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COLLEGE OF ENGINEERING
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Space Physics Research Laboratory

Scientific Report MS-2

THE THERMAL TRANSPIRATION EQUATION: TABLES OF F(s) vs. s

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LIST OF SYMBOLS

α: angle of attack

C: most probable thermal speed of particles: $C = \sqrt{\dfrac{2kT}{m}}$

$f(s) = e^{-s^2} + \sqrt{\pi}\ s\ (1 + \text{erf}s)$

k: Boltzmann's constant (1.3804×10^{-23} Joules/^o_K)

m: particle mass (Kg)

n: number density (Part/M^3

N_n: number density having a velocity U_n for a Maxwellian gas

Γ: number of particles crossing a unit area per unit time

$\lambda \quad = \dfrac{u_{n_O}}{C_O}$

P   = pressure

p   = density (kg/M^3)

$s \quad = \dfrac{V_n}{C_O}$

T: absolute temperature

u_n: velocity of molecules in the direction of normal to the orifice

$\vec{V}$:   drift velocity (relative velocity between chamber and gas)

$V_n \quad = V \cos \alpha$

Subscripts

     i: parameters inside gauge

     o: parameters outside gauge

OBJECTIVE

The purpose of this report is to present tables and graphs useful in the interpretation, in terms of ambient conditions, of pressure measurements made from pressure gauges mounted on sounding rockets and satellites.

INTRODUCTION

For a pressure gauge mounted on sounding rockets and satellites, the measured pressure values usually include the effects of the drift velocity of the moving gauge. Such a method was successfully employed by this Laboratory in the TP and Pitot-static series of atmospheric measurements.

The basic relationship which holds for a pressure gauge immersed in a gas, in which the mean free path of the molecules is long compared with the dimensions of the gauge, is the well known "thermal transpiration" equation. This equation, with drift velocity considerations, has been derived by various authors[1,2] and is presented here for completeness.

DERIVATION OF f(s)

Consider a chamber with an orifice of area A immersed in a gas which has a drift velocity $\vec{V}$ (Fig 1.). Parameters outside the chamber are denoted by subscripts "o" and parameters inside the chamber by subscripts "i". The angle between the drift velocity vector $\vec{V}$ and the normal to the orifice is denoted by α, and will henceforth be referred to as the angle of attack. For the condition of dynamic equilibrium, the number of gas molecules entering the chamber from the outside must be equal to the number of molecules leaving the chamber. If we assume that the mean free path of the molecules, both inside and outside the chamber, is large compared with the dimensions of the orifice, there will be no interaction between the incoming and exiting flux of particles. It is therefore possible to consider the flows separately and equate the equilibrium relationships.

The quantity $d\Gamma_i$, the incremental number of molecules leaving the chamber per unit time per unit area and having velocities in the range u_{n_i} and $u_{n_i} + du_{n_i}$, is given by

$$d\Gamma_i = u_{n_i}\, dN_{n_i}$$

or

$$d\Gamma_i = u_{n_i}\, n_i\, f(u)\, du_{n_i} \tag{1}$$

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where

$$f(u) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m\,u^2}{2kT}}$$

is the Maxwell-Boltzmann velocity distribution function. In Cartecian coordinates $f(u)$ is given by

$$f(u_x u_y u_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m(u_x^2+u_y^2+u_z^2)}{2kT}} \tag{2}$$

For the case of molecules crossing an orifice, only one component need be considered. Consider the x-component. We can disregard molecules having velocities in the y and z directions (u_y, u_z) because they do not cross the orifice and therefore do not contribute to the overall flow. Adapted to our present notations, the one-dimensional Maxwell-Boltzmann velocity distribution function becomes

$$f(u_{n_i}) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m\,u_{n_i}^2}{2kT}} \tag{3}$$

Substituting Eq. (3) into Eq. (1), we have Γ_i, the number of molecules leaving the chamber per unit time per unit area:

$$\Gamma_i = \int_{u_{n_i}=0}^{u_{n_i}=\infty} n_i \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m\,u_{n_i}^2}{2kT}} u_{n_i}\, du_{n_i} \tag{4}$$

Equation (4) can be further simplified by substituting C_i for $\sqrt{\frac{2kTi}{m}}$, which gives

$$\Gamma_i = \int_0^\infty \frac{n_i}{\sqrt{\pi}} u_{n_i} e^{-\frac{u_{n_i}^2}{C_i^2}} d\left(\frac{u_{n_i}}{C_i}\right) \tag{5}$$

where C_i is the most probable thermal speed of the molecules inside the chamber. Integrating Eq. (5), we have

$$\Gamma_i = \frac{n_i\, C_i}{2\sqrt{\pi}} \int_0^\infty e^{-\left(\frac{u_{n_i}^2}{C_i^2}\right)} d\left(\frac{u_{n_i}}{C_i}\right)^2$$

$$\Gamma_i = \frac{n_i \, C_i}{2\sqrt{\pi}} \left[-e^{-u_{ni}^2/C_i^2} \right]_0^\infty$$

$$\Gamma_i = \frac{n_i \, C_i}{2\sqrt{\pi}} \tag{6}$$

Next, consider the flux of particles entering the chamber from the out-side. In computing this flux, the total velocity of the gas molecules will be the sum of the thermal velocity and the drift velocity of the chamber with respect to the gas. Thus $d\Gamma_o$, the incremental particle flux crossing the orifice from the outside per unit time per unit area and having a vel-ocity range between u_{n_o} and $u_{n_o} + du_{n_o}$, is given by

$$d\Gamma_o = (u_{n_o} + V_n) dN_{n_o} \tag{7}$$

where V_n is the component of drift velocity in the direction of normal to the orifice. Next substitute for the one-dimensional case of the Maxwell-Boltzmann velocity distribution function, or

$$dN_{n_o} = \frac{n_o}{\sqrt{\pi}} e^{-\left(\frac{u_{n_o}}{C_o}\right)^2} d\left(\frac{u_{n_o}}{C_o}\right)$$

into Eq. (7). Upon integration this gives the particle flux entering the chamber from the outside per unit time per unit area:

$$\Gamma_o = \int_{(u_{n_o}+V_n)=o}^{(u_{n_o}+V_n)=\infty} \frac{n_o}{\sqrt{\pi}} (u_{n_o}+V_n) \, e^{-\left(\frac{u_{n_o}}{C_o}\right)^2} d\left(\frac{u_{n_o}}{C_o}\right) \tag{8}$$

To simplify the notation further, let the ratio of the drift velocity in the direction of the normal to the orifice V_n, to the ambient most prob-able thermal velocity C_o, be denoted by s; and let the ratio of the com-ponent of the molecules velocity in the direction of the normal to the ori-fice, to the most probable thermal velocity, be denoted by λ:

$$\frac{V_n}{C_o} = s \quad ; \quad \frac{u_{n_o}}{C_o} = \lambda$$

Equation (8) can then be written

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$$\Gamma_O = \frac{n_O C_O}{\sqrt{\pi}} \int_{-s}^{\infty} \lambda \ e^{-\lambda^2} \ d\lambda + \frac{n_O V_n}{\sqrt{\pi}} \int_{-s}^{\infty} e^{-\lambda^2} \ d\lambda \tag{9}$$

