THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Electrical Engineering
Space Physics Research Laboratory

Scientific Report No. NS-1

THEORY AND IMPLEMENTATION OF THE
PITOT-STATIC TECHNIQUE FOR UPPER ATMOSPHERIC MEASUREMENTS

J. J. Horvath
R. W. Simmons
L. H. Brace

ORA Projects 04673 and 03514

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CONTRACT NO. NASr-54(01)
WASHINGTON, D.C.

and

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE RESEARCH DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 19(604)-6124

administered through:

OFFICE OF RESEARCH ADMINISTRATION
ANN ARBOR

March 1962
PREFACE

The Department of Electrical Engineering of The University of Michigan has been, for many years, actively engaged in various aspects of atmospheric research by rocket-borne techniques. Early funding of the research effort was by the Geophysics Research Directorate of the Air Force Cambridge Research Center. One of the early experiments* was based on the physics of the supersonic flow field surrounding a right circular cone. The pitot and the cone-wall pressures, measured by radioactive ionization gages, were interpreted in terms of pressure, temperature, and density of the atmosphere. This effort continued through the IGY when approximately 10 payloads of this type were launched aboard Aerobee and Nike-Cajun vehicles at Ft. Churchill, Manitoba.

On the basis of the IGY launchings the experimenters have concluded that the ram and cone-wall measurements, as used on the Aerobee rocket, are very satisfactory and relatively easily accomplished. On the other hand, the upper altitude limit of the cone-wall measurement appears to be approximately 90 km, the system is not well suited to the measurement of winds, and the rocket angle of attack must be small. The latter is a serious limitation since it restricts the technique to the Aerobee rocket or other very stable and generally expensive vehicles.

In view of this, an alternative system, the pitot-static method of Ainsworth and LaGow, which suffers less from these limitations, has been adopted for ambient-pressure, temperature, density, and wind determination. This report reviews the theory of the technique and the implementation of the experiment at the Space Physics Research Laboratory. The research is sponsored by the National Aeronautics and Space Administration under contract NASr-54(01).

The results of a test launching of the pitot-static payload carried out at Ft. Churchill in October 1960 are presented as an appendix to this report. This launching and the associated research was done with the support of AFCRL Geophysics Research Directorate.

<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>ix</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xi</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>2.0 THEORY OF THE PITOT-STATIC MEASUREMENT</td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Outline of the Theory of Measurement</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Ambient density from the Ram-Pressure Measurement</td>
<td>6</td>
</tr>
<tr>
<td>2.4 Ambient-Pressure Measurement</td>
<td>8</td>
</tr>
<tr>
<td>2.5 Ambient Temperature</td>
<td>9</td>
</tr>
<tr>
<td>2.6 Atmospheric Wind from Spin-Pressure Measurement</td>
<td>9</td>
</tr>
<tr>
<td>2.7 Measurement Redundancy</td>
<td>12</td>
</tr>
<tr>
<td>3.0 IMPLEMENTATION OF THE EXPERIMENT</td>
<td>15</td>
</tr>
<tr>
<td>3.1 Radioactive Ionization Cage—Densatron</td>
<td>15</td>
</tr>
<tr>
<td>3.2 Supporting Rocket-Borne Equipment</td>
<td>23</td>
</tr>
<tr>
<td>3.3 Densatron Calibration</td>
<td>23</td>
</tr>
<tr>
<td>4.0 REFERENCES</td>
<td>29</td>
</tr>
<tr>
<td>APPENDIX—Nike-Cajun AA 6.340</td>
<td>31</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The pitot-static probe.</td>
<td>4</td>
</tr>
<tr>
<td>2. Phase-shift wind technique.</td>
<td>11</td>
</tr>
<tr>
<td>3. Phase-amplitude wind method.</td>
<td>13</td>
</tr>
<tr>
<td>4. Densatron ion current vs. pressure.</td>
<td>16</td>
</tr>
<tr>
<td>5. Densatron block diagram.</td>
<td>17</td>
</tr>
<tr>
<td>6. Densatron schematic complete.</td>
<td>18</td>
</tr>
<tr>
<td>7. Ionization gage exploded view.</td>
<td>19</td>
</tr>
<tr>
<td>8. Densatron exploded view.</td>
<td>20</td>
</tr>
<tr>
<td>9. Assembled Densatron electronics.</td>
<td>21</td>
</tr>
<tr>
<td>10. Assembled Densatron.</td>
<td>22</td>
</tr>
<tr>
<td>11. Pitot-static probe and rocket vehicle.</td>
<td>24</td>
</tr>
<tr>
<td>12. Pitot-static probe assembly diagram.</td>
<td>25</td>
</tr>
<tr>
<td>13. Calibration vacuum system.</td>
<td>27</td>
</tr>
<tr>
<td>16. Ambient pressure measured directly and calculated from ambient density, AA 6.340.</td>
<td>35</td>
</tr>
<tr>
<td>17. Ambient temperature calculated from the directly measured and integrated ambient pressures, AA 6.340.</td>
<td>36</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\( C_m \)  most probable particle speed given by \( \sqrt{\frac{2 k T}{m}} \)

