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A SURVEY OF PROGRESS

1944-1951

by

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And The Project Supervisors

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A report on the origin and progress of current research projects of the Column Research Council, prepared by Bruce G. Johnston* with the assistance of various council members, for presentation at the 1951 Annual Convention of the Structural Engineers Association of California.

Objectives, Origin, and Organization of the Council

The Column Research Council, hereinafter referred to as "CRC", has prepared a statement of its general purposes, as follows:

a. "To organize, maintain, and administer a national forum where problems relating to the design and behavior of columns and other compression elements in metal structures can be presented and pertinent structural research problems can be proposed for investigation, with the assurance of an evaluation of all problems proposed and of support for those projects adjudged important.

b. "To digest critically the world's literature on structural behavior involving compression elements and the properties of

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**Superceded by a number of research committees that previously were subcommittees of the Committee on Research.

metallic materials available for their construction, and make the results widely available to the engineering profession.

c. "To organize and administer cooperative research projects in the field of compression elements.

d. "To stimulate, aid, and guide column research projects on the foregoing problems in the engineering colleges, endowed laboratories, and other research laboratories.

e. "To study the application of the results of this program to the design of compression elements.

f. "To develop a comprehensive and consistent set of formulas or rules covering their design.

g. "To promote the widest possible adoption of such formulas by designers and specification-writing bodies.

h. "To publish and disseminate original research information within the field of the Council."

To date, as will be seen from this report, progress has been primarily in the first four and the last of the foregoing items.

For some years prior to 1941 as a result of an original suggestion of Jonathan Jones the "Committee on Design of Structural Members", of the American Society of Civil Engineers, had been working on metal column problems, and had formulated certain research proposals. Certain individuals, especially Jonathan Jones, Shortridge Hardesty, and F. H. Frankland, saw the need for putting this work on a broader base, so as to gain for it more general acceptance and at the same time raise funds to make feasible the needed research. CRC thereby came into being; it was formally organized in 1944, and now, with 26 participating organizations having from 1 to 4 officially designated representatives each, it is beginning to fulfill its original objectives. Much remains to be done, and it is still a growing organization. Credit for the sound and steady development of CRC is due to its chairman and acting director, Mr. Shortridge Hardesty, who has ably guided its work from the beginning to the present time. Much credit is also due to Lynn Beedle, of Lehigh University, part-time secretary of CRC, for his enthusiastic and loyal attention to the many details of its work during the past several years when the research projects and growth of the council were most active. Initial administrative and financial support by the Engineering Foundation brought the council into being, and this support, still continued, is now supplemented by major contributions from the Association of American Railroads, the American Institute of Steel Construction, the Aluminum Company of America, the Office of Naval Research, Research Corporation, the Bureau of Public Roads, various State Highway departments, and a number of other associations and individuals.

The organization of CRC is shown in Fig. 1. The various Research Committees, organized and developed initially under a general Committee on Research, now report directly to the Executive Committee, the latter functioning in both a technical and administrative capacity. The category groups are not organized as such but are listed to represent the various classes of structure for which adequate column design information is needed. Although the basic fundamentals of the problem are the same in all structures, the listing of the last category group, "Air Craft", does not mean that CRC intends to propose design rules for aircraft construction. It was felt, however, that affiliation with this field was desirable and that earthbound structures could well profit by the rapid strides made in theory, research, and design prodedures as applied to aircraft.

The Committee on Recommended Practice has been set up with the purpose of considering ways and means of applying results of research to the actual design of structures. It is expected that the activity of this committee will increase rapidly in the near future, now that the research committees are beginning to produce results having potential application in design.

Development of the Research Program

Prior to the proper planning of a structural research program the following questions should be answered:

(1) What are the problems to which structural engineers are seeking a solution?

(2) To what extent does existing information provide solution to these problems? Not infrequently there is a lapse of many years before applicable theory and test results bring about improvement in engineering specifications and design procedures.

(3) What is currently being done about the problems? There is a great deal of duplication in research. Some duplication is desirable, but it is essential, in a broad program, to undertake the difficult task of finding out what parallel work is in progress.

(4) Finally, what projects are needed, in the light of past and current research, to give the engineer the answers he needs?

The research program of CRC has been developed strictly in accordance with the foregoing plan. One of its first steps was to circulate a questionnaire to leading design engineers requesting detailed information as to what were considered important structural problems involving stability against buckling. Suggestions for research were also requested and the questionnaire closed with the query "What metals do you expect to use in future structural designs?" The returns to this questionnaire were analyzed by a special committee,* and a summary report was issued in 1947. It was evident that attention should be given first to the column as part of a real structure in which end bending moments and restraint of adjacent framing were involved,

> ". . . giving consideration not only to individual members but also to complete frames or trusses . . . . The per cent of the total construction tonnage affected by these problems is high, indicating that improvements in structural design methods for these problems would have a wide beneficial effect."

Some research was also recommended on questions of lateral-torsional buckling. Considerable interest was expressed in problems of local buckling, but the committee recommended that research on these matters be delayed until after the results of tests and analyses carried out during the war, for aircraft, be assessed and made generally available. Special interest was expressed in the design of the compression flange of through girder bridges and the top chords of through trusses which had no top bracing members. As to materials other than ASTM A7 structural steel, interest was expressed in structural silicon steel (ASTM-A94), low-alloy structural steel (ASTM-A242), and structural aluminum alloys.

In carrying out the second phase of program development, that is, to survey existing information, CRC received major initial support from the David Taylor Model Basin of the U. S. Navy Department. Quoting from the April 14, 1947, semiannual report of CRC, the late

> "Dr. D. F. Windenburg, Chief Physicist of the David Taylor Model Basin, advised that his bureau was ready to enter in a contract . . . for preparing a critical bibliography on the strength of structural members under compressive loading."

An initial allocation of $30,000 was provided for this work. The complete study will shortly appear in book form as one of the Engineering Monograph Series, a fitting memorial to its author, the late Dr. Friedrich Bleich,** and to Dr. Windenburg, in whose mind the plan had originally been conceived. In the foreword to this book, Rear Admiral C. O. Kell, USN, states, in part;

> "To obtain pertinent information and assemble it in a single publication, the Bureau of Ships, through the David Taylor Model Basin, and Frankland and Lienhard, Consulting Engineers of New York, has sponsored this excellent work by Dr. Friedrich Bleich. Dr. Bleich, with the cooperation of the Column Research Council,

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*E. E. Lundquist, Chairman, with J. Jones, H. C. Tammen, and B. G. Johnston.

**The book is being edited by Dr. Hans Bleich, of Columbia University, son of Dr. Friedrich Bleich. Dr. Hans Bleich also wrote two chapters of the book as originally planned.

has assembled all available data on this subject, has appraised it and selected the most useful and applicable methods of analysis for inclusion in this book, and has supplemented this work by suggestions and recommendations of members of the staff of the David Taylor Model Basin and the Bureau of Ships to provide this up to date treatise on the buckling strength of metal structures."

The third phase of program development, to ascertain current programs or those recently completed, was accomplished by the circulation of a second questionnaire, in this case, of world wide scope. Planned also as an aid to the preparation of Dr. Bleich's book, the detailed reporting of the continuing returns from this questionnaire has been done by Mr. Lynn Beedle, Secretary of CRC and Assistant to the Director of Fritz. Engineering Laboratory, Lehigh University. The third and fourth interim reports, issued by CRC in 1948 and 1950, respectively, list the recent unpublished reports and project statements on current research, together with any available details concerning the program. One further result of the survey is a report prepared by Professor M. Ros, Zurich, Switzerland, which summarized research over the past 25 years in the Swiss Federal Institute. This report originally in French, has been translated and printed by CRC.

The final and fourth step in the development of the research program--the actual planning of projects in the light of engineering needs and surveys of existing information--was carried out by the Committee on Research. This committee was active from the beginning of the CRC's organization, and its various subcommittees did the planning of actual projects. These subcommittees were finally seven in number, as follows:

A: Mechanical Properties of Materials
B: Initial Eccentricities of Compression Elements
C: Local Buckling of Compression Elements
D: Columns in Structural Frames
E: Torsional Instability of Structural Elements
F: Stability of Structural Elements under Dynamic Loading
G: Stability of Laterally Unsupported Bridge Chords.