Rearranging the second integral, we can write Eq. (9) as

$$\Gamma_O = \frac{n_O C_O}{\sqrt{\pi}} \int_{-s}^{\infty} \lambda \ e^{-\lambda^2} \ d\lambda + \frac{n_O C_O}{\sqrt{\pi}} \ s \left[\int_{-s}^{0} e^{-\lambda^2} \ d\lambda + \int_{0}^{\infty} e^{-\lambda^2} \ d\lambda \right] \tag{10}$$

The above are standard integrals.

$$\frac{n_O C_O}{\sqrt{\pi}} \int_{-s}^{\infty} \lambda \ e^{-\lambda^2} \ d\lambda = \frac{n_O C_O}{2 \sqrt{\pi}} \left[-e^{-\lambda^2} \right]_{-s}^{\infty}$$

$$\frac{n_O C_O}{\sqrt{\pi}} \int_{-s}^{\infty} \lambda \ e^{-\lambda^2} \ d\lambda = \frac{n_O C_O}{2 \sqrt{\pi}} \ e^{-s^2} \tag{11}$$

$$\frac{n_O C_O}{\sqrt{\pi}} \ s \int_{-s}^{0} e^{-\lambda^2} \ d\lambda = \frac{n_O C_O \ s}{\sqrt{\pi}} \int_{0}^{s} e^{-\lambda^2} \ d\lambda$$

$$= \frac{n_O C_O}{2} \ s \left[\frac{2}{\sqrt{\pi}} \int_{0}^{s} e^{-\lambda^2} \ d\lambda \right]$$

$$\frac{n_O C_O}{\sqrt{\pi}} \ s \int_{-s}^{0} e^{-\lambda^2} \ d\lambda = \frac{n_O C_O \ s}{2} \ \text{erfs} \tag{12}$$

$$\frac{n_O C_O}{\sqrt{\pi}} \ s \int_{0}^{\infty} e^{-\lambda^2} \ d\lambda = \frac{n_O C_O \ s}{2} \left[\frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-\lambda^2} \ d\lambda \right]$$

$$\frac{n_O C_O \ s}{\sqrt{\pi}} \int_{0}^{\infty} e^{-\lambda^2} \ d\lambda = \frac{n_O C_O \ s}{2} \tag{13}$$

Therefore

$$\Gamma_O = \frac{n_O C_O}{2 \sqrt{\pi}} \left[e^{-s^2} + s\sqrt{\pi} \ (1 + \text{erfs}) \right] \tag{14}$$

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For equilibrium to exist, it is necessary that $\Gamma_i = \Gamma_o$. Equating Eqs. (6) and (14) yields

$$n_i C_i = n_o C_o \, f(s) \tag{15}$$

where

$$f(s) \overset{\Delta}{=} e^{-s^2} + s \sqrt{\pi} \, (1 + \text{erf} s) \tag{16}$$

Therefore, a relation between inside parameters and outside parameters of a pressure gauge can be obtained as a function of $s = V_n/C_o$.

Tables I and II give

$$f(s) = \left[e^{-s^2} + s \sqrt{\pi} \, (1 + \text{erf} s) \right]$$

versus s.

Figure 2 shows $f(s)$ in graphic form. From the graph it can be seen that $f(s)$ becomes linear for large s, and examination of Eq. (16) shows that for large s

$$f(s) = 2 \sqrt{\pi} \, s \tag{17}$$

Figure 3 shows $f(s)$ for large negative s. From the graph it can be seen that $f(s)$ becomes a straight line on the semi-log plot, indicating that for large negative s, $f(s)$ approaches an exponential variation. The above is also seen to be true upon examination of Eq. (16). Hence for large negative s

$$f(-s) = e^{-s^2} \tag{18}$$

Equation (15) can be interpreted in terms of pressure and temperature.

Since

$$P = nkT$$

and

$$C = \sqrt{\frac{2kT}{m}} \, ,$$

where P = pressure

        k = Boltzmann's constant

        T = Absolute temperature
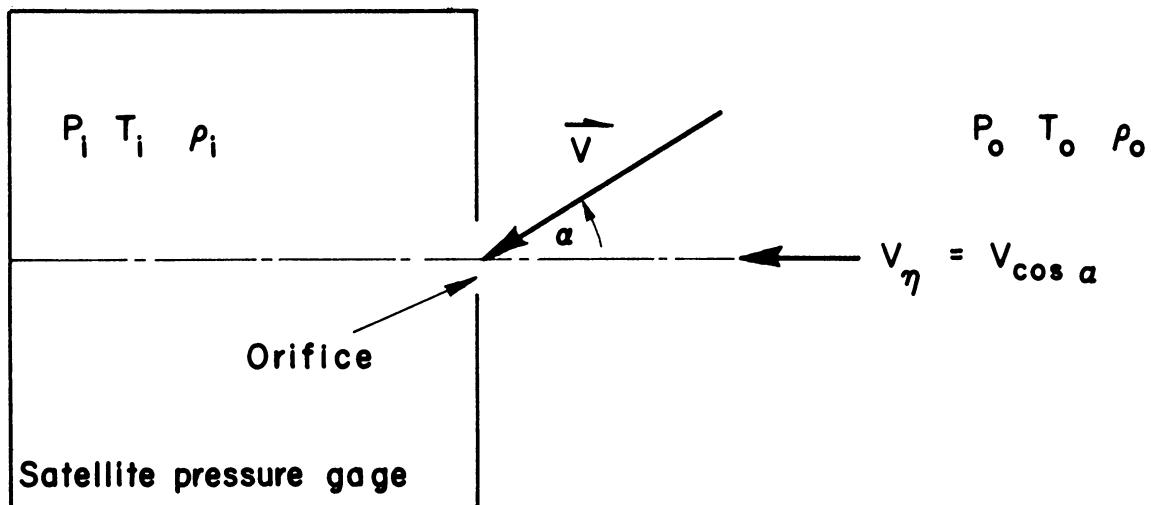
        m = Molecular mass:

$$\frac{n_i C_i}{n_o C_o} = \left(\frac{P_i}{kT_i}\sqrt{\frac{2kT_i}{m}}\right)\left(\frac{kT_o}{P_o}\sqrt{\frac{m}{2kT_o}}\right)$$