\( g \)  acceleration due to gravity

\( h \)  altitude

\( k \)  Boltzmann constant \( 1.380 \times 10^{-23} \) joules/°K

\( K \)  constant given by \( \frac{\gamma + 1}{2\gamma} \left[ \frac{(\gamma + 1)M^2}{4\gamma M^2 - 2\gamma + 2} \right] \frac{1}{\gamma - 1} \)

\( m \)  mean molecular mass

\( M \)  mach number

\( P \)  pressure

\( P_i \)  gage-measured pressure

\( R \)  gas constant \( 2.151 \frac{\text{mm} M^2}{\text{kg} \ °K} \) for air

\( S \)  velocity parameter \( \frac{V}{C_{m0}} \cos \alpha \)

\( T \)  temperature

\( \bar{V} \)  velocity

\( v \)  speed of sound

\( \bar{W} \)  wind velocity

\( \alpha \)  angle of attack

\( \gamma \)  ratio of specific heats \( C_p/C_v = 1.4 \) for air

\( \rho \)  mass density

\( \delta \)  wind phase angle

\( \Delta p_s \)  peak-to-peak spin pressure amplitude
<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>ambient</td>
</tr>
<tr>
<td>H</td>
<td>horizontal</td>
</tr>
<tr>
<td>N</td>
<td>normal</td>
</tr>
<tr>
<td>d</td>
<td>descent</td>
</tr>
<tr>
<td>u</td>
<td>ascent</td>
</tr>
</tbody>
</table>
ABSTRACT

The pertinent characteristics of the grenade, sodium vapor, falling sphere, and pitot-static atmospheric measurement techniques are outlined. The pitot-static probe being implemented by investigators at The University of Michigan is described and the theory is discussed. The results of a test launching in October 1960 are presented to demonstrate the use of the pitot-static technique. It is concluded that the accuracy and versatility of the pitot-static probe and portability of its associated ground support system make it an extremely useful tool for the measurement of pressure, temperature, density, and winds up to 120 km in the earth's atmosphere, particularly at remote sites where other techniques are not feasible.
1.0 INTRODUCTION

The importance of gaining an understanding of the physics of the upper atmosphere* is most effectively demonstrated by the magnitude of the effort dedicated to the task of developing direct-measurement techniques for this region. Several rocket-borne techniques have proved useful in the direct measurement of the atmosphere above balloon altitudes. Among the more successful of these is the grenade experiment,¹ in which sounds from a series of grenade explosions propagate through the atmosphere to an array of microphones carefully positioned down range of the rocket launch site. A detailed analysis of the arrival times of the sounds from the explosions leads to a measurement of atmospheric temperatures and winds over the range of altitudes bounded by the first and last grenade.

A second technique, which has been highly successful and is well adapted to the measurement of winds, is the sodium-vapor experiment in which a ground-based array of cameras photographs a sodium vapor trail behind an ascending rocket.

A third technique measures atmospheric density by determining the drag on a sphere which is ejected from an ascending rocket and is allowed to fall freely back to earth. The measurements are made either by an internally-contained accelerometer (active falling sphere) or by radar tracking of a passive reflecting sphere.

A fourth technique, the pitot-static probe, which is to be described here, uses pressure measurements at suitable points on a rocket surface to obtain the ambient pressure, density, temperature, and winds.

These techniques cannot, in general, be considered substitutes for each other since their primary measurement, range of altitudes, and ground-support requirements differ widely. Table I gives a point-by-point comparison of the techniques discussed above. A study of the table shows a number of relative advantages and limitations exhibited by each technique.

**Measured Parameters.**—The falling-sphere, grenade, and sodium-vapor experiments are well suited to the direct measurement of a single physical parameter which is related to the parameters of the upper atmosphere as compared to the three parameters which are measured directly by the pitot-static method.

**Altitude Ranges.**—The falling-sphere and grenade techniques are limited

---

*Upper atmosphere refers to the altitudes above approximately 30 km, which may be reached by balloons.
to the continuous flow region below 90 km where the drag coefficient is well defined and where sound will propagate efficiently. The falling-sphere method may be extended to higher altitudes as the coefficient drag becomes better understood there. The sodium-vapor method encounters no basic altitude limitations other than photographic resolution and tracking. One aspect of the pitot-static method, the ram-pressure measurement, is not well defined in the transition region between 85 and 90 km, but becomes valid again above this altitude. The ultimate altitude of the pitot-static technique is limited to approximately 120 km by the pressure sensitivity of the present radioactive ionization gages.

