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THE RANDOM OPTICS OF PARTICLE BEAMS*

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Abstract

Multiple Coulomb Scattering, which is generally regarded as an undesirable dirt effect in high energy physics, can be used to focus or deflect beams of energetic particles. For example, a parallel beam of protons passing through a "lens" of lead, made so that the path length in lead increases as the off-axis distance squared will produce a focal spot of two or more times the intensity of the beam in the absence of the lens. Other configurations of scattering material can produce a net deflection of the beam. By using a high Z material and high energy beams (>1 Gev) multiple scattering effects become relatively more important than energy loss or nuclear interactions. The application of these ideas to handling beams of muons and electrons is also explored. Corresponding effects such as Compton scattering and nuclear diffraction scattering may be used to "focus" gamma-ray and neutron beams.

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I INTRODUCTION

In setting up the optics of intense beams of relativistic particles such as the external beam of the Brookhaven Cosmotron, it is convenient to photograph the beam focus using Polaroid film. In order to locate a reference point on the film a metal object is placed at a known position ahead of the film and its "shadow" recorded on the film. The curious properties of these shadows or (more properly) images, was the starting point for the thoughts pursued below. If the film is placed against the object, no image is seen; however, if the film is a few centimeters beyond the object, the image is very sharp and detailed. That no image appears in the former case is reasonable since the attenuation of a 3 Bev beam by 1/8 inch of brass is only a few percent. The image that develops with spacing is then due to scattering in the metal; in this case multiple coulomb scattering. If we cconsider the image due to a slab extending from $x=0$ to large x located at $z=0$ (with beam in the $+z$ direction) the intensity beyond the slab at $z=+z_1$ at $x=+x_1$, and $x=-x_1$ is the same (neglecting absorption), if $|x_1|/z_1$ is much greater than the multiple coulomb scattering angle. However, near $x=0$, the intensity will be reduced for x positive since some particles are scattered out of that region, and the intensity will be increased for x negative since no particles are scattered out, but some are scattered in. The situation is represented schematically in figure 1.

II. A MULTIPLE-SCATTERING LENS

Now it is attractive to consider whether this scattering, which is normally a nuisance, might be used to advantage. The essence of the idea is the observation above that a scattering object near but not in a direct line between a source and a detector can increase the flux at the detector.

In order to design a lens employing multiple scattering, consider a parallel beam in the $+z$ direction, where it is desired to increase the flux at $x=y=0, z=a$ over its value in the parallel beam. At $z=0$ we can place a ring of scattering material in the xy plane. This will increase the intensity at $z=a$ since some particles scatter into but none out of the zone defined by the inner radius of the ring. The next step is to add a sequence of rings with thickness increasing with radius so as to optimize the flux at $x=y=0$, $z=a$. Our intuitive guess, which turns out to be correct, is that the multiple-scattering material should be so contoured as to make the root-mean-square scattering angle always the angle subtended at $z=a$. Since θ_{rms} is proportional to l^{2} this results in a thickness of scatterer proportional to radius squared.

We shall derive an expression for the central intensity and show that the above choice is optimum. Consider all angles small so that $\sin\theta = \theta$, $\cos\theta = 1$. For a scattering slab, using $\theta_0 = \theta_{rms}$

$$P(\theta)\,d\theta = \frac{1}{\theta_0}\sqrt{\frac{2}{\pi}}\; e^{-\theta^2/2\theta_0^2}\, d\theta$$

$$\frac{d\sigma}{d\Omega}\,\delta\Omega = P(\theta)\,d\theta\,\frac{\theta\,\delta\phi}{2\pi\theta}\;,$$

$$\frac{d\sigma}{d\Omega} = \frac{P(\theta)}{2\pi\theta}\,.$$

For a lens, consider the geometry of figure 2 where Φ_i is the incident flux per unit area and Φ is the flux per unit area at the focus.

$$\Phi = \Phi_i\; 2\pi r\, dr\, \frac{d\sigma}{d\Omega}\,\frac{d\Omega}{dA}\;,$$

$$\frac{r}{a} = \theta\;,\quad \frac{d\Omega}{dA} = \frac{1}{a^2}$$

$$d\Phi = \Phi_i\,\frac{r\,dr}{a^2}\,\frac{P(\theta)}{\theta}$$

$$d\Phi = \Phi_i\, P(\theta)\, d\theta = \Phi_i \int_{\theta_{MIN}}^{\theta_{MAX}} \sqrt{\frac{2}{\pi}}\,\frac{1}{\theta_0}\, e^{-\theta^2/2\theta_0^2}\, d\theta\,.$$

