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USE OF A LARGE THERMOCOUPLE JUNCTION TO LOCATE TEMPERATURE DISTURBANCES

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USE OF A LARGE THERMOCOUPLE JUNCTION TO LOCATE TEMPERATURE DISTURBANCES¹

ABSTRACT

A temperature disturbance within a large thermocouple junction produces a voltage which decreases with distance from the disturbance. With proper junction geometry, resulting voltage residues may be compared at two or more points on the junction. Positions and magnitudes of temperature disturbances are thereby determined. Useful measurement applications result.

1 INTRODUCTION

This report describes a unique use of a common sensing device—the thermocouple. Unlike the customary thermocouple junction, which measures temperature at a point, the large thermocouple junction locates temperature disturbances within the junction. A temperature disturbance at a point located on a large thermocouple junction will produce currents which fall off with distance from disturbance. Residual potential differences between selected leads indicate the location of the disturbance.

The voltage residues so produced can be used advantageously for a number of measuring applications. These include measurement of the magnitude and position of discrete heat sources, liquid level, fluid mixing, velocity gradients, and temperature equilibrium over a continuum. A large thermocouple may also be used to measure the atmospheric turbulence which deteriorates infrared and radar information in combat surveillance.

Analysis of the voltage potentials in a large thermocouple junction and confirming experiments are presented in this paper.

2 ANALYSIS

A special case of the large thermocouple junction is shown in Figure 1. This is a long junction, electrically equivalent to a transmission line.

¹The author appreciates the help of W. C. Meecham and V. L. Larrowe for discussions and suggestions concerning this work.

The transmission-line equation

$$E_{O} = E_{X} \operatorname{sech} \sqrt{RG} x \tag{1}$$

where $\mathbf{E}_{\mathbf{v}}$ is voltage generated

 E_{o} is output voltage at end of junction

R is resistance per unit length of element

x is the distance from source to end of junction

G is conductance per unit length between elements

describes the voltage decrement from the disturbance to the end of a long junction.

A large thermocouple junction may be viewed as a number of nonadditive point thermocouple junctions placed along a transmission line. A temperature disturbance causes a voltage to be generated at one of these points. This voltage decreases with distance along the junction due to current leakage between elements.

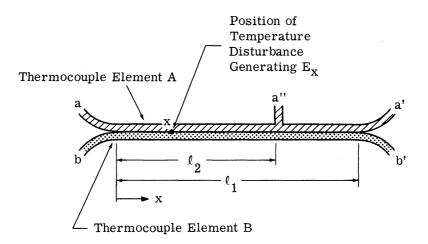


FIGURE 1. CONSTRUCTION OF LONG THERMOCOUPLE

Figure 1 illustrates a long thermocouple junction constructed by twisting, soldering or fusing two wires together. Five fixed leads are shown in Figure 1: a, a', a'', b, b', four located at the junction ends and one at a distance from an end. If a temperature disturbance occurs at point x, generating a voltage E_x , the two leads from the left end of the junction will produce a voltage difference of E_x sech \sqrt{RG} x.

Since elements A and B may have different values of resistance per unit length, a reference voltage level of $\frac{R_b}{R_a + R_b}$ E_x above element B at point x is taken (see Appendix A). R_a is

resistance per unit length for element A, and R_b for element B. Letting $\sqrt{RG} = K$, the voltages E_a , E_a , E_a , E_b , E_b , at locations indicated by the subscript, follow:

$$E_{a} = \frac{R_{a}}{R_{a} + R_{b}} E_{x} \operatorname{sech} Kx$$
 (2)

$$E_{a'} = \frac{R_a}{R_a + R_b} E_x \operatorname{sech} K(\ell_1 - x)$$
(3)

$$E_{a''} = \frac{R_a}{R_a + R_b} E_x \operatorname{sech} K(\ell_1 - x) \cosh K(\ell_1 - \ell_2) \qquad x < \ell_2$$
 (4)

$$E_{a''} = \frac{R_a}{R_a + R_b} E_x \operatorname{sech} Kx \cosh K\ell_2 \qquad x > \ell_2$$
 (4a)

$$E_b = -\frac{R_b}{R_a + R_b} E_x \text{ sech } Kx$$
 (5)

$$E_{b'} = -\frac{R_b}{R_a + R_b} E_x \operatorname{sech} K(\ell - x)$$
 (6)

