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Final Report

WAVE HINDCASTS VS. RECORDED WAVES

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TABLE OF CONTENTS

	Page
LIST OF TABLES	v
LIST OF FIGURES	vii
ABSTRACT	ix
1. INTRODUCTION	1
2. WAVE HINDCASTING METHODS	2
3. WAVE HINDCASTS AT THE LAKE MICHIGAN RESEARCH TOWER	9
A. General Considerations	9
B. Wind Data	10
C. Wave Data	13
D. Results	14
4. SUMMARY AND CONCLUSIONS	20
REFERENCES	21
ACKNOWLEDGMENTS	22
APPENDIX A. AUGUST DATA AND RESULTS	23
APPENDIX B. SEPTEMBER DATA AND RESULTS	29
APPENDIX C. OBSERVED SPECTRA	37

LIST OF TABLES

Table	Page
1 Neumann and Pierson-Moskowitz Spectra	5
2 Correlation Coefficient Comparison of Wind Data	12
3 Student's t Comparison of Wind Data	12
A.1 Wind Conditions in August Hindcast Period	24
A.2 Hindcasts for August Hindcast Period	25
A.3 Comparison of Hindcast and Observed Wave Values for August Hindcast Period	27
B.1 Wind Conditions in September Hindcast Period	30
B.2 Hindcasts for September Hindcast Period	31
B.3 Comparison of Hindcast and Observed Significant Heights for September Hindcast Period	33
B.4 Comparison of Hindcast and Observed Periods of Maximum Amplitude for September Hindcast Period	34
B.5 Comparison of Hindcast and Observed Largest Wave in September Hindcast Period	35

LIST OF FIGURES

Figure		Page
1.	Wave hindcasting flow chart.	8
2.	Calibration curve for spectra.	15
3.	Location of Lake Michigan Research Tower.	16
4.	Hindcast and observed significant heights: August.	18
5.	Hindcast and observed significant heights: September.	19
C.1.	Observed spectra: August.	38
C.2.	Observed spectra: September.	46
C.3.	Correction for hydrodynamical attenuation.	53

ABSTRACT

Wave heights at the Lake Michigan Research Tower are hindcast for periods in August and September of 1964. The hindcast is compared with wave heights measured by the U. S. Lake Survey. Hindcasts made with the Neumann energy spectrum prove to be superior to those made with the Pierson-Moskowitz spectrum and are in satisfactory agreement with observations. Results are presented in both tabular and graphical form.

1. INTRODUCTION

The research program "Wave Hindcasts vs Recorded Waves", Contract DA-20-064-CIVENG-65-6, calls for wave hindcasts at the Lake Michigan Research Tower for the month of October, 1964, and for the comparison of these hindcasts with wave heights recorded by the U.S. Lake Survey. The collapse of the tower during the storm of September 23-24 required a change in the hindcast period to August 1 through August 10 and September 13 through September 23, during which times necessary data was available. This report presents the results of the work carried out under the above contract.

2. WAVE HINDCASTING METHODS

The statistical theory of water waves was set forth in the early post-war years by a number of authors and has been the subject of review articles by Pierson (1955) and Longuet-Higgins (1962). As the theory is well known, only major results will be presented here.

Under the assumption that ocean waves are essentially random in character, the sea surface $\zeta(t)$ at a fixed point in space is represented by the sum

$$\zeta(t) = \sum_n a_n \cos(\mu_n t + \epsilon_n) , \quad (1)$$

where the phases ϵ_n are chosen at random. The sum of the squares of the amplitudes in a small element of frequency $d\mu$ is

$$\sum_{d\mu} a_n^2 = A^2(\mu) d\mu , \quad (2)$$

$A^2(\mu)$ being the energy spectrum as defined by Pierson (op.cit.). The number E , defined by

$$E = \int_0^{\infty} A(\mu) d\mu , \quad (3)$$

is related to ζ by

$$E = \sum_n a_n^2 = 2 \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^{\tau} \zeta^2(t) dt . \quad (4)$$

Thus $(E/2)^{1/2}$ is the root mean square wave height.

The properties of a process such as that given by equation (1) were investigated by Rice (1944, 1945) in connection with the problem of noise in electric circuits. Rice showed that if the frequency band of ζ is relatively narrow, the probability $\underline{p}(r)dr$ that the wave amplitude a (one half the crest to trough height) lies between r and $r + dr$ is

$$\underline{p}(r)dr = \frac{2r}{E} e^{-r^2/E} dr \quad (5)$$

and that the expected number $\epsilon(n)$ of waves occurring per unit time is

$$\epsilon(n) = \frac{1}{2\pi} \left\{ \frac{1}{E} \int_0^{\infty} \mu^2 A^2(\mu) d\mu \right\}^{1/2} \quad (6)$$

Longuet-Higgins (1952) showed that the probability distribution (5) can be used to find the mean amplitude of the highest pN waves in a record with N waves, that this amplitude is

$$a^{(p)} = \sqrt{E} \left\{ \left(\ln \frac{1}{p} \right)^{\frac{1}{2}} + \frac{1}{p} \int_{\left(\ln \frac{1}{p} \right)^{\frac{1}{2}}}^{\infty} e^{-\theta^2} d\theta \right\}, \quad (7)$$

and that for large N the expected value $\epsilon(a_{\max})$ of the largest wave amplitude is

$$\epsilon(a_{\max}) = \sqrt{E} \left\{ \left(\ln N \right)^{\frac{1}{2}} + .289 \left(\ln N \right)^{-\frac{1}{2}} \right\}. \quad (8)$$

An approximate result for the crest to trough height H is obtained by doubling (7) and (8). The mean height \bar{H} is

$$\bar{H} = 2 \times a^{(1)} = 1.77 \sqrt{E} \quad (9a)$$

while

$$H_{1/3} = 2.83 \sqrt{E} \quad (9b)$$

$$H_{1/10} = 3.60 \sqrt{E} \quad (9c)$$

$$\epsilon(H_{\max}) = 2 \sqrt{E} \left\{ \left(\ln N \right)^{\frac{1}{2}} + .289 \left(\ln N \right)^{-\frac{1}{2}} \right\}. \quad (9d)$$

Equation (9d) is especially useful in conjunction with (6), which relates the frequency of waves to the wave spectrum. The results given in equation (9) were compared with observations by Longuet-Higgins (1952) and proved to be quite accurate.

In order to forecast waves, the energy spectrum must be related to causal factors, such as the wind. A number of authors have proposed spectra for fully developed seas, i.e., wave systems arising from constant or slowly changing winds

with large fetch and duration. Two such spectra are given in Table 1. In each the spectrum is related to the wind speed at only one elevation, the anemometer height of the research vessel.

The Neumann spectrum was one of the first to be developed and has been used by Pierson, Neumann, and James in their wave forecasting manual, H.O. Pub. No. 603, Practical Methods for Observing and Forecasting Ocean Waves. The dependence of wave height on the 2.5 power of velocity is a somewhat controversial feature of this spectrum; many other scientists believe the height to be proportional to the square of the velocity. Pierson (1964) has shown that consideration of the variation of the wind speed with elevation brings the Neumann, Pierson-Moskowitz, and other spectra into better agreement, though the results definitely diverge at high wind speeds. It is still not clear which of the proposed spectra is most nearly correct.

An energy spectrum derived for a fully developed sea in which all frequencies are present can be adapted for use in seas which are not fully developed. The predominant opinion is that in such cases the spectrum is modified by application of a filter which only allows a certain range of frequencies to be present. One such filter is used in cases in which the sea is either fetch or duration limited. For these cases there is an upper period T_u such that $A^2 = 0$ for $T > T_u$, or $\mu < 2\pi/T_u$. In H.O. Pub. 603 curves are plotted which show the effect of this type of filter on wave heights. For a given wind speed there may be limitations due to either the fetch or the duration, with an empirically obtained T_u for each case. It is recommended that when waves are both fetch and duration limited the lower value of T_u is used, so that

$$E = \int_{\frac{2\pi}{T_u}}^{\infty} A^2(\mu) d\mu \quad (10)$$

is the smaller of the two possible values.

A second type of filter must be used when the wind has stopped or decreased greatly. If F is the distance from an observation point to the windward edge of the fetch and t is the time after the wind shift, waves propagating with the deep water group velocity

$$C_g = \left(\frac{g}{4\pi}\right) T$$

from the windward edge of the fetch reach the observation point at

$$t = F/C_g$$

TABLE 1

NEUMANN AND PIERSON-MOSKOWITZ SPECTRA

Neumann Spectrum (Pierson, 1955)

$$A^2(\mu) = \frac{C}{\mu^6} e^{-2(g/\mu V)^2},$$

where $C = 51.7 \text{ ft}^2 \text{ sec}^{-5}$ and

$V =$ wind speed at 7.5 m ;

$$E = \int_0^{\infty} A^2(\mu) d\mu = C \frac{\sqrt{3}\pi}{2\sqrt{2}} \left(\frac{V}{g}\right)^5 ;$$

$$\frac{H_1}{3} = 2.83 \sqrt{E} = 1.39 \left(\frac{V}{10}\right)^{2.5} \text{ ft.},$$

where V is measured in knots ;

$$\begin{aligned} \bar{T} &= \text{Average period} = 1/\epsilon(n) \\ &= \sqrt{3} \pi (V/g) = 2.85 (V/10) \text{ sec}, \end{aligned}$$

where V is measured in knots;

$$\begin{aligned} T_m &= \text{Period of maximum amplitude of } A^2 \\ &= \sqrt{6} \pi (V/g) = 4.03 \left(\frac{V}{10}\right) \text{ sec}, \end{aligned}$$

where V is measured in knots.

Pierson-Moskowitz Spectrum (Pierson and Moskowitz, 1964)

$$A^2(\mu) = \frac{\alpha g^2}{\mu^5} e^{-\beta(g/\mu U)^4},$$

where $\alpha = .0162$, $\beta = 0.74$, and

$U =$ wind speed at 19.5 m ;

$$E = \int_0^{\infty} A^2(\mu) d\mu = \frac{\alpha}{4\beta} U^4/g^2$$

$$\frac{H_1}{3} = 2.83 \sqrt{E} = 1.82 \left(\frac{U}{10}\right)^2 \text{ ft.},$$

where U is measured in knots;

$$\begin{aligned} T &= \text{Average period} = 1/\epsilon(n) \\ &= \frac{2\pi}{(\pi\beta)^{1/4}} U/g = 2.67 \left(\frac{U}{10}\right) \text{ sec}, \end{aligned}$$

where U is measured in knots;

$$\begin{aligned} T_m &= \text{Period of maximum amplitude of } A^2 \\ &= 2\pi \left(\frac{5}{4\beta}\right)^{1/4} U/g = 3.75 \left(\frac{U}{10}\right) \text{ sec}, \end{aligned}$$

where U is measured in knots.

Hence in this case

$$T_u = (4\pi/g)(F/t)$$

since waves with higher periods have already passed the observation point. Therefore the decay of sea into swell is described by the law

$$E = \int_{gt/2F}^{\infty} A^2(\mu) d\mu. \quad (11)$$

A number of filters applying to other situations are discussed in H.O. Pub. 603. They each serve to provide a period band which can then be used in conjunction with the energy spectrum to calculate E and the desired wave heights and periods.

The process of wave prediction can thus be summarized as follows:

- (1) A wind field is obtained either from isobar analysis or from reported ship winds. These winds must be corrected by empirical rules to provide the wind at whatever elevation is called for by the spectrum to be used.

(2) The fetch and duration are estimated. If the sea is fetch or duration limited, an empirical filter is applied to cut out the lower frequencies. E can then be calculated for either fully developed or fetch or duration limited seas.

(3) The past history of the wind system is studied to see if swell is present. If so, the contribution to E from swell is added on to the contribution from the local sea.

(4) The composite spectrum of sea plus swell can be used to calculate the average period, the period range in which most (say 9/10) of the energy is located, and the period of maximum energy.

(5) The composite value of E is used to calculate the various average heights. The expected maximum amplitude in a record of any given duration can also be calculated.

A flow chart illustrating the method is shown in Figure 1.

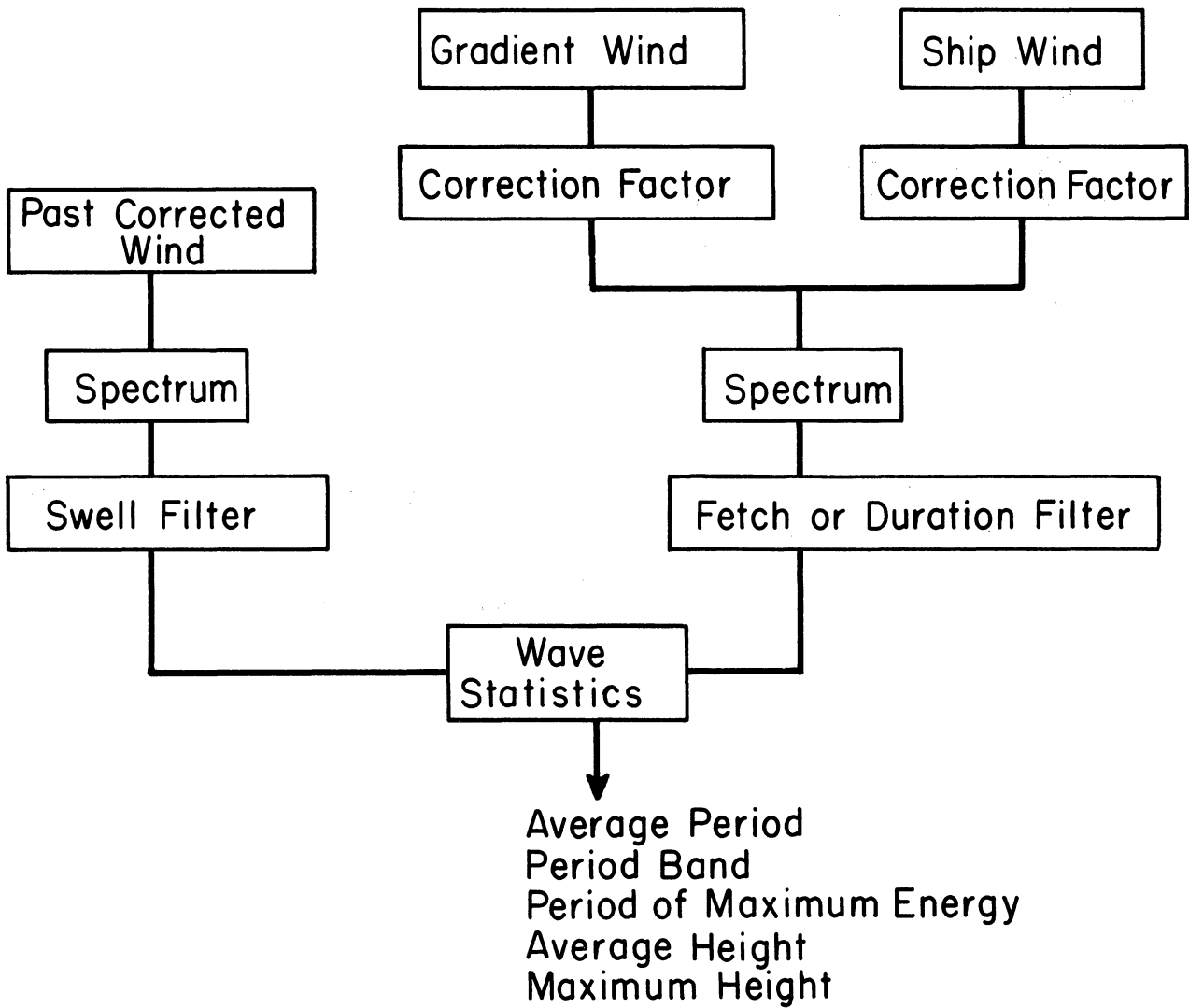


Figure 1. Wave hindcasting flow chart.

3. WAVE HINDCASTS AT THE LAKE MICHIGAN RESEARCH TOWER

A. GENERAL CONSIDERATIONS

The wave climatology of Lake Michigan is affected by its relatively small dimensions, about 275 nautical miles in length and 100 nautical miles in breadth, with a maximum depth of 923 feet and a mean depth of 276 feet (University of Michigan, 1963). The oceans, of course, are much larger and deeper. Since wind generated waves are fetch limited, with the minimum fetch necessary for a fully developed sea an increasing function of wind velocity, waves generated by strong winds have lower amplitudes on the lake than they would have for the same winds blowing over the oceans. The periods and wavelengths are also lower than in the oceans.

An example of this effect is the case of a 44 knot wind, which for a fetch of 1000 nautical miles would produce waves with a significant height ($H_{1/3}$) of 56.5 feet, an average period of 12.5 seconds, and an average wave length of 533 feet. The same wind blowing along a Milwaukee to Muskegon line, with a fetch of 100 nautical miles, would produce waves at Muskegon with a significant height of only 18 feet, an average period of 6.6 seconds, and an average wavelength of 210 feet.

Depth effects are less important than might be expected. One's first inclination is to suppose that waves in Lake Michigan might have to be treated as shallow rather than deep water waves. In order to treat this possibility, an idealized problem was worked. This consisted of supposing that long crested waves of period T and deep water amplitude a_0 advance into shallow water of depth h with depth contours parallel to the wave crests. The object is to calculate the shallow water wave amplitude a . This problem ignores the effects of wave ray convergence but is still useful in deciding if the deep water theory is adequate.

The result of the calculation is that the amplitude ratio (a/a_0) as a function of the non-dimensional number gT^2/h is approximately constant and equal to unity for $gT^2/h \leq 10$, decreases to 0.91 at $gT^2/h = 41$ and then increases, coming back to unity at $gT^2/h = 100$. As $h \rightarrow 0$,

$$(a/a_0) \rightarrow \frac{1}{2\sqrt{\pi}} \left(\frac{gT^2}{h} \right)^{1/4},$$

and the wave breaks.

Of interest to the present study is the fact that for $h = 50$ feet, the water depth at the Lake Michigan Research Tower, $gT^2/h = 100$ corresponds to

$T = 12.5$ seconds. Since higher wave periods are seldom if ever found in Lake Michigan, the amplitude ratio is affected only slightly by bottom effects. By contrast, the higher period waves found in the oceans would be strongly affected by bottom effects in water of depth 50 feet. There may also be frictional and percolative effects, but not enough is known about these to make a quantitative estimate of how these modify wave heights.

