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A DIAGNOSTIC STUDY OF THE MAINTENANCE OF STATIONARY DISTURBANCES IN THE ATMOSPHERE

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ABSTRACT

The maintenance of kinetic energy and vorticity balance in the stationary disturbances over the Northern Hemisphere is investigated on the basis of wind statistics data, as published by Crutcher (1959). It is found that these disturbances feed kinetic energy to the large-scale transient disturbances at a rate which is of the same order of magnitude as the rate of energy dissipation due to small-scale friction. The effect of kinematic as well as mechanical (mountain torque) interaction between the stationary disturbances and the basic zonal current appear to be relatively small. The main process compensating for the energy loss due to transient disturbances and small-scale friction seems to be the conversion of available potential energy into kinetic energy.

Observational analysis of the time-averaged vorticity equation shows that in the free atmosphere an air particle moving along with the time-mean flow tends to conserve its absolute vorticity. At the same time, however, the kinematic forcing effect of large-scale transient eddies is important in the vorticity equation and largely determines the vertical

velocity field in the stationary disturbances. Non-linear terms in the vorticity equation for stationary disturbances are relatively small compared with the linear terms except at high latitudes.

1. INTRODUCTION

This report is concerned with the so-called stationary disturbances, which together with the zonally averaged mean motion form the time-mean flow pattern in the atmosphere. The existence of these disturbances can formally be ascribed to three principal factors: the deflecting effect of mountains on the zonal current, the longitudinally non-uniform stationary heating and the forcing effect of large-scale transient disturbances. The relative importance of these factors is not yet established and despite several theoretical studies (e.g. Charney and Eliassen, 1949; Smagorinsky, 1953; Saltzman, 1963) an adequate theory of these disturbances is still missing.

One way of getting insight into the maintenance of a certain mode of motion in the atmosphere is to make diagnostic observational studies of important dynamic quantities, such as energy, momentum and vorticity, for that mode. This can be done by evaluating from the data the different terms in the relevant equations. On this line, Murakami (1963) investigated the energetics of stationary disturbances. Using the data from the year 1950, he found that stationary disturbances fed kinetic energy into both the transient disturbances and the zonally averaged mean motion and were maintained by baroclinic processes converting available potential energy into kinetic energy. Saltzman (1962), also using the data for 1950, computed empirical forcing functions to represent the effects of transient disturbances on the maintenance of the potential vorticity field of the stationary disturbances. These functions appeared to be of the same magnitude as those required to account for the observed conditions and therefore should be included, along with the mechanical and thermal forcings, in the general theory of stationary disturbances.

In the present study, the observational approach is again used. In section 2 the maintenance of kinetic energy of stationary disturbances is discussed with a few additional aspects compared to the paper by Murakami (1963). Section 3 deals with the dynamics of these disturbances as revealed by the analysis of the vorticity budget. The source of wind data for the present investigation has been the "Upper Wind Statistics Charts of the Northern Hemisphere" (Crutcher, 1959), which in most areas is based upon wind observations from a period of five years or more; temperature data were taken from Goldie et al. (1957) and supplemented for the lower troposphere by isobaric height data from Jacobs (1958) and Hennig (1958).

The most steady mean flow in the atmosphere is probably obtained by taking a time-average of different quantities over an ensemble of several years. Therefore, the main discussion in the present report deals with the stationary disturbances as observed on such an annual mean flow. However, attention is also given to normal conditions in winter (January-February) and summer (June-August), for which the steady-state assumption also roughly applies.

The following notation is used:

A = horizontal area

a = radius of the earth

$\mathbf{F}$ = frictional force per unit mass

f = Coriolis parameter

g = acceleration of gravity

h = elevation of the earth's surface

k = kinetic energy per unit mass

$\mathbf{k}$ = a unit vector in the vertical direction

p = pressure

p_o = pressure at the earth's surface

t = time

u = zonal wind component

v = meridional wind component

$\mathbf{V}$ = horizontal wind vector

λ = longitude

φ = latitude

Φ = geopotential

$$\zeta = \frac{1}{a \cos \varphi}\left(\frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \varphi} \cos \varphi\, u\right) = \text{relative vorticity}$$

$$\omega = \frac{dp}{dt}$$

$$[x] = \frac{1}{2\pi}\int_0^{2\pi} x \, d\lambda = \text{zonal average of } x$$

$$x^* = x - [x] = \text{deviation from the zonal average}$$

$$\bar{x} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x \, dt = \text{time average of } x$$

$$x' = x - \bar{x} = \text{deviation from the time average}$$

∇ = a horizontal del-operator in pressure coordinates

2. KINETIC ENERGY BALANCE

In this section, the amount and distribution of kinetic energy of stationary disturbances in the atmosphere over the Northern Hemisphere is discussed briefly. The emphasis, however, is put on an observational analysis of different physical factors which affect this kinetic energy.

The horizontal wind velocity at an arbitrary point in space and time can formally be expressed as

$$\mathbb{V} = [\bar{\mathbb{V}}] + \bar{\mathbb{V}}^* + \mathbb{V}',$$

where the terms on the right-hand side denote the velocity associated with the zonally averaged mean motion, stationary disturbances and transient disturbances, respectively. The kinetic energy of horizontal motion, averaged zonally and with respect to time, can then be expressed as

$$k = 1/2[\overline{V^2}] = k_Z + k_S + k_T,$$

where

$$k_Z = 1/2([\bar{u}]^2 + [\bar{v}]^2),$$

$$k_S = 1/2([\bar{u}^{*2}] + [\bar{v}^{*2}])$$

and $$k_T = 1/2([\overline{u'^2}] + [\overline{v'^2}]).$$

Here and through the rest of the paper, the subindices Z, S and T are used to refer to the energy of the zonally averaged motion, stationary disturbances and transient disturbances, respectively.

The average values of k, k_Z, k_S and k_T north of 15°N and between 100 mb and 1000 mb are given in Table 1, which also shows the partitioning of these energies into contributions from the zonal and meridional wind component. It is seen that the kinetic energy associated with the stationary disturbances is relatively small, being only about four per cent of the total kinetic energy in the annual mean conditions and eight per cent in winter and in summer. Most of

Table 1. Average amount of kinetic energy and its different components in the atmosphere between 100 mb and 1000 mb north of 15°N. (1) total, (2) contribution from the zonal wind component and (3) contribution from the meridional wind component. Unit: joules kg^{-1}.

	k			k_Z			k_S			k_T		
	1	2	3	1	2	3	1	2	3	1	2	3
Annual mean	147	105	42	48	48	0	6	4	2	93	53	40
Normal winter (December-February)	213	159	54	95	95	0	17	12	5	101	52	49
Normal summer (June-August)	83	54	29	20	20	0	6	4	2	57	30	27

the kinetic energy of the stationary disturbances and practically all of the energy of the zonally averaged mean motion is associated with the zonal wind component while an approximate equipartitioning of energy between the two wind components prevails in the transient disturbances.

The meridional distribution of the kinetic energy of the stationary disturbances for annual mean conditions is shown in Fig. 1. The maximum of energy is found close to 40°N and is due mainly to the zonal wind component. The kinetic energy of the meridional wind component in these disturbances is seen to have a primary maximum between 50°N and 55°N and a secondary maximum close to 20°N.

The planetary scale of the stationary disturbances is clearly seen from Fig. 2, which shows, for annual mean conditions, the kinetic energy in these disturbances as a function of zonal wave number: the first three wave numbers account for about 90 per cent of the total kinetic energy in the six analyzed harmonics. The kinetic energy has relative maxima at wave numbers one and three. This feature arises primarily from the zonal component of motion; the energy associated with meridional wind component is seen to have only one maximum at wave number three.

