

**SOLSTICE:
An
Electronic
Journal of
Geography
and
Mathematics.**

(Major articles are
refereed;
full electronic archives
available)



**SOLSTICE, VOLUME XVIII, NUMBER 2;
DECEMBER, 2007.**

**Front matter: December, 2007.
Editorial Board, Advice to Authors, Mission Statement.**

**Awards
(contains recent listing of "blue ribbons" from the "Best of the Google 3D
Warehouse")**

SPECIAL ISSUE ON PROJECTIVE GEOMETRY CONSTRUCTIONS

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(reviewed)

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The Animated Pascal

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Desargues's Two-Triangle Theorem

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ANNOUNCEMENT and INVITATION

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IN MEMORIAM

[ALLEN K. PHILBRICK](#)

[NORTON S. GINSBURG](#)

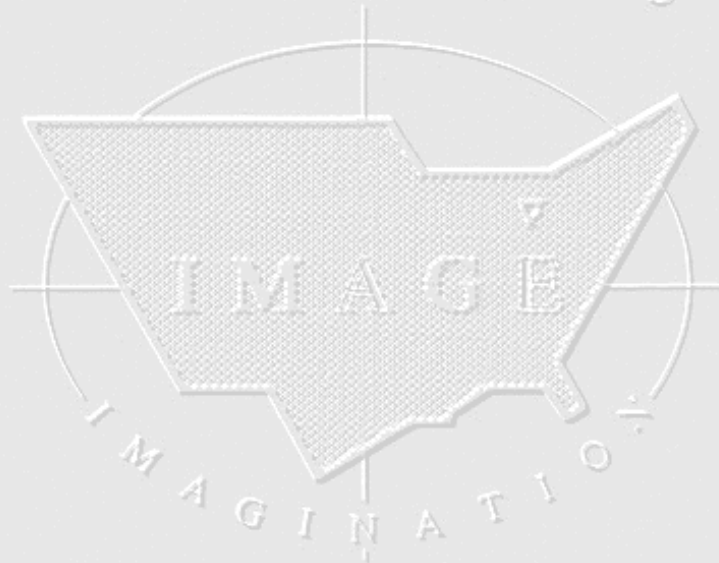
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SOLSTICE: AN ELECTRONIC JOURNAL OF GEOGRAPHY AND MATHEMATICS

<http://www.imagenet.org>

December, 2007

VOLUME XVIII, NUMBER 2

ANN ARBOR, MICHIGAN

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MISSION STATEMENT

The purpose of Solstice is to promote interaction between geography and mathematics. Articles in which elements of one discipline are used to shed light on the other are particularly sought. Also welcome are original contributions that are purely geographical or

purely
mathematical. These may be prefaced (by editor or author) with commentary suggesting directions that might lead toward the desired interactions.
Individuals wishing to submit articles or other material should contact an editor, or send e-mail directly to sarhaus@umich.edu.

SOLSTICE ARCHIVES

Back issues of Solstice are available on the WebSite of the Institute of Mathematical Geography, <http://www.imagenet.org> and at various sites that can be found by searching under "Solstice" on the World Wide Web. Thanks to Bruce Long (Arizona State University, Department of Mathematics) for taking an early initiative in archiving Solstice using GOPHER.

PUBLICATION INFORMATION

To cite the electronic copy, note the exact time of transmission from Ann Arbor, and cite all the transmission matter as facts of publication. Any copy that does not superimpose precisely upon the original as transmitted from Ann Arbor should be presumed to be an altered, bogus copy of *Solstice*. The oriental rug, with errors, serves as the model for creating this weaving of words and graphics.



Awards and Recognition

(See [Press Clippings](#) page for other.)

