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# DESIGN MEMORANDUM
# ON
# BEARING CAPACITY OF SPREAD FOOTINGS
# ON COHESIVE SOIL

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# BEARING CAPACITY OF SPREAD FOOTINGS ON COHESIVE SOIL

In engineering practice the ability of a mass of cohesive soil or plastic clay to support loads distributed over an area is a problem of primary importance. The problem of this type most familiar to practicing engineers is the bearing capacity of spread footings or mats which serve as the substructure for buildings or other structures. There have been presented a considerable number of theoretical formulas for bearing capacity developed on the basis of varying assumptions during the formulative stage of soil mechanics and many of them are in use at the present time. With a large number of formulas to choose from, it has become difficult for engineers and even soil mechanics specialists to compare the assumptions upon which they are based to determine which formula is applicable to any specific case or in the case of apparent disparity in results, to decide which formula is valid. One of the primary objectives of this discussion, aside from presenting a rational development of bearing capacity, will be to compare the various formulas which are available.

## Stress Reactions in the Compression Cone

In Figure 1 is shown in simplified form the manner in which applied pressure on a spread footing is transmitted to the supporting soil (See Figure 33 in Notes on Pressure Distribution)[11]. A portion of the applied pressure at the bottom of the footing is distributed laterally within the compression cone which may be designated by the 1 to 1 angle of spread within which a uniform average pressure, $n_1$, is produced at a depth, h, below the footing. The amount of load which may be so distributed is limited by the shearing resistance of the soil acting on vertical planes through the edge of the bearing area. The rest of the load which may be supported by the soil mass is transmitted directly down the central column as a concentration of pressure. Stress reactions supplied by the soil which enter into such pressure distribution are shown in Figure 1 with the magnitude of the various pressure components indicated. In case the footing is at a depth, $h_o$, below the surface of the ground, there are certain stress reactions available above the loading plane which are also shown in Figure 1.

The development of a formula for the total supporting capacity will consist of evaluating each of these increments of pressure as a stress component depending upon the conditions of static equilibrium for each supporting element of mass within the compression cone. These supporting elements of mass are designated as Element 1 and Element 2. Element 1 is the compression block immediately below the bearing area with a lateral dimension equal to the lateral dimension of the footing. Element 2 is a similar cubical element on any side of Element 1 which supplies lateral support and plays a part in the lateral distribution of vertical pressure.

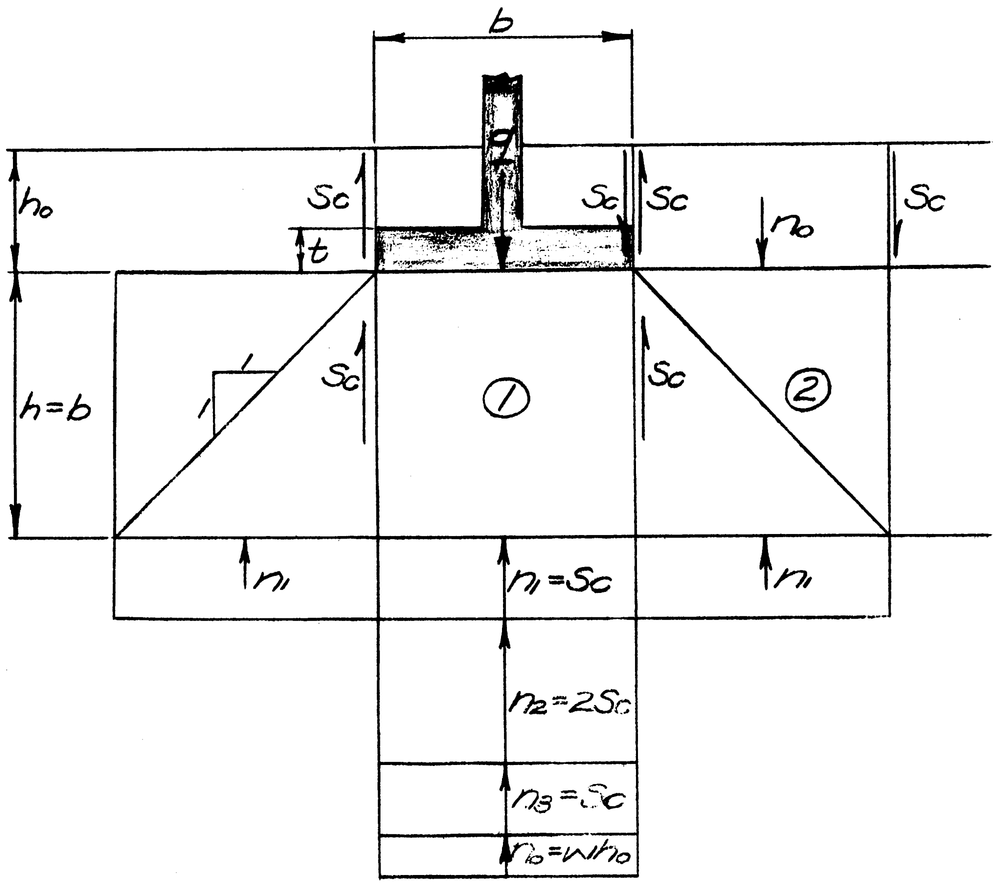
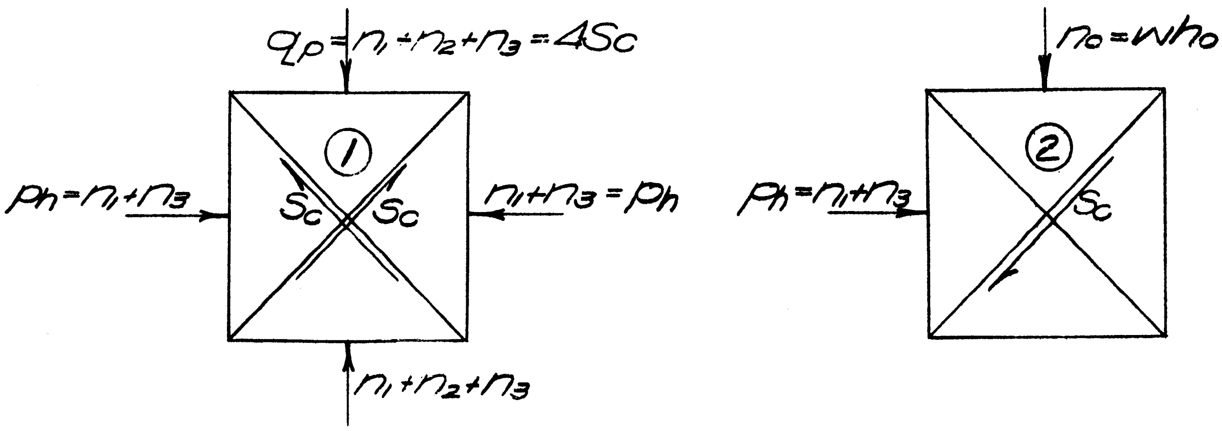



Fig. 1 Stress Reactions in the Compression Cone

Bearing Capacity Due to Developed Pressure




$q_p - p_h = 2S_c$

$n_2 = 2S_c$

$p_h = n_o + 2S_c$

When $h_o = 0 \quad n_o = 0$

and $p_h = n_1 + n_3 = 2S_c$

When $h_o = 0$

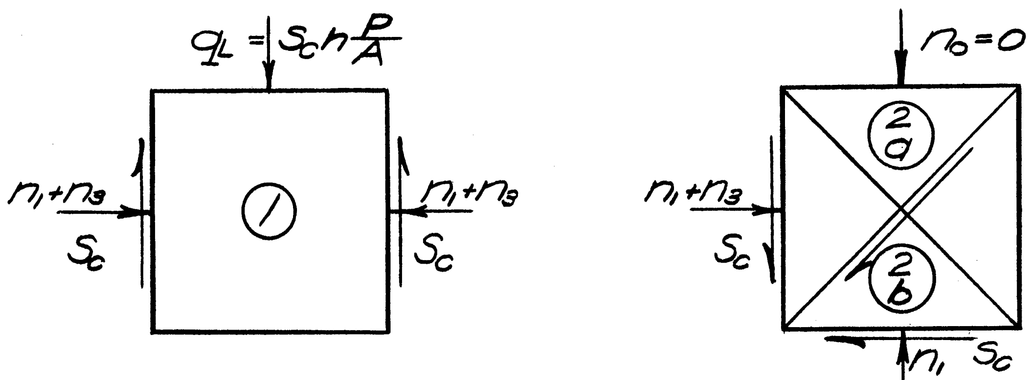
$$q_p = n_1 + n_2 + n_3 = 4S_c \qquad (1)$$

Fig. 2 Stress Reactions in Developed Pressure

The first increment of bearing capacity is designated as developed pressure, $q_p$, and the pressure components or stress reactions in the soil which make up developed pressure are illustrated in Figure 2. Element 1 acts as a compression block subjected to three pressure components shown as $n_1$, $n_2$ and $n_3$. Pressure components, $n_1$ and $n_3$, originate as lateral pressure or support from adjacent Element 2. They produce cubical compression on Element 1 which is transmitted as a fluid pressure resulting in no shearing stresses within that element. The vertical pressure component, $n_2$, acting on Element 1 is the block compressive strength or difference in principal pressures equal to twice the shearing resistance of the soil. Referring to Element 2 in Figure 2 the limit of lateral pressure which this block can supply without displacement is also equal to the maximum difference in principal pressures ~~of~~ or twice the shearing resistance of the soil. Thus the developed pressure, $q_p$, for a footing at the surface of the ground, when $h_o$ is zero, is equal to four times the shearing resistance of the soil due to cohesion.

Bearing Capacity Due to Lateral Distribution Below the Loading Plane

The next source of bearing capacity to be considered is lateral distribution below the loading plane, $q_L$, which provides support for the bearing area through lateral distribution of the vertical pressure outside the central column. This reaction originates in vertical shearing resistance acting on the boundaries of the central column as perimeter shear. In general form, this reaction on Element 1 may be expressed as the vertical shearing resistance multiplied by the perimeter-area ratio of the bearing area and the depth over which this vertical shearing resistance is mobilized. The equilibrium conditions on Element 1 under lateral distribution are shown in Figure 3.