In summary, the small horizontal dimensions of Lake Michigan reduce wave heights and periods, but in so reducing the periods they diminish the importance of the small vertical dimension. Hence the waves may be treated as deep water waves except near the shores.

B. WIND DATA

For predictions made during the August period there were four possible sources of wind data: (1) measurements at the research tower, a mile offshore at Muskegon, (2) reported winds from ships, (3) corrected gradient winds, and (4) winds measured at shore weather stations. Wind measurements were measured at shore weather stations. Wind measurements were not made at the tower during the September period due to instrument malfunctions.

The tower data consists of winds measured at 16, 8, 4, and 2 meters above the water surface. The 16 meter wind was used for hindcasts made with the Pierson-Moscowitz spectrum and the 8 meter wind for hindcasts with the Neumann spectrum, since the speeds should be close to those at 19.5 and 7.5 meters respectively.

The ship winds were measured at elevations of from 16 to 22 meters and were used without correction with the Pierson-Moscowitz spectrum. For use with the Neumann spectrum they were multiplied by the ratio V/U as determined from tower data taken in 1963 and 1964 (Elder, 1965), and the result was rounded off to the nearest knot. The scatter in this data was reduced considerably by separating the measurements into stability classes, according to the value of $\Delta T = (T_{\text{water}} - T_{\text{air}})$. The ratios as computed from Elder's data are

$$\begin{aligned} \frac{V}{U} &= \frac{\text{Wind speed at 7.5 m.}}{\text{Wind speed at 19.5 m.}} \\ &= .85 \quad \Delta T < -5^\circ\text{F} \\ &= .95 \quad -5^\circ\text{F} < \Delta T < +5^\circ\text{F} \\ &= 1.00 \quad \Delta T > +5^\circ\text{F} \end{aligned} \tag{12}$$

Since most of the ships report ΔT as well as the wind at anemometer height, equation (12) is applied without difficulty.

The value V/U for neutral conditions, 0.95, is slightly larger than that given by other authors (c.f. Pierson, 1964) for the range of speeds measured by Elder, 5 to 25 knots, but is accurate enough for the purposes of the present study. Even under the assumption that the other ratios in (12) are correct, it is at best a crude approximation to assume that the wind speed at only one elevation provides all the information necessary for relating the waves to the wind. However, this is the best assumption one can make at the present time.

The method used to find the wind speed from isobar analysis consisted of re-analyzing the maps provided by the Weather Bureau by plotting isobars at 1 mb. intervals, calculating the gradient wind using standard methods, and then correcting the gradient wind to its value in the boundary layer over the water. Mr. Al Strong of the University of Michigan Great Lakes Research Division has gathered data relating the gradient wind to the wind as measured at anemometer height on ships and has separated the data according to stability classes. Strong's best straight line fit of his data together with the ratios given in (12) yield

$$\begin{aligned}
 V &= \text{wind speed at 7.5 m in knots} \\
 &= 7.9 + .28 V_g \quad \Delta T < -5^\circ\text{F} \\
 &= 9.5 + .27 V_g \quad -5^\circ\text{F} < \Delta T < +5^\circ\text{F} \\
 &= 13.1 + .31 V_g \quad \Delta T > 5^\circ\text{F} ,
 \end{aligned}
 \tag{13}$$

where V_g is the gradient wind in knots. According to Strong, these results are most reliable when the gradient wind is greater than 10 knots.

The fourth source of data is shore weather stations. It is unclear a priori how representative winds measured at shore are of conditions at sea.

In order to test the validity of the sources of wind data, a number of correlation coefficients were computed and are given in Table 2. The sample in each case represents those occasions in the period August 1 through August 10 when the indicated comparison could be made, and the quantity compared is the wind at 7.5 m.

The correlation coefficients exhibited in Table 2 indicate that ship and corrected gradient winds leave something to be desired as sources of data, but are still acceptable. They also seem to be superior in accuracy to the Muskegon weather station winds. In order to test the latter conclusion, the null hypothesis was made that the ship winds and Muskegon weather station winds are equally valid for use at the research tower and the hypothesis was tested using Student's t distribution. Similar tests were made comparing the corrected gradient winds and the weather station winds, and also the squares of the wind speeds. This latter comparison was made because it is the square or some higher power of V which actually enters into the calculation of wave heights. The results are given in Table 3.

TABLE 2

CORRELATION COEFFICIENT COMPARISON OF WIND DATA

Correlation coefficient of	Sample size	Correlation coefficient
1. V_{grad} and V_S	23	0.72
2. V_M and V_S	23	0.18
3. V_T and V_S	7	0.72
4. V_{grad} and V_T	13	0.69
5. V_M and V_T	13	0.23

V_{grad} = corrected gradient wind
 V_S = ship wind
 V_M = Muskegon weather station wind
 V_T = research tower wind

TABLE 3

STUDENT'S t COMPARISON OF WIND DATA

Quantities Compared	t	Degrees of Freedom	Probability of t Occuring as a result of Chance
1. V_S and V_M	1.35	6	.25
2. V_{grad} and V_M	.79	12	.45
3. $(V_S)^2$ and $(V_M)^2$	1.32	6	.25
4. $(V_{grad})^2$ and $(V_M)^2$	1.25	12	.25

The probabilities in the last column are not low enough to state definitely that the weather station winds are inferior, but there is an inference in that direction. This is true despite the fact that the Muskegon weather station is so close to the research tower that if winds at sea are ever the same as shore winds, they are in this case. It was therefore decided to reject winds measured at shore weather stations for use in making hindcasts.

C. WAVE DATA

Wave heights were recorded during the August period by a pressure sensor maintained by the U. S. Lake Survey. During the September period a staff gage was also in operation. The output from these systems was analyzed by personnel of the U. S. Army Coastal Research and Engineering Laboratories using the electronic analyzer developed by the Beach Erosion Board. The properties of the analyzer have been described by Caldwell and Williams (1961) and its use in computing the spectra of waves from Hurricane Donna by Bretschneider (1961). The spectral curves give the frequency distribution of linear average and square average wave heights taken over a twenty minute period with a filter band width of 0.027 cps.

Thirty-eight such spectral curves have been put at the author's disposal by the Lake Survey. Of these, fourteen represent analyses of waves recorded by the pressure sensor in August, twelve of waves recorded at the same times in September by the staff gage. In addition, continuous wave records from both systems were available.

This data was used in the following way. The variance of the staff gage wave record was computed using University of Michigan facilities, the result being a continuous record of the variance for the preceding twenty minutes of real time. Since the various average wave heights are proportional to the square root of the variance, the computation in effect provides a continuous record of the significant height and the other average wave heights. This record was used as the source of observed wave heights for the September period, while the spectral curves provide information on the periods of the waves. It should be noted that analysis of the staff gage record indicated no appreciable set-up.

The wave record measured by the pressure sensor cannot be analyzed so simply because these waves experience a frequency-dependent hydrodynamical attenuation. However, spectra computed from the pressure sensor's output can be corrected simply for this effect. Accordingly, it was decided to calibrate the spectra computed from pressure sensor data and to use these spectra as the source of observed wave heights for the August period.

The calibration was effected by correcting the spectra for attenuation effects and then numerically computing for each curve the integral over frequency of linear average wave height less linear noise. The significant wave height should be proportional to this integral (Bretschneider, op.cit.).

The significant height as computed from the variance of the staff gage record was then plotted against integrated linear average and the points were fit using the method of least squares. One spectral curve was unusable for this purpose.

The resultant calibration curve, shown in Figure 2 and given analytically by

$$H_{\frac{1}{3}} = (17.1) \int (\text{Linear Average}) d\left(\frac{1}{T}\right) + 1.3 \text{ ft.}, \quad (14)$$

intercepts the $H_{1/3}$ axis at a positive value. This may be due to lack of response by the pressure sensor to low amplitude waves or to nonlinear calibration for low amplitudes.

D. RESULTS

The sources of wind data described in Section 3-B were used in making hindcasts for the periods August 1 - August 10 and September 13 - September 23 of 1964. For each synoptic time during the hindcast periods gradient winds and reported ship winds were entered directly onto a map of Lake Michigan. These winds were then corrected to provide winds at the desired anemometer heights. In each case a fetch was estimated by eye and the winds were then averaged over the fetch. When winds from the research tower were available these were used in preference to the gradient and ship winds, and hindcasts were made at the time of measurement of tower winds rather than at synoptic times. Forty hindcasts were made during the August period and forty-five during the September period. The location of the research tower is shown in Figure 3.

The wind conditions in August are given in Table A.1 in the Appendix. The winds during this period were generally light, from ten to twenty knots, and often were offshore. One would thus expect small amplitude waves.

The results of the hindcast are given in Table A.2. The quantities tabulated are the significant height, the period band, and the period at which the theoretical spectra reach their maximum amplitude. For cases in which the waves are fetch or duration limited, only predictions from the Neumann spectrum are given. For the range of wind speeds observed during this period higher waves are hindcast when using the Pierson-Moskowitz spectrum.

In Table A.3 the hindcast significant heights and periods of maximum amplitude are tabulated along with the observed values taken from the spectra shown in Figure C.1. Both of the hindcast methods predict higher wave heights than those observed, with the waves hindcast using the Neumann spectrum in better agreement. For the thirteen occasions on which the observed spectra could be integrated to obtain the significant height, the average observed

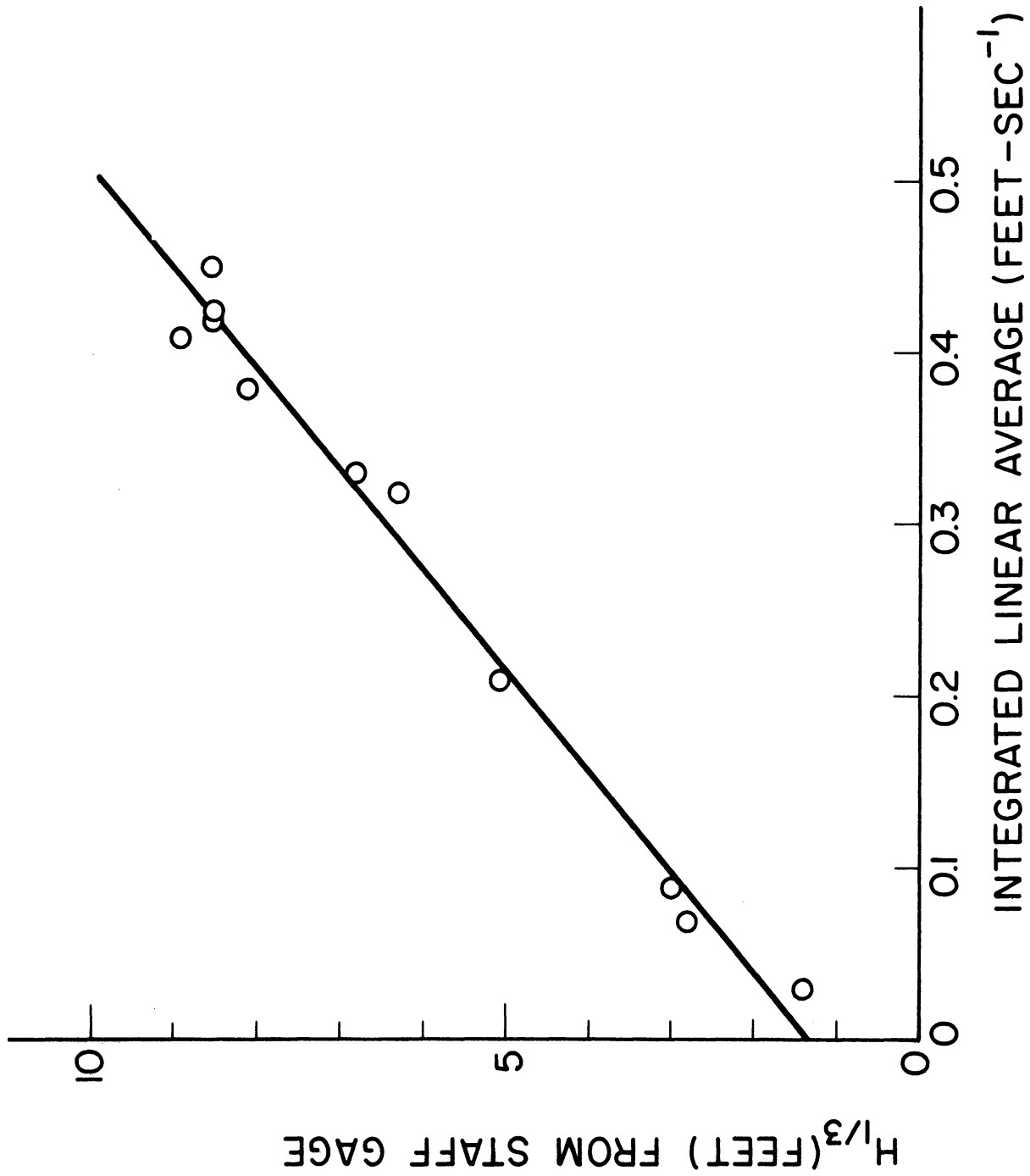


Figure 2. Calibration curve for spectra.

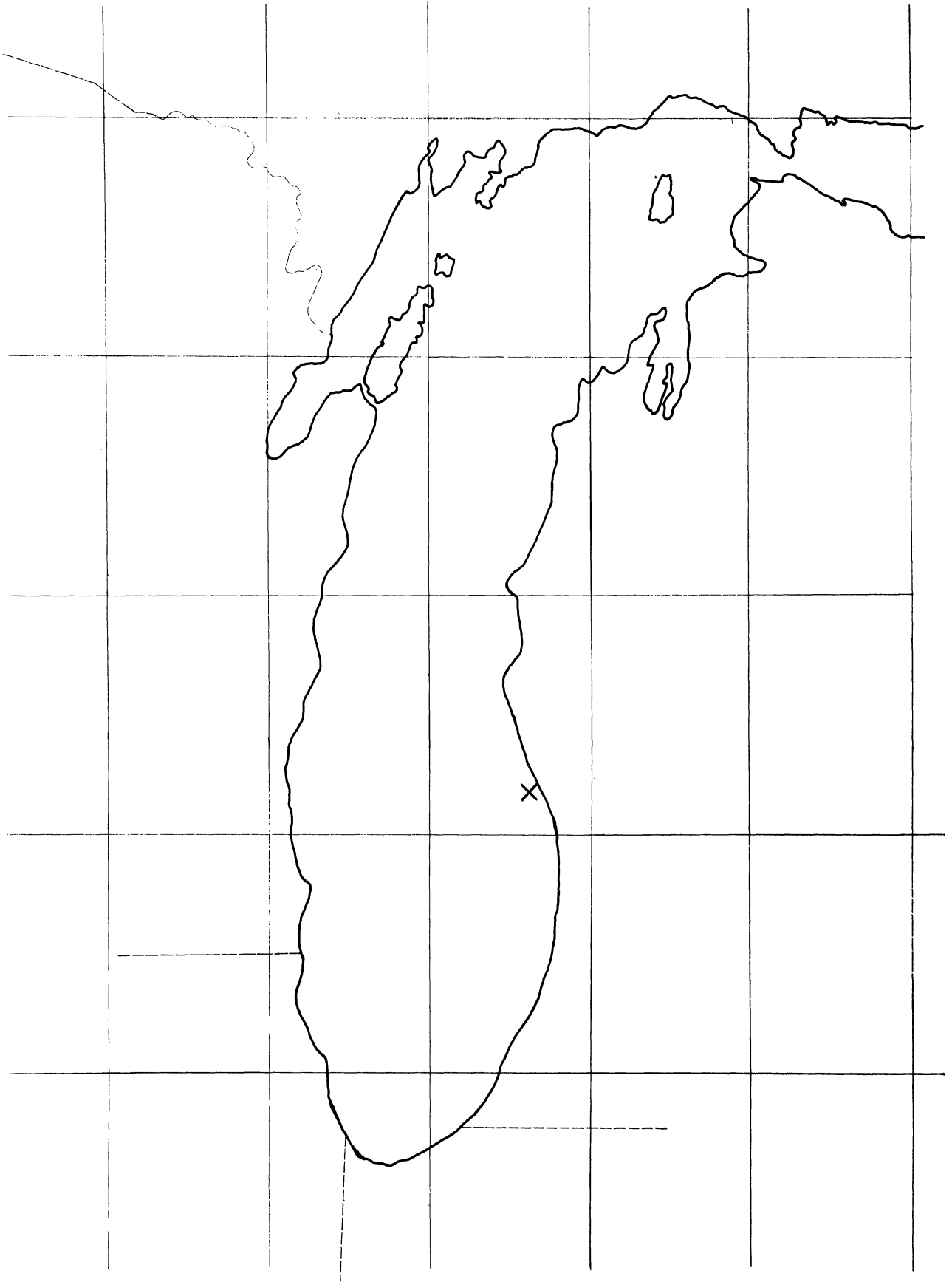


Figure 3. Location of Lake Michigan Research Tower.

significant height was 1.7 feet and the average significant height hindcast using the Neumann spectrum 2.5 feet.

The hindcast and observed significant heights are plotted as functions of time in Figure 4. Given the variability of the wave heights, it appears that the agreement is fair.

The September period was somewhat more interesting because of the stronger winds and accompanying higher waves. Fortunately, it was possible to make a more detailed comparison of hindcast and observed wave heights during this period.

The wind conditions in September are given in Table B.1 in the Appendix. At the start of the hindcast period the sea was clam. The winds gradually freshened, reached a peak on the morning of the 14th, and then shifted and became offshore. From early afternoon of the 14th to the morning of the 20th the winds were either weak or offshore. During the next few days the winds increased and then decreased. In the early hours of the 23rd a front passed over the lake and the winds rapidly increased, reaching a peak of 39 knots in the evening. At midnight the research tower collapsed.

The result of the hindcast are given in Table B.2. As in the August period, higher waves are forecast when using the Pierson-Moskowitz spectrum.