A normalized energy spectrum (energy in each harmonic

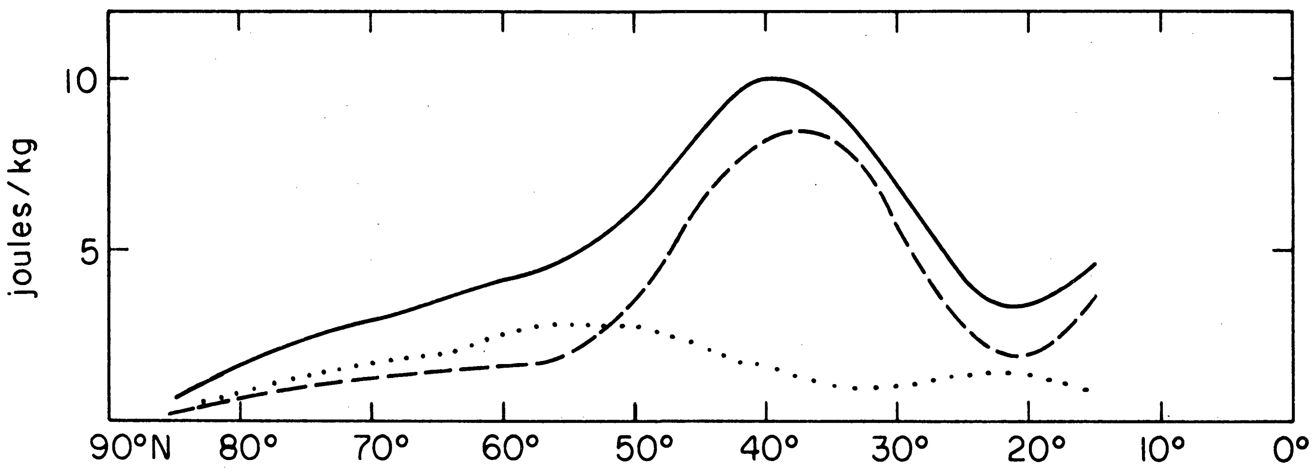

Fig. 1. The meridional distribution of the average kinetic energy of the stationary disturbances (heavy line) between 100 mb and 1000 mb for annual mean conditions. The contributions from the zonal and meridional wind components are given by the dashed and dotted lines, respectively.

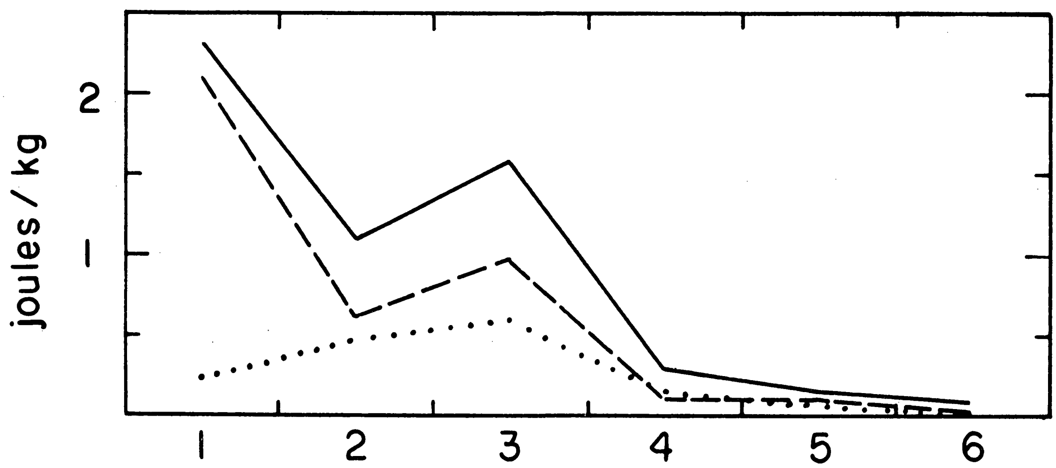

Fig. 2. The average kinetic energy of the stationary disturbances (annual mean conditions) between 100 mb and 1000 mb north of 15°N as a function of the zonal wave number. The contributions from the zonal and meridional wind components are given by the dashed and dotted lines, respectively.

divided by the sum of energies in all the six harmonics) for stationary disturbances in normal winter and summer conditions is given in Fig. 3. It is seen that in winter the wave numbers one and three dominate in the spectrum whereas in summer the kinetic energy decreases smoothly with increasing zonal wave number.

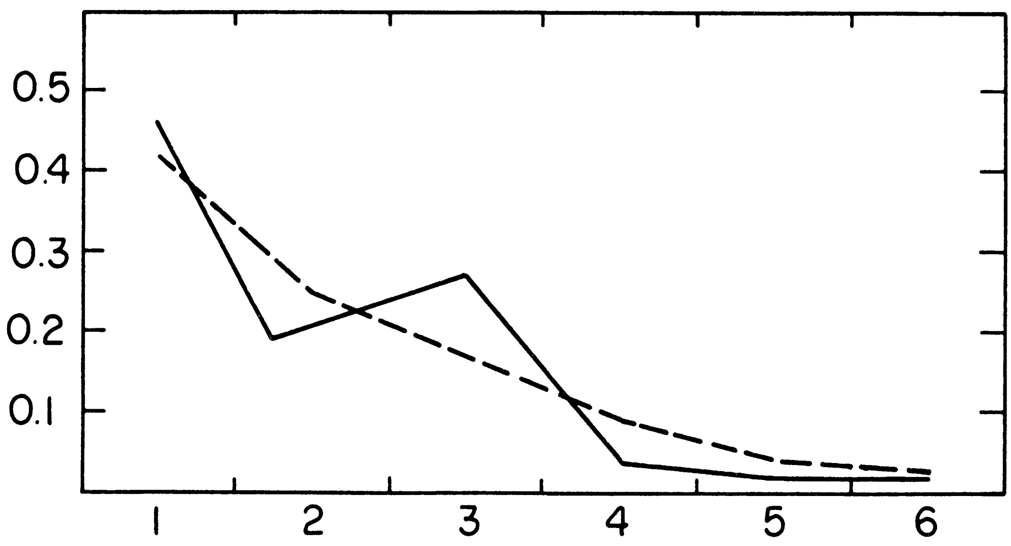

Fig. 3. The normalized spectrum of the kinetic energy of stationary disturbances (average between 100 mb and 1000 mb, north of 15°N) for winter (heavy line) and for summer (dashed line). The abscissa is the zonal wave number.

The equation for the balance of kinetic energy in stationary disturbances is given in equation (2). The corresponding equations for kinetic energy of the zonally averaged motion and of the transient disturbances are written down as equation (1) and equation (3), respectively. All these equations as well as the expressions (4) to (16) refer to the mass M of the atmosphere north of $\varphi_1 = 15°N$ and between the pressure surfaces $p_1 = 100$ mb and $p_2 = 1000$ mb.

$$\frac{\partial K_Z}{\partial t} = 0 = F_Z - C_{ZS} - C_{ZS}^{M} - C_{ZT} + W_Z + D_Z \qquad \ldots(1)$$

$$\frac{\partial K_S}{\partial t} = 0 = F_S + C_{ZS} + C_{ZS}^{M} - C_{ST} + W_S + D_S \qquad \ldots(2)$$

$$\frac{\partial K_T}{\partial t} = 0 = F_T + C_{ZT} + C_{ST} + W_T + D_T \qquad \ldots(3)$$

where

$$K_Z = \int_M k_Z dm$$

$$K_S = \int_M k_S dm$$

and

$$K_T = \int_M k_T dm$$

$$F_Z = \frac{2\pi\ a \cos\varphi_1}{g} \int_{p_1}^{p_2} (1/2[\bar{v}]^2[\bar{v}])_{\varphi_1} dp \qquad \ldots(4)$$

$$F_S = \frac{2\pi\ a \cos\varphi_1}{g} \int_{p_1}^{p_2} (1/2[\bar{v}^{*2}][\bar{v}] + 1/2[\bar{v}^{*2}\bar{v}^*]$$

$$+ [\bar{u}][\bar{u}^*\bar{v}^*] + [\bar{v}][\bar{v}^*\bar{v}^*])_{\varphi_1} dp \qquad \ldots(5)$$

$$F_T = \frac{2\pi\ a \cos\varphi_1}{g} \int_{p_1}^{p_2} (1/2[\overline{v'^2}][\bar{v}] + 1/2[\overline{v'^2}\bar{v}^*] + 1/2[\overline{v'^2 v'}]$$

$$+ [\bar{u}][\overline{u'v'}] + [\bar{v}][\overline{v'v'}] + [\bar{u}^*\overline{u'v'}] + [\bar{v}^*\overline{v'v'}])_{\varphi_1} dp$$

$$\ldots(6)$$

$$C_{ZS} = \int_M \{[\bar{u}](\frac{1}{a\cos^2\varphi} \frac{\partial}{\partial\varphi} \cos^2\varphi[\bar{u}^*\bar{v}^*] + \frac{\partial}{\partial p}[\bar{u}^*\bar{\omega}^*])$$