$n_1 + n_3 = 2S_c$ $\qquad\qquad$ $n_1 = S_c$, $n_3 = S_c$

Fig. 3 Stress Reactions from Lateral Distribution

$$q_L = S_c h \frac{P}{A} \qquad (2)$$

P = Perimeter in feet.
A = Area in square feet.

$$h = b$$

Strip Footings - $\frac{P}{A} = \frac{2}{b}$ $\quad q_L = S_c h \times \frac{2}{b} = 2S_c \qquad (2a)$

Round Footings - $\frac{P}{A} = \frac{\pi b}{\frac{\pi b^2}{4}} = \frac{4}{b}$

Square Footings - $\frac{P}{A} = \frac{4b}{b^2} = \frac{4}{b}$

Round and Square Footings - $q_L = S_c h \times \frac{4}{b} = 4S_c \qquad (2b)$

The perimeter-area ratio is inversely proportional to the width or diameter of the bearing area being equal to $\frac{4}{b}$ for round and square footings and $\frac{2}{b}$ for strip footings. Any pressure component or bearing capacity term in which the perimeter-area ratio is a factor would ordinarily vary as the size of the bearing area and would be progressively smaller as the size of the bearing area increases. However, evaluating the ultimate bearing capacity, as in the present case, the depth to which lateral displacement of the soil mass takes place is equal to the width of the bearing area. Consequently, when the depth, h, is taken equal to b and substituted in the formula for bearing capacity due to lateral distribution, the bearing capacity term is no longer inversely proportional to the width of the bearing area but is the same for all sizes of bearing area.

Justification for assuming that the vertical shearing resistance on the perimeter plane is mobilized for a depth equal to the footing width is provided from the equilibrium conditions on Element 2 under lateral distribution. As shown in Figure 2 and already discussed in connection with developed pressure, the maximum lateral support available from Element 2 is established by the maximum difference in principal pressures equal to twice the shearing resistance. However, only a portion of the pressure transmitted to Element 2 is involved in the downward transmission of pressure within the compression cone.

The pressure transmitted downward on Element 2 is designated in Fig. 3 as $n_1$ and cannot exceed the vertical shearing resistance, $S_c$, acting on the perimeter plane. Thus under the lateral distribution of vertical pressure within the compression cone, the 45-degree portion of Element 2 designated as 2b is in equilibrium under a combination of normal pressure, $n_1$, and shearing resistance acting on the horizontal and vertical planes bounding that wedge.

The additional lateral pressure component, $n_3$, while available insofar as lateral displacement is concerned is not transmitted downward as a distributed pressure within the compression cone. It is, however, maintained as a concentration of pressure in the central column and transmitted downward by Element 1 as shown in Figure 1. Under the combination of stresses illustrated in Fig. 3, the bearing capacity available from lateral distribution then reduces to four times the shearing resistance for round and square footings and twice the shearing resistance for strip footings.

Bearing Capacity Due to Static Head or Flotation

In Fig. 1 provision was made for including in the bearing capacity of a spread footing, reactions which may be mobilized above the loading plane when the footing is at a depth, $h_o$, below the surface. The first and most important of such factors is the bearing capacity due to static head or flotation which may be designated as $q_h$. Such overburden pressure may be regarded as cubical compression transmitted through Elements 1 and 2 to the bottom of the footing. When described as flotation, it is implied that the soil mass is capable of transmitting fluid pressures proportional to the static head with an active pressure or upward reaction on the bearing area. Such a pressure component causes no shearing stresses within the soil mass and being independent of shearing stress plays no part in the lateral distribution of pressure within the central column but merely adds to the concentration of pressure which may be transmitted downward by Element 1.

Another way of viewing static head is to consider that when a footing is placed in an excavation an applied pressure equal to the weight of soil excavated may be imposed on the soil mass without causing any shearing stresses within the mass. As indicated, the bearing capacity due to static head, $q_h$, is equal to the weight of the soil in pounds per cubic foot multiplied by the depth of overburden, $h_o$.

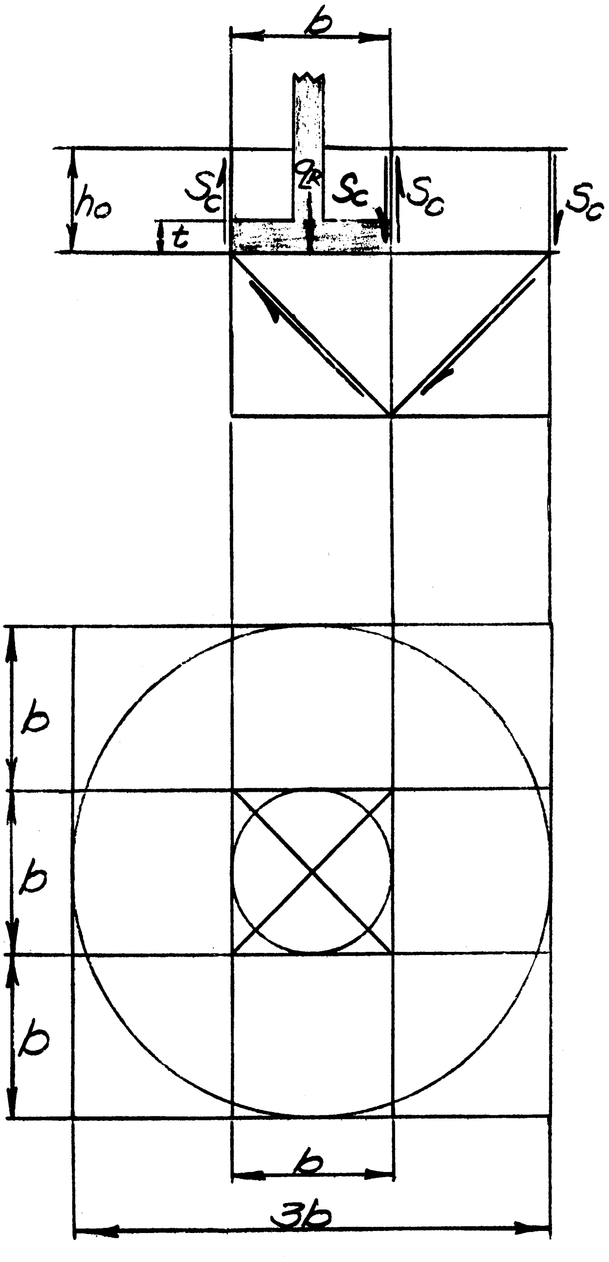
$$q_h = n_o = wh_o \tag{3}$$

Bearing Capacity Due to Resistance to Upheaval

Before complete lateral displacement under a bearing area can take place, the surrounding overburden may exert confining pressures in excess of the static head originating in vertical shearing resistance on boundaries of the displaced columns of earth. This confining pressure has been designated as resistance to upheaval. Such shearing reactions were indicated in Fig. 1 but are presented in more detail in Fig. 4. As applied to a strip footing the upward movement of a column of earth on either side of a bearing area would be resisted by downward shearing forces on two vertical planes. The total downward shearing force is then distributed as an equivalent pressure over a width of element, b. The increment of bearing capacity, $q_r$, is then equal to twice the shearing resistance times the depth, h, divided by the width, b.

In connection with this expression it may be noted that full shearing resistance is assumed on the perimeter planes through the edge of the footing which may or may not be true depending upon the manner in which the backfill may be placed. If shearing resistance in the backfill is to be considered negligible, shearing resistance on this perimeter plane would be neglected. In other cases the footing may be poured in contact with undisturbed soil and even though the backfill is not a source of resistance, shearing resistance may be mobilized for the thickness of the footing, t.

In round or square footings, resistance to upheaval must be evaluated as a three-dimensional problem in which such resistance is acting on all sides of a footing. Expressions for these cases are developed in connection with Fig. 4 and it is pointed out that shearing resistance available around the perimeter plane of the bearing area acts on a much smaller area than the reaction on the outside boundary of the columns subjected to upheaval. When the total vertical shearing force mobilized is distributed over the area on




**Strip Footings**

$$q_r = \frac{2S_c h_o}{b} \qquad (4a)$$

**Round Footings**

Total Vertical Shear

$$S_c h_o \pi b + S_c h_o 3\pi b$$

Equivalent Pressure Area

$$\frac{9\pi b^2}{4} - \frac{\pi b^2}{4} = 2\pi b^2$$

Equivalent Pressure, $q_R$

$$q_R = \frac{S_c h_o \pi b}{2\pi b^2} + \frac{3S_c h_o \pi b}{2\pi b^2}$$

$$q_R = \frac{1S_c h_o}{2b} + \frac{3S_c h_o}{2b} = \frac{2S_c h_o}{b} \qquad (4b)$$

**Square Footings**

Total Vertical Shear

$$4S_c h_o b + 12S_c h_o b$$

Equivalent Pressure Area $= 8b^2$

Equivalent Pressure, $q_R$

$$q_R = \frac{4S_c h_o b}{8b^2} + \frac{12S_c h_o b}{8b^2}$$

$$q_R = \frac{1S_c h_o}{2b} + \frac{3S_c h_o}{2b} = \frac{2S_c h_o}{b} \qquad (4b)$$

Fig. 4 Resistance to Upheaval

General Equation $\quad q_R = S_c h_o \frac{P}{A} \qquad (4)$

which it acts as an equivalent pressure, it is found that the shearing surface on the perimeter of the bearing area supplies only one-quarter of the total reaction while the shearing resistance mobilized on the outside boundary of the column subjected to upheaval supplies three-quarters of the total reaction. For round and square footings, the bearing capacity increment due to resistance to upheaval is given by Eq. (4b). When shearing resistance is mobilized for the full depth on all shear surfaces, resistance to upheaval in three dimensions, which is a function of the group capacity of columns on all sides of the bearing area, becomes the same as the expression for the two-dimensional case or strip footing. However, the subdivision of resistance to upheaval into two terms is desirable to retain because of the varying conditions which may be encountered in design which may eliminate shearing resistance on some of the surfaces involved.

Bearing Capacity Due to Perimeter Shear Forces Above the Loading Plane

When shearing resistance is mobilized on vertical planes through the edge of the bearing area as discussed in connection with resistance to upheaval, it follows that the opposing shear force or reaction acts on the free body which includes the footing itself. The upward shear forces act on the entire perimeter of the bearing area and are available to carry applied pressure in addition to pressure acting along the bottom of the footing. Following equations give the magnitude of these reactions in terms of bearing capacity for strip footings, square and round footings.