Hindcast significant wave heights and observed significant heights are given in Table B.3, and wave heights hindcast with the Neumann spectrum and observed wave heights in Figure 5. Except for the period of light and/or offshore winds, the agreement is excellent. The decay of sea into swell starting at noon on the 14th was followed closely by the hindcast, as was the development of the sea during the storm on the 23rd. On a number of occasions the hindcast waves developed more rapidly than observed; presumably this was due to overestimating the duration of the wind system.

The periods of maximum amplitude as interpolated from the hindcasts and those taken from the spectra shown in Figure C.2 are tabulated in Table B.4. The agreement is good here also.

Another point of interest is the maximum expected wave. This was hindcast for the first two days and last four days of the hindcast period and the results compared with observed maximum waves taken directly from the wave record. The values are given in Table B.5. The excellent agreement serves as confirmation of the statistical theory.

In passing it should be noted that the waves which caused the collapse of the tower were fetch and duration limited and had period bands much narrower than would ordinarily be expected for winds of this magnitude. Whether or not this concentration of energy in a narrow period band was responsible for the tower's collapse is a moot point.

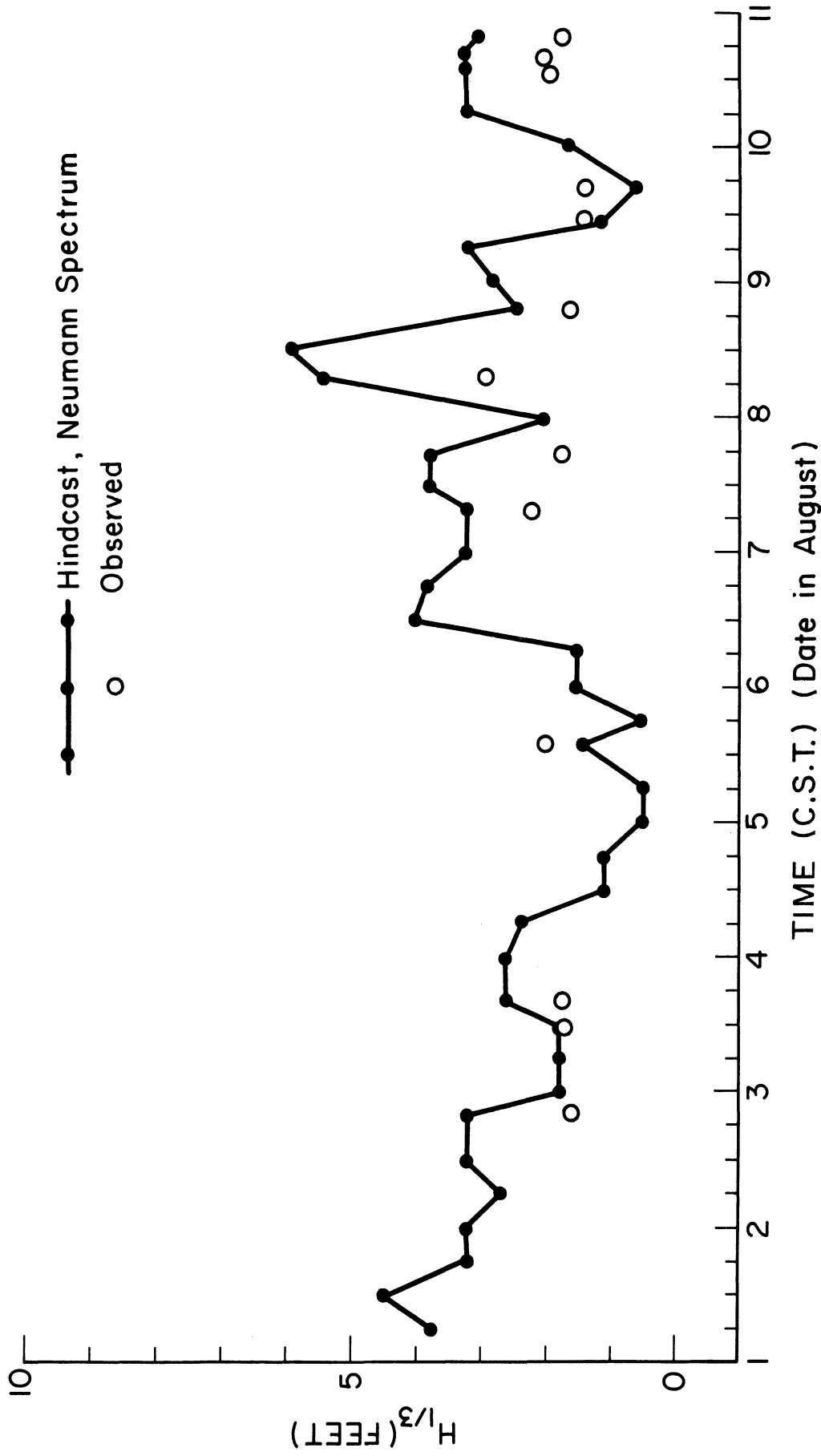


Figure 4. Hindcast and observed significant heights: August.

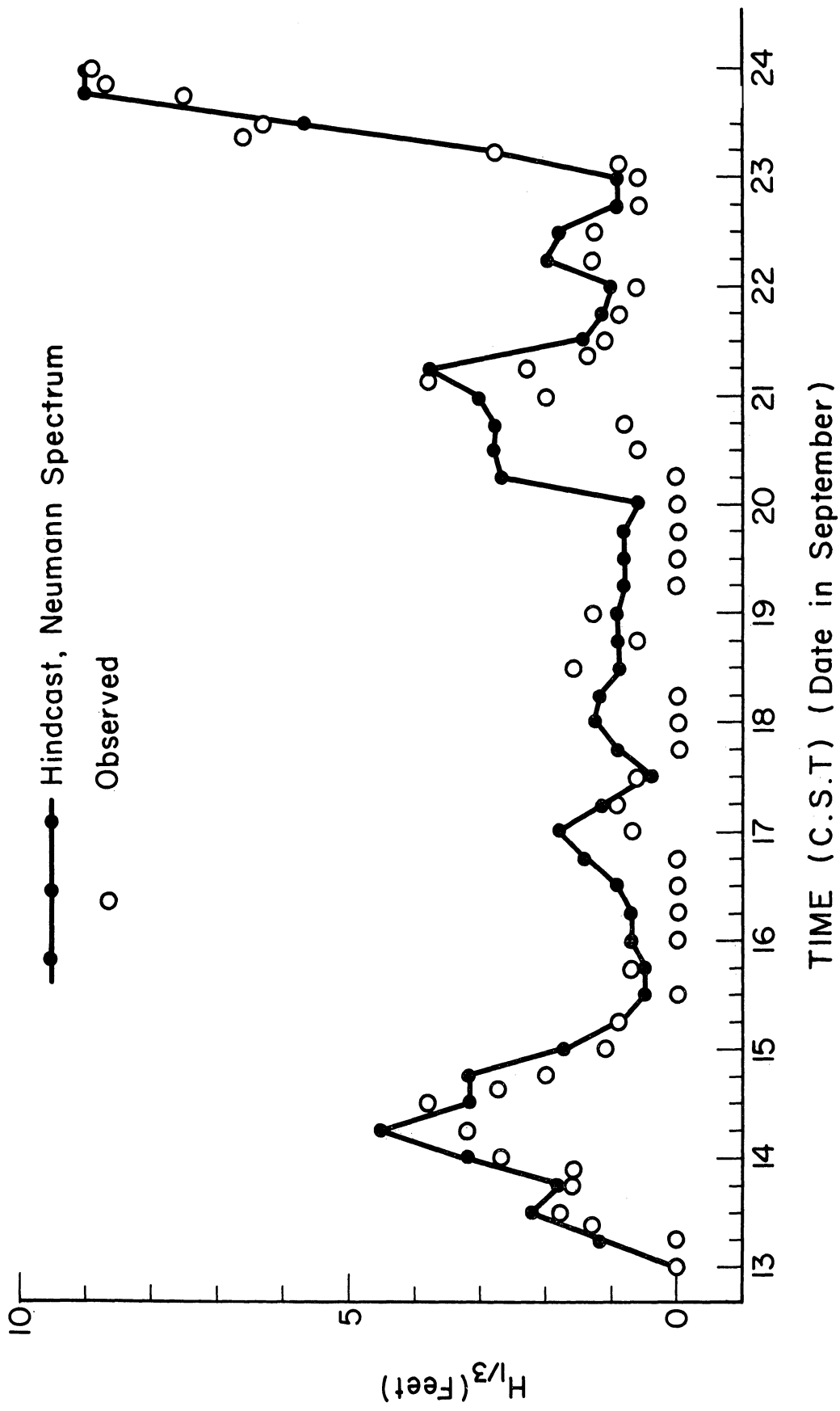


Figure 5. Hindcast and observed significant heights: September.

4. SUMMARY AND CONCLUSIONS

Hindcasts for the August period were made using corrected gradient winds, ship winds, and winds measured at the Lake Michigan Research Tower as input to the Neumann and Pierson-Moskowitz spectra. Due to the relatively small number of measurements at the research tower, the availability of tower data was less useful than had been anticipated.

An examination of the sources of wind data reveals that the corrected gradient winds are more representative of conditions at sea than winds measured at shore weather stations. The sample on which this conclusion is based is fairly small, and a comprehensive study is needed on this matter. It might be possible to average the winds reported by all weather stations around the lake to get representative values. Such a procedure would be useful, since a large amount of labor is involved in computing the gradient winds.

If reported ship winds are available these are definitely preferred. It is advisable to correct the ship winds to the anemometer height called for by whatever energy spectrum is used. Since this correction depends on the stability, as does the correction for gradient winds, the air-sea temperature difference should be obtained whenever possible.

For the thirteen occasions in August on which hindcast and observed waves could be compared, the average significant height hindcast using the Neumann spectrum was 2.5 feet and the average observed significant height 1.7 feet. The correlation coefficient between observed and hindcast significant heights was 0.75. The agreement for this hindcast period is fair. For the forty-five occasions in September on which comparisons were made, the average significant height hindcast using the Neumann spectrum was 2 feet and the average observed significant height, 1.4 feet. However, during this period there was much better agreement when the waves were large, particularly during the storm preceding the collapse of the tower. This is reflected by a high value of the correlation coefficient, 0.96. The agreement between observed waves and waves hindcast using the Pierson-Moscowitz spectra was not as good.

In view of the above facts, it appears that corrected gradient and ship winds should be used in preference to winds measured at shore weather stations. It also appears that acceptable hindcasts can be made using the Neumann spectrum and the procedures given in H.O. Pub. 603.

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APPENDIX A

AUGUST DATA AND RESULTS

TABLE A.1

WIND CONDITIONS IN AUGUST HINDCAST PERIOD

V = Wind speed at 7.5 meters

U = Wind speed at 19.5 meters

Time(C.S.T.)	V(Knots)	U(knots)	<(°)	Time(C.S.T.)	V(Knots)	U(Knots)	<(°)
1:0600	15	16	19-20	6:0600	18	19	15
1:1200	16	17	20	6:1200	18	19	16-20
1:1800	14	15	19-22	6:1800	15	16	18-23
2:0000	10	12	18-21	7:0000	14	15	20-25
2:0600	13	15	23	7:0726	10.9	12.9	21
2:1200	14	16	25	7:1200	15	16	28-33
2:1926	9.8	11.3	23-25	7:1736	14.8	16	30
3:0000	11	13	26	8:0000	10	10	00
3:0600	9	11	32	8:0740	17.6	17.7	30
3:1136	6.7	7.7	30	8:1130	21.5	22.4	35-03
3:1636	12.8	15	32	8:2030	Calm		
4:0000	8	9	03	9:0000	14	15	32
4:0600	9	10	31-33	9:0600	Calm		
4:1200	9	9	34	9:1130	9.4	10.7	35
4:1800	5	6	03-10	9:1630	7.1	8.7	35
5:0000	17	18	02-08	10:0000	14	15	15-18
5:0600	15	16	02	10:0600	14	15	20
5:1406	16.6	17.5	35-04	10:1330	6.2	7.3(est.)	20
5:1800	15	16	04-05	10:1530	6.8	8.0(est.)	16
6:0000	14	15	15	10:1950	8.8	10.4(est.)	12

TABLE A.2

HINDCASTS FOR AUGUST HINDCAST PERIOD

$H_1/3$ = Significant height

P.B. = Period band containing 90% of wave energy

T_m = Period of maximum amplitude of energy spectrum

Abbreviations for sea conditions: FD, fully developed, FL, fetch limited,
DL, duration limited, Sw, swell.

Time (C.S.T.)	Sea	$H_1/3$ (ft.)	Hindcasts (Neumann Spectrum) P.B.(sec.)	T_m (sec.)	Hindcasts (Pierson-Moskowitz Spectrum) $H_1/3$ (ft.)	P.B.(sec.)	T_m (sec.)
1:0600	FD	3.8	1.8-8.3	6.1	4.7	2.4-7.5	6.0
1:1200	FD	4.5	2.0-8.8	6.5	5.3	2.5-8.0	6.4
1:1800	FD	3.2	1.5-7.8	5.7	4.1	2.2-7.0	5.6
2:0000	Sw	3.2	1.5-7.8	5.7	4.1	2.2-7.0	5.6
2:0600	FD	2.7	1.2-7.4	5.3	4.1	2.2-7.0	5.6
2:1200	FD	3.2	1.5-7.8	5.7	4.7	2.4-7.5	6.0
2:1926	Sw	3.2	1.5-7.8	5.7	4.7	2.4-7.5	6.0
3:0000	FD	1.8	1.0-6.5	4.4	3.1	1.6-5.2	4.1
3:0600	Sw	1.8	1.0-6.5	4.4	3.1	1.6-5.2	4.1
3:1136	Sw	1.8	1.0-6.5	4.4	3.1	1.6-5.2	4.1
3:1636	FD	2.6	1.2-7.4	5.2	4.1	2.2-7.0	5.6
4:0000	Sw	2.6	1.2-7.4	5.2	4.1	2.2-7.0	5.6
4:0600	Sw	2.4	1.2-5.5	5.5	3.4	2.2-5.5	5.5
4:1200	FD	1.1	1.0-4.0	3.6	1.5	1.3-4.2	3.6
4:1800	Sw	1.1	1.0-4.0	3.6	1.5	1.3-4.2	3.6
5:0000	FL	0.5	2.3-2.5	2.5			
5:0600	FL	0.5	1.5-2.5	2.5			
5:1406	FL	1.4	2.0-3.0	3.0			
5:1800	FL	0.5	1.4-2.5	2.5			
6:0000	FL	1.5	1.2-4.5	4.5			

TABLE A.2 (Concluded)

Time (C.S.T.)	Sea	Hindcasts (Neumann Spectrum)		Hindcasts (Pierson-Moskowitz Spectrum)	
		$H_1/3$ (ft.)	P.B.(sec.)	$H_1/3$ (ft.)	P.B.(sec.)
			T_m (sec.)		T_m (sec.)
6:0600	FL	1.5	2.5-3.0		3.0
6:1200	FL	4.0	2.5-5.8		5.8
6:1800	FD	3.8	1.8-8.3	4.7	2.2-7.0
7:0000	FD	3.2	1.5-7.8	4.1	2.2-7.0
7:0726	Sw	3.2	1.5-7.8	4.1	2.2-7.0
7:1200	FD	3.8	1.8-8.3	4.7	2.4-7.5
7:1736	FD	3.8	1.8-8.3	4.7	2.4-7.5
8:0000	Sw	2.0	1.8-4.2	2.7	2.4-4.2
8:0740	DL	5.4	2.4-9.0		
8:1130	Sw	5.9	2.4-9.8		
8:1930	Sw	2.4	2.4-4.0	2.4	2.8-4.0
9:0000	DL	2.8	1.5-5.8		
9:0600	Sw	3.2	1.5-7.8	4.1	2.2-7.0
9:1130	FL	1.1	1.0-3.8		
9:1360	FD	0.6	1.0-4.0	1.4	1.3-4.1
10:0000	FL	1.6	1.5-7.8		
10:0600	FD	3.2	1.5-7.8	4.1	2.2-7.0
10:1330	Sw	3.2	1.5-7.8	4.1	2.2-7.0
10:1530	Sw	3.2	1.5-7.8	4.1	2.2-7.0
10:1950	Sw	3.0	1.5-6.8	4.0	2.2-6.8

TABLE A.3

COMPARISON OF HINDCAST AND OBSERVED WAVE VALUES FOR AUGUST HINDCAST PERIOD

 $H_1/3$ = Significant height T_m = Period of maximum amplitude of energy spectrum

Time (C.S.T.)	Hindcast (Neumann Spectrum) $H_1/3$ (ft.)	T_m (sec.)	Hindcast (Pierson-Moskowitz Spectrum) $H_1/3$ (ft.)	T_m (sec.)	Observed $H_1/3$ (ft.)	T_m (sec.)
2:1926	3.2	5.7	4.7	6.0	1.6	4.2
3:1136	1.8	4.4	3.1	4.1	1.7	4.2
3:1636	2.6	5.2	4.1	5.6	1.7	3.6
5:1406	1.4	3.0			2.0	3.6
7:0726	3.2	5.7	4.1	5.6	2.2	4.6
7:1736	3.8	6.1	4.7	6.0	1.7	4.6
8:0740	5.4	7.1			2.9	4.6
8:1930	2.4	4.0	2.4	4.0	1.6	5.0
9:1130	1.1	3.8			1.4	6.2
9:1630	0.6	2.9	1.4	3.3	1.4	5.6
10:1330	3.2	5.7	4.1	5.6	1.9	3.8
10:1530	3.2	5.7	4.1	5.6	2.0	4.2
10:1950	3.0	5.7	4.0	5.6	1.7	5.0