$$+ [\bar{v}](\frac{1}{a\cos\varphi} \frac{\partial}{\partial\varphi} \cos\varphi[\bar{v}^{*2}] + \frac{\partial}{\partial p}[\bar{v}^*\bar{\omega}^*]$$

$$+ \frac{[\bar{u}^{*2}]}{a} \tan\varphi)\}dm \qquad \ldots(7)$$

$$C_{ZS}{}^M = \int_A \frac{[\bar{u}_o]}{a\cos\varphi}[\bar{p}_o \frac{\partial h}{\partial\lambda}]dA \qquad \ldots(8)$$

$$C_{ZT} = \int_M \{[\bar{u}](\frac{1}{a\cos^2\varphi} \frac{\partial}{\partial\varphi} \cos^2\varphi[\overline{u'v'}] + \frac{\partial}{\partial p}[\overline{u'\omega'}])$$

$$[\bar{v}](\frac{1}{a\cos\varphi} \frac{\partial}{\partial\varphi} \cos\varphi[\overline{v'^2}] + \frac{\partial}{\partial p}[\overline{v'\omega'}]$$

$$+ \frac{[\overline{u'u'}]}{a} \tan\varphi)\}dp \qquad \ldots(9)$$

$$C_{ST} = \int_M \{[\bar{u}^*(\frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} \overline{u'u'} + \frac{1}{a \cos^2 \varphi} \frac{\partial}{\partial \varphi} \cos^2 \varphi \, \overline{u'v'}$$

$$+ \frac{\partial}{\partial p} \overline{u'\omega'}) + \bar{v}^*(\frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} \overline{u'v'}$$

$$+ \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \cos \varphi \, \overline{v'v'} + \frac{\partial}{\partial p} \overline{v'\omega'} + \frac{\overline{u'u'}}{a} \tan \varphi)]\}dp \qquad ...(10)$$

$$W_Z = -\int_M [\bar{v}]\frac{1}{a} \frac{\partial[\bar{\Phi}]}{\partial \varphi} \, dm \qquad ...(11)$$

$$W_S = -\int_M [\bar{\mathbb{V}}^* . \nabla\bar{\Phi}^*]dm \qquad ...(12)$$

$$W_T = -\int_M [\bar{\mathbb{V}}' . \nabla\bar{\Phi}']dm \qquad ...(13)$$

$$D_Z = \int_M [\bar{\mathbb{V}}] . [\bar{\mathbb{F}}]dm \qquad ...(14)$$

$$D_S = \int_M [\bar{\mathbb{V}}^* . \bar{\mathbb{F}}^*]dm \qquad ...(15)$$

$$D_T = \int_M [\bar{\mathbb{V}}' . \bar{\mathbb{F}}']dm . \qquad ...(16)$$

In equations (1) to (3), which are obtained from the equations of motion, F-terms represent the flux of kinetic energy northward across the latitude 15°N. (The vertical flux of kinetic energy across the upper and lower boundary has been assumed zero). C-terms represent the conversion between two different energy forms, denoted by the subindices. W and D stand for the work done by pressure forces and friction forces, respectively. The equations are essentially the same as those derived by Murakami (1963) except for the C_{ZS}^{M}-term, which has not been considered in the earlier investigation. This conversion term, which comes into equation (1) from the first equation of motion, is taken to be the product of the mean zonal component of the surface wind ($[\bar{u}_o]$) and the net zonal pressure force ($[\frac{p_o}{a \cos \varphi} \frac{\partial h}{\partial \lambda}]$) which arises from pressure differences between the western and eastern sides of mountains, at the same elevation. In order to keep the total kinetic energy of the time-mean motion unaffected by mountains, C_{ZS}^{M} has been introduced in equation (2) with a sign opposite to that in equation (1). This way of defining a new energy transformation function is somewhat arbitrary. It makes sense, however, when compared with the way in which the effect of mountains is incorporated in theoretical models of stationary disturbances. In these models (e.g. Saltzman, 1965) the lower boundary is assumed to be an isobaric surface just above all mountain tops. The energetics of the model is then affected by the mountains through a forced energy flux across the lower boundary. This flux has essentially the same expression as C_{ZS}^{M} and may therefore, in the light of the present formulation, be considered as a conversion of the kinetic energy of the zonally averaged motion into that of the stationary disturbances. This mechanical conversion occurs in the lower troposphere in contrast to the corresponding kinematic conversion C_{ZS}, which primarily takes place in the upper troposphere.

According to equation (2), the kinetic energy of stationary disturbances north of 15°N is affected by the energy flux (F_S) across this latitude, by the kinematic interaction between the zonally averaged mean motion and the stationary disturbances (C_{ZS}), by the mountain forcing (C_{ZS}^{M}), by the interaction between stationary and transient disturbances (C_{ST}) and by work done by pressure forces (W_S) and friction forces (D_S). Our goal is to evaluate from the available data these processes and as many as possible of the other quantities in (4) to (16). In the following the procedure and the results of this evaluation are discussed term by term.

The calculation of F_S, C_{ZS} and C_{ST} was made from the following approximate expressions:

$$F_S = \frac{2\pi a \cos \varphi_1}{g} \int_{p_1}^{p_2} ([\bar{u}][\bar{u}^* \bar{v}^*])_{\varphi_1} dp \qquad (17)$$

$$C_{ZS} = \int_M \frac{[\bar{u}]}{a \cos^2\varphi} \frac{\partial}{\partial\varphi} \cos^2\varphi [\bar{u}^* \bar{v}^*] dm \qquad \ldots(18)$$

$$C_{ST} = \int_M \{[\bar{u}^* (\frac{1}{a \cos\varphi} \frac{\partial}{\partial\lambda} \overline{u'u'} + \frac{1}{a \cos^2\varphi} \frac{\partial}{\partial\varphi} \cos^2\varphi\ \overline{u'v'})$$

$$+ \bar{v}^* (\frac{1}{a \cos\varphi} \frac{\partial}{\partial\lambda} \overline{u'v'} + \frac{1}{a \cos\varphi} \frac{\partial}{\partial\varphi} \cos\varphi\ \overline{v'v'}$$

$$+ \frac{\overline{u'^2}}{a} \tan\varphi)]\} dm. \qquad \ldots(19)$$

These equations are obtained from equations (4), (7) and (10) by neglecting supposedly small terms containing mean meridional velocity, vertical velocity or triple correlation of eddy quantities of purely stationary or transient mode. Expressions (17) to (19) were evaluated with data from "Upper Wind Statistics Charts of the Northern Hemisphere" (Crutcher, 1959) which, among other things, includes charts of the mean zonal wind component $\bar{u}$, the mean meridional wind component $\bar{v}$, standard deviations ($\sqrt{\overline{u'^2}}$ and $\sqrt{\overline{v'^2}}$) of the zonal and meridional wind components and the correlation coefficient ($\overline{u'v'}/(\sqrt{\overline{u'^2}}.\sqrt{\overline{v'^2}})$) between the two wind components for the 100, 200, 300, 500, 700 and 850 mb pressure levels and separately for four seasons of the year. Data were read from the maps at every five degrees of latitude and every ten degrees of longitude. The calculations were made for winter

(December-February), for summer (June-August) and for annual mean conditions for which data were obtained by combining data for the four seasons. The vertical integration in equations (17) to (19) was made by using the trapezoidal rule and by assuming that the integrands vanish at the 1000 mb level. This assumption as well as an artificial interpolation of data in the lower troposphere across mountains has little significance for the results, because the largest contribution to the integrals comes from the upper troposphere.

The computed values of F_S, C_{ZS} and C_{ST}, divided by the area north of 15°N, are given in Table 2. For comparison,

Table 2. Energy conversions and horizontal boundary fluxes (see equations (17) to (21)) for the atmosphere north of 15°N as evaluated from upper wind statistics. Unit: watts m^{-2}.