Ground Support, Portability.—Comparison of the ground-support requirements of the four techniques gives insight into the relative portability of each method. Both the grenade and sodium-vapor experiments require extensive and well-surveyed ground-support systems for the placement of microphones and cameras. Thus, the use of these techniques is financially feasible only at well-established launch sites. The pitot-static and active falling-sphere techniques enjoy freedom from such complex ground-support systems, however, and thus may be made completely portable with no sacrifice in system accuracy. The pitot-static technique requires only a single station DOVAP for velocity determination, and telemetry receiver and tape recorder for recording of data. Such a system can be installed in a trailer and carried nearly anywhere. In the past, the falling-sphere experiment has been successfully carried out on shipboard where a simple radar is sufficient to assign altitudes to the data. The possibility of shipboard launching of the pitot-static probe is particularly intriguing.

Redundancy.—Table I also shows the degree of redundancy found in each of the four techniques. As can be seen, the falling-sphere, sodium-vapor, and grenade methods have no inherent redundancy where the pitot-static technique exhibits extensive redundancy. That is, the density may be obtained from each of two independent measurements: the spin pressure and the stagnation pressure. The ambient temperature may be obtained from two independent measurements, the ram and ambient pressures. The ambient pressure may be obtained from the same two independent measurements. And finally, the wind may be calculated by each of three different methods. Therefore, in the opinion of the authors, greater confidence can be had in the atmospheric parameters obtained by the pitot-static technique owing to the redundancy inherent in the measurement system.

*Redundancy refers to the ability of the experiment to obtain the same parameter from more than one independent measurement.
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Directly Measured Parameter</th>
<th>Primary* Deduced Atmospheric Parameters</th>
<th>Secondary** Deduced Atmospheric Parameters</th>
<th>Present Maximum Altitude</th>
<th>Ground Support</th>
<th>Portability</th>
<th>Redundancy</th>
<th>Other Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Falling sphere</td>
<td>Drag</td>
<td>Density</td>
<td>Pressure, temperature</td>
<td>80 km</td>
<td>Tracking, telemetry receiver, data recorder</td>
<td>Good</td>
<td>None</td>
<td>Uncertainty in coefficient of drag at higher altitudes</td>
</tr>
<tr>
<td>2 Grenade</td>
<td>Arrival times of sound bursts</td>
<td>Temperature, wind</td>
<td></td>
<td>90 km</td>
<td>Tracking, sound receiver, data recorder, surveyed launch site, and balloon data</td>
<td>Poor</td>
<td>None</td>
<td>Relatively few data points available from each flight</td>
</tr>
<tr>
<td>3 Sodium vapor</td>
<td>Mass motion and rate of diffusion of a vapor trail</td>
<td>Wind</td>
<td></td>
<td>200 km</td>
<td>Ballistic cameras, surveyed launch site</td>
<td>Poor</td>
<td>None</td>
<td>Limited to dusk firing and clear skies</td>
</tr>
<tr>
<td>4 Pitot-static</td>
<td>Ambient pressure, ram pressure, wind modulated pressure</td>
<td>Pressure, density, temperature, and wind</td>
<td>Pressure, density, temperature, wind</td>
<td>120 km</td>
<td>Tracking, telemetry receiver, data recorder</td>
<td>Good</td>
<td>Extensive</td>
<td>Uncertainty in ram pressure interpretation in the region of transition (85 km to 90 km)</td>
</tr>
</tbody>
</table>

*Primary refers to a direct solution from a measured parameter.

**Secondary refers to a solution from a primary deduced parameter.
Fig. 1. The pitot-static probe.
2.0 THEORY OF THE PITOT-STATIC MEASUREMENT

2.1 INTRODUCTION

The pitot-static technique is an aerodynamic approach to the measurement of pressure, temperature, and density in the upper atmosphere. The technique is a simple implementation of basic well-known aerodynamic theory. The word "pitot-static" implies two of the primary measurements which are inherent in the technique: the pitot or ram pressure and the static or ambient pressure. In addition to these two primary measurements, the system has been extended to include a spin-pressure measurement which leads to a determination of the upper-atmospheric wind.

The pitot-static probe technique has been extensively studied and developed by Ainsworth of the Goddard Space Flight Center. The probe design and measurement technique discussed here is based on this earlier work. The mechanical pressure sensors used by Ainsworth, however, have been replaced with radioactive ionization gages developed at this laboratory for use in rocket-borne probes during the IGY.

Figure 1 illustrates the configuration used. The raw data consist of the continuous measurement of pressure in the three chambers within the pitot-static probe: the ram pressure chamber, the ambient chamber, and the wind chamber. The single orifice of the ram and wind chambers, and the many orifices of the manifold ambient chamber are shown in the figure.

2.2 OUTLINE OF THE THEORY OF MEASUREMENT

The aerodynamic theory of the experiment is considered in the three flow regions which will be encountered by the probe in a high altitude flight. These are the continuous-flow region (below approximately 85 km), the transition region (between 85 and 90 km), and the free-molecular-flow region (above 90 km). These regions are defined by the dimension of the probe relative to the mean free path of the atmospheric particles and thus may be specified as ranges in altitude for any given probe.