APPENDIX B

SEPTEMBER DATA AND RESULTS

TABLE B.1

WIND CONDITIONS IN SEPTEMBER HINDCAST PERIOD

V = Wind speed at 7.5 meters

U = Wind speed at 19.5 meters

Time(C.S.T.)	V(Knots)	U(Knots)	<(°)	Time(C.S.T.)	V(Knots)	U(Knots)	<(°)
13:0000		Calm		18:1800	17	18	12-15
13:0600	10	10	23-27	19:0000	15	16	13-14
13:1200	12	13	18-25	19:0600	10	11	12-15
13:1800	11	13	21-23	19:1200	8	9	13-18
14:0000	14	15	21-24	19:1800	10	11	12-14
14:0600	16	17	23-25	20:0000	8	9	13-14
14:1200	14	15	27	20:0600	13	14	17-20
14:1800	21	22	03-06	20:1200	15	16	16
15:0000	21	21	05-09	20:1800	14	15	17
15:0600	18	18	07-09	21:0000	16	18	16
15:1200	14	15	07-09	21:0600	15	16	18-20
15:1800	13	14	03-05	21:1200	10	11	17-18
16:0000	12	12	13-18	21:1800	9	10	12-19
16:0600	12	13	12-15	22:0000	9	9	19-20
16:1200	9	9	15-18	22:0600	13	14	30-34
16:1800	10	11	16-19	22:1200	11	12	27-32
17:0000	11	11	18-23	22:1800	12	13	11-13
17:0600	9	10	15-25	23:0000	12	13	14
17:1200	6	7	18	23:0600	23	24	25-30
17:1800	14	15	14	23:1200	19	21	24
18:0000	15	16	15	23:1800	37	39	26
18:0600	13	14	15	24:0000	24	26	27
18:1200	17	18	12-15				

TABLE B.2

HINDCASTS FOR SEPTEMBER HINDCAST PERIOD

$H_1/3$ = Significant Height
 P.B. = Period band containing 90% of wave energy
 T_m = Period of maximum amplitude of energy spectrum

Abbreviations for sea conditions: FD, fully developed, FL, fetch limited,
 DL, duration limited, Sw, swell

Time(C.S.T.)	Sea	Hindcasts (Neumann Spectrum)		Hindcasts (Pierson-Moskowitz Spectrum)			
		$H_1/3$ (ft.)	P.B.(sec.)	T_m (sec.)	$H_1/3$ (ft.)	P.B.(sec.)	T_m (sec.)
13:0000		0			0		
13:0600	FD	1.2	1.0-6.0	3.8	1.6	1.4-4.4	3.5
13:1200	FD	2.2	1.0-7.0	4.8	3.1	1.9-6.1	4.9
13:1800	FD	1.8	1.0-6.5	4.4	3.1	1.9-6.1	4.9
14:0000	FD	3.2	1.5-7.8	5.3	4.1	2.2-7.0	5.6
14:0600	FD	4.5	2.0-8.8	6.5	5.3	2.5-8.0	6.4
14:1200	FD	3.2	1.5-7.8	5.7	4.1	2.2-7.0	5.6
14:1800	Sw	3.2	1.5-7.8	5.7	4.1	2.2-7.0	5.6
15:0000	Sw	1.7	1.5-3.9	3.9	2.0	2.2-3.9	3.9
15:0600	Sw	.9	1.5-2.6	2.6	.9	2.2-2.6	2.6
15:1200	FL	.5	1.5-2.0	2.0			
15:1800	FL	.5	1.2-2.0	2.0			
16:0000	FL	.7	1.0-2.4	2.4			
16:0600	FL	.7	1.0-2.4	2.4			
16:1200	FL	.9	1.0-3.8	3.8			
16:1800	FD	1.4	1.0-6.0	4.0	2.0	1.5-5.0	4.4
17:0000	FD	1.8	1.0-6.5	4.4	2.2	1.6-5.2	4.1
17:0600	FD	1.1	1.0-5.5	3.6	1.6	1.4-4.4	3.0
17:1200	FD	.4	.8-3.0	2.4	.9	.9-2.8	2.3
17:1800	FL	.9	1.5-2.5	2.5			
18:0000	FL	1.3	1.8-3.0	3.0			
18:0600	FL	1.2	1.2-3.0	3.0			

TABLE B.2 (Concluded)

Time(C.S.T.)	Sea	Hindcasts (Neumann Spectrum)		Hindcasts (Pierson-Moskowitz Spectrum)	
		$H_1/3$ (ft.)	P.B.(sec.)	$H_1/3$ (ft.)	P.B.(sec.)
			T_m (sec.)		T_m (sec.)
18:1200	FL	.9	2.3-2.5		
18:1800	FL	.9	2.3-2.5		
19:0000	FL	.9	1.8-2.5		
19:0600	FL	.8	1.0-2.5		
19:1200	FD	.8	.9-3.5	1.5	1.3-4.2
19:1800	FL	.8	1.0-2.5		3.4
20:0000	FL	.6	.9-2.4		
20:0600	FD	2.7	1.0-7.0	3.6	1.9-6.1
20:1200	FL	2.8	1.8-5.5		4.9
20:1800	FL	2.8	1.7-5.9		
21:0000	FL	3.0	2.0-5.0		
21:0600	FD	3.8	1.8-8.3	4.7	2.4-7.5
21:1200	FD	1.4	1.0-6.0	2.2	1.6-5.2
21:1800	FD	1.1	1.0-6.0	1.6	1.4-4.4
22:0000	FD	1.0	1.0-6.0	1.5	1.3-4.2
22:0600	DL	2.0	1.2-4.8		
22:1200	FL	1.8	1.0-6.5		
22:1800	FL	.9	1.0-2.5		
23:0000	FL	.9	1.0-2.5		
23:0600	DL	2.8	3.5-4.0		
23:1200	DL	5.7	2.8-7.0		
23:1800	FL	9.0	5.8-6.0		
24:0000	FL	9.0	3.7-8.5		

TABLE B.3

COMPARISON OF HINDCAST AND OBSERVED SIGNIFICANT HEIGHTS FOR
SEPTEMBER HINDCAST PERIOD $H_{1/3}$ = Significant height

N: Neumann spectrum

P-M: Pierson-Moskowitz spectrum

O: Observed

Time (C.S.T.)	$H_{1/3}$ (ft.)			Time (C.S.T.)	$H_{1/3}$ (ft.)		
	N	P-M	O		N	P-M	O
13:0000	0	0	0	19:0600	.8		0
13:0600	1.2	1.6	0	19:1200	.8	1.5	0
13:0900			1.3	19:1800	.8		0
13:1200	2.2	3.1	1.8	20:0000	.6		0
13:1800	1.8	3.1	1.6	20:0600	2.7	3.6	0
13:2100			1.6	20:1200	2.8		.6
14:0000	3.2	4.1	2.7	20:1800	2.8		.8
14:0600	4.5	5.3	3.2	21:0000	3.0		2.0
14:1200	3.2	4.1	3.8	21:0300			3.8
14:1500			2.7	21:0600	3.8	4.7	2.3
14:1800	3.2	4.1	2.0	21:0900			1.4
15:0000	1.7	2.0	1.1	21:1200	1.4	2.2	1.1
15:0600	.9	.9	.9	21:1800	1.1	1.6	.9
15:1200	.5		0	22:0000	1.0	1.5	.6
15:1800	.5		.7	22:0600	2.0		1.3
16:0000	.7		0	22:1200	1.8		1.3
16:0600	.7		0	22:1800	.9		.6
16:1200	.9		0	23:0000	.9		.6
16:1800	1.4	2.0	0	23:0300			.9
17:0000	1.8	2.2	.7	23:0600	2.8		2.8
17:0600	1.1	1.6	.9	23:0900			6.6
17:1200	.4	.9	.6	23:1200	5.7		6.3
17:1800	.9		0	23:1800	9.0		7.5
18:0000	1.3		0	23:2100			8.7
18:0600	1.2		0	23:2330			8.9
18:1200	.9		1.6	24:0000	9.0		
18:1800	.9		.6				
19:0000	.9		1.3				

TABLE B.4

COMPARISON OF HINDCAST AND OBSERVED VALUES OF PERIOD OF
MAXIMUM AMPLITUDE FOR SEPTEMBER HINDCAST PERIOD

T_m = Period of maximum amplitude of energy spectrum
 N: Neumann spectrum
 P-M: Pierson-Moskowitz spectrum
 O: Observed

Time(C.S.T.)	T_m (sec.)		
	N	P-M	O
21:0100	5.2		4.8
21:0500	5.9		5.6
21:0900	5.4		4.8
21:1400	3.9		4.9
23:0700	4.5		5.7
23:1100	6.5		7.0
23:1500	6.5		7.8
23:1900	6.4		8.2
23:2000	6.8		8.3
23:2100	7.3		8.6
23:2200	7.7		8.7
23:2300	8.1		8.9

TABLE B.5

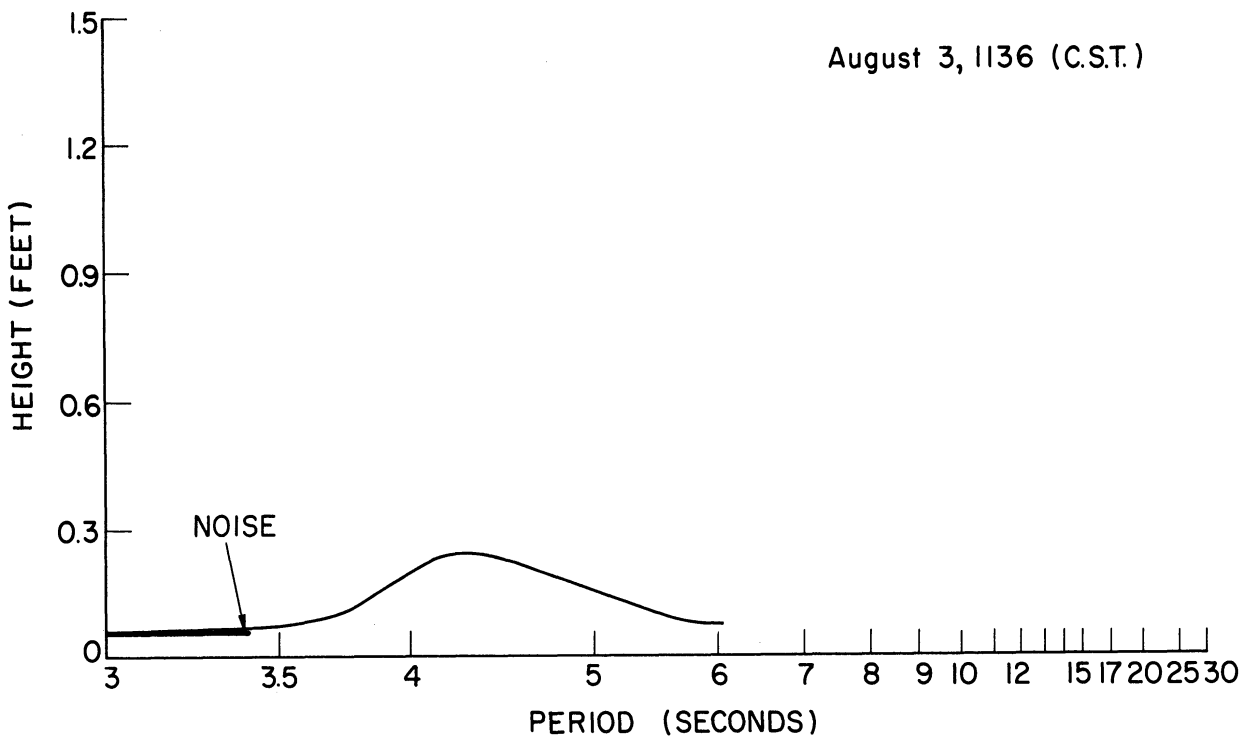
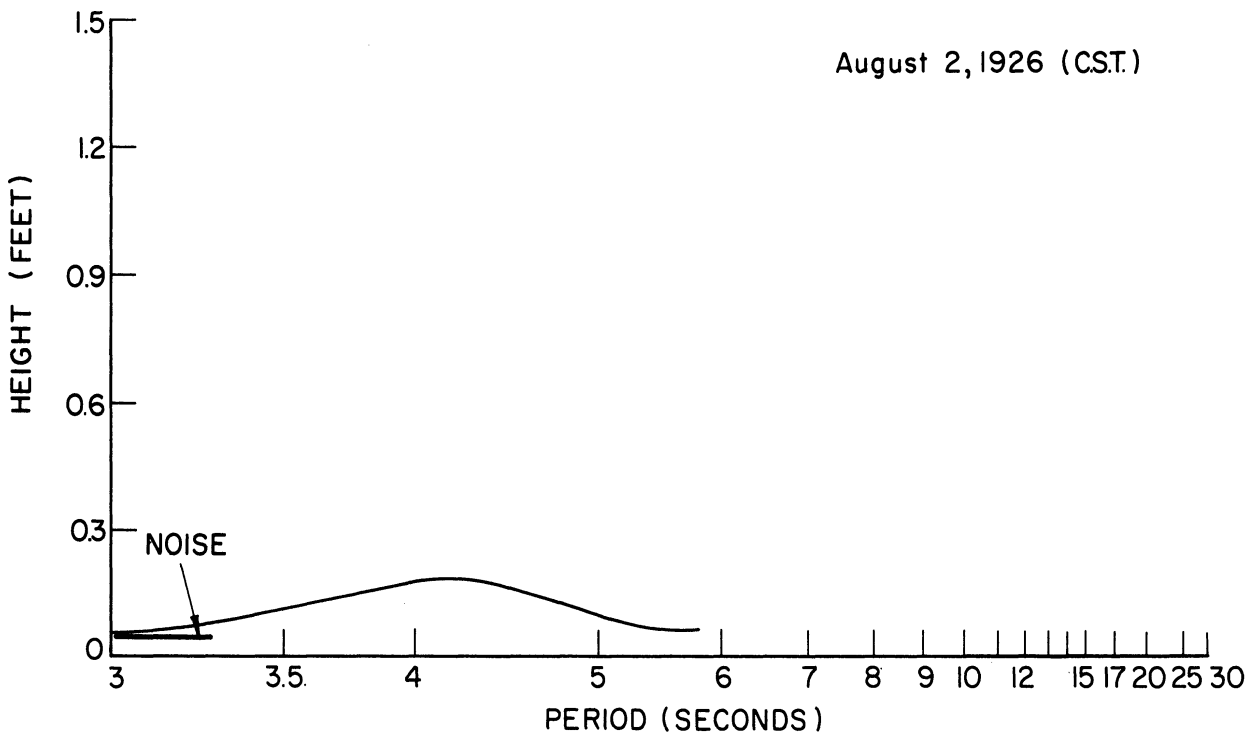
COMPARISON OF HINDCAST AND OBSERVED LARGEST WAVE IN SEPTEMBER
HINDCAST PERIOD

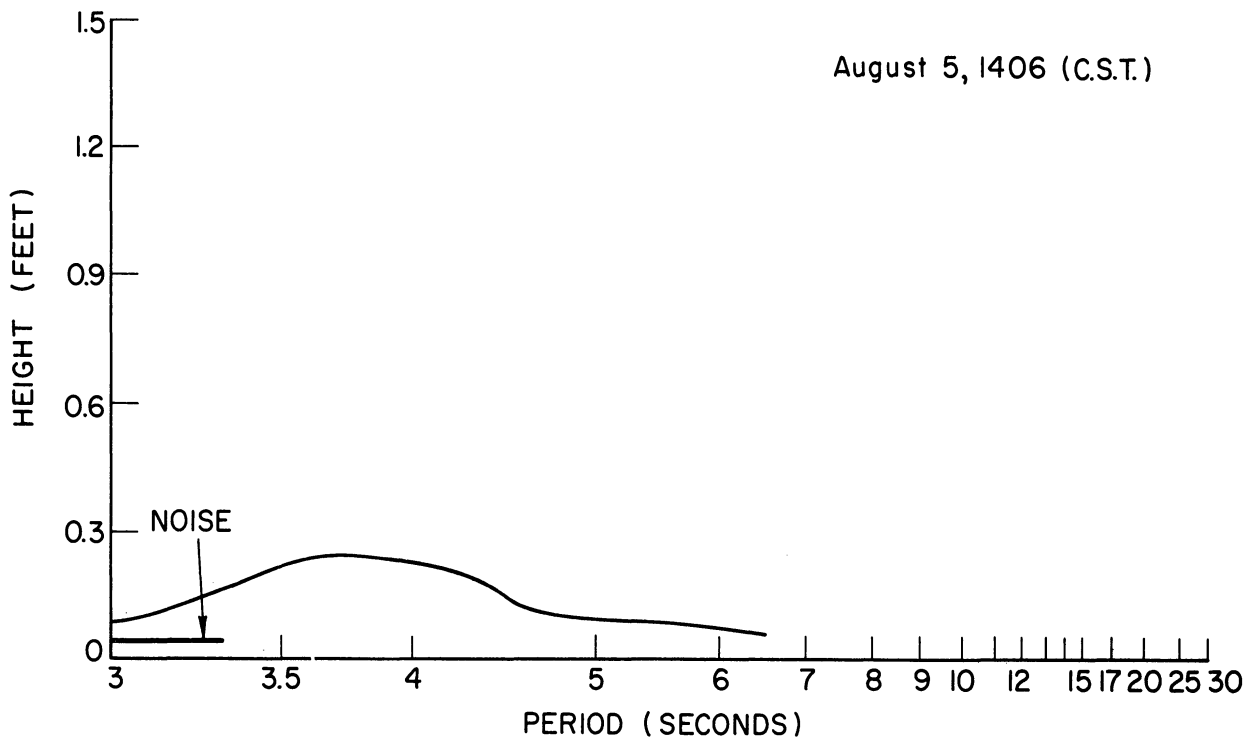
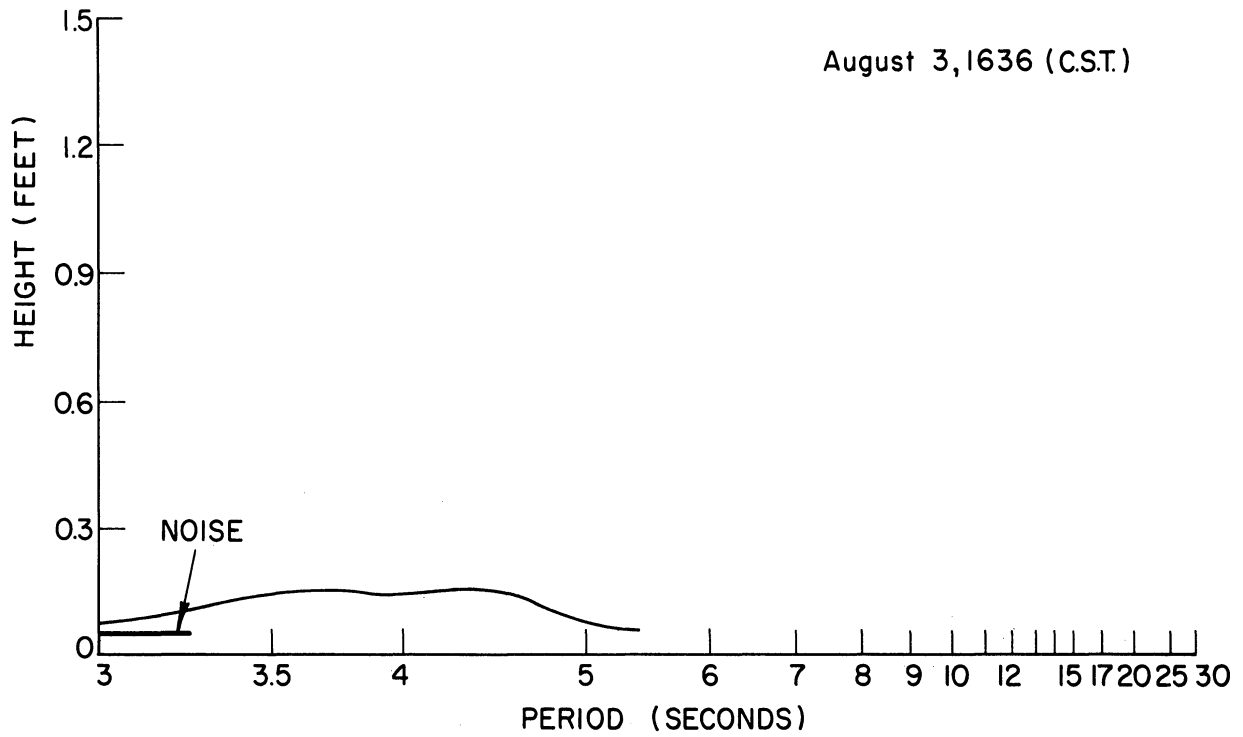
H_m = Largest wave in twenty minute period preceding given time
 N: Neumann spectrum
 O: Observed