- 2007: Best of 3D Warehouse awards (blue ribbons); these buildings come up default in all free downloads of Google Earth when the "3d buildings" checkbox is checked. They are designed for planning, rather than for architectural, purposes; file size is kept small. What is important is giving the "impression" of the building rather than giving large amounts of detail. View the associated .kmz files in Google Earth to understand the context; they are attached to the linked pages below. Be sure to turn on the "terrain" switch, otherwise buildings made in older software (older versions of Google SketchUp) will float above the surface.
 - Archimedes's models (S. Arlinghaus is "Archimedes").
 - Campus models of Arlinghaus: [1](#) (Alumni Center), [2](#) (Angell Hall), [3](#) (Angell Hall Complex), [4](#) (Art Museum, first model), [5](#) (Art Museum, second model), [6](#) (Bagnoud Building), [7](#) (Biomedical Sciences Building), [8](#) (Bursley Hall), [9](#) (C. C. Little Building), [10](#) (Chemistry Building), [11](#) (Clements Library, first model), [12](#) (Clements Library, second model), [13](#) (Crisler Arena), [14](#) (Dennison Building, first model), [15](#) (Dennison Building, second model), [16](#) (East Hall, first model), [17](#) (East Hall, second model), [18](#) (Frieze Building), [19](#) (Hatcher Library North), [20](#) (Hatcher Library South), [21](#) (Haven Hall), [22](#) (Hill Auditorium, first model), [23](#) (Hill Auditorium, second model), [24](#) (Kraus Natural Science Building), [25](#)

- (Michigan League, first model), [26](#) (Michigan League, second model), [27](#) (Literature, Science, and the Arts Building), [28](#) (Mason Hall), [29](#) (Michigan Stadium), [30](#) (Modern Language Building), [31](#) (Northwood IV), [32](#) (Pharmacy College), [33](#) (Power Center), [34](#) (Rackham Building, first model), [35](#) (Rackham Building, second model), [36](#) (Randall Laboratory), [37](#) (Schembechler Hall), [38](#) (Shapiro Library), [39](#) (Tappan Hall, second model), [40](#) (Tisch Hall), [41](#) (University Hospitals), [42](#) (West Hall, first model), [43](#) (West Hall, second model).
- DDA models of Arlinghaus: [1](#), [2](#), [3](#), [4](#), [5](#), [6](#), [7](#), [8](#), [9](#), [10](#), [11](#), [12](#)
 - [YouTube](#) version of all, above.
- Build Your Campus competition models--student participants each won at least one blue ribbon, as a Best of 3D Warehouse award
 - Lauren Leigh Hoffman: [Dana Building](#)
 - Juan Sergio Ponce de Leon: [Yost Arena](#), [South Quad](#)
 - Andrew Walton: [Golf Course Clubhouse](#)
- 2007: University of Michigan models of about 300 buildings included in the online folder resulting from the "Build Your Campus" competition.
 - 2007: Archimedes selected by Google as a "Featured Modeler."
 - 2006: Google 3D Warehouse, "Google Picks" then go to "Cities in Development" <http://sketchup.google.com/3dwarehouse/> to see textured models of downtown Ann Arbor buildings.
 - 2006: *3D Atlas of Ann Arbor, Version 2*. Google Earth Community, ranked a "Top 20 Rated Post" on Entrance page, December 8, 2006.
 - 2006: *3D Atlas of Ann Arbor, Version 2*. [Rated](#) a 5 globe production (top score) in Google Earth Community, November 2006.
 - 2004: Sandra L. Arlinghaus and William C. Arlinghaus, Spatial Synthesis Sampler, *Solstice*, Summer 2004. Semi-Finalist, [Pirelli](#) 2003 INTERNETional Award Competition.
 - 2004: Sandra Lach Arlinghaus, recipient, The President's™ Volunteer Service Award, March 11, 2004.
 - 2003: Jeffrey A. Nystuen, won the 2003 Medwin Prize in Acoustical Oceanography given by the [Acoustical Society of America](#). The citation was "for the innovative use of sound to measure rainfall rate and type at sea". It is awarded to a young/mid-career scientist whose work demonstrates the effective use of sound in the discovery and understanding of physical and biological parameters and processes in the sea.
 - 2002: [Sandra L. Arlinghaus](#), William C. Arlinghaus, and Frank Harary. *Graph Theory and Geography: an Interactive View (eBook)*, published by John [Wiley](#) and Sons, New York, April 2002. Finished as a Finalist in the 2002 Pirelli INTERNETional Award Competition (in the top 20 of over 1200 entries worldwide).
 - 2001: *Solstice*, Semi-Finalist, Pirelli 2001 INTERNETional Award Competition in the Environmental Publishing category.
 - 1992: *Solstice*, article about it by Ivars Peterson in *Science News*, 25 January, 1992..
 - 1991: *Solstice*, article about it by Joe Palca, *Science* (AAAS), 29 November, 1991.