$$\frac{n_i C_i}{n_o C_o} = \frac{P_i}{P_o}\sqrt{\frac{T_o}{T_i}}$$

Therefore

$$\frac{P_i}{P_o} = \sqrt{\frac{T_i}{T_o}}\, f(s) \tag{19}$$

$\overrightarrow{V}$ = Drift velocity of gas with respect to the pressure gage
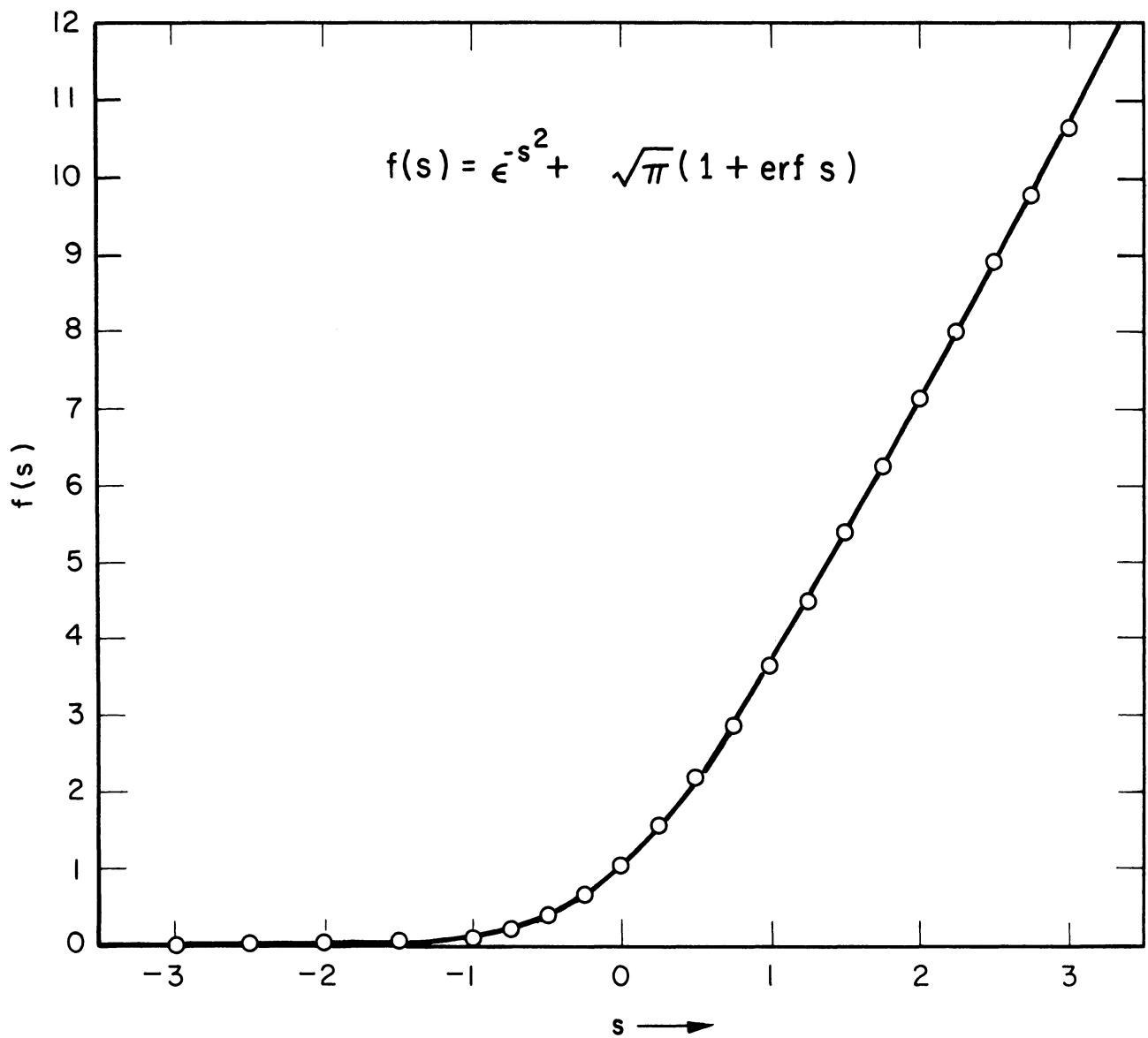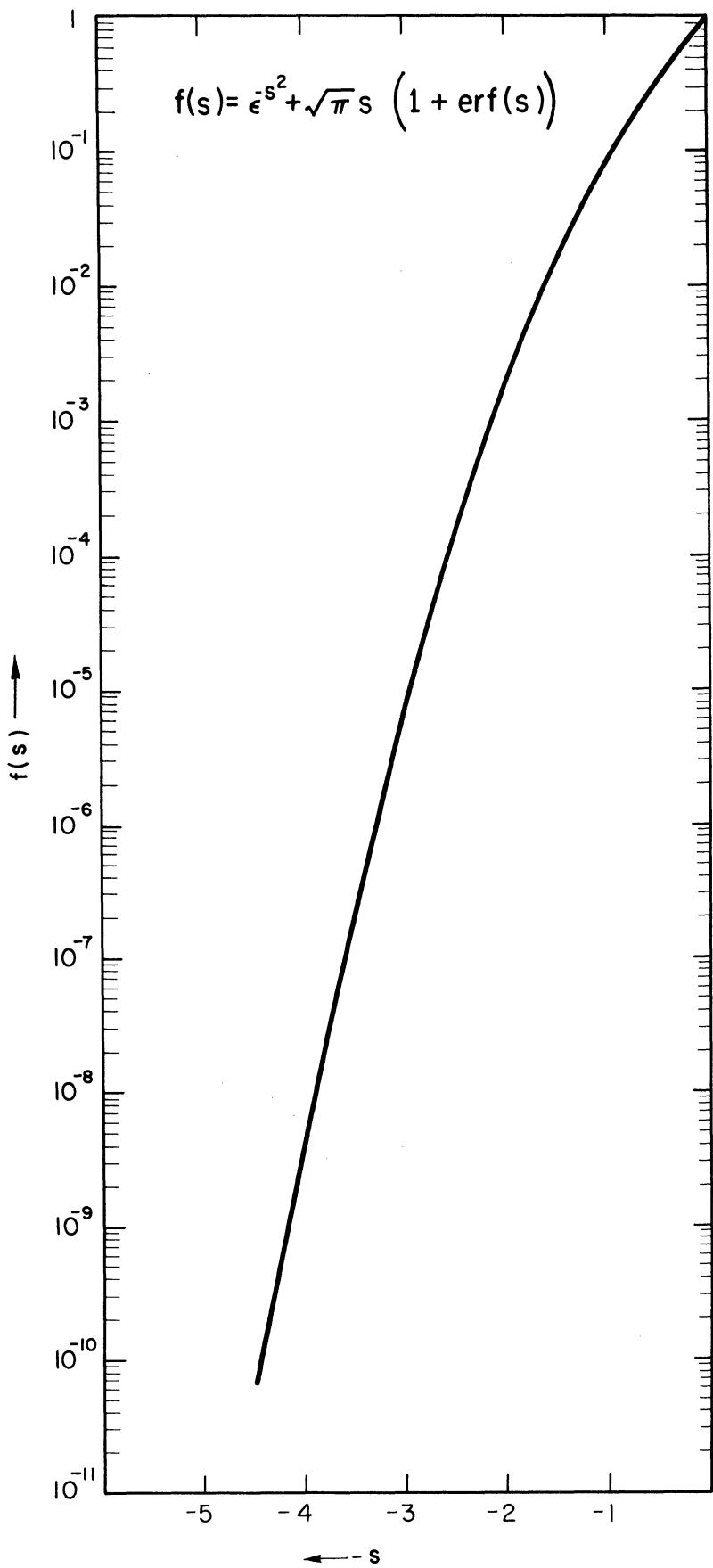
Fig. 1