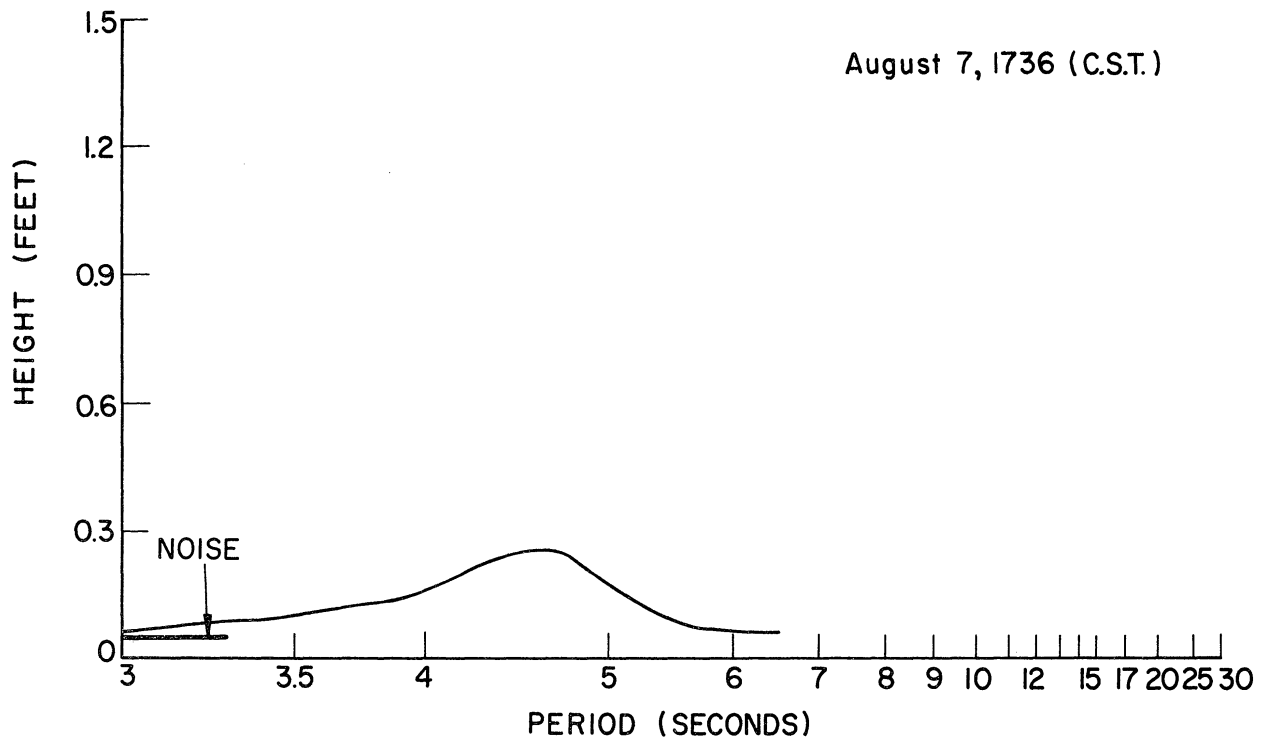
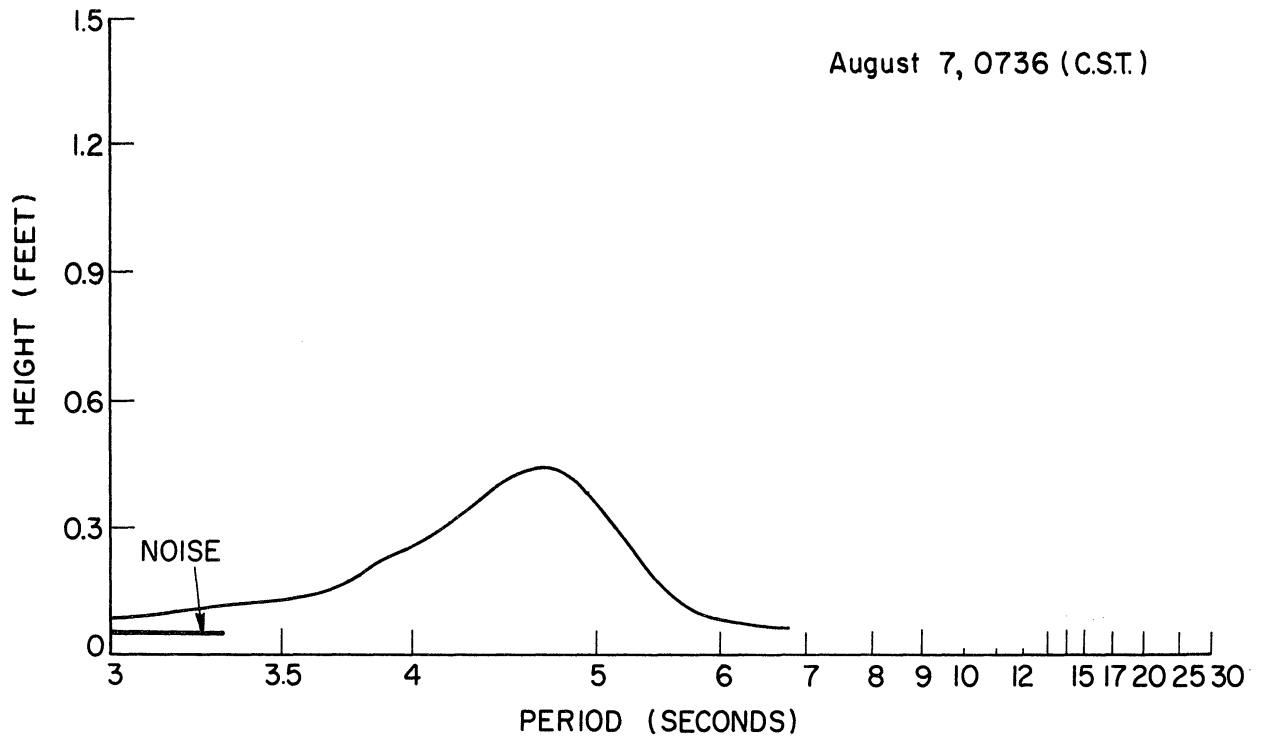
Time (C.S.T.)	H_m (feet)	
	N	O
13:1200	3.9	3.8
13:1800	3.2	3.5
14:0000	5.7	5.0
14:0600	7.8	6.4
14:1200	5.7	7.1
14:1800	5.7	3.8
15:0000	3.1	2.2
15:0600	1.4	2.2
20:1200	5.0	1.4
20:1800	5.0	1.5
21:0000	5.4	3.4
21:0600	6.7	4.9
21:1200	2.5	2.3
21:1800	2.0	1.9
22:0000	1.8	.8
22:0600	3.6	2.6
22:1200	3.2	2.6
22:1800	1.6	1.9
23:0000	1.6	1.1
23:0600	5.1	5.2
23:1200	10.1	11.2
23:1800	14.0	14.2
24:0000	15.9	15.7

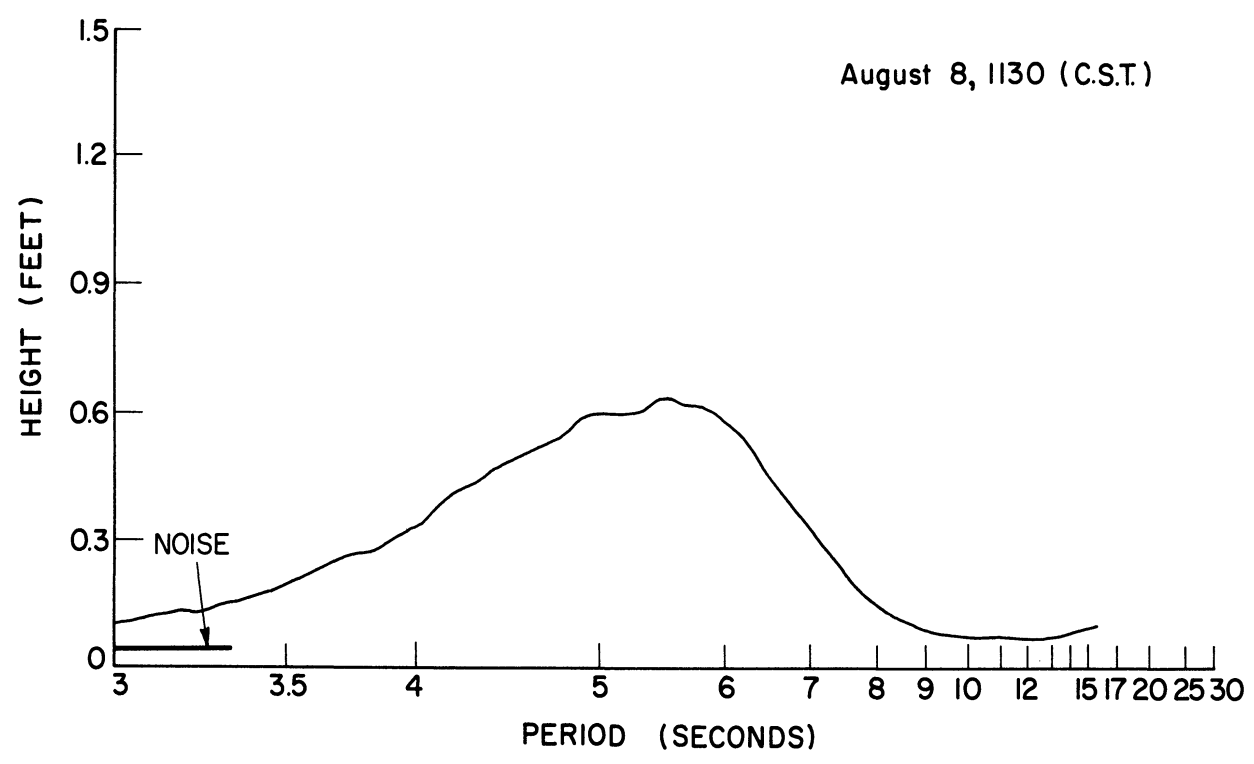
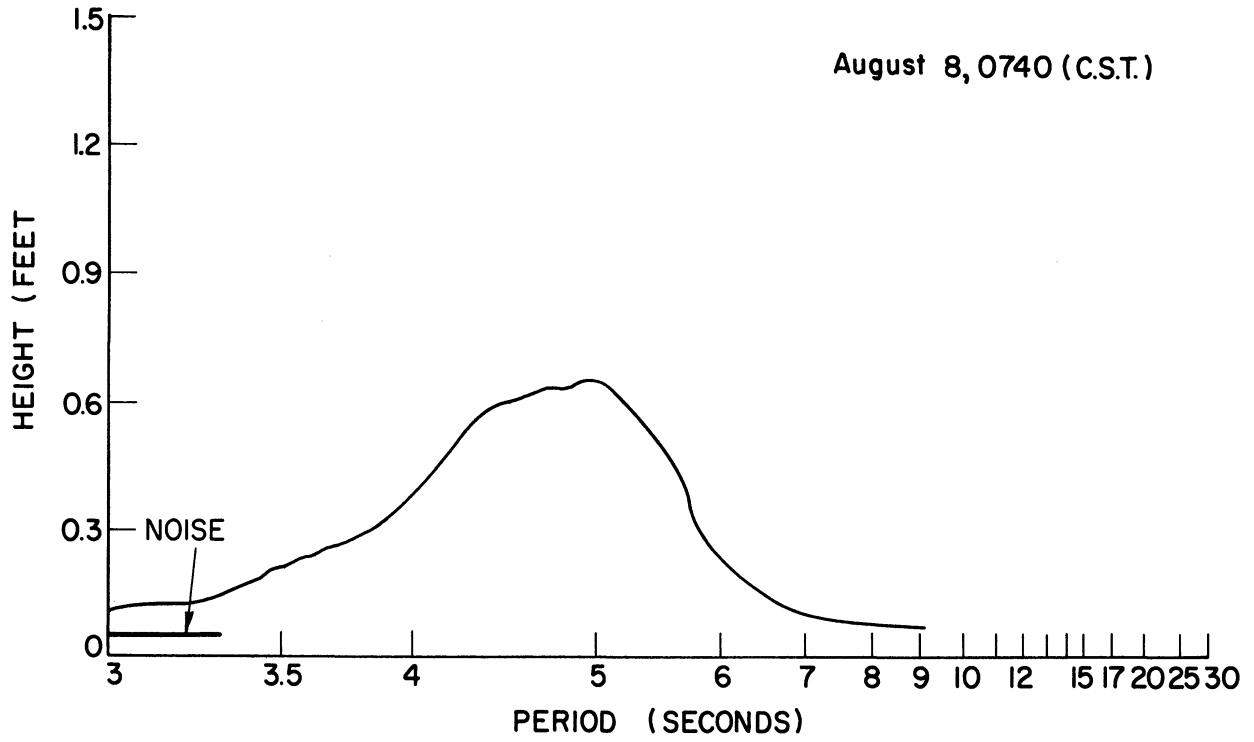
APPENDIX C
OBSERVED SPECTRA