The condition that the lens be parabolic corresponds to letting $\theta_0 = \alpha\theta$

$$\Phi = \Phi_i \int_{\theta_{MIN}}^{\theta_{MAX}} \sqrt{\frac{2}{\pi}}\; e^{-\theta^2/2\alpha^2\theta^2}\,\frac{d\theta}{\alpha\theta}\;,$$

$$\Phi = \Phi_i\,\sqrt{\frac{2}{\pi}}\; e^{-1/2\alpha^2}\,\frac{1}{\alpha}\,\ln\frac{\theta_{MAX}}{\theta_{MIN}}$$

To optimize the flux,

$$\Phi/\Phi_i = \left(\sqrt{\tfrac{2}{\pi}}\ \ln\frac{\theta_{MAX}}{\theta_{MIN}}\right)\frac{1}{\alpha}\ e^{-1/2\alpha^2},$$

$$\frac{d\,(\Phi/\Phi_i)}{d\,\alpha} = \left(\sqrt{\tfrac{2}{\pi}}\ \ln\frac{\theta_{MAX}}{\theta_{MIN}}\right)\frac{1}{\alpha}\ e^{-1/2\alpha^2}\left[\frac{1}{\alpha^2}-1\right],$$

$$\frac{d\,(\Phi/\Phi_i)}{d\,\alpha} = 0 \quad \text{for} \quad \alpha = 1, \quad \text{and}$$

$$\frac{d^2(\Phi/\Phi_i)}{d\alpha^2} = -2 \quad \text{at} \quad \alpha = 1.$$

Therefore, the optimum flux at some point on the axis of the lens is given by $\Phi = \Phi_i \sqrt{\frac{2}{\pi e}}\ \ln\left(\frac{r_{MAX}}{r_{MIN}}\right)$,
where $r$max is the outer radius of the lens and r min is some inner radius. The logarithmic divergence for $r$min = 0 reflects the fact that the incident beam was taken as perfectly parallel (zero phase-space volume) and that we have only evaluated the flux at the geometric on-axis point. The integrated flux over a finite area in the focal plane would be finite of course. In practice it seems reasonable to use for r min a value corresponding to the radius of the desired focal spot. For $r$max, the value is probably best set by the maximum material one can place in the beam path before attenuation losses or impurities become prohibitive. With a hole of radius r min in the center of the lens, the flux at the focal spot is given by $\Phi = \Phi_i\left[1 + \sqrt{2/\pi e}\ \ln\left(\frac{r_{MAX}}{r_{MIN}}\right)\right]$.

Consider a proton beam of 2.1 Bev/c momentum focused by a lead lens. We will limit the path length in lead to 1/5 of a collision mean free path and ask for a 1 cm diameter focal spot. For lead, $\ell_{collision}$ = 13.8 cm, ℓ_{rad} = 0.51. We will make the lens 20 cm diameter, and (from the above) 2.76 cm thick at the edge. The thickness, ℓ, will be given by $\ell = .0276/r^2$ cm, except for a one cm diameter hole in the center. The focal length, a, is given from the multiple scattering formula (for θ, not $\theta_{proj.}$)

$$\frac{r_{MAX}}{a} = \theta_0 = \frac{21.2}{pv}\sqrt{\frac{\ell_{MAX}}{\ell_{rad}}}$$

$$a = r_{MAX}\ \frac{pv\,(Mev)}{21.2}\sqrt{\frac{\ell_{rad}}{\ell_{MAX}}}\ .$$

For the above numbers, $a = 10^3\sqrt{\frac{0.51}{2.76}} = 430\text{ cm}.$

The central flux would be $\Phi = \Phi_i\,(1 + 0.485 \ln 20) = 2.45\,\Phi_i$.

This does not include the attenuation near the periphery of 20%, but noting the behavior of Φ with r_{max} suggests that the resulting intensity loss will be very small (e.g. <5%). The maximum energy loss is only about 40 Mev, or 2%.

III. OTHER GEOMETRICS

Consider a thin lead strip inclined at an angle comparable to θ_{rms} to the beam direction, as in figure 3. A particle scattering to the right would scatter out of the strip, corresponding to a shorter path length in the strip. A particle scattering to the left would have a longer path length in the strip and would ultimately scatter over a broader angular range. Thus, there should be a resultant deflection of the beam direction to the right.

