

# DURABLES IN OPEN ECONOMY MACROECONOMICS

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To my parents

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## CHAPTER I

### Introduction

This dissertation studies the roles that durable goods play in open economy macroeconomics. The three essays examine if and how durable goods improve the ability to understand business cycle features.

Chapter II shows how investment behavior and international risk sharing, two business cycle features that many models have failed to explain, can be addressed in a single model. The key feature of the two-country model in this chapter is the use of structures in addition to the use of equipment in production. Structures are nontraded goods, which dampen the mobility of capital and explain the positive co-movement of investment across countries. The ability to use nontraded goods in investment increases the wealth effect from country-specific shocks and explains the low correlation between relative consumption and the real exchange rate, i.e., the lack of international risk sharing.

Chapter III incorporates trade in durable goods in an otherwise standard two-country, two-good real business cycle model. Because traded goods are durable, terms of trade movements cause reallocations of purchases across time as well as across goods. The model explains why a deterioration of the terms of trade may not lead immediately to a switch away from a relatively more expensive imported good

toward a domestic good. In addition to providing consumption risk sharing, asset markets play an important role in facilitating the reallocation of durable purchases across time.

Chapter IV documents that the United States and other industrialized economies are more open to international trade and that a growing fraction of goods traded are capital goods. This chapter presents a two-country model with variable capital utilization in order to study the impact of these changes on business cycles. The main finding is that these changes by themselves do not strongly affect the volatility of output. Greater trade openness has not been, at least not directly, a contributing factor in the moderation of business cycles since the mid-1980s. A potentially more important effect is the improved ability to allocate resources across borders to achieve higher efficiency in production.

## CHAPTER II

### Investment in Open Economy

#### 2.1 Introduction

Investment has the very important role of facilitating the consumption-smoothing behavior of households in the economy. Although explaining the volatility and the cyclical behavior of investment in a closed economy setting has become a relatively routine practice, explaining the behavior of investment in an open economy setting is still a challenge. It is well understood that in two-country models where international trade is frictionless and business cycles are driven by productivity shocks (e.g., Baxter and Crucini (1995); Backus, Kehoe, and Kydland (1995)), the desire to smooth consumption induces households in the country that receives a favorable shock to increase investment while inducing households in the other country to reduce investment. As a result, models tend to predict a negative cross-country correlation of investment, which is rejected by the data. Furthermore, investment's role in consumption smoothing implies that investment should have a bearing effect on international risk sharing—the collaboration of households in different countries to achieve the households' desired consumption path. However, international risk sharing itself is a puzzle. Models cannot explain the perceived low degree of international risk sharing in the data, which is represented by the negative correlation

between relative consumption and the real exchange rate.<sup>1</sup>

This paper aims to address the behavior of investment and the issue of international risk sharing in a unified framework. The two-country real business cycle model developed here features the nontraded good, whose price relative to the price of the traded good is an important determinant of the real exchange rate. The key feature of the model is that households can use the nontraded good in investment, if they wish, as well as the traded goods. It is not difficult to justify this setting. The stock of the productive capital of a country includes equipment (e.g., computers, machinery, robots) and structures (e.g., factory buildings, warehouses). Production of goods and services usually requires services from both types of capital. Although equipment is traded internationally, structures are not.<sup>2</sup> To make the difference in tradability explicit in the model, investment in equipment is represented by purchases of traded goods while investment in structures is represented by purchases of the nontraded good.

Introducing the nontraded good in investment can correct the behavior of investment that is driven by the consumption-smoothing motive in the presence of productivity shocks. The complementarity between equipment and structures means that it is costly to adjust the quantity of one type of capital without adjusting the other type of capital.<sup>3</sup> The fact that structures need to be produced domestically makes it extremely difficult to adjust the quantity of structures quickly. By making deviations from the steady state mix of equipment and structures costly, the com-

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<sup>1</sup>The theoretical relationship between relative consumption and the real exchange rate was first brought to attention in Backus and Smith (1993). The key result is that when asset markets are complete, i.e. when risk sharing is perfect, the correlation is equal to 1.

<sup>2</sup>While real business cycle models often make a simplifying assumption that there is only one type of capital, there are many ways in which these two types of capital differ in reality. Examples of important differences include the fact that equipment has a higher rate of depreciation than structures and the fact that production of equipment has seen more technological advances than production of structures. Greenwood, Hercowitz, and Krusell (1997) call advances in equipment production “investment-specific technological change.”

<sup>3</sup>For example, factories have machinery as well as buildings. The cost may arise due to congestion if one tries to add more and more machines into the existing buildings.

plementarity can reduce the gain from reallocating investment in equipment across countries enough to make the cross-country correlation of investment positive.

The ability to use the nontraded good in investment provides an explanation for the low degree of international risk sharing. Specifically, this ability enables country-specific productivity shocks to generate a bigger relative wealth shift in favor of the country receiving the shocks by raising the value of that country's output relative to the value of the other country's output. When productivity improves in the nontraded sector, the nontraded output is abundant. Since the shock is temporary, the desire to smooth consumption implies that the value of each unit of the nontraded good, the price of nontraded good relative to the traded good, is adversely affected. The ability to use the nontraded good in investment, by allowing the households to defer consumption, limits the extent of this adverse effect. The model that has equipment and structures generates a correlation between relative consumption and the real exchange rate close to zero while the model that has equipment as the only capital generates a substantially higher correlation.

The paper is organized as follows. Section 2 describes basic facts about investment in equipment and structures. Section 3 discusses some evidence for international risk sharing and the explanations of the lack of risk sharing in the literature. Section 4 develops a two-country model with equipment and structures as factors of production. Section 5 discusses the parameterizations of the models and their implications for the steady state. Section 6 reports the results and the interpretation of the model in section 3 as well as the model's variants. Section 7 performs a sensitivity analysis on the degree of complementarity between equipment and structures. Section 8 provides conclusions.

Table 2.1: Investment in equipment and structures

Country	Investment in Equipment (% of GDP)	Investment in Structures (% of GDP)
Australia	8.50	8.09
Austria	8.78	7.28
Canada	6.82	6.80
Denmark	7.86	5.29
Finland	7.64	7.13
France	5.94	5.21
Germany	8.10	5.94
Italy	8.94	5.48
Japan	10.34	10.25
Korea	12.38	13.04
Netherlands	6.89	5.64
Spain	7.07	7.35
United Kingdom	7.78	4.86
United States	6.78	5.23

The shares reported are averages of the shares for the period 1981 to 2004. Data on investment and GDP are from the Organization for Economic Co-operation and Development (OECD) Annual National Account Database. Investment in equipment is Gross Fixed Capital Formation (GFCF) in metal products and machinery, transport equipment, and other products. Data series on investment in nonresidential structures are not available. Investment in structures is approximated by GFCF in other constructions.

## 2.2 Investment in equipment and structures

While housing is typically categorized as a structure, investment in structures in this paper refers to investment in nonresidential structures only. Table 2.1 provides a first glance at the significance of the two types of investment for a number of OECD countries.<sup>4</sup> An interesting feature is that the share in GDP of investment in equipment and the share in GDP of investment in structures tend to be correlated: countries that invest more in equipment also invest more in structures. This is not

<sup>4</sup>The OECD Annual National Account decomposes gross fixed capital formation (GFCF) in 6 categories: Product of agriculture, forestries, fisheries and aquaculture; Metal products and machinery; Transport equipment; Housing; Other constructions; and Other products. Data on GFCF in metal products and machinery, and transport equipment are used to represent investment in equipment. Data on GFCF in other constructions is used to represent investment in structures.

Table 2.2: Properties of investment in equipment and structures

Country	Correlations with GDP		Standard deviations relative to GDP	
	Investment in Equipment	Investment in Structures	Investment in Equipment	Investment in Structures
Australia	0.84	0.72	3.76	4.61
Austria	0.62	0.59	4.69	4.99
Canada	0.80	0.57	3.71	2.34
Denmark	0.84	0.69	3.86	5.34
Finland	0.95	0.86	3.54	3.15
France	0.88	0.80	4.22	5.20
Germany	0.70	0.56	4.34	2.22
Italy	0.91	0.76	4.43	4.82
Japan	0.82	0.81	3.15	3.04
Korea	0.93	0.59	4.75	2.33
Netherlands	0.58	0.83	3.77	3.56
Spain	0.89	0.84	4.40	4.22
United Kingdom	0.81	0.28	4.81	3.25
United States	0.80	0.74	4.01	3.00

Data on investment and GDP covering the period 1981 to 2004 are from the Organization for Economic Co-operation and Development (OECD) Annual National Account Database. Investment in equipment is Gross Fixed Capital Formation (GFCF) in metal products and machinery, transport equipment, and other products. Data series on investment in nonresidential structures are not available. Investment in structures is approximated by GFCF in other constructions. All series have been logged and Hodrick-Prescott filtered with a smoothing parameter of 100.

necessarily the case for production.<sup>5</sup> The investment shares are quite similar across countries, except for Japan and Korea. The fact that Japan is a major producer of equipment helps to explain the high share of investment in equipment. Because Korea is growing at a fast pace during the sample period, it is not surprising to see high shares of investment. Within a country, investment in equipment tends to be larger than investment in structures.

Table 2.2 reports business cycle properties of investment in equipment and structures. The similarity of the cyclical behaviors across countries are quite remarkable. Both types of investment are highly procyclical and highly volatile. The high correlations with GDP imply that the two types of investment also tend to be highly correlated over the business cycle. Thus, the positive co-movement is a regularity at

<sup>5</sup>See appendix C.

both low and high frequencies.

### 2.3 International risk sharing

Backus and Smith (1993) show in an endowment economy that the correlation between relative consumption and real exchange rate should equal 1 if asset markets are complete; in other words, if risk sharing is perfect. Thus, this correlation is a measure of the degree of international risk sharing. The big discrepancy between the observed value and the predicted values based on various models is an important puzzle in international finance. The inability to explain the low correlation is often referred to as the Backus-Smith puzzle or the consumption-real exchange rate anomaly.

#### Data

Table 2.3 reports this correlation for pairs of several OECD countries, including Australia, Belgium, Canada, Finland, France, Italy, Netherlands, Norway, Spain, the United Kingdom, and the United States. Data on consumption volumes, nominal exchange rates, and consumer price indices are taken from the OECD database. The sample covers 1970 to 2003. For any country pair, relative consumption is  $\frac{C_1}{C_2}$  and the real exchange rate is  $\frac{SP_{c,2}}{P_{c,1}}$ , where  $C$  denotes consumption,  $S$  denotes the bilateral nominal exchange rate, and  $P$  denotes the consumer price index. To calculate the correlation, data series have been logged and either differenced or Hodrick-Prescott filtered with a smoothing parameter of 100.

#### Model

In recent years, the ability of models to generate wealth effect has been the focus of papers on the Backus-Smith puzzle. Some researchers have found ways to generate a bigger wealth effect from a productivity improvement in the traded good sector.



Table 2.3: Backus-Smith correlations

Correlations	Max	Min	Median	Average
<i>First Difference</i>				
All pairs	0.52	-0.65	-0.15	-0.15
Others vs. USA.	0.09	-0.45	-0.21	-0.21
<i>H-P Filter</i>				
All pairs	0.56	-0.78	-0.26	-0.23
Others vs. USA.	0.16	-0.59	-0.29	-0.27

Corsetti, Dedola, and Leduc (2004) argue that one possibility is to study the response of the terms of trade because it represents the value of domestic output relative to foreign output. In order for a supply shock to generate a big wealth effect, the terms of trade have to appreciate. To get this result, the authors assume that the elasticity of substitution between traded goods is so low that the demand for country 1's traded good relative to country 2's traded good is decreasing in the terms of trade (i.e., an upward sloping demand curve). This implies that the terms of trade must appreciate after a productivity improvement in country 1 just to clear the world market. Others consider a risk sharing contract with limited commitment. Bodenstein (2004) and Kang (2007) show that a bigger wealth effect is derived from the notion that the constrained optimal contract should allow more current and future consumptions to the country with a better shock to prevent that country from deviating to autarky.

The analysis of international risk sharing in this paper is similar to previously mentioned studies to the extent that it is considering the wealth effect. Unlike those studies, however, this paper focuses on the wealth effect that arises from shocks to nontraded good productivity.

Other papers that study this puzzle include Chari, Kehoe, and McGrattan (2002), Salaive and Tuesta (2003), Ghironi and Melitz (2005), and Benigno and Thoenissen

(2006).

## 2.4 An open economy model with equipment and structures

The world consists of two countries of the same size. Each country produces a traded good and a nontraded good. To produce these goods, a country needs services from two types of capital, namely equipment and structures, labor, and technology. Households accumulate capital and rent them to perfectly competitive goods producers. Households can use goods for consumption or to generate equipment and structures. Equipment is a composite good: it is combination of the domestically-produced traded good and the imported good. A unit of structures is a unit of nontraded good. Households in each country own the capital stock of that country. Neither labor nor the capital stocks is internationally mobile.

In each period  $t$  the economy experiences one event  $s_t \in S$  where  $S$  is a finite set of events. Let  $s^t$  be the history of events up to and including date  $t$ , the probability at date 0 of any particular history  $s^t$  is  $\pi(s^t)$ .

Let  $h$  and  $f$  be traded goods produced by country 1 and 2, respectively, and  $n$  be the nontraded good. The production of traded and nontraded goods require services from equipment  $k_e$ , structures  $k_d$ , labor  $l$ , and exogenous technology  $z$ . Let the superscript  $j = \{h, n\}$  denote traded and nontraded sectors in country 1, respectively, production takes the following form

$$(2.1) \quad y_1^j(s^t) \equiv F(z_1^j(s^t), k_{e,1}^j(s^{t-1}), k_{d,1}^j(s^{t-1}), l_1^j(s^t)) = e^{z_1^j(s^t)} K_1^j(s^{t-1})^{\alpha^j} l_1^j(s^t)^{1-\alpha^j},$$

where  $K^j$  denotes the aggregate services of capital defined as a combination of services of equipment and structures:

$$(2.2) \quad K_1^j(k_{e,1}^j(s^t), k_{d,1}^j(s^t)) = \left[ \nu^{\frac{1}{\sigma}} k_{e,1}^j(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\nu)^{\frac{1}{\sigma}} k_{d,1}^j(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

The parameter  $\sigma$  is the elasticity of substitution between the services, and  $1 - \nu$  is the share of services of structures. Note that the complementarity between the services makes it costly to deviate from the steady state mix of equipment and structures. Let  $w_1^j(s^t)$ ,  $R_{e,1}^j(s^t)$  and  $R_{d,1}^j(s^t)$  be the wage and the rental rate on equipment and structures in units of the good produced in sector  $j$  in country 1. The maximization problem of perfectly competitive good producers in country 1 is given by

$$\begin{aligned} \max_{k_{e,1}^j(s^{t-1}), k_{d,1}^j(s^{t-1}), l_1^j(s^t)} & \{F(z_1^j(s^t), k_{e,1}^j(s^{t-1}), k_{d,1}^j(s^{t-1}), l_1^j(s^t)) \\ & - w_1^j(s^t)l_1^j(s^t) - R_{e,1}^j(s^t)k_{e,1}^j(s^{t-1}) - R_{d,1}^j(s^t)k_{d,1}^j(s^{t-1})\}. \end{aligned}$$

Capital services and labor hours in each sector are determined by

$$(2.3) \quad F'(k_{e,1}^j(s^{t-1})) = R_{e,1}^j(s^t)$$

$$(2.4) \quad \left( \frac{\nu k_{d,1}^j(s^{t-1})}{1 - \nu k_{e,1}^j(s^{t-1})} \right)^{\frac{1}{\sigma}} = \frac{R_{e,1}^j(s^t)}{R_{d,1}^j(s^t)}$$

$$(2.5) \quad F'(l_1^j(s^t)) = w_1^j(s^t),$$

where  $F'(x)$  denotes the derivative of  $F$  with respect to  $x$ . It is convenient to think of the demand for capital service as a mix of capital services that minimizes the cost of  $K_1^j$ .

Households in country 1 accumulate and own the stock of equipment and the stock of structures used in country 1. These capital stocks are sector-specific, which means that it is not possible to reallocate them across sectors (the rental rates need not be the same across sectors). The accumulation equations are given by

$$(2.6) \quad k_{e,1}^j(s^t) = (1 - \delta_e)k_{e,1}^j(s^{t-1}) + x_{e,1}^j(s^t)$$

$$(2.7) \quad k_{d,1}^j(s^t) = (1 - \delta_d)k_{d,1}^j(s^{t-1}) + x_{d,1}^j(s^t),$$

where  $\delta_e$  and  $\delta_d$  are depreciation rates of equipment and structures, respectively; and  $x_{e,1}^j(s^t)$  and  $x_{d,1}^j(s^t)$  are investment in equipment and investment in structures (in

sector  $j$ ), respectively. While a unit of structures is a unit of the nontraded good, an equipment is a combination of the domestically produced traded good and the imported good:

$$(2.8) \quad x_{e,1}(h_{x,1}(s^t), f_{x,1}(s^t)) = \left[ \psi_x^{\frac{1}{\phi_x}} h_{x,1}(s^t)^{\frac{\phi_x-1}{\phi_x}} + (1 - \psi_x)^{\frac{1}{\phi_x}} f_{x,1}(s^t)^{\frac{\phi_x-1}{\phi_x}} \right]^{\frac{\phi_x}{\phi_x-1}},$$

where  $\phi_x$  is the elasticity of substitution between goods  $h$  and  $f$  in equipment investment;  $1 - \psi_x$  is the share of imported goods in equipment investment; and  $h_{x,1}(s^t)$  and  $f_{x,1}(s^t)$  are domestically produce traded and imported goods used in equipment investment, respectively. Note that the newly created equipment can be allocated freely between the two sectors:

$$(2.9) \quad x_{e,1}(s^t) = x_{e,1}^h(s^t) + x_{e,1}^n(s^t).$$

Let  $p_{f,1}(s^t)$  and  $P_{e,1}(s^t)$  be the price of  $f$  and the price of equipment in country 1, both in units of good  $h$ . The following maximization problem gives the demand for traded goods in investment:

$$\max_{h_{x,1}(s^t), f_{x,1}(s^t)} \{P_{e,1}(s^t)x_{e,1}(h_{x,1}(s^t), f_{x,1}(s^t)) - h_{x,1}(s^t) - p_{f,1}(s^t)f_{x,1}(s^t)\}.$$

The use of  $h$  and  $f$  in equipment investment in country 1 is determined by

$$(2.10) \quad P_{e,1}(s^t)x'_{e,1}(h_{x,1}(s^t)) = 1$$

$$(2.11) \quad \left( \frac{\psi_x}{1 - \psi_x} \frac{f_{x,1}(s^t)}{h_{x,1}(s^t)} \right)^{\frac{1}{\phi_x}} = \frac{1}{p_{f,1}(s^t)}.$$

Note that  $P_{e,1}(s^t) = [\psi_x + (1 - \psi_x)(p_{f,1}(s^t))^{1-\phi_x}]^{\frac{1}{1-\phi_x}}$  and that  $p_{f,1}(s^t)$  is also the terms of trade, which is defined as the price of imports into country 1 relative to exports from country 1. Aggregate investment expenditure is defined as the sum of expenditure on equipment and structures:

$$(2.12) \quad x_1(s^t) = P_{e,1}(s^t)x_{e,1}(s^t) + p_{n,1}(s^t)(x_{d,1}^h(s^t) + x_{d,1}^n(s^t)),$$

where  $p_{n,1}$  is the price of the nontraded good in units of the traded good produced in country 1.

Households derive utility from consuming a combination of the domestically produced traded good, the imported good, and the nontraded good. Let  $c_{t,1}(s^t)$  be the traded goods bundle in consumption. This bundle is defined as

$$(2.13) \quad c_{t,1}(h_{c,1}(s^t), f_{c,1}(s^t)) = \left[ \psi_c^{\frac{1}{\phi_c}} h_{c,1}(s^t)^{\frac{\phi_c-1}{\phi_c}} + (1 - \psi_c)^{\frac{1}{\phi_c}} f_{c,1}(s^t)^{\frac{\phi_c-1}{\phi_c}} \right]^{\frac{\phi_c}{\phi_c-1}},$$

where  $\phi_c$  is the elasticity of substitution between goods  $h$  and  $f$  in the traded goods bundle (in consumption);  $1 - \psi_c$  is the share of imported goods in the traded goods bundle; and  $h_{c,1}(s^t)$  and  $f_{c,1}(s^t)$  are domestically produce traded and imported goods used in consumption, respectively. Let  $P_{ct,1}(s^t)$  be the price of the traded goods bundle in units of the traded good produced in country 1, the following maximization problem gives the demand for traded goods in consumption:

$$\max_{h_{c,1}(s^t), f_{c,1}(s^t)} \{ P_{ct,1}(s^t) c_{t,1}(h_{c,1}(s^t), f_{c,1}(s^t)) - h_{c,1}(s^t) - p_{f,1}(s^t) f_{c,1}(s^t) \}.$$

The use of  $h$  and  $f$  in consumption in country 1 is determined by

$$(2.14) \quad P_{ct,1}(s^t) c'_{t,1}(h_{c,1}(s^t)) = 1$$

$$(2.15) \quad \left( \frac{\psi_c}{1 - \psi_c} \frac{f_{c,1}(s^t)}{h_{c,1}(s^t)} \right)^{\frac{1}{\phi_c}} = \frac{1}{p_{f,1}(s^t)}.$$

Note that  $P_{ct,1}(s^t) = [\psi_c + (1 - \psi_c)(p_{f,1}(s^t))^{1-\phi_c}]^{\frac{1}{1-\phi_c}}$ . Given the definition of the traded goods bundle, the consumption bundle that households in country 1 consume is

$$(2.16) \quad c_1(c_{t,1}(s^t), c_{n,1}(s^t)) = \left[ \omega^{\frac{1}{\rho}} c_{t,1}(s^t)^{\frac{\rho-1}{\rho}} + (1 - \omega)^{\frac{1}{\rho}} c_{n,1}(s^t)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}},$$

where  $\rho$  is the elasticity of substitution between the traded goods bundle and the nontraded good in consumption;  $1 - \omega$  is the share of the nontraded good; and

$c_{n,1}(s^t)$  is the nontraded good used in consumption. Let  $P_{c,1}(s^t)$  be the price of the consumption bundle in units of the traded good produced in country 1. The following maximization problem gives the demand for traded and nontraded components in consumption:

$$\max_{c_{t,1}(s^t), c_{n,1}(s^t)} \{P_{c,1}(s^t)c_1(c_{t,1}(s^t), c_{n,1}(s^t)) - P_{ct,1}(s^t)c_{t,1}(s^t) - p_{n,1}(s^t)c_{n,1}(s^t)\}.$$

The demand for traded and nontraded components in consumption in country 1 are determined by

$$(2.17) \quad P_{c,1}(s^t)c_1'(c_{t,1}(s^t)) = P_{ct,1}(s^t)$$

$$(2.18) \quad \left( \frac{\omega c_{n,1}(s^t)}{1 - \omega c_{t,1}(s^t)} \right)^{\frac{1}{\rho}} = \frac{P_{ct,1}(s^t)}{p_{n,1}(s^t)}.$$

Note that  $P_{c,1}(s^t) = [\omega(P_{ct,1}(s^t))^{1-\rho} + (1 - \omega)(p_{n,1}(s^t))^{1-\rho}]^{\frac{1}{1-\rho}}$ .