General Equation $$q_s = S_c h_o \frac{P}{A} \tag{5}$$

Strip Footings $$q_s = S_c h_o \frac{2}{b} = \frac{2S_c h_o}{b} \tag{5a}$$

Square Footings $$q_s = S_c h_o \frac{4b}{b^2} = \frac{4S_c h_o}{b} \tag{5b}$$

Round Footings $$q_s = \frac{S_c h_o \pi b}{\frac{\pi b^2}{4}} = \frac{4S_c h_o}{b} \tag{5b}$$

Summation of all Resistance Factors Included in the Ultimate Bearing Capacity

The ultimate bearing capacity of a spread footing on cohesive soil is the summation of all the factors of resistance which have been described and includes developed pressure, lateral distribution below the loading plane, static head, resistance to upheaval and perimeter shear forces. This ultimate bearing capacity can be most concisely stated in the form of the following general equations which apply to strip footings, round and square footings as designated. Equations (6a) and (7a) represent the expressions which are generally used which include a combination of like terms which may be desirable for brevity although such combinations may have the disadvantage of making it more difficult to identify the sources of resistance from which these terms originate as given in Eqs. (6) and (7).

Strip Footings - Two-Dimensional

$$q = 4S_c + 2S_c + wh_o + \frac{2S_c h_o}{b} + \frac{2S_c h_o}{b} \qquad (6)$$

$$q = 6S_c + wh_o + \frac{4S_c h_o}{b} \qquad (6a)$$

$$\text{When } h_o = 0 \quad q = 6S_c \qquad (6b)$$

Round and Square Footings - Three-Dimensional

$$q = 4S_c + 4S_c + wh_o + \frac{2S_c h_o}{b} + \frac{4S_c h_o}{b} \qquad (7)$$

$$q = 8S_c + wh_o + \frac{6S_c h_o}{b} \qquad (7a)$$

$$\text{When } h_o = 0 \quad q = 8S_c \qquad (7b)$$

## REVIEW OF OTHER BEARING CAPACITY FORMULAS

As previously stated there are a number of formulas for evaluating the ultimate bearing capacity of cohesive soils with which it is desirable to correlate the expressions which have been presented above. In their general form, most of these other formulas have been developed on the assumption that shearing resistance of soil is a function of the angle of internal friction, $\phi$. However, for a purely cohesive soil which is generally taken to be representative of plastic clays, $\phi$ is equal to zero so these formulas may be reduced to a simplified form for direct comparison.

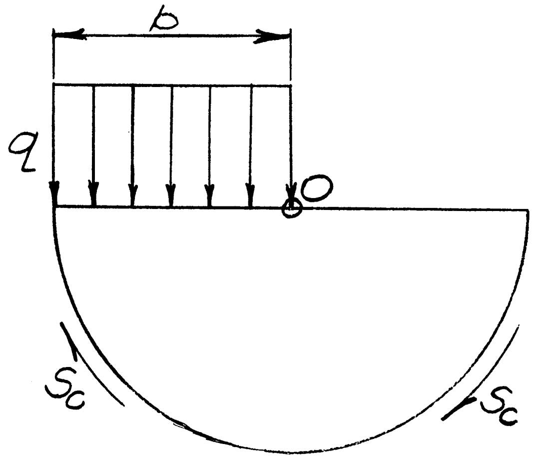
Tschebotarioff,[1] Taylor,[2] and Capper and Cassie[3] have all reviewed a number of the available formulas. From these and other references listed at the end of this memorandum, all of the available and fairly well known formulas for bearing capacity of a cohesive soil have been taken and compared under the conditions designated. In correlating these formulas, there are several factors which entered into both the writer's development and the derivation of the other formulas which must be subjected to careful analysis before a direct comparison can be made.

<u>Other Bearing Capacity Formulas</u>

Assumptions - $h_o = 0$ $\quad \theta = 45 - \frac{\phi}{2}$ $\quad \phi = 0$

<u>Strip Footings</u>

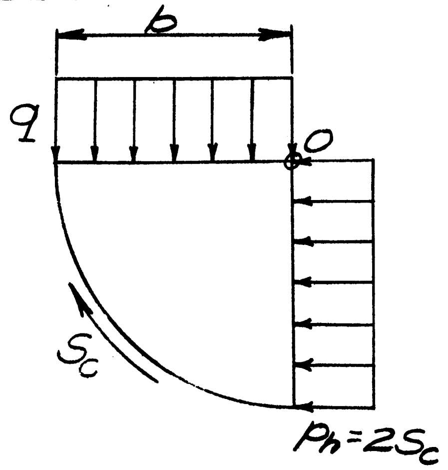
| | | |
|---|---|---|
| <u>Bell</u> | $q = 4S_c$ | (Reference 3, 4) |
| <u>Krey</u> | $q = 6S_c$ | (Reference 3) |
| <u>Prandtl</u> | $q = 5.14\ S_c$ | (References 1, 2, 3, 6) |
| <u>Fellenius</u> | $q = 5.52\ S_c$ | (References 1, 2, 3) |
| <u>Terzaghi</u> | | |
| | General Shear $q = 5.7\ S_c$ | (References 1, 2, 3, 6) |
| | Local Shear $q = 3.8\ S_c$ | (References 2, 6) |
| <u>Tschebotarioff</u> | | (Reference 1) |




$$\frac{qb^2}{2} = S_c \pi b$$

General Shear $q = 6.28\ S_c$

Tschebotarioff (continued)




$$\frac{qb^2}{2} = S_c \pi \frac{b}{2} b + 2S_c \frac{b^2}{2}$$

$$q = 3.14\ S_c + 2S_c$$

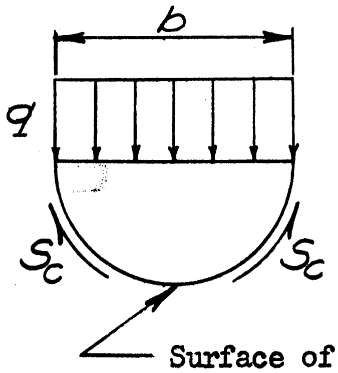
With passive lateral pressure mobilized

General Shear $q = 5.14\ S_c$

Without passive lateral pressure

Local Shear $q = 3.14\ S_c$

Michell




Max. $f_s = \frac{q}{\pi} = S_c$ (References 3, 8)

$q = \pi S_c$

Surface of Maximum Shearing Stress, $f_s$.

Local Shear $q = 3.14\ S_c$

Round or Square Footings

Krey

$q = 4S_c$ (References 5a, 5b)

Hencky

$q = 5.64\ S_c$ (Reference 3)

Terzaghi

$q = 7.4\ S_c$ (Reference 1, 6, 7)

Tschebotarioff

$q = 7.95\ S_c$ (Reference 1)

## Summary of Other Formulas

### Strip Footings

Local Shear - q varies (3.14 to 3.8) $S_c$

General Shear - q varies (4.0 to 6.28) $S_c$

### Round or Square Footings

General Shear - q varies (4 to 7.95) $S_c$

## Discussion of Other Formulas

There are two major sources of variation in the values of ultimate bearing capacity given by the formulas under discussion. In the first place the conditions designated as local shear and general shear produce the widest range of difference in the bearing capacity values. In the second place, the question of whether or not lateral distribution has been taken into consideration also produces considerable variation particularly between the two-dimensional and three-dimensional solutions.

Terzaghi was apparently the first to give consideration to local shear and general shear as a means of differentiating between different ranges of bearing capacity. Terzaghi's definition of local shear and general shear is rather difficult to relate definitely to specific factors of resistance but it is essential to do so for intelligent comparison with the other formulas. Terzaghi(6) describes these two types of failure on the basis of typical load-settlement diagrams, one of which is characteristic of an incompressible soil mass while the other is characteristic of a relatively compressible soil.

If the soil is relatively incompressible all of the resistance factors involved in general shear failure accompanying lateral displacement are mobilized at a relatively small amount of settlement. On the other hand, in a compressible soil, the settlement becomes relatively large and may progress with volume displacement being absorbed by volume change within the compression cone without mobilizing the resistance to lateral displacement within any acceptable range of settlement, if at all.

Failure by local and general shear as defined by Terzaghi, may be identified in terms of the resistance factors employed in several of the other formulas which have been cited. For example, Tschebotarioff[1] develops an expression for the ultimate bearing capacity under general shear failure by considering the rotation of a cylindrical volume about a center taken at the edge of the footing. This solution results in a value of ultimate bearing capacity of 6.28 times the shearing resistance which is the highest given by any of the formulas for a strip footing under the assumed conditions. Tschebotarioff feels that this value is somewhat too high and suggests that it be reduced to a value approximating that obtained by Fellenius of 5.52 times the shearing resistance. Such an adjustment is purely a matter of judgment and must be done empirically so that from the standpoint of the principles involved, the higher value should be considered.

Tschebotarioff has also presented a solution in which the passive lateral pressure is included as a separate factor making it possible to separate it from the rotational resistance on a cylindrical surface immediately below the bearing area. Assuming that the latter resistance is comparable to local shear failure produces the two values shown in the review of other formulas of 3.14 times the shearing resistance for local shear failure and 5.14 times the shearing resistance for general shear failure.

Interesting confirmation of the lower bearing capacity value for local shear can be obtained from the Michell[8] solution for stresses under a uniformly loaded strip. (See Figs. 3 and 4 in Notes on Pressure Distribution.)[11] In the Michell solution the maximum shearing stress occurs on a cylindrical surface with the center of the cylinder at the center of the bearing area. Equating this shearing stress which is $\frac{q}{\pi}$ to the shearing resistance of the soil produces the same value for local shear failure of 3.14 times the shearing resistance given by Tschebotarioff's solution. Thus for strip footings carrying a uniform load the ultimate bearing capacity under local shear ranges from 3.14 to 3.8 times the shearing resistance. The higher value which is given by Terzaghi is based on empirical factors taken from experiment to provide an approximate value for the bearing capacity.