The interpretation of the pitot or ram pressure in terms of ambient density follows from the theory in a straight-forward manner. In the continuous region, the ambient density is obtained from the measured ram pressure through the use of the Rayleigh equation and the equation of state. In the transition region, the present theory is not adequate to interpret properly the measured ram pressure in terms of the ambient density. But as pointed out previously, the loss of interpretable ram-pressure information occurs over a very small altitude range (between 85 and 90 km) and is not detrimental to the measure-
ment above and below this region. In the free-molecular-flow region, the continuum theory does not hold, and the density must be deduced from the molecular point of view by means of a modified thermal transpiration equation.\(^3\)

The static or ambient pressure is given directly by the manifold gage\(^4\) in the continuous-flow region. In the free-molecular-flow region, the measured pressure is corrected for the thermal transpiration effect to obtain the ambient pressure.

Having obtained the ambient pressure and density from two independent measurements, the ambient temperature is computed using the equations of state.

The atmospheric wind measurement in all three flow regions is based on the roll modulated ambient pressure in the wind chamber. A straight-forward analysis of the peak-to-peak amplitude and phase of the modulated pressure combined with the measured ambient density, rocket aspect, and trajectory leads to a determination of the local atmospheric wind velocity and direction.\(^2,5\)

A detailed statement of the theory outlined above is given in the following sections.

2.3 AMBIENT DENSITY FROM THE RAM-PRESSURE MEASUREMENT

As indicated in Section 2.2, the interpretation of the ram pressure in terms of ambient density differs in the continuum and free-molecular-flow regions. This is detailed in the following sections:

2.3.1 Continuum Region.—In the continuous-flow region, the equations used in deducing the ambient density, \(\rho_a\), from the measured ram pressure, \(P_i\), are the well-known Rayleigh-pitot-static tube equation (1) and the equation of state (2).

\[
\frac{P_i}{P_a} = \frac{\gamma + 1}{2} M^2 \left[ \frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2\gamma + 2} \right]^{\frac{1}{\gamma - 1}}
\]  

(1)

which holds for small angles of attack, \(\alpha \leq 0\), and

\[
\frac{P_a}{P_a} = \frac{RT}{P_a}
\]

(2)

To obtain the ambient density, Eq. (1) and Eq. (2) are multiplied with the following result:
\[
\frac{p_i}{\rho_a} = \frac{\gamma + 1}{2} \frac{M^2}{RT_a} \left[ \frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2\gamma + 2} \right]^{\gamma-1}
\] (3)

Recall from basic thermodynamics that

\[
v^2 = \gamma RT_a
\] (4)

where \(v\) is the speed of sound, and the definition of the Mach number is

\[
M^2 = \frac{V^2}{v^2},
\] (5)

and \(V\) is the velocity of the vehicle to which the Mach number refers. Then Eq. (3, 4 and 5) may be combined to obtain the ambient density

\[
\rho_a = \frac{p_i}{Kv^2}
\] (6)

where

\[
K = \frac{(\gamma + 1)}{2} \left[ \frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2\gamma + 2} \right]^{\gamma-1}
\] (7)

\(K\) is found to be a weak function of Mach number and, for a given small range of velocity, may be approximated by a known constant. With \(p_i\) in mm Hg and \(V\) in ft/sec, the density becomes

\[
\rho_a = \frac{1521.1 \ p_i}{v^2} \text{ kg/m}^3, \ 3.5 \leq M \leq 7.2
\] (8)

Thus the ram pressure leads directly to the ambient density in the continuous-flow region.

2.3.2 Free-Molecular-Flow Region. — In the free-molecular-flow region (above 90 km), the interpretation of the ram pressure is modified owing to the change in flow conditions, related to the disappearance of shock phenomena. In this region, the thermal transpiration effect is given by Eq. (9).

\[
\frac{p_i}{p_a} = \left( \frac{T_i}{T_a} \right) F(S)
\] (9)
where \( F(S) = e^{-S^2} + S \sqrt{\pi} (1 + \text{erf } S) \)

\[
S = \frac{V \cos \alpha}{C_m o}, \text{ the ratio of the component of rocket velocity normal to the orifice to the most probable particle velocity, and}
\]

\( \alpha = \text{rocket angle of attack.} \)

Combining Eqs. (9 and 2) and using the appropriate simplifying assumption made by Ainsworth,\(^2\) leads again to the ambient density.

\[
\rho_a = \frac{P_i}{\sqrt{\frac{2\pi k T_i}{m} |V| \cos \alpha}} \quad (10)
\]

### 2.4 AMBIENT-PRESSURE MEASUREMENT

In the continuous-flow region, the ambient pressure is measured directly at the manifold ambient chamber located as shown in Fig. 1. Wind tunnel investigations by Laurmann\(^4\) show that the surface pressure at this position on the tube is indeed ambient, and that the manifold chamber minimizes deviations from the ambient pressure caused by bow-shock perturbations, boundary-layer interference due to the expansion wave at the flare, and non-axial flow about the probe. In support of Laurmann's findings, the probe measurement we carried out at Ft. Churchill, October 1960, indicated no angle of attack dependence in the ambient-pressure measurement (see Appendix).