Assume that the only asset traded internationally is a non-state-contingent bond. Let  $B_1(s^t)$  be the quantity of bonds purchased by households in country 1 after history  $s^t$  that pay one unit of good  $h$  in period  $t + 1$  irrespective of the state of the economy in  $t + 1$ , and  $Q(s^t)$  be the price in units of good  $h$  of these bonds. The maximization problem of the representative household in country 1 is given by

$$\begin{aligned}
& \max \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ \frac{1}{\gamma} [c_1(s^t)^\mu (1 - l_1(s^t))^{1-\mu}]^\gamma \right. \\
& + \lambda_1(s^t) \left[ R_{e,1}^h(s^t) k_{e,1}^h(s^{t-1}) + R_{d,1}^h(s^t) k_{d,1}^h(s^{t-1}) + w_1^h(s^t) l_1^h(s^t) \right. \\
& \quad + p_{n,1}(s^t) (R_{e,1}^n(s^t) k_{e,1}^n(s^{t-1}) + R_{d,1}^n(s^t) k_{d,1}^n(s^{t-1}) + w_1^n(s^t) l_1^n(s^t)) \\
& \quad + B_1(s^{t-1}) - Q(s^t) B_1(s^t) - \frac{\tau}{2} (B_1 - B_1(s^t))^2 - P_{c,1}(s^t) c_1(s^t) \\
& \quad \left. \left. - P_{e,1}(s^t) (x_{e,1}^h(s^t) + x_{e,1}^n(s^t)) - p_{n,1}(s^t) (x_{d,1}^h(s^t) + x_{d,1}^n(s^t)) \right] \right. \\
& + \Omega_{e,1}^h(s^t) \left[ -k_{e,1}^h(s^t) + (1 - \delta_e) k_{e,1}^h(s^{t-1}) + x_{e,1}^h(s^t) - \frac{\phi}{2} k_{e,1}^h(s^{t-1}) \left( \frac{x_{e,1}^h(s^t)}{k_{e,1}^h(s^{t-1})} - \delta_e \right)^2 \right] \\
& + \Omega_{d,1}^h(s^t) \left[ -k_{d,1}^h(s^t) + (1 - \delta_d) k_{d,1}^h(s^{t-1}) + x_{d,1}^h(s^t) - \frac{\phi}{2} k_{d,1}^h(s^{t-1}) \left( \frac{x_{d,1}^h(s^t)}{k_{d,1}^h(s^{t-1})} - \delta_d \right)^2 \right] \\
& + \Omega_{e,1}^n(s^t) \left[ -k_{e,1}^n(s^t) + (1 - \delta_e) k_{e,1}^n(s^{t-1}) + x_{e,1}^n(s^t) - \frac{\phi}{2} k_{e,1}^n(s^{t-1}) \left( \frac{x_{e,1}^n(s^t)}{k_{e,1}^n(s^{t-1})} - \delta_e \right)^2 \right] \\
& + \Omega_{d,1}^n(s^t) \left[ -k_{d,1}^n(s^t) + (1 - \delta_d) k_{d,1}^n(s^{t-1}) + x_{d,1}^n(s^t) - \frac{\phi}{2} k_{d,1}^n(s^{t-1}) \left( \frac{x_{d,1}^n(s^t)}{k_{d,1}^n(s^{t-1})} - \delta_d \right)^2 \right] \\
& \left. + \eta_1(s^t) [l_1(s^t) - l_1^h(s^t) - l_1^n(s^t)] \right\},
\end{aligned}$$

where  $\beta$  is the discount rate;  $1 - \gamma$  is the risk aversion parameter;  $\mu$  is the consumption share;  $\tau$  is the portfolio adjustment cost;  $\phi$  is the investment adjustment cost;  $\lambda_1(s^t)$  is the marginal utility of a unit of good  $h$ ; and  $\Omega_{e,1}^j(s^t)$  and  $\Omega_{d,1}^j(s^t)$  are the shadow prices of equipment and structures in sector  $j$ , respectively.<sup>6</sup> Let  $U(s^t) \equiv \frac{1}{\gamma} [c_1(s^t)^\mu (1 - l_1(s^t))^{1-\mu}]^\gamma$  denote the household's utility. The first order

<sup>6</sup>The investment adjustment cost may be necessary to prevent too much reallocations of investment because households are free to allocate investments across sectors. A small portfolio adjustment cost is included to ensure that the model has a unique steady state. See Schmitt-Grohé and Uribe (2003) for details.

conditions of the household's problem when  $\tau = \phi = 0$  are given by

$$(2.19) \quad \lambda_1(s^t) = \frac{U'(c_1(s^t))}{P_{c,1}(s^t)}$$

$$(2.20) \quad -\frac{U'(l_1(s^t))}{U'(c_1(s^t))} = \frac{w_1^h(s^t)}{P_{c,1}(s^t)}$$

$$(2.21) \quad w_1^h(s^t) = p_{n,1}(s^t)w_1^n(s^t)$$

$$(2.22) \quad Q(s^t) = \beta \sum_{s^{t+1}} \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)}$$

$$(2.23) \quad \lambda_1(s^t)P_{e,1}(s^t) = \Omega_{e,1}^h(s^t)$$

$$(2.24) \quad \lambda_1(s^t)p_{n,1}(s^t) = \Omega_{d,1}^h(s^t)$$

$$(2.25) \quad \lambda_1(s^t)P_{e,1}(s^t) = \Omega_{e,1}^n(s^t)$$

$$(2.26) \quad \lambda_1(s^t)p_{n,1}(s^t) = \Omega_{d,1}^n(s^t)$$

$$(2.27) \quad \Omega_{e,1}^h(s^t) = \beta(1 - \delta_e)\Omega_{e,1}^h(s^{t+1}) + \beta\lambda_1(s^{t+1})R_{e,1}^h(s^{t+1})$$

$$(2.28) \quad \Omega_{d,1}^h(s^t) = \beta(1 - \delta_d)\Omega_{d,1}^h(s^{t+1}) + \beta\lambda_1(s^{t+1})R_{d,1}^h(s^{t+1})$$

$$(2.29) \quad \Omega_{e,1}^n(s^t) = \beta(1 - \delta_e)\Omega_{e,1}^n(s^{t+1}) + \beta\lambda_1(s^{t+1})p_{n,1}(s^{t+1})R_{e,1}^n(s^{t+1})$$

$$(2.30) \quad \Omega_{d,1}^n(s^t) = \beta(1 - \delta_d)\Omega_{d,1}^n(s^{t+1}) + \beta\lambda_1(s^{t+1})p_{n,1}(s^{t+1})R_{d,1}^n(s^{t+1}).$$

Equation (2.20) is the labor supply condition. Equation (2.21) shows that wages in units of the traded good must be the same across sectors because labor is freely mobile. Equations (2.22) shows the trade off between consumption and saving in bonds. Equations (2.23)-(2.26) show the shadow prices of the two types of capital in units of the marginal utility of the traded good. Finally, equations (2.27)-(2.30) show the trade off between consumption and investment in equipment and structures in each sector.

Households in country 2 face similar optimization problems. It is convenient to still write country 2's optimization in units of country 1's traded good when the law of one price holds for traded goods. Consequently, the price indices for country 2's



consumption bundles are

$$P_{ct,2}(s^t) = [\psi_c(p_{f,1}(s^t))^{1-\phi_c} + (1 - \psi_c)]^{\frac{1}{1-\phi_c}}$$

$$P_{c,2}(s^t) = [\omega(P_{ct,2}(s^t))^{1-\rho} + (1 - \omega)(p_{n,2}(s^t))^{1-\rho}]^{\frac{1}{1-\rho}}$$

The real exchange rate, defined as the price of consumption in country 2 relative to the price of consumption in country 1, is given by

$$(2.31) \quad rx(s^t) = \frac{P_{c,2}(s^t)}{P_{c,1}(s^t)}.$$

One way to quantify how much countries engage in international risk sharing is to measure the extent to which country-specific productivity shocks affect the countries' relative wealth position. This approach has a useful benchmark case of complete asset markets, in which countries achieve perfect international risk sharing essentially by pooling their resources. This arrangement means that country-specific supply shocks have no effect on relative wealth. In other words, the marginal utility of a traded good must be the same in both countries given that the law of one price holds for traded goods. Let  $rw(s^t)$  denote the ratio of the marginal utility of a unit of good  $h$  (the numeraire good) in country 1 relative to country 2:

$$(2.32) \quad rw(s^t) = \frac{\lambda_1(s^t)}{\lambda_2(s^t)}.$$

When asset markets are complete,  $rw$  is equal to 1 in all states of the world.<sup>7</sup> When the only asset available is a non-contingent bond, country-specific shocks can change relative wealth. Specifically, a shift in wealth in favor of country 1 is represented by

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<sup>7</sup>To see this, let  $B_i(s^t, s_{t+1})$  be the quantity of bonds purchased by households in country  $i$  after history  $s^t$  that pay one unit of good  $h$  in period  $t+1$  if and only if the state of the economy is  $s_{t+1}$ , and  $Q(s^t, s_{t+1})$  be the price in units of  $h$  of these bonds. The first order condition of the household's optimization problem with respect to bonds is

$$Q(s^t, s_{t+1}) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)} = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_2(s^{t+1})}{\lambda_2(s^t)}.$$

By iterating this condition, it can be shown that  $\lambda_1(s^t) = \kappa \lambda_2(s^t)$ , where  $\kappa$  is a constant. When the two country are symmetric,  $\kappa$  can be set to 1 without loss of generality.

a decline in  $rw(s^t)$  while a shift in wealth in favor of country 2 is represented by a rise in  $rw(s^t)$ . To see why, one can start by assuming that the marginal utility of  $h$  in country 2 is fixed. A decline in the numerator  $\lambda_1 = \frac{U'(c_1)}{P_{c,1}}$  implies that consumption increases at the time when price increases, which means that households in country 1 must experience an increase in wealth. Thus, an increase in relative wealth in favor of country 1 is a situation when households in country 1 increase their consumption more than households in country 2 even though the price of consumption in country 1 increases more than the price of consumption in country 2.

In the equilibrium, goods markets and bond markets clear:

$$(2.33) \quad h_{c,1}(s^t) + h_{x,1}(s^t) + h_{c,2}(s^t) + h_{x,2}(s^t) = y_1^h(s^t)$$

$$(2.34) \quad c_{n,1}(s^t) + x_{d,1}^h(s^t) + x_{d,1}^n(s^t) = y_1^n(s^t)$$

$$(2.35) \quad f_{c,1}(s^t) + f_{x,1}(s^t) + f_{c,2}(s^t) + f_{x,2}(s^t) = y_2^f(s^t)$$

$$(2.36) \quad c_{n,2}(s^t) + x_{d,2}^f(s^t) + x_{d,2}^n(s^t) = y_2^n(s^t)$$

$$(2.37) \quad B_1(s^t) + B_2(s^t) = 0.$$

#### Additional variables of interest

Let  $Y_i(s^t)$  denote Gross Domestic Product (GDP) in country  $i$  in units of the final consumption good. Define GDP as the sum of domestic production:

$$(2.38) \quad \begin{aligned} Y_1(s^t) &= \frac{1}{P_{c,1}(s^t)} (Y_1^h(s^t) + p_{n,1}(s^t) Y_1^n(s^t)) \\ Y_2(s^t) &= \frac{1}{P_{c,2}(s^t)} (p_{f,1}(s^t) Y_1^f(s^t) + p_{n,2} Y_1^n(s^t)). \end{aligned}$$

Let  $nx(s^t)$  denote net exports as a fraction of GDP in country 1:

$$(2.39) \quad nx(s^t) = \frac{(h_{c,2}(s^t) + h_{x,2}(s^t)) - p_{f,1}(s^t)(f_{c,1}(s^t) + f_{x,1}(s^t))}{Y_1(s^t)}.$$

Let  $rc(s^t)$  denote the ratio of country 1's consumption to country 2's consumption:

$$(2.40) \quad rc(s^t) = \frac{c_1(s^t)}{c_2(s^t)}.$$

## 2.5 Calibration

Table 2.4 reports the parameter values in the benchmark model. Most of the parameters unrelated to equipment and structures are taken directly from existing international business cycle literature. The discount rate ( $\beta$ ) is 0.95, the value normally used for the calibration of annual frequency. The risk aversion parameter ( $1 - \gamma$ ) is  $-1$ , which implies that the elasticity of intertemporal substitution is 0.5. The consumption share ( $\mu$ ) is chosen so that one-third of the time endowment is devoted to working. Consistent with Stockman and Tesar (1995), the share of the nontraded good in consumption ( $1 - \omega$ ) is 0.5; the elasticity of substitution between traded and nontraded goods in consumption ( $\rho$ ) is 0.44; and capital shares in traded and nontraded sectors ( $\alpha^h$  and  $\alpha^n$ ) are 0.39 and 0.44, respectively. Consistent with Bernard, Eaton, Jensen, and Kortum (2003), the elasticity of substitution between the domestically-produced traded good and the imported good ( $\phi_c$  and  $\phi_x$ ) is 4.<sup>8</sup> The investment adjustment cost parameter ( $\phi$ ) is chosen so that the inverse of the elasticity of investment-capital ratio with respect to Tobin's  $q$  is  $1/15$ , the value used in Baxter and Crucini (1993). The exogenous productivity process is taken from Corsetti, Dedola, and Leduc (2004).

Certain parameters governing the behavior of equipment and structures have counterparts in the closed economy literature. The rates of depreciation are taken from Greenwood, Hercowitz, and Krusell (1997), whose estimates are 0.124 for  $\delta_e$  and 0.056 for  $\delta_d$  based on data of the United States over the 1954-1990 period. The same authors use a unit elasticity of substitution between equipment and structures in the production function ( $\sigma=1$ ).

The remaining parameters; the share of equipment in capital service ( $\nu$ ), and the

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<sup>8</sup>See Obstfeld and Rogoff (2000) for a brief survey of papers that estimate this parameter.

Table 2.4: Benchmark parameter values

Description	Parameter value
<i>Preference</i>	
Discount rate	$\beta = 0.95$
Consumption share	$\mu = 0.36$
Risk aversion	$1 - \gamma = 2$
<i>Production</i>	
Capital share	$\alpha^h = 0.39$
	$\alpha^n = 0.44$
Depreciation rate	$\delta_e = 0.124$
	$\delta_d = 0.056$
Elasticity of substitution between equipment and structures	$\sigma = 1$
traded bundle and nontraded in consumption	$\rho = 0.44$
traded goods in consumption	$\phi_c = 4$
traded goods in investment	$\phi_x = 4$
Share of structures in capital service	$1 - \nu = 0.4$
nontraded good in consumption	$1 - \omega = 0.5$
imports in traded bundle in consumption	$1 - \psi_c = 0.15$
imports in traded bundle in investment	$1 - \psi_x = 0.3$
Investment adjustment cost	$\phi = 1/15$
<i>Technology</i>	
$z(s^t) = \rho_z z(s^{t-1}) + e(s^t)$ , where $z = [z_1^h, z_2^f, z_1^n, z_2^n]$	
$\rho_z =$	$\begin{pmatrix} 0.820 & 0.001 & 0.068 & 0.180 \\ 0.001 & 0.820 & 0.180 & 0.068 \\ -0.003 & 0.033 & 0.961 & -0.011 \\ 0.033 & -0.003 & -0.011 & 0.961 \end{pmatrix}$
$\text{var}(e(s^t)) =$	$\begin{pmatrix} 0.057 & 0.028 & 0.010 & 0.008 \\ 0.028 & 0.057 & 0.008 & 0.010 \\ 0.010 & 0.008 & 0.008 & 0 \\ 0.008 & 0.010 & 0 & 0.008 \end{pmatrix}$

shares of imported good in the traded goods bundle in consumption ( $1 - \psi_c$ ) and investment ( $1 - \psi_x$ ); are calibrated using the model and data of the United States over the 1970-2006 period. These parameters are set so that the model displays the following features observed in the data: (i) investment in equipment (information processing equipment, industrial equipment and transport equipment) is twice the size of investment in structures (nonresidential structures); (ii) imports of goods in investment (imports of durable goods) is 6.2 percent of GDP; and (iii) imports is 12

percent of GDP.

The share of equipment in capital service is 0.6. From equations (2.4), (2.6) and (2.7), it is clear that a higher  $\nu$  implies a higher ratio of investment in equipment to investment in structures. This value of the share is roughly similar to the one in Greenwood, Hercowitz, and Krusell (1997).<sup>9</sup> The share of imported good in the traded goods bundle is 0.15 in consumption and 0.3 in investment. The bigger share of imported good in investment reflects the fact that despite accounting for roughly half of imports, investment is much smaller than consumption as a share of GDP.

## 2.6 Results

The model is solved using a linearization method. The statistics from the models are the averages of 100 simulations. The statistics of quantity variables, such as GDP, consumption and investment, are calculated at steady state price levels.

### 2.6.1 Benchmark model

#### Investment

Column “Benchmark” in table 2.5 reports the business cycle properties of investment for the benchmark model. Components of investment in the model are as volatile as the data counterparts. However, contrary to the US evidence, the model predicts that investment in equipment will be more volatile than investment in structures.<sup>10</sup> While aggregate investment and investment in equipment are highly procyclical in the model, investment in structures is countercyclical. Although the data suggests that investment in structures is also less procyclical than the other com-

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<sup>9</sup>In their calibration, the capital share in production is 0.3. The share of equipment in production is 0.17, which implies that the share of equipment in capital service is 0.57. Appendix A discusses how to calibrate these shares directly from the Input-Output tables.

<sup>10</sup>For the US, the volatility of investment in equipment is not always lower than that of investment in structures. See, for example Stock and Watson (1999), whose study uses data from 1953 to 1996.

Table 2.5: Properties of investment in the model

Statistics	Data	Benchmark	Variations on the benchmark economy		
			Complete Markets	Only Equipment	CES Investment
<i>Standard deviations relative to GDP</i>					
Investment, total	3.55	2.78	2.97	3.53	2.46
Investment, equipment	3.77	4.98	4.99	3.53	2.67
Investment, structures	4.02	3.62	3.45	n.a.	2.08
<i>Correlations with GDP</i>					
Investment, total	0.87	0.86	0.83	0.78	0.95
Investment, equipment	0.79	0.79	0.78	0.78	0.95
Investment, structures	0.43	-0.20	-0.11	n.a.	0.95
<i>Cross-country correlations</i>					
Investment, total	0.25	-0.23	-0.39	-0.36	0.33
Investment, equipment	0.24	0.09	0.01	-0.36	0.36
Investment, structures	0.004	0.68	0.67	n.a.	0.23
<i>Autocorrelations</i>					
Investment, total	0.59	0.32	0.32	0.33	0.36
Investment, equipment	0.61	0.38	0.38	0.33	0.36
Investment, structures	0.62	0.64	0.66	n.a.	0.36

Annual data series from 1970 to 2004 of the United States are used to compute standard deviations, correlations with GDP, and autocorrelations. Investment in equipment is Gross private domestic fixed investment in equipment. Investment in structures is Gross private domestic fixed investment in nonresidential structures. Cross-country correlations are averages of correlations between the US with Canada, France, Germany, Italy, Japan, and the United Kingdom. All series have been logged and Hodrick-Prescott filtered with a smoothing parameter of 100. The statistics from the model are the averages of 100 simulations.

ponent, its correlation with output is still significantly larger than zero. The model's predictions for cross-country correlations of aggregate investment and investment in equipment are too low while its prediction for a cross-country correlation of investment in structures is too high. In fact, it predicts that aggregate investment will be negatively correlated across countries. Finally, the model predicts a relatively low degree of persistence of investment, except for that of investment in structures.

The high volatility and the low cross-country correlation of investment in equipment are standard features of business cycles driven by productivity shocks. Suppose the productivity improvement occurs in country 1. This improvement raises the

marginal products of capital and, all else equal, the demand for services of equipment and structures. Country 1 can quickly increase investment in equipment by exporting less of the domestically produced traded good and importing more of the foreign traded good. From the perspective of country 2, the reallocation of traded goods to country 1 means a reduction in the capacity to create new equipment and thus a reduction in investment in equipment. Hence, international trade in goods used in investment is the reason for the high volatility and the negative cross-country correlation of investment in equipment. In addition, the ability to quickly increase or decrease investment in equipment explains the low autocorrelation.

The countercyclicality of investment in structures is a result of the reduction in the demand for service from structures. From equation (2.4), firms want less structures if their rental rate has increased relative to the rental rate of equipment. The change in the relative rental rate reflects the change in the relative price of investment goods. To see the relationship, it is convenient to look at the log-linearized version of equations (2.27) and (2.28). Suppose, for simplicity, that  $\delta_e = \delta_d = \delta$ :

$$(2.41) \quad (1 - \delta)r\tilde{p}(s^{t+1}) - \frac{1}{\beta}r\tilde{p}(s^t) = \bar{R}(\tilde{R}_{e,1}^h(s^{t+1}) - \tilde{R}_{d,1}^h(s^{t+1})),$$

where  $\tilde{x} \equiv \frac{\Delta x}{x}$ ,  $rp = \frac{\Omega_{d,1}^h}{\Omega_{e,1}^h} = \frac{p_{n,1}}{P_{e,1}}$  is the relative price of structures and equipment, and  $\bar{R} = \frac{1}{\beta} - 1 + \delta$  is the steady state rental rate. It is clear that the rental rate of structures is higher than that of equipment if  $r\tilde{p}(s^t) > r\tilde{p}(s^{t+1})$ , that is, if the relative price of structures is temporally high (and falling). Intuitively, when the relative price of structures is temporarily high, households may find it optimal to delay investment in structures and use their resources to purchase and accumulate equipment. Moreover, households can also finance their purchase of equipment by decumulating structures. Thus, in order for households to maintain a large enough stock of structures, the rental rate of structures must increase relative to the rental

rate of equipment to compensate for the price movements. Because an improvement in the productivity of the traded good sector causes the relative price of the nontraded good to increase, it is the shock that is responsible for the countercyclical movement of investment in structures.

Equation (2.4) also shows that the effect of the rental rate is increasing in the elasticity of substitution between services of equipment and structures. This means that if the services of different types of capital are highly complementary, the difference in the rental rate will not have a big effect on the desired relative amount of service. In such a case, investment in structures can be procyclical. The sensitivity analysis later on confirms this intuition.

#### **Other variables**

Column “Benchmark” in table 2.6 reports other business cycle properties of the benchmark model. The model predicts similar volatilities of consumption and employment to the data; however, its predictions for the volatilities of the terms of trade and the real exchange rate are too low. This discrepancy, termed the “price anomaly” by Backus, Kehoe, and Kydland (1995), is a well known property of real business cycle models. Unlike most models, this model does generate a high volatility of net exports. This result is due to the high volatility of investment in equipment discussed earlier. The model also does remarkably well at replicating the cyclicalities of quantity variables, including net exports, but it does not do well for price variables. The positive correlation between the terms of trade and output is a common feature in models with productivity shock. The model also suffers from the “quantity anomaly,” the term used by Backus, Kehoe, and Kydland (1995) to represent the inability of models to replicate the ranking of the cross-country correlations in output and consumption. The low cross-country correlation of output and the high cross-



Table 2.6: Properties of other variables in the model

Statistics	Data	Benchmark	Variations on the benchmark economy		
			Complete Markets	Only Equipment	CES Investment
<i>Standard deviations relative to GDP</i>					
Consumption	0.67	0.55	0.51	0.47	0.50
Employment	0.68	0.32	0.38	0.36	0.36
Terms of trade	1.63	0.45	1.56	0.40	0.39
Real exchange rate	3.59	0.38	0.34	0.49	0.54
<i>Standard deviations</i>					
Net exports/GDP	0.59	0.56	0.64	1.12	0.18
<i>Correlations with GDP</i>					
Consumption	0.89	0.96	0.88	0.86	0.88
Employment	0.89	0.95	0.92	0.93	0.93
Net exports/GDP	-0.49	-0.12	-0.04	-0.18	0.18
Terms of trade	-0.22	0.48	0.51	0.44	0.51
Real exchange rate	0.23	-0.08	0.01	-0.10	-0.15
<i>Cross-country correlations</i>					
GDP	0.50	0.32	0.21	0.28	0.39
Consumption	0.38	0.73	0.85	0.63	0.71
Employment	0.31	-0.07	-0.43	0.11	0.13
Backus-Smith correlation	-0.27	0.06	0.82	0.32	0.17

Annual data series from 1970 to 2004 of the United States are used to compute standard deviations and correlations with GDP. These data, except the real exchange rate, are from the Bureau of Economic Statistics (BEA). The terms of trade is import price index divided by export price index. The real exchange rate is the inverse of real effective exchange rate series from the OECD Main Economic Indicators database. Cross-countries correlations are averages of correlations between the US with Canada, France, Germany, Italy, Japan, and the United Kingdom. The Backus-Smith correlation is the average of the correlations between relative consumption and the real exchange rate (US versus other OECD countries). All series have been logged and Hodrick-Prescott filtered with a smoothing parameter of 100. The statistics from the model are the averages of 100 simulations.

country correlation of consumption reflect the extent of cross-country productivity spill-over. When the spill-over is positive, country 2 wants to increase consumption immediately due to a higher expected income, but it wants to delay its investment and production to wait for the spill-over. The same reasoning also explains the low cross-country correlation of employment.

The last statistic in table 2.6 is the Backus-Smith correlation, which is the correlation between relative consumption  $rc(s^t)$  and the real exchange rate  $rx(s^t)$ . Given

the definition of the real exchange rate, the negative correlation observed in the data means that country 1 has more consumption than country 2 when consumption in country 1 is more expensive than consumption in country 2. The correlation between relative consumption and the real exchange rate predicted by the benchmark model is 0.06. While it is still high relative to the data, a value this low is a major accomplishment because it is well known that most models, regardless of their asset market assumption, will predict that this correlation should be very high; in fact, close to one.<sup>11</sup>

One explanation of the low Backus-Smith correlation is the lack of international risk sharing. Specifically, incomplete asset markets allow country-specific shocks to affect the relative wealth position of these two countries such that the country that experiences a favorable shock receives a disproportionately larger positive wealth effect. As discussed in section 2.3, households in that country will demand more consumption than households in the other country even when their consumption bundle has become relatively more expensive. To understand this finding, figure 2.1 shows the responses of the relative wealth, relative consumption, and the real exchange rate when country 1 experiences a 1 percent improvement in productivity.

In response to a productivity improvement in the traded good sector, country 1 receives a bigger positive wealth effect than country 2 as shown by the negative response of  $rw(s^t)$ . This increase in wealth is due to the ability to smooth consumption through capital accumulation. Moreover, the rise in the relative price of the nontraded good  $p_n$ , by raising the value of country 1's output, adds to the wealth effect that country 1 receives.<sup>12</sup> Note that country 2 also receives a positive wealth effect because of a favorable movement in the terms of trade. The increase in wealth

<sup>11</sup>See, for example, Chari, Kehoe, and McGrattan (2002).

<sup>12</sup>An increase in the relative price of nontraded good following a productivity improvement in the traded good sector is often referred to as the Balassa-Samuelson effect.

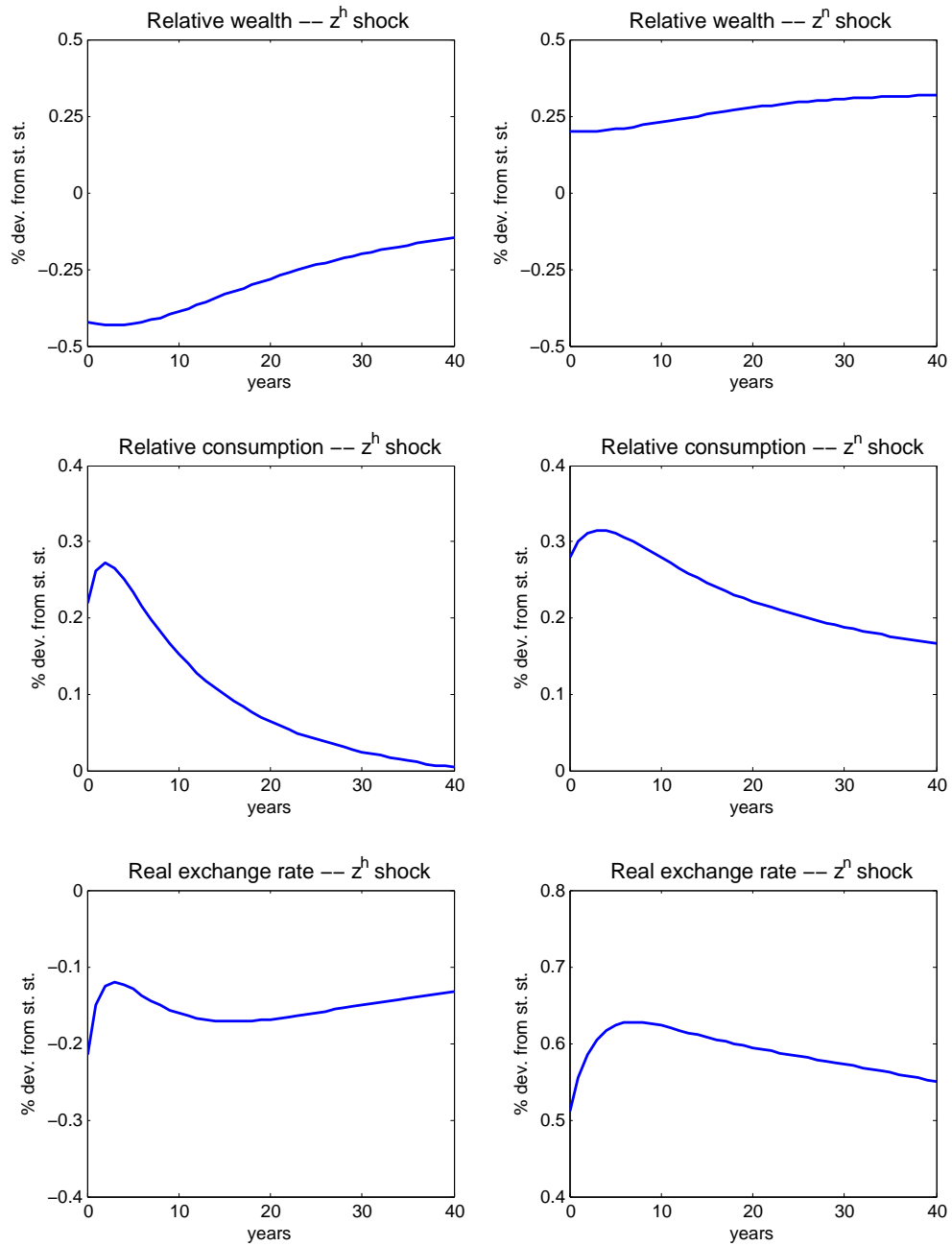


Figure 2.1: Relative wealth, relative consumption, and real exchange rate

explains why relative consumption rises even though the consumption bundle is relatively more expensive (the real exchange rate appreciation). The appreciation of the real exchange rate is primarily driven by an increase in the price of the nontraded good relative to the price of the traded good. If the only shock in the model was a shock to traded good productivity, the Backus-Smith correlation would be negative.

