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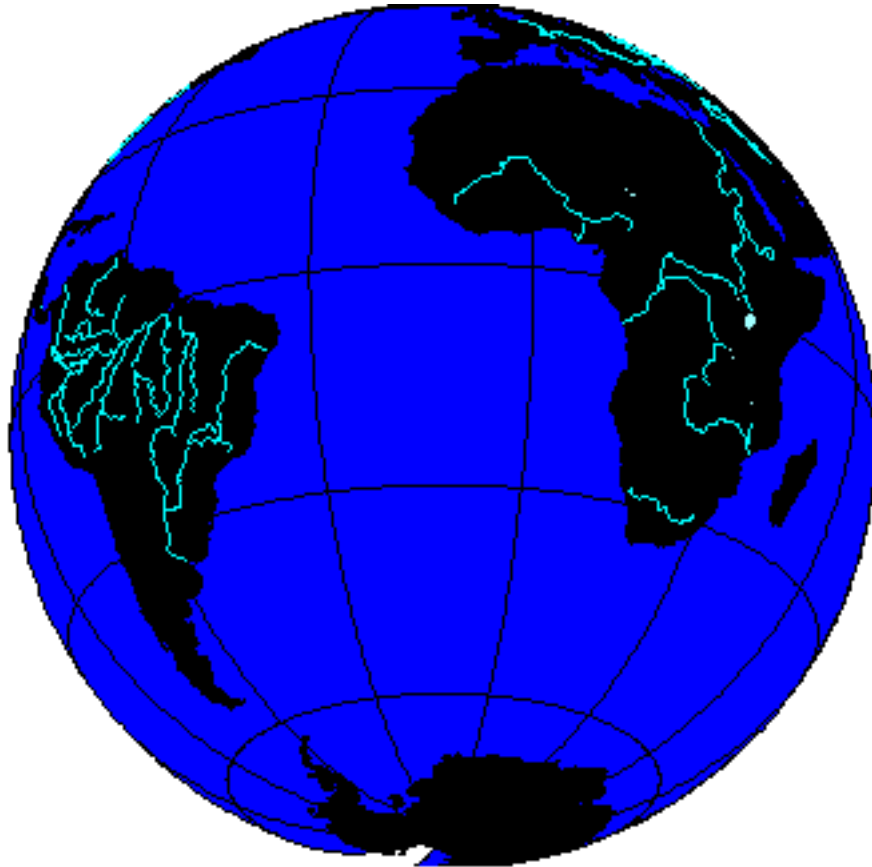**Sandra Lach Arlinghaus**
Bisectors, Buffers, and Basemaps

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SOLSTICE:

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Earth: with 23.5 degrees south latitude as the central parallel.

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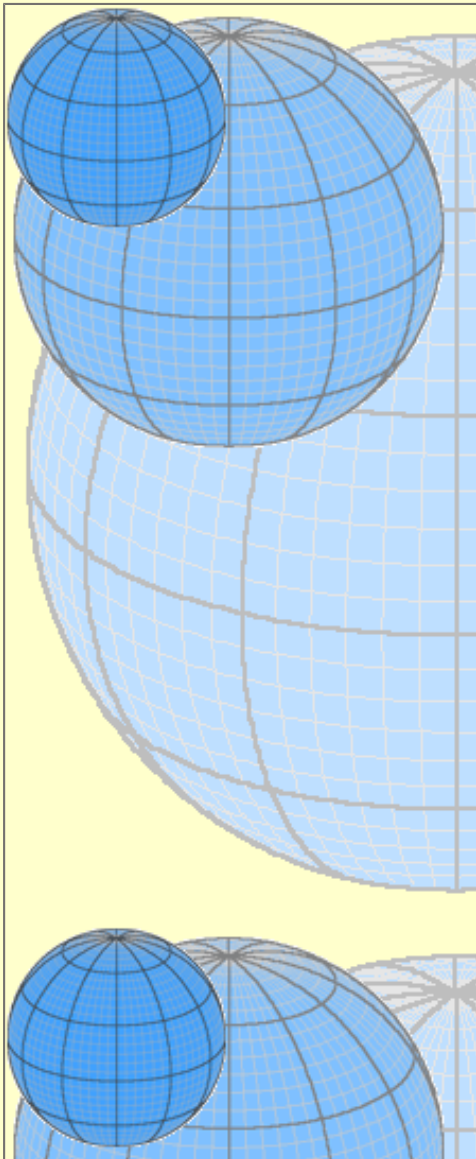
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Spherical Measures without Spherical Trigonometry

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htm

A common misconception suggests that it is necessary to understand spherical trigonometry in order to calculate properties of a spherical surface. That this is false can be demonstrated by deriving the formula for distance and direction on a sphere without any knowledge of spherical trigonometry. However, some knowledge of the properties of a sphere is required.