The voltage differences between leads are readily found by subtracting the value at one lead from the value at another. For instance, the voltage difference between lead a and lead a' is

$$E_{a} - E_{a'} = \frac{R_{a}}{R_{a} + R_{b}} E_{x} \left[\operatorname{sech} Kx - \operatorname{sech} K(\ell_{1} - x) \right]$$
 (7)

A voltage difference of zero occurs at $x = \frac{1}{2} \ell_1$.

For the two-dimensional junction of the voltage decrement is described by the equation

$$E_0 \simeq E_x \sqrt{\frac{2}{\pi \sqrt{rg}}} e^{-\sqrt{rg}} x \tag{8}$$

for large value of x when edge effects are ignored (see Appendix B). r is resistance per unit square, g conductance per unit area. Thus, if the junction in Figure 2 extends sufficiently beyond the edges shown, the voltage difference between two leads on element A at the corners c, d is

$$E_{c} - E_{d} = \sqrt{\frac{2}{\pi \sqrt{rg}}} E_{x} \left(\frac{e^{-\sqrt{rg} x_{c}}}{\sqrt{x_{c}}} - \frac{e^{-\sqrt{rg} x_{d}}}{\sqrt{x_{d}}} \right)$$
(9)

Also,

$$E_{e} - E_{f} = \sqrt{\frac{2}{\pi \sqrt{rg}}} E_{x} \left(\frac{e^{-\sqrt{rg} x_{e}}}{\sqrt{x_{e}}} - \frac{e^{-\sqrt{rg} x_{f}}}{\sqrt{x_{f}}} \right)$$
(9a)

Thus a location on a plane may be found.

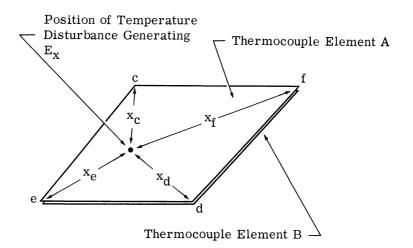


FIGURE 2. CONSTRUCTION OF A LARGE THERMOCOUPLE OF TWO FLAT ELEMENTS. Distance from point x to leads c, d, e, f shown at corners of element A.

3 EXPERIMENTS

Five experiments were performed to confirm the basic idea of using a large junction for position measurement, and to explore its applicability to several measurement applications. These experiments were preliminary in the sense that they were only intended to explore the measurement possibilities of a large thermocouple junction, and they do not exhaust the possible applications of this device. Ultimate sensitivity and accuracy were not investigated, nor were refinements for particular applications. Therefore, such techniques as linearizing by varying resistance or conductance in the junction, using many leads along a junction, using focused radiation for a heat source, or varying the impedances of the measuring instruments were beyond the scope of this effort.

Measurements were made with a Leeds and Northrup K-potentiometer and a 2430-A galvanometer in all but the fluid-mixing experiment, for which a Consolidated Electrodynamics Model 5-116 oscillograph with a 5-315 galvanometer was employed.

3.1. DISCRETE HEAT SOURCE APPLIED TO AN UNHEATED LONG JUNCTION

A discrete heat source was applied at various positions along junctions approximately 10 cm long. Voltage residue differences between pairs of the four end leads were plotted against the positions of the heat source. These experiments were performed to determine the dependence of voltage outputs on heat position and to give a basis to the above theory.

Long thermocouples were constructed of iron-constantan elements by twisting or soldering two parallel wires. Three wire sizes were used. The junction rested upon a large aluminum block insulated by 0.001-inch Mylar tape. Heat was conducted to a small portion of the couple through a wire attached to a 60-watt soldering iron. A lathe bed was used to position the heat source.