	F_S	C_{ZS}	C_{ST}	F_T	C_{ZT}
Annual mean conditions	0.00	-0.02	0.09	0.02	-0.32
Normal winter (December-February) conditions	-0.02	-0.10	0.16	0.04	-0.36
Normal summer (June-August) conditions	-0.03	-0.03	-0.01	-0.01	-0.20
Year 1950 (after Murakami, 1963)		-0.04	0.06		-0.24

the table also includes the values of F_T and C_{ZT}, which were computed from the wind statistics using the following approximations to equations (6) and (9):

$$F_T = \frac{2\pi\ a \cos \varphi_1}{g} \int_{p_1}^{p_2} ([\bar{u}][\bar{u}'\bar{v}'])_{\varphi_1} dp \qquad \ldots(20)$$

$$C_{ZT} = \int_M \frac{[\bar{u}]}{a \cos^2 \varphi} \frac{\partial}{\partial \varphi} \cos^2 \varphi [\overline{u'v'}] dm. \qquad \ldots(21)$$

The values obtained by Murakami (1963) for the year 1950 are given on the lowermost line in Table 2.

The first thing to be noticed in the results is that for the annual mean conditions the boundary fluxes are very small compared to the conversion terms and accordingly the polar cap north of 15°N can be considered as a kinematically closed system from the standpoint of the stationary and transient disturbances. This situation is seen to be valid for normal winter conditions, but during summer F_S is of the same magnitude as C_{ZS} and C_{ST}.

The negative values of C_{ZS} show that the stationary disturbances feed energy by kinematic interaction into the zonally averaged mean motion. The value for annual mean conditions, -0.02 watts m^{-2}, is smaller than the one obtained by Murakami (1963) for the year 1950. Because an average over an ensemble of many years obviously defines better steady-state conditions than an average over only one year, the value obtained in the present study should be more representative of the atmosphere than that of Murakami. At this point it may be noticed that the magnitude of C_{ZS} is much smaller than that of C_{ZT}. The latter is always negative which conforms with the already well-established fact that the transient eddies in the atmosphere feed energy into the mean zonal current.

From the standpoint of the present study the most important quantity in Table 2 appears to be C_{ST}, which represents the conversion of kinetic energy of stationary disturbances into that of transient disturbances. Considering annual mean conditions this conversion is seen to be positive. This means that the large-scale transient disturbances act as an energy sink for the stationary disturbances. From the stand-point of transient disturbances this energy conversion, 0.09 watts m^{-2} is rather insignificant, because energy generation and dissipation in these disturbances (W_T and D_T in equation (3)) are of the order of a few watts m^{-2}. However, it turns out to be very important in the kinetic energy balance of stationary disturbances. The meridional distribution of the conversion integrand is shown in Fig. 4. It is seen that the stationary disturbances feed energy into transient disturbances in the

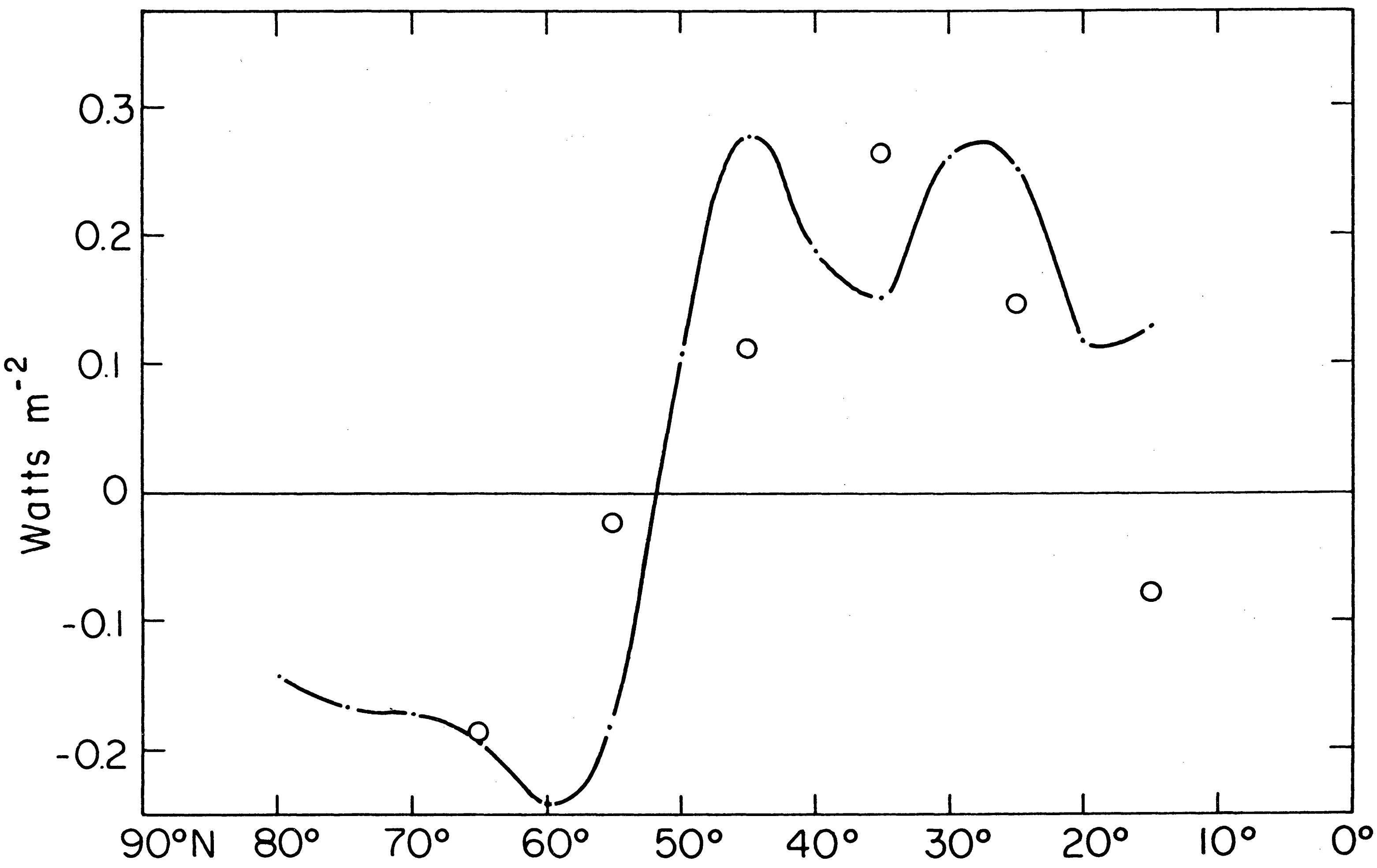

Fig. 4. The meridional distribution of the rate (in the annual mean conditions) of the conversion of the kinetic energy of stationary disturbances into that of transient disturbances. The circles denote the values obtained by Murakami (1961) for the year 1950. Unit: watts m^{-2}.

latitudes south of 50°N, whereas the opposite is true further north. The values obtained by Murakami for the year 1950, which are also shown in Fig. 4, agree reasonably well with those of the present study.

In order to get a better idea about the relative importance of F_S, C_{ZS} and C_{ST} in the maintenance of the kinetic energy of stationary disturbances, an attempt was made to evaluate also the forcing effect of mountains (C_{ZS}^M), the rate of energy dissipation due to small-scale turbulence (D_S) and generation of kinetic energy due to work done by pressure forces (W_S). All these processes are difficult to estimate in any way and therefore the following discussion is necessarily semi-qualitative.

To evaluate the C_{ZS}^M-term, the only data available with regard to the mountain torque seem to be those published by White (1949). These were used together with the mean zonal component of the surface wind. For the latter, data were obtained from Tucker (1957) and also by extrapolation from the free atmosphere. (In this extrapolation, data for the earth's topography were taken from Berkofsky and Bertoni (1955)). For the annual mean conditions the calculations gave:

$$C_{ZS}^M = 0.02 \text{ watts m}^{-2}.$$

Thus there is in the Northern Hemisphere, due to the presence of mountains, a small drainage of kinetic energy of mean zonal motion into that of stationary disturbances. The rate of this energy conversion is small compared to the conversion between stationary and transient disturbances. This result does not give support to the normal expectations: in many theoretical studies of the stationary disturbances the transient eddy forcing has been neglected and the forcing due to mountains retained.