The subcommittees formulated problem statements which, except in the case of pre-existing projects, were generally broadcast to a number of universities. Proposals to do the work were studied, and recommendations for project awards were then made to the Executive Committee. With the subcommittees well established and with various projects under way, the need for a coordinating committee on research disappeared, and the various research project committees now report directly to the Executive Committee.

The subject of the column as a part of a framed structure had been designated as of greatest interest by structural engineers who replied to the initial questionnaire. Work on this subject is now well along and CRC was able to hold its first Technical Session in May of 1951, at which time detailed reports of progress on the framed column programs were presented.

Research Projects

A resume of the research work on CRC will concern its activity in relation to the following problems:

1. The relation between material properties and column strength
2. The local buckling of compression elements
3. The column as part of a structural frame
4. The lateral-torsional buckling of beams and columns
5. The behavior of columns under dynamic loads
6. The stability of the compression chord of a bridge truss without top bracing.

There is an obvious similarity between these subjects and the various research committee designations as previously listed.

The Relation between Material Properties and Column Strength

The study of the basic relationship between the compressive stress-strain curve of a material and the strength of an ideal column is a proper preliminary to a consideration of the modifying factors that affect the strength of a real column in an engineering structure. The first and greatest step in the development of this relationship was provided by Euler[1] nearly 200 years ago. Subsequent refinements to take account more properly of inelastic behavior in the Euler column formula have been reviewed by your own F. R. Shanley,[2] who himself has made a most significant contribution to a better understanding of the inelastic bending and buckling of the ideal column.

A carefully prepared statement by Research Committee A* describes in detail the proper procedure for obtaining compressive stress-strain curves of a metal and defines the relationship between such curves and the strength of an ideal column. For this purpose, an ideal column may be defined by a set of assumptions, as follows:

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[1]Euler, Leonhard, "Sur la Force des colonnes", Academie des Sciences de Berlin, Mem. 13:252 (1757). English translation by J. A. van den Broek, Am. J. Phy. 15:309-18 (1947).

[2]Shanley, F. R., "Applied column theory", Proc. ASCE 75:759-88, 1949 disc. 1085-8, 1224-8, 1397-1400.

*W. R. Osgood, Chairman.

Assumptions as to material:

1. Same compressive stress-strain properties throughout
2. No initial internal stresses due to welding, cooling after rolling, etc.

Assumptions as to shape and end conditions:

3. Column is straight and of uniform cross section
4. The load resultant and the centroidal axis of the column coincide up to the load at which the column begins to bend
5. The end conditions must be determinate; that is, they must lead to a definite equivalent slenderness ratio of the column.

Assumptions as to buckling behavior:

6. Ordinary bending theory for small deflections may be used, and shear may be neglected
7. The cross section does not twist and does not change in shape during bending.

The statement of committee A reads in part as follows:

"Compressive stress-strain curves should be plotted with stress as ordinate and strain as abscissa to as large a scale as the accuracy of the data justifies. The individual values of stress and strain should also be reported. When applying the above procedures to material which is suspected of showing a considerable variation in properties, the specimens should be taken from a sufficient number of locations to define the extent of the variation in properties.

"It is quite generally accepted that the column strength can be determined with satisfactory accuracy by the use of the tangent-modulus method applied to a compressive stress-strain curve for the material, if the material throughout the cross section of the column has reasonably uniform properties and the column does not contain appreciable residual stresses. The strength of a column may be expressed by

$$\frac{P}{A} = \frac{\pi^2 E_T}{\left(\frac{KL}{r}\right)^2}, \qquad (1)$$

in which $\frac{P}{A}$ = average stress in the column,

E_T = tangent modulus (slope of stress-strain curve) at stress P/A,

$\frac{KL}{r}$ = equivalent slenderness ratio of the column.

This equation may be written

$$\frac{KL}{r} = \pi \sqrt{\frac{E_T}{\frac{P}{A}}} \qquad (2)$$

"Fig. 2 [originally 1] shows a typical compressive stress-strain curve of a high-strength aluminum alloy. Lines have been drawn tangent to this curve at different values of stress (P/A). The slopes of these lines define the corresponding tangent modulus (E_T). Values for stress and tangent modulus are thus obtained"

. . for substitution in Equation 2, by means of which values of KL/r are determined for corresponding values of average column stress, P/A. As shown in Fig. 3, these coupled values may then be plotted to provide the basic column-strength curve of average column stress at which bending commences plotted against the equivalent slenderness ratio. The five limiting cases to which this formula may then be applied are shown in Fig. 4, which tabulates the length factor "K". K may also be evaluated for more complex conditions of restraint. It is assumed that the column remains in one plane and that no twisting occurs during buckling.

The statement of Committee A continues, in part,

"It has been shown both experimentally and analytically that where considerable amounts of residual stress are present in a shape, the tangent-modulus column curve based on the stress-strain curves from coupons cut from the section will not necessarily closely approximate the actual column strengths."

The recognition of the role of residual stress is in itself an important contribution of CRC. The tangent-modulus formula (Equation 1) is most directly applicable to the treatment of aluminum alloys, stainless steels, heat-treated steels, and other materials that have decidedly nonlinear characteristics in the vicinity of the yield strength level. Structural steels, on the other hand, have nearly linear stress-strain characteristics up to a sudden yield point, whereupon, strains ten to twenty times the elastic strain develop suddenly with no increase in stress level. The relatively minor deviations in linearity of the stress-strain curve near the yield point are overshadowed by variations in yield stress level over the cross section and by the possible effects of residual, or initial stresses, induced either by welding or by cooling after rolling. An example of measured values and distribution of residual stress in a wide flange beam is shown in Fig. 5.[3]

[3]Taken from report by Luxion, W. W., and Johnston, B. G., "Plastic Behavior of Wide Flange Beams"--Progress Report No. 1, Welded Continuous Frames and their Components, Welding Journal 27: 538s (1948).

Exploratory work on the question of residual stress in structural steel columns has been in progress at Lehigh University for the past two years as part of a broad program of work on the strength of welded frames. CRC has acted in an advisory capacity, the project being sponsored by the Welding Research Council. A very recent progress statement furnished by Mr. Lynn Beedle, under date of September 17, presents results of pilot tests as shown in Figs. 6 and 7, which illustrate the importance of residual stress in determining the critical buckling load of a structural steel column. In this pilot program a 20-inch length of an 8WF31 member was tested with flat ends. An average stress-strain diagram was obtained which was used to compute a column curve according to the tangent modulus concept.

The following paragraphs (with changes in figure numbers) are quoted from the Lehigh Memorandum:

> "In Fig. 6 is shown the stress-strain diagram as determined by strain measurements on a small coupon as compared with the average stress-strain curve for the full 8WF31 cross section containing residual stress. Measurements for the latter were made with a simple dial gage technique, readings being taken at each of the four corners over a gage length equal to that of the specimen.
>
> "In Fig. 7 are shown column curves determined from the tangent modulus of the curves shown in Fig. 6. Plotted upon the curve are the critical loads of three tests at slenderness ratios of approximately 28, 42, and 56.
>
> "This study of a short compression specimen resulting in the curve of Fig. 7 is only a pilot investigation. However present evidence indicates the following:
>
> "(1) A satisfactory column curve cannot be obtained on the basis of tests of small samples cut from various places in the cross section.
>
> "(2) From Fig. 7, as expected, the percentage reduction is greatest in the range L/r = 90. No tests fell in this range and they should be done.
>
> "(3) Good agreement with the column curve is obtained for T11 and T18. T15 carries less load than predicted on the basis of the average curve determined from the 'residual stress specimen.'"

Although the Lehigh pilot tests indicate the importance of residual stress in structural-steel columns and the possible value of compressive tests of complete short sections, it is to be noted that the columns as tested were free to bend in their strong direction and that there was thus an averaging effect of the residual stress across the flange. Had bending been freely permitted in the weak direction, the adverse effect of residual stress would probably have been even greater. This problem has been studied theoretically for the special case of a structural steel WF shape with a symmetrical residual-stress pattern similar to that shown in Fig. 5.[4] A generalized statement of the residual-stress column problem for any stress-strain relation and residual stress pattern has been presented by Wm. R. Osgood,[5] Chairman of CRC Research Committee A, which had been asked to investigate the matter.