Results are shown in Figures 3 and 4 in the form of millivolt outputs between various pairs of leads plotted as functions of heat source position. In each case, the output is seen to be dependent upon the position of the applied heat source. In Figure 3, the voltage between two end leads of the same element ($E_a - E_a$, of Figure 1) is of similar magnitude and opposite polarity when heat is applied at opposite ends of the couple. A null voltage occurs when the temperature disturbance is near the center. This corresponds to Equation 7. In Figure 4, the voltage between leads of opposite elements at one end of the long couple ($E_a - E_b$ of Figure 1) is at a maximum when heat is applied at this end. As predicted in Equations 2 and 3, the voltage decreases as the heat source moves toward the opposite end. Figure 4 also shows the voltage between opposing elements at opposite ends of the junction ($E_a - E_b$, of Figure 1). With heat at one end, the voltage is relatively large. Its lowest value is near the center, and its maximum value is at $x = \ell_1$. Since element B (constantan) has a higher resistance than element A (iron), this result is expected considering Equations 2 and 6.

3.2. SMALL COOLING JET APPLIED TO A HEATED LONG JUNCTION

A long junction was heated for the purpose of locating heat conduction anomalies. This use of a heated junction extends the measuring possibilities to environments where no temperature disturbance normally exists. A small jet of air was placed on the junction at various distances from the ends. The cooled spot from the jet resulted in thermoelectric voltages of opposite polarities from those found in Type 1 experiments.