The range of bearing capacity values for general shear on strip footings varies from 4 to 6.28 times the shearing resistance. In this connection, it should be pointed out that in the development of those formulas presented by Bell, Krey, Prandtl and Fellenius, there was no specific reference to the conditions of local or general shear but it must be presumed that they refer to general shear only. The Bell formula, $q = 4S_c$, is based on plain surfaces of failure and amounts simply to evaluating the maximum difference in principal pressures on two mutually supporting elements of mass. The Krey formula, $q = 6S_c$ is cited by Capper and Cassie but no development is given and this value could not be confirmed from the available references to the original Krey formula. The values given by Prandtl and Fellenius, falling in the middle of the range of values cited, are those most frequently referred to in writing on this subject. The Prandtl formula evaluates the full resistance to general shear considering that the surfaces of shear failure are most closely approximated by a logarithmic spiral. Fellenius assumed rotation of the soil mass under the footing on a circular surface with a center of rotation determined by trial and not coinciding with the edge of the footing.

Turning to the three-dimensional problem of round or square footings there were four formulas found in available references with values of ultimate bearing capacity ranging from 4 to 7.95 times the shearing resistance.

The Krey formula for this case was cited in the first reference in the War Department Engineering Manual for Civil Works and in the form presented reduces to a bearing capacity value of four times the shearing resistance for a cohesive soil. There was no indication of whether this referred to the condition of local or general shear but it must be presumed that it does refer to general shear and obviously includes no recognition of lateral distribution of vertical pressure. The Hencky formula, $q = 5.64\ S_c$, was given by Capper and Cassie without development but must also be presumed to refer to general shear.

In connection with the Terzaghi value of 7.4 times the shearing resistance the author states that for computing the bearing capacity of spread footings with square or circular bases not even an approximate theory is available. On the basis of experiments a semi-empirical equation has been derived which, for a fairly dense or stiff soil produces the value that has been given. Terzaghi also indicates that if the supporting soil is fairly loose or soft the values of the empirical factors must be adjusted in a way that would presumably take into consideration the condition of local shear. However, no specific bearing capacity values are indicated for local shear on a square or round footing.

Tschebotarioff presents a solution in which the resistance to rotation of a cylindrical soil volume includes resistances on the ends of the cylinder at the edges of the footing. When the length of the cylinder is equal to the width of the footing, the total resistance evaluated in this way amounts to 7.95 times the shearing resistance including minor empirical factors by which the theoretical value has been reduced.

Thus the available formulas for the ultimate bearing capacity of a round or square footing produce values ranging from 4 to 7.95 times the shearing resistance. It may be noted that all of these formulas refer to general shear and that no specific values were presented for local shear failure. In this connection Capper and Cassie have noted that, "experiments on square and circular footings have shown ultimate values 25 to 30 per cent greater than for strip loading". Applying such an increase to the range of values shown for local shear failure on a strip footing would produce values of bearing capacity, for a round or square footing under local shear approaching 5 times the shearing resistance of the soil.

## CONCLUSIONS AND RECOMMENDATIONS

Summarizing the review of the various bearing capacity formulas that have been discussed, the resistance factors associated with local and general shear failure may be identified with stress reactions in the compression cone as illustrated in Fig. 1. The resistance factors available in connection with local shear failure may be conceived to be those associated with lateral distribution of vertical pressure within the compression cone prior to concentration of pressure in the central column. These factors would include the vertical shearing resistance on the perimeter planes and that component of lateral pressure identified as $n_1$ necessary to the equilibrium of Elements 1 and 2 as required to produce the uniform average distribution of pressure over the full width of the compression cone. Bearing capacity available at this stage of loading would then be equal to 3 times the shearing resistance for the strip footing or two-dimensional case and 5 times the shearing resistance for the square or round footings in the three-dimensional case.

In the case of general shear failure for a strip footing, ultimate bearing capacity ranges from 4 to 6.28 times the shearing resistance in the other formulas that have been reviewed as compared to 6 times the shearing resistance in the development presented by the writer. For the three-dimensional case of the round or square footing bearing capacity values in the other formulas range from 4 to 7.95 times the shearing resistance as compared to 8 times the shearing resistance in the formulas presented by the writer including all factors of resistance involved in developed pressure and lateral distribution of vertical pressure.

Consequently it is concluded that Eq. (6b) for strip footings and Eq. (7b) for round or square footings are substantially confirmed by the range of values found in other bearing capacity formulas that have been reviewed and particularly so when the factors of resistance which entered into development of the formulas have been carefully analyzed. It should be emphasized at this point that the comparison that has been made applies to footings at the surface of the ground when the static head or overburden surrounding the footing is equal to zero. Inclusion of static head, resistance to upheaval and perimeter shear forces mobilized around the edge of the footing as given in either Eqs. (6) and (6a) for the strip footing or Eqs. (7) and (7a) for the round or square footing are all factors that can be logically justified, both by theory and practical experience.

While these factors have not been completely presented in any one formula known to the writer, some of them are generally recognized and all of them have been referred to upon occasion by other investigators. Static head or flotation has been universally accepted as a factor in the bearing capacity of any footing at some depth beneath the surface of the ground. Terzaghi and Peck[(7)] discuss resistance to upheaval in the bearing capacity of cylindrical piers but consider this resistance only at the cylindrical surface of the pier itself while neglecting the downward shearing force on vertical boundaries of the columns subjected to upheaval. Capper and Cassie[(3)] have noted that Skempton[(9)] suggests that the additional support afforded by cohesion between the soil and the sides of the footing should be allowed for by the addition of a term equal to the area of the side of the footing times the cohesion. The writer[10] has previously used all of these resistances in evaluating ultimate bearing capacity and obtained confirmation by field observation that they were actually mobilized in the sinking of large caissons.

In connection with the perimeter shear forces as shown in Eqs. (6) and (7) it should be noted that under ordinary practical conditions, the vertical shearing resistance may be mobilized only for the depth of the footing itself. In this connection, it has been generally recognized as good engineering practice to excavate for footings in cohesive soils to the actual size of the footing and pour the concrete in direct contact with undisturbed soil. This insures that the perimeter shear forces will be mobilized for the thickness of the footing and may in the case of smaller footings produce a substantial factor in the bearing capacity.

Eqs. (6) and (7) in their final form are recommended as a basis for evaluating the ultimate bearing capacity of spread footings including all factors of resistance that may be mobilized in a general shear failure producing complete lateral displacement of the supporting soil mass. With each factor of resistance definitely identified it should not be difficult to use such an equation intelligently under the variable conditions encountered in the field in which some of these factors may not be mobilized.

Segregation of those factors which are identified with local shear leads directly to the problem of allowable bearing capacity and a number of other considerations which enter into the design of spread footings and control the factor of safety which may be provided. This is a matter that will be considered in a discussion of factors of safety and design procedures but otherwise local shear must generally not be confused with the ultimate bearing capacity of a footing.

## THE USE OF BEARING CAPACITY EQUATIONS IN THE DESIGN OF SPREAD FOOTINGS

In the design of spread footings the use of the equations for ultimate bearing capacity presented in the preceding discussion involves a number of general considerations of great fundamental importance. The primary consideration is the selection of a factor of safety to be used in design. Under current design procedures there are four general methods by which the factor of safety may be controlled and which may be outlined as follows:

1. An applied pressure which will produce a selected allowable settlement which is held constant for all sizes of loaded area.

2. A factor of safety applied to the shearing resistance value to be used in computation of the bearing capacity, it being presumed that the shearing resistance is to be measured by some precise testing procedure.

3. A numerical ratio or selected proportion of the ultimate bearing capacity.

4. An applied pressure made up of those factors of resistance which are first mobilized by the supporting soil mass as, for example, in local shear failure as compared to general shear failure.

Before the relation between these various methods of determining the factor of safety can be intelligently discussed, it is necessary to review several fundamental relationships involving bearing capacity, settlement and size of loaded area. There has been considerable confusion among practicing engineers in attempting to interpret certain statements of such fundamental relationships in soil mechanics which might appear to be quite conflicting. For example, Taylor, after reviewing the various equations for ultimate bearing capacity of cohesive soil, states that these equations, "are the first presentation herein of another fundamental relationship which may be expressed as follows: In a highly cohesive soil the ultimate bearing capacity is a constant and is independent of the breadth of the footing". On the other hand, Tschebotarioff, in discussing certain fallacies which have arisen in soil mechanics practice states as follows: "We now know that under the same unit pressure on the same ground a foundation of larger area will tend to settle more than a foundation of smaller area."

Both of these statements are correct under the conditions to which they referred, although they may seem to be contradictory to a practicing engineer who does not fully comprehend the relationship between ultimate bearing capacity and settlement. Any contradiction disappears when it is considered that a uniform pressure proportional to the ultimate bearing capacity applied to all sizes of footings would produce a settlement which increases as the size of the footing increases. Conversely if it is desired to produce a substantially uniform settlement in all sizes of loaded area, it would be necessary to reduce the applied pressure as the size of the bearing area increases.

The classical statement of this relationship between load, settlement and size of bearing area, which has been widely accepted, is known as "diameter rule". This relationship is based on the assumption of a perfectly elastic supporting medium and makes no provision for resistances brought into play by plastic readjustments in the supporting mass which supplies an essential or even major part of the bearing capacity of cohesive soils.

The application of the diameter rule to plastic solids such as a cohesive clay, produced one of the major fallacies in soil mechanics practice to which the writer directed attention in 1928 in connection with the interpretation of field loading tests.[12] The consequences of applying the diameter rule to loaded areas varying in size are illustrated in Fig. 5 and Fig. 6.

In Fig. 5 is shown the relationship between settlement and applied pressure on two circular loaded areas with diameters of 12 and 30 inches, which are representative of sizes used in field loading tests. For the same applied pressure per unit area the settlement is directly proportional to the respective diameters according to the diameter rule. It may be noted that if this relationship was extended to practical sizes of spread footings or concrete mats the settlement predicted would become unreasonable. For example, a square footing 10 feet in width would produce a settlement of 5 inches and a concrete mat 100 feet in width would produce a settlement of 50 inches.