Suppose one wants to know the great circle distance between two locations given by their names in latitude and longitude. Call these P_1 (ϕ_1, λ_1) and P_2 (ϕ_2, λ_2).
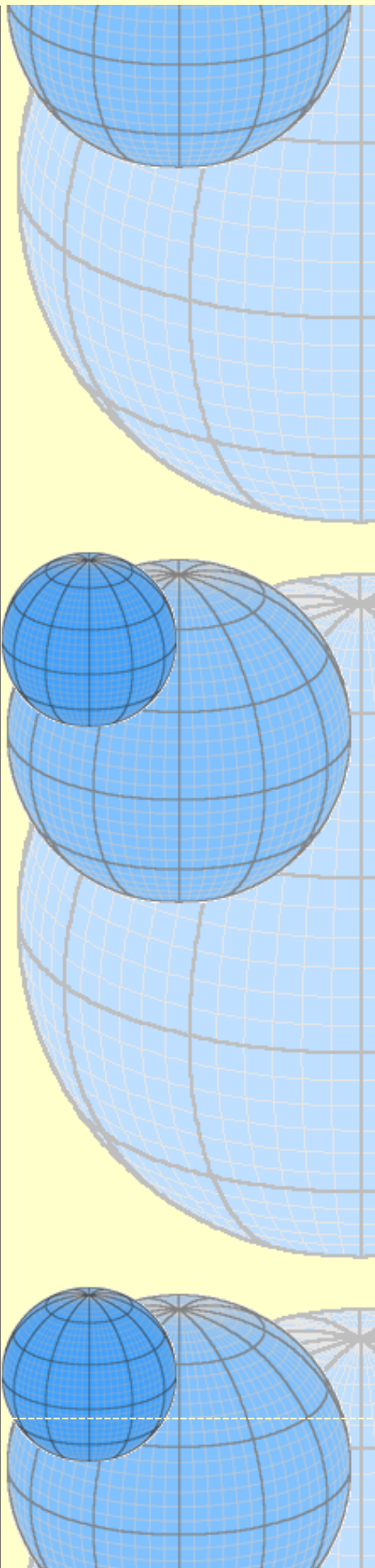
Using plane trigonometry one can compute the location of these points in a three dimensional rectangular (Euclidean) coordinate system, centered at the origin of the sphere. This yields the locations as vectors in space as V_1 (X_1, Y_1, Z_1) and V_2 (X_2, Y_2, Z_2). The straight line distance between these two points is given by the well known formula: $D_{12} = [(X_1-X_2)^2 + (Y_1-Y_2)^2 + (Z_1-Z_2)^2]^{1/2}$. This is the chord distance between the two points, penetrating the sphere. The chord distance between the two points can be considered to be one side of a triangle, with origin at the center of a circle, whose other two sides have a length equal to the radius of the sphere. Take this radius to be one unit, and call it R. Divide the triangle into two equal parts, by constructing the perpendicular to the chord from the center. Now we know two sides of a right triangle, and the length of the third side (from the center to the chord) can be calculated. From this the angle at the center can be calculated, using any of the trigonometric formulae relating points in a right triangle. Twice this angle is the angle that spans the distance between the two points, that is, it is the angular separation of the points on the sphere.

In more detail, use standard spherical coordinates:
$X_1 = R \cos \phi_1 \cos \lambda_1$

$Y_1 = R \cos \phi_1 \sin \lambda_1$

$Z_1 = R \sin \phi_1$

$X_2 = R \cos \phi_2 \cos \lambda_2$

$Y_2 = R \cos \phi_2 \sin \lambda_2$

$Z_2 = R \sin \phi_2$

The length, L, of the line from the center of the circle to the perpendicular to the chord is, using C as the length of the chord, $L = [R^2 - (C/2)^2]^{1/2}$. Now, from elementary trigonometry, $C/2L = \tan \alpha$, $L/R = \cos \alpha$, $C/2R = \sin \alpha$. Doubling α gives the angle subtended at the center of the circle, and at the center of the sphere. Use the simplest (the third) of these relations to calculate the distance on a sphere.

Example:

Near Santa Barbara, CA: $\phi_1 = 34°$, $\lambda_1 = -120°$.

Near Zürich, CH: $\phi_2 = 47.5°$, $\lambda_2 = 8.5°$

$X_1 = R \cos \phi_1 \cos \lambda_1 = -0.414529$

$Y_1 = R \cos \phi_1 \sin \lambda_1 = -0.717978$

$Z_1 = R \sin \phi_1 = 0.559193$

$X_2 = R \cos \phi_2 \cos \lambda_2 = 0.668169$

$Y_2 = R \cos \phi_2 \sin \lambda_2 = 0.099859$

$Z_2 = R \sin \phi_2 = 0.737277$

$C = [(-.414529 - .668169)^2 + (-.717978 - .099859)^2 + (.559193 - .737277)^2]^{1/2} = 1.368491$, L = 0.729252 but is not really needed, and continue to use R = 1. Then, from $\alpha = \sin^{-1}(C/2)$, $\alpha = 43.176$ degrees of arc. Twice this value gives 86.35 degrees for the length of the arc on the circle. Now take R= 6378 kilometers to approximate the radius of the sphere. Then the distance between Santa Barbara and Zürich, on the spherical assumption, is $\pi*\alpha*R/90$ or 9612 km.

No spherical trigonometry has been used. But if one is familiar with vector algebra the result can be obtained more immediately, namely by calculating the angle between the two vectors directly as

$2\alpha = \cos^{-1}(V_1 \cdot V_2 / |V_1||V_2|)$.

For the direction of the second point from the first recall that directions are measured between great circles. In this case we need the direction of the great circle from Santa Barbara to Zürich with respect to the reference direction from North. The plane passing through Santa Barbara, the North Pole, and the origin, cuts the sphere along the North-South great circle and has equation -.717978 X + .414529 Y = 0, from the three-point equation of a plane. The plane spanning the region between Santa Barbara, Zürich, and the origin intersects the sphere along the connecting great circle and has equation -.5851891 X + .6792581 Y + .4383362 Z = 0. The angle between these two planes is 31.9 degrees East of North. (see C. Oakley, 1954, "Analytic Geometry", Barnes & Noble, pages 179, 185). The area of a polygon on a sphere is equal to $R^2\{\pi(n-2)+\Sigma\beta_i\}$ where the sum is taken over the n interior angles. Again, no spherical trigonometry has

been used.

What's At Home?
Shelter for the Poor in Low Income Cities

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Figure 1. Table of Parameters

Figure 2. People by the Billions

Figure 3. Homeless family in Bombay.

Finding shelter for the poor in low-income cities is a problem for *now* and for the *future*. The twentieth century saw huge growth in human population. This population is now entering the twenty-first century with enormous and growing needs for sustenance and shelter. Millions of new families are created each year all seeking ways to sustain life, to nurture, and to shelter their children. In the new century, most of the population growth will be in cities. Most of these cities will be poor because their already poor economies simply cannot grow at rates needed to raise the level of living while accommodating their own population growth. In addition these cities receive huge waves of poor, unskilled immigrants who not only are destitute but who are often refugees fleeing oppressive regimes. How do these people live? All these people need shelter. How, in the past seventy years, have four billion more people found shelter? The parameters of this process are migration and growth, poverty, homelessness, and rule of law (Figure 1).

