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ON SHOCK-WAVE VELOCITY IN DROPLET IMPACT PHENOMENA

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ABSTRACT

A relationship between the shock wave and impact velocities in high-speed liquid-solid impact is determined from the fundamental equations (continuity, momentum, and state). Very good agreement is found with available experimental data.

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INTRODUCTION

The pressure developed in the high speed impact between a liquid and a solid surface is of interest and concern in the problems of turbine blades operating in moist vapors and of supersonic aircraft and missiles flying in rain.

The pressure which acts on the material surface and causes the damage is related to the "water-hammer pressure", $p_o = \rho_o c v_o$, where v_o is the impact velocity and $\rho_o c$ is the mass flux, assuming that a shock front can be regarded as a discontinuity and that a shock front has negligible volume.

The velocity of propagation of the pressure or shock wave, c, can be approximated by the acoustic velocity, c_o, in the undisturbed liquid if the impact is just a small disturbance. However, for high velocity impact, the deviation from c_o becomes significant, and c must then be taken as an approximate shock wave velocity satisfying the continuity and momentum equations with an appropriate equation of state.

It would be desirable to have an expression for c as a function of v, which could then be used to calculate p directly. Most of the fundamental studies [2, 3, 4, 5, 6] have tabulated shock front velocities, c, as a function of pressure, p_o. Heymann [1] has proposed, as an approximate relationship between c and v, $c = c_o + kv$. However, no direct analytical formulation has been given, as yet, to justify this approximation. Such a relationship is developed in the present paper.

II. FUNDAMENTAL RELATIONS

If a shock front is discontinuous and its thickness is negligible, the governing equations applying to such a shock front impinging upon a rigid boundary, derived from continuity and momentum considerations, are:

$$\rho_o c = \rho (c - v) \tag{1}$$

$$p - p_o = \rho_o c v \tag{2}$$

where c is the shock wave velocity,

v is the impact velocity,

ρ is the density of liquid in the compressed state,

ρ_o is the density of the undisturbed liquid, and

$p-p_o$ is the impulse of the net force per unit area across the shock front.

The equation of state of water should provide the additional relation necessary for solving the problem, namely

$$\frac{p + B}{p_o + B} = \left(\frac{\rho}{\rho_o}\right)^n \tag{3}$$

where B is a function of the thermal state (B has the value of 3.047 kilobars at 20^{o}C and 1 bar. Though somewhat sensitive to temperature, it is relatively insensitive to pressure.), and

n is approximated as a constant equal to 7.15 for pressures up to 25 kilobars.

Solving equations (1) and (3) for ρ and equating, we obtain

$$\frac{\rho_o c}{c-v} = \rho_o \left[\frac{p+B}{p_o+B} \right]^{1/n} \tag{4}$$

Raising both sides to the power n, and replacing p from eq. (2), eq. (4) becomes

$$\left(\frac{\rho_o c}{c-v}\right)^n = \rho_o^{\,n} \left[1 + \frac{\rho_o c v}{p_o+B} \right] \tag{5}$$

Rearranging eq. (5) yields

$$\frac{c^2}{\left(\frac{p_o+B}{\rho_o}\right)} = \frac{1-(1-\frac{v}{c})^n}{(\frac{v}{c})(1-\frac{v}{c})^n} \tag{6}$$

where $\frac{v}{c} = \frac{\rho-\rho_o}{\rho} < 1$ always.

Equation (6) now gives the shock wave vel ocity, c, directly in terms of $\frac{v}{c}$. The value of c is then used to determine impact velocity v. It is more desirable to have explicit relations for c directly in terms of v. One such relation, we propose here, is

$$\frac{c}{c_o} = 1 + a\left(\frac{v}{c_o}\right) + b\left(\frac{v}{c_o}\right)^2 \tag{7}$$

In the present case, constants a = 1.925 and b = -0.083 are determined by a least square fit computer program for $\frac{v}{c_o}$ up to 3 from the curve resulting from eq. (6). Within this range predictions for c from eq. (6) and (7) agree to within $\pm$ 1%.

III. RESULTS

Results calculated from eq. (6) agree extremely well with the whole spectrum of experimental data [7, 8, 9, 10] as shown in Fig. 1. The asymptotic value of acoustic velocity, c_o, calculated from eq. (6), is about 4850 ft/sec which is slightly lower than the 4900 ft/sec that we adopted from Heymann's paper for calculation of v/c_o.

The relations

$$\frac{c}{c_o} = 1 + k\frac{v}{c_o} \quad , \quad k = 2.0 \; , \tag{8}$$

and

$$\frac{c}{c_o} = 1 + a\left(\frac{v}{c_o}\right) + b\left(\frac{v}{c_o}\right)^2 , \quad \begin{array}{l} a = 1.925 \\ b = -0.083 \end{array} \tag{9}$$

have been evaluated and compared with the numerical results of

$$\frac{c^2}{\left(\dfrac{p_o + B}{\rho_o}\right)} = \frac{1 - \left(1 - \dfrac{v}{c}\right)^n}{\left(\dfrac{v}{c}\right)\left(1 - \dfrac{v}{c}\right)^n}$$

Table I shows that the percentage deviation of $\frac{c}{c_o} = 1 + 2\frac{v}{c_o}$ increases as $\frac{v}{c_o}$ increases. It is about 6% at $\frac{v}{c_o} = 1.2$, 11% at $\frac{v}{c_o} = 2.0$, and 16% at $\frac{v}{c_o} = 3.0$. On the other hand the percentage deviation of $\frac{c}{c_o} = 1 + 1.925\frac{v}{c_o} - 0.083\left(\frac{v}{c_o}\right)^2$ is less than 1.2% over the whole range of $\frac{v}{c_o}$ up to 3.