$$f(s) = \epsilon^{-s^2} + \sqrt{\pi}\,(1 + \mathrm{erf}\ s)$$

Fig. 2

$$f(s) = \epsilon^{-s^2} + \sqrt{\pi}\, s \left(1 + erf(s)\right)$$

Fig. 3

TABLE I

-s	F(-s)	-s	F(-s)
-3.0000	.00000595	-2.5000	.00012722
-2.9900	.00000634	-2.4900	.00013463
-2.9800	.00000675	-2.4800	.00014245
-2.9700	.00000721	-2.4700	.00015070
-2.9600	.00000771	-2.4600	.00015939
-2.9500	.00000824	-2.4500	.00016856
-2.9400	.00000876	-2.4400	.00017821
-2.9300	.00000939	-2.4300	.00018841
-2.9200	.00001000	-2.4200	.00019912
-2.9100	.00001066	-2.4100	.00021042
-2.9000	.00001135	-2.4000	.00022233
-2.8900	.00001211	-2.3900	.00023485
-2.8800	.00001290	-2.3800	.00024803
-2.8700	.00001377	-2.3700	.00026191
-2.8600	.00001466	-2.3600	.00027652
-2.8500	.00001561	-2.3500	.00029191
-2.8400	.00001665	-2.3400	.00030808
-2.8300	.00001771	-2.3300	.00032508
-2.8200	.00001886	-2.3200	.00034298
-2.8100	.00002008	-2.3100	.00036178
-2.8000	.00002137	-2.3000	.00038157
-2.7900	.00002274	-2.2900	.00040234
-2.7800	.00002421	-2.2800	.00042417
-2.7700	.00002573	-2.2700	.00044713
-2.7600	.00002739	-2.2600	.00047121
-2.7500	.00002911	-2.2500	.00049653
-2.7400	.00003094	-2.2400	.00052311
-2.7300	.00003290	-2.2300	.00055101
-2.7200	.00003495	-2.2200	.00058029
-2.7100	.00003716	-2.2100	.00061104
-2.7000	.00003948	-2.2000	.00064327
-2.6900	.00004192	-2.1900	.00067710
-2.6800	.00004449	-2.1800	.00071259
-2.6700	.00004725	-2.1700	.00074980
-2.6600	.00005015	-2.1600	.00078881
-2.6500	.00005323	-2.1500	.00082972
-2.6400	.00005651	-2.1400	.00087258
-2.6300	.00005995	-2.1300	.00091749
-2.6200	.00006359	-2.1200	.00096454
-2.6100	.00006743	-2.1100	.00101384
-2.6000	.00007149	-2.1000	.00106546
-2.5900	.00007580	-2.0900	.00111953
-2.5800	.00008037	-2.0800	.00117611
-2.5700	.00008515	-2.0700	.00123536
-2.5600	.00009025	-2.0600	.00129736
-2.5500	.00009559	-2.0500	.00136222
-2.5400	.00010125	-2.0400	.00143008
-2.5300	.00010725	-2.0300	.00150105
-2.5200	.00011355	-2.0200	.00157527
-2.5100	.00012021	-2.0100	.00165287

TABLE I (Continued)

-s	F(-s)	-s	F(-s)
-2.0000	.00173400	-1.5000	.01528635
-1.9900	.00181877	-1.4900	.01589787
-1.9800	.00190737	-1.4800	.01653108
-1.9700	.00199995	-1.4700	.01718668
-1.9600	.00209665	-1.4600	.01786531
-1.9500	.00219764	-1.4500	.01856768
-1.9400	.00230309	-1.4400	.01929450
-1.9300	.00241317	-1.4300	.02004645
-1.9200	.00252810	-1.4200	.02082428
-1.9100	.00264803	-1.4100	.02162876
-1.9000	.00277317	-1.4000	.02246063
-1.8900	.00290374	-1.3900	.02332066
-1.8800	.00303989	-1.3800	.02420968
-1.8700	.00318192	-1.3700	.02512846
-1.8600	.00332998	-1.3600	.02607787
-1.8500	.00348434	-1.3500	.02705874
-1.8400	.00364523	-1.3400	.02807194
-1.8300	.00381291	-1.3300	.02911836
-1.8200	.00398759	-1.3200	.03019887
-1.8100	.00416957	-1.3100	.03131442
-1.8000	.00435908	-1.3000	.03246591
-1.7900	.00455647	-1.2900	.03365432
-1.7800	.00476195	-1.2800	.03488058
-1.7700	.00497586	-1.2700	.03614573
-1.7600	.00519847	-1.2600	.03745075
-1.7500	.00543012	-1.2500	.03879662
-1.7400	.00567114	-1.2400	.04018444
-1.7300	.00592184	-1.2300	.04161524
-1.7200	.00618258	-1.2200	.04309009
-1.7100	.00645369	-1.2100	.04461010
-1.7000	.00673557	-1.2000	.04617635
-1.6900	.00702854	-1.1900	.04779001
-1.6800	.00733301	-1.1800	.04945219
-1.6700	.00764938	-1.1700	.05116409
-1.6600	.00797806	-1.1600	.05292683
-1.6500	.00831945	-1.1500	.05474169
-1.6400	.00867398	-1.1400	.05660983
-1.6300	.00904209	-1.1300	.05853249
-1.6200	.00942426	-1.1200	.06051095
-1.6100	.00982091	-1.1100	.06254646
-1.6000	.01023253	-1.1000	.06464029
-1.5900	.01065962	-1.0900	.06679379
-1.5800	.01110268	-1.0800	.06900823
-1.5700	.01156221	-1.0700	.07128497
-1.5600	.01203875	-1.0600	.07362538
-1.5500	.01253283	-1.0500	.07603079
-1.5400	.01304504	-1.0400	.07850263
-1.5300	.01357588	-1.0300	.08104227
-1.5200	.01412600	-1.0200	.08365115
-1.5100	.01469595	-1.0100	.08633069

TABLE I (Concluded)