In contrast, a productivity improvement in the nontraded good sector leads to a positive response of  $rw(s^t)$ , which means that country 1 receives a smaller positive wealth effect than country 2. Here country 1 receives a small wealth effect because of a decline in the value of its output caused by a fall in the relative price of the nontraded good. The lack of wealth effect implies that households in country 1 want to consume more than households in country 2 only if their consumption bundle is relatively cheaper. In other words, the effect of an increase in the supply dominates the effect of an increase in the demand. Conditional on this shock, relative consumption and the real exchange rate move in the same direction. As a result, the Backus-Smith correlation should be positive if the only shock in the model were a shock to traded good productivity.

Since shocks to traded and nontraded sectors randomly hit the economy in the stochastic simulations, the low Backus-Smith correlation is realized because the wealth effect from the shock to traded good productivity is strong enough to dominate the lack of wealth effect from the shock to nontraded good productivity.

### 2.6.2 Complete markets

When asset markets are complete, households can perfectly insure against country specific risks so that productivity shocks do not affect relative wealth. It is clear from table 2.5 and table 2.6 that the only major difference from the benchmark model is

the Backus-Smith correlation. Specifically, the model with complete asset markets can no longer replicate the low correlation. To understand this result, recall that the marginal utility of good  $h$  is always the same in both countries. Using the definition of the marginal utility, this means

$$\frac{U'(c_1)}{P_{c,1}} = \frac{U'(c_2)}{P_{c,2}}.$$

This expression shows the tight link between relative consumption and the real exchange rate: the increase in relative consumption (the fall in  $\frac{U'(c_1)}{U'(c_2)}$ ) must be matched by a depreciation of the real exchange rate (the rise in  $\frac{P_{c,2}}{P_{c,1}}$ ).<sup>13</sup> Intuitively, without the wealth effect, country 1 will only have a higher consumption level than country 2 if the consumption good is cheaper in country 1.

Because relative consumption and the real exchange rate now move in the same direction regardless of the origin of the shock, differences between the model with complete asset markets and the benchmark model originate from the responses to a productivity shock in the traded good sector. Specifically, relative consumption must now fall in response to this shock because of the increase in the price of the nontraded good. The fall in relative consumption means that the increase in consumption by country 1 is much smaller than in the benchmark case. For country 1, a smaller response of consumption means that the increased traded good output must be channeled toward investment or export. It is clear from tables 2.5 and 2.6 that the higher volatility of investment, the less countercyclical net exports, and the higher volatility of the terms of trade are consistent with this intuition. Since country 1 must share its good fortune equally with country 2, the wealth effect that country 2 receives is larger than in the benchmark model. Furthermore, the larger wealth effect to country 2 means that its households will consume more and work

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<sup>13</sup>The non-separability between consumption and leisure in the utility function is the only reason why the Backus-Smith correlation is not perfectly equal to 1 when asset markets are complete.

less, which explains a higher cross-country correlation of consumption and smaller cross-country correlations of output and employment.

### 2.6.3 Only equipment

Introducing nontraded goods in consumption alone allows open economy models to account for the several empirical facts, such as the deviation from Purchasing Power Parity and the low cross-country correlation of consumption. To date, the literature has ignored the presence of nontraded goods in investment. In terms of this model, it means that the economy only uses one type of capital, equipment, in production and that all the nontraded output must be consumed. Column “Only Equipment” in tables 2.5 and 2.6 report properties of this model. Compared with the benchmark model, two cyclical behaviors of investment are noticeably different: investment in equipment is less volatile and is less correlated across countries. In addition, net exports are more volatile and the Backus-Smith correlation is significantly higher. Except for the volatility of investment in equipment, these statistics suggests that the benchmark model is more consistent with the data than the model with only equipment.

The changes in the behavior of investment are due to the fact movements in the relative price of the nontraded good no longer influence the demand for capital. When there were two types of capital, a productivity improvement in the traded good sector causes the price and the rental rate of structures to increase temporarily relative to those of equipment and thus encourages firms to substitute toward equipment. Moreover, international trade implies that the price of equipment also falls relative to the price of structures in the other country. Without this substitution, investment in equipment is less volatile and is less positively correlated across countries.

The higher volatility of net exports is largely due to the fact that trade as a share of GDP has increased. Because the nontraded output is smaller, the share of trade is now 14% instead of 11% in the benchmark model. The volatilities of export and import themselves, however, do not change by much.

The higher Backus-Smith correlation represents the greater extent of international risk sharing. To see why, it is useful to examine the responses of relative wealth to productivity shocks in country 1 shown in figure 2.2. Following a shock to traded good productivity, the response of  $rw(s^t)$  closely resembles that of the benchmark model. Consistently, this response means that country 1 receives a larger positive wealth effect than country 2. Following a shock to nontraded good productivity, however, the response of  $rw(s^t)$  is significantly larger than that of the benchmark model. The large positive response of  $rw(s^t)$  means that country 1 receives a much smaller positive wealth effect than country 2 does. From this figure, it can be concluded that the improved risk sharing is explained by the lack of wealth effect in response to a shock to nontraded good productivity.

Intuitively, the absence of structures from the economy implies that it is no longer possible to turn the nontraded output into capital. The desire to smooth consumption implies that households do not want to alter their consumption path. Without the ability to invest, the extra nontraded output generated by the productivity improvement has little value. A big reduction in the value of their output explains why the wealth effect received by households in country 1 is small. In the model, the relative price of the nontraded good needs to fall sharply following a productivity improvement in the nontraded sector so that the demand for consumption is tilted toward the nontraded good (see equation (2.18)) to clear the market. Unlike the benchmark model, an increase in consumption here is associated with a minimal

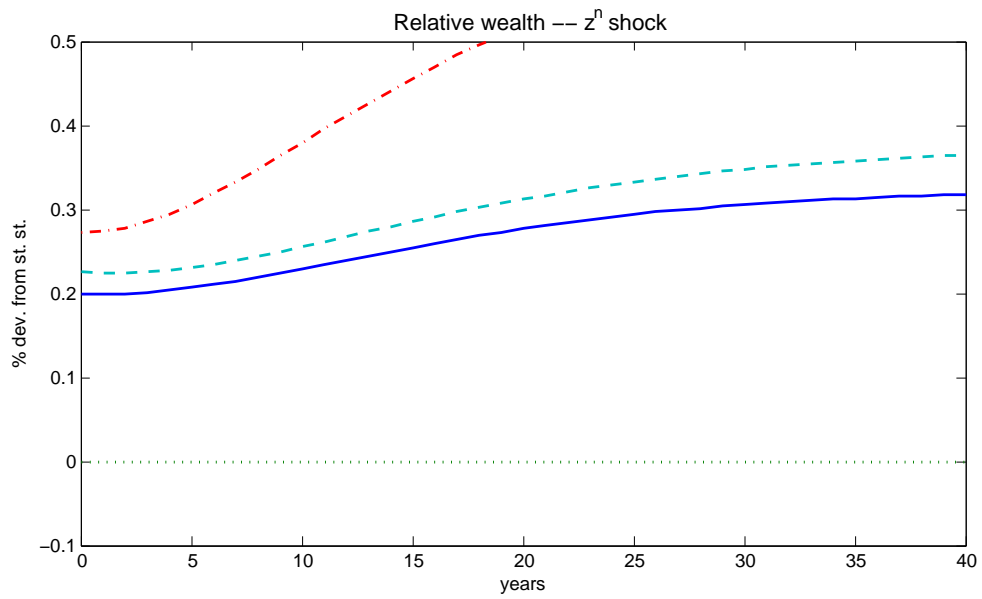
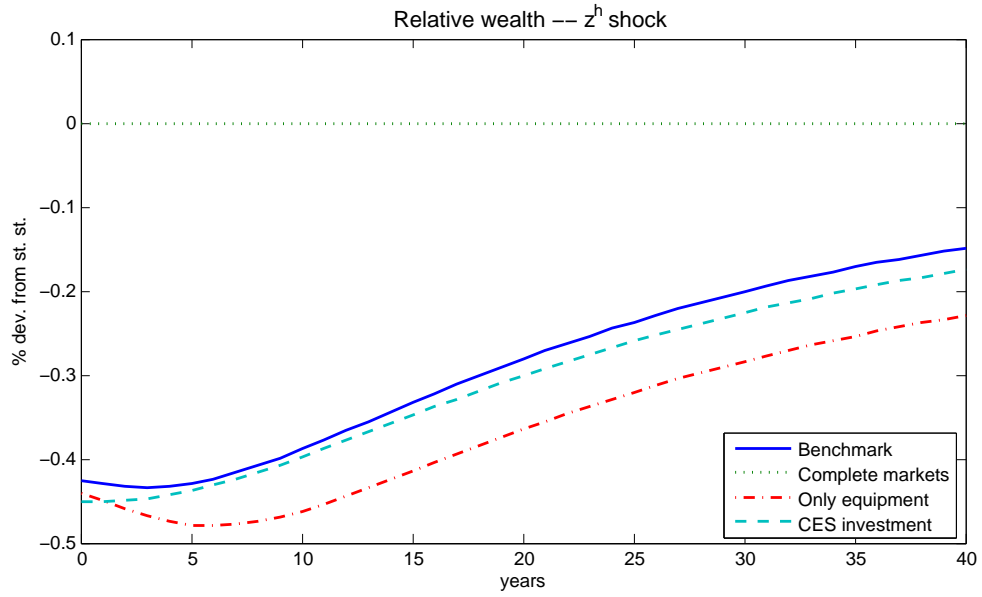


Figure 2.2: Relative wealth in variations of the model



gain in wealth.<sup>14</sup>

#### 2.6.4 CES Investment

In the benchmark model, production requires services from the stock of equipment and structures. However, real business cycle models often assume that investment, like consumption, is a composite good (see, for example, Backus, Kehoe, and Kydland (1995)). The appeal to this approach is the way the final investment good is defined, which is conceptually similar to how the input-output use table is presented. In the input-output use table, private fixed investment is “produced” using several commodity inputs including manufacturing, which is traded, and construction, which is nontraded.<sup>15</sup> The final investment good  $x_1(s^t)$  is given by the following Constant Elasticity of Substitution (CES) aggregator:

$$x_1(x_{e,1}(s^t), x_{d,1}(s^t)) = \left[ \nu^{\frac{1}{\sigma}} x_{e,1}(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\nu)^{\frac{1}{\sigma}} x_{d,1}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where  $\sigma$  is the elasticity of substitution between purchases of new equipment and purchases of new structures;  $1-\nu$  is the share of structures;  $x_{e,1}(s^t)$  is defined as in equation (2.8); and  $x_{d,1}(s^t)$  is the amount of nontraded output devoted to investment. The final investment good can be freely allocated within the border, that is, between traded and nontraded good sectors. The capital stocks, which are not internationally mobile, are given by the following accumulation equations

$$\begin{aligned} k_1^h(s^t) &= (1-\delta)k_1^h(s^{t-1}) + x_1^h(s^t) - \frac{\phi}{2}k_1^h(s^{t-1}) \left( \frac{x_1^h(s^t)}{k_1^h(s^{t-1})} - \delta \right)^2 \\ k_1^n(s^t) &= (1-\delta)k_1^n(s^{t-1}) + x_1^n(s^t) - \frac{\phi}{2}k_1^n(s^{t-1}) \left( \frac{x_1^n(s^t)}{k_1^n(s^{t-1})} - \delta \right)^2, \end{aligned}$$

<sup>14</sup>One can think of a productivity improvement in the nontraded sector in the absence of structures as an improvement that is biased toward a consumption good. Kimball (1994) shows that permanent productivity shocks to the production of nondurable consumption have little transition dynamics because the new steady state can be reached immediately. Specifically, a permanent productivity improvement leads to a proportional increase in nondurable consumption and a proportional decrease in its price, while there are no changes in labor supply or investment.

<sup>15</sup>See appendix A for actual data on the construction commodity in the United States input-output table.

where  $\delta$  is the depreciation rate of this composite capital good, and  $x_1^h(s^t) + x_1^n(s^t) = x_1(s^t)$ . Since the final investment good bundle is defined analogously to the final consumption good bundle, it is easy to show that the demands for equipment and structures are given by

$$\begin{aligned} P_{x,1}(s^t)x_1'(x_{e,1}(s^t)) &= P_{e,1}(s^t) \\ \left(\frac{\nu}{1-\nu}\frac{x_{d,1}(s^t)}{x_{e,1}(s^t)}\right)^{\frac{1}{\sigma}} &= \frac{P_{e,1}(s^t)}{p_{n,1}(s^t)}. \end{aligned}$$

Note that  $P_{x,1}(s^t) = [\nu(P_{e,1}(s^t))^{1-\sigma} + (1-\nu)(p_{n,1}(s^t))^{1-\sigma}]^{\frac{1}{1-\sigma}}$ . The CES assumption has a strong implication that the only factor affecting investment in equipment relative to investment in structures is the current period relative price. This assumption ignores the fact that expected future price changes can be powerful factors for durable goods like equipment and structures. For example, households may want to buy more structures than equipment today albeit the high relative price of structures in the current period if the relative price of structures is rising.<sup>16</sup>

The share  $\nu$  is set at 0.67 so that, as in the benchmark model, investment in equipment is twice the size of investment in structures. The depreciation rate  $\delta$  is set at 10 percent. The elasticity of substitution  $\sigma$  is set at 0.58 based on an estimate using the US data for the period 1970-2006.<sup>17</sup>

Business cycle properties of the model with this investment specification are reported in tables 2.5 and 2.6 under ‘‘CES Investment.’’ Unlike the benchmark model, the behavior of investment in equipment and the behavior of investment in structures are similar. This result is not surprising given the high degree of complementarity

<sup>16</sup>Using equation (2.41), the rising relative price of structures means that  $\tilde{r}\tilde{p}(s^t) < \tilde{r}\tilde{p}(s^{t+1})$ . This condition implies that investment in structures will be larger than investment in equipment. The rationale is analogous to the paragraph following the same equation.

<sup>17</sup>The estimation is of the form

$$\Delta \log \frac{E_t}{S_t} = (1-\sigma)\Delta \log \frac{P_{E,t}}{P_{S,t}} + \epsilon_t,$$

where  $E$  and  $S$  are gross private domestic investment in equipment and nonresidential structures at current price;  $P_E$  is the price index for equipment and softwares; and  $P_S$  is the price index for nonresidential structures. The estimate of  $1-\sigma$  is 0.42 with the standard deviation of 0.2. Data series are obtained from the BEA.

between the two components. Because it is costly to deviate from the steady state mix of investments, it is not efficient to purchase more equipment without also purchasing more structures. Furthermore, an improvement in the productivity of the traded good sector generates a smaller boom in equipment investment. This result explains why the cross-country correlation of investment is high and why net exports is procyclical.

The model still does a good job at replicating the lack of international risk sharing. Figure 2.2 shows that the responses of relative wealth are similar to those of the benchmark model. Clearly, the higher Backus-Smith correlation relative to the benchmark case is explained by a smaller wealth effect following a shock to the nontraded good sector. This finding is related to the fact that the complementarity between purchases of equipment and structures is a friction that makes it less attractive to increase investment in structures alone. In particular, an improvement in the productivity of the nontraded good sector generates a smaller boom in investment in structures and a smaller wealth effect.

## 2.7 Sensitivity analysis

Given the uncertainty surrounding the elasticity of substitution parameter, table 2.7 reports the results for different values of  $\sigma$ .

For the benchmark model, lowering  $\sigma$  clearly improves its ability to replicate behavior of investment. Most noticeably, investment in structures is now procyclical and cross-country correlations of investment are now positive. As  $\sigma$  gets lower, it is more and more costly to deviate from the steady state mix of services from the stock of equipment and structures. Although the rental rate of structures is temporarily high after an improvement in traded good productivity, firms still need to keep the

Table 2.7: Varying the elasticity of substitution

Statistics	Data	Benchmark economy			CES investment		
		$\sigma = 0.1$	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 0.1$	$\sigma = 0.5$	$\sigma = 1.0$
<i>Correlations with GDP</i>							
Investment, total	0.87	0.93	0.87	0.86	0.95	0.95	0.95
Investment, equipment	0.79	0.86	0.79	0.79	0.95	0.95	0.95
Investment, structures	0.43	0.81	0.15	-0.20	0.96	0.96	0.94
Net exports/GDP	-0.49	-0.004	-0.09	-0.12	0.21	0.18	0.14
<i>Cross-country correlations</i>							
Investment, total	0.25	0.08	-0.15	-0.23	0.35	0.33	0.31
Investment, equipment	0.24	0.15	0.03	0.09	0.36	0.36	0.36
Investment, structures	0.004	0.16	0.51	0.68	0.33	0.25	0.10
Backus-Smith correlation	-0.27	0.05	0.02	0.06	0.18	0.17	0.16

Annual data series from 1970 to 2004 of the United States are used to compute correlations with GDP. Investment in equipment is Gross private domestic fixed investment in equipment. Investment in structures is Gross private domestic fixed investment in nonresidential structures. Cross-country correlations are averages of correlations between the US with Canada, France, Germany, Italy, Japan, and the United Kingdom. All series have been logged and Hodrick-Prescott filtered with a smoothing parameter of 100. The statistics from the model are the averages of 100 simulations.

ratio of equipment to structures close to the level prior to the shock. This condition implies that there is a strong demand for structures, which explains why investment in structures is procyclical. The cross-country correlation of investment is higher because the benefit of reallocating investment in equipment across countries is lower, similar to what was described in the CES investment model. While the lower  $\sigma$  makes net exports become less countercyclical, it does not have a significant effect on the Backus-Smith correlation.

For the CES investment model, lowering  $\sigma$  does not have a significant effect on the behavior of investment. However, it exacerbates the counterfactual implication for the behavior of net exports.<sup>18</sup>

<sup>18</sup>Pakko (2003) shows that introducing investment adjustment costs in a two-country model can induce positive investment co-movement; however, it has the side-effect of reversing the cyclical behavior of net exports. Thus, the complementarity between equipment and structures can be considered an example of microeconomic-founded investment adjustment costs.

## 2.8 Conclusion

This paper presents a unified framework to address two business cycle regularities that are not well replicated by previous models: the behavior of investment and the extent of international risk sharing. The model developed here has a unique feature that the nontraded good, in addition to the traded goods, is used in investment, which is supported by the fact that equipment and structures are factors of production. There are two ways in which the presence of structures improves the model's performance. First, the complementarity between equipment and structures reduces the gain from reallocating investment in equipment across countries. This property enables the model to replicate the positive co-movement of aggregate investment across countries observed in the data. Second, investment in structures is a mechanism for households to smooth consumption and to increase the wealth effect following shocks to nontraded output. This property reduces the extent of international risk sharing. The model predicts a low correlation between relative consumption and the real exchange rate when asset markets are incomplete. Thus, the model also offers a solution to one of the important puzzles in international finance.

Shortcomings of this model include the fact that both price and quantity anomalies are present. As long as some investment goods are traded, a tendency to shift resources to wherever the productivity is high will make the predicted cross-country correlation of output lower than that of consumption. While models without nominal rigidities like this one are not expected to generate much volatility of international relative prices, Heathcote and Perri (2002) show that it is possible if countries are not allowed to borrow or lend. The consideration of nominal rigidities or other forms

of asset market imperfections are left for future research.

## 2.9 Appendix

### 2.9.1 Appendix A: Calibrating the shares of nontraded goods and imported goods in investment from Input-Output tables

(Some of the ideas in following discussion are based on a note provided to the author by Miles Kimball.)

The Input-Output (I-O) Use tables offer another way to calibrate the shares of the nontraded good and the imported good in investment. The rows of the Use tables represent commodity inputs in the “production” of commodities and composite outputs (consumption, investment, government purchases and exports) which are the columns of the Use table. While exports can be viewed as inputs for getting imports, imports of a commodity can be viewed as inputs in the production of that commodity.

#### **The share of nontraded goods in investment**

In section 2.4, the share of structures is calibrated using the information from the NIPA table that investment in nonresidential structures is about half the size of investment in equipment. From the I-O Use table, private fixed investment uses a commodity “construction” as one of the inputs. Construction is a nontraded commodity because its trade share is zero (see table 2.8). Private fixed investment can be thought of as a composite good, which is produced by other commodities including construction. One can say that the commodity inputs are complementary at the time of production of fixed investment.

Furthermore, table 2.8 shows that nonresidential construction commodity is worth 251,521 million dollars. In that same year, manufacturing commodity (which represents equipment) as an input in private fixed investment is worth 554,541 million

Table 2.8: The use of commodities by industries

Commodity/Industry	Private Fixed Investment	Exports of Goods and Services
<i>Construction</i>		
New residential 1-unit structures, nonfarm	172,282	0
New multifamily housing structures, nonfarm	23,673	0
New residential additions and alterations, nonfarm	53,349	0
New farm housing units and additions and alterations	5,429	0
Manufacturing and industrial buildings	24,833	0
Commercial and institutional buildings	141,096	0
Highway, street, bridge, and tunnel construction	721	0
Water, sewer, and pipeline construction	3,000	0
Other new construction	54,538	0
Maintenance and repair of farm and nonfarm residential structures	23,495	6
Maintenance and repair of nonresidential buildings	0	90
Maintenance and repair of highways, streets, bridges, and tunnels	0	0
Other maintenance and repair construction	3,838	0
Sum of commodity input	506,254	96
Total Industry output	1,319,909	898,584
Commodity input as a share of Industry output	0.38	0

The unit is millions of dollars. This table use data from 1997 benchmark Input-Output Use table of the Bureau of Economic Analysis.

dollars. This means that nonresidential construction commodity is about half the size of manufacturing commodity. This ratio is similar to the ratio of investment in nonresidential structures to investment in equipment. Thus, a calibration of this share using the I-O table yields approximately the same value as the calibration strategy in section 2.4.

#### **The share of imported goods in investment**

In the model, equipment is a composite good. Its components are the domestically-produced traded good and the foreign-produced traded good. The following discussion shows how to calculate the import share in equipment. Manufactured goods (code 3) are mostly traded goods. They are commodity inputs in the production of private fixed investment and private consumption. With the idea that international trade is a technology that turns exports into imports, some of these inputs are actually imports. Total imports as a share of total manufacturing commodity output is approximately 30 percent. However, it is not appropriate to assume that consumption and investment have the same import share because the degree of home bias may differ. The benchmark Use table for 1997 contains about 320 commodities under manufacturing. The majority of commodities under codes 31 and 32 are consumption goods as their use shares under private fixed investment are mostly zeroes. For the same reason, commodities under code 33 are investment goods. For each commodity, the share of imports is calculated by dividing imports by total commodity output. The average of the import shares of consumption goods is 17, while the average of the import shares of investment goods is 28. These numbers are very close to the 15 and 30 percent values derived in section 2.4.

In conclusion, the calibration in the paper is consistent with the implications of the I-O table.



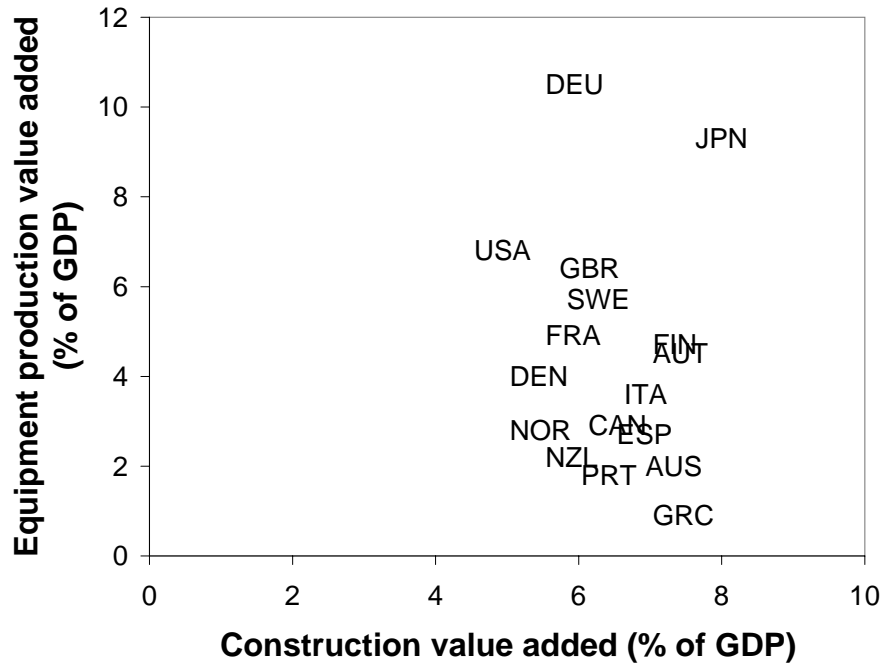


Figure 2.3: Production of equipment and structures

### 2.9.2 Appendix B: Production of equipment and structures

While a country that has a high level of investment in equipment tends to have a high level of investment in structures, the same statement cannot be made for production of equipment and structures. Eaton and Kortum (2001) document that production of equipment is concentrated in a few countries, far more concentrated than production of manufactured goods. In the data, the share of equipment production in GDP varies greatly across countries even when non-OECD countries are excluded. Figure 2.3 plots the share of equipment production in GDP against the share of construction, which is used to proxy for the production of structures. For each country, equipment production is represented by the value added of the equipment producing industries reported by Eaton and Kortum (2001) for 1985. Shares of construction are the value added shares of construction, also for 1985, reported by the OECD STAN Structural Analysis database.