Although it is too early for sweeping conclusions, all of the evidence at this time points to the desirability of further research on residual stress with the possibility indicated that such studies may permit great improvement in predicting the column strength curves for structural steel columns by means of a tangent modulus column theory which incorporates the effect of residual stress. It should be noted that residual stress has relatively minor effect in the case of columns with very short or very long equivalent lengths.

The effect of initial end eccentricity on column strength is under study at Purdue University, where tests are being conducted for CRC by Professor L. T. Wyly. Members of Research Committee B, under the chairmanship of G. M. Magee, have expressed the opinion that improvements in specifications relative to end milling tolerances will result from this work.

The Local Buckling of Compression Elements

Structural steel columns are usually designed so as not to buckle locally, but it is essential to consider this possibility in order to develop safe rules of design that will insure that the column will develop its full integral buckling strength prior to such local failure. Local buckling is of special concern in aircraft construction, where thin-walled sections of aluminum alloys are frequently used and where, in the aircraft skin, wrinkling is sometimes permitted--at least in the case of subsonic speeds. A similar situation exists in the rapidly expanding field of light gage steel structures used in building construction. Dr. Eugene E. Lundquist, formerly chairman* of Research Committee C, suggested the preparation of a summary report that would bring together all the significant results of recent research at the National

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[4]Yang, Ching Huan, Beedle, Lynn S., and Johnston, B. G. "Residual Stress and the Yield Strength of Steel Beams", Progress Report No. 5, "Welded Continuous Frames and Their Components", to be published in the March, 1952, Welding Journal.

[5]Osgood, Wm. R., "Residual Stresses in Columns", presented at the June, 1951, meeting of the First U. S. National Congress of Applied Mechanics.

*Dr. George Winter is now Chairman of Committee C, with Dr. Lundquist as Co-chairman.

Advisory Committee for Aeronautics Research Laboratories at Langley Field. This was considered a desirable preliminary to any extensive new work on this subject. This report has just been published[6] and while not directly sponsored by CRC was prepared as a contribution to the further planning of its work.

CRC also undertook partial financial support of a project already in progress under the guidance of Professors S. Timoshenko and Jack R. Benjamin of Stanford University. Planned for completion by September, 1952, Professor Benjamin states that the investigation covers

> "analytical studies of two types of built-up columns, latticed, and batten plate or perforated cover-plate. Comparison with experimental data is being made at every opportunity. . . The entire project has been aimed at developing methods of analysis capable of application in a normal design office by practicing engineers. The final report will have two main parts of interest to the practicing engineer:
>
> "1. Practical simplified methods of analysis for stresses, deflections, and buckling loads. This will enable more rational design to be made of any column requiring such attention.
>
> "2. Analysis of current design specifications and such overall conclusions as can be made based on an elastic analysis."

A project on local buckling of flange elements is being sponsored without financial support at Lehigh University.

The Column as Part of a Structural Frame*

> "One of the most unsatisfactory features of the column problem is the question of end conditions. Here the practical designer is left with very scanty aid from either theory or experiment. . . The indefinite nature of the practical conditions would be a serious addition to the difficulties of the column problem, were it not for the fact that most practical columns are relatively short."

The foregoing remarks were published thirty years ago as a result of Salmon's[7] overall study and summary of column theory and experiment that he had carried

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[6]Stowell, E. Z., Heimerl, G. J., Libove, D., and Lundquist, E. E., "Buckling Stresses for Flat Plates and Sections", ASCE Proc. Separate No. 77, July, 1951.
[7]Salmon, E. H., Columns, Frowde, Hodder, and Stoughton, London 1921.
*Under cognizance of Research Committee D, with Dr. N. M. Newmark as Chairman.

out as a student between 1906 and 1915. His observations were based on the fact, still pertinent today, that our ideas regarding column strength are determined to too great an extent by tests of laboratory specimens which have either flat or simulated pin ends and in which the ends are position-fixed against sidesway. The current research projects under sponsorship of CRC are throwing new light on the behavior of the framed column, representing a potent factor in this area of development.

In Great Britain, J. F. Baker[8] and his associates have done work for the past ten years in which small model frames were tested and the behavior of columns evaluated as a part of the frame. In this country, CRC has been affiliated with three projects on the framed column at Cornell University, Pennsylvania State College, and Lehigh University.

The problem of the framed column is complicated by the fact that the end restraints and/or applied end moments, resulting from the action of adjacent framing, do not in general remain constant during increase of axial load in any particular column. An adjacent tension member will provide increased restraint against sidesway or end rotation of the column in question. An adjacent compression member, on the other hand, will provide decreasing restraint with increasing load, until finally it may even reverse its behavior and apply moment or shear to the column in place of the restraint it previously provided. These facts necessitate a consideration of the entire frame or a significantly large portion of its weaker regions.

Stimulated by the interest of CRC in the framed column, Dr. George Winter, at Cornell University, undertook work at an early date and has published a bulletin[9] on this subject. Referring to this bulletin and to more recent work which has been sponsored jointly by CRC and the U. S. Bureau of Public Roads, at Cornell University, Dr. Winter states:

"(a) It [Reference 9] reviews the field of buckling of rigid-joint structures, develops a new method for trusses by the end restraint method and, in particular, presents the first reasonably practical method of investigating frames with sidesway by moment distribution. It shows the buckling strength of such structures to be considerably below that obtained when sidesway is prevented. It is believed that this should eventually be reflected in design codes, since the effective length* of columns in such frames, far from being 0.75L as assumed in most specifications, may be considerably larger

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[8]Baker, J. F., Horne, M. R., and Roderick, J. W., "The Behavior of Continuous Stanchions", Proc. Roy. Soc., London 198: 493-509, 1949.

[9]Winter, G., Hsu, P. T., Koo, B., and Loh, M. H., "Buckling of Trusses and Rigid Frames", Cornell University Engineering Experiment Station Bulletin No. 36, April, 1948.

*See Fig. 8.

even than the real length. It presents also tables and charts which facilitate not only stability computations but also the determination of moments and stresses at subcritical loads.

"(b) The first Progress Report[10] of the project proper, by P. P. Bijlaard, deals specifically with buckling of trusses and other structures without sidesway. Existing methods involved lengthy successive approximations impractical for design. By a systematic subdivision of the structure by appropriate hinges, and by a method of "split rigidities" the analysis of such a structure is now no more involved than an ordinary moment distribution analysis.

"(c) The second Progress Report[10] recently distributed deals with the problem of <u>elastically restrained, eccentrically loaded members</u>, i.e., the most frequent, practical type. The only previously available information was the treatment of one single special case by Chwalla[11] by means of a very tedious, semi-graphical method. Professor Bijlaard's work, in contrast, covers equal, equal and opposite, and unequal eccentricities, i.e., the whole practical range. The method is approximate, to keep it within practical limits of required time. Some significant results are illustrated as follows. First, a standard column curve for steel has to be assumed as a basis. In the one chosen for the report the proportional limit was assumed to be 10 ksi below the lower yield point. This may not be sufficiently general, but if some other standard curve is agreed upon, the method can easily be modified accordingly. Some significant numerical results are given on Fig. 9, which gives the buckling loads of the various loadings and configurations shown on top of the figure. The second column contains Chwalla's results for the few cases for which they are available. They may be regarded as "exact" and hence as a standard of comparison. The third and fourth columns give, respectively, the results of Bijlaard's "almost exact" and of his much shorter approximate method. Agreement between them and also with Chwalla is seen to be excellent. The fifth column refers to more standard, American structural steel and shows that the differences in detail of stress-strain curves are not very significant if the

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[10]Bijlaard, P. P., "First Progress Report on Investigation of Buckling of Rigid Frame Structures", George Winter, ed., "Second Progress Report on Investigation of Buckling of Rigid Frame Structures", June, 1951, Cornell University. (These reports, distributed by CRC, are at present available only in mimeographed form.)