Migration and Growth
In 1930, there were two billion people on earth. It had taken 120 years to grow from one billion. Forty-five years later,
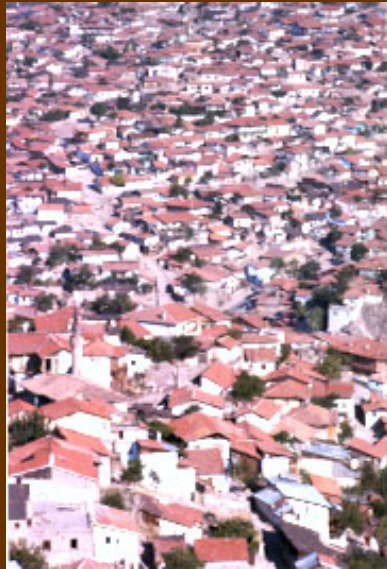
Figure 4. A collage of squatter settlements.

in 1975, the population had doubled to four billion people. Twenty-four years later, the population had grown again by two billion people. In the first quarter of the 21st century, another two billion will be added to the total, at least, according to the optimistic forecast by the United Nations, which sees a decline in world population growth but an increasing growth in the urban population. The number of people has increased by the billions (Figure 2).

Poverty
Basic human needs must be met or people die. Food and health are basic. The first need is to be fed. For poor people, most of their income goes toward finding food. Shelter comes next. Those with extremely low incomes are homeless (Figure 3). Generally, opportunities for making a living have been better in urban areas than in rural regions. The consequence has been a vast rural to urban migration. However, city economies are not able to keep pace. Opportunities for making a living are meager. Many people are homeless or live in spontaneous shelters, that is, self-built, squatter settlements or shantytowns squeezed into marginal spaces in and around the city (Figure 4). Such shelters have different names in different places: bustee in India, gegucondu in Turkey, favelas in Brazil. To meet the housing needs in poor cities, appropriate technologies for self-built housing must be utilized. Hong Kong's population grew eight-fold since 1931. Most of this population has been housed in high-rise apartment buildings (Figure 5). Hong Kong is among the wealthy cities of the world and its

Figure 5. Hong Kong high-rise apartments;
30,000 people live in this complex.

Figure 6. Table of Cities of over Ten Million

Figure 7. Squatter homes.

Figure 8. What's at Home

economy is buoyant. The Hong Kong Housing Authority has provided for seventy percent of the housing demand through a building program financed by loans from the city government. These loans have been or are being paid back on time and with interest. This payback with interest is possible because the occupants, whose incomes have steadily risen, can pay sufficient rent to meet capital and upkeep costs. This process is not an option for cities where the majority of the people live in poverty.

At the turn of the twentieth century, most of the world's poor could not afford to allocate the recommended twenty-five percent or more of their income to pay for shelter. These huge concentrations of poor people are a legacy of the last century. In 1975, five cities had ten million inhabitants or more. Two of these cities, Tokyo and New York were in affluent nations. In the year 2000, nineteen cities had ten million inhabitants or more with four of them in affluent nations (Tokyo, New York, Los Angeles and Osaka). The rest are in low-income nations (Figure 6). By 2015 the estimate is for 23 cities over 10 million. Most of their inhabitants live in self-built housing constructed by people with very low incomes and skills who must rely on building technologies appropriate for those circumstances. Consequently, affordable shelter is frequently inadequate in the extreme. The shelters are likely to provide insecure and inadequate protection against the elements and intrusions. They lack access to urban services, and are likely to occupy land illegally (Figure

Figure 9. Calcutta, India, squatter settlement along active rail lines: not a good place to raise children.

Figure 10. Bombay street children.

Figure 11. Nairobi, Kenya, Mathes Valley house with lockable door, two windows, and corn crop that must be guarded.

Figure 12. Outdoor kitchen of a home in the Rapki Valley, Nepal

7). They are hazardous to life. People constantly strive to improve their housing as a way to improve the quality of their lives.

Homelessness

What is missing when you have no home? Shelter is a complex mix of factors each of which contribute to quality of life (Figure 8). We all seek to create a secure and comfortable home place. In fact, most animals do the same by creating nests or dens in which to raise offspring. To protect children is a specie imperative no less for humans than for animals. Factors that are important for a home place fall into the categories *site* and *situation*, terms that are familiar to geographers. *Site* attributes refer to in-place or inside characteristics such as the design and type of building materials used for buildings, the slope and drainage of the building lot terrain, the temperature range or number of days of sunshine. *Situation* refers to the position of the home place relative to other locations. A home place must have access to community services such as utilities, schools, and generally to connections to the larger society. All these characteristics should be considered when assessing the viability of home places or when planning aid in building human habitats.

Locational (Access) Needs. Three types of outside connections are needed for a home place to function effectively. They are (i) access to physical services, (ii) access to community, and (iii) access to status.

Access to Physical Services A modern American home is serviced by several physical

Figure 13. De facto ownership leads to
steady improvement.

Figure 14. Durban, South Africa,
city-installed outdoor privys for sanitation.
Metered electric power supplied
direct to houses.

Figure 15. Calcutta, India, public water tap
with continuous flow when water is available.

links such as a motorable road, electric power line, and water and sewer lines. There are information links too for mail, newspapers, telephone, and radio/TV. Access to information usually requires a fixed home address and/or fixed receiver equipment, e. g., street address/mailbox, phone jack, or cable TV. Mobile receivers, laptop computers and homepages on the Internet have introduced new spatial dynamics to information exchange by adding a virtual home to the physical home. Sometimes, depending upon local conditions and availability, services can be provided on-site, such as, well water or a septic tank and drainage field. Such facilities do have neighborhood or locational implications depending upon soil type, aquifer capacity, and nearby housing density. Many of these attributes are absent in the squatter settlements of low-income cities.