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INTRODUCTION TO THE SPECIAL ISSUE*

Projective geometry is a non-Euclidean geometry that sits atop all other non-Euclidean geometries. It is the most general geometry and possesses complete symmetry. The infinite is no different from the ordinary. Two points determine a line; two lines determine a point. Indeed, "parallel" lines intersect at a point at infinity (at least to our Euclidean-trained minds). There is complete duality.

Because the academic curriculum is focused almost entirely on Euclidean geometry, the constructions of projective geometry, which are quite beautiful, remain hidden from most. They appear "unnatural" and "non-intuitive." The extra capability of the internet and related software permits animating difficult to visualize projective scenes and the instantaneous sharing of these across a wide range of locales.

This issue of Solstice shares several important projective constructions with readers:

- Harmonic conjugacy
- Constructions associated with conics in the projective plane.
- Desargues's Two-Triangle theorem.

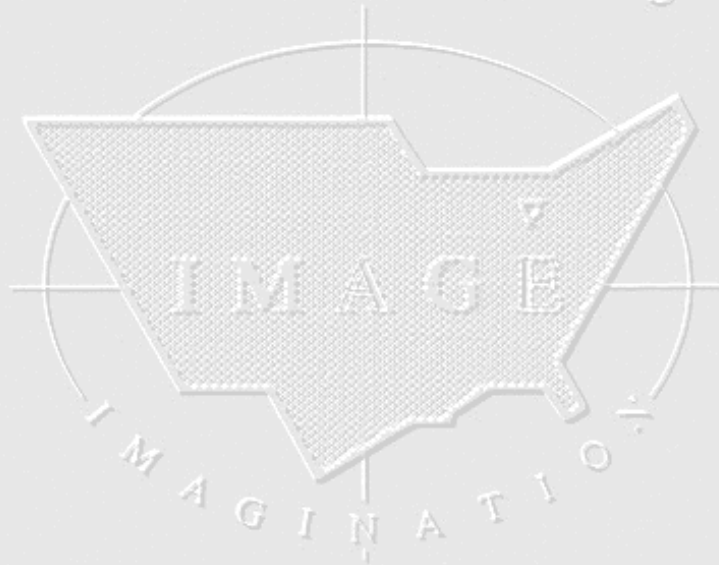
Read about harmonic conjugacy in association with true perspective projections of the globe to a mapping plane. Learn how all perspective mapping is captured by this projective geometric construction!

**Dedicated to the memory of Professor H.S.M. Coxeter, 1907-2003.*

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Geography -- Visual Unity

Sandra Lach Arlinghaus

Benoit Mandelbrot has shown us the power of computer-generated images to make mathematical concepts come alive that had otherwise been tucked away in mathematics classrooms [Mandelbrot, 1982]. What is accurate in terms of definition/proof presentation may not penetrate the minds of those less gifted in mastering the mathematical notation and its underlying system of logic. Beautiful images, conceived by those who understand both the abstract concepts and their representation using current technological capability, can go a long way toward making heretofore inaccessible concepts become accessible.

A recent example of such transformation in accessibility appeared in YouTube this past summer [Arnold and Rogers, 2007]. Imagine being able to see Möbius transformations come alive as an animated sequence showing their relations to each other in the plane and then, unifying those representations by lifting them into the third dimension on a sphere. Take a look at this award-winning work, now. Use the link above to view it at low resolution on YouTube. Original Source Douglas N. Arnold and Jonathan Rogness, June 2007: <http://www.youtube.com/watch?v=jX3VmDqiFnY>.