-s	F(-s)	-s	F(-s)
1.0000	.08908233	-.5000	.35386777
-.9900	.09190755	-.4900	.36244480
-.9800	.09480783	-.4800	.37117914
-.9700	.09778468	-.4700	.38007231
-.9600	.10083956	-.4600	.38912583
-.9500	.10397404	-.4500	.39834120
-.9400	.10718961	-.4400	.40771990
-.9300	.11048785	-.4300	.41726337
-.9200	.11387031	-.4200	.42697309
-.9100	.11733855	-.4100	.43685044
-.9000	.12089417	-.4000	.44689683
-.8900	.12453876	-.3900	.45711365
-.8800	.12827395	-.3800	.46750225
-.8700	.13210131	-.3700	.47806393
-.8600	.13602250	-.3600	.48880000
-.8500	.14003916	-.3500	.49971178
-.8400	.14415290	-.3400	.51080047
-.8300	.14836542	-.3300	.52206731
-.8200	.15267837	-.3200	.53351351
-.8100	.15709341	-.3100	.54514024
-.8000	.16161223	-.3000	.55694862
-.7900	.16623649	-.2900	.56893977
-.7800	.17096791	-.2800	.58111478
-.7700	.17580817	-.2700	.59347469
-.7600	.18075896	-.2600	.60602053
-.7500	.18582200	-.2500	.61875328
-.7400	.19099901	-.2400	.63167391
-.7300	.19629167	-.2300	.64478333
-.7200	.20170171	-.2200	.65808243
-.7100	.20723085	-.2100	.67157207
-.7000	.21288079	-.2000	.68525305
-.6900	.21865324	-.1900	.69912620
-.6800	.22454993	-.1800	.71319222
-.6700	.23057257	-.1700	.72745185
-.6600	.23672289	-.1600	.74190579
-.6500	.24300256	-.1500	.75655466
-.6400	.24941331	-.1400	.77139904
-.6300	.25595685	-.1300	.78643955
-.6200	.26263485	-.1200	.80167668
-.6100	.26944902	-.1100	.81711093
-.6000	.27640104	-.1000	.83274276
-.5900	.28349260	-.0900	.84857260
-.5800	.29072535	-.0800	.86460079
-.5700	.29810096	-.0700	.88082770
-.5600	.30562110	-.0600	.89725363
-.5500	.31328737	-.0500	.91387880
-.5400	.32110143	-.0400	.93070347
-.5300	.32906492	-.0300	.94772782
-.5200	.33717939	-.0200	.96495195
-.5100	.34544650	-.0100	.98237599

TABLE II

s	F(s)	s	F(s)
.0000	1.00000000	.5000	2.12626773
.0100	1.01782398	.5100	2.15329444
.0200	1.03584795	.5200	2.18047532
.0300	1.05407181	.5300	2.20780885
.0400	1.07249546	.5400	2.23529339
.0500	1.09111880	.5500	2.26292732
.0600	1.10994162	.5600	2.29070905
.0700	1.12896369	.5700	2.31863689
.0800	1.14818478	.5800	2.34670928
.0900	1.16760460	.5900	2.37492457
.1000	1.18722275	.6000	2.40328097
.1100	1.20703891	.6100	2.43177697
.1200	1.22705266	.6200	2.46041080
.1300	1.24726354	.6300	2.48918077
.1400	1.26767103	.6400	2.51808524
.1500	1.28827463	.6500	2.54712251
.1600	1.30907378	.6600	2.57629082
.1700	1.33006783	.6700	2.60558850
.1800	1.35125621	.6800	2.63501385
.1900	1.37263818	.6900	2.66456521
.2000	1.39421304	.7000	2.69424072
.2100	1.41598004	.7100	2.72403878
.2200	1.43793841	.7200	2.75395766
.2300	1.46008730	.7300	2.78399560
.2400	1.48242588	.7400	2.81415096
.2500	1.50495327	.7500	2.84442192
.2600	1.52766851	.7600	2.87480688
.2700	1.55057067	.7700	2.90530413
.2800	1.57365875	.7800	2.93591183
.2900	1.59693174	.7900	2.96662840
.3000	1.62038858	.8000	2.99745214
.3100	1.64402820	.8100	3.02838132
.3200	1.66784948	.8200	3.05941430
.3300	1.69185127	.8300	3.09054938
.3400	1.71603243	.8400	3.12178481
.3500	1.74039175	.8500	3.15311906
.3600	1.76492797	.8600	3.18455043
.3700	1.78963989	.8700	3.21607727
.3800	1.81452622	.8800	3.24769786
.3900	1.83958562	.8900	3.27941069
.4000	1.86481678	.9000	3.31121406
.4100	1.89021841	.9100	3.34310648
.4200	1.91578905	.9200	3.37508622
.4300	1.94152734	.9300	3.40715176
.4400	1.96743186	.9400	3.43930152
.4500	1.99350116	.9500	3.47153395
.4600	2.01973379	.9600	3.50384751
.4700	2.04612827	.9700	3.53624061
.4800	2.07268310	.9800	3.56871176
.4900	2.09939677	.9900	3.60125944

TABLE II (Continued)

s	F(s)	s	F(s)
1.0000	3.63388228	1.5000	5.33248621
1.0100	3.66657859	1.5100	5.36734378
1.0200	3.69934708	1.5200	5.40222186
1.0300	3.73218614	1.5300	5.43711966
1.0400	3.76509452	1.5400	5.47203696
1.0500	3.79807070	1.5500	5.50697273
1.0600	3.83111328	1.5600	5.54192662
1.0700	3.86422086	1.5700	5.57689810
1.0800	3.89739215	1.5800	5.61188656
1.0900	3.93062568	1.5900	5.64689147
1.1000	3.96392021	1.6000	5.68191236
1.1100	3.99727440	1.6100	5.71694881
1.1200	4.03068680	1.6200	5.75200003
1.1300	4.06415635	1.6300	5.78706592
1.1400	4.09768170	1.6400	5.82214582
1.1500	4.13126153	1.6500	5.85723931
1.1600	4.16489470	1.6600	5.89234596
1.1700	4.19857991	1.6700	5.92746526
1.1800	4.23231608	1.6800	5.96259683
1.1900	4.26610190	1.6900	5.99774039
1.2000	4.29993623	1.7000	6.03289539
1.2100	4.33381802	1.7100	6.06806147
1.2200	4.36774594	1.7200	6.10323840
1.2300	4.40171903	1.7300	6.13842571
1.2400	4.43573630	1.7400	6.17362303
1.2500	4.46979648	1.7500	6.20883000
1.2600	4.50389862	1.7600	6.24404633
1.2700	4.53804165	1.7700	6.27927172
1.2800	4.57222444	1.7800	6.31450570
1.2900	4.60644615	1.7900	6.34974831
1.3000	4.64070576	1.8000	6.38499892
1.3100	4.67500222	1.8100	6.42025745
1.3200	4.70933467	1.8200	6.45552349
1.3300	4.74370223	1.8300	6.49079680
1.3400	4.77810377	1.8400	6.52607703
1.3500	4.81253856	1.8500	6.56136423
1.3600	4.84700572	1.8600	6.59665781
1.3700	4.88150430	1.8700	6.63195777
1.3800	4.91603357	1.8800	6.66726375
1.3900	4.95059252	1.8900	6.70257550
1.4000	4.98518050	1.9000	6.73789293
1.4100	5.01979661	1.9100	6.77321595
1.4200	5.05444014	1.9200	6.80854392
1.4300	5.08911031	1.9300	6.84387702
1.4400	5.12380636	1.9400	6.87921494
1.4500	5.15852755	1.9500	6.91455746
1.4600	5.19327319	1.9600	6.94990450
1.4700	5.22804254	1.9700	6.98525578
1.4800	5.26283491	1.9800	7.02061117
1.4900	5.29764974	1.9900	7.05597061