Access to Community A home place needs access to social and economic exchange. Principal among these is proximity to work if income is earned outside the home. The home should also be conveniently located relative to other social services, retail stores, government offices, and homes of relatives and friends: in sum, the urban matrix. Schools are important. In Seoul, families move to school districts that have the best schools because students are assigned to school by place of residence. Residential land values in the best districts have soared due to this demand.
In the giant, low-income cities, the people in poverty

Figure 16. High occupancy "shotgun house" in low-income neighborhood in the Desire district of New Orleans, Louisiana, USA.

Figure 17. Ankara, Turkey. Well-cared for home with fruit trees.

will crowd into marginal places despite possibly terrible site conditions in order to get access to social and economic opportunities (Figure 9). Low cost, subsidized public transportation is a necessity in large urban areas with a high proportion of the population in poverty. Even modest fares may be a burden to the very poor. They must walk and must therefore crowd housing into places within walking distances of places of opportunity.

Access to Status A permanent home address is often a condition of citizenship. You have to have a permanent address in order to vote. In some places, home ownership is a requirement for voting on property tax proposals. You need to be a resident for your children to attend the public schools or to be eligible for welfare or social services. Children of migrant farm workers in the United States are often denied access to local schools and social services, which only adds to the difficulties in obtaining an education or sustaining health due to their short tenure in any one place.
People of means are inclined to invest far more in their home than is necessary for mere shelter. The home is used for displaying wealth and power to gain or affirm high status in the community. Ostentatious megahouses are characteristic of the nouveau riche in the communities of Silicon Valley and elsewhere across the United States at the end of the last century. The middle class behaves similarly by investing more than is prudent into too large a house and lot. Too much invested in housing leaves

assets too concentrated and debt service too high in relation to annual income. The desire to own a large single-family house on a big lot is a major component in urban sprawl.

At the beginning of the last century, Americans believed that a woman's place was in the home. At the end of the century, that attitude had changed with over half the women in the work force outside the home. Still, in many parts of the world, a woman's status is defined by her role in the home. If she is not a family member located in a home place, she is an outcast subject to harassment and danger.

Site Needs. The home place is built or arranged to provide for (i) restoration, (ii) health, and (iii) security. Our home is our castle.

Restoration We have physiological needs that are periodic; foremost among them is the need to sleep. We can sleep anywhere but we find great comfort in returning to our own beds at night (Figure 10). This desire creates the great diurnal movement from homeplace into and back from the urban matrix characteristic of urban life. Other periodic needs are for food and drink. Again, these needs can be met elsewhere but it is efficient and comforting to have a place at home to meet these needs. A place for toiletry and bathing meets a daily need for grooming as we set out for the day. Finally, it is comforting to have a place to relax, to retreat from the alertness necessary in public and/or strange places.

Health The dwelling provides a roof overhead for protection from the elements, rain,

snow, cold and heat. It can also be equipped to protect against hazards such as wind, fire, and earthquakes. Keeping the homeplace clean and fresh protects against disease and disease vectors. These elements are deficient to various degrees in squatter settlements. Crowded, unsanitary conditions, flimsy construction, lack of safe water and accumulation of wastes create hazards that threaten the entire urban area with spreading infectious disease or catastrophic fire, wind, or earthquake damage.

Security Strong walls and secure locks can provide for personal safety for one's self and family. A secure home also affords protection of possessions and wealth (Figure 11). Physical barriers work best when attended to by vigilant concern for who is coming and going. Protection is more readily sustained where private, protected space is buffeted from open public space by semi-public areas. In larger houses, strangers are customarily welcome at the foyer/lobby or in the living room but not the kitchen or bedrooms. In some places, an outside courtyard shared with immediate neighbors can act as a semi-public buffer where strangers are immediately recognized if they enter unannounced (Figure 12). A sense of security has much deeper roots than mere physical protection. The home place is where your roots are located. It is steeped in memories, memories of childhood, events, commitment, artifacts, landscapes not by sight alone but with memories of smells, sounds, tastes and kinetic senses. People love their homes. When separated in distant places they long for

home. They will fight for and die for their home.

The sentiment of home arises from symbolic, shared meaning. Yi-Fu Tuan said that, "A place is a pause in movement…The pause makes it possible for a locality to become a center of felt value" (Tuan, page 138). The repeated returns to pause at home creates its value. Mostly, however, this is a collective achievement. Sharing the place with family and friends is what makes the place a home. The lonely hotel room or the empty house after the children have left and the spouse has died may be a source of bitterness and sorrow rather than joy. To some the homeplace helps to define an individual's place or the community's place in the cosmos. Divine or supernatural blessings of the home may enhance a sense of security. In Thailand and Indonesia small house temples are placed at the corner of the home lot to invite spirits to protect the inhabitants. Sacred places are usually not in dwelling places but are nearby in the region accessible for periodic visits, if not daily, perhaps annually. Jerusalem is a Holy City; it is sacred for at least three major religions and is, unfortunately, a highly contested place. People will die to maintain control over it.

Rule of Law

An interesting observation is that squatter settlements are more of a problem for democratic regimes where rule of law is respected than they are for totalitarian regimes that rule by terror. The former Soviet Union and its Eastern European satellite states did not have squatter settlement problems because

whenever people located housing in places not permitted by the state, the housing was forcibly removed by police or military action. State terror ruled. In contrast, regimes that are more law-abiding do not forcibly move against their citizens unless permitted through court action and approval. Gegucandu means *mushroom house* in Turkish. This refers to the practice of extremely rapid construction of a house, literally overnight like a growing mushroom. The house is illegally built on state owned land and is immediately occupied by a family. Officials can issue a stop order when a house is observed under construction on state land. However, if the house is already occupied, individual rights being observed, it takes a court order to have it removed. Istanbul and Ankara as well as other Turkish cities have thousands of spontaneously built houses on state lands. The courts are so far behind that once occupied, the inhabitants have *de facto* ownership despite having no title to the land (Figure 13). India, the world's largest democracy, also has thousands of squatter settlements that are not arbitrarily removed by state force.