As discussed in Section 2.2, the measured pressure in the free-molecular-flow region will differ in magnitude from the ambient pressure owing to the thermal transpiration effect as given by

\[
\frac{P_i}{P_a} = \sqrt{\frac{T_i}{T_a}} \quad (11)
\]

Thus the ambient pressure cannot be found by using the manifold pressure until the ambient temperature is known, however an alternate method employs the previously obtained density and the hydrostatic equation

\[
P_a = P_{ref} + g \int_{h_2}^{h_1} \rho \, dh \quad (12)
\]

which is integrated numerically from a chosen low pressure, \( P_{ref} \) at some higher altitude \( h_2 \), to higher pressures at lower altitudes, \( h_1 \). It can be shown that
the choice of $P_{ref}$, the starting point of the integration, is relatively unimportant and Eq. (12) will rapidly converge to the true ambient pressure. A poor choice of $P_{ref}$ is readily apparent by its incompatibility with the resulting pressure profile, and another value which gives more rapid convergence is selected.

2.5 AMBIENT TEMPERATURE

The ambient temperature may be calculated by each of two methods. The more simple method is by direct substitution of the measured ambient density and pressure (as found in Section 2.4) in the equation of state, Eq. (2) which is then solved for the ambient temperature. A second method, described in detail by Ainsworth, et al.,\textsuperscript{2} is an iterative method that results in a set of derived temperature, pressure, and density profiles consistent through the equation of state. The correct temperature profile is recognized as that which produces the best possible fit between the derived pressure and density profile and the measured pressure and density profiles along the entire region of measurement. The criteria of best possible fit is that the first and second derivatives (slopes and slope changes) of the derived pressure and density profiles match the corresponding derivatives of the measured pressure and density profiles. This method is time-consuming and, thus, costly owing to the many profiles which must be computed, but should yield very accurate temperature data to verify the temperatures obtained by the simpler method described above.

2.6 ATMOSPHERIC WIND FROM THE SPIN-PRESSURE MEASUREMENT

2.6.1 Introduction.—Figure 1 shows the position of the spin-pressure orifice which permits measurement of the ambient-pressure modulation due to the non-axial flow resulting from the horizontal rocket velocity, the angle of attack of the probe, and the atmospheric wind.

Consider the pitot-static probe of Fig. 1 coasting in the measurement region and precessing with a small cone angle. In the absence of wind, the spin-pressure maximum coincides with the direction of the component of rocket velocity normal to the probe axis, which is essentially fixed throughout the flight.

Now consider the same probe in the presence of wind. The direction of the spin-pressure maximum will be shifted by an angle determined by the magnitude and direction of the wind vector. Coupling the rocket aspect and trajectory with the direction of spin-pressure maximum, the phase shift, $\phi$, between the wind and the no-wind conditions can be determined at any given altitude. If the following simplifying assumptions are made, (1) that the wind is horizontal, and (2) that the zenith angle of the probe is sufficiently small so that the measured phase angle can be transferred to the horizontal plane with little error, then the planar vector equation,
\[ \overline{V_H} = \overline{V_H} + V_N^{5,6} \] (12)

can be solved with two sets of trajectory and phase angle data to obtain the wind direction and magnitude at any given altitude.

Three procedures exist for determining the wind from Eq. (12), two of which are applicable in all three flow regions, and the other is valid only in free molecular flow. These methods are described in detail below.

2.6.2 Ascent-Descent Phase-Shift Method.\textsuperscript{5}—This method was suggested by Horovitz and LaGow and successfully used by Ainsworth.\textsuperscript{2} As stated previously, the phase shift between the wind and no-wind condition can be obtained from the spin-pressure measurement. Since these data are available for both the ascent and descent, two directions of spin-pressure maxima with respect to the no-wind condition may be obtained at a selected altitude. By using the ascent and descent trajectory (\( \overline{V_{H_u}} \) and \( \overline{V_{H_d}} \)) with the two phase shift angles (\( \delta_u \) and \( \delta_d \)), Eq. 12 may be solved for the wind vector at the selected altitude.

The solution of this equation is most easily carried out graphically as shown in Fig. 2. The ascent and descent data, \( V_{H_u} \), \( V_{H_d} \), \( \delta_u \) and \( \delta_d \) are plotted, and the lines 0-b and 0-c are constructed. These lines represent the directions of the spin-pressure maxima (direction of \( \overline{V_{N_u}} \) and \( \overline{V_{N_d}} \)) on ascent and descent, respectively, and their lengths have no particular significance. Following the rules of vector addition, lines d-c and f-g are constructed parallel to 0-b and 0-c, respectively. Their intersection then defines the direction and magnitude of the wind, \( \overline{W_H} \).