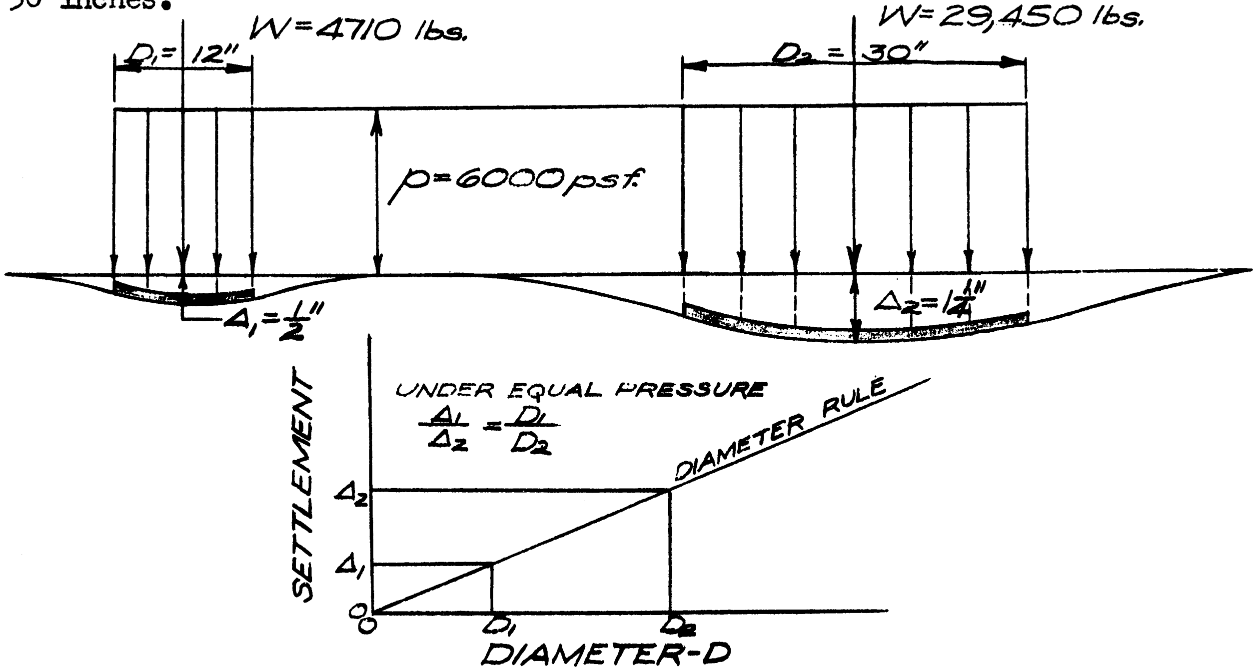



Fig. 5 Relation Between Settlement and Size of Area Under the Diameter Rule

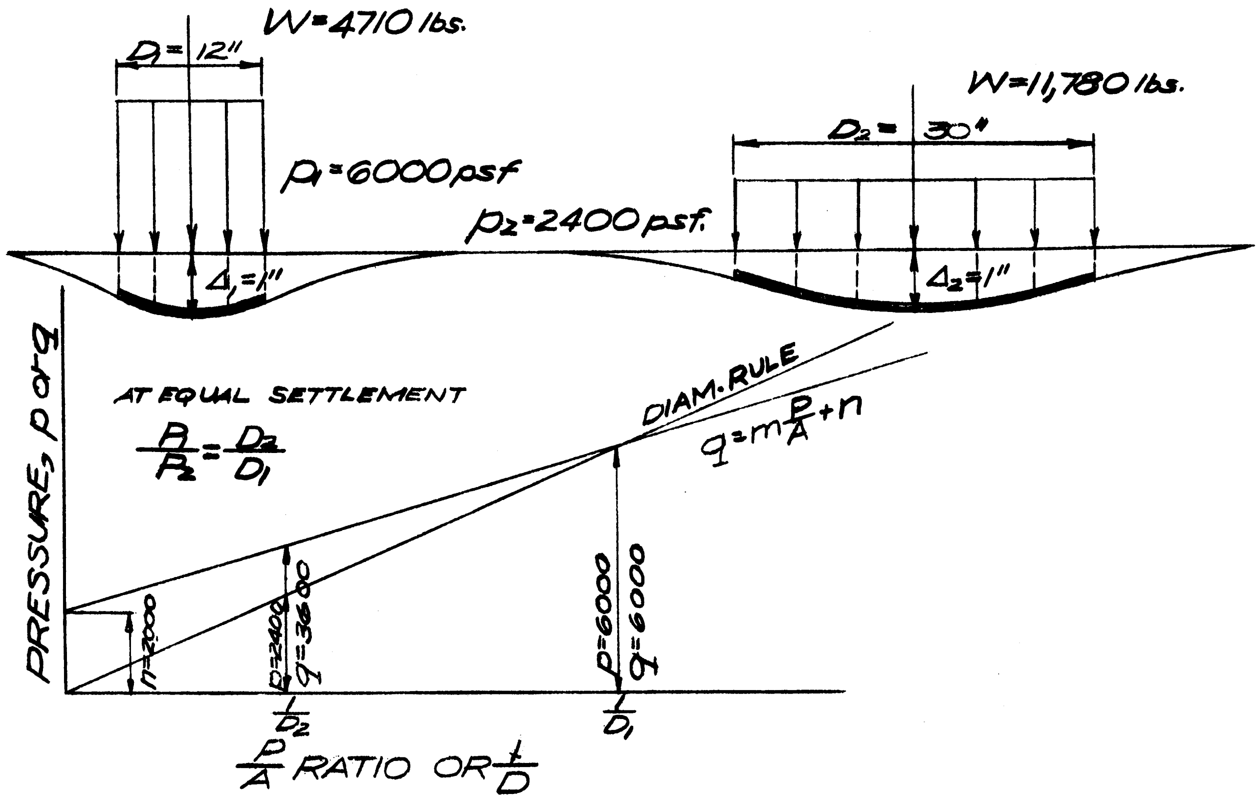



Fig. 6 Variation in Applied Pressure at Constant Settlement

In Fig. 6 is illustrated the result of applying the diameter rule to loaded areas varying in size and assuming that the settlement is to be held constant. According to the diameter rule, the applied pressure per unit area at constant settlement would vary inversely as the diameter or width of the footing, or in proportion the perimeter-area ratio. Under the condition of equal settlement the applied pressure supported by the larger area, having a diameter of 30 inches, is greatly reduced from that supported by the area with a diameter of 12 inches. From a practical standpoint it is of much greater importance that under the diameter rule, when the perimeter-area ratio approaches zero for areas representative of spread footings or large concrete mats, the applied pressure which could be supported at an equal settlement also approaches zero or in other words becomes negligible.

This fallacy or misconception which controverts practical experience is the one which was corrected by the writer's introduction of the following linear equation for bearing capacity illustrated in Fig. 6.

$$q = m \frac{P}{A} + n \qquad (8)$$

$q$ = bearing capacity in pounds per square foot

$m$ = perimeter shear in pounds per lineal foot of perimeter

$n$ = developed pressure in pounds per square foot

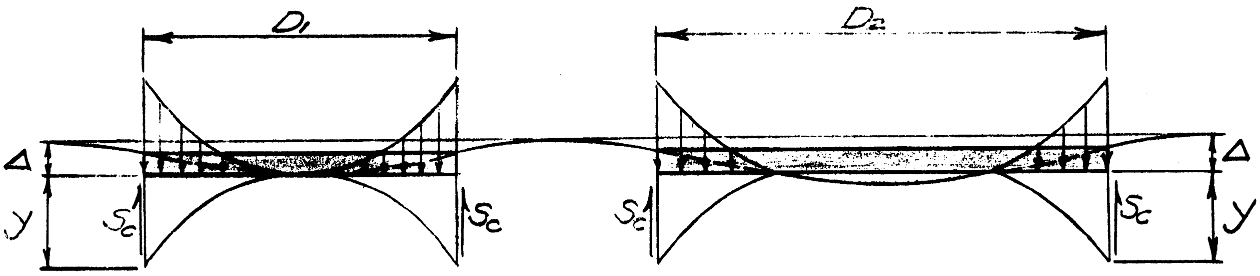
$\frac{P}{A}$ = perimeter-area ratio in feet per square foot

In this linear equation the bearing capacity, $q$, is a function of the perimeter-area ratio multiplied by the perimeter shear but also including a factor in the bearing capacity which is independent of the size of the bearing area and has generally been designated as developed pressure.

The linear equation, $q = m \frac{P}{A} + n$, may be correlated directly with the development of the equations for ultimate bearing capacity summarized in Fig. 1 of this memorandum. Those pressure components or resistance factors designated as developed pressure and static head which are represented by the term, n, in the linear equation, are independent of the size of bearing area and are available at equal settlement in the large as well as the small bearing areas. Those resistance factors designated as resistance to upheaval, perimeter shear forces and the lateral distribution effect are included in the perimeter shear, m, of the linear equation and are modified by the perimeter-area ratio. Thus the resistance factors included in the latter group vary with the size of the bearing area and for all practical purposes become negligible for larger spread footings and concrete mats. However, they are not negligible in the sizes of loaded areas involved in field loading tests and must be taken into consideration to accurately extrapolate load tests results into practical sizes of spread footings.

In correlating the linear equation for bearing capacity with the formulas for ultimate bearing capacity of spread footings which have been developed as part of this memorandum, there is one additional relationship which requires further explanation. In connection with the bearing capacity originating in lateral distribution of vertical pressure it was assumed in Eq. (6) and Eq. (7) that the shearing resistance was mobilized for the full depth, h, equal to the width of the bearing area. As previously pointed out, this assumption eliminated variation in bearing capacity with the size of the area insofar as lateral distribution is concerned. It is now necessary to bring this variation back into the discussion.