TABLE II (Continued)

s	F(s)	s	F(s)
2.0000	7.09133387	2.5000	8.86212695
2.0100	7.12670076	2.5100	8.89756787
2.0200	7.16207105	2.5200	8.93300915
2.0300	7.19744480	2.5300	8.96845102
2.0400	7.23282188	2.5400	9.00389302
2.0500	7.26820201	2.5500	9.03933537
2.0600	7.30358505	2.5600	9.07477796
2.0700	7.33897120	2.5700	9.11022091
2.0800	7.37435991	2.5800	9.14566410
2.0900	7.40975136	2.5900	9.18110752
2.1000	7.44514537	2.6000	9.21655130
2.1100	7.48054165	2.6100	9.25199509
2.1200	7.51594031	2.6200	9.28743935
2.1300	7.55134135	2.6300	9.32288373
2.1400	7.58674443	2.6400	9.35832822
2.1500	7.62214947	2.6500	9.39377296
2.1600	7.65755659	2.6600	9.42921793
2.1700	7.69296563	2.6700	9.46466291
2.1800	7.72837639	2.6800	9.50010812
2.1900	7.76378900	2.6900	9.53555357
2.2000	7.79920310	2.7000	9.57099915
2.2100	7.83461887	2.7100	9.60644484
2.2200	7.87003613	2.7200	9.64189065
2.2300	7.90545487	2.7300	9.67733669
2.2400	7.94087487	2.7400	9.71278262
2.2500	7.97629637	2.7500	9.74822891
2.2600	8.01171899	2.7600	9.78367507
2.2700	8.04714286	2.7700	9.81912136
2.2800	8.08256781	2.7800	9.85456789
2.2900	8.11799395	2.7900	9.89001453
2.3000	8.15342128	2.8000	9.92546105
2.3100	8.18884933	2.8100	9.96090770
2.3200	8.22427869	2.8200	9.99635458
2.3300	8.25970876	2.8300	10.03180146
2.3400	8.29513979	2.8400	10.06724823
2.3500	8.33057165	2.8500	10.10269535
2.3600	8.36600423	2.8600	10.13814235
2.3700	8.40143764	2.8700	10.17358935
2.3800	8.43687189	2.8800	10.20903659
2.3900	8.47230661	2.8900	10.24448371
2.4000	8.50774205	2.9000	10.27993107
2.4100	8.54317832	2.9100	10.31537843
2.4200	8.57861495	2.9200	10.35082579
2.4300	8.61405206	2.9300	10.38627303
2.4400	8.64949000	2.9400	10.42172062
2.4500	8.68492830	2.9500	10.45716798
2.4600	8.72036707	2.9600	10.49261534
2.4700	8.75580645	2.9700	10.52806306
2.4800	8.79124618	2.9800	10.56351042
2.4900	8.82668626	2.9900	10.59895813

TABLE II (Continued)

s	F(s)	s	F(s)
3.0000	10.63440573	3.5000	12.40680003
3.0100	10.66985321	3.5100	12.44224787
3.0200	10.70530081	3.5200	12.47769570
3.0300	10.74074841	3.5300	12.51314366
3.0400	10.77619624	3.5400	12.54859185
3.0500	10.81164396	3.5500	12.58403981
3.0600	10.84709156	3.5600	12.61948764
3.0700	10.88253927	3.5700	12.65493584
3.0800	10.91798723	3.5800	12.69038367
3.0900	10.95343494	3.5900	12.72583175
3.1000	10.98888278	3.6000	12.76127970
3.1100	11.02433050	3.6100	12.79672766
3.1200	11.05977833	3.6200	12.83217585
3.1300	11.09522629	3.6300	12.86762357
3.1400	11.13067412	3.6400	12.90307176
3.1500	11.16612184	3.6500	12.93851972
3.1600	11.20156980	3.6600	12.97396779
3.1700	11.23701763	3.6700	13.00941563
3.1800	11.27246535	3.6800	13.04486370
3.1900	11.30791330	3.6900	13.08031178
3.2000	11.34336114	3.7000	13.11575973
3.2100	11.37880898	3.7100	13.15120757
3.2200	11.41425705	3.7200	13.18665576
3.2300	11.44970500	3.7300	13.22210371
3.2400	11.48515284	3.7400	13.25755167
3.2500	11.52060068	3.7500	13.29299974
3.2600	11.55604863	3.7600	13.32844770
3.2700	11.59149659	3.7700	13.36389565
3.2800	11.62694454	3.7800	13.39934361
3.2900	11.66239262	3.7900	13.43479156
3.3000	11.69784045	3.8000	13.47023952
3.3100	11.73328841	3.8100	13.50568771
3.3200	11.76873648	3.8200	13.54113567
3.3300	11.80418432	3.8300	13.57658362
3.3400	11.83963215	3.8400	13.61203170
3.3500	11.87508023	3.8500	13.64747977
3.3600	11.91052806	3.8600	13.68292761
3.3700	11.94597614	3.8700	13.71837568
3.3800	11.98142409	3.8800	13.75382376
3.3900	12.01687205	3.8900	13.78927171
3.4000	12.05232000	3.9000	13.82471967
3.4100	12.08776796	3.9100	13.86016762
3.4200	12.12321591	3.9200	13.89561570
3.4300	12.15866387	3.9300	13.93106365
3.4400	12.19411194	3.9400	13.96651173
3.4500	12.22955990	3.9500	14.00195968
3.4600	12.26500797	3.9600	14.03740764
3.4700	12.30045605	3.9700	14.07285571
3.4800	12.33590400	3.9800	14.10830367
3.4900	12.37135196	3.9900	14.14375174