## CHAPTER III

# Trade in Durable Goods in an International Real Business Cycle Model

### 3.1 Introduction

Changes in the terms of trade affect the demand for traded goods. In general, a deterioration of the terms of trade is expected to induce a substitution away from imported goods toward domestic goods because a deterioration represents an increase in the relative price of imported goods. At business cycle frequency, the correlation between the terms of trade and the import ratio tells us the extent of the substitution.<sup>1</sup> Although the correlation is generally less than zero, it differs significantly across groups of goods. Specifically, the correlation is  $-0.48$  for consumption goods, while it is  $-0.06$  for investment goods. One can examine the two-country, two-good real business cycle model in Backus, Kehoe, and Kydland (1995) to see the relationship between the terms of trade and the import ratio. In that model, however, the assumption that the imported good and the domestic good have a constant elasticity of substitution in consumption and investment implies that changes in the import ratio and changes in the terms of trade are proportional, that is, the correlation between the import ratio and the terms of trade is exactly  $-1$ .

The implication of changes in the import ratio and changes in the terms of trade

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<sup>1</sup>The import ratio is defined as the expenditure on imported goods divided by the expenditure on domestically produced goods.

being proportional is that future changes in the terms of trade have no effect on the demand for traded goods today. This condition fails to capture the influence of expected future price changes on the demand for durable goods.<sup>2</sup> Specifically, a deterioration of the terms of trade can actually induce a substitution toward imported goods if the deterioration is expected to continue, which means that the price of the imported good in the current period is temporarily low. For this reason, it is plausible that the correlation between the terms of trade and the import ratio of durable goods is far from  $-1$ . Because most investment goods are durable goods while most consumption goods are nondurable goods, expected price changes are more important for investment goods.

This paper develops a model to evaluate whether the effect of expected price changes on purchases of durable goods can explain why terms of trade movements affect the demand for traded goods in consumption differently from those in investment. This model has a unique feature in that production requires services from both domestic and imported capitals.<sup>3</sup> Because capital goods are durable, the flow of capital goods across borders represents trade in durable goods. While services from the two types of capital have a constant elasticity of substitution, purchases of goods for investment do not, which means that changes in the import ratio and changes in the terms of trade do not need to be proportional. The capital service model is in fact quite similar to the equipment-structure closed economy model introduced by Greenwood, Hercowitz, and Krusell (1997). For the open economy setting, Eaton and Kortum (2001) introduce an empirical model of trade in capital goods in which services from a continuum of different types of capital are required in production.

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<sup>2</sup>Burda and Gerlach (1992) also study the effect of intertemporal price changes on purchases of durable goods. They focus on explaining the behavior of trade balance in the 1980s.

<sup>3</sup>An airline is an example of such production. To provide transportation services, an airline needs a fleet of Boeing planes *and* a fleet of Airbus planes. The complementarity between the fleets enables the airline to cover a wide range of destinations.

The results of the capital service model are compared against those of a standard two-country, two-good model.

The model with trade in durable goods replicates the observed business cycle property that the correlation between the terms of trade and the import ratio of investment goods is different from that of consumption goods. The model's distinguishing feature is that the deterioration of the terms of trade following a productivity shock immediately raises the demand for the more expensive imported capital good relative to the cheaper domestic capital good. The substitution toward the cheaper good takes place only over time. Moreover, this model can generate the very high volatility of trade quantities (imports and exports) observed in the data.

The key mechanism behind the results is indeed the intertemporal effect of the terms of trade movements. The smaller deterioration of the terms of trade at the date of the productivity shock than its peak level has effects that are similar to a temporary drop in the relative price of imported capital. With the ability to borrow in asset markets, that country may find it economical to increase the stock of imported capital immediately. Hence, this intertemporal effect leads to large inflows of imported capital goods and few expenditure switchings. Eventually, the deterioration of the terms of trade winds down, and the demand for domestic capital increases relative to that of imported capital. While the volatility of trade quantities is sensitive to the degree of complementarity between capital services, the substitution toward the expensive imported capital is not. This model also provides a good example of how the ability to borrow or lend internationally has benefits beyond the facilitation of consumption risk sharing.

The paper is organized as follows. Section 2 develops the model with trade in capital goods and contrasts it with a typical two-country, two-good model. Sec-

tion 3 discusses the parameterizations of the models and their implications for the steady state. Section 4 reports the results and interpretation. Section 5 discusses the strengths and weaknesses of the model with trade in durable goods. Section 6 provides conclusions.

### 3.2 The models

The notation used in the model setup is based on Heathcote and Perri (2002). The world consists of two countries of the same size. Country 1 specializes in the production of intermediate good  $a$ , while country 2 specializes in the production of intermediate good  $b$ . The law of one price holds for intermediate goods. Households in both countries demand goods  $a$  and  $b$  for consumption and investment. Households in country 1(2) supply labor and rent capital to perfectly competitive firms producing good  $a(b)$ . Households in each country own the capital stock of that country. Neither labor nor the capital stock is internationally mobile.

In each period  $t$  the economy experiences one event  $s_t \in S$  where  $S$  is a finite set of events. Let  $s^t$  be the history of events up to and including date  $t$ , the probability at date 0 of any particular history  $s^t$  is  $\pi(s^t)$ .

#### 3.2.1 A model of trade in durable goods

This model is referred to as “the capital service model” hereafter. The production of intermediate goods in country  $i = \{1, 2\}$  requires services from domestically produced capital, imported capital, and labor. Let  $k_i^a(s^{t-1})$  and  $k_i^b(s^{t-1})$  be the stocks of capital, and  $n_i(s^t)$  be labor hours in period  $t$ . The production function takes the following form

$$(3.1) \quad F(z_i(s^t), k_i^a(s^{t-1}), k_i^b(s^{t-1}), n_i(s^t)) = e^{z_i(s^t)} K_i (s^{t-1})^\alpha n_i(s^t)^{1-\alpha},$$

where  $\alpha$  is the capital share,  $z_i(s^t)$  is an exogenous technology shock and  $K_i(s^{t-1})$  is the aggregate services of capital defined as

$$(3.2) \quad K_i(k_i^a(s^t), k_i^b(s^t)) = \begin{cases} \left[ \theta^{\frac{1}{\rho}} k_i^a(s^t)^{\frac{\rho-1}{\rho}} + (1-\theta)^{\frac{1}{\rho}} k_i^b(s^t)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, & i = 1 \\ \left[ (1-\theta)^{\frac{1}{\rho}} k_i^a(s^t)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}} k_i^b(s^t)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, & i = 2, \end{cases}$$

where  $\rho$  is the elasticity of substitution between services of capital types  $a$  and  $b$ , and  $1 - \theta$  is the share of service of imported capital. In this model, the complementarity between services of capital makes it costly to deviate from the steady state mix of the stocks of capital. Let  $w_i(s^t)$ ,  $R_i^a(s^t)$  and  $R_i^b(s^t)$  be the wage and rental rate on capital of types  $a$  and  $b$ , respectively in country  $i$  in terms of the intermediate good produced in country  $i$ . The maximization problem of perfectly competitive intermediate good producers in country  $i$  is given by

$$\begin{aligned} \max_{k_i^a(s^{t-1}), k_i^b(s^{t-1}), n_i(s^t)} \quad & \{ F(z_i(s^t), k_i^a(s^{t-1}), k_i^b(s^{t-1}), n_i(s^t)) \\ & - w_i(s^t)n_i(s^t) - R_i^a(s^t)k_i^a(s^{t-1}) - R_i^b(s^t)k_i^b(s^{t-1}) \}. \end{aligned}$$

For country 1, the demand for capital services and the demand for labor are given by

$$(3.3) \quad F'(k_1^a(s^{t-1})) = R_1^a(s^t)$$

$$(3.4) \quad \left( \frac{\theta k_1^b(s^{t-1})}{1 - \theta k_1^a(s^{t-1})} \right)^{\frac{1}{\rho}} = \frac{R_1^a(s^t)}{R_1^b(s^t)}$$

$$(3.5) \quad F'(n_1(s^t)) = w_1(s^t),$$

where  $F'(x)$  denotes the derivative of a function  $F$  with respect to  $x$ . It is convenient to think of the demand for capital service as a mix of capital services that minimizes the cost of  $K_1$ .

Households in country  $i$  accumulate and own two types of capital used in country

$i$ . The accumulation equations are given by

$$(3.6) \quad k_i^a(s^t) = (1 - \delta)k_i^a(s^{t-1}) + a_{ix}(s^t)$$

$$(3.7) \quad k_i^b(s^t) = (1 - \delta)k_i^b(s^{t-1}) + b_{ix}(s^t),$$

where  $\delta$  is the depreciation rate, and  $a_{ix}(s^t)$  and  $b_{ix}(s^t)$  are the amount of intermediate good of each type devoted to investment in country  $i$ . In this setup, international trade in goods  $a$  and  $b$  for investment represents trade in durable goods.

The final consumption good is a composite good, which is produced with intermediate goods  $a$  and  $b$  as inputs by a CES production technology:

$$(3.8) \quad c_i(a_{ic}(s^t), b_{ic}(s^t)) = \begin{cases} \left[ \omega^{\frac{1}{\sigma}} a_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} + (1 - \omega)^{\frac{1}{\sigma}} b_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 1 \\ \left[ (1 - \omega)^{\frac{1}{\sigma}} a_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 2, \end{cases}$$

where  $\sigma$  is the elasticity of substitution between goods  $a$  and  $b$ , and  $1 - \omega$  is the share of imported goods. Let  $q_i^a(s^t)$  and  $q_i^b(s^t)$  be the prices of goods  $a$  and  $b$  in country  $i$  in units of the final consumption good produced in country  $i$ , the following maximization problem gives the demand for each intermediate good:

$$\max_{a_{ic}(s^t), b_{ic}(s^t)} \{c_i(a_{ic}(s^t), b_{ic}(s^t)) - q_i^a(s^t)a_{ic}(s^t) - q_i^b(s^t)b_{ic}(s^t)\}.$$

For country 1, the demand for intermediate goods in consumption is determined by

$$(3.9) \quad c'(a_{1c}(s^t)) = q_1^a(s^t)$$

$$(3.10) \quad \left( \frac{\omega}{1 - \omega} \frac{b_{1c}(s^t)}{a_{1c}(s^t)} \right)^{\frac{1}{\sigma}} = \frac{q_1^a(s^t)}{q_1^b(s^t)}.$$

The terms of trade, defined as the price of imports into country 1 relative to exports from country 1, is given by

$$(3.11) \quad tot(s^t) = \frac{q_1^b(s^t)}{q_1^a(s^t)}.$$

Note that while changes in the ratio of imported good to domestic good in consumption are proportional to changes in the terms of trade due to the constant elasticity of substitution specification, there is no such direct relationship between the ratio of imported good to domestic good in investment. This distinction is unique to the capital service model and has a nice implication for its business cycle properties.

Assume that asset markets are complete. Let  $B_i(s^t, s_{t+1})$  be the quantity of bonds purchased by households in country  $i$  after history  $s^t$  that pay one unit of good  $a$  in period  $t + 1$  if and only if the state of the economy is  $s_{t+1}$ , and let  $Q(s^t, s_{t+1})$  be the price in units of good  $a$  of these bonds. The maximization problem of the representative household in country 1 is given by

$$\begin{aligned} & \max_{c_1(s^t), n_1(s^t), B_1(s^t), k_1^a(s^t), k_1^b(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ \frac{1}{\gamma} [c_1(s^t)^\mu (1 - n_1(s^t))^{1-\mu}]^\gamma \right. \\ & + \lambda_1(s^t) \left[ q_1^a(s^t) (R_1^a(s^t) k_1^a(s^{t-1}) + R_1^b(s^t) k_1^b(s^{t-1}) + w_1(s^t) n_1(s^t)) + q_1^a(s^t) B_1(s^{t-1}) \right. \\ & \quad \left. \left. - q_1^a(s^t) \sum_{s^{t+1}} Q(s^t, s_{t+1}) B_1(s^t, s_{t+1}) - c_1(s^t) - q_1^a(s^t) a_{1x}(s^t) - q_1^b(s^t) b_{1x}(s^t) \right] \right. \\ & + \Omega_1^a(s^t) \left[ -k_1^a(s^t) + (1 - \delta) k_1^a(s^{t-1}) + a_{1x}(s^t) - \frac{\phi}{2} k_1^a(s^{t-1}) \left( \frac{a_{1x}(s^t)}{k_1^a(s^{t-1})} - \delta \right)^2 \right] \\ & \left. + \Omega_1^b(s^t) \left[ -k_1^b(s^t) + (1 - \delta) k_1^b(s^{t-1}) + b_{1x}(s^t) - \frac{\phi}{2} k_1^b(s^{t-1}) \left( \frac{b_{1x}(s^t)}{k_1^b(s^{t-1})} - \delta \right)^2 \right] \right\}, \end{aligned}$$

where  $\beta$  is the discount rate;  $1 - \gamma$  is the risk aversion parameter;  $\mu$  is the consumption share;  $\lambda_1(s^t)$  is the marginal utility of consumption;  $\Omega_1^a(s^t)$  and  $\Omega_1^b(s^t)$  are the shadow prices of capital of types  $a$  and  $b$ , respectively. Let  $U(s^t) \equiv \frac{1}{\gamma} [c_i(s^t)^\mu (1 - n_i(s^t))^{1-\mu}]^\gamma$  denotes the household's utility. Labor supply and the Euler equations for consump-



tion and investment when the investment adjustment cost  $\phi$  is 0 are given by

$$(3.12) \quad -\frac{U'(n_1(s^t))}{U'(c_1(s^t))} = q_1^a(s^t)w_1(s^t)$$

$$(3.13) \quad \lambda_1(s^t)q_1^a(s^t) = \Omega_1^a(s^t)$$

$$(3.14) \quad \frac{q_1^b(s^t)}{q_1^a(s^t)} = \frac{\Omega_1^b(s^t)}{\Omega_1^a(s^t)}$$

$$(3.15) \quad \Omega_1^a(s^t) = \beta(1 - \delta)\Omega_1^a(s^{t+1}) + \beta\lambda_1(s^{t+1})q_1^a(s^{t+1})R_1^a(s^{t+1})$$

$$(3.16) \quad \Omega_1^b(s^t) = \beta(1 - \delta)\Omega_1^b(s^{t+1}) + \beta\lambda_1(s^{t+1})q_1^a(s^{t+1})R_1^b(s^{t+1})$$

$$(3.17) \quad Q(s^t, s_{t+1}) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)} \frac{q_1^a(s^{t+1})}{q_1^a(s^t)}.$$

Equation (3.12) is the labor supply condition. Note that  $q_1^a(s^t)w_1(s^t)$  is the real wage in units of the final consumption good. A unit of intermediate good can either be consumed or invested. Equations (3.13) and (3.14) show the shadow prices of investment goods, which are prices in utility units. Note that, without the investment adjustment cost, the ratio of the shadow prices is just the terms of trade. Equations (3.15) and (3.16) are the Euler equations showing the trade off between consumption and investments in  $k_1^a$ , and  $k_1^b$ , respectively. Using equations (3.17), it is easy to show that the marginal utility of consumption in country 2 relative to the marginal utility of consumption in country 1 is equal to  $\frac{q_1^a(s^t)}{q_2^a(s^t)}$ , the real exchange rate.<sup>4</sup> This is a well-known implication of the complete asset market assumption. Let  $rx(s^t)$  denote the consumption based real exchange rate:

$$(3.18) \quad rx(s^t) = \frac{q_1^a(s^t)}{q_2^a(s^t)}.$$

Investment expenditure  $x_1(s^t)$  is defined as the sum of expenditure on intermediate

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<sup>4</sup>Regardless of the asset market assumption, the consumption based real exchange rate is defined as the price of consumption in country 2 relative to consumption in country 1. Using the law of one price for intermediate goods,

$$rx(s^t) = \frac{q_1^a(s^t)}{q_2^a(s^t)} = \frac{q_1^b(s^t)}{q_2^b(s^t)}.$$

goods devoted to investment:

$$(3.19) \quad x_1(s^t) = q_1^a(s^t)a_{1x}(s^t) + q_1^b(s^t)b_{1x}(s^t).$$

In the equilibrium, goods markets and bond markets clear:

$$(3.20) \quad a_{1c}(s^t) + a_{1x}(s^t) + a_{2c}(s^t) + a_{2x}(s^t) = F(z_1(s^t), K_1(s^{t-1}), n_1(s^t))$$

$$(3.21) \quad b_{1c}(s^t) + b_{1x}(s^t) + b_{2c}(s^t) + b_{2x}(s^t) = F(z_2(s^t), K_2(s^{t-1}), n_2(s^t))$$

$$(3.22) \quad B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0, \forall s_{t+1} \in S.$$

### 3.2.2 A typical two-country model

This model is referred to as “the final good model” hereafter. As in Heathcote and Perri (2002), intermediate goods  $a$  and  $b$  are used to produce the final good, which can be consumed or invested. The final good production in country  $i$  is represented by

$$G_i(a_i(s^t), b_i(s^t)) = \begin{cases} \left[ \omega^{\frac{1}{\sigma}} a_i(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_i(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 1 \\ \left[ (1-\omega)^{\frac{1}{\sigma}} a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_i(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 2 \end{cases}$$

Let  $x_i(s^t)$  be the amount of the final good devoted to investment. The capital accumulation equation is given by

$$(3.23) \quad k_i(s^t) = (1-\delta)k_i(s^{t-1}) + x_i(s^t),$$

where  $k_i(s^t)$  the only one type of capital. The production of intermediate good in country  $i$  takes the following form

$$(3.24) \quad F(z_i(s^t), k_i(s^{t-1}), n_i(s^t)) = e^{z_i(s^t)} k_i(s^{t-1})^\alpha n_i(s^t)^{1-\alpha}.$$

The maximization problem of perfectly competitive intermediate good producers is given by

$$\max_{k_i(s^{t-1}), n_i(s^t)} \{ F(z_i(s^t), k_i(s^{t-1}), n_i(s^t)) - w_i(s^t)n_i(s^t) - R_i(s^t)k_i(s^{t-1}) \}.$$

For country 1, the demand for capital service and the demand for labor are determined by

$$(3.25) \quad F'(k_1(s^{t-1})) = R_1(s^t)$$

$$(3.26) \quad F'(n_1(s^t)) = w_1(s^t).$$

To facilitate comparisons with the capital service model, rewrite the final good production so that there is a distinction between the demands for intermediate goods in consumption and those in investment:

$$(3.27) \quad c_i(a_{ic}(s^t), b_{ic}(s^t)) = \begin{cases} \left[ \omega^{\frac{1}{\sigma}} a_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 1 \\ \left[ (1-\omega)^{\frac{1}{\sigma}} a_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 2 \end{cases}$$

$$(3.28) \quad x_i(a_{ix}(s^t), b_{ix}(s^t)) = \begin{cases} \left[ \omega^{\frac{1}{\sigma}} a_{ix}(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} b_{ix}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 1 \\ \left[ (1-\omega)^{\frac{1}{\sigma}} a_{ix}(s^t)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} b_{ix}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 2 \end{cases}$$

In this model, the complementarity between intermediate goods in the production of the final investment good makes it costly to deviate from the steady state mix of the flow of purchases of intermediate goods. Furthermore, international trade no longer includes durable goods because the capital goods,  $x_1$  and  $x_2$ , in the model are country-specific.

Each country maximizes the production of final good given its preference over the intermediate goods and their prices:<sup>5</sup>

$$\begin{aligned} & \max_{a_{ic}(s^t), b_{ic}(s^t)} \left\{ c_i(a_{ic}(s^t), b_{ic}(s^t)) - q_i^a(s^t) a_{ic}(s^t) - q_i^b(s^t) b_{ic}(s^t) \right\} \\ & \max_{a_{ix}(s^t), b_{ix}(s^t)} \left\{ x_i(a_{ix}(s^t), b_{ix}(s^t)) - q_i^a(s^t) a_{ix}(s^t) - q_i^b(s^t) b_{ix}(s^t) \right\}. \end{aligned}$$

<sup>5</sup>Note that the price of  $x$  in units of  $c$  is equal to 1 by assumption. Hence, there is no need to include the relative price of investment in the maximization problem for the final investment good.

Consequently, the demand for intermediate goods in country 1 is determined by

$$(3.29) \quad c'(a_{1c}(s^t)) = q_1^a(s^t)$$

$$(3.30) \quad \left( \frac{\omega}{1-\omega} \frac{b_{1c}(s^t)}{a_{1c}(s^t)} \right)^{\frac{1}{\sigma}} = \frac{q_1^a(s^t)}{q_1^b(s^t)}$$

$$(3.31) \quad x'(a_{1x}(s^t)) = q_1^a(s^t)$$

$$(3.32) \quad \left( \frac{\omega}{1-\omega} \frac{b_{1x}(s^t)}{a_{1x}(s^t)} \right)^{\frac{1}{\sigma}} = \frac{q_1^a(s^t)}{q_1^b(s^t)}.$$

Note that the substitution between domestic and imported goods at time  $t$  is a function of the terms of trade at time  $t$ . This result is true for the investment bundle as well as the consumption bundle.

The representative household in country 1 maximizes utility:

$$\begin{aligned} & \max_{c_1(s^t), n_1(s^t), B_1(s^t), k_1(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ \frac{1}{\gamma} [c_1(s^t)^\mu (1 - n_1(s^t))^{1-\mu}]^\gamma \right. \\ & + \lambda_1(s^t) \left[ q_1^a(s^t) (R_1(s^t) k_1(s^{t-1}) + w_1(s^t) n_1(s^t)) + q_1^a(s^t) B_1(s^{t-1}) \right. \\ & \quad \left. \left. - q_1^a(s^t) \sum_{s^{t+1}} Q(s^t, s_{t+1}) B_1(s^t, s_{t+1}) - c_1(s^t) - x_1(s^t) \right] \right. \\ & \left. + \Omega_1(s^t) [-k_1(s^t) + (1 - \delta) k_1(s^{t-1}) + x_1(s^t)] \right\}. \end{aligned}$$

Labor supply and the Euler equations for consumption and investment are given by

$$(3.33) \quad -\frac{U'_{n_1(s^t)}}{U'_{c_1(s^t)}} = q_1^a(s^t) w_1(s^t)$$

$$(3.34) \quad \lambda_1(s^t) = \Omega_1^a(s^t)$$

$$(3.35) \quad Q(s^t, s_{t+1}) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)} \frac{q_1^a(s^{t+1})}{q_1^a(s^t)}$$

$$(3.36) \quad \Omega_1(s^t) = \beta(1 - \delta) \Omega_1(s^{t+1}) + \beta \lambda_1(s^{t+1}) q_1^a(s^{t+1}) R_1(s^{t+1})$$

Because the final consumption good and the final investment good are produced using the same technique, the marginal utility of consumption and the shadow price of capital are equal.

In the equilibrium, goods markets and bond markets clear:

$$(3.37) \quad a_{1c}(s^t) + a_{1x}(s^t) + a_{2c}(s^t) + a_{2x}(s^t) = F(z_1(s^t), k_1(s^{t-1}), n_1(s^t))$$

$$(3.38) \quad b_{1c}(s^t) + b_{1x}(s^t) + b_{2c}(s^t) + b_{2x}(s^t) = F(z_2(s^t), k_2(s^{t-1}), n_2(s^t))$$

$$(3.39) \quad B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0, \forall s_{t+1} \in S.$$

#### Additional variables of interest

Let  $y_i(s^t)$  denote Gross Domestic Product (GDP) in country  $i$  in units of the final consumption good. For the capital service model,

$$(3.40) \quad y_i(s^t) = \begin{cases} q_i^a(s^t)F(z_i(s^t), k_i^a(s^{t-1}), k_i^b(s^{t-1}), n_i(s^t)), & i = 1 \\ q_i^b(s^t)F(z_i(s^t), k_i^a(s^{t-1}), k_i^b(s^{t-1}), n_i(s^t)), & i = 2, \end{cases}$$

while for the final good model,

$$(3.41) \quad y_i(s^t) = \begin{cases} q_i^a(s^t)F(z_i(s^t), k_i(s^{t-1}), n_i(s^t)), & i = 1 \\ q_i^b(s^t)F(z_i(s^t), k_i(s^{t-1}), n_i(s^t)), & i = 2. \end{cases}$$

Let  $nx(s^t)$  denote net exports for country 1 as a fraction of GDP in country 1:

$$(3.42) \quad nx(s^t) = \frac{q_1^a(s^t)(a_{2c}(s^t) + a_{2x}(s^t)) - q_1^b(s^t)(b_{1c}(s^t) + b_{1x}(s^t))}{y_1(s^t)}$$

Let  $irc(s^t), irx(s^t), ir(s^t)$  denote import ratio of goods in consumption, import ratio of goods in investment, and import ratio of goods, respectively. For country 1, these import ratios measured at steady state prices are defined as

$$(3.43) \quad irc(s^t) = \frac{b_{1c}(s^t)}{a_{1c}(s^t)}, \quad irx(s^t) = \frac{b_{1x}(s^t)}{a_{1x}(s^t)}, \quad ir(s^t) = \frac{b_1(s^t)}{a_1(s^t)},$$

where  $b_1(s^t) = b_{1c}(s^t) + b_{1x}(s^t)$  and  $a_1(s^t) = a_{1c}(s^t) + a_{1x}(s^t)$ .