In this connection there is shown in Fig. 7 the considerations which are involved when perimeter shear, m, is considered as a constant for all sizes of bearing area as in the linear equation. In reinforced concrete footings, the footing itself is generally more rigid than the supporting soil mass. Consequently the footing tends to bridge the elastic depression formed by the applied load and to develop higher intensities of pressure at




$$m = S_c y$$

$$q_{L1} = m\frac{P_1}{A_1} = \frac{4m}{D_1} \qquad q_{L2} = m\frac{P_2}{A_2} = \frac{4m}{D_2}$$

$$\frac{q_{L1}}{q_{L2}} = \frac{D_2}{D_1}$$

Fig. 7 Stress Reactions on Rigid Footings at Constant Settlement

the edges of the bearing area resulting from the perimeter shear being developed in the early stages of settlement. Under such a sequence in the development of stress reactions from the supporting soil, the perimeter shear forces involved in lateral distribution are mobilized for some constant distance below the loading plane rather than for the full depth of the element which was taken equal to the width of the bearing area. It is the constant settlement which controls the depth to which the shear is mobilized and the constant edge reaction must be distributed over an increasing width of bearing area in order to express its contribution to the bearing capacity in terms of an equivalent pressure. Thus it is that the bearing capacity due to lateral distribution becomes a function of the perimeter-area ratio and at constant settlement varies with the size of bearing area. It should be emphasized, however, that this variation is much less extreme than in the case of the diameter rule which involves a decrease in bearing capacity for all factors of resistance and becomes absurd when extended to practical sizes of footings.

## DESIGN BASED ON CONSTANT SETTLEMENT

On the basis of the preceding discussion of the relationship between settlement, applied pressure and the size of bearing area, it is possible to establish consistent design procedures, following any one of the methods of selecting a factor of safety which were outlined. From a theoretical standpoint the most desirable method of designing substructures consisting of spread footings, would be to provide for constant settlement for all sizes of footings involved in a structure. Theoretically there are two methods of providing for constant settlement.

### Consolidation Theory

One approach to the problem of evaluating settlement under a loaded area presumes that settlement is due primarily to consolidation of the supporting soil. The consolidation theory which has been vigorously promoted and gained wide acceptance conceives that settlement due to consolidation is caused by squeezing water out of the voids of a saturated soil under the applied pressure. The consolidation theory which postulates that the movement of moisture is caused by pore water pressure or excess hydrostatic pressure as distinguished from pressure components originating in shearing resistance due to cohesion, cannot be applied to compressible soils in which the voids are not filled with water.

The experimental procedure followed in applying the consolidation theory is to obtain relatively large undisturbed samples which are brought into the laboratory and subjected to a consolidation test. In this test the sample is placed between porous stones and completely confined in a test cylinder in which it is subjected to applied pressure in sufficient magnitude to squeeze the water out of the sample. These laboratory tests are then translated into consolidation settlement under practical conditions by a coefficient of consolidation involving a change in the void ratio of the soil mass and modified by the permeability of the soil in order to obtain predicted settlements under field conditions.

For a number of years the Soil Mechanics Laboratory at the University of Michigan conducted such consolidation tests but they have been abandoned as part of their routine soil testing procedure due primarily to the fact that settlement predictions based on these tests have frequently proved to be quite unreliable. This experience has also been confirmed by numerous examples in the engineering literature in which the settlement predictions based upon consolidation tests have failed by wide margins as a prediction of the actual settlement that has been experienced.

The inaccuracy in settlement predictions has occurred in two ways. In the first place, when the applied pressures are substantially less than the ultimate bearing capacity of the soil with respect to displacement, settlement experienced in the field has been very much less than that which was predicted from the laboratory consolidation tests. In the second place, when the applied pressure exceeds the ultimate bearing capacity of the soil, progressive settlement under plastic flow generally continues without any noticeable decrease due to the presumed consolidation of the soil. The latter experience is of the greatest practical importance because it illustrates the danger of overemphasis on consolidation as a source of settlement. This has led practicing engineers in many notable cases to ignore the danger of exceeding the ultimate bearing capacity of the soil which has resulted in total mass displacement.

There may be several reasons for the unsatisfactory experience in predicting settlement by the consolidation theory. To begin with, the theory is not applicable to unsaturated soils with unfilled void space characterizing most of the compressible soils encountered in practice. In connection with saturated clays in which the settlement observed has been substantially less than that predicted from consolidation tests, Terzaghi and Peck (7) account for these discrepancies as a secondary time effect due to the lag in the reaction of clay to a change in stress as noted in the following quotations:

> "These delays in the reaction of clay to a change in stress, like the secondary time effect and the influence on $c_v$ (coefficient of consolidation) of the magnitude of the load increment, cannot be explained by means of the simple mechanical concept on which the theory of consolidation is based. Their characteristics and conditions for occurrence can be investigated only by observation."

> "It is obvious that the results of a settlement computation are not even approximately correct unless the assumed hydraulic boundary conditions are in accordance with the drainage conditions in the field. Every continuous sand or silt seam located within a bed of clay acts like a drainage layer and accelerates the consolidation of the clay, whereas lenses of sand and silt have no effect. If the test boring records indicate that a bed of clay contains partings of sand and silt, the engineer is commonly unable to find out whether or not these partings are continuous. In such instances the theory of consolidation can be used only for determining an upper and lower limiting value for the rate of settlement. The real rate remains unknown until it is observed."

These statements touch upon the writer's primary misgivings as to the practical applicability of the consolidation theory. In his opinion the conditions under which an isolated sample in the laboratory is tested depart so far from the conditions under which the soil mass is loaded in the field that there is little reason to expect that such test would provide a reasonable basis for predicting settlement. Aside from the obvious difficulty of reproducing the actual drainage conditions in the laboratory, the sample is completely confined in the test cylinder so that there is no opportunity to observe the weakness of the soil with respect to displacement which becomes a controlling factor under actual field conditions.

This is the source of the major weakness in the practical application of the consolidation test which has been referred to above as the second and more important source of inaccuracy in settlement predictions. In summarizing the writer's position on the consolidation theory it is concluded that this approach does not provide an acceptable basis of designing footings for constant settlement and it is not recommended.

Field Loading Tests

The second approach to the problem of designing spread footings for constant settlement is the use of field loading tests. When it becomes necessary to proportion footings to produce constant settlement within more precise limits than ordinarily required, it is the writer's opinion that such field loading tests provide the only practicable method of procedure. Inasmuch as bearing capacity is a function of the size of the bearing area as illustrated by Eq. (8), it is obvious that such tests must be conducted on several sizes of bearing area in order to determine the size effect. To produce acceptable results control of testing conditions in the field, including the rate of loading and measurement of load and settlement must be carried out with more than ordinary precision. The observation must then be carefully analyzed and properly interpreted in order to extrapolate the results of the field loading tests into the size range of spread footings. The interpretation of field loading tests is a subject that has been presented elsewhere and cannot be exhaustively treated in this memorandum[13][14]. A general equation for settlement of a loaded area and the derivation of Eq. (8) in these terms is given in Appendix A for convenient reference.

There have been a number of examples given of the successful application of field loading tests in which the accuracy of predicted settlements has been demonstrated[13][14]. Suffice it to say for the present discussion that Eq. (8) which is predicated upon equal settlement is recommended as the basis of design for constant settlement. Analysis of a series of loading tests determines the limiting values of the soil resistance coefficients given in Appendix A and the limiting value of the stress reactions, perimeter shear, m, and developed pressure, n. Selection of an allowable settlement may be based either on a permissible deformation in the supported structure or a reasonable margin below the limiting value of supporting capacity.

Limitations of field loading tests are well recognized but should not be taken as a basis for their rejection when conditions require design for constant settlement with more than ordinary precision. When such tests are properly conducted as is necessary for reliable results, they are both time-consuming and expensive. When nonuniform soil conditions are encountered involving significant changes in soil characteristics within the site and particularly with reference to stratification within a depth equal to the lateral dimension of the loaded areas, field loading tests may be deceiving. Under such nonuniform soil conditions a complete series of loading tests must be conducted for each significant variation in soil conditions and the results must be interpolated in terms of the distribution of load through the various strata from the various sizes of footings involved in the design phase of the problem. For these reasons a comprehensive series of field loading tests may frequently become prohibitive and consequently much effort has been devoted to developing other methods of evaluating bearing capacity in engineering design.

## DESIGN BASED ON SHEARING RESISTANCE TESTS

The computation of the bearing capacity of spread footings on cohesive soil from shearing resistance values determined by some precise test procedure and involving an adequate factor of safety is the second method of design which has been developed. This method requires the use of the equations for ultimate bearing capacity which have been presented as the primary objective of this memorandum. In this respect, Eq. (6) and Eq. (7) have been recommended for computing the ultimate bearing capacity of spread footings. Equation (8) has been presented as the basis of modifying the ultimate bearing capacity and to serve as a guide in reducing or discarding certain terms in Eqs. (6) and (7).

Laboratory Shear Tests

The first problem to which attention must be directed is the determination of shearing resistance values to which a factor of safety may be applied. There are two types of shearing resistance tests in general use, both of which are conducted on undisturbed samples obtained from borings at the site of the proposed construction. The University of Michigan Soil Mechanics Laboratory has developed a transverse or ring shear test which represents the routine procedure used by that and some other laboratories. A number of other soil mechanics laboratories use the unconfined compression test as the basis for determining the shearing resistance of a cohesive soil. For some years the Soil Mechanics Laboratory at the University of Michigan has been running both of these tests in parallel in order to establish correlation between the results.

The ring shear test is a measure of what may be called the static yield value or shear stress greater than which the soil will suffer progressive deformation. In these tests observations are made of the rate of shearing deformation for each load increment applied. From this may be determined by extrapolation the actual load at which progressive deformation occurs. The final results as shown are thus independent of the dynamic resistance and represent that applied stress which may be sustained in static equilibrium.

The shearing resistance as determined by unconfined compression tests conducted in accordance with generally accepted procedures has not been corrected for dynamic effects so the test may be termed a rapid shear test. The load is applied at a continuous rate until failure is produced and in a much shorter period of time (five-minute loading period). Over several thousands of parallel tests it has been found that shear values obtained from this unconfined compression test are very close to four times those obtained from the ring shear test in the case of plastic clays of the truly cohesive type or in the case of saturated clays.