Because this method requires no evaluation of the absolute magnitude of the spin pressure, it is valid in all flow regions. Only the direction of spin-pressure maximum is required. A disadvantage of this method is the necessity of assuming identical ascent and descent winds at each altitude.

2.6.3 Precession Phase-Shift Method.\textsuperscript{5}—This technique, like that of the previous section, is valid at all altitudes, but depends on a change in probe aspect (rocket precession) to produce the required change in the measured phase shift angle during a series of successive spin periods. One assumes a constant wind in the region traversed during the necessary change in aspect. The normal precession rate of the rocket is adequate to satisfy this condition. The wind is deduced following the procedure of the previous section, where the ascent and descent components are replaced by the successively measured components.

2.6.4 Phase-Amplitude Method.—In the free-molecular-flow region, where the measurement of wind is of particular importance, a third technique for determining the wind is available. This approach, also successfully used by Ainsworth,\textsuperscript{2} does not require descent or successive spin period data, but rather
Fig. 2. Phase-shift wind technique.
the phase angle and peak-to-peak amplitude of the spin-pressure modulation. These data are used in Eq. (13) derived by Ainsworth, et al.2

\[ \Delta p_s = \rho_a \sqrt{\pi} C_{m_1} |\vec{V}_N| \]  

(13)

where \( \Delta p_s \) is the spin pressure peak-to-peak amplitude,

\( \rho_a \) is the measured density,

\( C_{m_1} \) is the most probable particle speed determined from the gage temperature

and

\( |\vec{V}_N| \) is the magnitude of the velocity vector normal to the spin pressure orifice at spin pressure maximum.

From Eq. (13) \( |\vec{V}_N| \) can be computed

\[ |\vec{V}_N| = \frac{\Delta p_s}{\rho C_{m_1} \sqrt{\kappa}} \]  

(14)

Using the simplifying assumptions of Section 2.6.1, Eq. (12) is then solved graphically for the magnitude and direction of the wind, as shown in Fig. 3.

Reviewing the quantities depicted in Fig. 3, the vector \( \vec{V}_H \), the horizontal velocity component of the rocket, is obtained from the trajectory. The angle \( \beta_H \), is the spin-pressure phase shift which defines the direction of the vector \( \vec{V}_N \) and is determined from the trajectory and rocket aspect, while the magnitude of \( \vec{V}_N \) is calculated from Eq. (14), valid in free molecular flow. And finally, the horizontal wind vector \( \vec{W}_H \) is the sum of \( \vec{V}_H \) and \( \vec{V}_N \).

This method requires the accurate measurement of the spin-pressure magnitude and direction unlike the two previous methods which required only direction; however, it requires only a single measurement, as opposed to two measurements in the previous techniques.

2.7 MEASUREMENT REDUNDANCY

A prime advantage found in the pitot-static technique is the redundancy inherent in the multiple sensing system, which permits each of the atmospheric parameters to be obtained by more than one independent method. If self-consistency exists among the data resulting from the various methods, increased confidence can be had in the measured values. Redundancy also helps maintain minimum data loss in the event of failure or improper operation of independent parts of the system.
Fig. 3. Phase-amplitude wind method.
Ambient Pressure.—As discussed in Section 2.4, the ambient pressure is measured directly at the manifold chamber in the continuous-flow region, and is found by integration of the ambient density in the free-molecular-flow region as well as the continuous-flow region.

Ambient Density.—The ambient density is deduced primarily from the ram-pressure measurement, as discussed in Section 2.2. The density may also be obtained in the free-molecular-flow region from the spin-pressure measurement alone. Using the wind as deduced from the methods discussed in Section 2.6.2 and Section 2.6.3, a value of $|\nabla n|$ is obtained. Combining $|\nabla n|$ and the peak-to-peak amplitude of the spin pressure, $\Delta p_s$, Eq. (13) is solved for the ambient density. Thus, in free-molecular-flow, the density may be obtained from two independent measurements.

Wind.—The three techniques available for the determination of the wind were discussed in Section 2.6. Two of these methods are valid in all flow regions, and the third is valid in free molecular flow. Thus a high degree of redundancy exists in this measurement which permits added confidence in the wind profiles obtained.
3.0 IMPLEMENTATION OF THE EXPERIMENT

3.1 RADIOACTIVE IONIZATION GAGE—DENSATRON

The pressure transducers (Densatrons) utilized with the pitot-static probe combine a radioactive ionization chamber with a multi-range electrometer amplifier to form a device capable of resolving the pressures found in the region of 30 to 120 km.