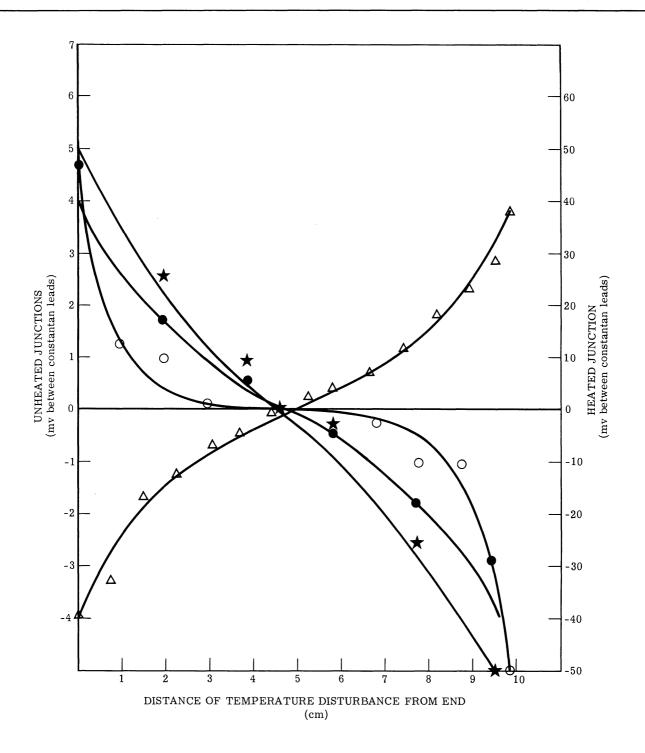


FIGURE 3. LONG IRON-CONSTANTAN THERMOCOUPLE VOLTAGE DIFFERENCES BETWEEN CONSTANTAN LEADS AT EACH END

Heated by
metal contact

Cooled by 0.5cm-diameter air jet

Cooled by 1.25-amp, 1000-cps current

Cooled by 1.25-amp, 1000-cps current

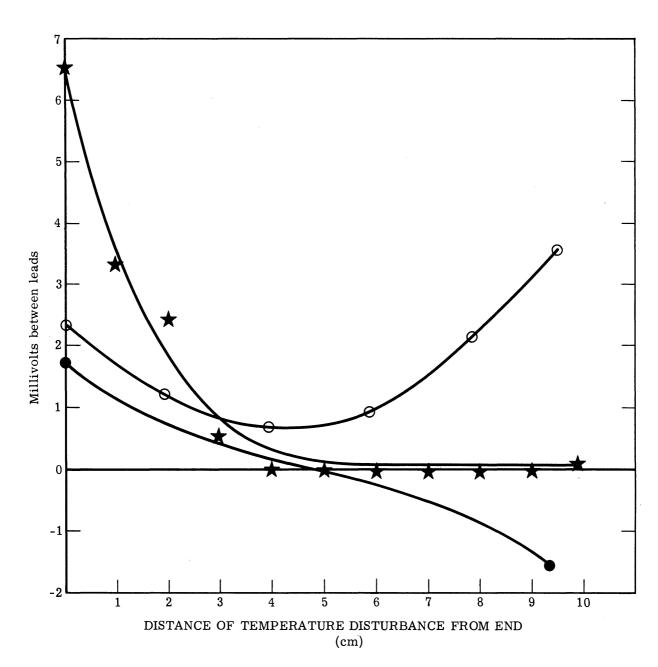


FIGURE 4. LONG IRON-CONSTANTAN THERMOCOUPLE VOLTAGE DIFFERENCES BETWEEN VARIOUS LEADS

- ★ 30 B & S gauge, twisted, unbonded, iron against constantan at end x = 0
- O 24 B & S gauge, twisted, unbonded, iron lead at x = 0, constantan lead at $x = \ell$
- 20 B & S gauge, twisted, unbonded, iron lead at each end

Joule law heating (1.25 amp at a frequency of 1000 cps) was applied across the thermocouple junction. The air jet—0.5 cm in diameter with a velocity of 6.5 x 10^2 cm/sec—was positioned with a micrometer actuated lathe bed.

Figure 3 shows the output across the two constantan end leads as a function of position.

These results are predicted by Equation 7, and indicate that the long thermocouple junction is capable of measuring other than temperature disturbances—in this case a fluid flow disturbance.

3.3. MIXING OF BOILING AND FREEZING WATER

The large thermocouple is particularly adapted to measuring gross temperature balance.

To illustrate such an application, boiling and freezing water were stirred together until temperature equilibrium resulted.

Figure 5 is the reduction of an oscillograph time history of the temperature balance in the mixture. Two constantan end leads of a 30-gauge iron-constantan thermocouple 10 cm long were used. The mixing container was a $12 \times 10 \times 13/4$ -inch plastic tray. Temperature equilibrium was reached in 70 seconds. The temperature variations along the junction were of the same frequency as the mixing strokes.

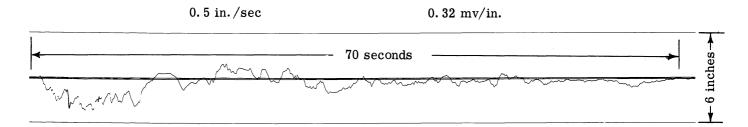


FIGURE 5. OSCILLOGRAPH TIME HISTORY OF LONG THERMOCOUPLE OUTPUT IN MIXING OF BOILING WITH $0^{\rm o}$ C WATER

Thus the degree of temperature balance over a continuum is shown. Its frequency, magnitude, polarity, and damping are indicated from a time history.

3.4. MEASUREMENT OF LIQUID LEVEL

A large thermocouple may be used to locate an interface between two substances. To illustrate this application, the water level in a container was measured by means of a heated long junction. Measurement was possible because the water-immersed portion of the junction cools faster than that remaining in air.

Again, Joule law heating (1.25 amp at a frequency of 1000 cps) was applied to the junction. The heated 30-gauge iron-constantan junction was supported vertically, and the water level was raised and lowered. Also, the results of minute water-level changes were observed. The resolution was limited by air currents in the room.