There is no very satisfactory way of evaluating the rate of frictional loss of kinetic energy in the atmosphere. If we assume that most of the dissipation takes place in the boundary layer, we have from the classical Ekman-Taylor boundary layer theory the following expression for D, the rate of total energy dissipation (cf. e.g. Brunt, 1941):

$$D = \sqrt{\frac{f\kappa}{2}}\,\rho_o \sin 2\alpha_o V^2, \qquad \ldots(22)$$

where κ is the coefficient of eddy viscosity, ρ_o the air density at the surface, α_o the angle between surface wind and geostrophic wind and V is the speed of the geostrophic wind, assumed constant with height. If we now suppose that the assumptions of the above theory (balance between pressure, Coriolis and friction forces in the boundary layer, constant geostrophic wind and eddy viscosity) hold at any time and that the time variation of the parameters κ and α_o is small, the theory should be valid also for the time-mean conditions. Consequently, we would have

$$D_S = \sqrt{\frac{f\kappa}{2}}\,\rho_o \sin 2\alpha_o V_S^{\,2}, \qquad \ldots(23)$$

where V_S is the velocity of the time-mean geostrophic wind in the boundary layer. If this is taken to be the vector mean wind speed ($\sqrt{\bar{u}^2+\bar{v}^2}$) at the 850 mb level, equation (22) gives (with $f = 10^{-4}$ sec^{-1}, $\kappa = 10^5$ cm^2 sec^{-1}, $\rho_o = 1.25$ gm cm^{-3} and $\alpha_o = 20°$) for annual mean conditions north of 15°N:

$$D_S = 0.11 \text{ watts m}^{-2}.$$

(The corresponding value of the annual mean total dissipation as obtained from equation (22) turns out to be 1.7 watts m^{-2}.) Even if its numerical value may not be accurate, the order of magnitude of D_S should be correct. Comparing the above value of D_S with the earlier computed value of $C_{ST} = 0.09$ watts m^{-2} we can say that the small-scale turbulence and large-scale transient disturbances are equally important in destroying the kinetic energy of stationary disturbances.

The most difficult term in equation (2) to estimate directly is W_S, the work done by pressure forces in stationary disturbances. In light of the above results for the other terms in equation (2), there should be a generation of kinetic energy in the stationary disturbances due to this process, i.e. W_S should be positive. It is easy to show that for a closed system the expression for W_S can be written as

$$W_S = -\int_M [\bar{V}^* \cdot \nabla\bar{\Phi}^*]dm = -\int_M [\bar{\alpha}^*\bar{\omega}^*]dm,$$

where α denotes specific volume. (This second expression for W_S is normally called the rate of conversion of available potential energy to kinetic energy in the stationary disturbances.) Thus, in addition to the fields of mean wind and specific volume which are roughly known, a direct evaluation of W_S would require a knowledge of either the time-mean ageostrophic winds or the vertical velocity field. Reliable estimates of either of them are difficult to obtain on a hemispheric scale. In his study of the stationary disturbances, Murakami (1963) computed the mean vertical velocity for the year 1950 from the observed mean winds with the aid of the continuity equation. When using this method, which is very sensitive to errors in the wind field, the mass balance requirements (ω = 0 at the top and the bottom of the atmosphere) cannot usually be satisfied. Depending on whether he assumed $\bar{\omega}$ to vanish at 1000 mb or 100 mb, Murakami obtained

$W_S = 0.65$ watts m^{-2} or $W_S = 0.31$ watts m^{-2}, respectively. When the same method was applied in the present study, the data of which cover at least four other years in addition to 1950, the corresponding values turned out to be 0.19 watts m^{-2} and 0.03 watts m^{-2}. Even if all these values are of the right sign, the magnitude of W_S is not fixed with any certainty. This is probably just an indication of the fact that reliable estimates of mean divergence cannot be obtained from the mean wind field by a direct method.

One of the indirect ways of getting approximate values of divergence and vertical velocity is to make use of the vorticity equation. This approach was applied in the present study to the time-mean conditions (see next section) and by using the values of $\bar{\omega}$ so obtained, the $-[\bar{\alpha}^*\bar{\omega}^*]$ covariance was computed. The resulting values were generally positive in the middle and high latitudes, where the vorticity method of computing vertical velocities is best applicable. The less reliable values for lower latitudes were negative and large and made also W_S negative.

The best estimate of W_S can at the present time probably be obtained by computing it as a residual term from equation (2). Having evaluated all the other terms, the value of W_S which balances this equation is

$$W_S = 0.20 \text{ watts m}^{-2}$$

for the annual mean conditions. This value is larger than any of the other terms in equation (2) which means that the conversion of available potential energy into kinetic energy plays the dominant role in the kinetic energy balance of stationary disturbances. At this point it may be noted that the above value of W_S is an order of magnitude smaller than the rate of the total conversion of available potential energy into kinetic energy normally obtained for the atmosphere (see e.g. Oort, 1964).

A convenient way of summarizing the results of this section is to present them in the form of an energy flow diagram, schematically shown on the left hand side of Fig. 5. In this diagram only processes affecting the kinetic energy of stationary disturbances are indicated. The numerical values for the annual mean conditions are given on the right-hand side of the same figure. The physical picture arising from the numbers is as follows: the main source of kinetic energy for the stationary disturbances is the available potential energy. The dissipative mechanisms are the small-scale turbulence as well as the large-scale transient eddies; the combined effect of these two processes is such that the residence time of kinetic energy in the stationary

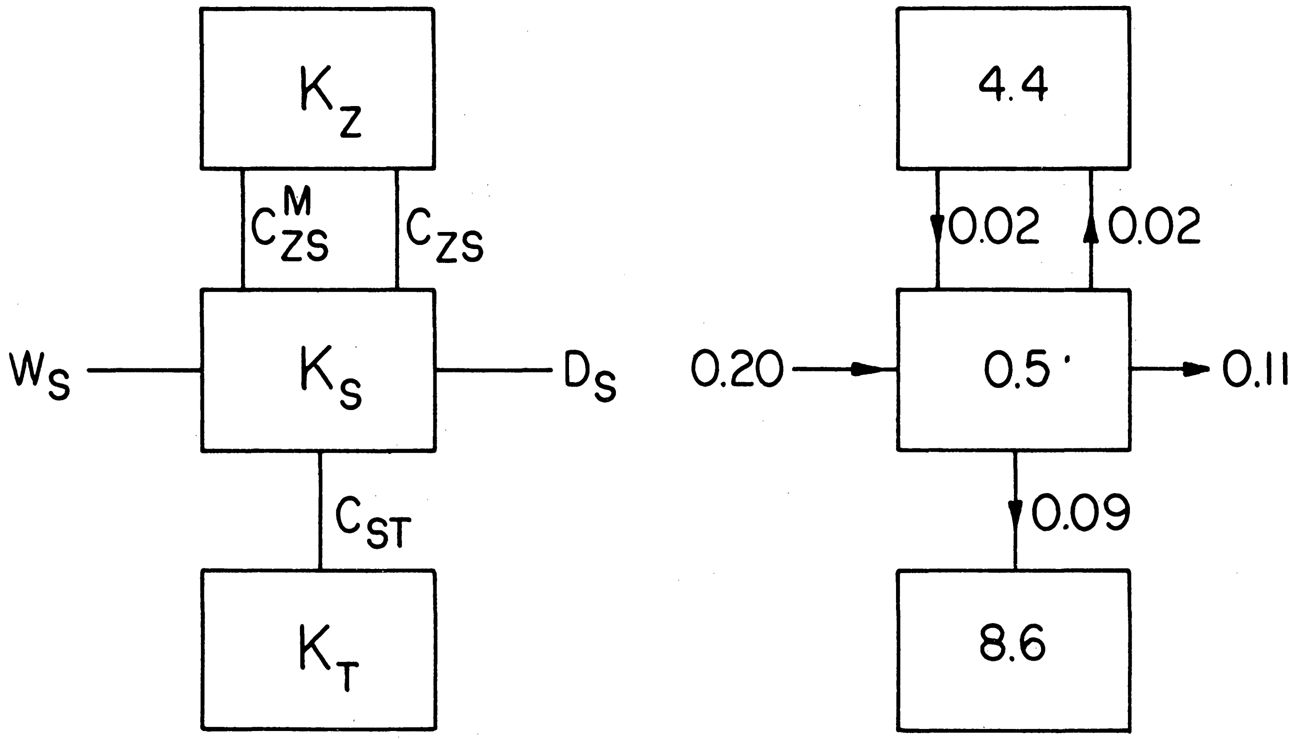

Fig. 5. Scheme of the kinetic energy balance in the stationary disturbances (left) and the numerical values obtained from annual mean conditions (right). The unit of energy per unit area is 10^5 joules m^{-2} and that of the energy changes is watts m^{-2}.

disturbances $(K_S/(D_S + C_{ST}))$ is only about three days. The kinematic as well as mechanical (mountain torque) interaction between stationary disturbances and the zonally averaged mean motion are relatively small.