City officials are often at odds with people who construct inexpensive, self-built houses. Squatter settlements are often hazardous. Public safety is always at issue. Settlements may be located on land unsuitable for housing such as in floodways or on very steep slopes: places unworthy for standard housing. Cheap, self-built structures are built without housing codes or subdivision standards. The properties

may not be accessible by motor vehicle, which means fire trucks and other emergency vehicles cannot reach residences. They lack public services such as potable water or sewer and waste disposal or treatment. Health hazards that affect the whole city are the result. City officials are motivated to address such issues because of political pressure from more affluent citizens living in other parts of town (Figure 14). Low-income districts may be overcrowded with too many people per room occupancy. Houses located on property to which there is no title are not eligible for public services. Some of the people living in sub-standard housing may be illegal aliens who shun contact with any officials due to their lack of standing under the law. Financing improvements in squatter settlements is extremely difficult. The economy of the low-income city is not only weak but the government has difficulty in finding funds to provide necessary public services. Urban public transit needs to be subsidized because the clients are too poor to support the system with fare box payments. If public services cannot be provided directly to private properties, it is difficult to recover costs through service charges. For example, if there is only the capacity to provide widely spaced public potable water stands; charging for the water is not possible. The water is free to all users (Figure 15). When this is the condition, water is usually available for only a short time at each stand, perhaps not even daily. The water is distributed piece-meal district by district to avoid complete loss of pressure in the system. Land taxes are not a major source

of income for low-income cities. In New Orleans land tax was assessed by linear front foot; hence, the rise of the long, narrow "shotgun house" (Figure 16). The problem is lack of cadastral surveys assigning property to landowners and lack of spatial information management capacity in assessor's offices.

The public policy approach to these issues shapes the way the urban fabric develops. Millions of people have moved to the large, poor cities of the world and more are coming. The formal economies of these cities cannot cope with the growth. Some accommodation by public officials to the informal economies and to the capacity for self-help exhibited by those with little means is called for. Tapping the energy of poor but able people has been tried through site and service programs in which the city lays out streets and property boundaries and allows the occupants to build their own houses, sometimes with some simple building restrictions such as minimum wall heights. Ownership title is then given to the occupant. When this happens the house and lot are usually continually improved. In time a very suitable home emerges (Figure 17).

References

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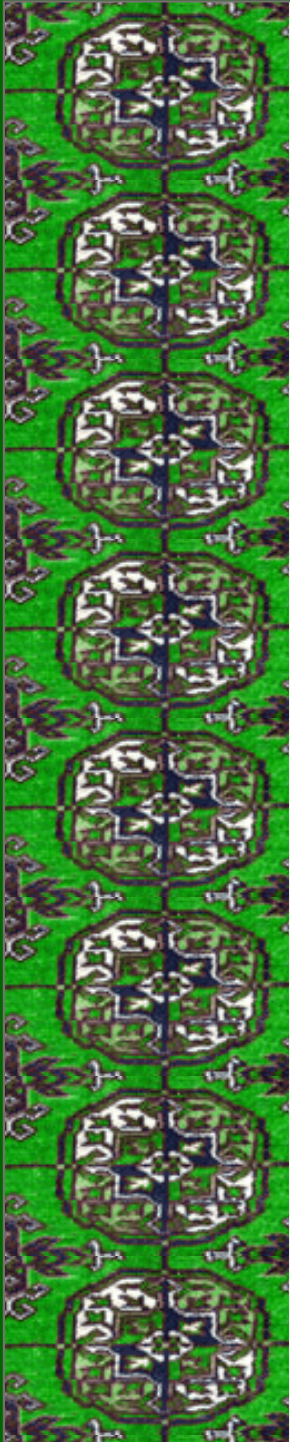
United Nations, World Urbanization Prospects: The 1999 Revision, Table 5. http://www.un.org/esa/population/pubsarchive/urbanization/urbanization.pdf (downloaded Nov. 1, 2001).