This effect suggests other forms of lenses, using contours of scatterer of constant thickness. For example a lead sheet paraboloid appears from such qualitative arguments to be able to "focus" particles from a point source into a more nearly "parallel" beam as in figure 4. Here the path length in the scatterer increases with θ, and also scattering into the beam is favored over scattering out of the beam for the reasons above.

The same configuration in reverse might also work as a lens to "focus" a parallel beam to a point by running the rays backwards, however, it is important to emphasize that this is a non-Liouvillian system. For example in this case, the change of pathlength with scattering angle decreases the effectiveness in focusing a parallel beam to a point but increases the effectiveness in focusing a point to a parallel beam.

Now, as a practical matter, this random, or multiple scattering optics is obviously much less favorable than magnetic quadrupole optics. Surprizingly, the focal lenth of a multiple scattering lens is close to that of a typical quadrupole doublet of the same aperture and (if we fix the maximum interaction loss in the lens as above) has the same dependance on momentum and aperture.

IV NEUTRINO BEAMS

There are several features of scattering lenses that bear comment, however. First, there is no dependence on particle charge. Second, such lenses would be cheap, expendable, and require no servicing. These properties make the lens possibly worth consideration for neutrino beams where high radioactivity levels in the external proton beam are serious. In particular, if a small (1 meter) bubble chamber is located 50 meters from the proton target, a scattering lens of 5 meter focal length with a 10 cm hole on its axis, a maximum radius of 20 cm and a thickness of about 8 cm might increase the flux of 1 Bev neutrinos by about 60% (neglecting the emmission angle between the pion and neutrino). A more useful scatterer would be a simple lead cylinder of constant thickness (about 5 or 10 cm) with a hole in the center subtending the same angle as the neutrino detector. Higher energy pions would scatter less but are emitted at smaller angles to the beam axis so that the flux gain would be somewhat comparable over a range of pion (e.g. neutrino) energies.

V MUON AND ELECTRON LENSES

The constraint on maximum thickness due to nuclear interactions is removed for muons and electrons. For a muon beam, the absober used to purify the beam could be contoured to "focus" the muons, where now virtually any geometry could be exploited.

For electrons, the competing process is, of course, bremsstrahlung, and for this reason the lens material is arbitrary. To avoid serious radiation losses, keeping $\ell < 0.1\,\ell_R$, such electron lenses would have very long focal lengths (small f-numbers). For $\ell_{max} = 0.1\,\ell_R$, $a = r\,E/7$ where E is in Mev.

VI GAMMA-RAY AND NEUTRON LENSES

Perhaps the only application of random optics would be for neutral particles; neutrons and γ rays. Photons would be focused by Compton scattering, or multiple Compton scattering. This would be most useful at energies from 5 to 100 Mev using a very low Z material (e.g. liquid hydrogen) where compton scattering exceeds pair production. This has not been evaluated, but qualitatively the same arguements hold as above for the charged particle case. This could serve as a useful beam concentrator for the X-ray beams from betatrons and electron synchrotrons.

Neutrons in the multi Bev energy range undergo nuclear diffraction elastic scattering presumably. This has not been explored experimentally, but it is probably that the parameters characteristic of proton-proton scattering are applicable. Following the proton-proton case,

