.

At least two simultaneous asymptotic processes should be considered that

give quite different weighting functions. The first one operates by close

binary encounters and is a discrete random process of well-known type. The

influx to a certain volume element in phase space because of its density

deficiency is largely determined by the distance of the element from the center

of gravity of the distribution density functions. In this case the weighting

function accounts for the impact parameter and the angle between the relative

velocities of the two electrons with respect to their common center of gravity.

The second process is the result of interaction between a large

number of electrons at considerably larger distances. The weighting function

in this case permits only very small changes of momentum. The interaction

current density across a certain surface in momentum space is consequently

determined chiefly by a relation between the actual density gradient at this

surface and the equilibrium density gradient there.

It has been shown[8] that when the interaction is due to Coulomb forces

neither one of these two processes can in general be neglected in comparison

with the other.

In enumerating the factors that determine the steady state we should begin with the boundary conditions at the cathode. It is natural to assume that the mass and thermal capacity of the cathode are so large that the temperature and distribution of the emitted electrons are independent of the temperature and distribution of the returning electrons, that is to say, the energy and momentum of the electron cloud are not conserved at the cathode. We made this assumption previously in order to arrive at the thermal equilibrium distribution, Eq 20. If momentum were at least partly conserved, the center of gravity of the distribution could not fall on the line $p_x = - eBy$ but somewhere between this line and $p_x = 0$. The assumption made here appears satisfactory as long as the emission is strictly thermionic; if secondary emission is appreciable, the boundary conditions become much more difficult to state.

Between the cathode and the potential minimum the inaccessible part of the phase space is likely to be small and not to include dense regions of the distribution. The potential and space-charge density then drops approximately according to Eq 21 until roughly the Brillouin density is reached.

The most interesting region in real space is between the potential minimum and the edge of the cloud. The boundary of the secular volume in phase space on the cathode side is likely to be almost parallel to the y-axis here, since the potential distribution is probably not very different from that of the Brillouin solution (see Fig. 6). The space-charge density in the secular volume obviously must be such that the net number of electrons entering from the right equals the net number leaving at the left boundary. This density is obviously smaller than the one required for ideal thermal equilibrium; the center of gravity of the distribution must therefore be expected to be to the right of the line $p_x = - eBy$, that is, closer to the right-hand boundary than to the left-hand one. The space-charge density required for a steady state is therefore closer to the density at the right boundary than to the

one at the left, which is close to zero. To calculate this space-charge

distribution is evidently very difficult, but it should be clear that its

density is by no means negligible as far as calculation of space-charge

distribution in real space is concerned.

Twiss maintains that a double-stream motion with considerably more

than thermal energy exists here in the cathode-accessible volume of phase

space. The square of the y-directed momentum of an electron can be written

(Eq 31)

$$p_y^2 = p_{yo}^2 + 2mE - e^2B^2y^2 - 2p_x\ eBy \cong p_{yo}^2 - 2p_x\ eBy \qquad (32)$$

The orbits of the electrons in the p_y-y-plane are parabolas, and

since eBy p_x, p_y may be of a different order of magnitude than p_{yo} for

negative values of p_x. The orbits do not extend to the anode, because outside

the edge of the space-charge swarm $e^2B^2y^2$ is considerably larger than $2mE$, so

that the electrons turn back into the cloud.

At first sight these conclusions seem to be inconsistent with the

view presented in section IV, that the initial energy distribution at any point

is an incomplete Maxwell-Boltzmann distribution of cathode temperature. The

answer is that as the electrons move in the positive y-direction and gain kinetic

energy from the electric field, they also move towards the high-energy

fringes of a distribution that, if it were complete, would have a much higher

space-charge density than at the cathode.

The high-energy electrons will be much closer to the center of

gravity of the actual, considerably distorted distribution than to the center

of gravity of the Maxwell-Boltzmann distribution of which they form a small

part. Consequently, the average energy or "temperature" of the actual distri-

bution will be much higher than at the cathode.

This increase of the average electron energy with y no doubt contains the clue to a number of magnetron problems. Considering the cut-off magnetron as a resistor, we should expect the increased electron temperature to increase the noise output. Twiss has attributed the noise level to a "noise amplification" caused by the double-stream interaction in the space-charge cloud. The points of view may not be equivalent, but they are certainly related.

The difference between the space-charge distribution discussed here and the Brillouin or Twiss solution is probably not very large when the volume of the secular region in phase space is small, i.e., when the tube is operated close to the cut-off voltage. The anode current will be appreciably larger than the direct convection current from cathode to anode, however, because of the diffusion through the secular region.

When the anode voltage is far below the cut-off voltage, on the other hand, the secular volume is large, and the total space charge there may be large enough to affect the space-charge density in real space and the potential distribution appreciably.

So far it has been assumed that space-charge-limited conditions prevail in the magnetron, so that a potential minimum exists a finite distance from the cathode.. Sufficient emission is then available to maintain a space-charge density in the cloud roughly equal to the Brillouin density. Under temperature-limited conditions this is not the case (Fig. 7), initially at least, although the possibility should be considered that the accumulation of space charge in the secular region could modify the potential distribution in such a way that Fig. 7 changed into something like Fig. 3. This is very likely to be approximately true when the magnetron is operated far below the cut-off voltage, so that the anode current is infinitesimally small. There can be no potential minimum, however, and this part may be significant. Twiss

has described a process whereby the shot noise from the cathode is increased by the space-charge effects at the potential minimum instead of being reduced as in a space-charge-limited diode. Twiss's suggestion that actual oscillations, similiar to plasma oscillations, may take place between the cathode and the potential minimum seems less plausible.

The mathematical difficulties discourage any attempt to predict numerically the space-charge distribution in a cut-off magnetron; we must therefore look to experimental investigations for quantitative information. Reverdin[9] has described an interesting electron-optical method to explore the magnetron space charge. His results show that the steady-state formation is sensitive to cathode geometry and temperature, but his data are too scanty to yield any definite quantitative information as to the effect of the various parameters of the problem.

It is to be hoped that the exploring-beam experiments under way at Columbia Radiation Laboratory and at the University of Michigan will throw more light on the subject.

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