TABLE II (Continued)

s	F(s)	s	F(s)
4.0000	14.17919970	4.5000	15.95159984
4.0100	14.21464777	4.5100	15.98704779
4.0200	14.25009573	4.5200	16.02249575
4.0300	14.28554368	4.5300	16.05794382
4.0400	14.32099164	4.5400	16.09339166
4.0500	14.35643959	4.5500	16.12883973
4.0600	14.39188755	4.5600	16.16428757
4.0700	14.42733574	4.5700	16.19973588
4.0800	14.46278369	4.5800	16.23518372
4.0900	14.49823165	4.5900	16.27063179
4.1000	14.53367972	4.6000	16.30607963
4.1100	14.56912768	4.6100	16.34152770
4.1200	14.60457563	4.6200	16.37697554
4.1300	14.64002383	4.6300	16.41242385
4.1400	14.67547172	4.6400	16.44787169
4.1500	14.71091974	4.6500	16.48331976
4.1600	14.74636781	4.6600	16.51876760
4.1700	14.78181577	4.6700	16.55421567
4.1800	14.81726372	4.6800	16.58966351
4.1900	14.85271192	4.6900	16.62511182
4.2000	14.88815987	4.7000	16.66055965
4.2100	14.92360783	4.7100	16.69600773
4.2200	14.95905578	4.7200	16.73145580
4.2300	14.99450374	4.7300	16.76690364
4.2400	15.02995169	4.7400	16.80235171
4.2500	15.06539989	4.7500	16.83779979
4.2600	15.10084784	4.7600	16.87324786
4.2700	15.13629580	4.7700	16.90869570
4.2800	15.17174375	4.7800	16.94414377
4.2900	15.20719171	4.7900	16.97959161
4.3000	15.24263966	4.8000	17.01503968
4.3100	15.27808762	4.8100	17.05048752
4.3200	15.31353581	4.8200	17.08593583
4.3300	15.34898376	4.8300	17.12138367
4.3400	15.38443172	4.8400	17.15683174
4.3500	15.41987979	4.8500	17.19227958
4.3600	15.45532775	4.8600	17.22772765
4.3700	15.49077570	4.8700	17.26317549
4.3800	15.52622390	4.8800	17.29862380
4.3900	15.56167185	4.8900	17.33407164
4.4000	15.59711981	4.9000	17.36951971
4.4100	15.63256776	4.9100	17.40496778
4.4200	15.66801572	4.9200	17.44041562
4.4300	15.70346367	4.9300	17.47586370
4.4400	15.73891187	4.9400	17.51131177
4.4500	15.77435982	4.9500	17.54675984
4.4600	15.80980778	4.9600	17.58220768
4.4700	15.84525573	4.9700	17.61765575
4.4800	15.88070369	4.9800	17.65310359
4.4900	15.91615164	4.9900	17.68855166

TABLE II (Continued)

s	F(s)	s	F(s)
5.0000	17.72399974	5.5000	19.49639988
5.0100	17.75944781	5.5100	19.53184772
5.0200	17.79489565	5.5200	19.56729579
5.0300	17.83034372	5.5300	19.60274363
5.0400	17.86579156	5.5400	19.63819170
5.0500	17.90123963	5.5500	19.67363954
5.0600	17.93668771	5.5600	19.70908761
5.0700	17.97213578	5.5700	19.74453568
5.0800	18.00758386	5.5800	19.77998376
5.0900	18.04303169	5.5900	19.81543159
5.1000	18.07847977	5.6000	19.85087967
5.1100	18.11392760	5.6100	19.88632774
5.1200	18.14937568	5.6200	19.92177558
5.1300	18.18482375	5.6300	19.95722365
5.1400	18.22027183	5.6400	19.99267173
5.1500	18.25571966	5.6500	20.02811980
5.1600	18.29116774	5.6600	20.06356764
5.1700	18.32661557	5.6700	20.09901571
5.1800	18.36206365	5.6800	20.13446355
5.1900	18.39751172	5.6900	20.16991186
5.2000	18.43295979	5.7000	20.20535970
5.2100	18.46840763	5.7100	20.24080777
5.2200	18.50385571	5.7200	20.27625561
5.2300	18.53930354	5.7300	20.31170368
5.2400	18.57475162	5.7400	20.34715152
5.2500	18.61019969	5.7500	20.38259983
5.2600	18.64564776	5.7600	20.41804767
5.2700	18.68109584	5.7700	20.45349574
5.2800	18.71654367	5.7800	20.48894358
5.2900	18.75199175	5.7900	20.52439165
5.3000	18.78743958	5.8000	20.55983973
5.3100	18.82288766	5.8100	20.59528756
5.3200	18.85833573	5.8200	20.63073587
5.3300	18.89378381	5.8300	20.66618371
5.3400	18.92923164	5.8400	20.70163178
5.3500	18.96467972	5.8500	20.73707962
5.3600	19.00012755	5.8600	20.77252769
5.3700	19.03557563	5.8700	20.80797553
5.3800	19.07102370	5.8800	20.84342384
5.3900	19.10647178	5.8900	20.87887168
5.4000	19.14191961	5.9000	20.91431975
5.4100	19.17736769	5.9100	20.94976759
5.4200	19.21281552	5.9200	20.98521566
5.4300	19.24826360	5.9300	21.02066350
5.4400	19.28371167	5.9400	21.05611181
5.4500	19.31915975	5.9500	21.09155965
5.4600	19.35460782	5.9600	21.12700772
5.4700	19.39005566	5.9700	21.16245556
5.4800	19.42550373	5.9800	21.19790363
5.4900	19.46095157	5.9900	21.23335171

TABLE II (Continued)

s	F(s)	s	F(s)
6.0000	21.26879978	6.5000	23.04119968
6.0100	21.30424786	6.5100	23.07664776
6.0200	21.33969569	6.5200	23.11209559
6.0300	21.37514377	6.5300	23.14754367
6.0400	21.41059160	6.5400	23.18299174
6.0500	21.44603968	6.5500	23.21843958
6.0600	21.48148751	6.5600	23.25388765
6.0700	21.51693583	6.5700	23.28933573
6.0800	21.55238366	6.5800	23.32478380
6.0900	21.58783174	6.5900	23.36023164
6.1000	21.62327957	6.6000	23.39567971
6.1100	21.65872765	6.6100	23.43112755
6.1200	21.69417548	6.6200	23.46657562
6.1300	21.72962379	6.6300	23.50202370
6.1400	21.76507163	6.6400	23.53747177
6.1500	21.80051970	6.6500	23.57291961
6.1600	21.83596778	6.6600	23.60836768
6.1700	21.87141562	6.6700	23.64381552
6.1800	21.90686369	6.6800	23.67926359
6.1900	21.94231176	6.6900	23.71471167
6.2000	21.97775984	6.7000	23.75015974
6.2100	22.01320767	6.7100	23.78560758
6.2200	22.04865575	6.7200	23.82105565
6.2300	22.08410358	6.7300	23.85650373
6.2400	22.11955166	6.7400	23.89195156
6.2500	22.15499973	6.7500	23.92739987
6.2600	22.19044781	6.7600	23.96284771
6.2700	22.22589564	6.7700	23.99829578
6.2800	22.26134372	6.7800	24.03374362
6.2900	22.29679155	6.7900	24.06919169
6.3000	22.33223963	6.8000	24.10463953
6.3100	22.36768746	6.8100	24.14008760
6.3200	22.40313578	6.8200	24.17553568
6.3300	22.43858361	6.8300	24.21098375
6.3400	22.47403169	6.8400	24.24643159
6.3500	22.50947976	6.8500	24.28187966
6.3600	22.54492760	6.8600	24.31732750
6.3700	22.58037567	6.8700	24.35277557
6.3800	22.61582375	6.8800	24.38822365
6.3900	22.65127182	6.8900	24.42367172
6.4000	22.68671966	6.9000	24.45911980
6.4100	22.72216773	6.9100	24.49456763
6.4200	22.75761557	6.9200	24.53001571
6.4300	22.79306364	6.9300	24.56546354
6.4400	22.82851171	6.9400	24.60091186
6.4500	22.86395979	6.9500	24.63635969
6.4600	22.89940763	6.9600	24.67180777
6.4700	22.93485570	6.9700	24.70725560
6.4800	22.97030354	6.9800	24.74270368
6.4900	23.00575161	6.9900	24.77815151