In correlating the results of the ring shear test and unconfined compression test, it has been the practice to express the results of the unconfined or rapid shear test in terms of an equivalent static shearing resistance which is one-quarter of the actual measured value. The shearing resistance in unconfined compression is taken as one-half of the unconfined compressive strength. The reduction by the ratio of one-quarter gives an equivalent shearing resistance which compares very closely with the static shearing resistance or yield value from the ring shear test in the plastic or truly cohesive types of clay. In more highly consolidated clays and those containing a substantial amount of granular material, equivalent shearing resistance values from unconfined compression are generally higher than the comparable values in the ring shear test. It has become the practice to use the higher values as representative of the mechanical strength of the more highly consolidated soil. In this connection it is considered that the granular structure dictates the plane of failure which is steeper than 45 degrees and that the true shearing resistance due to cohesion acts on this steeper angle of failure and accounts for the higher equivalent shearing resistance. The relation between the shearing resistance due to cohesion and the equivalent shearing resistance from unconfined compression is expressed by the following equation:

$$S_{uc} = \frac{S_c}{\sin 2\theta}$$

In highly consolidated soils where internal stability of the granular structure plays a major role, the ratio between the transverse shearing resistance, $S_c$, and the shearing resistance from unconfined compression, $S_{uc}$, is used as a basis of evaluating the angle of pressure transmission, $\theta$. However, this practice is somewhat tentative at the present time and should only be used when there are a sufficient number of comparative test results to establish a reliable statistical average between these two shearing resistance values.

Before closing the discussion of comparative shear values, one other exception to the general rule should be mentioned. In flocculated clays of abnormally high moisture content there has been a general trend for the equivalent shearing resistance from unconfined compression to run consistently lower than the static yield value from the ring shear test. Such clays having a flocculated structure are identified as having a high degree of sensitivity characterized by a loss of shearing resistance when disturbed. The remolded shearing strength is substantially less than the shearing resistance in the undisturbed state and there is evidence that the recovery of the original shearing strength due either to molecular reorientation of adsorbed moisture or consolidation can be questioned. In dealing with such materials in design it is suggested that the lower value of equivalent shearing resistance from unconfined compression should be used in design computations.

### Overload Ratio and Factors of Safety

In present soil mechanics practice the shear value upon which the design is based is necessarily predicated upon the type of laboratory test that has been used. A number of laboratories use the unconfined compression test and assume the shearing resistance as being one-half the unconfined compression test without reduction for the dynamic effect of the rapid rate of loading. This test value is defined as the ultimate shearing resistance and a factor of safety is applied to this figure.

It has been the writer's practice to use the static yield value from the ring shear test or the equivalent shearing resistance from unconfined compression as outlined above, as a basis for computing the supporting capacity of a cohesive soil mass. In this connection it is considered that any permanent structure should be in static equilibrium and not subjected to progressive settlement in any amount. In comparison with building construction practice within the writer's experience such a criterion has resulted in a substantial reduction in the nominal allowable bearing values which have been in use. It does not, however, provide a factor of safety comparable to that used in other fields of structural design employing steel and concrete and other building materials. However, there is provided a considerable range of load in which the progressive settlement which does take place may be permissible within the life of the structure involved and the probability of a sudden mass movement or slide is eliminated.

Under certain practical conditions where slow progressive movement is permissible, the writer has employed an overload ratio as a design factor by which the static shearing resistance or yield value may be increased. The overload ratio is defined as the ratio obtained by dividing the shearing stress imposed upon the supporting soil mass by the static shearing resistance or yield value of the soil.

In recent years since the relationship between these two types of shearing resistance tests has been quite clearly established, it is possible to compare design criterion based either on an overload ratio or a safety factor applied to the ultimate shearing resistance and arrive at approximately the same basis of design. Tschebotarioff[1] has presented a comparison between these two methods of design which is reproduced below.

TABLE I

The Average Relationship Between Housel's Overload Ratio and Factors of Safety, $F_s$, Based on the Ultimate Shearing Resistance of Clays

| | Values Suggested by Housel | | Values Suggested by Terzaghi and Peck |
|---|---|---|---|
| | Overload Ratio | Factor of Safety $F_s$ | |
| Permanent Structures | 1.00<br>1.33 | 4.00<br>3.00 | Normal |
| Temporary Structures* | 2.00<br>2.50 | 2.00<br>1.66 | Minimum |
| Failure Condition | 4.00 | 1.00 | Failure |

*Refers to excavation and temporary loading conditions that will presumably not extend over a period of several months at the most.

Tschebotarioff concludes that an overload ratio of unity is somewhat too conservative even for permanent structures but it is the writer's recommendation that the static shearing resistance or yield value equivalent to an overload ratio of unity be accepted as the controlling value for the design of permanent structures. It is considered that any exception to this general rule takes on the character of a calculated risk and must be done with a full realization of the consequences of progressive settlement or, in the case of overload ratios higher than normal, the increasing possibility of rapid progressive settlement or sudden mass movement.

As noted by Tschebotarioff the writer has suggested that for temporary loading conditions such as excavations during the period of construction overload ratios as high as 2.0 or 2.5 may be employed without serious danger of slides. In addition there are other conditions frequently encountered in practice where considerable progressive settlement may be permitted and where overload ratios as high as 2.0 or 2.5 may also be accepted as calculated risks. Particular reference is made to mass storage of materials such as ore, coal and building materials in which complete flexibility is involved with no rigid or semirigid substructure to be seriously damaged.

When the allowable shearing resistance value based either on an overload ratio or factor of safety as the case may be, has been selected, this value may be used in Eq. (6) or Eq. (7) including all terms of resistance which are applicable to the specific field conditions involved in the project under consideration.

The only qualification which must be made in this recommendation has to do with the fact that these equations for ultimate bearing capacity, even though they may be reduced to allowable values by the use of a conservative shear value, still involve a settlement that varies with the size of the loaded area. As pointed out in the discussion of the relationship between the linear equation for bearing capacity and the equations for ultimate bearing capacity, this statement applies only to those factors of resistance which are a function of the perimeter-area ratio. When control of settlement is not required as is frequently the case in mass storage over large areas, no further reduction in the allowable bearing capacity will be required.

## ADDITIONAL SAFETY FACTORS IN DESIGN

On the other hand, in the design of footings which support rigid structures that would be damaged by differential settlement, it may be necessary to make further reductions in the allowable bearing capacity in order to control the settlement within acceptable limits. Such reductions may be made by either Methods 3 or 4 which were outlined on Page 20 of this memorandum. Method 3 involves an arbitrary reduction in the overall bearing capacity as computed by the formulas and does not take into consideration the relative magnitude of the various sources of resistance which have been evaluated or the sequence in which these resistance factors may be developed. Furthermore, the additional safety factor is entirely a matter of engineering judgment and the degree of conservatism which it is desired to introduce into the design.

Keeping in mind that the primary objective of an additional factor of safety beyond that provided by a conservative shear value is to limit the settlement or control the differential settlement, it would appear that Method 4 provides the more logical procedure. By this method certain factors in the bearing capacity equations can be eliminated depending upon the sequence in which they are developed by the supporting soil mass.

This method may be compared to accepting local shear failure as the final criterion in design, with the qualification that bearing capacity due to static head or flotation should always be included. Justification for accepting the bearing capacity due to static head as the first in the order of availability comes from the fact that until the applied pressure replaces the weight of the soil removed in placing the footing, there are no stresses imposed on the supporting soil mass.

Next in the order of availability without excessive settlement are those components of pressure associated with developed pressure and lateral distribution under local shear failure (See Page 17). These factors of resistance amount to three times the shearing resistance for a strip footing and five times the shearing resistance for a round or square footing. At this stage of mobilization of supporting capacity which corresponds most closely to local shear failure the following bearing capacity equations would be used:

Strip Footings $\quad q = 3S_c + wh_o$

Round or Square Footings $\quad q = 5S_c + wh_o$

It may be pointed out that in these equations the ultimate bearing capacity has been reduced by eliminating all resistance to upheaval, that portion of the passive lateral pressure designated as $n_3$ not associated with lateral distribution of vertical pressure and the vertical pressure component, $n_2$, originating in the maximum difference in principal pressures on the compression block, Element 1, which directly supports the bearing area (See Fig. 1).

It is the writer's opinion from consideration of all factors involved in bearing capacity that the pressure component, $n_2$, or the maximum difference in principal pressureson the supporting block immediately under the bearing area could be included in the bearing capacity without defeating the objective of eliminating excessive settlement in compressible soil masses. When this is done the equations for allowable bearing capacity would be as follows:

| | | |
|---|---|---|
| Strip Footings | $q = 5S_c + wh_o$ | (10) |
| Round Footings | $q = 7S_c + wh_o$ | (11) |

Equation (10) and Eq. (11) represent the writer's recommendation for an additional reduction in allowable bearing capacity, based on equations for ultimate bearing capacity which have been presented in this memorandum for the purpose of reducing the total settlement. This is with the understanding that the shearing resistance value to be used in these equations is the static yield value or equivalent shearing resistance from unconfined compression. Under these conditions it is felt that applied pressures of the prescribed magnitude would not produce excessive settlement in relatively compressible soil masses. It must be recognized, however, that the use of such an equation does not completely eliminate the possibility for differential settlement of footings which vary considerably in size. There are still in Eqs. (10) and (11) certain bearing capacity factors arising from lateral distribution which imply differential settlement.

In this connection it may be desirable to supplement these equations in order to provide a technique for controlling differential settlement within a range of footing sizes that may be encountered in any supported structure. Such a technique may be devised from the linear equation for bearing capacity and while it involves a somewhat different approach its validity is supported by the successful application of load test results in designing footings for constant settlement. As has been previously pointed out the developed pressure, n, in Eq. (8) includes those factors independent of the size of the bearing area designated as developed pressure and static head. On the other hand, perimeter shear, m, becomes a constant in Eq. (8) only because the settlement is constant and vertical shearing resistance is mobilized for a constant depth in all sizes of footings. Following this line of reasoning it is suggested that any structure in which it is desired to vary the allowable bearing capacity for different sizes of footings, that the perimeter shear, m, be computed as the product of the shearing resistance due to cohesion, $S_c$, for a depth equal to the width of the smallest footing and then be taken as a constant for all other sizes of footings. This procedure is illustrated by the following equations:

$$q = m \frac{P}{A} + n \qquad (8)$$

Let $D_1$ = width of smallest footing

$D_2$ = width of a larger footing

$n$ = developed pressure and static head from Eqs. (10) and (11).

$$n = 3S_c + wh_o$$

$$m = S_c D_1$$

Strip Footings

$$q_1 = S_cD_1 \times \frac{2}{D_1} + 3S_c + wh_o$$

$$q_1 = 5S_c + wh_o \qquad (10)$$

$$q_2 = S_cD_1 \times \frac{2}{D_2} + 3S_c + wh_o$$

$$q_2 = 2S_c \frac{D_1}{D_2} + 3S_c + wh_o \qquad (10a)$$

Square or Round Footings

$$q_1 = S_cD_1 \times \frac{4}{D_1} + 3S_c + wh_o$$

$$q_1 = 7S_c + wh_o \qquad (11)$$

$$q_2 = S_cD_1 \times \frac{4}{D_2} + 3S_c + wh_o$$

$$q_2 = 4S_c \frac{D_1}{D_2} + 3S_c + wh_o \qquad (11a)$$

In conclusion it should be pointed out that in all of the preceding development in this memorandum it has been assumed that the supporting soil mass is substantially uniform for a depth at least equal to the width of the bearing areas under consideration. In practice, it is generally true that there are substantial variations in shearing resistance with the depth which present a new set of conditions and problems. Segregation of the various factors of resistance in the bearing capacity equations will make it easier to take into consideration variations in shearing resistance with depth, particularly in connection with shearing resistance above and below the loading plane.