[11]Chwalla, E., "Aussermittig gedruckte Baustahlstabe mit elastisch eingespannten Enden und verschieden grossen Angriffshebeln, <u>Stahlbau</u>", 1937, Nos. 7-8.

yield point is about the same. The last column, computed by methods developed here by Dr. Lee[12] in a thesis also distributed by CRC, gives those loads at which outer fiber yielding starts, i.e., the usual criterion for the secant formula. Comparison of the first and last column shows the tremendous effect of eccentricity on "carrying capacity" if the yield criterion is used. The next-to-last column, however, shows that the actual carrying capacity is much larger than the yield load and about midway between the latter and the concentric buckling load. Local buckling and residual stresses are not accounted for and may affect this picture adversely. Once these added influences are clarified, results of this type should be important in deciding whether secant-type formulas should be retained in codes.

"(d) An experimental investigation is almost completed on the influence of riveted gusset plates on buckling loads. It is clear that welded gusset plates increase buckling loads considerable as compared to these for the same member without them. For riveted connections slip may weaken this effect. Column tests were made on members shown on Fig. 10. Results, though not yet completely available, seem to show that the strengthening effect is much less for riveted connections than the theoretical effect which would obtain if gusset plates were monolithic or welded. The latter effect can be computed by methods given in the First Progress Report.

"(e) An experimental investigation is about to start to verify the analysis on eccentric, restrained columns. The jig to be used provides elastic restraint by heavy coil springs. Eccentricity is produced by offsetting the column with respect to the knife edge. A square section and an I-section are to be tested, for various degrees of eccentricity, and various L/r."

As another of its three projects in this field, and the first to be completed, CRC has sponsored a contribution at nominal cost of printing, four reports by Dr. T. C. Kavanagh, at the Pennsylvania State College, that were an outgrowth of earlier work[13] carried on at New York University. Dr. Kavanagh has prepared a statement which is here quoted:

"A complete study of the determination of, and importance of, the effective length of compression members in trusses and frames

[12] Lee, Annabel Yuen-Wai, A Study on Column Analysis, Cornell University doctoral thesis, June, 1949.

[13] Wessman, H. E., and Kavanagh, T. C., "End Restraints on Truss Members", Trans. ASCE 115:1135 (1950).

with rigid and/or semi-rigid joints, has been released in four reports[14] on a research project completed in 1950 at The Pennsylvania State College.

"The nature of the problem is most simply illustrated by Fig. 8, [Fig. 1 of Report No. 4]. In a building column, such as is shown, the effective length may be considerably greater than the actual length or story height, and the use of the actual column length in a column may lead to a column design which would be seriously in error.

"In connection with this project, an original and generalized moment-distribution procedure has been developed enabling the exact solution of stability problems in sidesway, such as in the case of the bent illustrated previously; this process allows sway to take place simultaneously with rotation. For more complex, multi-story frames, approximation methods and formulas have been derived which yield entirely satisfactory results for design purposes.

"Among the results of this research project, the following are worth noting:

"(a) In steel truss design, with fixed loads and employing the usual ratios of factor of safety for compression and tension members, all compression members should be considered as hinged; tensile restraints merely become "plastic hinges" before buckling occurs. It appears that for steel roof trusses and possibly some lightly loaded highway trusses, it is quite possible that very little, if any, restraint will be offered by the tension chords to the web members, even in part-span loadings when present factors of safety are used.

"(b) For steels in the usual range of slenderness ratios as found on bridge and building construction trusses, the critical buckling loads lie extremely close to the yield point of the steel, and hence the use of involved stability analysis is not warranted.

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[14] Kavanagh, T. C., "Report No. 1: Approximate Analysis of Frameworks without Translation", CRC 1949; "Report No. 2: Design of Columns in Trusses and Frames without Translation", CRC 1950; "Report No. 3: Buckling of Frameworks with Semi-Rigid Joints", (with J. H. Moore) CRC 1950; "Report No. 4: Analysis and Design of Columns in Frames Subject to Translation". CRC 1950 (The four reports were reproduced and distributed by CRC to a limited list.)

"(c) For materials other than steel, or for the analysis of trusses in general, approximations based on the consideration of small groups of members instead of the structure as a whole, yield satisfactory accuracy.

"(d) A method of determining the critical loading for a given structure from only one trial loading (rather than the several heretofore necessary) has been developed.

"(e) The effective modulus in tension has been verified by a new testing method.

"(f) The effect of semi-rigid joints, of the type ordinarily found in practice, has been found in trusses not to vary appreciably from the critical loads found under usual assumption of rigid joints. Where sidesway is involved, as in rigid frames, the effect of semi-rigid joints may, on the other hand, be found quite appreciable.

"(g) The statement of the well-known "series criterion" and "stiffness criterion" for stability of frameworks has been generalized for frameworks with and without sidesway.

"All the above comments are based upon the solution for stability as a characteristic or "eigenvalue" problem. The introduction of secondary effects of moments or eccentricities at the joints, etc., changes the nature of the problem to a stress problem which is under investigation elsewhere.

"For more detailed comments, reference is made to the four reports cited."

The third framed column project, for which Research Committee D of CRC acts in an advisory capacity, is being carried out at Lehigh University under the sponsorship of the Welding Research Council and has been referred to previously in connection with the discussion of residual stress. The column portion of this program on "Welded Continuous Frames and their Components"* had been started in 1946 by the American Institute of Steel Construction as a resumption of their earlier Lehigh program on columns loaded with combined axial force and end moments. The equipment used in the current investigation has been described in a previously published report[15] and has the unique feature

*The overall project receives financial support through WRC and ONR from AISC, AISI, BuShips, and Bu. Y and D.

[15]Beedle, Lynn S., Ready, Joseph A., and Johnston, B. G., "Tests of Columns under Combined Thrust and Moment", Progress Report No. 2 on the "Ultimate Strength of Welded Continuous Frames and their Components", Soc. for Exp. Stress An., 8:No. 1:109, also WRC Bul. 8, 1950.

of permitting independent application of axial force and end moment by which the column at pre-buckling loads can be made to bend into any one of the configurations shown in Fig. 11. These configurations are caused by applied end-moments and should be differentiated from the similar shapes shown in Fig. 4, which are the buckled forms of concentrically loaded columns as determined by existing end-constraints. The objectives of this project were:

"(a) to determine the plastic behavior of steel WF columns of the type used in building frames with continuous connections

"(b) to determine carry-over and stiffness factors."

Both theory and experiment are being carried forward simultaneously in the program, which emphasizes the behavior of rolled WF sections tested in the as-delivered condition in commercially available sizes. One of the principal results of this work at Lehigh has been to indicate the tendency toward lateral buckling in the framed column, aggravated by the presence of residual stress. In continuous frames where bending strength is of primary importance, WF members are usually bent in their strong plane, in which case the possibility of lateral-torsional buckling is present. In the truss the primary function of a column is to transmit axial thrust, and secondary bending moments usually cause bending in the weak plane, in which case the possibility of lateral buckling is absent. Further mention of the Lehigh project will be made in the section on the lateral-torsional buckling of beams and columns.

The Lateral-Torsional Buckling of Beams and Columns

Another major endeavor of CRC is to develop satisfactory design formulas with respect to the lateral-torsional buckling behavior of unsupported beams and columns.

A program on beams is being sponsored at the University of Washington, under the direction of Dr. R. A. Hechtman, who explains the purpose of the project in the introduction to a recent progress report:[16]

> "The recently adopted AISC Formula for the lateral buckling strength of the unsupported compression flange of plain-carbon structural steel beams of I-shaped cross section is based upon the slenderness ratio $\ell d/bt$ rather than the ratio ℓ/b as used in the prior formula. The symbols ℓ, d, b, and t refer to the unsupported length of the compression flange, the beam depth, the flange width, and the flange thickness, respectively. The derivation of the formula assumes a straight beam of constant moment of inertia, simple-beam end support with lateral restraint of the compression flange to prevent tipping

[16] Hechtman, R. A., Tiedemann, J. L., and Miles, J. J., "Lateral Buckling of Rolled Steel Beams", First Progress Report, distributed by CRC, May, 1951.

of the end cross sections of the beam, and uniform load over the span length. The program of tests. . .has as its primary purpose the determination of the reliability of the new AISC Formula.

"Most of the test equipment has been fabricated, and the investigation is in the early stages of making the beam tests."