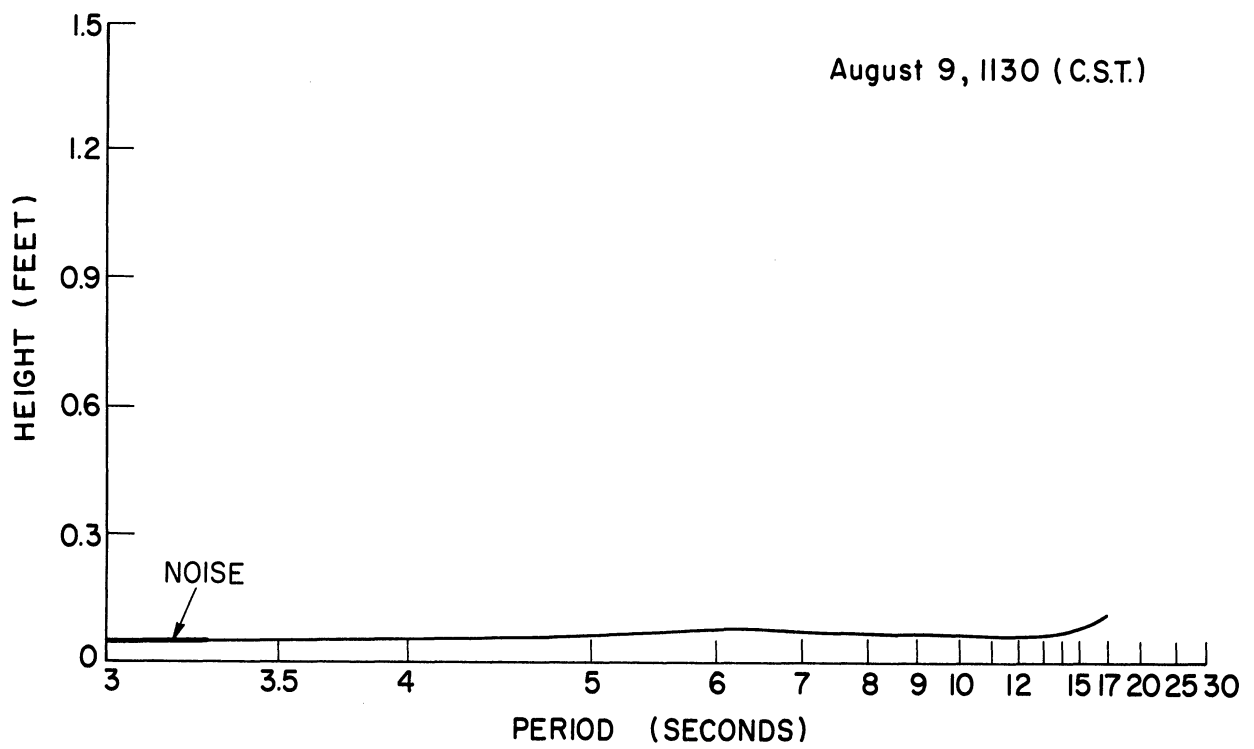
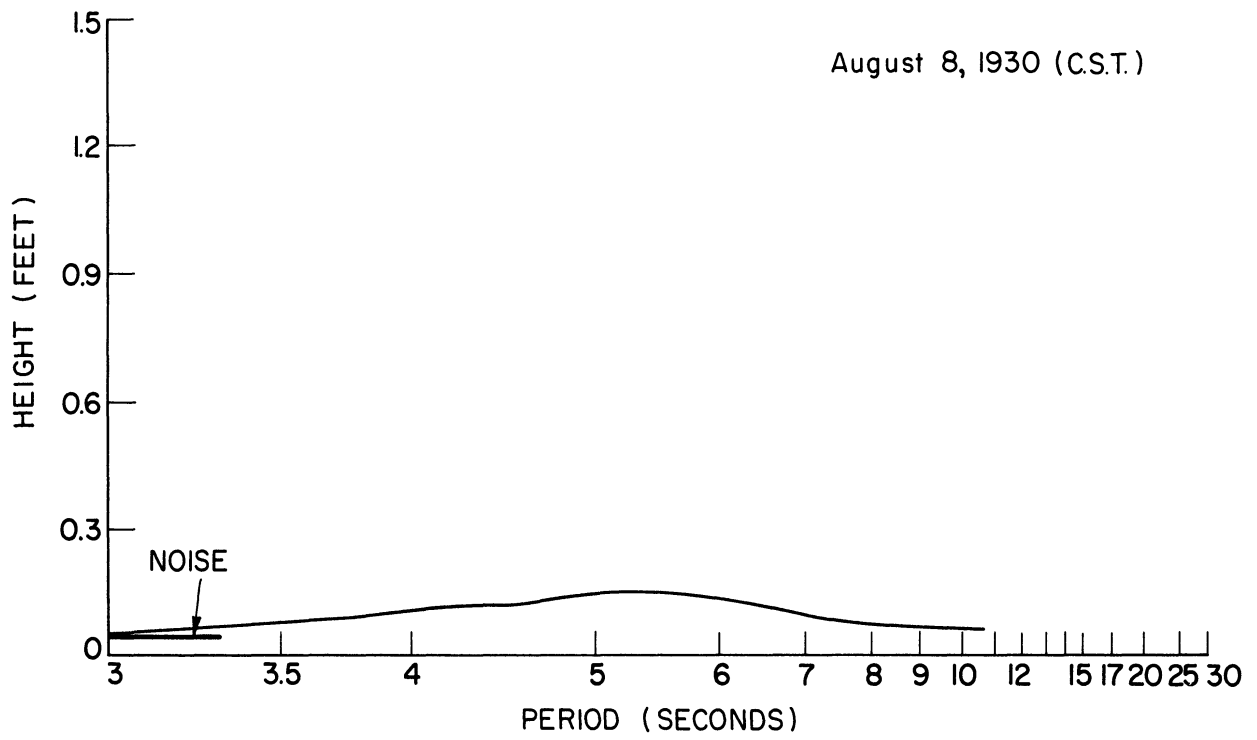
Figure C.1. Observed spectra: August.

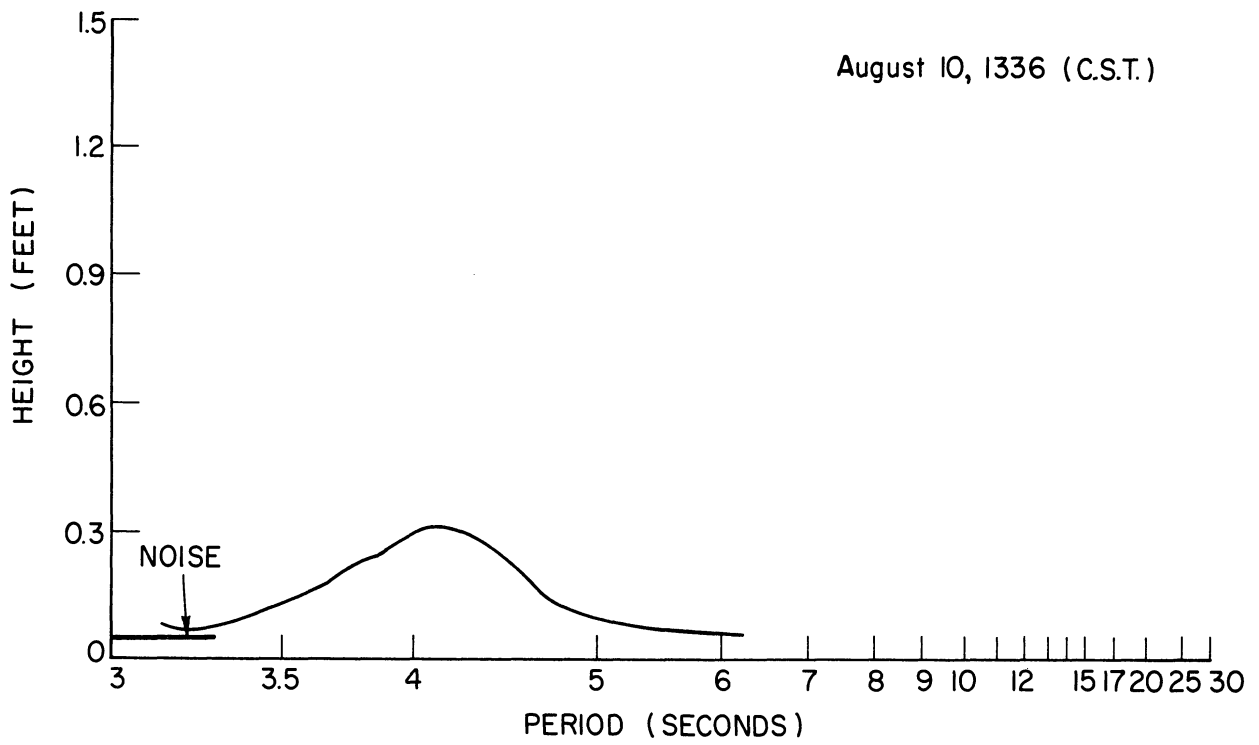
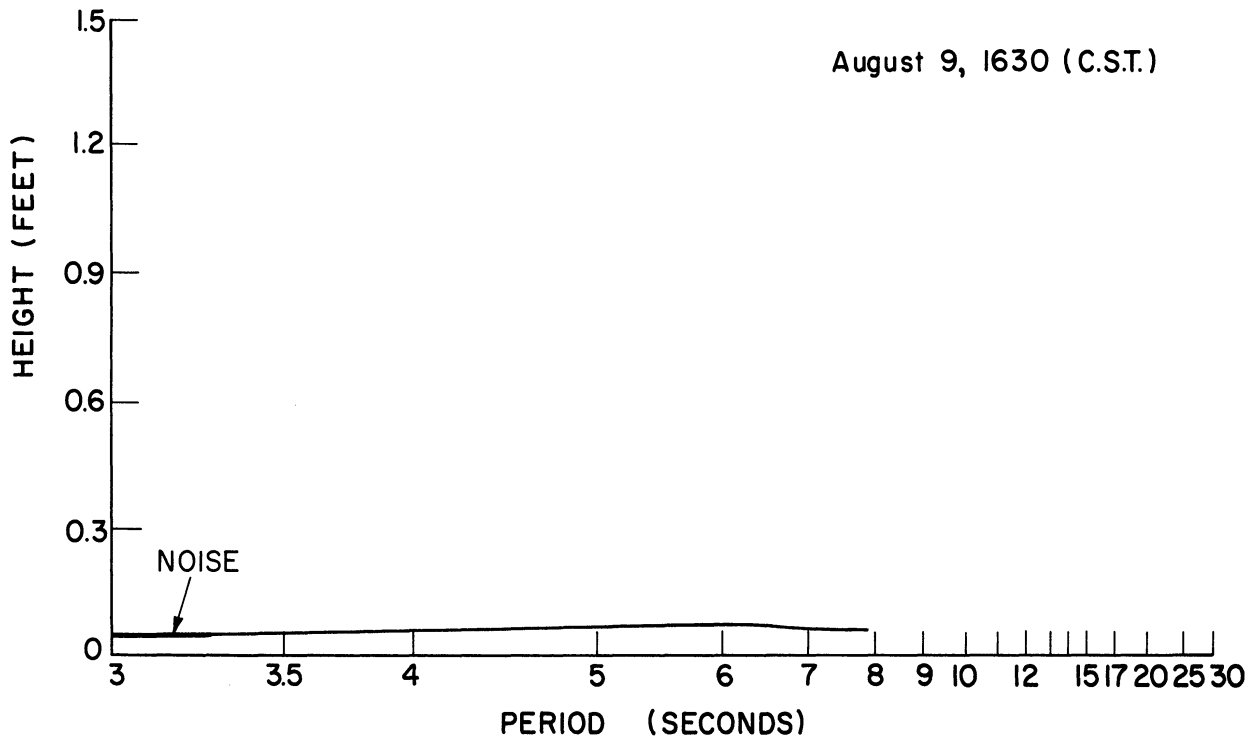