The results in Figure 6 show that the voltage output is a function of water level. A voltage output of the form $\frac{R_a}{R_a + R_b} E_x [1 - \operatorname{sech} K (\ell - x)]$ is caused when end ab is immersed (see

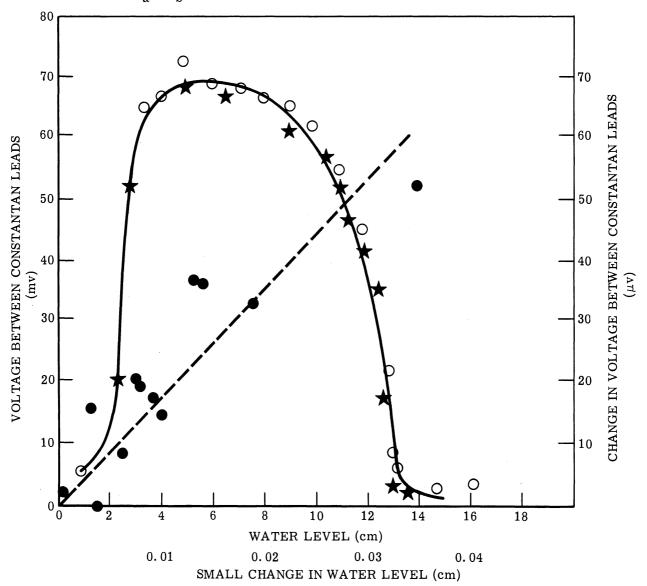


FIGURE 6. LONG IRON-CONSTANTAN THERMOCOUPLE WATER-LEVEL INDICATION

- ____ O Water level increased
- ★ Water level decreased
- ---• Small change in water level (μv changes)

Figure 1). x is the position of the interface. Lead a (Figure 1) always responds as if the temperature disturbance were at x = 0, because the end of the junction is the same temperature as that close to the interface.

3.5. DISCRETE HEAT-SOURCE LOCATION IN TWO DIMENSIONS

In this experiment temperature disturbances in two dimensions were investigated to demonstrate that the first four types of measurements can be extended to a surface from a length. In this case a rectangular plane was employed, but the type of surface does not appear limited. For example, a spherical or cylindrical surface could be employed.

A 1 x 0.9-inch rectangle of 0.001-inch Nichrome was taped to a commercial-grade aluminum plate. Copper-wire connections were taped to each corner of the Nichrome element. A sharp-ened 60-watt soldering-iron point was then placed at the indicated positions (Figure 2). The voltage outputs across the two opposite pairs of corners are recorded in Table I.

TABLE I. VOLTAGES ACROSS THE TWO DIAGONALS OF A 1 x 0.9-INCH NICHROME-ALUMINUM THERMOCOUPLE

	0.9	A:32 B: -128.0	+ 3.85	+ 5.76 - 6.72	+ 12.2	+ 30.8	+ 160.0
	0.8	A: - 4.50 B: -80.0	- 7.05 - 23.2	+ 2.63 - 6.4	+ 12. 2 - 0.64	+ 18.0	+ 128.0 + 3.22
INCHES FROM CORNER	0.6	A: - 1.28 B: + 1.93	+ 0.64 - 12.2	+ 1.28	+ 3.85	+ 8.00	+ 4.50 + 1.24
	0.4	A: - 3.80 B: - 6.73	- 2.30 - 3.80	0 - 5.10	+ 2.30	+ 4.17 + 0.64	+ 9.62 + 2.63
	0.2	A: - 12.8 B: - 0.96	- 10.25 - 3.52	- 4.50 - 0.64	+ 1.93 + 1.28	0 + 3.70	+ 1.93 + 8.66
	0	A: - 48.2 B: + 3.80	- 20. 0	- 10.25 + 3.80	- 11.0 + 9.60	- 4.80 + 27.0	+ 3.20 + 145.0
		0	0.2	0.4	0.6	0.8	1.0

INCHES FROM CORNER

Boxes indicate position of applied heat. A is voltage between 0,0 and 1,0.9. B is voltage between 0,0.9 and 1,0.