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Bisectors, Buffers, and Base Maps
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Introduction
In 1911, Thiessen and Alter [21] wrote on the analysis of rainfall using polygons surrounding rain gauges. Given a scatter of rain gauges, represented abstractly as dots, partition the underlying plane into polygons containing the dots in such a way that all points within any given polygon are closer to the rain gauge dot within that polygon than they are to any other gauge-dot. The geometric construction usually associated with performing this partition of the plane into a mutually exclusive, yet exhaustive, set of polygons is performed by joining the gauge-dots with line segments, finding the perpendicular bisectors of those segments, and extracting a set of polygons with sides formed by perpendicular bisectors. It is this latter set of polygons that has come to be referred to as "Thiessen polygons" (and earlier names such as Dirichlet region or Voronoi polygon, see Coxeter [4]). The construction using bisectors is tedious and difficult to execute with precision when performed by hand. Kopec (1963) [11] noted that an equivalent construction results when circles of radius the distance between adjacent points are used. Indeed, that construction is but one case of a general construction of Euclid. Like Kopec, Rhynsburger (1973) [20] also sought easier ways to construct Thiessen polygons: Kopec through knowledgeable use of geometry and Rhynsburger through the development of computer algorithms. The world of the Geographical Information System (GIS) software affords an opportunity to combine both.
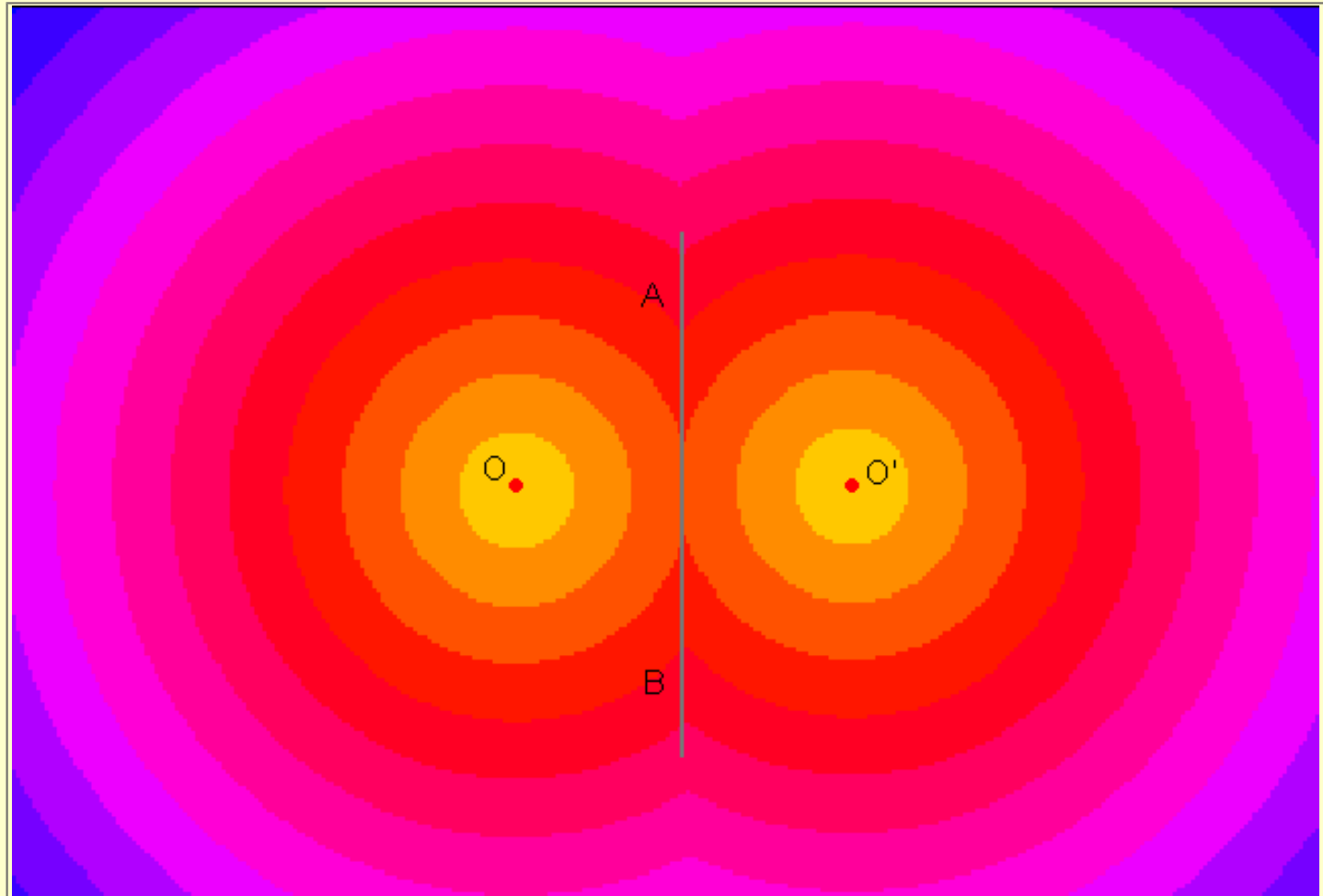
Bisectors
A theorem/construction of Euclid shows how to draw a perpendicular bisector separating any pair of distinct points in the Euclidean plane. The animation in Figure 1 illustrates this procedure:

- Given O and O' in the plane.
- Draw a segment joining O and O'
- Construct two circles, one centered on O and the other centered on O', each of radius greater than half the distance between O and O'. The radii are the same.
- Label the intersection points of the circle as A and B. Draw a line through A and B. This line is the perpendicular bisector of |OO'|.

In the final frame of the animation in Figure 1, the highly colorful one, the use of a GIS displays clearly that radius length produces the same position for |AB| independent of choice (greater than 0.5*|OO'|). The last frame was produced in ArcView 3.2 (with Spatial Analyst Extension enabled) using the "calculate distance" feature. It shows the general result, of which Kopec used one element. One need not be limited to choosing the distance between adjacent points--any distance greater than half that distance will produce the same result.

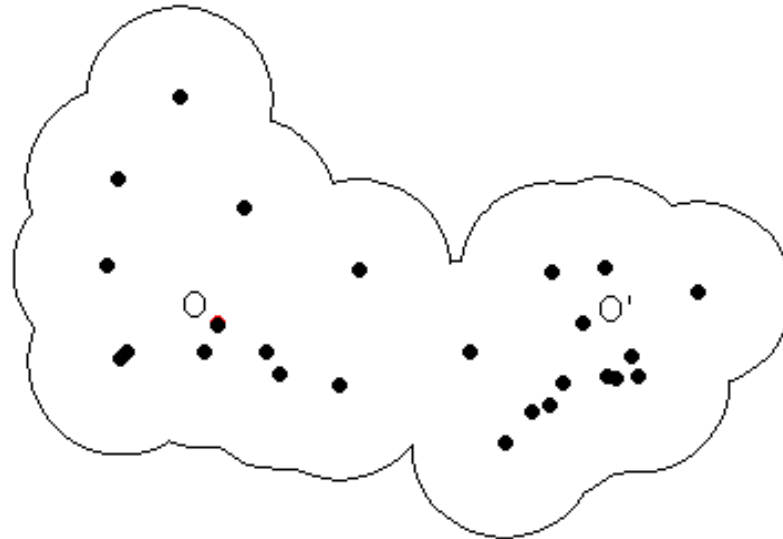**Figure 1.** Animation showing construction of perpendicular bisector, AB, of |OO'|.

Buffers

Traditionally one might have used a drawing compass and a straightedge to construct a perpendicular bisector between two points. It is an easy matter to do so, however, using a GIS, as suggested above. If there are more than two points, the matter can become quickly tedious. Again, the GIS offers a quick and accurate way to calculate positions (Figure 2).