### 3.3 Calibration

The benchmark parameter values calibrated at a quarterly frequency are reported in table 3.1. To facilitate the comparisons, both models are calibrated to have identical steady states. Most parameter values for the utility function, the production

Table 3.1: Benchmark parameter values

Description	Parameter value
<i>Preference</i>	
Discount rate	$\beta = 0.99$
Consumption share	$\mu = 0.34$
Risk aversion	$1 - \gamma = 2$
<i>Production</i>	
Capital share	$\alpha = 0.36$
Depreciation rate	$\delta = 0.025$
Elasticity of substitution between	
capital services	$\rho = 0.9$
intermediate goods	$\sigma = 0.9$
Share of imported	
capital services	$1 - \theta = 0.15$
intermediate goods	$1 - \omega = 0.15$
Investment adjustment cost	$\phi = 1/15$
<i>Technology</i>	
Productivity transition matrix <sup>a</sup>	$A = \begin{bmatrix} 0.970 & 0.025 \\ 0.025 & 0.970 \end{bmatrix}$
Std. of innovations to productivity	$\sigma_{\varepsilon 1} = \sigma_{\varepsilon 2} = 0.0073$
Corr. of innovations to productivity	$\text{corr}(\varepsilon_1, \varepsilon_2) = 0.290$

<sup>a</sup> The law of motion for the vector of technology shocks  $z(s^t) = [z_1(s^t), z_2(s^t)]$  is given by  $z(s^t) = Az(s^{t-1}) + \varepsilon(s^t)$ .

function of intermediate goods, and the aggregator of intermediate goods are taken from Heathcote and Perri (2002). Parameter values specific to the capital service model are discussed below.

The capital service model assumes that both types of capital have the same depreciation rate  $\delta$  equal to that of the aggregate capital stock in the final good model. The steady state hours  $\bar{n}$  is chosen so that the prices of intermediate goods in units of the final consumption good is equal to 1. The previous two conditions imply that both types of capital have the same steady state rental rate. Using equation (3.4), the steady state mix of capital services and the steady state mix of intermediate goods in investment are identical:

$$\left( \frac{\theta}{1 - \theta} \frac{\delta \bar{k}_1^b}{\delta \bar{k}_1^a} \right)^{\frac{1}{\rho}} = \left( \frac{\theta}{1 - \theta} \frac{\bar{b}_{1x}}{\bar{a}_{1x}} \right)^{\frac{1}{\rho}} = 1.$$

For the final good model, equation (3.32) shows that the steady state mix of intermediate goods in investment is

$$\left( \frac{\omega \bar{b}_{1x}}{1 - \omega \bar{a}_{1x}} \right)^{\frac{1}{\sigma}} = 1.$$

Thus, the two models will have the same steady state when choosing  $\theta = \omega$ . The import share of the intermediate good ( $1 - \omega$ ) is chosen so that intermediate good imports is 15 percent of the final good expenditure. The choices of the elasticity of substitution between types of capital ( $\rho$ ) and the elasticity of substitution between intermediate goods ( $\sigma$ ) do not affect the steady state. In the benchmark calibration, both elasticities are 0.9 which is the value of the elasticity of substitution between intermediate goods estimated by Heathcote and Perri (2002). The investment adjustment cost parameter  $\phi$  is chosen so that the inverse of the elasticity of investment-capital ratio with respect to Tobin's  $q$  is  $1/15$ , the value used in Baxter and Crucini (1993).

### 3.4 Results

The models are solved using a linearization method. The statistics from the models are the average of 100 simulations, each of which is 104 quarters long.

#### 3.4.1 Benchmark case

The business cycle statistics of the models are reported in table 3.2. Looking at the volatilities in panel (A), the capital service model predicts much larger volatilities of investment, imports, exports, and the import ratio than the final good model does. The difference between the volatilities of the import ratios in investment ( $irx$ ) in panel (D) suggests that the results in panel (A) are driven by trade in durable

Table 3.2: Properties of the benchmark models

(A) Volatilities								
	% std.	% std./% std $y$				% std.		
Economy	$y$	$c$	$x$	$n$	$ex$	$im$	$nx$	$ir$
US data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	4.30
Capital service model	1.23	0.50	4.45	0.35	3.26	3.25	0.95	2.45
Final good model	1.16	0.53	2.68	0.31	0.98	0.99	0.19	0.67

(B) Correlations with output								
	correlation between							
Economy	$c, y$	$x, y$	$n, y$	$ex, y$	$im, y$	$nx, y$	$tot, y$	$rx, y$
US data	0.86	0.95	0.87	0.32	0.81	-0.49	-0.24	0.13
Capital service model	0.93	0.71	0.96	0.00	0.41	-0.26	0.62	0.62
Final good model	0.96	0.96	0.97	0.56	0.89	-0.60	0.58	0.58

(C) Cross country correlations and international relative price volatility							
	correlation between				% std.		
Economy	$y1, y2$	$c1, c2$	$x1, x2$	$n1, n2$	$tot$	$rx$	
Data	0.58	0.36	0.30	0.42	2.99	3.73	
Capital service model	0.09	0.72	-0.75	-0.39	0.59	0.41	
Final good model	0.23	0.68	-0.24	-0.09	0.74	0.52	

(D) Trade components						
	correlation between			% std.		
Economy	$ir, tot$	$irc, tot$	$irx, tot$	$irc$	$irx$	
US data	-0.35	-0.48	-0.06	3.41	5.45	
Capital service model	-0.21	-1.00	-0.06	0.53	9.33	
Final good model	-1.00	-1.00	-1.00	0.67	0.67	

The data statistics for panels (A) except  $ir$ , (B), and (C) are taken from Heathcote and Perri (2002). The data statistics for  $ir$  and panel (D) are calculated from US time series for the period 1970Q1 to 2006Q4. Definition:  $ir = \frac{b1}{a1}$ ,  $irc = \frac{b1c}{a1c}$ ,  $irx = \frac{b1x}{a1x}$ ,  $b1$  = non-petroleum imports,  $b1c$  = non-petroleum non-durable imports + services imports,  $b1x$  = durable imports,  $a1$  = domestic absorption ( $C + I + G$ ) -  $b1$ ,  $a1c = (C + G) - b1c$ ,  $a1x = I - b1x$ .  $tot$  is the corresponding terms of trade. See data appendix for details.

goods. Although trade quantities have high volatility, the terms of trade and the real exchange rate have very low volatility. Backus, Kehoe, and Kydland (1995) call the latter property the “price anomaly.”

Panel (B) shows that both models predict similar comovements of variables with



output. As in the data, consumption, investment, and labor hours are highly procyclical, while net exports are counter-cyclical. Both models fail to replicate the negative correlation between the terms of trade and output in the data. This last feature is common among models with total factor productivity shocks.

The capital service model does not do as well as the final good model at replicating cross-country correlations in panel (C). Both models predict the wrong ranking of the cross-country correlations in consumption and output (Backus, Kehoe, and Kydland (1995) call this property the “quantity anomaly”). Furthermore, the capital service model predicts stronger negative cross-country correlations in investment and employment relative to the final good model. These features are not supported by the data.

Panel (D) shows that the capital service model is clearly better at replicating the correlations between import ratios and the terms of trade, especially for the import ratio in investment. Consistent with the data, the correlation between the import ratio in investment is much higher than that in consumption. It is also clear that the CES specifications in equations (3.10), (3.30) and (3.32) are responsible for the perfect negative correlations. Thus, while the constant elasticity of substitution between the intermediate goods assumption does a reasonable job at explaining how terms of trade movements affect the import ratio in consumption, it does a bad job explaining the ratio in investment.

In sum, the capital service model and the final good model have distinguishable business cycle properties. Relative to the final good model, the capital service model generates higher volatilities of investment and trade quantities. Productivity shocks create larger reallocations of investment, causing investment to be less correlated across countries. A novel feature of the capital service model is the ability to ac-

count for the empirical feature that terms of trade movements affect import ratios in consumption and investment differently.

A relevant question to ask at this point is whether the ability to generate a realistic relationship between  $irx$  and  $tot$  is robust. Specifically, will the capital service model behave just like the final good model if they generate similar volatilities of investment? Since the high volatility comes from large cross-border reallocations of investment, there are three possible ways to curb these flows. The first way is to make the capital stocks more complementary.<sup>6</sup> The second way is to increase the investment adjustment cost. The third alternative is to prohibit international lending and borrowing, i.e., exports must equal imports period by period. In this arrangement, the following conditions must be satisfied:

$$B_1(s^t, s_{t+1}) = B_2(s^t, s_{t+1}) = 0, \forall s_{t+1} \in S.$$

Without asset trades, countries are in financial autarky.<sup>7</sup>

### 3.4.2 The elasticity of substitution between capital services

Figure 3.1 shows the statistics of interest for different values of the elasticity of substitution between capital services,  $\rho$ . (In figures and tables hereafter, “CS” denotes the capital service model and “FG” denotes the final good model.) As expected, the volatility of investment is inversely related to this elasticity. It is clear that the ability to replicate the correlation between  $irx$  and  $tot$  is not very sensitive to the volatility of investment. A low value of  $\rho$  also improves other predictions of

<sup>6</sup>The complementarity between the capital stocks means that deviations from the steady state ratio of  $k^a$  to  $k^b$  are going to be small. However, deviations of the capital stocks from steady state are already small even if investments move a lot because of a big stock-to-flow ratio. As a result, deviations from the steady state ratio of  $a_{1x}$  to  $b_{1x}$  can be very large in the capital service model unless the complementarity between the capital stocks is very strong.

<sup>7</sup>Heathcote and Perri (2002) show that the ability to borrow and lend internationally is very important. They find that the incomplete asset market model with a one-period non-contingent bond being the only asset traded is more similar to the model with complete markets than to the model with financial autarky.

the capital service model.<sup>8</sup> From this figure, one can make a strong case that the elasticity of substitution between capital services should be significantly lower than the elasticity of substitution between intermediate goods. The capital service model matches the final good model in the volatility of investment when  $\rho$  is approximately 0.05.

To clarify the results in table 3.2 and figure 3.1, figure 3.2 shows the impulse responses of key variables for country 1 to a one percent positive productivity shock for  $\rho = \{0.05, 0.9\}$  as well as the benchmark final good model under complete asset markets. In all cases, the productivity improvement leads to an increase in output, consumption and investment, and a deterioration of net export and the terms of trade. Recall the results that net export and investment in the capital good model are more volatile than those in the final good model, and that the same variables are more volatile when  $\rho$  is larger. This figure shows that these differences originate from the responses of the imported good for investment,  $b_{1x}$ . The response of this variable can be especially large in the capital service model. More importantly, while the response  $b_{1x}$  is larger than that of  $a_{1x}$  in the capital service model for each value of  $\rho$ , the opposite is true in the final good model.

In the final good model, it is obvious that the deterioration of the terms of trade explains why the response of  $b_{1x}$  must be smaller than that of  $a_{1x}$ . The increase in the terms of trade means that the imported good is relatively more expensive than the domestic good. To minimize the cost of making a unit of the final good when the elasticity of substitution between intermediate goods is constant, the expenditure-switching motive tilts the demand towards good  $a$ . The same logic explains the

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<sup>8</sup>A low elasticity increases the cross-country correlation of investment by making reallocations more difficult. Moreover, a low elasticity implies a larger movement of the terms of trade. A larger depreciation of the terms of trade raises the value of good  $b$ , which gives the households in country 2 to work more. As a result, the cross-country correlation of employment is higher. Heathcote and Perri (2002) find that lowering the value of  $\sigma$  improves the performance of the final good model along the similar dimensions.

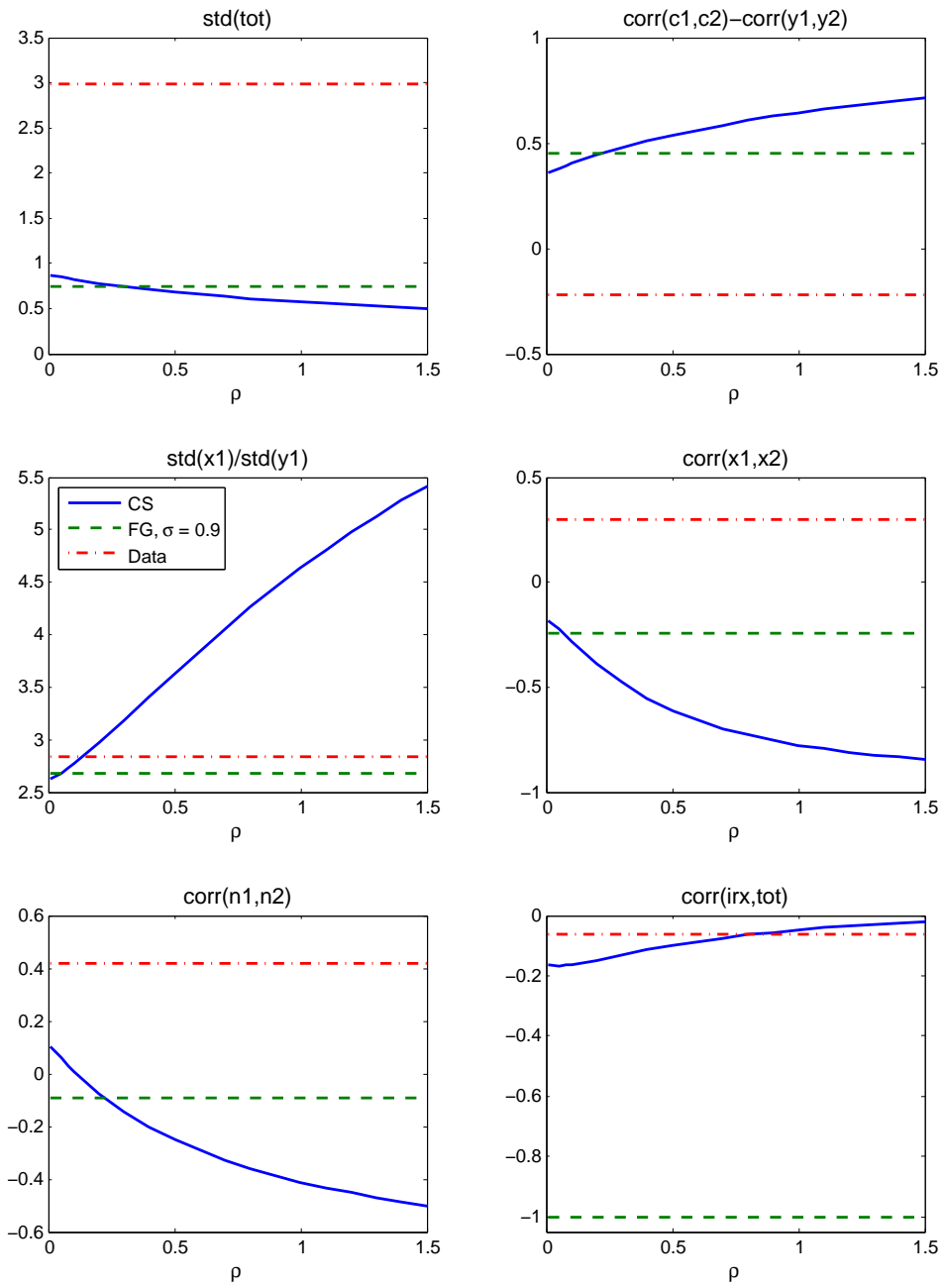


Figure 3.1: Varying  $\rho$ , the elasticity of substitution between capital services (complete markets).

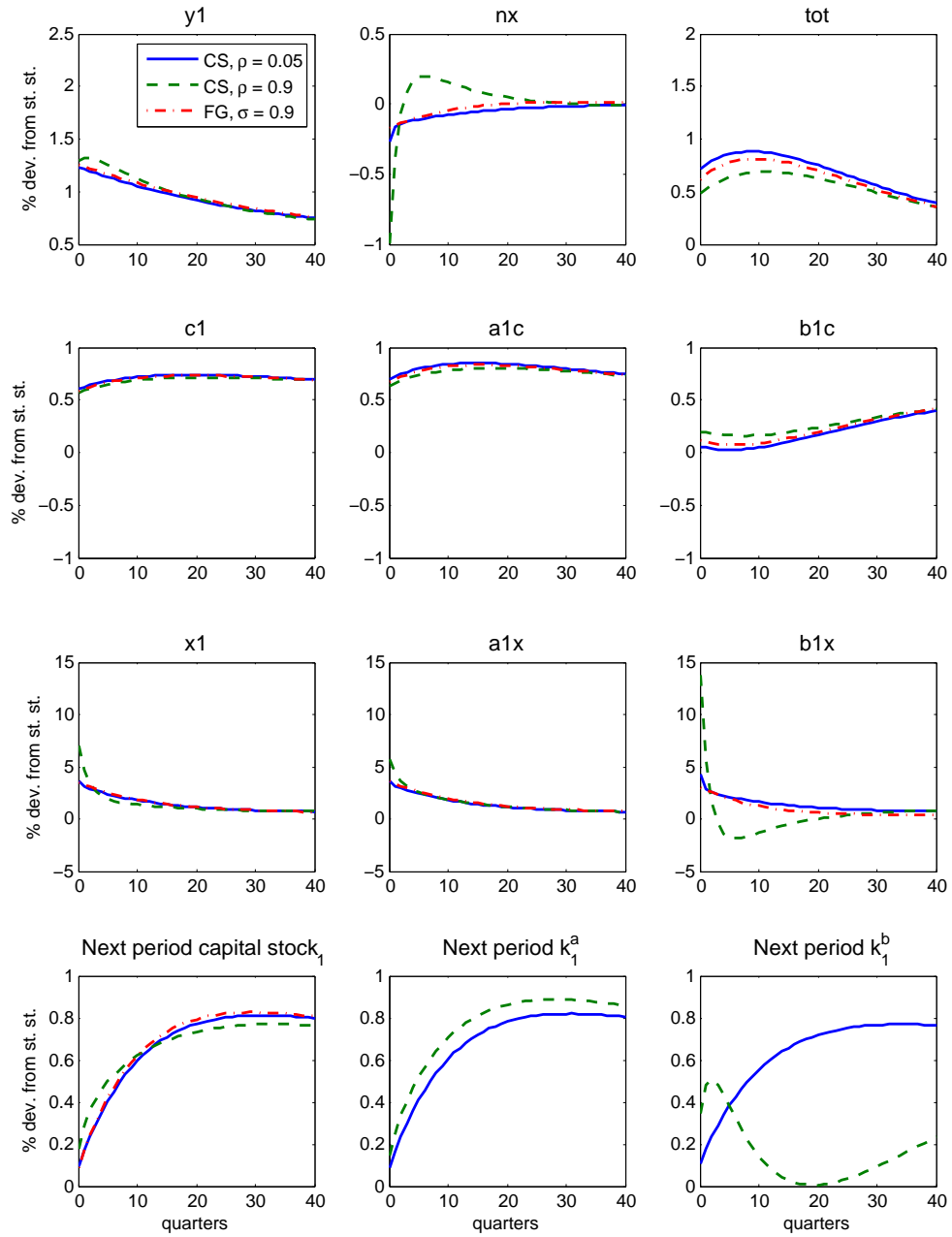


Figure 3.2: Impulse response functions (complete markets).

relationship between  $a_{1c}$  and  $b_{1c}$  for both models. In the capital service model, movements of the terms of trade also explain why the response of  $b_{1x}$  is larger than that of  $a_{1x}$ . Note from figure 3.2 that the terms of trade continues to rise for several quarters after the initial jump. The continued deterioration implies that the imported good will be even more expensive relative to the domestic good in the near future. Because the high productivity increases country 1's demand for capital, the temporarily low price of the imported capital induces country 1 to aggressively increase the purchase of good  $b$  for investment.<sup>9</sup> As a result, country 1 ends up buying more good  $b$  than good  $a$  even if good  $b$  is currently more expensive. The plots of  $k_1^a$  and  $k_1^b$  are consistent with this reasoning.

Formally, terms of trade movements affect the capital stocks through the rental rates. Using equations (3.15), (3.16) and (3.14):

$$(3.44) \quad (1 - \delta)\tilde{t}ot(s^{t+1}) - (1 + \bar{r})\tilde{t}ot(s^t) = \bar{R}(\tilde{R}_1^a(s^{t+1}) - \tilde{R}_1^b(s^{t+1})),$$

where  $\tilde{x} \equiv \frac{\Delta x}{x}$ ,  $1 + \bar{r} = \frac{1}{\beta}$ , and  $\bar{R} = \bar{R}^a = \bar{R}^b$  is the steady state rental rate. If the terms of trade is rising ( $\tilde{t}ot(s^{t+1}) > \tilde{t}ot(s^t)$ ), this equation says that the rental rate of  $k_1^a$  must exceed that of  $k_1^b$ . The higher rental rate of  $k_1^a$  is to ensure that there is no profitable arbitrage opportunity.<sup>10</sup> The change in the relative rental rates raises the desired level of  $k_1^b$  relative to  $k_1^a$  (equation 3.4).

The reason for the hump-shape response of the terms of trade is as follows. For households in country 2, the presence of productivity spill-over means that their

<sup>9</sup>In other words, the demands for durable goods depend on the user cost rather than the current price. Normalize the price of good  $a$  to 1. The user costs in units of good  $a$  are

$$\begin{aligned} u^a(s^t) &= 1 - \frac{1 - \delta}{1 + i(s^t)} \\ \frac{u^b(s^t)}{p^b(s^t)} &= 1 - \frac{1 - \delta}{1 + i(s^t)}(1 + \pi(s^t)), \end{aligned}$$

where  $u^j(s^t)$  is the user cost of durable good  $j$ ;  $1 + i(s^t)$  is the rate of return on an alternative asset;  $p^b(s^t)$  is the relative price of  $b$  which is also the terms of trade; and  $\pi(s^t)$  is the inflation rate of  $p^b(s^t)$ . Clearly, the user cost of  $b$  is lower when the inflation rate is positive.

<sup>10</sup>If the rental rates are the same, the households will sell their holdings of  $k_1^a$ , use the proceed to buy good  $b$  this period and sell them next period at a higher price.

future income is higher than their current income. The desire to smooth consumption encourages them to raise consumption today. At the date of the shock, however, they also want to lend as many resources to country 1 as possible to take advantage of country 1's high productivity level. As the productivity in country 1 starts to fall and spill over into country 2, the willingness to lend declines. Because consumption and investment in country 2 is biased toward good  $b$ , the decreased willingness to lend means that the price of good  $b$  relative to the price of good  $a$  is rising. After a while, production activity in country 2 begins to pick up. At this point, the path of the terms of trade changes.

The intertemporal effect of movements of the terms of trade in the capital service model has strong implications for the elasticity of substitution between traded goods and the correlation between import ratios and the terms of trade. Unlike the final good model, the elasticity of substitution between traded goods can change over time. In the early periods following a productivity shock, the elasticity is negative as the demand for  $b$  increases relative to  $a$  while the relative price of  $b$  is high. Later on, however, the elasticity is positive as the steep rise in the terms of trade winds down. This pattern explains why the correlation between  $irx$  and  $tot$  is much higher in the capital service model. Lowering the elasticity of substitution between capital services from 0.9 to 0.05 reduces, but does not eliminate, the impact of the intertemporal effect of the terms of trade.

### 3.4.3 The investment adjustment cost

Making the deviation from steady state investment to capital ratio more costly is a direct measure to reduce the volatility of investment. Figure 3.3 shows the business cycle statistics of the capital service model under complete asset markets

for larger values of the adjustment cost parameter  $\phi$  while maintaining  $\rho$  at 0.9. For comparison, the figure also shows the capital service model with  $\rho = 0.05$  and the benchmark adjustment cost, “CS,  $\rho = 0.05$ .”

The model with the benchmark value of  $\rho$  requires the adjustment cost  $\phi$  of about 0.7, ten times its benchmark value, to replicate the volatility of investment of the model with  $\rho = 0.05$ . Unlike lowering the elasticity of substitution between capital services, raising the adjustment cost significantly worsen the model’s prediction on the correlation between the import ratio and the terms of trade.

The impulse response functions in figure 3.4 illustrate the differences between the capital service model with a low degree of substitution between capital services and the capital service model with a high investment adjustment cost. At this high a value for the adjustment cost, the response of  $b_{1x}$  is smaller than that of  $a_{1x}$ . The hump-shape response of the terms of trade implies that the benefits from buying good  $b$  early still exist, however, the costs of changing the investment to capital ratio are simply too high. Consequently, the terms of trade deterioration induces a positive expenditure-switching effect both in the short run and in the long run. As a result, the correlation between  $irx$  and  $tot$  is close to  $-1$ .

Without the intertemporal effect of the terms of trade, net export is also less volatile. Furthermore, a relatively high value of the elasticity of substitution between capital services leads to a smaller deterioration of the terms of trade. Since a smaller deterioration of the terms of trade means that good  $a$  is relatively more valuable, households in country 1 have incentive to work more. The plot of GDP confirms this intuition.



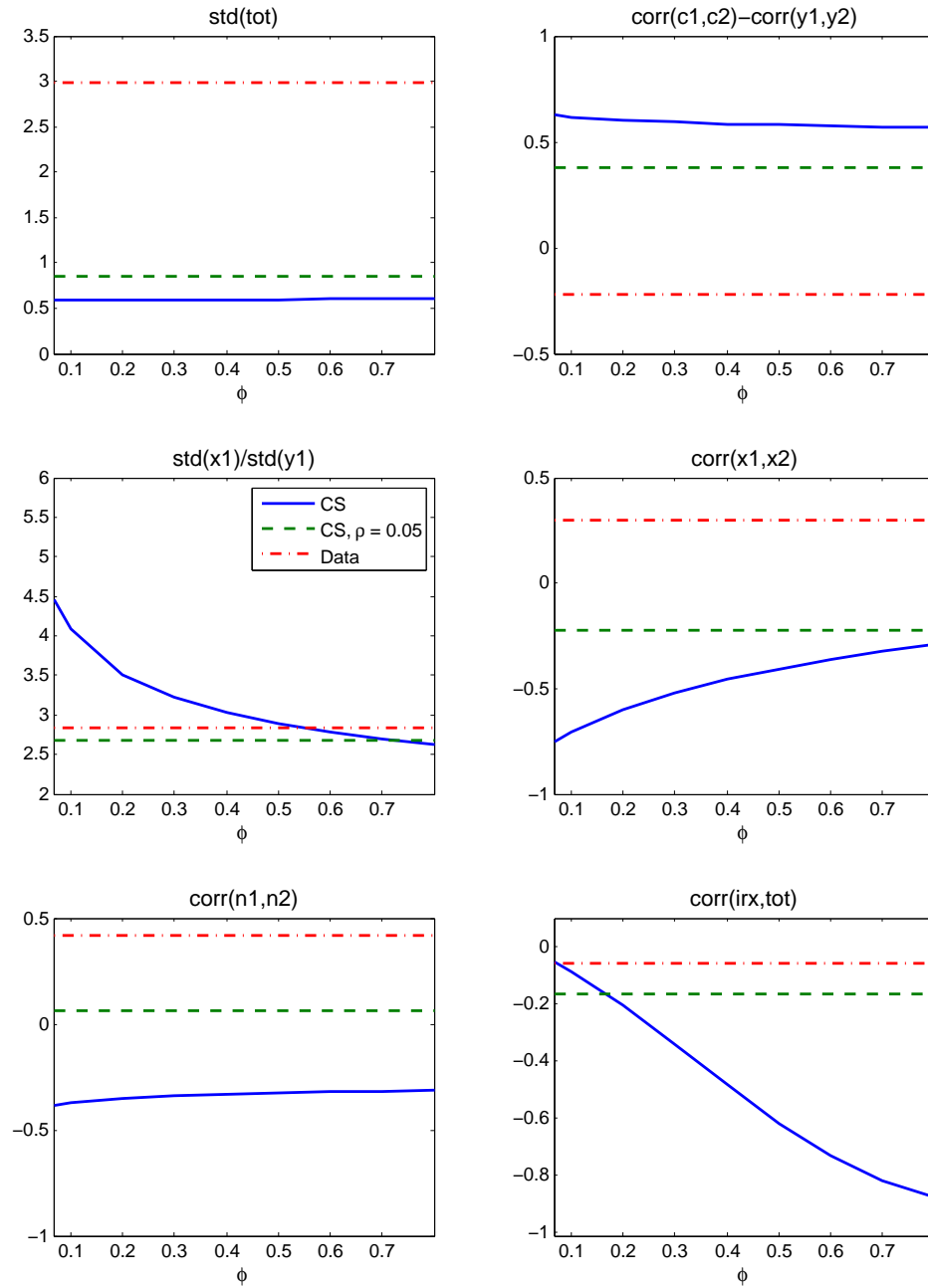


Figure 3.3: Varying  $\phi$ , the investment adjustment cost (complete markets).