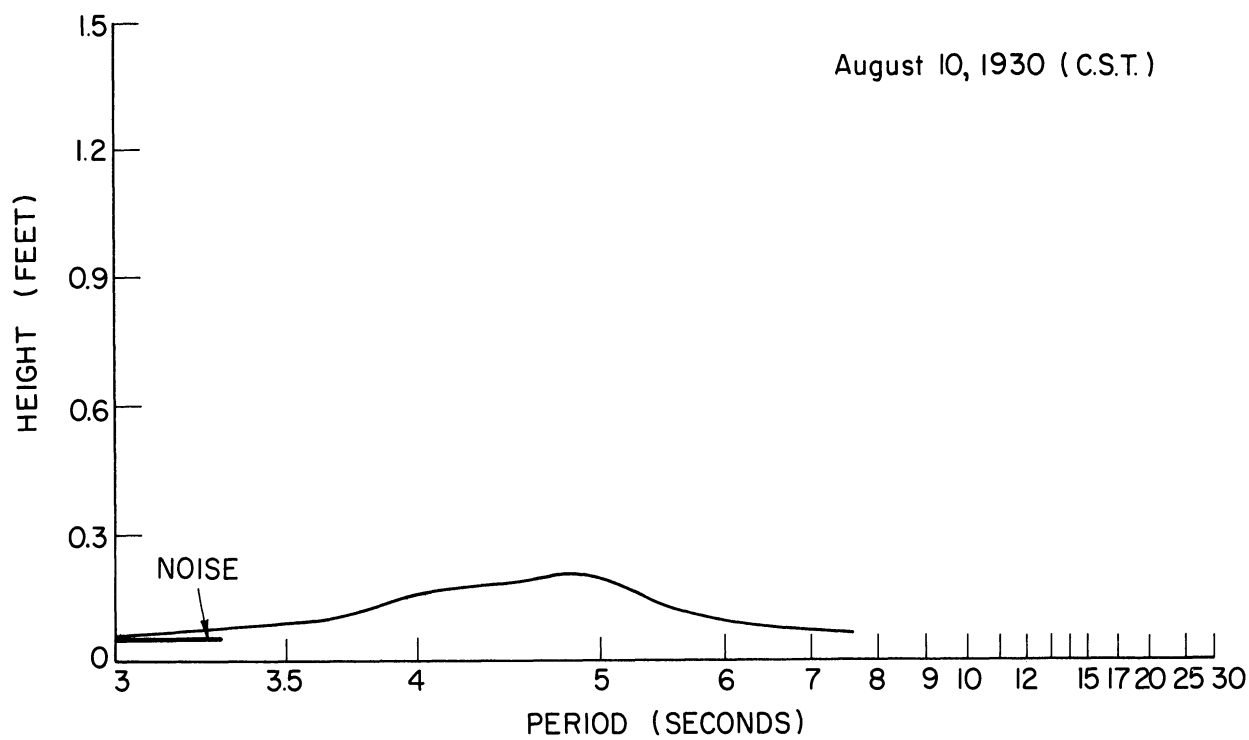
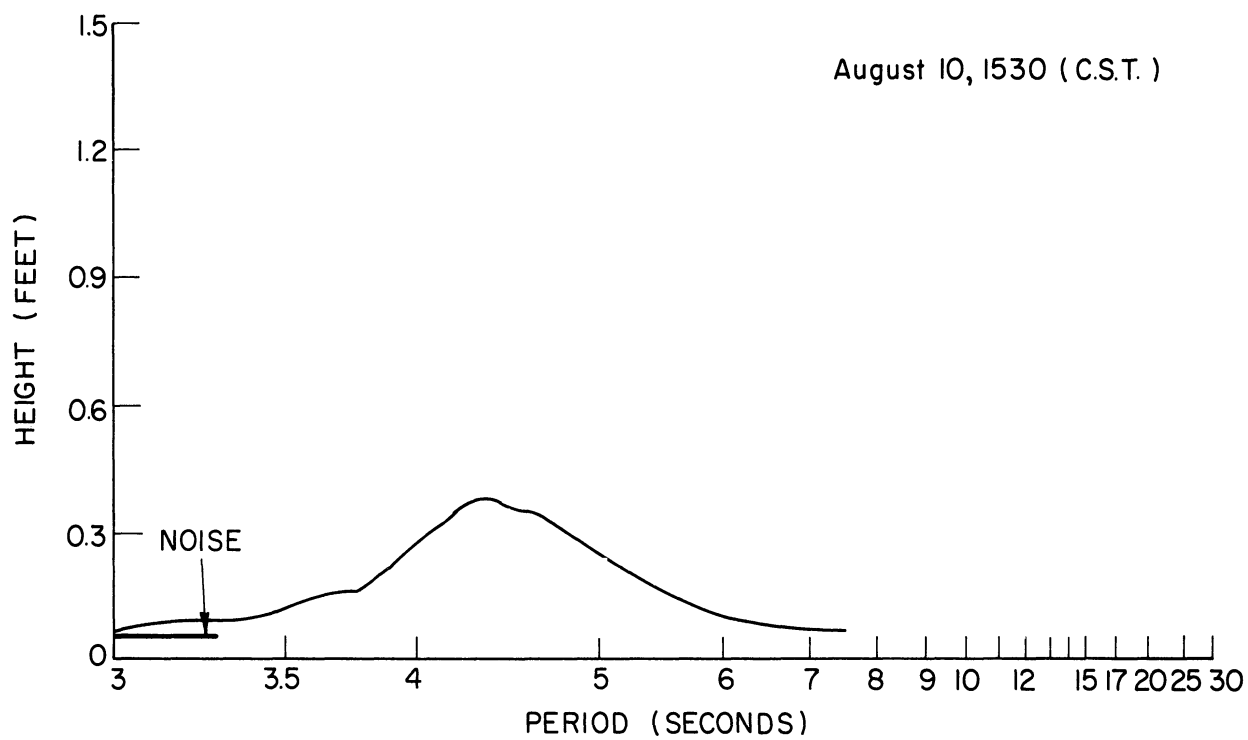
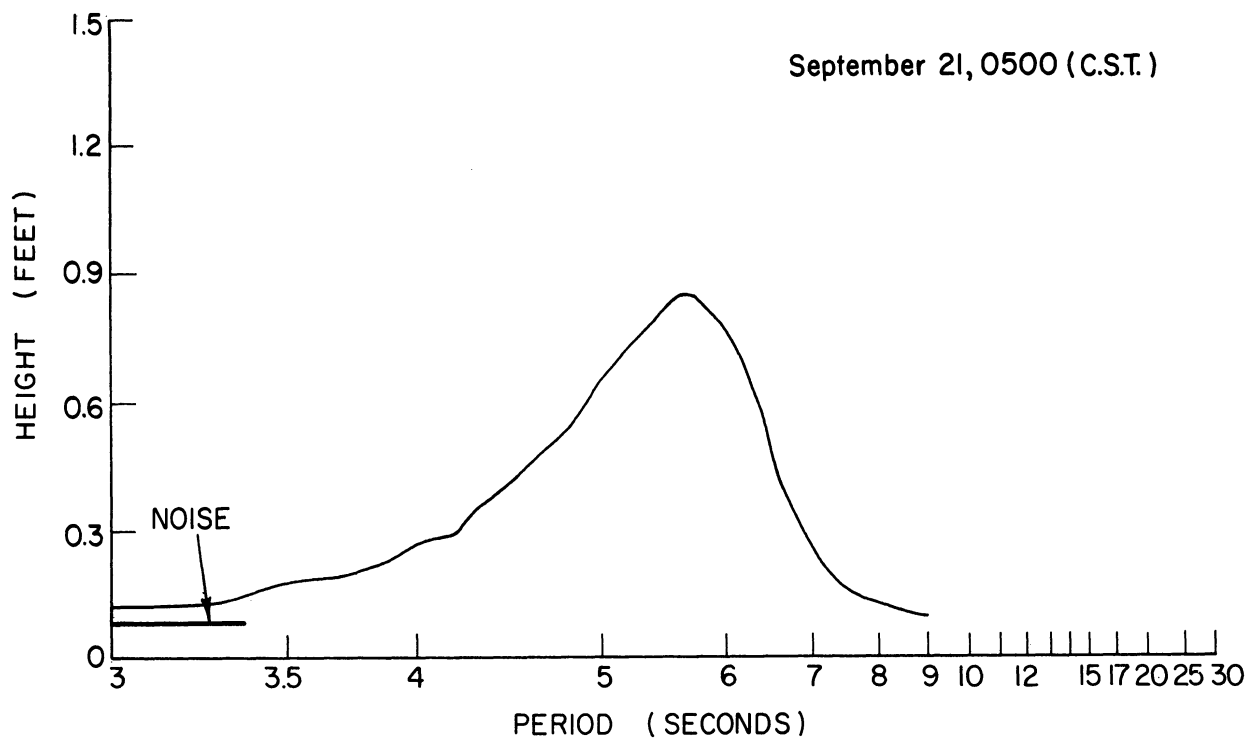
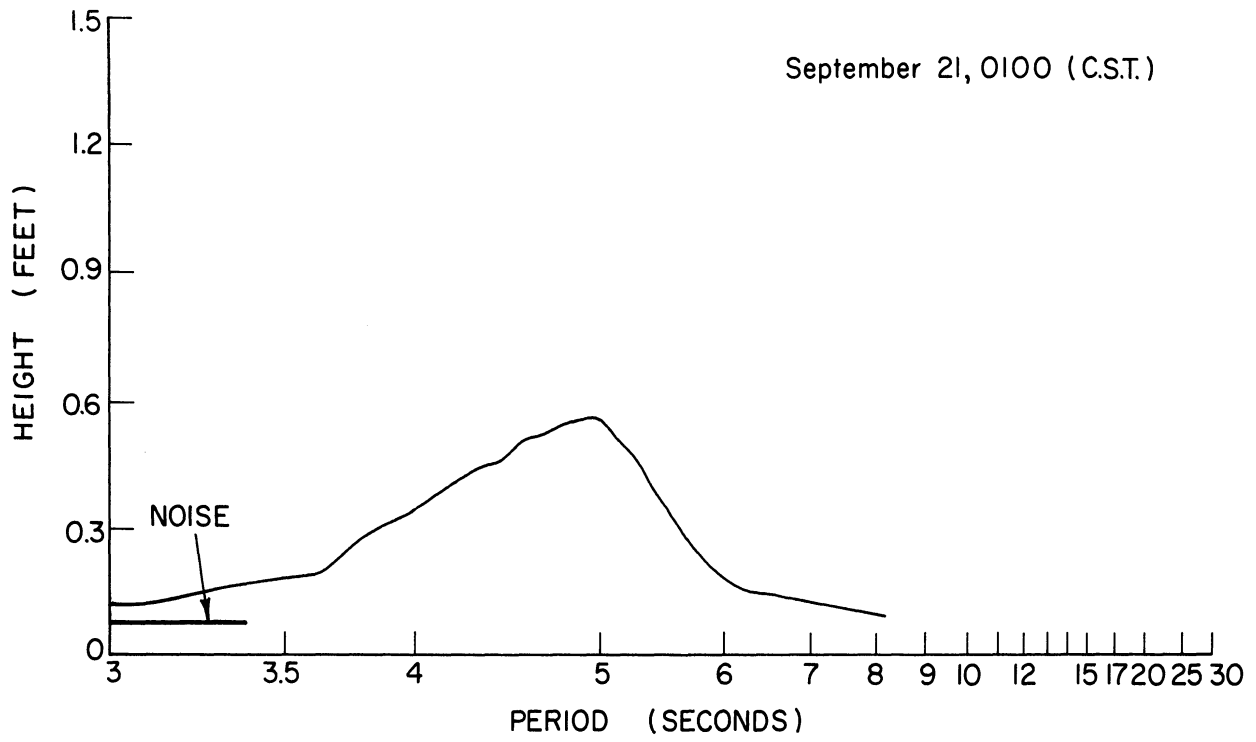
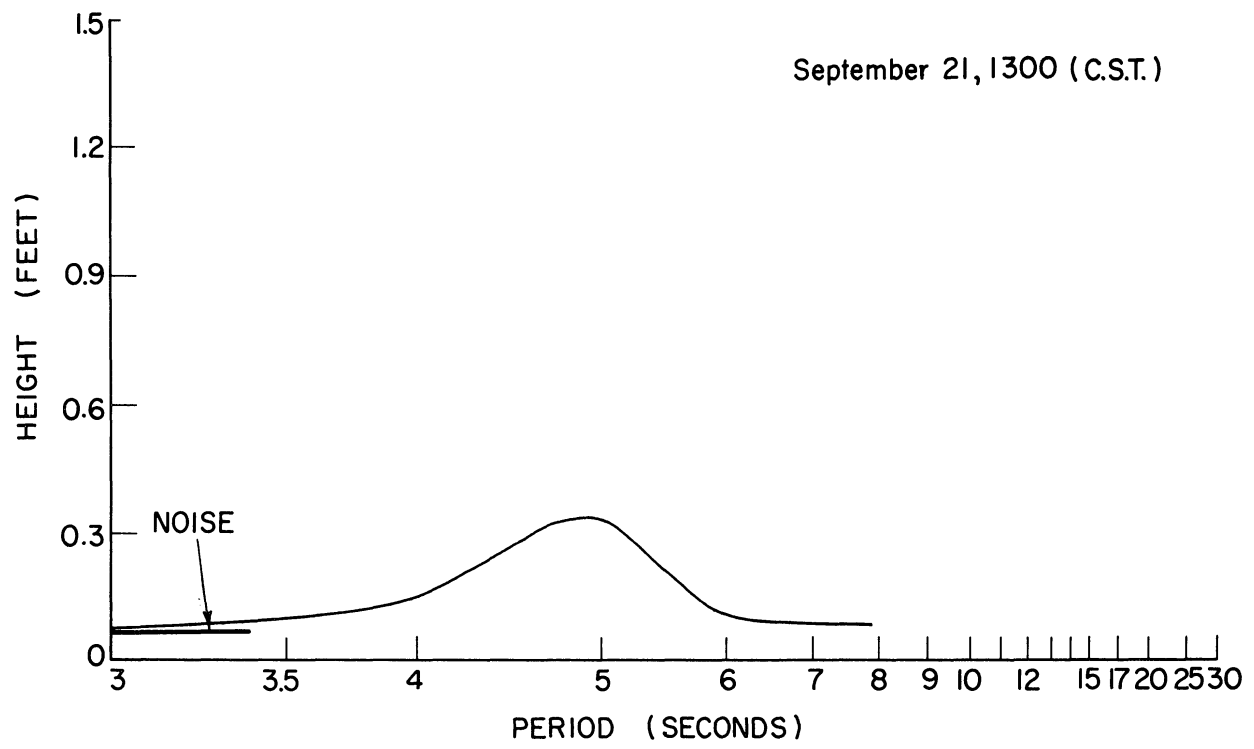
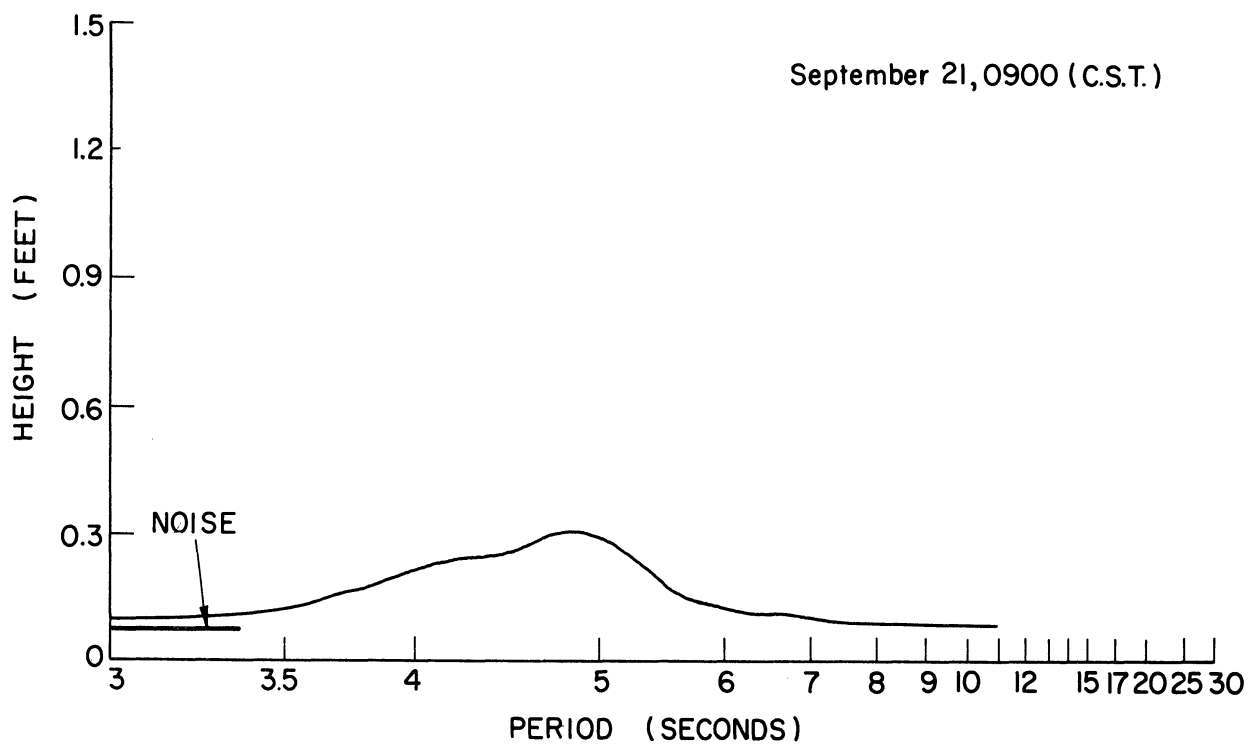
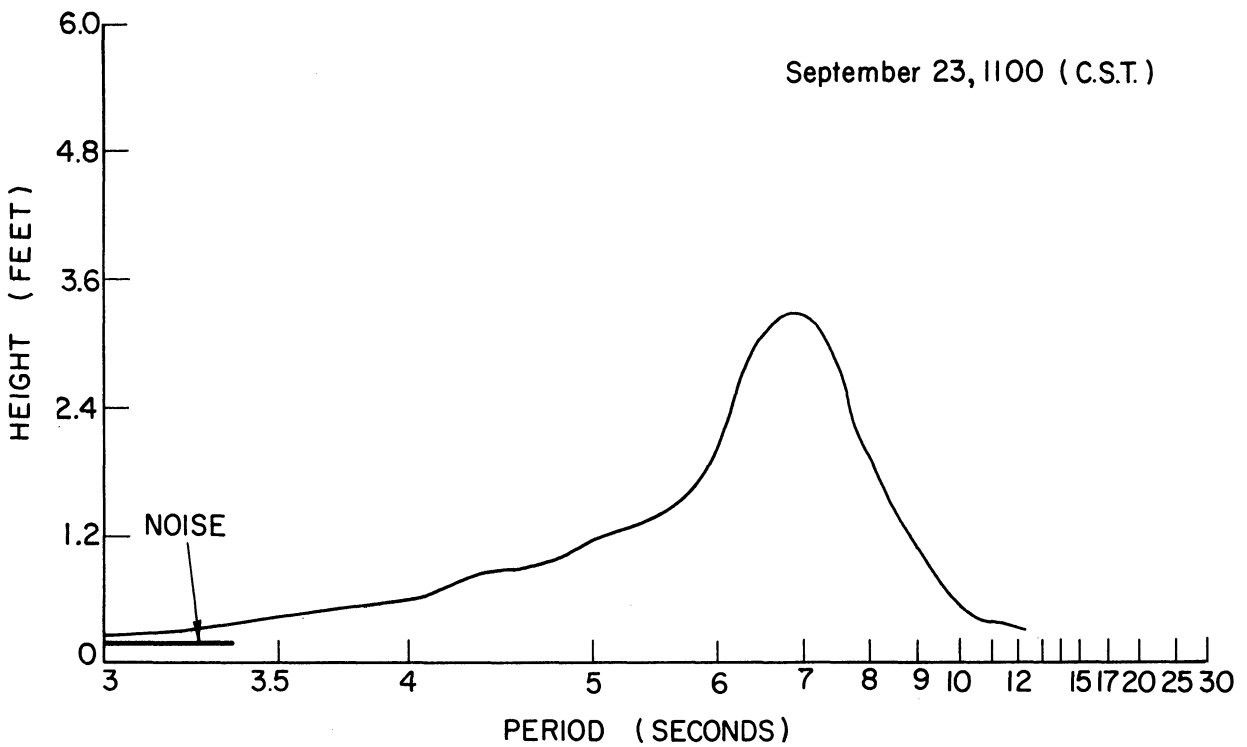
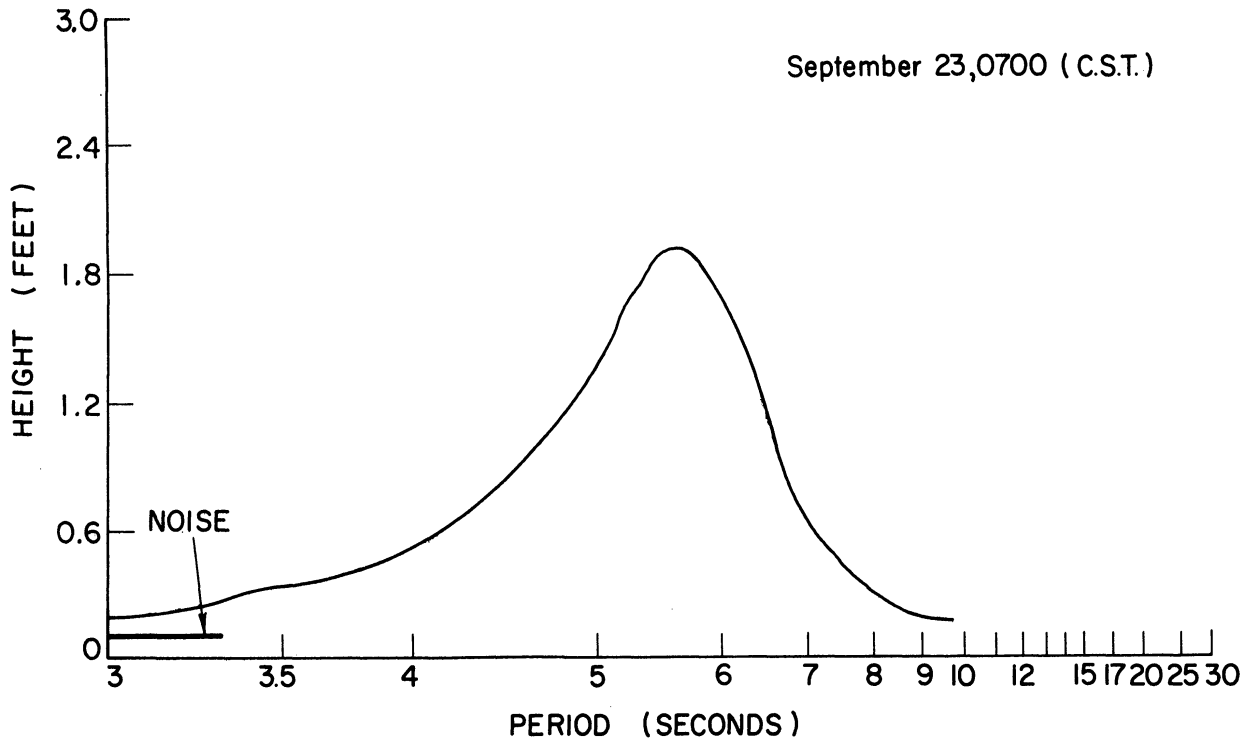
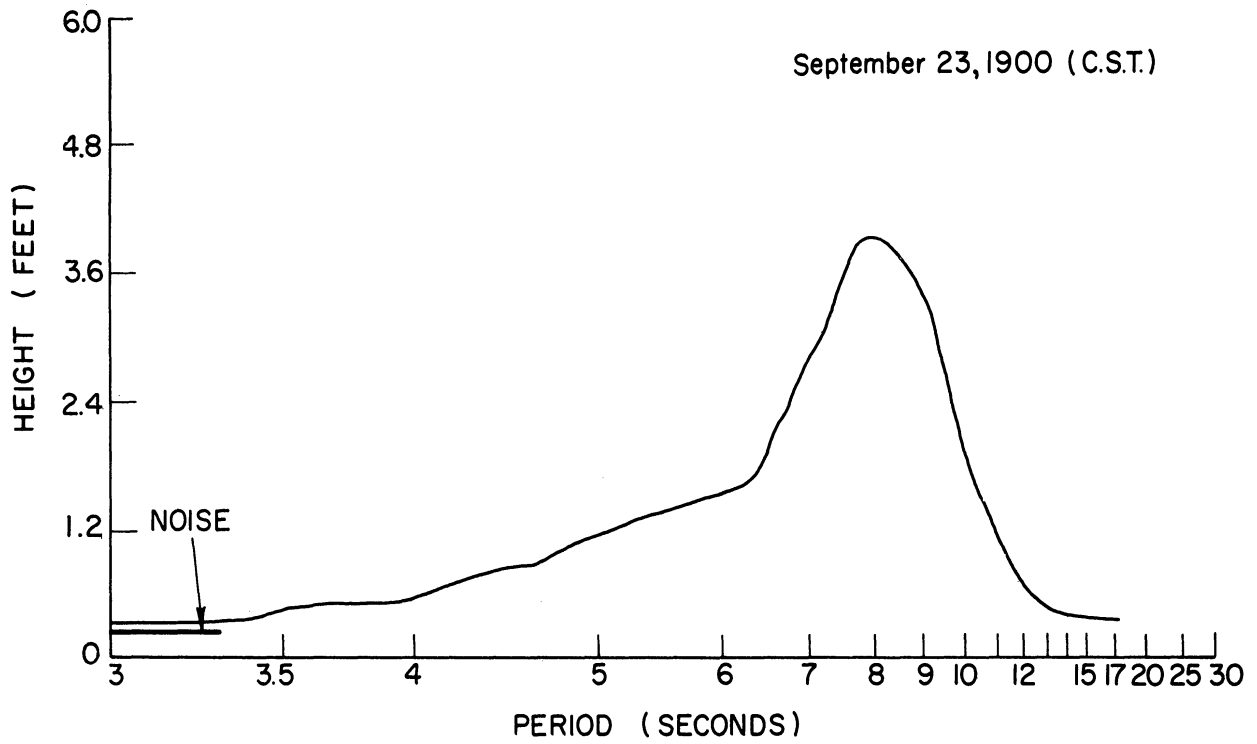
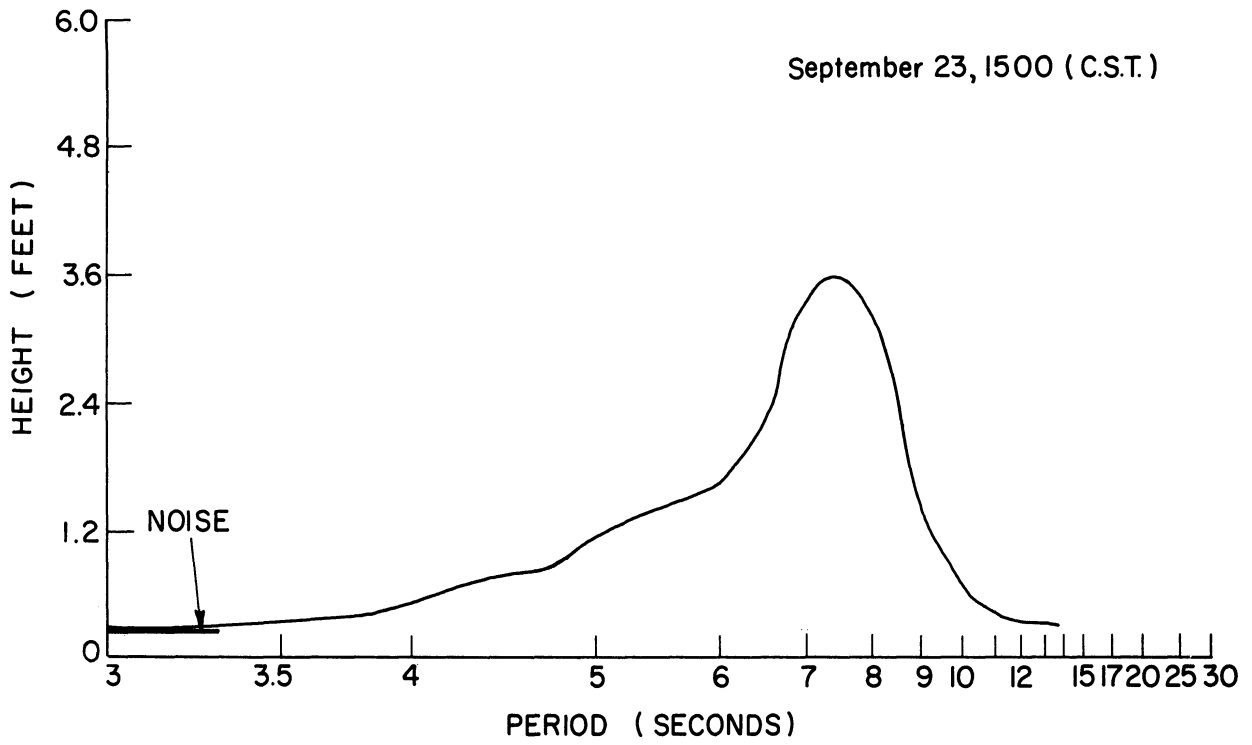