A program on eccentrically loaded columns has been sponsored* in its later stages (without cost to CRC) at the Research Laboratories of the Aluminum Company of America, where structural research of fundamental quality and broad scope has been carried out for many years. A report submitted to CRC under date of January 10, 1951, was the basis for a recent paper[17] which has been summarized for this report by Mr. Hill as follows:

"An experimental investigation has been made at Aluminum Research Laboratories on the behavior of I- and H-section aluminum-alloy columns, loaded eccentrically in the plane of the web, and which fail by lateral-torsional buckling. Some failures occurred at stresses within the elastic range of the material, while others involved plastic action. Fig. 12 shows the cross sections of the 3 types of specimens. The proportions were such as to exclude the possibility of local buckling.

"Fig. 13 shows that the critical loads were generally lower than those predicted by the solution of Timoshenko[18] for buckling of rectangular bars (curve labeled "equation No. 3") and the solution of J. N. Goodier[19] for I-section beam columns, including the torsional affect of the thrust (curve labeled "equation No. 4"). The tests showed good agreement, however, with a modification of Goodier's theory which takes approximate account of the effect of the deflection of the column on the bending moment (curve labeled "equation No. 6").

"Fig. 14 compares the test results with various simplified interaction relationships involving the ratio of applied bending moment to critical moment for pure bending (M/M') and the ratio of applied end load to critical axial load for failure in the lateral direction (P/P') or the ratio of applied end load to the critical axial load for buckling in any direction (P/P'''). As the figure indicates, the straight-line interaction formula is unconservative

[17]Hill, H. N., and Clark, J. W., "Lateral Buckling of Eccentrically Loaded I- and H-Section Columns", presented at the First U. S. National Congress of Applied Mechanics, June, 1951.

[18]Timoshenko, S., "Theory of Elastic Stability", McGraw Hill, 1st ed. (1936) p. 243.

[19]Goodier, J. N., "Flexural-Torsional Buckling of Bars of Open Section", Cornell University, Exp. Sta. Bul. 28, (1942).

*This Sponsorship was prompted by Mr. R. L. Templin of the Aluminum Company, who has been an active supporter of CRC since its beginning.

for some of the test results unless, as in part (c) of the figure, approximate account is taken of the deflection of the column by dividing the applied bending moment M by the quantity $(1 - \alpha)$, where α is the ratio of applied end load to the Euler critical load for buckling in the plane of the web.

"The formula given in part (c) of this figure is suggested as a basis for a design formula to cover failures of this type."

The previously mentioned program of column tests at Lehigh University has included bending in the strong plane, and this phase of the program necessarily has been concerned with the problem of lateral-torsional buckling of steel columns. In a statement[20] prepared for this report, Mr. Lynn Beedle writes, in reference to beams and columns bent initially in their strong plane:

"The three-dimensional interaction curve shown in Fig. 15 has been developed to provide a basic framework for discussing the theoretical and experimental work. For axially-loaded members with no applied end moments, the theoretical curve is the familiar column curve of load (or stress) plotted against slenderness ratio. The theoretical curve shown is based upon idealized structural steel. For members without axial load, the curves in the moment-length plane are beam curves. As the length increases, lateral buckling becomes a more serious condition.

"Between these two limits, curves in space define two interaction conditions:

(a) yield strength (M_{yc})
(b) collapse strength (M_{pc}).

For the former it is seen that there is a range for the particular loading condition shown in which the yield strength is theoretically unaffected by slenderness ratio. If generally confirmed by test, then for a particular range it would be unnecessary to apply correction to F_a and F_b. For comparison the surface formed by the AISC interaction formula is shown by the shaded portions.

"Equations have been developed and summarized[21] for predicting the yield strength for each of the loading conditions being used in the

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[20] As here quoted, this is a revision as given by Lehigh Project Staff, "Progress Report M on Welded Continuous Frames and Their Components", Fritz Laboratory Report 205.13, September 17, 1951.

[21] Ketter, R. L., and Beedle, L. S., "Interaction Curves for Columns: Progress Report L on Welded Continuous Frames and Their Components", Fritz Laboratory Report 205A4, April 20, 1951.

the program. To a more limited extent, methods have been outlined for predicting the collapse strength.

"The results of numerous tests on 8WF31 columns are shown in Fig. 16. This Figure amounts to a "cross-curve" of the previous figure at $L/r = 56$. Tests 3 and 4 are in reasonable agreement with the theory. But T5 collapsed at a load less than the predicted initial yield load.

"The short cross lines indicate loads at which "yield lines" were observed. They indicate that a residual stress of about 12 ksi was present. In the case of T5, this early yielding allowed the member to buckle sidewise. For comparison the allowable working loads according to the AISC formula would fall along the dot-dash line.

"The other tests at low P/P_{cr} values are for other load conditions in which the moment along the column is also a maximum at the ends.

"The results of column tests using the same slenderness ratio as in Fig. 16 are shown in Fig. 17. However, under the "single curvature" load condition the moment along the column is a maximum at the center. If, due to residual stress, yielding occurs at a lower load than predicted, failure occurs by lateral-torsional buckling. As shown by the tests, except at a very low value of P/P_{cr}, the failure load is considerably less than the predicted value of initial yield load.

"This investigation in the Fritz Laboratory at Lehigh University is still underway. However, the tests carried out up to the present time indicate:

"(a) There is a range of loading conditions in which steel columns will carry more load than predicted and another range in which columns collapse below the predicted yield strength. Since most column formulas are based upon stress as a criterion, this indicates that on the one hand the formulas are too safe and on the other hand they may not be over conservative.

"(b) Columns loaded in single curvature do not develop their yield strength unless the axial load is relatively low.

"(c) The influence of residual stress is more important than previously realized in reducing the strength of these columns bent about their strong axes. Yielding at a cross section markedly reduces lateral buckling strength.

"(d) For load conditions other than single curvature the influence of axial load may in some cases be ignored at low axial loads (f_a/f_b about 10 per cent).

"(e) After more analysis and test work, it may be possible to specify a range of L/r and P/P_{cr} in which the influence of L/r may be neglected.

"(f) An analytical solution to the inelastic lateral buckling problem is an important step in further research."

The straight-line interaction curve for design of columns that also transmit bending moment at pre-buckling loads is well known as a part of AISC Specifications. Both the Aluminum Company and Lehigh investigations have brought out the fact that this formula may be too conservative in some cases and, though less frequently, may not be conservative enough. Its use for all cases apparently cannot result in the maximum of economy in design.

In a nonexperimental investigation supported jointly by CRC and the Rhode Island Department of State Highways, at Brown University, Dr. D. C. Drucker and J. Zickel have undertaken a survey of all available information on interaction formulas and related experimental evidence. After reviewing the Aluminum Company work, Figs. 18 and 19, taken from their recent progress report[22], show interaction plots of tests conducted by Professor M. O. Withey, at Wisconsin[23,24], and later tests carried out at Lehigh University.[25] In both these programs Columns were loaded eccentrically either in the strong or weak bending directions, there being no tendency toward lateral buckling in the latter case. The scatter between such extremes of bending behavior is evident especially in Fig. 18, for columns in the very practical design range of L/r less than 80 and more than 40. In commenting on this work, Dr. Drucker observes:

"(a) Straight line interaction curves are not always safe.

"(b) Consideration of plastic deformation greatly alters concepts of factors of safety, sometimes not increasing the factor at all, sometimes but little and often increasing it a great deal. If the considerable increase is consciously or otherwise taken into account in specifications, low factors may result in some cases."

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[22]Zickel, J., and Drucker, D. C., "Investigation of Interaction Formula", Brown University Report No. 1 to CRC, April, 1951.

[23]ASCE, "Second Progress Report of the Special Committee on Steel Column Research", Trans. ASCE 95: 1152, 1931.

[24]ASCE, "Final Report of the Special Committee on Steel Column Research," Trans. ASCE 97: 1376, 1933.

[25]Johnston, B. G., and Cheney, L., "Steel Columns of Rolled Wide Flange Section, Progress Report No. 2", Am. Inst. Of Steel Const., November, 1942.

The work at Brown University has not as yet specifically evaluated the problem of lateral-torsional buckling but proposes to include this feature in the continuation of this project during the second fiscal year that is just commencing.