The projection "light" at the top of the sphere in the video sends points on the sphere to points in the plane via stereographic projection, one of a class of "true" perspective projections employed by map makers to send part of the surface of a spherical globe to a plane. Three members of this projection class are shown below (Figure 1). In all cases, the plane into which points are projected is tangent to the globe at S, the south pole of the globe. The projections are:

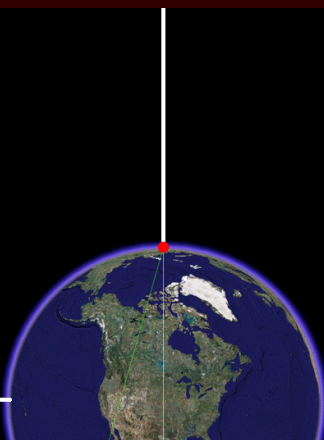
Stereographic Projection, in which the center of projection is at the north pole of the globe. A point P on the globe is sent to a point $P^{(stereo)}$ in the plane (Figure 1a).

Gnomonic Projection, in which the center of projection is at the center of the globe. A point P on the globe is sent to a point $P^{(gnomon)}$ in the plane (Figure 1b).

Orthographic Projection, in which the center of projection is at a point at infinity. A point P on the globe is sent to a point $P^{(ortho)}$ in the plane (Figure 1c).

Each of the first three images shows an animation suggesting how a point is sent from the globe to the plane. When a large number of points, outlining regions of various sorts, is sent from the globe to the plane, a geographic outline map is created.

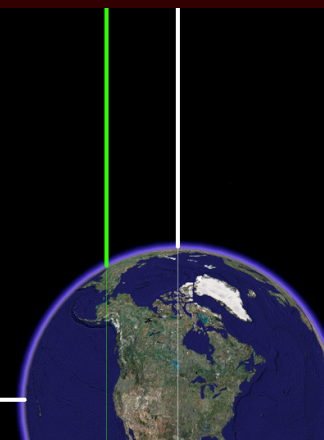
Stereographic
Projection



Gnomonic
Projection



Orthographic
Projection



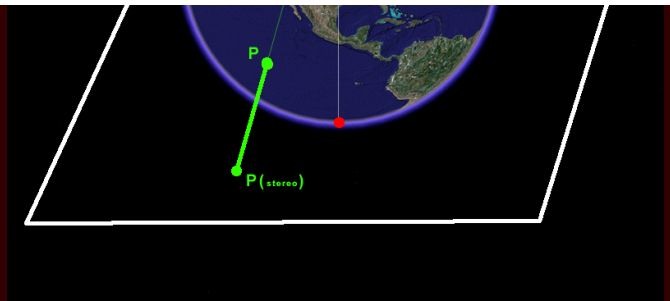


Figure 1a. Projection is from the north pole.

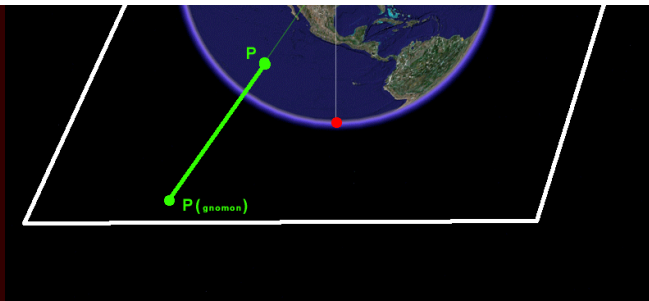


Figure 1b. Projection is from the center of the globe.

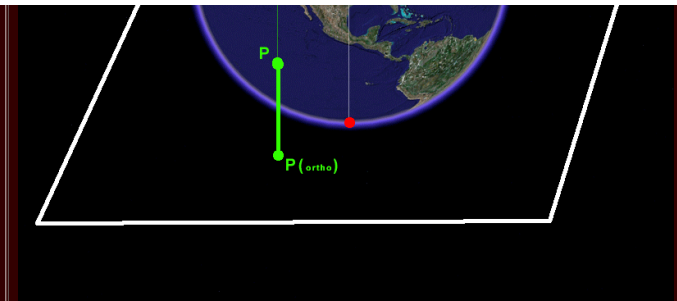


Figure 1c. Projection is from a point at infinity.