3. VORTICITY BALANCE

In this section the dynamics of the stationary disturbances are discussed in the light of the relative magnitude of different terms in the time-averaged vorticity equation.
At the same time an approximate picture of the normal vertical velocities required to maintain the vorticity balance is presented.

The vorticity equation can be written as

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla \zeta + f \nabla \cdot \mathbf{V} = \zeta \frac{\partial \omega}{\partial p} - \omega \frac{\partial \zeta}{\partial p} - \mathbf{k} \cdot \nabla \omega \times \frac{\partial \mathbf{V}}{\partial p} + \mathbf{k} \cdot \nabla \times \mathbf{F} \qquad \ldots (24)$$

For vorticity changes in the free atmosphere in the time-range of a few days the terms on the right-hand side of (24) have been found small, and, in fact, have been neglected in most numerical weather prediction models used so far. If we assume that these terms are small also in the time-averaged vorticity equation, we obtain from equation (24)

$$\bar{\mathbf{V}} \cdot \nabla \bar{\zeta} + \beta \bar{v} + f \nabla \cdot \bar{\mathbf{V}} = F \qquad \ldots (25)$$

where

$$F = \frac{1}{a \cos \varphi}\left(\frac{\partial Y}{\partial \lambda} - \frac{\partial X \cos \varphi}{\partial \varphi}\right),$$

$$X = -\left(\frac{1}{a \cos \varphi} \frac{\partial}{\partial \lambda} \overline{u'u'} + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \overline{u'v'} \cos \varphi - \frac{\tan \varphi}{a} \overline{u'v'}\right)$$

$$Y = -\left(\frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \overline{u'v'} + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \overline{v'v'} \cos \varphi + \frac{\tan \varphi}{a} \overline{u'u'}\right).$$

F represents the forcing effect of horizontal transient disturbances on the time-mean vorticity field and has been discussed by Saltzman (1962) and Saltzman and Rao (1963).

In equation (25) all the terms can be evaluated from the wind statistics data except the divergence term for which a value is obtained as a residual necessary to balance the equation. The calculations were made for the 100, 200, 300, 500, 700 and 850 mb levels. Instead of discussing results for individual pressure surfaces, results will be shown for vertically integrated values, which have an interpretation in terms of vertical velocity. Using the equation of continuity ($\nabla \cdot \bar{\mathbb{V}} + \frac{\partial \bar{\omega}}{\partial p} = 0$) and assuming that

$$\bar{\omega} = 0 \text{ at } p = 100 \text{ mb},$$

we obtain by integrating (25) with respect to pressure:

$$\bar{\omega} = \bar{\omega}_A + \bar{\omega}_B + \bar{\omega}_C, \qquad \ldots(26)$$

where

$$\bar{\omega}_A = -\frac{1}{f}\int_{p_1}^{p} \bar{\mathbf{V}}\cdot\nabla\bar{\zeta}\, dp$$

$$\bar{\omega}_B = -\frac{1}{f}\int_{p_1}^{p} \beta\bar{v}\, dp$$

$$\bar{\omega}_C = -\frac{1}{f}\int_{p_1}^{p} F\, dp$$

and $p_1 = 100$ mb.

Fig. 6a shows the longitudinal variation of $\bar{\omega}_A$, $\bar{\omega}_B$ and $\bar{\omega}_C$ for annual mean conditions at 500 mb at the latitudes 70°N, 50°N and 30°N. Table 3 gives the average magnitude of the different terms at each latitude. It is seen that all the computed terms are of the same order of magnitude. However, $\bar{\omega}_A$ and $\bar{\omega}_B$ which represent the advection of the mean relative vorticity and the earth's vorticity, respectively,

Table 3. Average magnitude (mean of the absolute values at 36 grid points) of the different terms in equation (26) at 500 mb (see Fig. 6a). Unit: 10^{-5} cb sec^{-1}.

	$\bar{\omega}_A$	$\bar{\omega}_B$	$(\bar{\omega}_A+\bar{\omega}_B)$	$\bar{\omega}_C$	$\bar{\omega}$
70°N	0.51	0.44	0.41	0.51	0.60
50°N	1.34	1.27	0.52	1.06	1.03
30°N	1.54	1.27	1.34	2.02	2.85

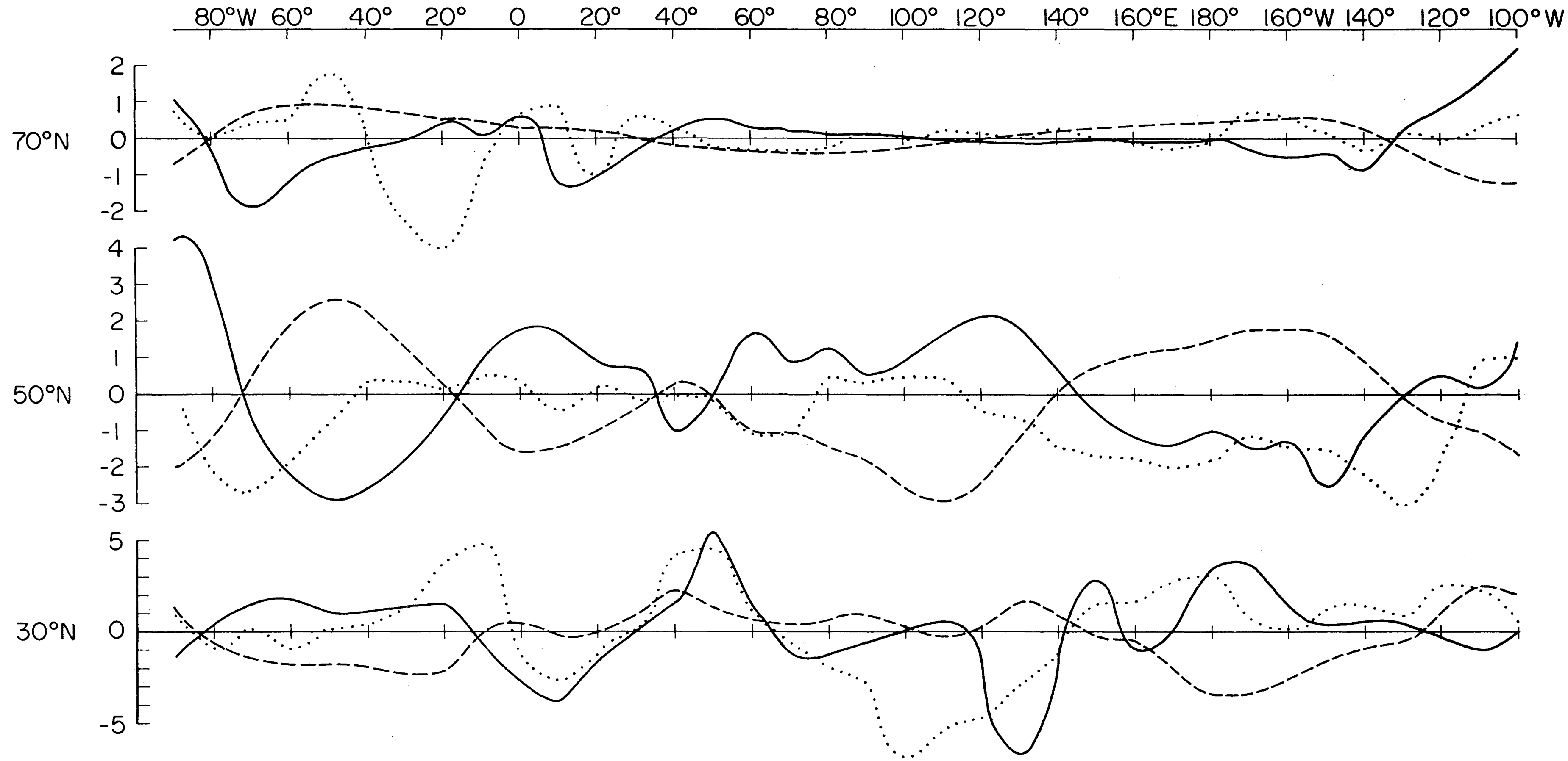

Fig. 6a. Longitudinal distribution of $\bar{\omega}_A$ (heavy line), $\bar{\omega}_B$ (dashed line) and $\bar{\omega}_C$ (dotted line) in annual mean conditions at 500 mb. Unit: 10^{-5} cb sec^{-1}.