TABLE II (Continued)

s	F(s)	s	F(s)
7.0000	24.81359982	7.5000	26.58599973
7.0100	24.84904766	7.5100	26.62144780
7.0200	24.88449574	7.5200	26.65689564
7.0300	24.91994357	7.5300	26.69234371
7.0400	24.95539165	7.5400	26.72779155
7.0500	24.99083948	7.5500	26.76323962
7.0600	25.02628756	7.5600	26.79868746
7.0700	25.06173563	7.5700	26.83413577
7.0800	25.09718370	7.5800	26.86958361
7.0900	25.13263178	7.5900	26.90503168
7.1000	25.16807961	7.6000	26.94047952
7.1100	25.20352769	7.6100	26.97592759
7.1200	25.23897552	7.6200	27.01137567
7.1300	25.27442384	7.6300	27.04682374
7.1400	25.30987167	7.6400	27.08227181
7.1500	25.34531975	7.6500	27.11771965
7.1600	25.38076758	7.6600	27.15316772
7.1700	25.41621566	7.6700	27.18861556
7.1800	25.45166349	7.6800	27.22406363
7.1900	25.48711181	7.6900	27.25951171
7.2000	25.52255964	7.7000	27.29495978
7.2100	25.55800772	7.7100	27.33040762
7.2200	25.59345555	7.7200	27.36585569
7.2300	25.62890363	7.7300	27.40130353
7.2400	25.66435170	7.7400	27.43675160
7.2500	25.69979978	7.7500	27.47219968
7.2600	25.73524761	7.7600	27.50764775
7.2700	25.77069569	7.7700	27.54309559
7.2800	25.80614376	7.7800	27.57854366
7.2900	25.84159160	7.7900	27.61399150
7.3000	25.87703967	7.8000	27.64943957
7.3100	25.91248751	7.8100	27.68488765
7.3200	25.94793582	7.8200	27.72033572
7.3300	25.98338366	7.8300	27.75578380
7.3400	26.01883173	7.8400	27.79123163
7.3500	26.05427957	7.8500	27.82667971
7.3600	26.08972764	7.8600	27.86212754
7.3700	26.12517548	7.8700	27.89757562
7.3800	26.16062379	7.8800	27.93302369
7.3900	26.19607162	7.8900	27.96847177
7.4000	26.23151970	7.9000	28.00391960
7.4100	26.26696754	7.9100	28.03936768
7.4200	26.30241561	7.9200	28.07481551
7.4300	26.33786368	7.9300	28.11026359
7.4400	26.37331176	7.9400	28.14571166
7.4500	26.40875983	7.9500	28.18115973
7.4600	26.44420767	7.9600	28.21660757
7.4700	26.47965574	7.9700	28.25205564
7.4800	26.51510358	7.9800	28.28750372
7.4900	26.55055165	7.9900	28.32295156

TABLE II (Concluded)

s	F(s)	s	F(s)
8.0000	28.35839987	8.5000	30.13079977
8.0100	28.39384770	8.5100	30.16624761
8.0200	28.42929578	8.5200	30.20169568
8.0300	28.46474361	8.5300	30.23714375
8.0400	28.50019169	8.5400	30.27259159
8.0500	28.53563952	8.5500	30.30803967
8.0600	28.57108760	8.5600	30.34348750
8.0700	28.60653543	8.5700	30.37893558
8.0800	28.64198351	8.5800	30.41438341
8.0900	28.67743134	8.5900	30.44983149
8.1000	28.71287942	8.6000	30.48527932
8.1100	28.74832749	8.6100	30.52072740
8.1200	28.78377533	8.6200	30.55617523
8.1300	28.81922364	8.6300	30.59162378
8.1400	28.85467172	8.6400	30.62707162
8.1500	28.89011955	8.6500	30.66251969
8.1600	28.92556763	8.6600	30.69796753
8.1700	28.96101570	8.6700	30.73341560
8.1800	28.99646354	8.6800	30.76886344
8.1900	29.03191161	8.6900	30.80431151
8.2000	29.06735945	8.7000	30.83975959
8.2100	29.10280752	8.7100	30.87520742
8.2200	29.13825536	8.7200	30.91065550
8.2300	29.17370343	8.7300	30.94610333
8.2400	29.20915127	8.7400	30.98155141
8.2500	29.24459982	8.7500	31.01699972
8.2600	29.28004766	8.7600	31.05244780
8.2700	29.31549573	8.7700	31.08789563
8.2800	29.35094357	8.7800	31.12334371
8.2900	29.38639164	8.7900	31.15879154
8.3000	29.42183948	8.8000	31.19423962
8.3100	29.45728755	8.8100	31.22968745
8.3200	29.49273562	8.8200	31.26513553
8.3300	29.52818346	8.8300	31.30058336
8.3400	29.56363153	8.8400	31.33603144
8.3500	29.59907937	8.8500	31.37147951
8.3600	29.63452744	8.8600	31.40692735
8.3700	29.66997528	8.8700	31.44237542
8.3800	29.70542383	8.8800	31.47782373
8.3900	29.74087167	8.8900	31.51327157
8.4000	29.77631974	8.9000	31.54871964
8.4100	29.81176758	8.9100	31.58416772
8.4200	29.84721565	8.9200	31.61961555
8.4300	29.88266349	8.9300	31.65506363
8.4400	29.91811156	8.9400	31.69051147
8.4500	29.95355940	8.9500	31.72595954
8.4600	29.98900747	8.9600	31.76140738
8.4700	30.02445531	8.9700	31.79685545
8.4800	30.05990338	8.9800	31.83230329
8.4900	30.09535146	8.9900	31.86775136

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