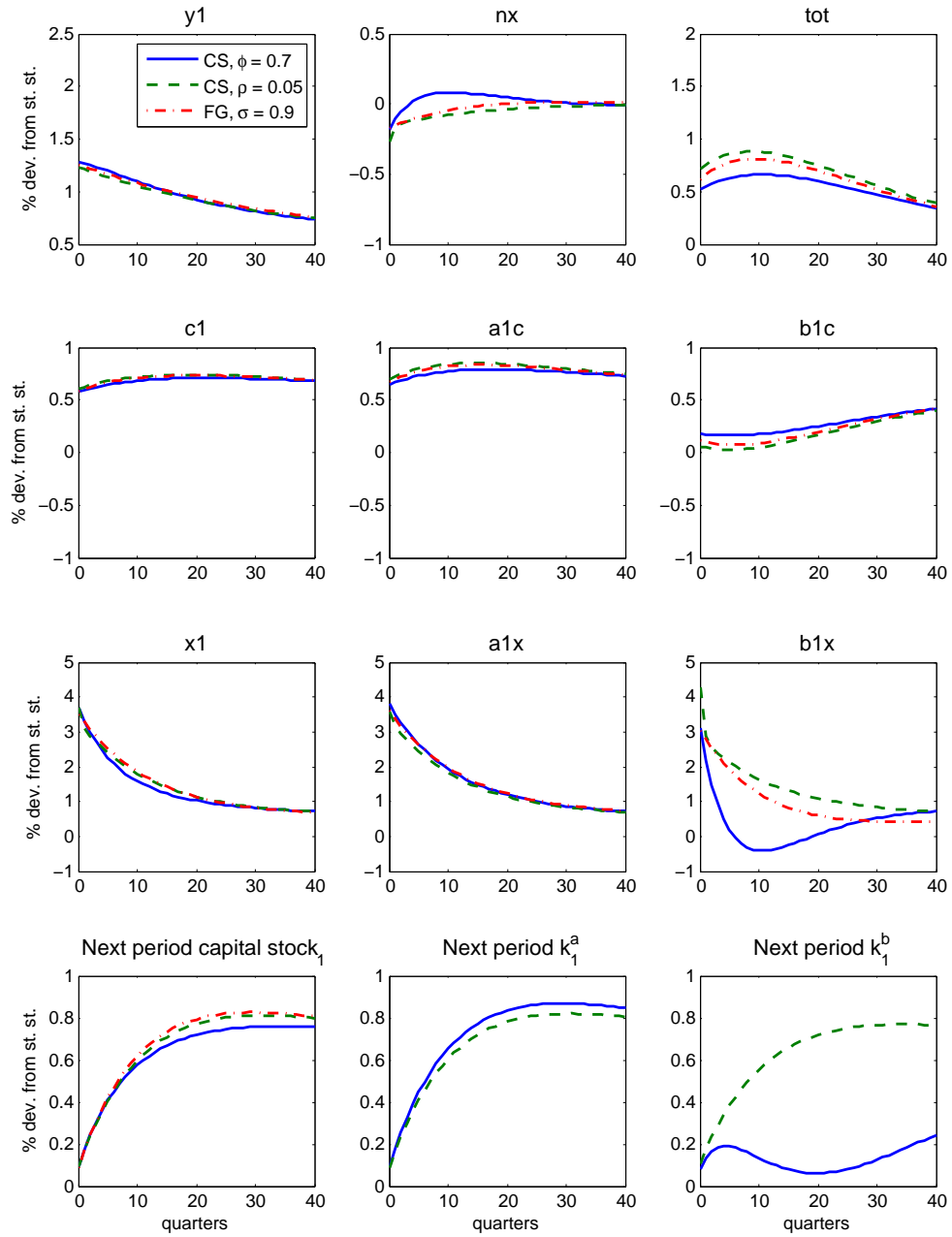


Figure 3.4: Impulse response functions: adjustment cost (complete markets).

### 3.4.4 Financial autarky

Figure 3.5 shows the business cycle statistic when these countries must maintain trade balance. The inability to borrow or lend reduces the volatility of investment by too much. In addition, the volatility of investment is not as sensitive to  $\rho$  as the benchmark case. The capital service model is no longer able to predict a high value of the correlation between  $irx$  and  $tot$ . Clearly, this inability is not due to the low volatility of investment because this correlation gets lower as the volatility of investment increases. It can be concluded that the ability to borrow or lend internationally is necessary for the model to explain the relationship between the import ratio in investment and the terms of trade.

Figure 3.6 shows the impulse responses of key variables for country 1 to a one percent positive productivity shock with  $\rho = \{0.05, 0.9\}$  as well as the final good model under financial autarky. For both values of  $\rho$ , the responses of  $a_{1x}$  are larger than that of  $b_{1x}$ . Thus, terms of trade deteriorations induce households to immediately switch to a cheaper good both in consumption and in investment.

The path of the terms of trade depends on the value of  $\rho$ . In both cases, however, it is not obvious whether the path of the terms of trade is still favorable for the substitution toward good  $b$  in investment. Referring to an intuition developed in the benchmark case, the absence of the asset market is the reason for the absence of the hump-shape response of the terms of trade. More importantly, country 1 still cannot run a current account deficit to finance large purchases of good  $b$  even if the responses of the terms of trade were to have that shape. Thus, the inability, or the lack of incentive, to economize purchases of the durable good can be considered a cost of missing asset markets.

Unlike previous figures, the initial response of the terms of trade is very sensitive

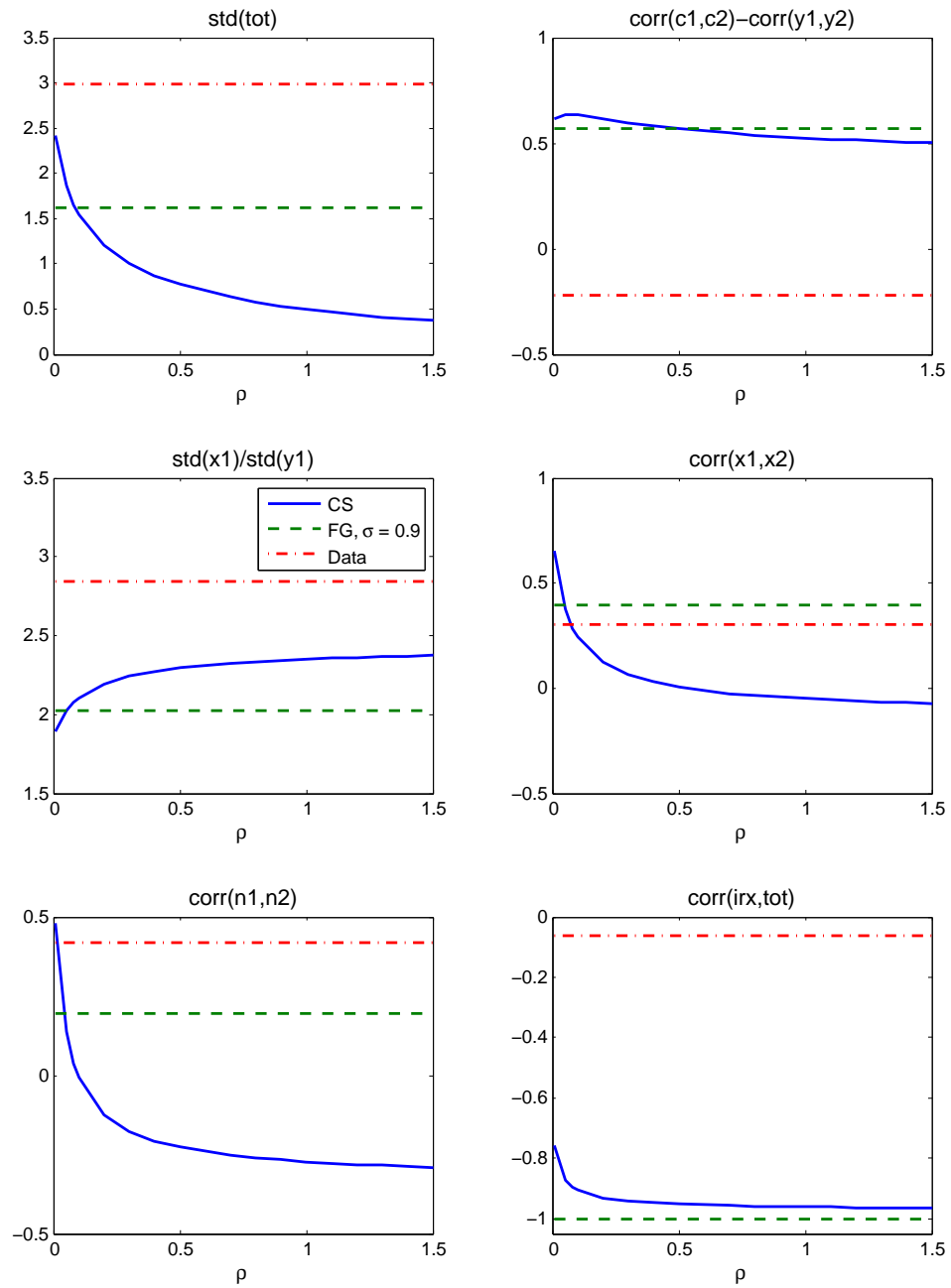


Figure 3.5: Varying  $\rho$ , the elasticity of substitution between capital services (financial autarky).

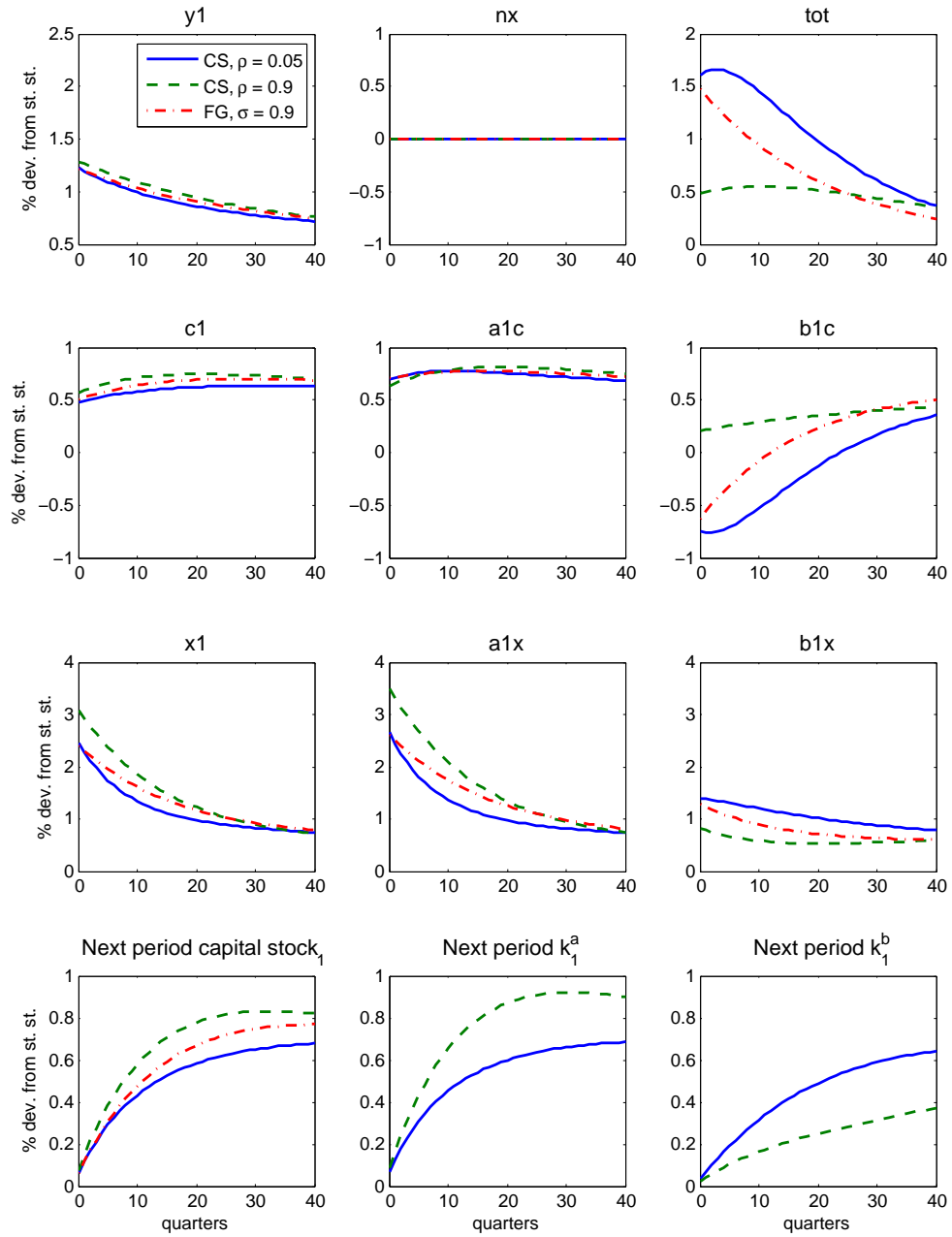


Figure 3.6: Impulse response functions (financial autarky).

to the elasticity of substitution between services of capital. The rise in the terms of trade reflects the scarcity of good  $b$  relative to good  $a$ . When the capital stocks are highly complementary, an increase in  $k_1^a$  induces a similar increase in  $k_1^b$ . As a result, investment in these two types of capital tend to move together. The need for good  $b$  means that country 1 is willing offer more units of good  $a$  in exchange, which explains the large terms of trade depreciation. In contrast, the need for good  $b$  is not so strong for larger values of  $\rho$ .

### 3.5 Trade in durable goods vs. trade in intermediate goods

Table 3.3 summarizes the results that facilitate the comparisons between the capital service model (with a low elasticity of substitution between capital services or with a high investment adjustment cost) and the final good model. Provided that both models generate a similar volatility of investment, the behavior of the models along other dimensions are also similar. However, the capital service model is able to explain why the correlation between the terms of trade and the import ratio of goods in investment is higher than the correlation between the terms of trade and the import ratio of goods in consumption. The intertemporal effect of the terms of trade, which exists when countries have access to asset markets, is clearly a novel feature of the capital service model. This effect is quantitatively significant for a wide range of parameter values. When asset markets are missing, the capital service model can still explain the ranking of the correlation between the terms of trade and import ratios, but only qualitatively. These findings are in favor of making trade in durable goods explicit.

Between the two methods to make the capital service model deliver a plausible volatility of investment, the model with a low elasticity of substitution between

Table 3.3: Complementary vs. adjustment cost: Complete markets

(A) Volatilities								
Economy	% std.	% std./% std $y$				% std.		
	$y$	$c$	$x$	$n$	$ex$	$im$	$nx$	$ir$
US data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	4.30
CS model, $\rho = 0.05$	1.14	0.55	2.68	0.29	1.04	1.04	0.24	0.64
CS model, $\phi = 0.7$	1.18	0.53	2.70	0.33	1.11	1.12	0.21	0.97
FG model	1.16	0.53	2.68	0.31	0.98	0.99	0.19	0.67

(B) Correlations with output								
Economy	correlation between							
	$c, y$	$x, y$	$n, y$	$ex, y$	$im, y$	$nx, y$	$tot, y$	$rx, y$
US data	0.86	0.95	0.87	0.32	0.81	-0.49	-0.24	0.13
CS model, $\rho = 0.05$	0.97	0.95	0.97	0.50	0.88	-0.55	0.57	0.57
CS model, $\phi = 0.7$	0.94	0.94	0.96	0.55	0.67	-0.36	0.61	0.61
FG model	0.96	0.96	0.97	0.56	0.89	-0.60	0.58	0.58

(C) Cross country correlations and international relative price volatility							
Economy	correlation between				% std.		
	$y1, y2$	$c1, c2$	$x1, x2$	$n1, n2$	$tot$	$rx$	
Data	0.58	0.36	0.30	0.42	2.99	3.73	
CS model, $\rho = 0.05$	0.26	0.64	-0.23	0.06	0.85	0.59	
CS model, $\phi = 0.7$	0.16	0.73	-0.32	-0.32	0.61	0.42	
FG model	0.23	0.68	-0.24	-0.09	0.74	0.52	

(D) Domestic and imported component						
Economy	correlation between			% std.		
	$ir, tot$	$irc, tot$	$irx, tot$	$irc$	$irx$	
US data	-0.35	-0.48	-0.06	3.41	5.45	
CS model, $\rho = 0.05$	-0.94	-1.00	-0.17	0.76	0.85	
CS model, $\phi = 0.7$	-0.93	-1.00	-0.82	0.55	2.37	
FG model	-1.00	-1.00	-1.00	0.67	0.67	

The data statistics for panels (A) except  $ir$ , (B), and (C) are taken from Heathcote and Perri (2002). The data statistics for  $ir$  and panel (D) are calculated from US time series for the period 1970Q1 to 2006Q4. Definition:  $ir = \frac{b1}{a1}$ ,  $irc = \frac{b1c}{a1c}$ ,  $irx = \frac{b1x}{a1x}$ ,  $b1$  = non-petroleum imports,  $b1c$  = non-petroleum non-durable imports + services imports,  $b1x$  = durable imports,  $a1$  = domestic absorption  $(C + I + G) - b1$ ,  $a1c = (C + G) - b1c$ ,  $a1x = I - b1x$ .  $tot$  is the corresponding terms of trade. See data appendix for details.

capital services generates a strong intertemporal effect of the terms of trade, while the model with the high investment adjustment cost does not. In addition, the former

is also better at accounting for observed cross-country correlation and international price volatility statistics. See appendix B for a plausible method of choosing both the elasticity of substitution between capital services and the investment adjustment cost parameters in one model.

### 3.6 Conclusion

A typical two-country, two-good model can be extended to include trade in durable goods by introducing complementarity between the services of domestically produced capital and imported capital in production of goods and services. The main advantage of the model with trade in durable goods is that this model allows movements in the terms of trade to have an intertemporal effect, in addition to the usual expenditure-switching effect. Although the intertemporal effect induces a switch toward a good that will become more expensive in the near future whether the good is currently cheap or expensive relative to its substitutes, the expenditure-switching induces a switch toward a currently cheaper good regardless of the path of the relative price. The intertemporal effect is needed to show that the way in which the terms of trade induces substitution between domestic and imported goods in consumption is different from the way in which it does in investment.

While the lack of data makes it difficult to directly estimate the elasticity of substitution between capital services, it is reasonable to conclude based on the results that this elasticity should be well below the elasticity of substitution between goods. Moving away from the benchmark calibration in certain directions are left for future research. One such direction is to relax the assumption that there is a home bias in both consumption and capital services. Home bias in capital services may not be a good assumption if the ratio of two capitals, e.g., robots and computers in an



assembly line, is roughly the same across countries. The other direction involves the assumption that the depreciation rates are the same for all capitals. For the United States, imports of durable goods have been increasing steadily and currently account for almost half of all imports of goods in recent years. It is difficult to know whether the increase in import is because of the lower home bias or because the U.S. no longer produce certain high depreciating types of capital such as computers and hence rely heavily on imports.

### 3.7 Appendix

#### 3.7.1 Appendix A: Data

Data on consumption, investment, government purchases, and trade and price indices by type of product are taken from the United States Bureau of Economic Analysis. The sample covers 1970 to 2006 at quarterly frequency.

Total import ( $b_1$ ) is imports of goods and services less imports of petroleum products. Expenditure on domestic product ( $a_1$ ) is the sum of consumption, investment and government purchases less total imports. Imports of goods for consumption ( $b_{1c}$ ) is imports of non-durable goods and services less imports of petroleum products. Consumption expenditure on domestic product ( $a_{1c}$ ) is the sum of consumption and government purchases less imports of goods for consumption. Imports of goods for investment ( $b_{1x}$ ) is imports of durable goods. Investment expenditure on domestic product ( $a_{1x}$ ) is investment less imports of durable goods.

The terms of trade is the ratio of price index for imports to price index for exports. The terms of trade for goods for consumption is the same as the terms of trade for all goods and services. The terms of trade for goods for investment is the terms of trade for durable goods.

Import ratios used in table 3.2 are in real dollars. To get real dollars ratios, nominal dollars ratios are divided by their corresponding terms of trade.

### 3.7.2 Appendix B: Choosing $\rho$ and $\phi$

Low values of  $\rho$  or high values of  $\phi$  allow the capital service to match the observed volatility of investment  $x$ . While a low  $\rho$  implies too little volatility of  $ir$ , a high value of  $\phi$  obstructs the intertemporal effect of the terms of trade and implies too low a correlation between  $irx$  and  $tot$ . This section of the appendix offers a criteria to pick  $\rho$  and  $\phi$  by defining an objective function.

Let  $L$  be the loss from the deviations from data statistics. Statistics of interests include  $std(x)/std(y)$ ;  $corr(irx, tot)$ ; and  $std(ir)$ . Values of  $\rho$  and  $\phi$  considered are between 0.05 and 0.9 (each increment is 0.05). While it is possible for the model to match the first two statistics, there are no values of  $\rho$  or  $\phi$  that enable the model to come close to matching the third statistic. The weights in the loss function will be chosen based on this information.

Let  $w_1$  and  $w_2$  be the weights on the first two moments, the loss function is

$$L = w_1 \left( \frac{std(x)}{std(y)} - 2.84 \right)^2 + w_2 (corr(irx, tot) - (-0.06))^2 + (1 - w_1 - w_2) (std(ir) - 4.30)^2$$

Using  $w_1 = 0.85$  and  $w_2 = 0.13$ , the combination  $(\rho, \phi) = (0.15, 0.05)$  minimizes the loss function. This result lends support to using a low value of  $\rho$  over a high value of  $\phi$  in the calibration. The conclusion that a low value of  $\rho$  is preferred is robust to different values of the weights as long as deviations from  $corr(irx, tot)$  is penalized.

## CHAPTER IV

# Trade Openness, Trade Composition and the Volatility of Output

### 4.1 Introduction

During the past few decades, most developed countries have become more open to international trade in goods and services. Furthermore, the composition of goods traded has also changed. For the United States, the shares of trade in GDP since 2000 are more than double those in the 1970s. The share of capital goods in merchandise trade is close to 50 percent today compared to just about 35 percent three decades ago. What are the effects of these changes on the business cycles? It has proved difficult to give a consistent answer to this question from the data.<sup>1</sup>

This paper begins by documenting the increase in trade openness and the change in the composition of trade for the G7 countries. To understand the effects of these changes, a two-country business cycle model with an emphasis on trade in capital goods is introduced. The production of goods in each country requires services from domestically produced and imported capital. Services from capital may be different from the stock of capital because the intensity at which a unit of capital is used (its

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<sup>1</sup>Empirical studies have found a mixed relationship between international trade and macroeconomic volatility. Cavallo and Frankel (2004) and Calvo, Izquierdo, and Mejia (2004) show that more open economies are less vulnerable to, and suffer less output loss from destabilizing shocks such as sudden stops and currency crashes. In addition, Cavallo (2005) shows that the net effect of trade openness on output volatility is negative after controlling for relevant endogeneity. On the other hand, Rodrik (1998) and Kose, Prasad, and Terrones (2006) provide evidence that increased trade openness leads to higher output volatility. These studies do not examine the role of trade composition.

utilization rate) is endogenously determined. Different calibrations of the import shares in consumption and in capital services enable the model to reflect either the increase in trade openness and/or the change in trade composition. Business cycles are generated by investment-specific technological changes, which directly influence the demand for capital goods.

The model predicts that the increase in openness and the change in trade should not have a big effect on the volatility of output because these two features have opposite effects on the intensive margin and the extensive margin of factor inputs. On one hand these features improve the ability to reallocate capital across countries, which makes the capital stock (the extensive margin) more responsive to shocks. On the other hand these features make the costs of inputs more procyclical, which reduces the desired utilization rate (the intensive margin). In many cases, the effect on the intensive margin is stronger than the effect on the extensive margin, which implies that output is less volatile when the economy is more open. The stabilizing effect would not be as strong had the increase in openness not been accompanied by a bias toward trade in capital goods. Although the effect on the volatility of output is small, the real benefit comes from the improved ability to reallocate factors of production to a country that is the most productive.

The analysis in this paper also contributes to the understanding of the moderation of the US business cycle: the volatility of the GDP since the mid-1980s has been much lower than that of the periods before that. In particular, the analysis suggests that the declines in the volatilities of exogenous shocks, investment-specific technological changes included, should be significant contributors to the observed volatility reduction. However, the analysis does not rule out the possibility that

Table 4.1: Trade openness

	Trade/GDP		
	1971-1980	1981-1990	1991-2004
Canada	0.48	0.52	0.73
France	0.38	0.45	0.48
Germany	0.39	0.48	0.57
Italy	0.40	0.41	0.46
Japan	0.24	0.23	0.20
United Kingdom	0.53	0.52	0.55
United States	0.16	0.19	0.23

Trade = Exports + Imports. Data on exports, imports and GDP are obtain from the OECD Annual National Account database.

other structural changes are important contributors as well.<sup>2</sup>

The paper is organized as follows. Section 2 documents the increase in trade openness and the shift in the composition of trade. Section 3 develops a two-country model to study the effects of trade in capital goods. Section 4 discusses the parameterizations of the model to study the effects of changes in the degree of openness and/or changes in the composition of trade. Section 5 reports and interprets the results. Section 6 examines the impact of different shock characteristics. Section 7 provides conclusions.