counterbalance each other to a large extent. The physical interpretation of this phenomenon, which is most pronounced at 50°N, is that an air particle moving along with the mean flow tends to conserve its absolute vorticity. This fact, noticed also by Wiin-Nielsen (1960), makes the sum of $\bar{\omega}_A$ and $\bar{\omega}_B$ generally smaller than $\bar{\omega}_C$, which represents the forcing effect of transient eddies. Accordingly, the distribution of $\bar{\omega}$ follows by and large that of $\bar{\omega}_C$, as is clearly seen in Fig. 6b. It can be concluded that the kinematic forcing effect of large-scale transient disturbances is very important and largely determines the field of vertical velocity in the stationary disturbances. At this point it must be noticed that at the lower boundary of the "free atmosphere" the vertical velocities, as determined from equation (26) should be compatible with the vertical velocities produced by friction in the boundary layer and by the forcing of mountains. Over oceans and flat continental areas the latter effect is absent. In such regions the boundary layer friction, which is difficult to evaluate by direct methods, can be determined from the values which are obtained from equation (26). These aspects will be discussed in more detail in a forthcoming paper.

Fig. 7 shows the distribution of the annual mean vertical velocity at the 500 mb level. There is no definite way of

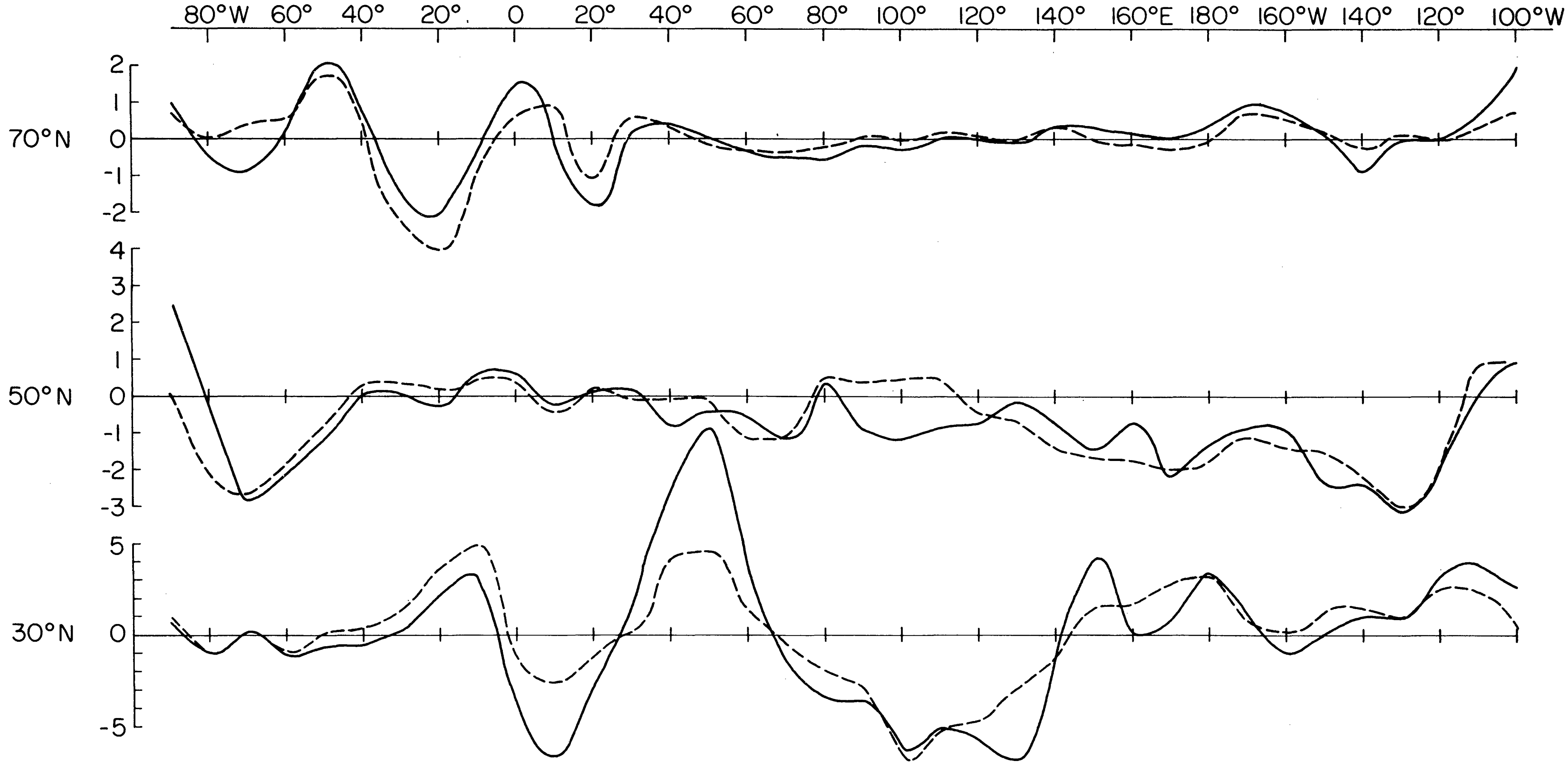

Fig. 6b. Longitudinal distribution of $\bar{\omega}$ (heavy line) and $\bar{\omega}_C$ (dashed line) in annual mean conditions at 500 mb. Unit: 10^{-5} cb sec^{-1}.