It seems evident, on the basis of the four cited investigations at the University of Washington, the Aluminum Company, Lehigh University, and Brown University, that significant improvements in design procedures may be expected as a result of this area of CRC endeavor. Still another investigation on torsional behavior is being started at the University of Illinois.

The Behavior of Columns under Dynamic Loads

CRC had hoped to start work on the subject of columns under dynamic loads that might be due to blast or shock resulting from atom bomb attack, earthquake, or other manner of suddenly applied load or movement. This work has been delayed until recently because of lack of funds to carry out the entire program of CRC.

It appears now that funds may be available in the near future to carry out a project of this type, in particular reference to the blast problem, for which the following statement has been prepared by CRC Research Committee F, under the chairmanship of Dr. N. J. Hoff, of Brooklyn Polytechnic Institute:

> "During a strong blast the peak pressure on a structure generally builds up practically instantaneously, then reduces to zero within one second; but the stress history in the members of the structure depends upon the geometry and inertia of the structure and the stress-strain relationship of its material of construction. The question to which an answer is urgently needed can be stated like this: In what structures subjected to strong blasts does one have to consider dynamics in analyzing the failure of compression members? If in all or most cases it would be possible to neglect the dynamics of buckling, then the problem of the failure of columns would be considerably simplified.
>
> "The objective of the investigation is to establish the conditions of failure of columns in buildings, and for this reason consideration of the inelastic properties of the material will be of paramount importance. Similarly, the dead weight of the structure and the soil properties may have a significant effect. The interaction between the various structural elements must be considered because it manifests itself in the end conditions to which the column is subjected. In particular, the amount of sidesway influences the failing load materially.

"It is realized that a complete answer to the problem can be had only through a lengthy investigation, including both analytical and experimental work, especially if the refraction of shock waves, the yielding of the connections and the soil, and the dynamic stress-strain relationship of the material are to be taken into account. It appears likely, however, that a qualified answer can be obtained in a much shorter time purely analytically, if the work of the investigation is limited to finding out whether the static failing load of the column represents a satisfactory engineering approximation to the actual failing load in a building subjected to a strong blast. The load history for the individual column as well as the time-dependent boundary conditions must be established approximately on the basis of available experimental data and the analytical methods of applied mechanics."

The Stability of Bridge Chords without Lateral Bracing

Research Committee G of CRC, under the chairmanship of Mr. Jonathan Jones, prepared a detailed problem statement on this subject, considered to be of considerable importance to the proper expanded use of this type of bridge of either simple or continuous span. After careful consideration of proposals from a number of universities, this project has been awarded to Pennsylvania State College, under the direction of Dr. T. C. Kavanagh, whose earlier project on framed columns previously has been described in this report. A preliminary report[26] that includes a literature survey recently has been given limited circulation. Dr. Kavanagh has furnished the following statement regarding this project, which is under the immediate supervision of Mr. E. C. Holt, Jr.

"A theoretical and experimental investigation is underway at the Pennsylvania State College to establish rational design procedures for "half-through" or "pony-truss" bridges.

"The top chord of such a truss is a compression member whose slenderness ratio greatly exceeds the limit required to prevent buckling of the chord in a direction perpendicular to the plane of the truss. Restraint against buckling is provided by the rigid cross frames formed by the floorbeams and the truss verticals. These cross frames and the chord itself must be so proportioned that the stability of the chord is assured. Primarily, the problem of the stability of the top chord of a pony truss is the same as that of the stability of a continuous beam column on elastic supports. This simplified picture, of course, is complicated by many secondary effects due to torsion, uncertainty as to the restraints at the ends of the bridge, bending of the floorbeams under the loads applied to them, and so forth.

[26] Holt, E. C., Jr., "The Continuous Beam-Column on Elastic Supports", A report reproduced and distributed by CRC, in cooperation with Pennsylvania State College, 1951.

"The program may be broken down into several parts. The first step has been to develop analytical procedures sufficiently general to indicate which of the many secondary effects may properly be neglected and which must be taken into account. In the inelastic range, it turns out that the most important things to consider are the floorbeam deflections and the restraints at the ends of the chord. The proper evaluation of the tangent modulus is also of great importance, but that is beyond the scope of this investigation.

"For the pony-truss problem, exact methods of analysis are far too complicated for office use, and it is necessary to develop a simplified design routine based on the more comprehensive analysis. Also included in the program is a critical evaluation of standard design specifications with a view to putting them on a more rational basis.

"The theoretical results will be checked experimentally, both by considering data on existing bridges and by model tests. The model tests will be performed on parts of structures and on complete bridges.

"On the basis of the results of the theoretical and experimental parts of the investigation, approximate limits will be established for the economical use of the pony-truss type of bridge. It is believed that present practice is unnecessarily conservative and that it is feasible to use unbraced chords for longer spans than are now considered practical."

Summary and Conclusions

The goal of the structural engineer, with respect to design procedures is threefold:

1: Safety in the resulting structure,
2: Economy, and
3: Simplicity.

Greater economy in certain areas of column design can be achieved by more accurate design calculations than have heretofore been used. This is especially true where increase in the accuracy is directed toward a more realistic representation of the real structure in design calculations and not only toward an increase in mathematical complexity. Nevertheless, in some cases, more complex procedures must be used if the maximum in economy is to be obtained. The CRC has as its goal the development of practical design procedures that are as simple as possible and yet consistent with accurate and safe predictions of structural strength.

The final test of the success of structural research is in its effect on the improvement of structural specifications so as to achieve greater economy in structural design without any sacrifice of safety. The following is a brief summary of the ways in which the current research of the CRC may be expected either to affect design specifications or produce needed design information.

(1) Provide a more logical basis for standard design formulas for concentrically loaded columns, based on tangent-modulus theory, and incorporating a statistical and theoretical evaluation of the effect of yield stress variation, shape of basic stress-strain curve, and the effect of residual stresses. The development of new design formulas for any new structural material will then be possible by straightforward and sound procedures, and will be adaptable to various conditions of end restraint if the effective length is known.

(2) Improve specifications for end-milling tolerances and other factors of column imperfection.

(3) Provide a better understanding of the problems of local buckling of rolled sections and built-up members, taking advantage of the great amount of recent research in the aircraft field.

(4) Provide a more rational basis for design of columns in trusses or continuous frames, in a manner that will take proper account of the entire frame action and the overall factor of safety with respect to buckling.

(5) Investigate the current specifications for the design of beam columns, which transmit both axial force and bending moment, with or without lateral support. The aim will be either to corroborate existing specifications or suggest improvements that will more consistently and accurately provide for safety with economy.

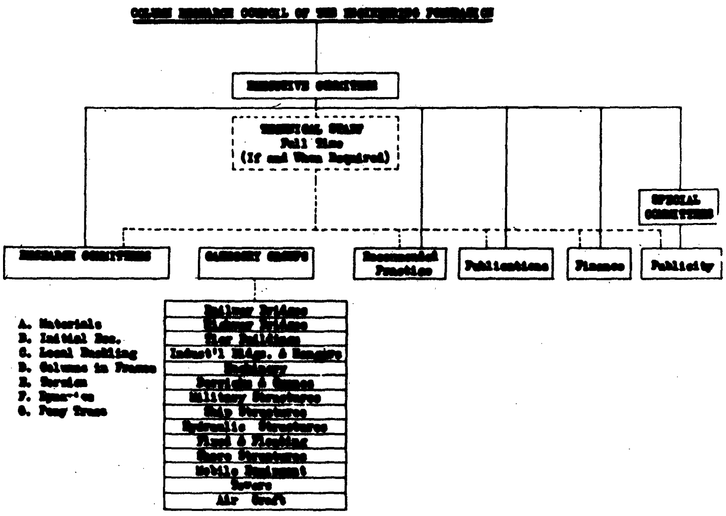
(6) To study the behavior of columns under dynamic load conditions and to determine to what extent, if at all, procedures applicable to static-load design may be applied.

(7) To develop simple design procedures for the top chords of bridges that have no top lateral bracing.