- Given a distribution of points in the plane (O and O' are now among this set).
- Create circular buffers around all the points, leaving the entire circle surrounding each point. It has become difficult to visualize the location of the set of perpendicular bisectors that are determined by this circular mass.
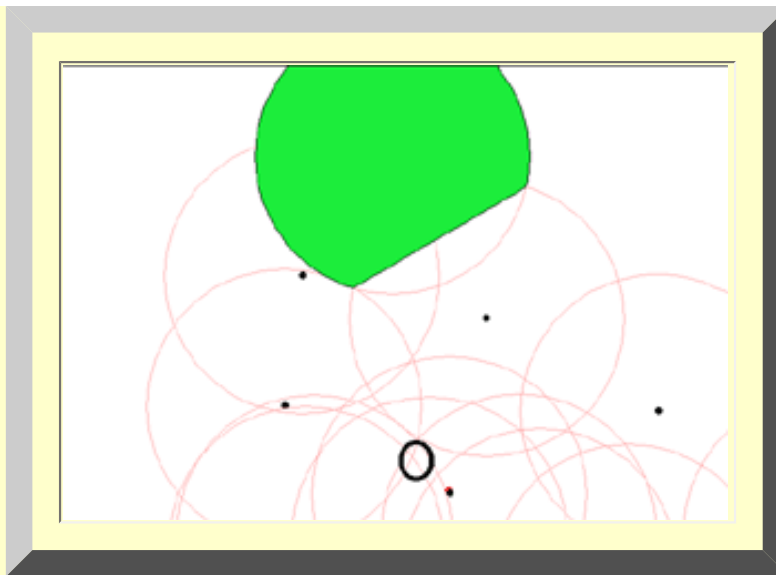
- Dissolve arcs within the circular mass. This procedure offers some help in visualizing where bisectors might be, but only a vague picture of bisector position is generated.

Figure 2. Circular buffers centered on a distribution of 25 points.

To actually position the lines of partition, or Thiessen polygon edges, in the GIS, use the "split polygon" feature available in ArcView or other GIS software, creating a sort of bubble foam (Figure 3 shows one split created in this manner). Numerous websites offer suggestions for use of Thiessen polygons ranging from rainfall regions, to hydrological modelling, to road centerline location (and others) [12, 13, 14].
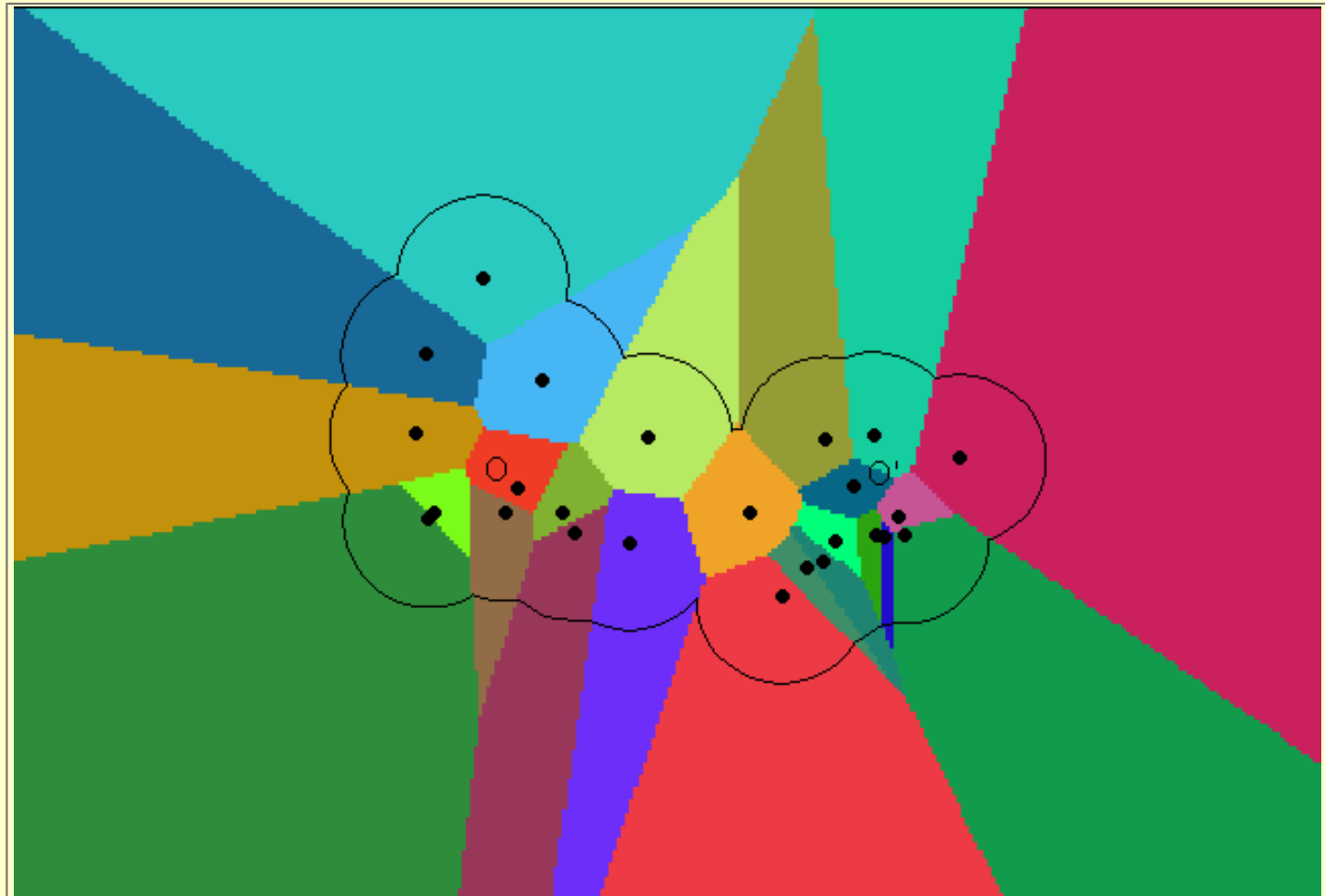
Figure 3. Use of the Polygon-split tool.

One contemporary website demonstrates the mechanics of this sort of approach using one buffer distance [6]. Others employ a variety of software to construct Thiessen polygons [22].

Again, the GIS is helpful: ArcView (Spatial Analyst extension) offers a single tool that quickly calculates Thiessen polygons. Use "Assign Proximity" to create zones around each point. Within each zone, all points are nearer to the distribution point in that zone than they are to any other point in the distribution. In Figure 4, the relationship between perpendicular bisector, buffer (construction of Euclid), proximity zone/Thiessen polygon becomes clear.