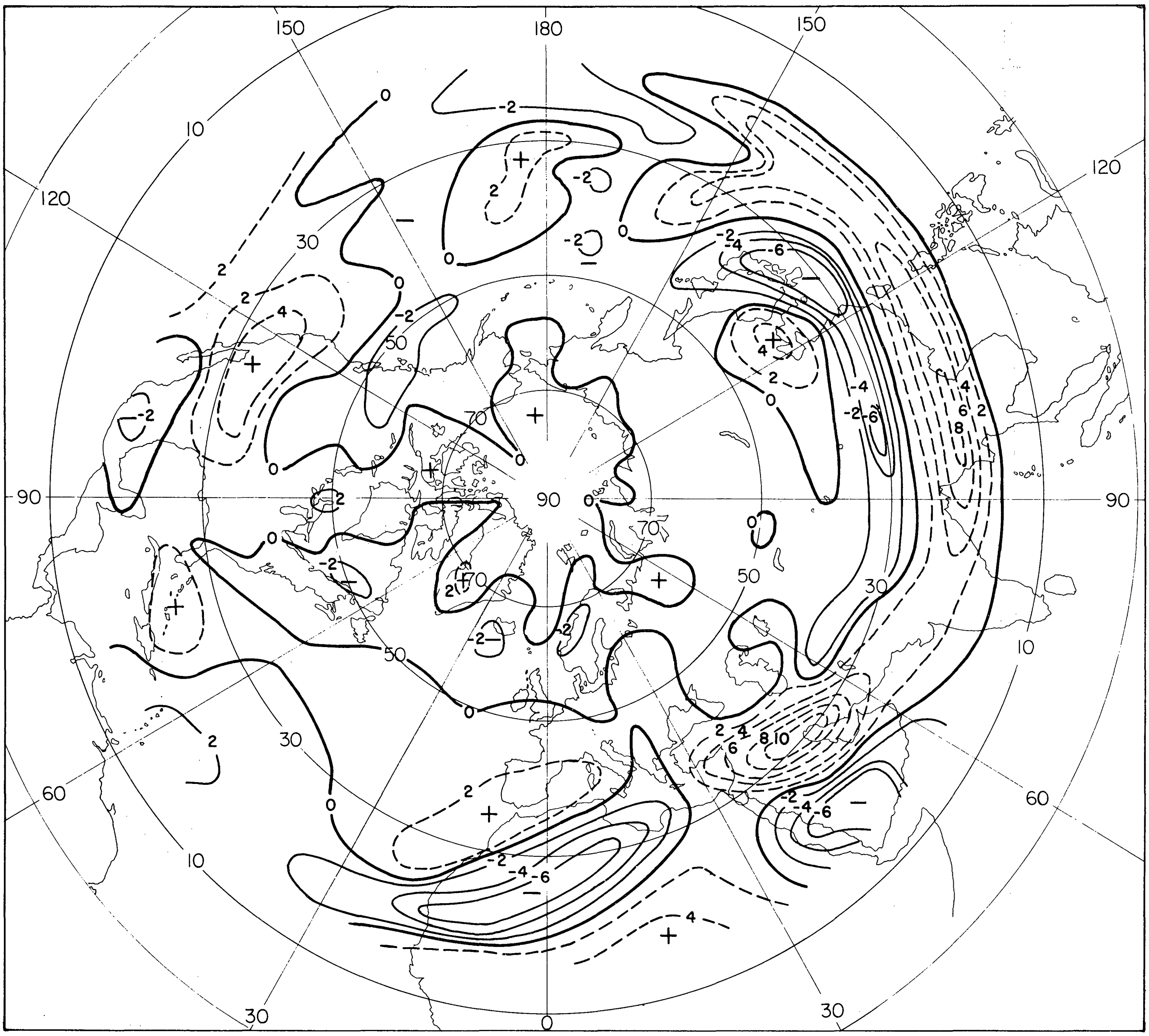

Fig. 7. The distribution of the annual mean vertical velocity $\bar{\omega}$ at the 500 mb level as computed from the vorticity equation. Unit: 10^{-5} cb sec^{-1}.

judging how correct the computed $\bar{\omega}$-values are but most features of the map in high and middle latitudes look realistic. A general downward motion is obtained in subpolar regions with a maximum, 2×10^{-5} cb sec^{-1} (0.3 cm sec^{-1}), occurring over Greenland. Upward mean motion is obtained along the east coast of North America and Asia and all over the northern Atlantic and northern Pacific. These are regions of strongest cyclonic activity and lows in the annual mean surface charts. The maximum of upward motion over western North America roughly coincides with the maximum of lifting caused by the Rocky Mountains. The large $\bar{\omega}$-values obtained south of about 30°N need not represent real vertical velocities because some of the assumptions underlying the present calculations are not valid in these latitudes; using the $\bar{\omega}$-values seen in Fig. 7 one can easily show, for example, that the twisting term and the term describing the vertical advection of mean vorticity cannot in these areas be neglected in comparison with those retained in equation (25).

An important question in theoretical studies of stationary disturbances in the atmosphere is the applicability of a linear model, i.e. whether these disturbances can be considered as small perturbations superimposed upon the basic zonal flow, in which case the terms involving products of perturbation quantities can possibly be neglected in the governing equations. In the vorticity equation the largest non-linear effects can be expected to arise from the horizontal advection of relative vorticity. For stationary disturbances, the linear vorticity advection is given by

$$\frac{[\bar{u}]}{a \cos \varphi} \frac{\partial \bar{\zeta}^*}{\partial \lambda} + \frac{\bar{v}^*}{a} \frac{\partial [\bar{\zeta}]}{\partial \varphi},$$

and the non-linear advection by

$$\bar{V}^* . \nabla \bar{\zeta}^* .$$

Fig. 8 shows the distribution of these terms as a function of latitude for normal winter conditions at 200 mb. (For this particular pressure level, data were read from the wind statistics using a denser grid than for other levels.) It is seen that the non-linear advection is at every latitude of smaller magnitude than the linear advection. The ratio of the average magnitude of these terms is 0.19, 0.24 and 0.47, for 30°N, 50°N and 70°N, respectively. In the light of these numbers it seems that the non-linear terms in the vorticity equation can be neglected when middle latitude conditions are of primary interest but should be included if the stationary disturbances at high latitudes are also to be simulated.

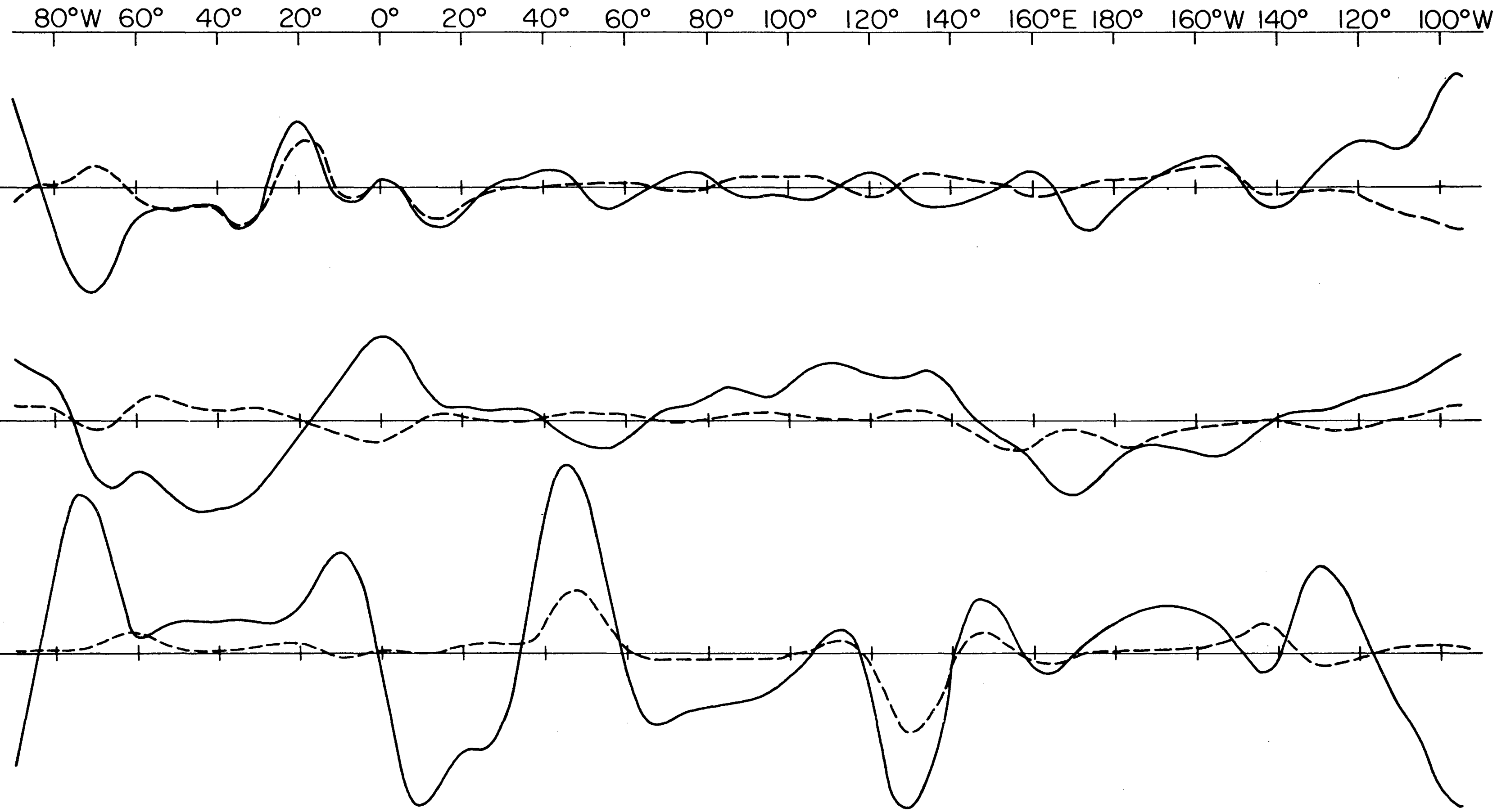

Fig. 8. Linear (heavy line) and non-linear (dashed line) horizontal vorticity advections in the stationary disturbances. Unit: $10^{-10}sec^{-2}$.

4. CONCLUDING REMARKS

The results of the present study show the importance of large-scale transient eddies for the maintenance of the stationary disturbances in the atmosphere. Contrary to normal expectations the direct mechanical effect of mountains on these "standing" waves seems to be relatively small.

In the present study only the kinetic energy and vorticity balance in the stationary disturbances have been discussed. A similar observational study of the thermal energy balance could possibly help to reveal the role of geographically fixed heat sources and sinks in the maintenance of these disturbances.

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