Acknowledgement

Special acknowledgement is due to Mr. Shortridge Hardesty, Chairman of CRC, for advice and cooperation in the preparation of this report; to the Executive Committee, for their helpful comments; to Messrs L. S. Beedle, W. R. Osgood, G. Winter, E. E. Lundquist, N. J. Hoff, J. Benjamin, T. C. Kavanagh,

R. Hechtman, D. Drucker, H. N. Hill, and H. Bleich, for quoted portions from earlier reports or for especially preparing statements here quoted. Without these specific contributions it would not have been possible to have prepared this report in the time available. Acknowledgement is also due the Civil Engineering Department of the University of Michigan, which, with the approval of Professor Earnest Boyce, Chairman, has provided funds and time required for the preparation and presentation.



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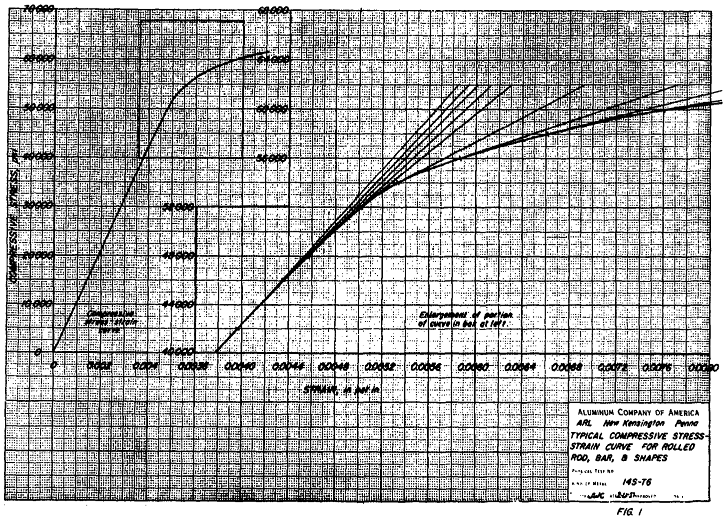

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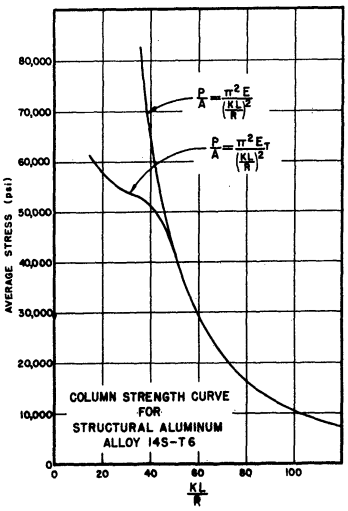

3

type of restraint	buckled form	condition at U		condition at L		Euler length factor "K"
		rotation	lateral position	rotation	lateral position	
1	U L	free	*fixed	free	*fixed	1.0
2	U L	free	*fixed	*fixed	*fixed	0.7
3	U L	*fixed	*fixed	*fixed	*fixed	0.5
4	U L	*fixed	free	*fixed	fixed	1.0
5	U L	free	free	*fixed	*fixed	2.0

* May also be elastically restrained, or, in laboratory tests may be a "flat-end" contact capable of transmitting only compression.

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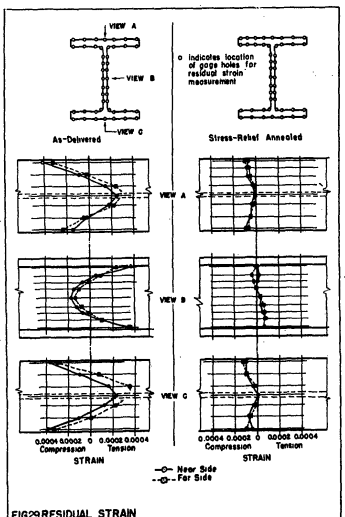

FIG29 RESIDUAL STRAIN IN 8 WF67 BEAM

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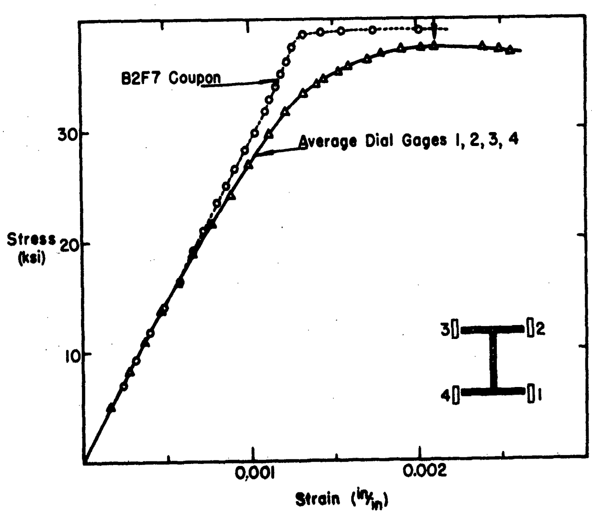

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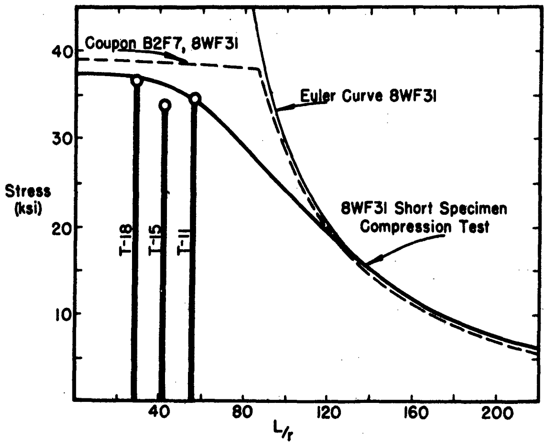

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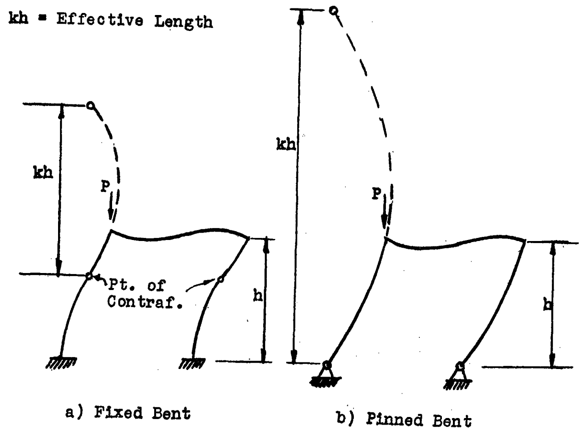

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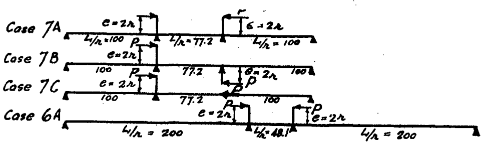

Buckling stresses in p.s.i.

Case	Steel used by Chwalla σ_y = 38,400 p.s.i.				Steel σ_y = 38,000 p.s.i.	Outer fiber yield criterion Lee
	Concentric loading	Exact Chwalla	Analytical Bijlaard	Approximate Bijlaard	Approximate Bijlaard	
7A	35,000	21,300	21,300	21,000	20,400	12,900
7B	35,000	-	28,400	27,700	27,000	11,000
7C	35,000	-	24,500	22,800	22,300	13,000
6A	37,000	21,300	21,900	21,000	-	10,100
7A I	35,000	-	-	-	21,000	11,800
7A I	35,000	-	-	-	20,800	15,900

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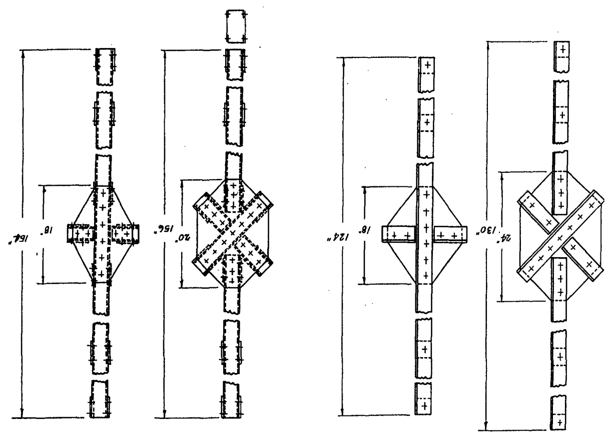