When significant changes in soil strata and shearing resistance values take place within a depth below the loading plane, equal to the width of the loaded area, there are two cases to be considered in extension of the bearing capacity equations to cover such variations in shearing resistance. In the first place, it has become common practice to use an average shearing resistance for the depth of footing computed as a weighted average taking into consideration the depth to which each respective shear value extends. In the second place, where the variation in shearing resistance involves a particularly weak stratum within the depth under consideration, the displacement in this soft layer must be investigated as a special problem.

In such cases the applied pressure at the footing level is distributed through the overlying layers by punching shear to determine the maximum pressure to which the soft layer is subjected. Displacement in the soft layer is then investigated in relation to the maximum difference in principal pressure on elements of mass taken equal to the thickness of the layer. The confining pressures exerted by overlying layers is based upon resistance to upheaval paralleling the relationship evaluated in similar terms in the equations for ultimate bearing capacity. Whether or not lateral distribution should be considered in the soft layer is debatable and the present practice is to include it only if the layer is within a depth equal to the width of the footing.

In practice there are frequently other special conditions which may be beyond those which have been discussed and to which the general principles illustrated in this memorandum can be extended. However, it is not considered practicable to attempt to anticipate all of these special conditions in the present memorandum.

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## A P P E N D I X   A

### General Equation for Settlement of a Loaded Area

A general equation which expresses the relation between settlement, size of bearing area, and load has been derived by integrating soil deformation within the compression cone. In Fig. 8 the relations included in this general equation are illustrated. The problem involves a finite bearing area which has been taken as a square of width b. The increase in the lateral dimension of the loaded area is given by r the tangent of the angle of spread. In a depth h according to the linear approximation of pressure distribution the total load W will be carried over an area with a lateral dimension of (b + 2rh). Combined triangular and rectangular distribution is selected as the most satisfactory representation of pressure distribution in that it properly portrays lateral distribution of the vertical load as a boundary phenomenon and provides a convenient basis for segregating the two different types of stress reaction involved. Accepting this pressure distribution the total load may then be represented as an equivalent uniform pressure over an area with a total lateral dimension of (b + rh), the area being $(b + rh)^2$.

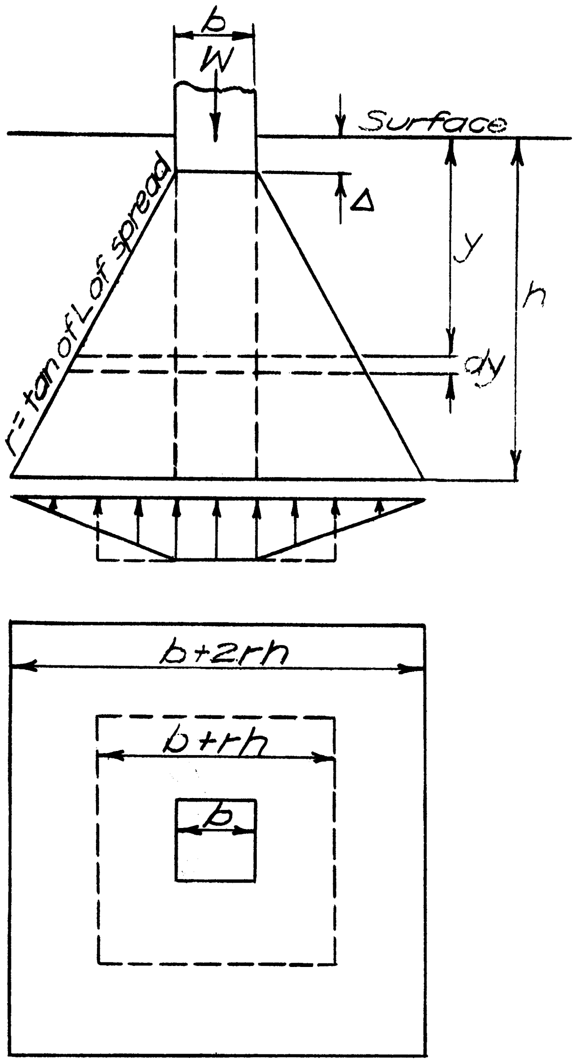


Fig. 8

Pressure Distribution and Settlement Under a Loaded Area

At any depth y below the surface an element of thickness (dy) will be subjected to an equivalent uniform pressure $p_y$. The modulus of incompressibility I is defined as the load per unit area divided by the settlement or deformation per unit depth.

$$I = \frac{p_y}{d\Delta/dy} = \frac{W/(b + ry)^2}{d\Delta/dy}$$

$\Delta$ = Total Settlement in feet.

$$d\Delta = \frac{W\,dy}{I(b + ry)^2}$$

$$\Delta = \int_0^h \frac{W\,dy}{I(b + ry)^2} = \frac{W}{I}\left[-\frac{1}{r(b + ry)}\right]_0^h$$

$$\Delta = \frac{Wh}{I(b^2 + brh)} \qquad (9a)$$

Reducing to terms of bearing capacity p and perimeter-area ratio

$$\Delta = \frac{pAh}{I(A + P\frac{rh}{4})} \qquad W = pA \quad P = 4b \quad A = b^2$$

Let

$$\frac{h}{I} = K_1 \qquad \frac{rh}{4} = K_2$$

$$\Delta = \frac{K_1 p}{1 + K_2 \frac{P}{A}} \qquad (9)$$

Equation (9a) is a general solution first obtained by C. C. Williams.[12] This form is subject to some limitation inasmuch as it contains quantities r, h, and I which are not subject to direct measurement under practical conditions. It is necessary further to express the general relation for settlement in terms of quantities that may be measured by test. This may be done by introducing two soil resistance coefficients $K_1$ and $K_2$ and establishing their relation to the straight-line equation (8). Equation (9) is a general expression which shows the relation between settlement, perimeter-area ratio, and bearing capacity. In order to show the relation between the straight-line equation (8) and equation (9) it is merely necessary to consider the settlement $\Delta$ as constant and express bearing capacity in terms of perimeter-area ratio.

$$K_1 p = \Delta + \Delta K_2 \frac{P}{A}$$

$$p = \frac{\Delta K_2}{K_1} \frac{P}{A} + \frac{\Delta}{K_1}$$

$$q = p = m \frac{P}{A} + n \qquad (8)$$

$$m = \frac{\Delta K_2}{K_1} \quad n = \frac{\Delta}{K_1}$$

$K_1 = \frac{\Delta}{n}$ $K_2 = \frac{m}{n}$ (Soil Resistance Coefficients).

From equation (9) it is seen that the only quantities involved in addition to the variables of load, settlement, and perimeter-area ratio are the two coefficients $K_1$ and $K_2$. From the direct relation to equation (8) it is apparent that $K_1$ and $K_2$ may be evaluated in terms of $\Delta$, m, and n which have been measured by bearing capacity tests. In the analysis m and n are determined for any given settlement and for the same conditions it follows that $K_1$ and $K_2$ are constants. The values of m and n, the stress reactions, will vary for different types of soil and for different ranges of load. The coefficients $K_1$ and $K_2$ will also vary with m and n and thus they become the coefficients which express the essential properties of the particular soil and on the basis of which the varying behavior of the material for different ranges of load may be definitely evaluated. $K_1$, which is the ratio of settlement divided by developed pressure, is defined as the <u>coefficient of settlement</u>. It analogous to the well-known coefficient of compressibility except that it expresses the total settlement as volume change in the body of soil included within the compression cone, rather than being expressed in deformation per unit volume. $K_2$ is the ratio of perimeter shear divided by developed pressure and is defined as the <u>stress reaction coefficient</u>. It expresses the relative importance of the two stress reactions involved in bearing capacity.

The bearing-capacity limit of the soil may be determined as the minimum value of $K_1$ or the maximum value of $K_2$ depending upon the sequence in which the two types of resistance are developed. For a relatively compressible soil the developed pressure is small for the lower range of loads and the major portion of the applied load is carried by perimeter shear. As the settlement increases and the bearing plate penetrates the surface, developed pressure increases and the values of $K_1$ decrease. The decreasing values of $K_1$ show that the resisting pressure is increasing faster than the settlement and indicate a margin of resistance which is available to bring the loaded area to equilibrium if no more load were to be added. The minimum value of $K_1$ defines the maximum developed pressure in the case of soils which are relatively compressible. Subsequent increasing values of $K_1$ in which the settlement increases more rapidly than the developed pressure show that increments of settlement are accumulating without proportional increase in resistance and signify the stage of progressive settlement. Meanwhile the values of $K_2$ are decreasing and show no evidence of critical changes in behaviour. The supporting capacity due to perimeter shear was available in the initial stage of loading and after having been fully utilized exerts no further influence on the transition to the stage of progressive settlement.

In the case of a relatively incompressible soil in which resistance to volume change is high the developed pressure is available as the initial resistance and the perimeter shear is developed as the settlement increases. As a result of reversing the sequence with which the stress reactions are developed, the coefficient of settlement $K_1$ increases throughout the entire range of the test and shows no critical value. The stress reaction coefficient $K_2$, however, increases during the initial stages and reaches a maximum value which represents the maximum amount of load which may be distributed to the body of soil by shear on the boundary surfaces. The maximum value of $K_2$ then becomes the criterion for the bearing-capacity-limit.