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TABLE I

Comparison of Values for Shock Wave Velocity as Function of Impact Velocity Calculated from Three Different Equations

Nondimensional Impact Velocity v/c_o	Nondimensional Shock Wave Velocity			Deviation %	
	Computed from eq. (1)* c_1/c_o	Computed from eq. (2)* c_2/c_o	Computed from eq. (3)* c_3/c_o	$\frac{c_2-c_1}{c_1}$	$\frac{c_3-c_1}{c_1}$
0.1	1.1924	1.2	1.1917	0.627	-0.065
0.5	1.9643	2.0	1.9418	1.808	-1.145
1.0	2.8547	3.0	2.8422	5.073	-0.439
1.5	3.6947	4.0	3.7012	8.263	0.177
2.0	4.5020	5.0	4.5186	11.06	0.370
2.5	5.2863	6.0	5.2945	13.50	0.159
3.0	6.0528	7.0	6.0290	15.65	-0.391

c_o = 4900 ft./sec. = speed of sound in the undisturbed liquid.

* eq. (1)

$$\frac{c_1^2}{\left(\frac{p_o+B}{\rho_o}\right)} = \frac{1-\left(1-\frac{v}{c_1}\right)^n}{\left(\frac{v}{c_1}\right)\left(1-\frac{v}{c_1}\right)^n}$$

ρ_o = 1.935 lb. sec.^{2}/ft.^{4}

B = 0.6367×10^7 psf

n = 7.15

eq. (2)

$$\frac{c_2}{c_o} = 1 + 2\,\frac{v}{c_o}$$

eq. (3)

$$\frac{c_3}{c_o} = 1 + 1.925\left(\frac{v}{c_o}\right) - 0.083\left(\frac{v}{c_o}\right)^2$$

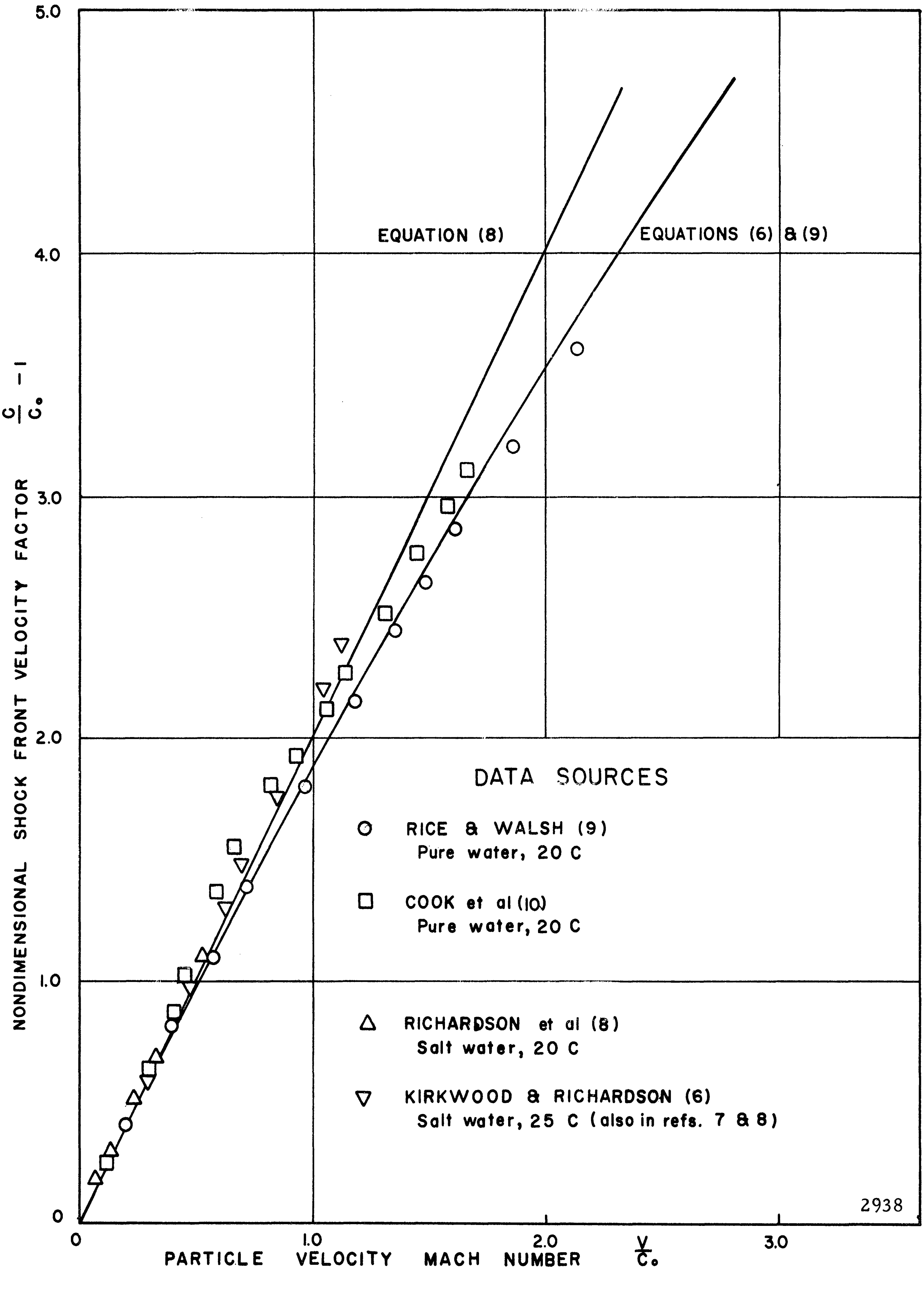

Figure 1. Shock Wave Velocity versus Particle Velocity in Water