This rough experiment indicates that position measurement is possible on a surface as well as on a length. It is thus concluded that a temperature disturbance corresponding to a point on the surface of a large thermocouple can be located by means of voltage residues.

4 DISCUSSION

These preliminary experiments introduce the kinds of measurements possible with the large thermocouple junction. However, they also expose several limitations and difficulties.

Because of the nature of the voltage decrement, linearity is difficult to achieve. Two approaches are possible. First, a relatively low resistance and low conductance in a junction allows the use of only a small portion of the decrement. Thus, as shown in Figure 2, the curve referring to the largest diameter of thermocouple (lowest resistance) is the closest to a linear curve. The second approach is to adjust the resistance or conductance so that the value $(RG)^{1/2}$ in Equation 1 varies with distance x in such a manner that sech $\sqrt{RG}x=1$ - Kx, where K is a constant.

The experiments also indicate that accuracy depends upon the type of junction and the uniformity of heat conduction to the junction. Thus, the twisted and unbonded junctions produce more erratic data points than the straight, soldered junction. The variations in conductivity between the elements and the difficulty of reproducing heater conduction to the twisted wires degrade the data.

Additional limitations are caused by the finite width of the heat source, conduction of heat along the junction, and the difficulty in maintaining constant temperature over the nondisturbed portion of the junction. These provide obvious obstacles to accuracy.

Appendix A VOLTAGE REFERENCE FOR A LONG THERMOCOUPLE

The voltage decrements are unequal in two elements with different resistivities. As distance from the generating point is increased, the voltage in each element approaches asymptotically a voltage level which serves as a reference voltage for the entire length of the junction. At the position of this reference voltage, current flows only perpendicularly from one element to the other. The voltage generated in element A is E_a , and in element B the voltage is E_b . As the reference voltage level is somewhere between E_a and E_b , E_a is positive, and E_b is negative when element A is positive with respect to element B.

The propagation constant for element A is $(R_a^G_a)^{1/2}$, and for element B is $(R_b^G_b)^{1/2}$ As the form of the voltage decrement is the same in each element, the following equality holds at the end of the junction:

$$\operatorname{sech}\left(R_{a}G_{a}\right)^{1/2}x = \operatorname{sech}\left(R_{b}G_{b}\right)^{1/2}x$$

where x is the distance to the temperature disturbance from the junction end.

From this,

$$R_a G_a = R_b G_b \tag{10}$$

The size of the voltage decrement in each element is determined by the magnitudes of \boldsymbol{E}_a and $\boldsymbol{E}_h.$

The change in current along the junction is due to conductance between the elements, and is therefore equal in magnitude in both elements. The current change is the voltage at x divided by the characteristic resistance:

$$dI_{x} = -\frac{E_{a} \operatorname{sech} \left(R_{a} G_{a}\right)^{1/2} x \cosh \left(R_{a} G_{a}\right)^{1/2} x dx}{\left(R_{a} / G_{a}\right)^{1/2}} = \frac{E_{b} \operatorname{sech} \left(R_{b} G_{b}\right)^{1/2} x \cosh \left(R_{b} G_{b}\right)^{1/2} x dx}{\left(R_{b} / G_{b}\right)^{1/2}}$$
(11)

where x is the distance of the temperature disturbance from the measuring end of the junction. The sech function represents the voltage decrease from x to the end of the junction, and the cosh function represents the voltage increase from the end of the junction to the variable distance x.