The Ionization Gage.—As with other ionization gages, neutral gas ionization results in a current which may be measured and, through laboratory calibration, interpreted in terms of gage pressure. The ionizing agent is in the form of a thin film of tritiated titanium on stainless steel foil, which is bonded to a flat disc serving as the ion chamber cover. The relatively low energy of the emitted beta particles makes the ion chamber completely free of external radiation. A disassembled ionization gage is shown in Fig. 11. Figure 4 illustrates the relationship between the neutral particle pressure and positive ion current. The nonlinearity at the high pressure (not shown) and low pressures is due to volume recombination and X-rays (dark current), respectively. The task of extending the low-pressure sensitivity is an ever-continuing effort.

The Electronic Circuits.—The electrometer amplifier and supporting circuits are shown in block diagram form in Fig. 5 and schematically in Fig. 6. To measure the wide range of pressure with optimum resolution, the electrometer amplifier has several linear ranges, each covering approximately one order of magnitude of current. The sensitivity of each range is determined by the value of hi-meg resistance at the input of the unity gain amplifier, and the amplifier output voltage is a direct measure of the gage current, thus the gage pressure.

The multi-range design of the amplifier requires that a range switching circuit be employed which will sense the amplifier output and insert the proper hi-meg resistor at the input. The switching is accomplished mechanically with a bi-directional synchromental motor, which is part of an electronic servo system. Because of the relatively large amount of power needed for motor operation, the system is designed to operate on the energy stored in a large capacitor.

The several voltages required for operation of the ionization gage and its associated electronics are developed in a DC-DC converter\(^7\) whose input power is nominally 5.9 volts ±10\% at 200 ma. Distribution of the secondary voltages is as shown in Fig. 6.

Figures 7 through 10 show various detail photographs of the Densatron.
Fig. 4. Densatron ion current vs. pressure.
Fig. 5. Densatron block diagram.
3.2 SUPPORTING ROCKET-BORNE EQUIPMENT

In addition to the various pressure measurements, the pitot-static payload includes an FM-FM telemetry system, a DOVAP transponder for trajectory, and an optical aspect system. This supporting equipment is consistent with the system goals of portability and low cost which will make the technique ideal for synoptic and/or remote launching sites.

DOVAP.—Owing to its relative portability and high degree of accuracy, a single station DOVAP system, with interferometer, is used to obtain the trajectory of the rocket. Such a system, now being used successfully at Wallops Island, can be placed in a single instrumentation van and carried to any remote launch site or placed on board a ship capable of launching small rockets.

Aspect System.—The optical aspect system which is used was developed at the Goddard Space Flight Center. A computer program will be available for data from this aspect system, which will greatly facilitate the reduction of the wind data from the pitot-static probe. This was a prime consideration in its choice.

Telemetry System.—As indicated above, an FM-FM system was selected for telemetry of probe and aspect data. To limit the cost of the system, much of the data will be commutated and placed on a single telemetry channel. The aspect system and wind Densatron will each have an analog channel.

Figure 11 is a perspective diagram of the probe attached to the Nike-Apache vehicle. The ram and ambient Densatrons are housed in the cylindrical section forward of the ambient and wind chamber, while the wind Densatron is mounted in the section aft of the wind chamber and extends into the flared section on which the turnstyle antenna is mounted. Aft of the flared section is the instrumentation housing containing the DOVAP, ASPECT system, and Telemetry system. The DOVAP antennae are mounted back along the rocket as has been the practice.

Figure 12 is an assembly diagram showing the chamber and Densatron mounting and the spaces provided for the DOVAP, Aspect, and Telemetry systems.

3.3 DENSATRON CALIBRATION

Vacuum System.—The Densatrons used in the probe are calibrated on a vacuum system built specifically for this purpose. The system, Fig. 13, consists of a main glass manifold which is connected at one end through a gate valve to the pumping equipment consisting of a water-cooled, three-stage fractionating oil diffusion pump in series with a mechanical fore pump. Attached to the other end is a second gate valve to a short metal manifold which permits one to three units to be mounted on the system for simultaneous calibration. The system is capable of pressures below $1 \times 10^{-6}$ mm Hg.
Fig. 11. Pitot-static probe and rocket vehicle.
Fig. 12. Pitot-static probe assembly diagram.
Pressure Standards.—A triple-range McLeod gage acts as the standard of pressure measurement in the range of $1 \times 10^{-4}$ to 5 mm Hg. For higher pressures, two Wallace and Tiernan aneroid gages, having a combined range of 1 to 800 mm Hg, are used. The low-pressure aneroid gage (0-50 mm Hg) is calibrated in the region where its range overlaps that of the McLeod gage (0-5 mm Hg). The small calibration correction which is established is extrapolated through its range. A similar technique is used to calibrate the high-pressure Wallace and Tiernan (0-1000 mb). To confirm the validity of this series of corrections, the high-pressure gage is calibrated at atmospheric pressure against a precision mercury manometer. An NRC Alphatron is used for approximate pressure indication as an accessory measurement.