## 4.2 International trade openness and its composition

International trade openness is measured by the ratio of the sum of exports and imports to GDP. Table 4.1 reports this ratio for each of the last 3 decades for G7 countries.

An interesting observation is that countries that are geographically close and are

<sup>2</sup>The literature about what causes the moderation is very large. The pioneering studies of McConnell and Perez-Quiros (2000) and Kahn, McConnell, and Perez-Quiros (2002) advocate improved inventory management hypothesis. Bernanke (2004) argues that better macroeconomic policy, especially monetary policy, is responsible for the moderation. This view is consistent with the evidence by Clarida, Gali, and Gertler (2000) that monetary policy has become more stabilizing after 1980. Dynan, Elmendorf, and Sichel (2006) and Peek and Wilcox (2006) point to the contribution of financial innovations. Finally, Stock and Watson (2002) and Ahmed, Levin, and Wilson (2004) argue that only good luck in forms of less frequent and smaller exogenous shocks can match the quantitative aspects.

Table 4.2: Trade composition

	Trade in Capital Goods/Merchandise Trade		
	1971-1980	1981-1990	1991-2004
Canada	0.40	0.44	0.44
France	0.28	0.31	0.42
Germany	0.33	0.37	0.44
Italy	0.26	0.28	0.35
Japan	0.31	0.42	0.50
United Kingdom	0.29	0.34	0.42
United States	0.34	0.41	0.46

Data on exports, imports of machinery and equipment as well as of total merchandise are obtained from the OECD ITCS International Trade by Commodities Statistics Database for the 1971-1990 period. Data for the 1991-2004 period are obtained from the WTO International Trade Statistic Database.

in trade or economic arrangements (Canada and the US; or France, Germany and Italy) have seen a steady growth in openness, while others have not. Some researchers find that business cycles within these groups have become more synchronized (see, for example Stock and Watson (2005)).

Table 4.2 reports the composition of goods in trade. It shows that the ratio of trade in machinery and equipment to total merchandise trade has been rising over this period. Since the share of investment in GDP has not changed by much, the increase in trade in capital goods, which are factors of production, reflects a greater interdependency between productions across countries.<sup>3</sup> The model developed in the next section examines how greater trade openness and the change in trade composition affect business cycle properties of GDP and its components.

### 4.3 A model of international trade and variable capital utilization

The world consists of two countries of the same size. Country 1 specializes in the production of intermediate good  $a$ , while country 2 specializes in the production

<sup>3</sup>Burstein, Kurz, and Tesar (2005) use the term “production sharing” to describe this interdependency.

of intermediate good  $b$ . Households in both countries demand goods  $a$  and  $b$  for consumption and investment. The law of one price holds for traded intermediate goods. Households in country 1(2) supply labor and rent capital, whose rate of utilization can vary, to perfectly competitive firms producing good  $a(b)$ . Households in each country own the capital stock of that country. Neither labor nor the capital stock is internationally mobile.

At time  $t$  the economy experiences one event  $s_t \in S$  where  $S$  is a finite set of events. Let  $s^t$  be the history of events up to and including date  $t$ , the probability at date 0 of any particular history  $s^t$  is  $\pi(s^t)$ .

The production of intermediate goods in country  $i = \{1, 2\}$  requires labor hours, and services from domestically produced and imported capitals:

$$(4.1) \quad y_i(s^t) = F(u_i(s^t)k_i^a(s^{t-1}), u_i(s^t)k_i^b(s^{t-1}), n_i(s^t)),$$

where  $u_i(s^t)$  is the utilization rate;  $k_i^a(s^{t-1})$  and  $k_i^b(s^{t-1})$  are the stocks of capital; and  $n_i(s^t)$  is labor hours at time  $t$ . Note that services from capital are the product of the utilization rate and the capital stock. In other words, the extensive margin is the capital stocks while the intensive margin is the utilization rate. Services from both types of capital are complementary in the production function. Let the aggregate capital services be the Constant Elasticity of Substitution (CES) aggregate of services from each capital. The assumption that both types of capital have the same utilization rate conveniently implies that the aggregate capital stock  $K_i(s^t)$  is also the CES aggregate of  $k_i^a(s^t)$  and  $k_i^b(s^t)$ , and that the aggregate services of capital

are the product of the utilization rate and the aggregate capital stock:

$$\begin{aligned}
& \left[ \theta_i^{\frac{1}{\rho}} (u_i(s^t) k_i^a(s^{t-1}))^{\frac{\rho-1}{\rho}} + (1 - \theta_i)^{\frac{1}{\rho}} (u_i(s^t) k_i^b(s^{t-1}))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\
&= u_i(s^t) \left[ \theta_i^{\frac{1}{\rho}} k_i^a(s^{t-1})^{\frac{\rho-1}{\rho}} + (1 - \theta_i)^{\frac{1}{\rho}} k_i^b(s^{t-1})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\
&= u_i(s^t) K_i(k_i^a(s^{t-1}), k_i^b(s^{t-1})),
\end{aligned}$$

where  $\rho$  is the elasticity of substitution between services of capital types  $a$  and  $b$ , and  $1 - \theta_i$  is the share of services of imported capital.<sup>4</sup> The production function takes the Cobb-Douglas form:

$$(4.2) \quad y_i(s^t) = (u_i(s^t) K_i(s^{t-1}))^\alpha n_i(s^t)^{1-\alpha},$$

where  $\alpha$  is the share of aggregate capital services in production.

Let  $w_i(s^t)$ ,  $R_i^a(s^t)$  and  $R_i^b(s^t)$  be the wage and the rental rate on capital services of types  $a$  and  $b$ , respectively in country  $i$  in terms of the intermediate good produced in country  $i$ . The maximization problem of perfectly competitive intermediate good producers in country  $i$  is given by

$$\begin{aligned}
\max_{u_i(s^t) k_i^a(s^{t-1}), u_i(s^t) k_i^b(s^{t-1}), n_i(s^t)} & \{ F(u_i(s^t) k_i^a(s^{t-1}), u_i(s^t) k_i^b(s^{t-1}), n_i(s^t)) - w_i(s^t) n_i(s^t) \\
& - R_i^a(s^t) u_i(s^t) k_i^a(s^{t-1}) - R_i^b(s^t) u_i(s^t) k_i^b(s^{t-1}) \}.
\end{aligned}$$

The demand for capital services and the demand for labor are given by

$$(4.3) \quad \alpha (u_i(s^t) K_i(s^{t-1}))^\alpha n_i(s^t)^{1-\alpha} K'(k_i^a(s^{t-1})) = R_i^a(s^t)$$

$$(4.4) \quad \left( \frac{\theta_i}{1 - \theta_i} \frac{k_i^b(s^{t-1})}{k_i^a(s^{t-1})} \right)^{\frac{1}{\rho}} = \frac{R_i^a(s^t)}{R_i^b(s^t)}$$

$$(4.5) \quad (1 - \alpha) (u_i(s^t) K_i(s^{t-1}))^\alpha n_i(s^t)^{-\alpha} = w_1(s^t),$$

<sup>4</sup>In addition, this assumption implies that the increase in the share of imported goods in investment is not driven by the increase in the utilization rate of the imported capitals. While it is possible that the US import more goods that need faster replacements (computers), a business cycle model with two symmetric economies is not equipped to handle this fact because it implies that the other country (computer producer) must see a fall in the share of imported goods in investment.



where  $f'(x)$  denotes the derivative of a function  $f$  with respect to  $x$ . It is convenient to think of demand as the mix of services that minimizes the cost of a unit of aggregate capital services.

Households accumulate and own the capital stocks. The accumulation of capital is represented by

$$(4.6) \quad k_i^a(s^t) = (1 - \delta_i(s^t))k_i^a(s^{t-1}) + q_i(s^t)a_{ix}(s^t)$$

$$(4.7) \quad k_i^b(s^t) = (1 - \delta_i(s^t))k_i^b(s^{t-1}) + q_i(s^t)b_{ix}(s^t).$$

Note that from the perspective of country 1 the capital stock  $k^a$  is the domestic capital because the domestically produced good  $a$  is the investment good. These accumulation equations differ from the standard one in two aspects: the depreciation rate is time-varying and a unit of intermediate good is transformed into investment at a time-varying rate. The first feature captures the fact that the utilization rate of capital is allowed to vary over the business cycle. As in Greenwood, Hercowitz, and Krusell (2000), the depreciation rate is an increasing and convex function of the utilization rate

$$(4.8) \quad \delta(u_i(s^t)) = \frac{\kappa}{\nu} u_i(s^t)^\nu, \quad \nu > 1.$$

This specification implies that the elasticity of the depreciation rate with respect to the utilization rate is  $\nu - 1$ . The higher the utilization rate, the faster capital depreciates. Although the rate is time-varying, both types of capital still have the same depreciation rate at every period. The second feature introduces the only source of fluctuation in this model, which is an exogenous investment-specific technological change (IST),  $q_i(s^t)$ . This shock is most suitable for the objective of this paper because it is the shock that directly affects the demands for capital goods.<sup>5</sup> In

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<sup>5</sup>The examination of the household's optimization problem provides some intuition for this point.

this model, the shock is country-specific rather than good-specific so that it directly affects the trade off between consumption and investment rather than the trade off between goods produced in different countries.

The final consumption good is produced using intermediate goods  $a$  and  $b$  as inputs in the following way

$$(4.9) \quad c_i(a_{ic}(s^t), b_{ic}(s^t)) = \left[ \omega_i^{\frac{1}{\sigma}} a_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} + (1 - \omega_i)^{\frac{1}{\sigma}} b_{ic}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where  $\sigma$  is the elasticity of substitution between goods  $a$  and  $b$ , and  $1 - \omega_i$  is the share of imported goods. Let  $P_{ic}(s^t)$  and  $p_i^b(s^t)$  be the prices of final consumption good and of  $b$  in country  $i$  in units of good  $a$  in country  $i$ , the following maximization problem gives the demand for each intermediate good:

$$\max_{a_{ic}(s^t), b_{ic}(s^t)} \{P_{ic}(s^t)c_i(a_{ic}(s^t), b_{ic}(s^t)) - a_{ic}(s^t) - p_i^b(s^t)b_{ic}(s^t)\}.$$

The demand for intermediate goods in consumption is determined by

$$(4.10) \quad c'(a_{ic}(s^t)) = \frac{1}{P_{ic}(s^t)}$$

$$(4.11) \quad \left( \frac{\omega_i}{1 - \omega_i} \frac{b_{ic}(s^t)}{a_{ic}(s^t)} \right)^{\frac{1}{\sigma}} = \frac{1}{p_i^b(s^t)}.$$

The terms of trade, defined as the price of imports into country 1 relative to exports from country 1, is given by

$$(4.12) \quad tot(s^t) = p_1^b(s^t).$$

Assume that asset markets are complete. Let  $B_i(s^t, s_{t+1})$  be the quantity of bonds purchased by households in country  $i$  after history  $s^t$  that pay one unit of good  $a$  in period  $t + 1$  if and only if the state of the economy is  $s_{t+1}$ , and let  $Q(s^t, s_{t+1})$  be the price in units of good  $a$  of these bonds. The maximization problem of the representative household in country 1 is given by

$$\begin{aligned}
& \max_{c_1(s^t), n_1(s^t), B_1(s^t), k_1^a(s^t), k_1^b(s^t), u_1(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ \frac{1}{\gamma} [c_1(s^t) - \varphi n_1(s^t)^\mu]^\gamma \right. \\
& + \lambda_1(s^t) \left[ R_1^a(s^t) u_1(s^t) k_1^a(s^{t-1}) + R_1^b(s^t) u_1(s^t) k_1^b(s^{t-1}) + w_1(s^t) n_1(s^t) + B_1(s^{t-1}) \right. \\
& \quad \left. \left. - \sum_{s^{t+1}} Q(s^t, s_{t+1}) B_1(s^t, s_{t+1}) - P_{ic}(s^t) c_1(s^t) - a_{1x}(s^t) - p_1^b(s^t) b_{1x}(s^t) \right] \right. \\
& + \Omega_1^a(s^t) \left[ -k_1^a(s^t) + (1 - \delta(u_1(s^t))) k_1^a(s^{t-1}) + q_i(s^t) a_{1x}(s^t) - \frac{\phi}{2} k_1^a(s^{t-1}) \left( \frac{q_i(s^t) a_{1x}(s^t)}{k_1^a(s^{t-1})} - \delta \right)^2 \right] \\
& \left. + \Omega_1^b(s^t) \left[ -k_1^b(s^t) + (1 - \delta(u_1(s^t))) k_1^b(s^{t-1}) + q_i(s^t) b_{1x}(s^t) - \frac{\phi}{2} k_1^b(s^{t-1}) \left( \frac{q_i(s^t) b_{1x}(s^t)}{k_1^b(s^{t-1})} - \delta \right)^2 \right] \right\},
\end{aligned}$$

where  $\beta$  is the discount rate;  $1 - \gamma$  is the risk aversion parameter; the inverse of  $\mu - 1$  is the Frisch labor supply elasticity;  $\delta$  is the steady state depreciation rate;  $\phi$  is the investment adjustment cost;  $\lambda_1(s^t)$  is the marginal utility of good  $a$ ; and  $\Omega_1^a(s^t)$  and  $\Omega_1^b(s^t)$  are the shadow prices of capital of types  $a$  and  $b$ , respectively.

The labor supply decision is governed by

$$(4.13) \quad \varphi \mu n_1(s^t)^{\mu-1} = \frac{w_1(s^t)}{P_{ic}(s^t)}.$$

For this utility function, which is used in Greenwood, Hercowitz, and Huffman (1988), only the real wage affects labor supply. This property helps simplify some of the analysis later on.

The first order with respect to the state-contingent bond is

$$(4.14) \quad Q(s^t, s_{t+1}) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)}$$

where

$$(4.15) \quad \lambda_1(s^t) = \frac{1}{P_{ic}(s^t)} [c_1(s^t) - \varphi n_1(s^t)^\mu]^{\gamma-1}$$

With complete asset markets, it is easy to show that the marginal utilities of good  $a$  (the numeraire good) must be the same in both countries at all times.

Assume for the moment that  $\phi = 0$ . The investment decision of the household is given by

$$(4.16) \quad \Omega_1^a(s^t) = \frac{1}{q_1(s^t)} \lambda_1(s^t)$$

$$(4.17) \quad \Omega_1^b(s^t) = \frac{p_1^b(s^t)}{q_1(s^t)} \lambda_1(s^t).$$

and

$$(4.18) \quad \Omega_1^a(s^t) = \beta \Omega_1^a(s^{t+1})(1 - \delta(u_1(s^{t+1}))) + \\ \beta \Omega_1^a(s^{t+1}) q_1(s^{t+1}) u_1(s^{t+1}) R_1^a(s^{t+1})$$

$$(4.19) \quad \Omega_1^b(s^t) = \beta \Omega_1^b(s^{t+1})(1 - \delta(u_1(s^{t+1}))) + \\ \beta \Omega_1^b(s^{t+1}) \frac{q_1(s^{t+1})}{p_1^b(s^{t+1})} u_1(s^{t+1}) R_1^b(s^{t+1}).$$

Equation (4.16) shows the difference between the price of good  $a$  in investment and its price in consumption. An increase in  $q$ , an improvement in IST, reduces the relative price of investment. Equation (4.17) shows the price of good  $b$  in investment in units of good  $a$  in consumption. In addition to IST, movements in the terms of trade,  $p^b$ , also affect this trade off. All else equal, a depreciation of the terms of trade raises the relative price of investment. The opposite is true for an appreciation of the terms of trade. Equations (4.18) and (4.19) are the Euler equations for investment. While the left hand side is the marginal cost of investment, the right hand side is the marginal benefit of investment. Both the cost and the benefits are in units of the investment goods. Everything else equal, an increase in IST raises the returns on investment (the product of  $q$  and  $R$ ) and demand for both  $a$  and  $b$ .<sup>6</sup> For the imported capital, a deterioration of the terms of trade reduces the returns and discourages investment.

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<sup>6</sup>Although IST shock raises investment demand like the usual total factor productivity (TFP) shock, IST shock is more similar to a demand shock than a supply shock because an increase in IST by itself does not lead to an immediate increase in the supply of the intermediate good.

In contrast, an appreciation of the terms of trade should encourage the accumulation of the imported capital.

The utilization rate is endogenously determined by

$$(4.20) \quad q_1(s^t)(R_1^a(s^t)k_1^a(s^{t-1}) + R_1^b(s^t)k_1^b(s^{t-1})) \\ = \delta'(u_1(s^t))(k_1^a(s^{t-1}) + p_1^b(s^t)k_1^b(s^{t-1})).$$

Changes in the utilization rate is the main determinant of services of capital in the short run. While the marginal benefit of a higher utilization rate is the extra output, the marginal cost is the increase in the rate of depreciation of capital ( $\delta' > 0$ ). When IST increases, the extra output can be transformed into the investment good at a higher rate, which makes it attractive to increase the utilization rate. It is important to notice that the terms of trade affects the desired utilization rate because it affects the cost of replacing the imported capital measured in units of the domestically produced good. For a given level of IST, a terms of trade deterioration discourages utilization while an appreciation encourages utilization. Furthermore, the effect of a given sized change in the terms of trade on the marginal cost of utilization is increasing in  $(1 - \theta_i)$ , the size of the imported capital relative to the aggregate capital.<sup>7</sup> To see the direct effect of the terms of trade (i.e., holding the utilization rate constant) on the marginal cost of utilization, one can log-linearize equation (4.21) and show that

$$\frac{\Delta MC}{MC} = \delta'(u) \left[ \frac{k^a}{k^a + p^b k^b} \frac{\Delta k^a}{k^a} + \frac{p^b k^b}{k^a + p^b k^b} \frac{\Delta k^b}{k^b} + \frac{p^b k^b}{k^a + p^b k^b} \frac{\Delta p^b}{p^b} \right],$$

where  $\frac{p^b k^b}{k^a + p^b k^b} = 1 - \theta_i$ . In addition, the fact that the import share of capital services is larger than the import share of consumption (the difference in the degree of home

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<sup>7</sup>One can also use equation (4.21) to see what would change if the utilization rates are not the same for these capitals. Movements in the terms of trade will have a bigger impact on the utilization rate of the imported capital. These movements also affect the utilization of the domestic capital through their effect on the marginal product of capital. Specifically, a deterioration of the terms of trade reduces the utilization of the imported capital, which reduces the marginal product of the domestic capital. As a result, the utilization rate of the domestic capital may decline as well.

bias) is not the reason why movements in the terms of trade affect utilization. Intuitively, the trade off is between units of the domestic intermediate good that can be produced with additional utilization versus units of the foreign intermediate good that are needed to replace the foreign capital. The appendix shows that the effect of the terms of trade is still proportional to the share of the imported good in a standard two-country, two-good model modified to include variable capital utilization. While it is well known that movements of the terms of trade have a larger effect in a more open economy through the aggregate demand channel, this model introduces another important channel which can be thought of as the aggregate supply channel.<sup>8</sup>

Investment expenditure  $x_1(s^t)$  is defined as the sum of expenditure on intermediate goods devoted to investment:

$$(4.21) \quad x_1(s^t) = a_{1x}(s^t) + p_1^b(s^t)b_{1x}(s^t).$$

In the equilibrium, goods markets and bond markets clear:

$$(4.22) \quad a_{1c}(s^t) + a_{1x}(s^t) + a_{2c}(s^t) + a_{2x}(s^t) = y_1(s^t)$$

$$(4.23) \quad b_{1c}(s^t) + b_{1x}(s^t) + b_{2c}(s^t) + b_{2x}(s^t) = y_2(s^t)$$

$$(4.24) \quad B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0, \forall s_{t+1} \in S.$$

Let  $Y_i(s^t)$  denote Gross Domestic Product (GDP) in country  $i$  in units of the final consumption good. Define GDP as the sum of domestic production:

$$(4.25) \quad \begin{aligned} Y_1(s^t) &= \frac{1}{P_{1c}(s^t)} y_1(s^t) \\ Y_2(s^t) &= \frac{p_1^b(s^t)}{P_{2c}(s^t)} y_2(s^t). \end{aligned}$$

Let  $nx(s^t)$  denote net exports for country 1 as a fraction of GDP in country 1:

$$(4.26) \quad nx(s^t) = \frac{(a_{2c}(s^t) + a_{2x}(s^t)) - p_1^b(s^t)(b_{1c}(s^t) + b_{1x}(s^t))}{y_1(s^t)}.$$

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<sup>8</sup>Finn (2000) introduces a closed economy model in which utilization of capital requires energy inputs and shows that an exogenous increase in energy prices, by reducing utilization, can lead to a recession. This result is analogous to how a deterioration of the terms of trade leads to a decline in utilization in this paper.

Table 4.3: Benchmark parameter values

Description	Symbol	Value	
		Pre-85	Post-85
<i>Parameters common for pre-85 and post-85 periods</i>			
Discount rate	$\beta$	0.99	
Firsch labor supply elasticity	$1/(\mu - 1)$	2	
Risk aversion	$1 - \gamma$	2	
Capital share	$\alpha$	0.36	
Steady state depreciation rate	$\delta$	0.025	
Elasticity of substitution between capital services	$\rho$	1	
intermediate goods	$\sigma$	4	
Elasticity of $\delta$ with respect to $u$	$\nu - 1$	0.404	
IST transition matrix <sup>a</sup>	$A$	$\begin{bmatrix} 0.97 & 0 \\ 0 & 0.97 \end{bmatrix}$	
<i>Period specific parameters</i>			
Share of imported capital services	$1 - \theta_1, \theta_2$	0.14	0.308
intermediate goods	$1 - \omega_1, \omega_2$	0.065	0.098
IST shock	$\Sigma$	$\begin{bmatrix} 0.007 & 0 \\ 0 & 0.007 \end{bmatrix} \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}$	

<sup>a</sup> The law of motion for the vector of investment-specific technology shocks  $\hat{q}(s^t) = [\ln q_1(s^t), \ln q_2(s^t)]$  is given by  $\hat{q}(s^t) = A\hat{q}(s^{t-1}) + \varepsilon(s^t)$ ,  $\varepsilon(s^t) \sim N(0, \Sigma/\Sigma)$ .

#### 4.4 Calibration

The US economy, similar to other economies in the world, has gradually become more open to international trade. The gradual nature of openness implies that there is no unique way to separate the economy into two subperiods to reflect the change in the degree of openness. This paper uses 1961-1984 (pre-85) as the first period and 1985-2006 (post-85) as the second period. The choice of the periods enables the model to answer whether international trade could have contributed to the recent moderation of the US (and other countries) business cycle. Table 4.3 reports the parameter values.

Most of the parameters governing households' preference and the production functions are standard in the business cycle literature. Preference parameters are the

same for both periods. The discount rate ( $\beta$ ) is 0.99, an appropriate value for quarterly simulation. The risk aversion parameter ( $1 - \gamma$ ) is 2, which implies an elasticity of intertemporal substitution of 0.5. The parameter  $\mu$  is 1.5, which corresponds to the Frisch labor supply elasticity of 2. The parameter  $\varphi$  is chosen so that the time endowment devoted to working is  $1/3$  in the steady state. The elasticity of substitution between intermediate goods ( $\sigma$ ) is set at 4 (see Bernard, Eaton, Jensen, and Kortum (2003)).

The share of services of capital in the production function ( $\alpha$ ) is 0.36. The steady state rate of depreciation of capitals ( $\delta(u)$ ) is 0.025, which is 10 percent per annum. The depreciation rate and the rate of return on capital together pin down the curvature parameter  $\nu$  of 1.404 and the steady state utilization rate. This value of  $\nu$  implies that the elasticity of the depreciation rate with respect to the utilization rate is 0.404, which is in the range suggested by the empirical work of Basu and Kimball (1997). The elasticity of substitution between capital services ( $\rho$ ) is set at 1.

For the pre-85 period, imports are 8 percent of GDP and 28 percent of imports are investment goods (imports of durable goods minus imports of consumer durable goods in the National Income Product Account (NIPA)). Given that fixed investment is about 20 percent of GDP less government purchases, the share of imported capital in the aggregate capital ( $1 - \theta_1$ , and  $\theta_2$ ) is 0.14 and the share of imported good in consumption ( $1 - \omega_1$ , and  $\omega_2$ ) is 0.065.

For the post-85 period, imports are 14 percent of GDP and 44 percent of imports are investment goods. With the expenditure share of investment remaining roughly unchanged at 20 percent, a similar calculation reveals that the share of imported capital in the aggregate capital is 0.308 and the share of imported good in consumption is 0.098. Thus, another way to describe the increase in trade openness and the



change in trade composition is that the degree of “home bias” in capital services has declined faster than that in consumption.

This paper follows the identification of IST proposed by Greenwood, Hercowitz, and Krusell (2000). They argue that an evidence supporting the presence of IST is the fact that there is a negative co-movement between price and quantity of equipment at both low frequency (the trends) and high frequency (the deviations from trends). Here, the relative price of equipment, which is the inverse of IST, is constructed as the ratio of the implicit price deflator for equipment and software to the average of the implicit price deflators for consumer nondurables and services. All data come from the NIPA tables. The correlation between the Hodrick-Prescott (H-P) filtered series is  $-0.56$ , which is similar to their finding with annual data. The process for IST is then estimated by the following equation:

$$(4.27) \quad \ln q_i(s^t) = \text{constant}_i + t \ln \text{growth rate}_i + \eta_i(s^t),$$

where

$$(4.28) \quad \eta_i(s^t) = \rho_i \eta_i(s^{t-1}) + \varepsilon_i(s^t), \quad 0 < \rho_i < 1, \quad \varepsilon_i(s^t) \sim N(0, \sigma_i).$$

For the pre-85 period, the degree of persistence ( $\rho_i$ ) is 0.97 and the volatility parameter ( $\sigma_i$ ) is 0.007. For the post-85 period, the degree of persistence is 0.94 and the volatility parameter is 0.005. Thus, the volatility of the exogenous shock has fallen by 28.6 percent.