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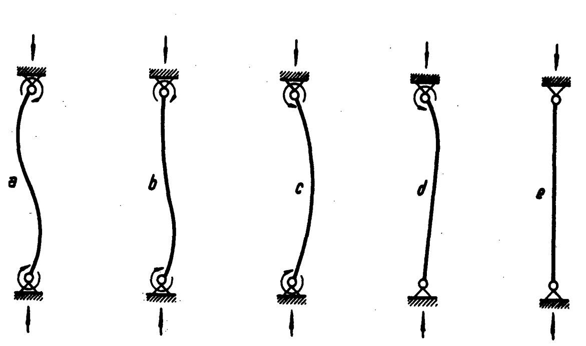

CONDITION OF TEST

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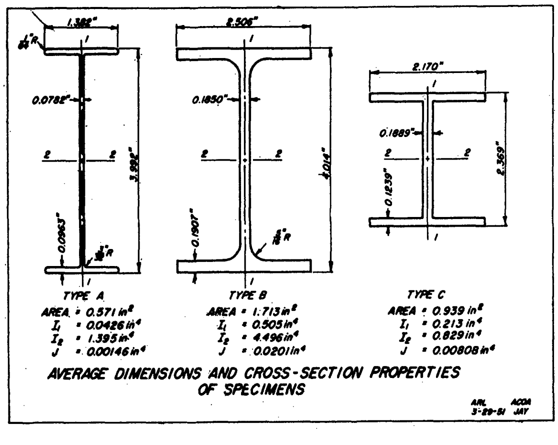

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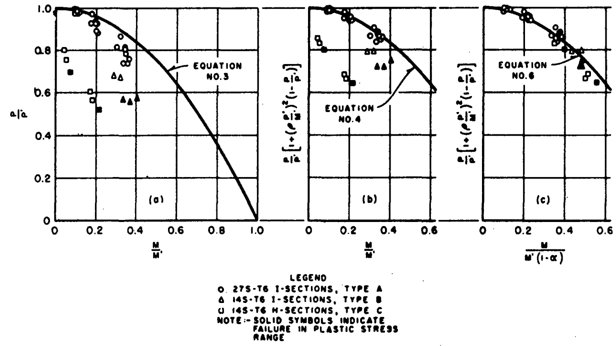

COMPARISON OF TEST RESULTS WITH THEORETICAL SOLUTIONS FOR LATERAL BUCKLING UNDER COMBINED BENDING AND COMPRESSION

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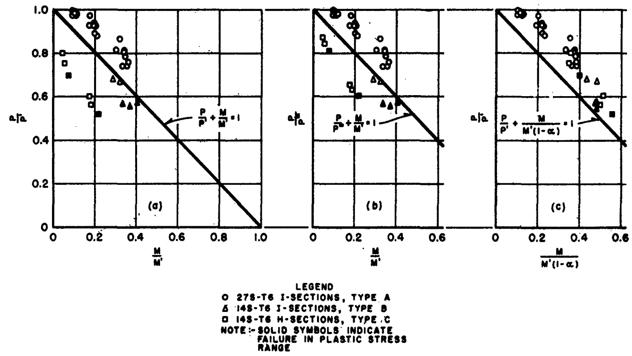

INTERACTION DIAGRAMS FOR LATERAL BUCKLING UNDER COMBINED BENDING AND AXIAL COMPRESSION

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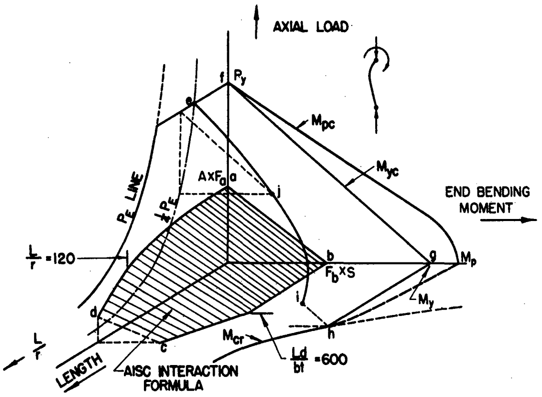

FIG. LOADING CONDITION "d" THREE DIMENSIONAL INTERACTION CURVE

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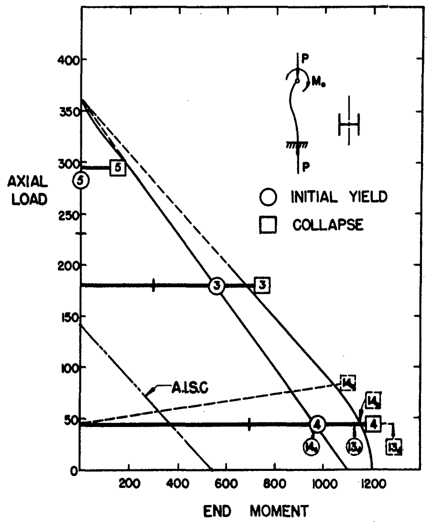

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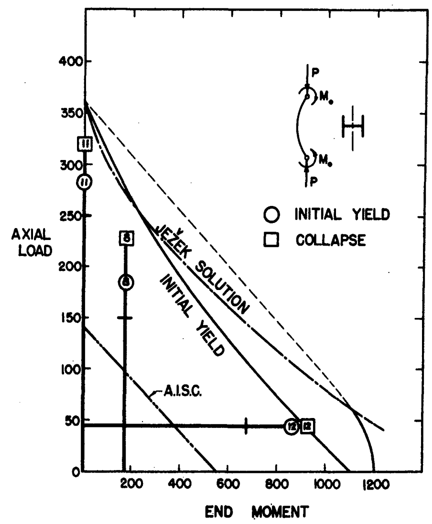

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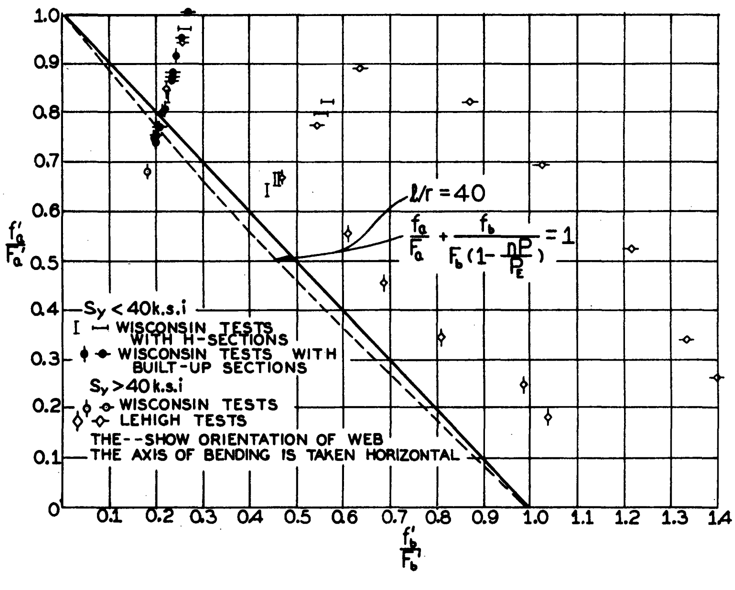

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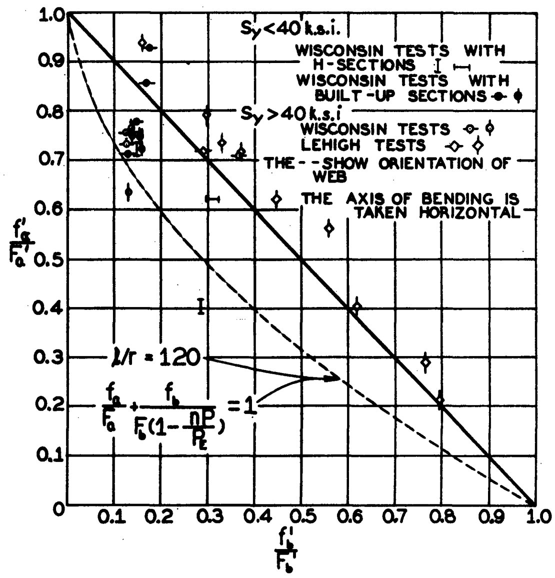

Experimental Data Interaction Diagram for $80 < \ell/r \leq 120$.

19