Through the differentiation of Equation 2 and the use of Equation 1 to remove the hyperbolic functions, it is found that

$$\frac{\left(R_{a}G_{a}\right)^{1/2}}{\left(R_{a}/G_{a}\right)^{1/2}}E_{a} = -\frac{\left(R_{b}G_{b}\right)^{1/2}}{\left(R_{b}/G_{b}\right)^{1/2}}E_{b}$$

and therefore,

$$\frac{E_a}{E_b} = -\frac{G_b}{G_a} = -\frac{R_a}{R_b}$$

As the generated voltage is

$$E_x = E_a - E_b = E_a \left(1 + \frac{R_b}{R_a} \right)$$

then

$$E_a = \frac{R_a}{R_a + R_b} E_x$$

and

$$E_b = -\frac{R_b}{R_a + R_b} E_x$$

Appendix B VOLTAGE DECREMENT FOR A LARGE FLAT THERMOCOUPLE

The voltage decrement between two infinite plates of resistance r per square and conductance g per unit area is derived as follows.

An annulus; with center at the voltage source position, with radial thickness Δx , and average circumference $2\pi x$, is considered. The resistance from the inner to the outer edge of the annulus is $r\Delta x/2\pi x$, giving a voltage drop of $\Delta E = -I(r\Delta x/2\pi x)$; so that, in the limit, $dE/dx = -Ir/2\pi x$, where I is current from the inner to the outer edge of the annulus. The conductance is $g\Delta x 2\pi x$, giving current between the plates of $\Delta I = Eg\Delta x 2\pi x$ in the annulus; in the limit, this becomes

$$\frac{dI}{dx} = - Eg2\pi x$$

as

$$x \frac{dE}{dx} = -\frac{Ir}{2\pi}$$

$$\frac{d}{dx}\left(x\frac{dE}{dx}\right) = -\frac{dI}{dx}\frac{r}{2\pi} = Rg xE$$

and

$$\frac{d^2E}{dx^2} + \frac{1}{x}\frac{dE}{dx} - rgE = 0.$$

Let

$$x' = i\sqrt{rg}x$$

$$dx = \frac{dx'}{i\sqrt{rg}}$$

where $i = \sqrt{-1}$. Then

$$-\operatorname{rg}\frac{d^{2}E}{dx'^{2}}-\frac{\operatorname{rg}dE}{x'dx'}-\operatorname{rg}E=0$$

When the common factor (- rg) is removed, the solutions of this equation are Bessel functions.

The solution for x large is found by the real part of the Hankel functions.

$$AH_0^{(1)} + BH_0^{(2)}$$

The real part of $H_0^{(1)}$ is

$$\begin{split} \mathrm{iH_0}^{(1)} &= \left[\mathrm{iJ_0}(\mathrm{i}\,\sqrt{\mathrm{rg}}\mathrm{x}) \,+\, \mathrm{iN_0}(\mathrm{i}\,\sqrt{\mathrm{rg}}\mathrm{x})\,\right] \\ &\simeq \mathrm{i}\left[\sqrt{\frac{2}{\pi\sqrt{\mathrm{rg}}\mathrm{i}\mathrm{x}}}\,\exp\,\left(\mathrm{ii}\,\sqrt{\mathrm{rg}}\mathrm{x}\right)\,\exp\,-\frac{\mathrm{i}\pi}{4}\right] \\ &\simeq \mathrm{i}\left[\sqrt{\frac{2}{\pi\sqrt{\mathrm{rg}}\mathrm{x}}}\exp\,\left(-\sqrt{\mathrm{rg}}\mathrm{x}\right)\,\frac{\exp\left(-\mathrm{i}\frac{\pi}{4}\right)}{\sqrt{\mathrm{i}}}\right] \end{split}$$

as

$$\frac{\exp\left(i\frac{\pi}{4}\right)}{\sqrt{i}} = \frac{1}{i}$$

$$iH_0^{(1)} \simeq \sqrt{\frac{2}{\pi \sqrt{rgx}}} \exp(-\sqrt{rgx})$$

and as $H_0^{(2)} = J_0(i\sqrt{rg}x) - iN_0(i\sqrt{rg}x)$ diverges, B = 0, and $A = E_0$. Therefore, $E_x = E_0\sqrt{\frac{2}{\pi\sqrt{rg}x}} \exp(-\sqrt{rg}x)$, for x large.

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