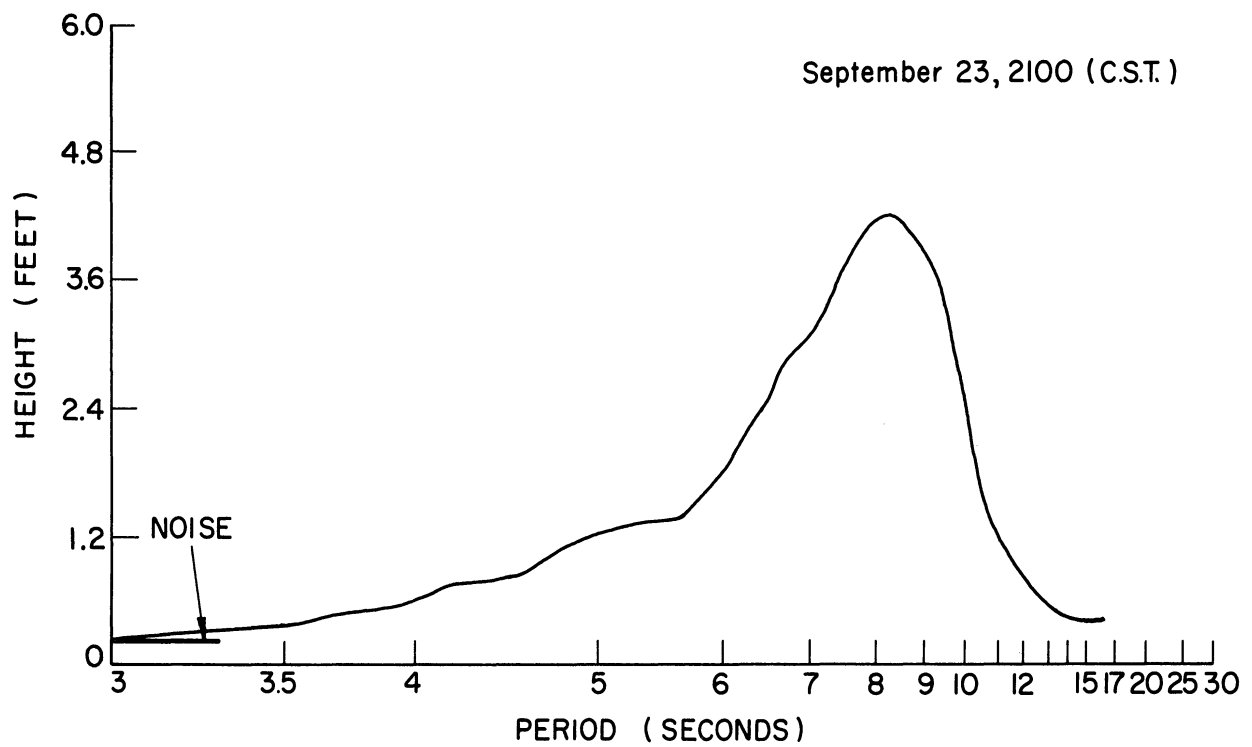
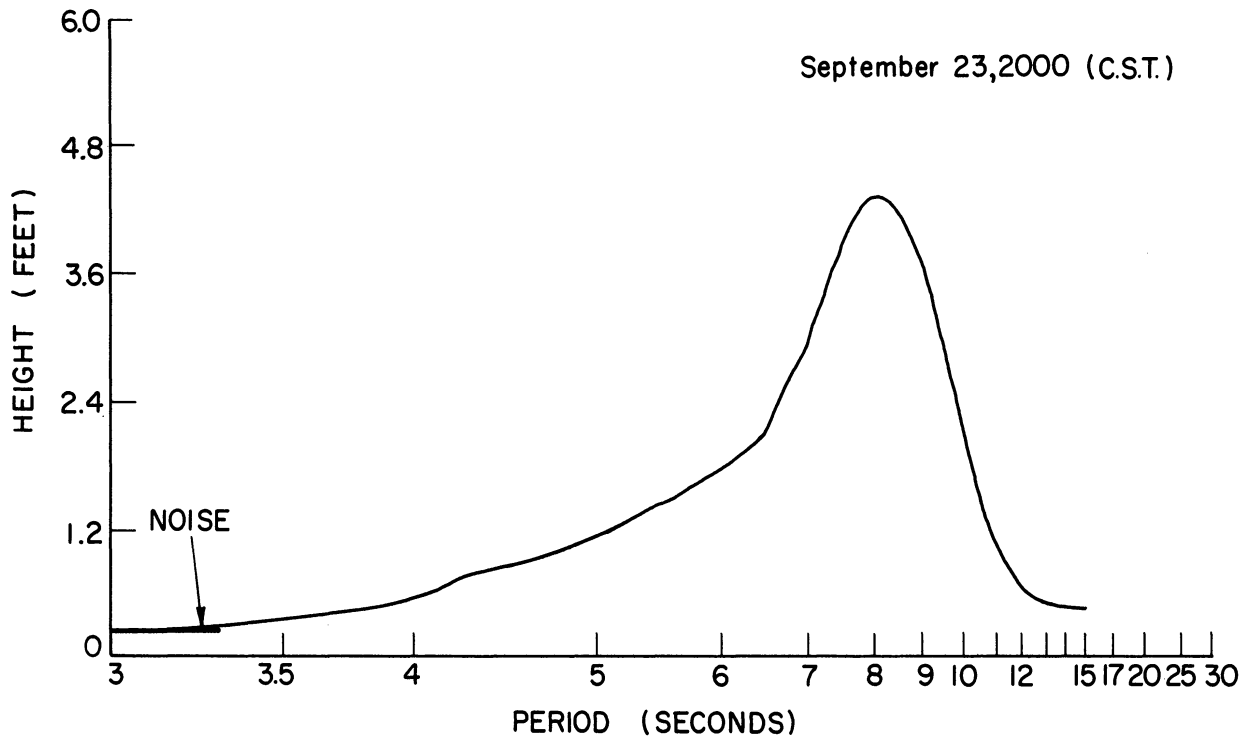
Figure C.2. Observed spectra: September.

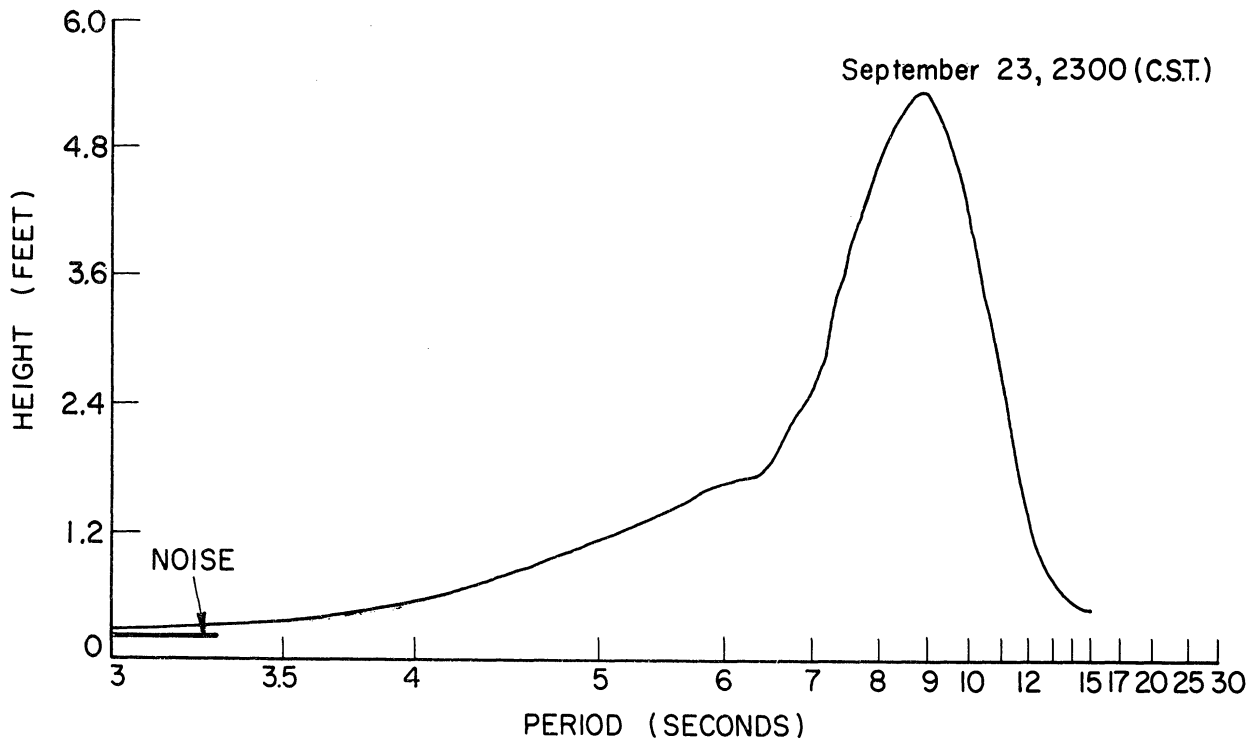
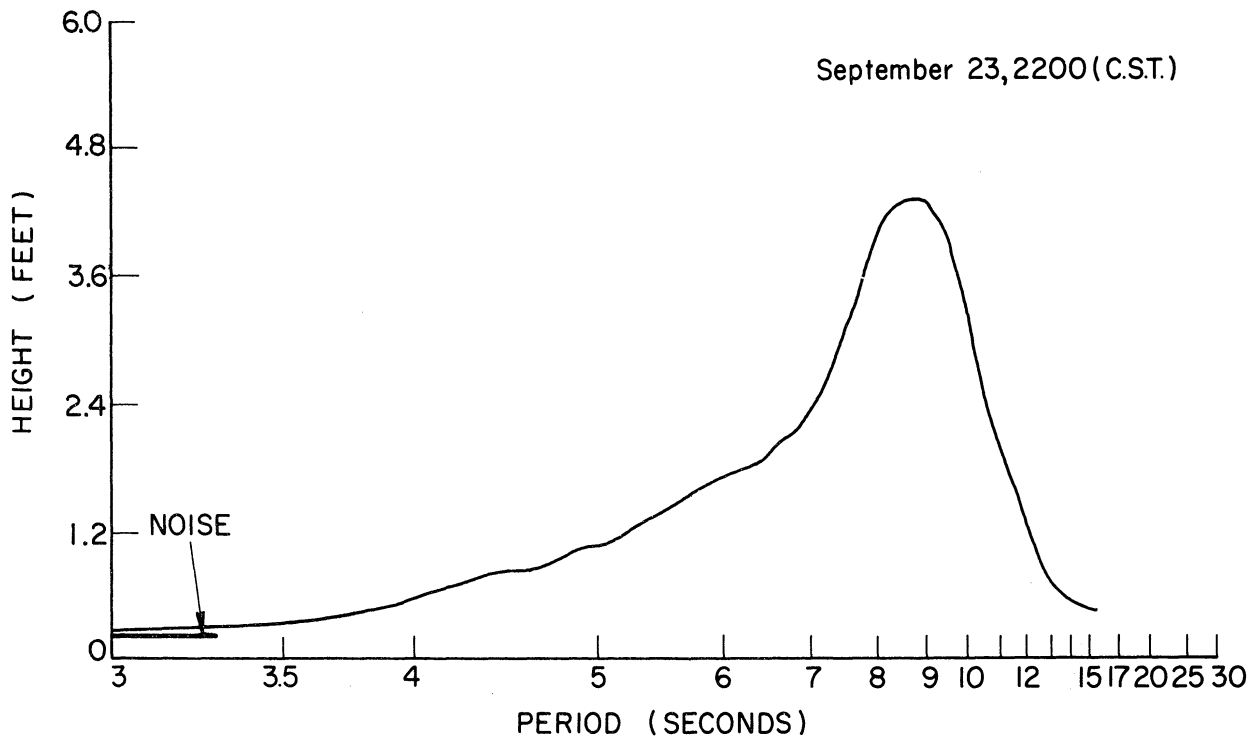












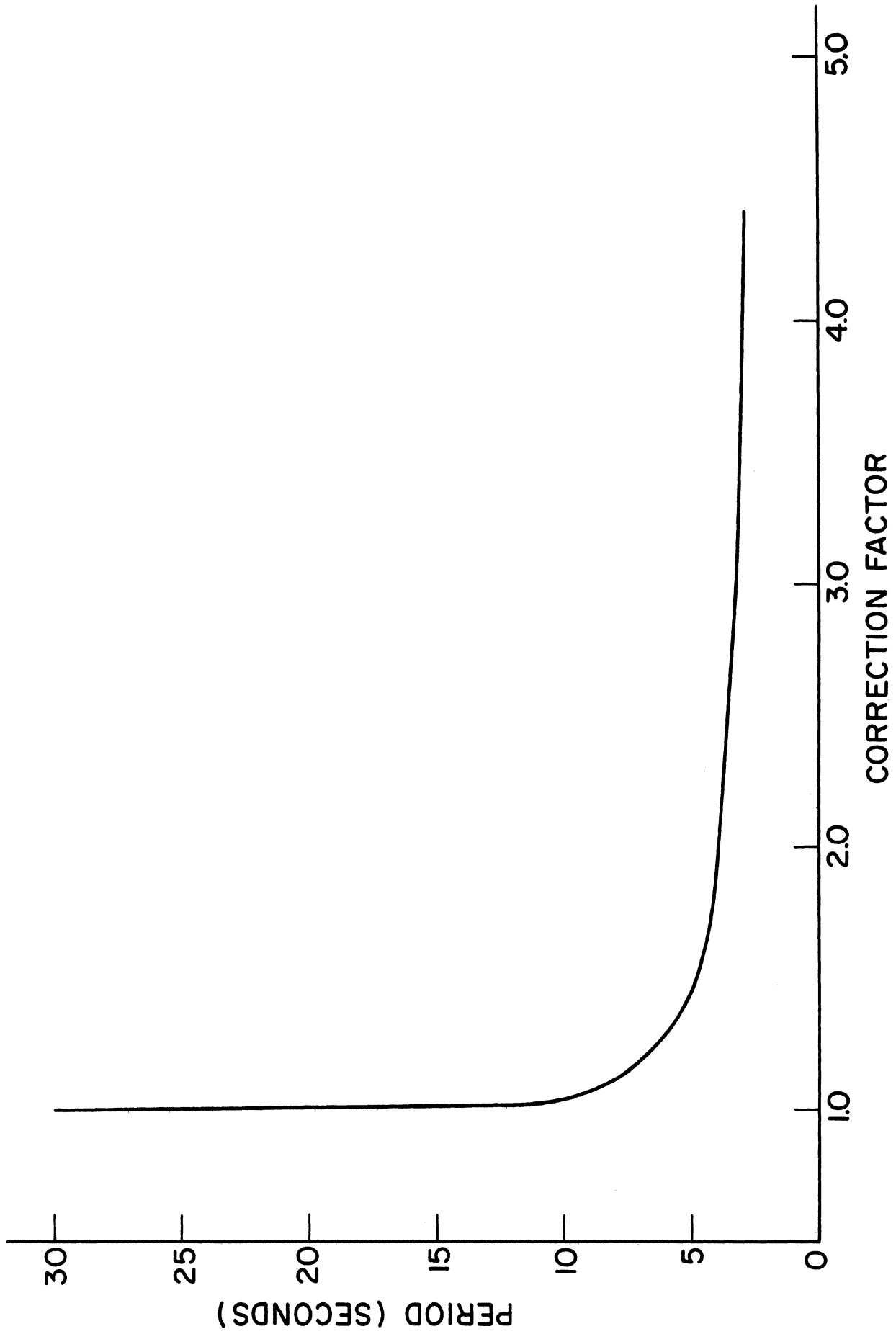


Figure C.3. Correction for hydrodynamical attenuation.

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