$$\sigma_{elastic} \cong 9\ \text{mb}$$

$$\sigma_{total} \cong 40\ \text{mb}$$

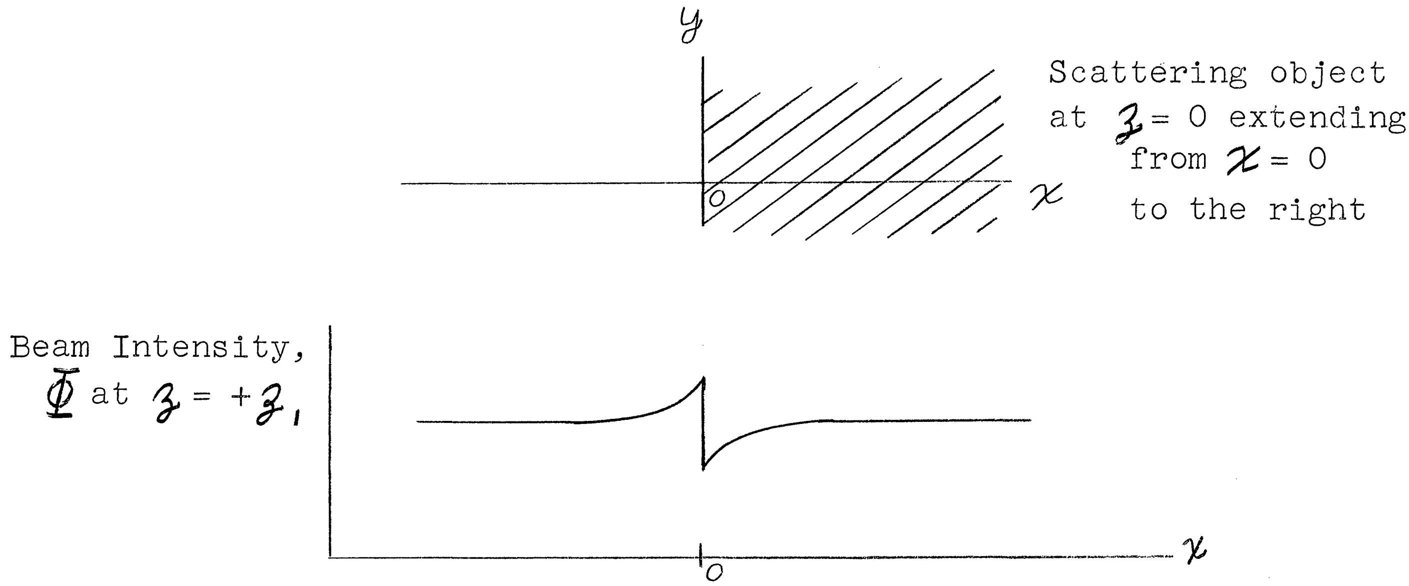
$$\frac{d\sigma_{el}}{d\Omega} \cong 9\, e^{-9 p^2 \theta^2} \qquad (\theta\ \text{small})$$

where p is in Gev/c. Nuclear interactions preclude much gain from multiple or plural scattering, so that a lens would be a cylinder of constant thickness, L, with an aperture along the axis. If the axial hole subtends and angle θ_1 from the focal spot and the outer radius subtends θ_2, then the flux per unit area at the target Φ, is given in terms of the incident flux per unit area Φ_i and the total and elastic cross sections, σ_t and σ_e by $\Phi = \Phi_i \left\{ 1 + \frac{\sigma_e}{\sigma_t} \left(1 - e^{-N\sigma_t L}\right)\left(e^{-9p^2\theta_1^2} - e^{-9p^2\theta_2^2}\right)\right.$.
Since $\sigma_e \cong (1/4)\,\sigma_t$, the maximum ratio of Φ to Φ_i is 1.25. These figures are for hydrogen. There are no cross section data from heavy nuclei for multi Bev neutrons; it is possible that $\sigma_{el} \cong \frac{1}{2}\sigma_t$ (as in classical optical diffraction) for heavy nuclei, but in any case the flux gain is not great.

For low energy neutrons, where in some materials $\sigma_{el} \cong \sigma_t$ lenses using plural scattering might be possible with intensity gain ratios greater than two.

VII CONCLUSION

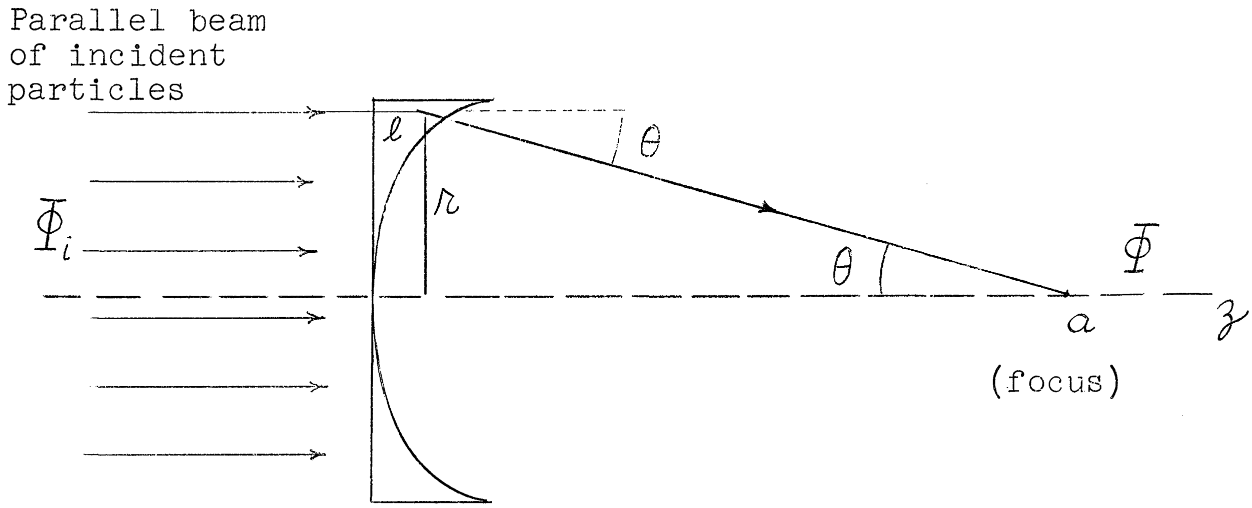
We have explored a novel class of devices for focusing fluxes of high energy particles, making use of the random rather than coherent scattering of the particles. It is shown that intensity gains of a factor of 2 or more are possible with charged particle beams and somewhat less with neutron beams. These figures are very modest, and we are not able to show any virtues of random optics over electro-magnetic optics for charged particle beams. However, the principle is amusing and experiments to explore some of these conjectures are planned. Possibly random optics contains surprizes and tricks; to our knowledge it is an unexplored field and this is sufficient motivation for our current investigation.