The data needed for the estimation of the same equation for an aggregate of foreign economies are not available. Among the non-US G7 countries, the OECD Main Economic Indicators database provides data that can be identified as price series of equipments for 4 countries.<sup>9</sup> This paper uses a weighted average of the

<sup>9</sup>Those countries are Canada (electrical machinery), Japan (machinery and equipment), Germany (investment

ratios of the price of equipment to the price of consumption, where the weights are the ratios of a country's purchasing power parity adjusted GDP in 2000 to the sum of the four countries' GDP. For the pre-85 period, the degree of persistence is 0.93 and the volatility parameter is 0.016. For the post-85 period, the degree of persistence is 0.98 and the volatility parameter is 0.008.

Due to the lack of quality data for non-US G7 countries, the IST process of the US is used as the process for both  $q_1(s^t)$  and  $q_2(s^t)$ . The simulations to follow also assume that the degree of persistence in the post-85 period remains the same as that of the pre-85 period. Part of a sensitivity analysis consider cases when the shocks are correlated and when there is a positive IST spill-over.

#### 4.5 Results

The model is solved using a linearization method. The statistics from the models are the averages of 100 simulations. The statistics of quantity variables, such as GDP, consumption and investment, are calculated at steady state price levels.

Table 4.4 reports business cycle properties of the model when it is calibrated to the pre-85 period. Note that the investment adjustment cost is chosen so that the model generates the observed volatility of investment. With the IST shock and endogenous capital utilization, the model explains close to 40 percent of the volatility of output and consumption. This number is slightly higher than what Greenwood, Hercowitz, and Krusell (2000) report for their closed economy model. Both consumption and investment are strongly procyclical as in the data. While the model overstates the volatility of net exports, it understates the volatility of the terms of trade. The model predicts, as in the data, that both net exports and the terms of trade are

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goods), and Great Britain (manufacturing inputs excluding foods). Furthermore, all series are price indices instead of implicit price deflators, which are more appropriate for the identification of IST. See Fisher (1999) for the discussion on the use of implicit price deflators.

Table 4.4: Properties of the model with variable utilization

Statistics	Data Pre-85	Model Pre-85
<i>Standard deviations</i>		
GDP	1.85	0.72
Consumption	1.07	0.36
Investment	5.85	5.85
Net exports/GDP	0.42	1.19
Terms of trade	3.04	0.45
<i>Correlations with GDP</i>		
Consumption	0.77	0.99
Investment	0.87	0.77
Net exports/GDP	-0.46	-0.59
Terms of trade	-0.22	-0.63

Quarterly data series from 1961 to 1984 of the United States are used to compute standard deviations and correlations with GDP. These data are from the Bureau of Economic Statistics (BEA). The terms of trade is import price index divided by export price index. Pre-85 is when imports are 8 percent of GDP and 28 percent of imports are investment goods. All series, except net exports, have been logged and H-P filtered with a smoothing parameter of 1600. The statistics from the model are the averages of 100 simulations.

countercyclical.<sup>10</sup> All features, except the countercyclicality of the terms of trade, are similar to those of models with Total Factor Productivity (TFP) shocks (e.g., Backus, Kehoe, and Kydland (1995); and Heathcote and Perri (2002)).

Intuitively, the countercyclicality of the terms of trade implies that the IST shock is more similar to a demand shock than a supply shock. Recall that an appreciation of the terms of trade means that the price of the domestically produced intermediate good rises relative to the price of the imported intermediate good. On the supply side, an IST shock by itself, unlike a TFP shock, is not a shock to the supply of intermediate goods. In the extreme case of a fixed utilization rate, an IST shock has almost no effect on supply in the very short run because the extensive margin of capital input is approximately fixed. Even when the utilization rate increases making the supply higher in the short run, the higher utilization rate itself further

<sup>10</sup>Because the terms of trade is defined as the price of imports relative to the price of exports, its decline means an increase in the value of domestic good, which is an appreciation of the terms of trade.

increases the demands for replacement investment. On the demand side, the increase in demand for intermediate goods is biased toward the domestic good because the share of domestic capital services is bigger than one half.<sup>11</sup>

The appreciation of the terms of trade amplifies the effect of an IST shock on investment and output. According to equation (4.19), an appreciation raises the return on investment in the foreign capital because extra outputs generated by the foreign capital can be exchanged for more units of the foreign intermediate good. For a given rate of utilization, the higher investment leads to more services of capital and output. With endogenous utilization, an appreciation, according to equation (4.21), also reduces the cost of replacing the foreign capital and thus increases the rate of utilization.

#### 4.5.1 Effects of trade openness and trade composition

There are three changes between the pre-85 and post-85 period: the degree of openness measured by the share of imports in GDP, the composition of trade measured by the share of investment goods in imports, and the size of the IST shock. Table 4.5 shows how each of the changes affect the volatility of output. The second column reports the combined effect of all changes. The experiment in the third and fourth columns abstract away from changes in the exogenous shock. The third column reports the effect of changes in the degree of openness and the composition, while the fourth column reports the effect of a change in the degree of openness alone.

Panel A shows the results of the model with endogenous utilization rate. When all three factors change (post-85 openness; post-85 shock), the model predicts that

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<sup>11</sup>The fact that both types of capital are utilized at the same rate makes this point clearer. Because consumption is also procyclical, the biased increase in the demand for domestic good also comes from the increase in consumption.

Table 4.5: Effects of changes in trade openness and its composition

(A) Endogenous Utilization				
Statistics	Pre-85 openness	Post-85 openness		
		Post-85 shock	Pre-85 shock	Pre-85 composition Pre-85 shock
<i>Standard deviations</i>				
GDP	<b>0.72</b>	<b>0.49</b>	<b>0.69</b>	<b>0.71</b>
Utilization	1.08	0.72	1.01	1.04
Consumption	0.36	0.25	0.36	0.37
Investment	5.85	6.39	8.94	8.18
Net exports/GDP	1.19	1.48	2.08	1.85
Terms of trade	0.45	0.11	0.15	0.25

(B) Fixed Utilization				
Statistics	Pre-85 openness	Post-85 openness		
		Post-85 shock	Pre-85 shock	Pre-85 composition Pre-85 shock
<i>Standard deviations</i>				
GDP	<b>0.29</b>	<b>0.31</b>	<b>0.43</b>	<b>0.39</b>
Consumption	0.19	0.19	0.27	0.25
Investment	5.33	5.97	8.36	7.67
Net exports/GDP	1.36	1.55	2.17	1.99
Terms of trade	0.55	0.14	0.20	0.31

Pre-85 openness is when imports are 8 percent of GDP and 28 percent of imports are investment goods. Post-85 openness is when imports are 14 percent of GDP and 44 percent of imports are investment goods. Imports are 14 percent of GDP and 28 percent of imports are investment goods in the last column. All series from the simulations, except net exports, have been logged and H-P filtered with a smoothing parameter of 1600. The statistics reported are the averages of 100 simulations.

output (GDP) should be less volatile. The ratio of the volatility post-85 to pre-85 is 0.68: a reduction of 32 percent. This reduction is slightly larger than the effect of the shock itself. The last two columns show that indeed greater openness and the change in trade composition have stabilizing effects because the volatilities of output are lower than the pre-85 level even if the size of the shock is unchanged. Moreover, because the volatility of GDP in column 3 is lower than that in column 4, the change in trade composition amplifies the stabilizing effects of openness. Notice that the pattern of output volatility follows the pattern of utilization volatility closely. Quantitatively, however, the stabilizing effects of the increased openness and the

change in composition are small. As a result, most of the reduction in the volatility of output is due to a smaller sized shock.

Although the volatility of output is not significantly influenced by more trade and the shift in the composition of trade toward investment goods, some variables have become more responsive to shocks. Both investment and net exports are significantly more volatile. If the shock size is as large as its pre-85 level, the increase in the volatilities of these variables will be larger than a factor of 1.5. In contrast, the model predicts that the terms of trade is significantly less volatile. The volatility of the terms of trade should reduce to about one third of the pre-85 level.

Panel B shows the results when the utilization rate of capital is not variable. Note that the volatilities of output are lower than the corresponding values in panel A simply because the utilization rate is procyclical. The predictions are very different: output should be significantly more volatile even when the size of the shock has fallen by almost 30 percent. In addition, the last two columns show that the change in the composition contributes to the increase in the volatility of output. All of these predictions are completely the opposite of those of the model with an endogenous utilization rate. However, the predictions for consumption, investment, net exports and the terms of trade are very similar both qualitatively and quantitatively.

When the endogeneity of the utilization rate is taken into consideration, one can draw two conclusions from table 3. First, changes in trade patterns over the past 50 years should not have a major impact on the volatility of output because, while they have a stabilizing effect through the utilization of capital, they enable the economy to adjust more quickly to take advantage of shocks. Second, the reduction in the volatility of IST shock is one example of the “good luck” behind the moderation of the business cycles.

### 4.5.2 Interpretation

To clarify how openness affects the volatility of output, figure 4.1 shows the impulse responses of key variables to a 1 percent increase in IST. It is clear that the volatility of output post-85 is smaller than the pre-85 level because the shock has a smaller impact on output during the first few periods after the date of the shock. Because output is produced using services from the aggregate capital stock and labor hours, the shock must have smaller effects on at least one of those factor inputs. For the services of capital, which is the intensive margin of the aggregate capital stock, the smaller effect is due to a decline in the response of the utilization rate because the aggregate capital stock itself respond more strongly.<sup>12</sup> The decline in the response of capital services also has a negative effect on real wage. Since real wage is the only factor determining labor hours (equation 4.15), labor hours respond less strongly. Thus, the effects of the utilization rate directly on capital services and indirectly on labor hours explain the reduction in the volatility of output.

The decline in the response of the utilization rate can be explained by a smaller appreciation of the terms of trade. Intuitively, it implies that a unit of the domestically produced intermediate good can be exchanged for less units of the imported intermediate good needed to replace the foreign capital. In other words, the cost of input following the shock is more procyclical in the post-85 period. According to equation (4.21), the desired rate of utilization should be smaller. The smaller appreciation of the terms of trade itself is due to the smaller degree of home bias in consumption and, especially, in capital services. The smaller home bias means that the increase in the demands for intermediate goods following an IST shock is

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<sup>12</sup>The increase in the responsiveness of the aggregate capital stock is because the reallocation of intermediate goods for investment between the two countries has a bigger impact. When the economy is nearly closed, i.e., when  $1 - \theta_1, \theta_2 \approx 0$ , the reallocations have negligible effects on the aggregate capital stock.

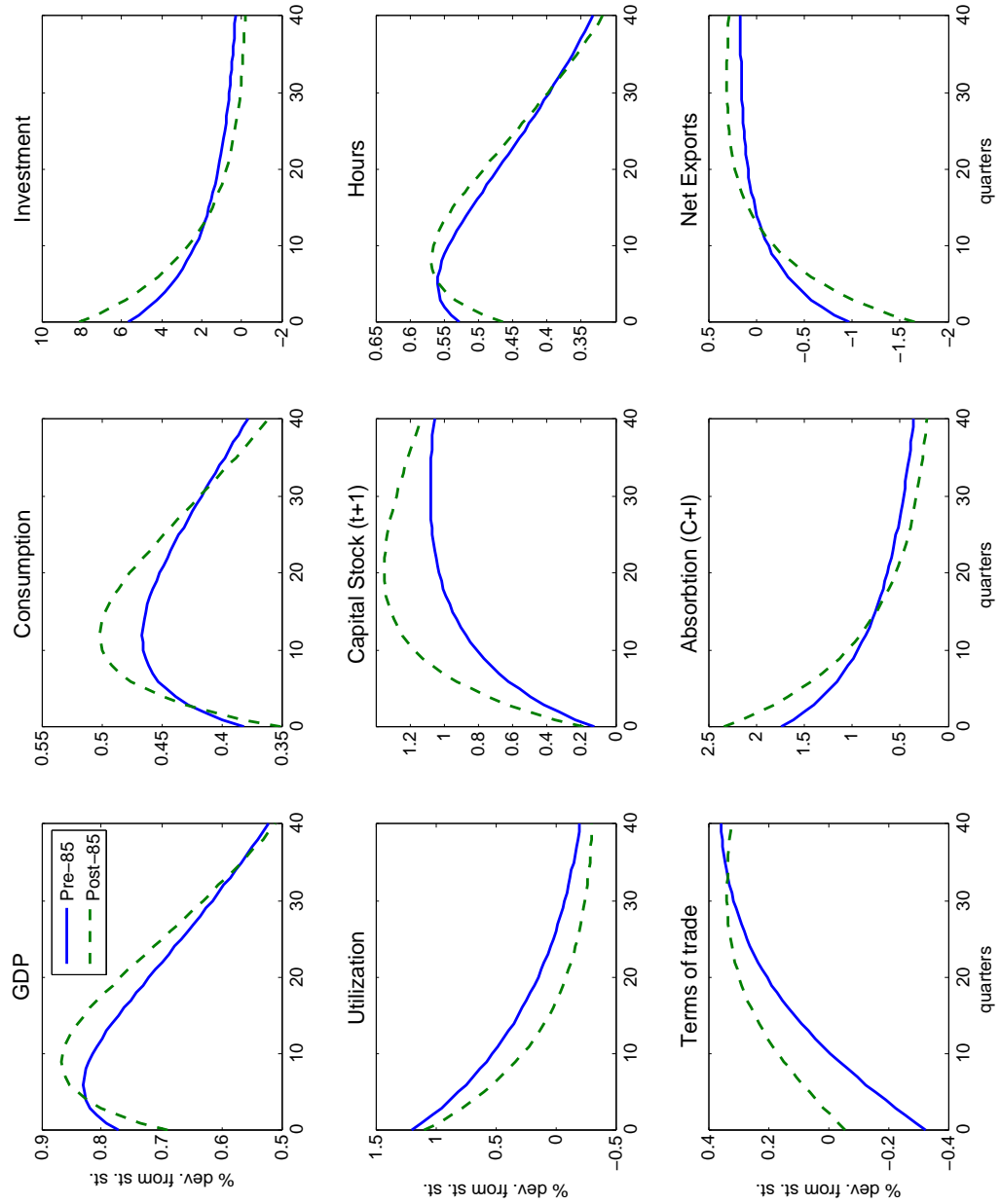


Figure 4.1: Impulse response functions to a 1% IST shock (Endogenous utilization)



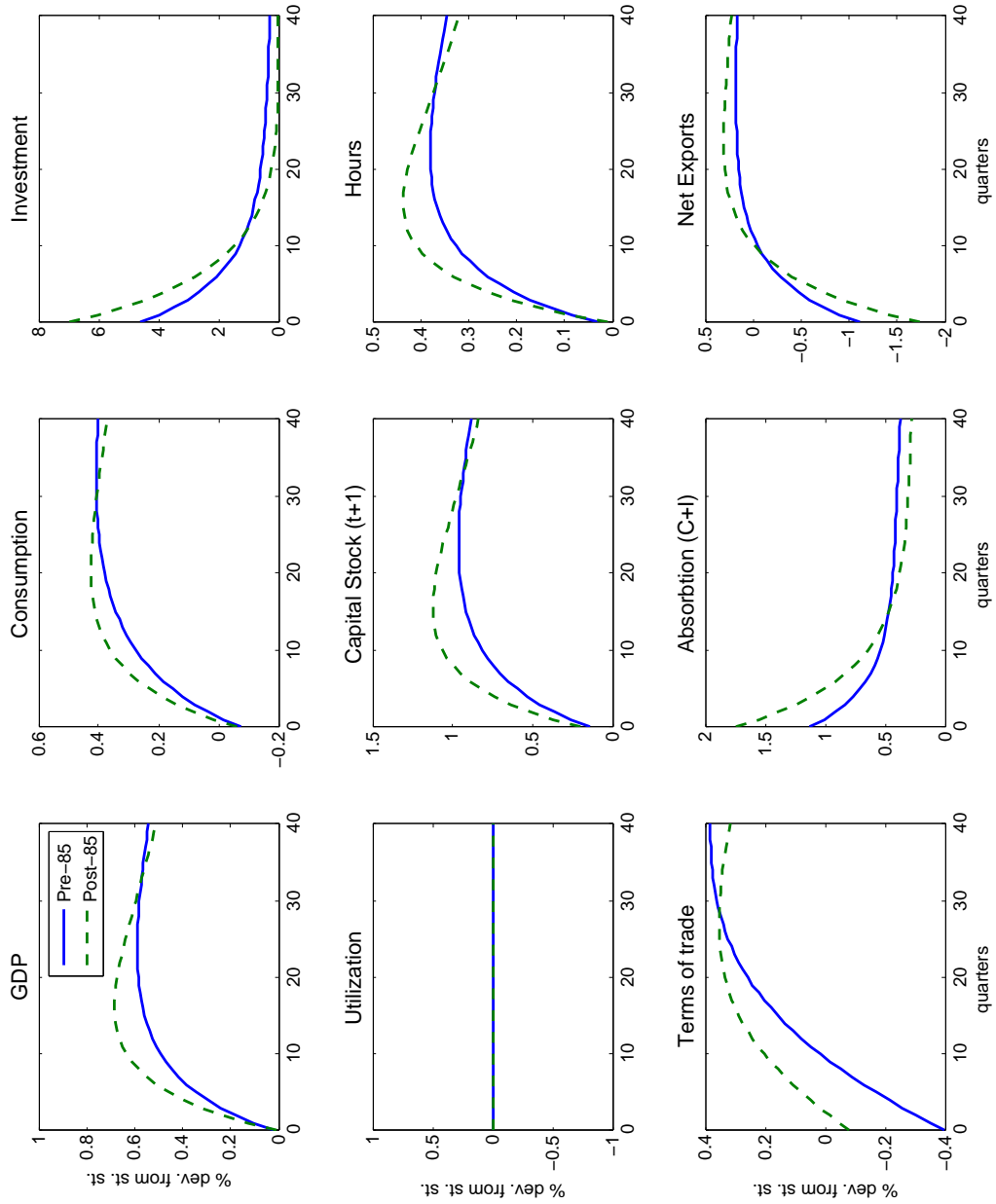


Figure 4.2: Impulse response functions to a 1% IST shock (Fixed utilization)

distributed more evenly between goods  $a$  and  $b$ .

Figure 4.2 shows the impulse responses to a 1 percent increase in IST of the economy with a fixed rate of utilization. The increase in the volatility of output is due to the greater responsiveness of both factor inputs in the post-85 period. Without variable utilization, capital services follow the path of the aggregate capital stock, which is more responsive. The increase in the response of capital has a positive effect on real wage, which causes labor hours to be more responsive. Note that the responses of the terms of trade is very similar to those in the model with variable utilization. This result can be explained in part by the facts that the shock affects investment in both types of capital symmetrically and that both types of capital are utilized at the same rate.<sup>13</sup>

It should be pointed out that greater openness improves the economies' ability to take advantage of a favorable shock. In both figures, the cumulative responses of output and consumption in the post-85 period are larger than their pre-85 counterparts. Some of the extra output, due to the risk sharing contract, will be sent to the other country in return for the resources used in production. Consequently, households in both countries enjoy higher consumption levels than the pre-85 period.

## 4.6 Sensitivity analysis

In the benchmark calibration, the exogenous shocks are uncorrelated across countries and a shock in one country does not affect IST in the other country. Table 4.6 reports the volatilities of output, utilization and the terms of trade when those two conditions are relaxed. In the first exercise, the correlation of the shock is 0.3. In the

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<sup>13</sup>These two figures suggest that output and the terms of trade should be less negatively related when the economy is more open. The data for the US supports this pattern because the correlation is  $-0.22$  in the pre-85 period, as reported in table 4.4, while it is  $0.11$  in the post-85 period. However, this result is only true when the terms of trade of all goods and services is used.

Table 4.6: Sensitivity analysis

Statistics	Pre-85 openness	Post-85 openness		
		Post-85 shock	Pre-85 shock	Pre-85 composition Pre-85 shock
Correlated shock				
<i>Standard deviations</i>				
GDP	<b>0.70</b>	<b>0.49</b>	<b>0.68</b>	<b>0.70</b>
Utilization	1.08	0.73	1.03	1.05
Terms of trade	0.38	0.09	0.13	0.21
Positive spillover				
<i>Standard deviations</i>				
GDP	<b>0.72</b>	<b>0.52</b>	<b>0.73</b>	<b>0.74</b>
Utilization	1.05	0.73	1.02	1.04
Terms of trade	0.72	0.25	0.35	0.48

Pre-85 openness is when imports are 8 percent of GDP and 28 percent of imports are investment goods. Post-85 openness is when imports are 14 percent of GDP and 44 percent of imports are investment goods. Imports are 14 percent of GDP and 28 percent of imports are investment goods in the last column. Correlated shock is the case when the correlation of the shock is 0.3 and the diagonal elements are 0.9. All series from the simulations, except net exports, have been logged and H-P filtered with a smoothing parameter of 1600. The statistics reported are the averages of 100 simulations.

second exercise, the off diagonal elements of the transition matrix  $A$  are 0.07 and the diagonal elements are reduced to 0.9 so that the largest eigenvalue of the transition matrix remains the same as in the benchmark calibration.

The top panel shows that the results are not significantly affected by the correlation of the shock. However, while the change in the composition still has stabilizing effects on utilization and output (comparing the last two columns), the increase in trade by itself has no effect on the volatility of output. The bottom panel shows that while the change in the composition has stabilizing effects on utilization and output, the increase in trade by itself can increase the volatility of output. The terms of trade behaves quite differently as well. Its volatility is higher than when there is no spillover. More importantly, the reduction in its volatility (comparing the first column to the last column) is considerably smaller compared to that in table 4.5.

Clearly, the smaller reduction in the volatility of the terms of trade explains why openness no longer has a stabilizing effect.

The positive spillover amplifies two effects. First, it makes households in country 2 want to lend more to country 1 and delay investment. Second, it implies that the wealth effect that country 2 receives from the shock is larger than when there is no spillover. These two effects have different impacts on the value of good  $b$  (relative to good  $a$ ). Because of home bias, the first effect reduces its value while the second effect increases its value. The fact that the terms of trade is more volatile when there is spillover means that the first amplification is stronger. The smaller reduction in the volatility of the terms of trade means that this amplification remains strong even after the home bias in capital services is smaller.

For a plausible range of the exogenous process, the model still predicts that changes in trade pattern over the past 50 years should not have a major impact on the volatility of output.<sup>14</sup>

## 4.7 Conclusion

This paper documents that most economies have become more open to international trade and that a growing fraction of trade is trade in durable goods. The paper develops a model to study how these changes affect the volatility of output and examines the impact of a shock that increases the ability of the economy to generate investment goods. When proper care is taken to account for the role of variable capital utilization, the model predicts that output should remain approximately as

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<sup>14</sup>Although they are not reported, the cross-country correlations of output, consumption and investment do change between the two period. These changes, however, are small if the shock correlation or the spill over do not change across periods. In the data, there exists some evidence that these international correlations have been much lower since the mid-1980s. Heathcote and Perri (2004) show that both a reduction in the correlation of shocks and an increase in asset trade are required to explain such a change. The model in this paper does not take into account the second ingredient because it assumes that asset markets are complete in both pre-85 and post-85 periods.

volatile as before unless the size of the shocks hitting the economy has changed. The findings in this paper support the notion that the moderation of the business cycle over the last few decades originated from a reduction in the size of shocks or the frequency at which they hit the economy. Although the effect on volatility is small, the greater openness allows the world as a whole to better take advantage of shocks by reallocating resources to the most productive region.

#### 4.8 Appendix: Utilization and the terms of trade in a standard two-country, two-good model

The model presented here is similar to those in Backus, Kehoe, and Kydland (1995) and Heathcote and Perri (2002). Intermediate goods  $a$  and  $b$  are combined using a constant elasticity of substitution technology to produce one final good  $G_i(s^t)$ , which can either be consumed or invested:

$$G_i(a_i(s^t), b_i(s^t)) = c_i(s^t) + x_i(s^t).$$

This specification means that the share of imported good in consumption is the same as that in investment. Without loss of generality, assume that the elasticity of substitution between intermediate goods is 1 so that the production of the final good is of the Cobb-Douglas form:

$$G_i(a_i(s^t), b_i(s^t)) = a_i(s^t)^{\omega_i} b_i(s^t)^{1-\omega_i}.$$

The aggregate capital stock is related to investment in the following way:

$$k_i(s^t) = (1 - \delta(u_i(s^t)))k_i(s^{t-1}) + q_i(s^t)x_i(s^t).$$

Assume that asset markets are complete. Let  $R_i(s^t)$  and  $P_i(s^t)$  be the rental rate of capital and the price of final good, respectively, in units of the intermediate good

produced in country  $i$ . The maximization problem of the representative household in country 1 is given by

$$\begin{aligned} & \max_{c_1(s^t), n_1(s^t), B_1(s^t), k_1(s^t), u_1(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ \frac{1}{\gamma} [c_1(s^t) - \varphi n_1(s^t)^\mu]^\gamma \right. \\ & + \lambda_1(s^t) \left[ R_1(s^t) u_1(s^t) k_1(s^{t-1}) + w_1(s^t) n_1(s^t) + B_1(s^{t-1}) \right. \\ & \quad \left. \left. - \sum_{s^{t+1}} Q(s^t, s_{t+1}) B_1(s^t, s_{t+1}) - P_1(s^t) (c_1(s^t) + x_1(s^t)) \right] \right. \\ & \left. + \Omega_1(s^t) [-k_1(s^t) + (1 - \delta(u_1(s^t))) k_1(s^{t-1}) + q_1(s^t) x_1(s^t)] \right\}. \end{aligned}$$

With  $\lambda_1(s^t) P_1(s^t) = \Omega_1(s^t)$ , the first order condition with respect to the utilization rate is given by

$$q_1(s^t) R_1(s^t) k_1(s^{t-1}) = P_1(s^t) \delta'(u_1(s^t)) k_1(s^{t-1}).$$

Note that  $P_1(s^t) = (p_1^b(s^t))^{1-\omega_1}$ . As a result, movements in the terms of trade  $p_1^b(s^t)$  still affect utilization. Consistent the analysis of equation (4.21), the effect of the terms of trade is increasing in the share of imported good.

## CHAPTER V

### Conclusion

This dissertation illustrates the usefulness of durable goods in explaining various features of the international business cycle. These include the positive cross-country correlation of investment, the lack of international risk sharing represented by the low correlation between relative consumption and the real exchange rate, and the expenditure-switching effect of the terms of trade. The study also discusses the implications of the increased international trade in durable goods.

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