- The initial frame shows the result of running the "assign proximity" feature of ArcView 3.2, Spatial Analyst Extension, on the scatter of 25 points. The colorful polygons separate the plane into Thiessen polygons. Within each polygon, all points are closer to the point from the 25 dot scatter that is in that polygon than they are to any other point in the 25 point scatter.
- The second frame superimposes circular buffers; thus, one sees how the Thiessen polygons are a direct consequence of the Theorem/Construction of Euclid.
- The third frame dissolves part of the circular buffer, exposing more clearly the relation between circular buffer and proximity zone as calculated by the computer algorithm in ArcView GIS.

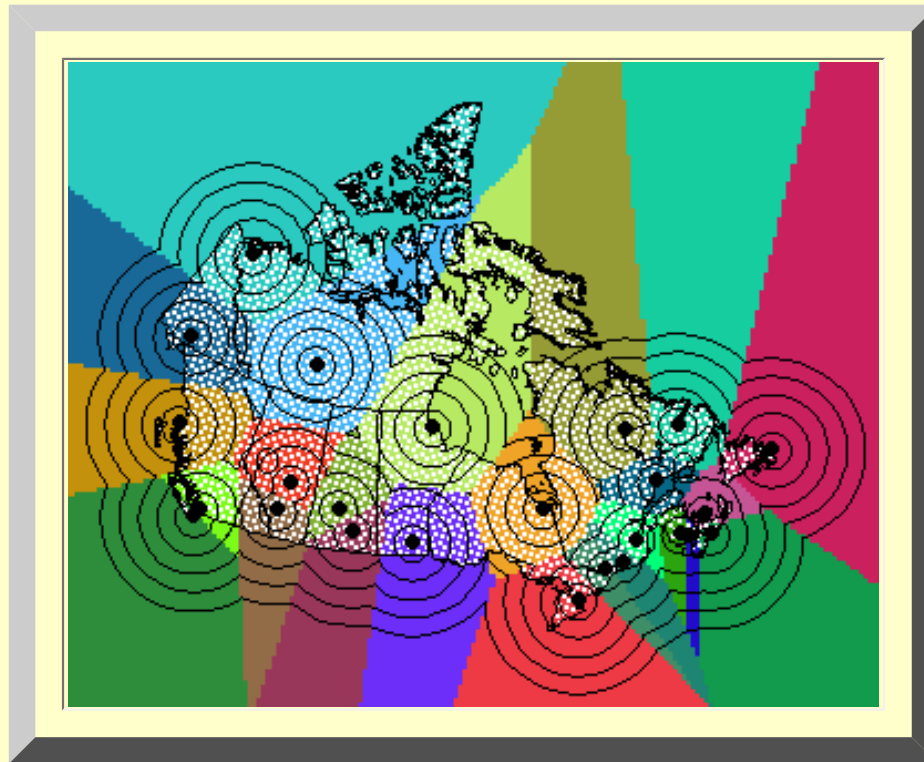**Figure 4.** Bisectors, buffers, and proximity zones (Thiessen polygons).

Whether one considers rail networks within sausage-like linear buffers, counts population in buffered bus routes, or selects minority groups from within a circular buffer intersecting census tracts, the buffer has long served, and continues to serve, as a basis for making decisions from maps. Buffers have a rich history in geographical analysis. Mark Jefferson [10, 2] rolled a circle along lines on a map representing railroad tracks to create line-buffers representing proximity to train service and suggested consequent implications for population patterns in various regions of the world. Julian Perkal and John Nystuen saw buffers in parallel with delta-epsilon arguments employed in the calculus to speak of infinitesimal quantities (reprint of Perkal, "An Attempt at Objective Generalization," Michigan Interuniversity Community of Mathematical Geographers, [16, 19]. Jefferson's mapping effort in 1928 was extraordinary; today, buffers of points, lines, or regions are trivial to execute in the environment of Geographic Information Systems software. To paraphrase Faulkner (1949), 'good

ideas will not merely endure, they will prevail' [7].

Base Maps

In a recent invited lecture (2001) to The University of Michigan Lecture Series in GIS Education, Arthur Getis noted [15] that he had used circular buffers around point observations and that he used a sequence of nested buffers to successively fill space to eventually include all individuals in the underlying point distribution gathered from field evidence. Thus, viewed abstractly, each set of buffers serves as a base map, with the sequence successively filling more space and including more individual observations in the analysis. A different view might see the Thiessen map as the base map (Figure 5). When it is calculated at the outset, it can serve as a standard against which to test more specialized views at varying buffer radii, on a continuing basis, as the research within buffers evolves. The Thiessen base map serves, therefore, as an "absolute" base map against which to view the "relative" base maps of varying local radii (and other configurations): it is a limit of a sequence of measures based on buffers that increasingly fill more space (but still leave gaps). Getis noted [15] that he and Ord had recently completed an article involving issues of global and local spatial statistical measures [18]. What is suggested here is the appropriate use of a geometric foundation: a use for a Thiessen, space-filling, base against which to test the results of sequences of successive measures in buffers that may not fill the underlying universe of discourse.

Figure 5. Distribution of 25 Canadian cities against Thiessen base map and circular buffer set. Files may be clipped to suit user needs. Note polygon sides in relation to pattern of intersecting circles (as in Figure 1 above) and space filling pattern of successive buffers closing in on individual Thiessen polygons in the background. Values within the buffers thus

approach, with increasing buffer radius, values associated with the underlying Thiessen polygons (as the buffers never fall outside the polygon, due to the construction of Euclid). In cases where clipping matches buffer boundaries, the buffer values converge to and attain the limiting values associated with the Thiessen polygons.

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Software used

- Adobe, Photoshop, 6.0.
- Environmental Systems Research Institute (ESRI), Redlands, CA.
  - ArcView 3.2
  - ArcView Spatial Analyst Extension
- Gamani, Movie Gear
- Macromedia, DreamWeaver, 3.0

- Netscape Composer

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