Beam of particles moving in the + direction

The nature of the scattering image of the edge of an object of constant thickness.

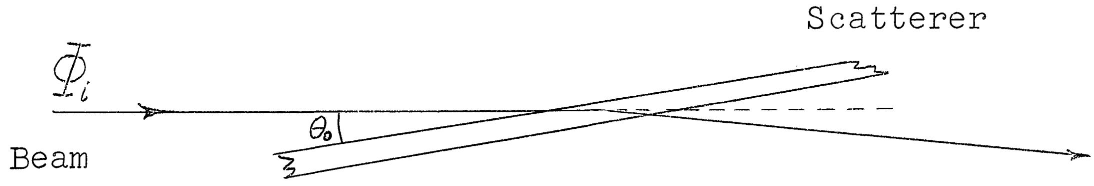
Figure 1



Multiple-scattering lens

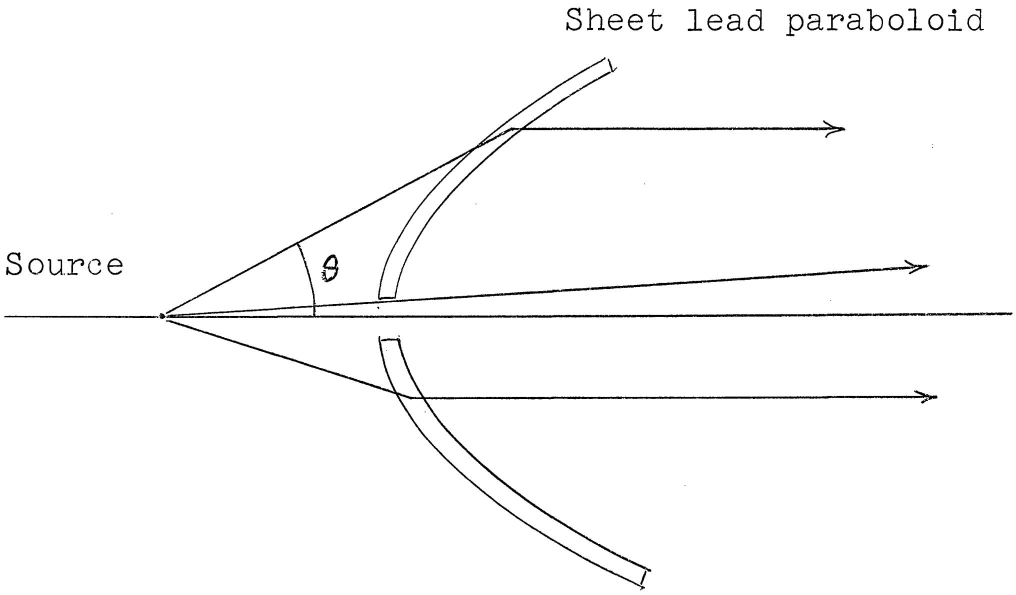
The geometry of a parabolic multiple-scattering lens.

Figure 2



The deflection of a beam by a scatterer inclined at grazing incidence.

Figure 3



A paraboloidal lens of constant thickness for focusing particles diverging from a point source.

Figure 4