Calibration Procedure.—The vacuum system is tested for leaks and outgassing by pumping it down to a minimum pressure (below $1 \times 10^{-6}$), closing the gate valve to the pumps, and then observing the rate of increase in pressure as indicated by the NRC Alphatron. The criterion of acceptable leak rate is that the pressure remains below $1 \times 10^{-4}$ mm Hg in the five-minute period after system seal-off. In preparation for calibration, one to three Densatrons are placed on the mounting manifold and maintained at the minimum pressure for 48 hours or more to insure adequate outgassing. The calibration then proceeds by increasing the system pressure from the minimum, in suitable increments, by bleeding in dry atmospheric air. At each calibration level, the pressure is measured by the appropriate standard and recorded along with each Densatron output voltage level. A series of such calibration points defines the pressure-voltage characteristic of each Densatron.

A calibration check is performed after the Densatrons have been pumped for an additional 48 hours or more. At least one point in each current range is checked. The unit is considered ready for flight if each check point lies within 2% of the original calibration.
4.0 REFERENCES


APPENDIX

NIKE-CAJUN AA 6.340

The pitot-static probe shown in Fig. 14 was flown on a Nike-Cajun vehicle from Ft. Churchill, Manitoba, Canada, October 17, 1960, at 3:04 p.m. CST. The research and this launching were supported by the Geophysics Research Directorate, Air Force Cambridge Research Laboratories. The primary purpose of the flight was to study the feasibility of the pitot-static technique using the low-cost Nike-Cajun vehicle. A secondary mission was the adaptation of the radioactive ionization gage, used in our IGY measurements, to use in the pitot-static probe. In these respects this payload may be considered the prototype of the probe discussed in this report, with the exception that no wind measurement was attempted.

The probe consisted of a stainless steel cylinder 3.5 inches in diameter and 42.5 inches long which housed the two Densatron units used to measure the ram and ambient pressures. The ram unit had a pressure range of $4 \times 10^2 \text{ mm Hg}$ to $6 \times 10^{-3} \text{ mm Hg}$, and the ambient unit had a pressure range of $1 \times 10^1 \text{ mm Hg}$ to $5 \times 10^{-3} \text{ mm Hg}$. A thermistor was located near each gage to determine the temperature rise of the hemispherical nose tip, the cylindrical skin, and the ambient pressure chamber. A DOWAP transponder was located in the cylindrical mating section aft of the flare and was used for both trajectory and telemetry. A radially mounted magnetometer was also located in the mating section for the purpose of determining the approximate aspect of the probe during flight. The total weight of the payload was 49.8 pounds.

The vehicle performed as predicted and reached a peak altitude of approximately 135 km. The instrumentation also performed normally and provided good ram-density data to over 90 km and ambient-pressure data from 30 km to 70 km, the limits being determined by the pressure range of the early Densatron units used. The angle of attack as indicated by the flight magnetometer did not exceed 10°, and did not appear to affect either the ram or ambient pressure measurement.

The preliminary data from the flight is presented in Figs. 15, 16 and 17. The density obtained from the ram pressure, Fig. 15, shows good agreement with the 1961 CIRA Standard Atmosphere. Also, the excellent agreement with balloon data near 30 km provides a high degree of confidence in the density measurement.
The two ambient-pressure profiles, Fig. 16, agree well with the CIRA model up to an altitude of approximately 65 km. Above this altitude the agreement in the directly measured pressure is not as good, although the measurement accuracy is judged to be adequate for the points shown. The gap in data between 70 km and 76 km was caused by loss in DOWAP signal owing to an antenna failure. For the same reason, trajectory information above this altitude may be slightly in error. The observed divergence in the two pressure profiles falls within the experimental error of the preliminary data reduction.

Figure 17 shows the temperature obtained from the two pressure profiles and the density profile through the equation of state. The general shape of the profiles agrees well with the CIRA Standard Atmosphere. The magnitude difference between the profiles is explained by the difference in the two pressure profiles used to calculate them. The temperature determined from the integrated pressure is favored in this flight, since it agrees with the balloon data at the low altitudes and with the model atmosphere at the higher altitudes. The divergence above 65 km of the temperature determined from the direct ambient pressure can be attributed to the error in the ambient pressure and the uncertainty in the gage temperature which is required for accurate data reduction. The point-to-point temperature spread arises from the experimental error in the individual density points. Had a smooth density profile been used, a smoother temperature profile would have resulted.

A final report on these data is forthcoming when further data reduction is completed.

CONCLUSION

This flight demonstrated the feasibility of the pitot-static technique for use on small rockets and indicated a need for: (1) extension of the Densatron pressure range down to $1 \times 10^{-3}$ mm Hg to increase the measurement altitude to above 120 km; (2) better gage-temperature measurement, and (3) accurate trajectory data through the entire flight. In the period since the flight, further refinement of the probe system has included the above improvements.
Fig. 15. Ambient density above Ft. Churchill, 17 October 1960, AA 6.340.
Fig. 16. Ambient pressure measured directly and calculated from ambient density, AA 6.340.
Fig. 17. Ambient temperature calculated from the directly measured and integrated ambient pressures, AA 6,340.