Clearly, there is an infinite number of choices available, up and down the white vertical line, for centers of projection. Figure 2 shows one relationship among the three projections of Figure 1. Perspective projections are only one style of projection; there are an infinity of other styles possible and within most of those, another infinity of ways to create maps. Readers wishing to discover more about the characteristics of individual projections, and classes of projections, are referred to the vast literature on the subject of cartography, the science and art of making maps [Snyder, 1993; Yang, Snyder, Tobler, 2000].

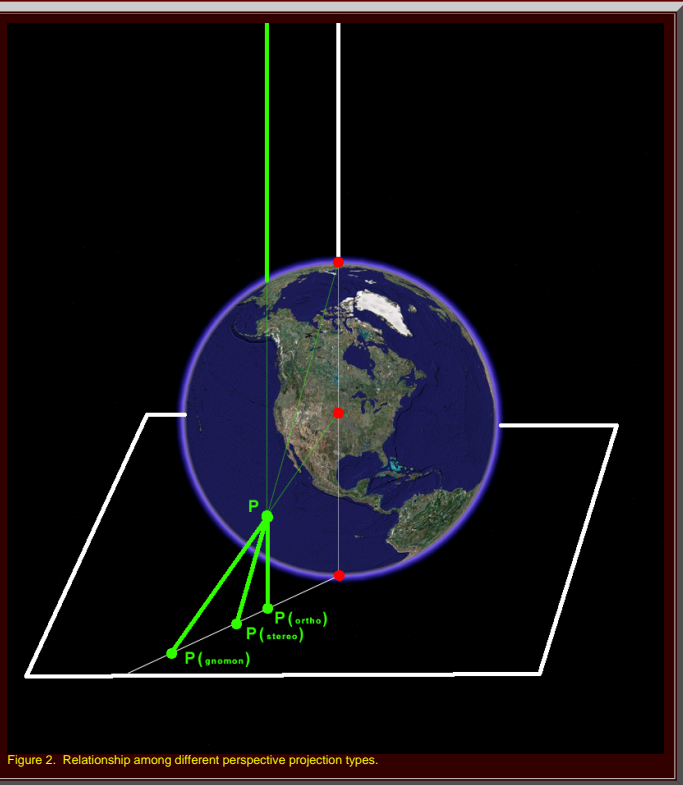


Figure 2. Relationship among different perspective projection types.

As Arnold and Rogness suggest, there may be more to learn by looking at the relationship between plane and spherical expressions. To consider this idea, turn to the underlying mathematics...that of projective geometry which explores the theory of perspectivity and related concepts [Coxeter, 1961]. Perspective projection through P , in the sequence below, shows how one might find, given three collinear points A , B , and C , a fourth point C' that is independent of arbitrary choices made during the process of construction. The points C and C' are harmonic conjugates with respect to A and B . Follow the animations in Figures 3a and 3b to understand the process.

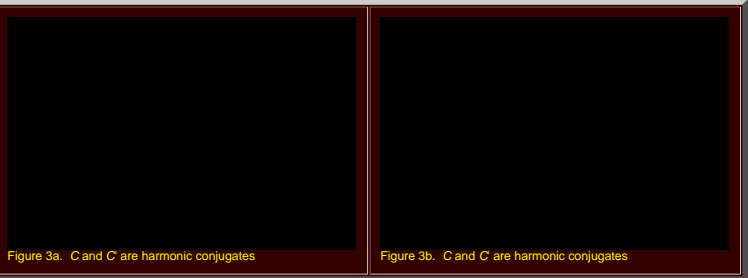


Figure 3a. C and C' are harmonic conjugates

Figure 3b. C and C' are harmonic conjugates

In 1986, an essay appeared couched in the language of projective geometry in which a theorem, linking harmonic conjugacy to perspective map projection, was proved to show the following (Harmonic Map Projection Theorem):

centers of projection that are inverses in relation to the poles of a sphere are harmonic conjugates in the projection plane in relation to the projected images of the poles of the sphere.
 as a special case of the observation above, it follows that gnomonic and orthographic projections, with inverse centers of projection in the sphere, are composed of points that are harmonic conjugates of each other in the plane [Arlinghaus, 1986].

The animation in Figure 4 illustrates both of these points and it does so more clearly than did the original text which used notation and static grayscale images only.

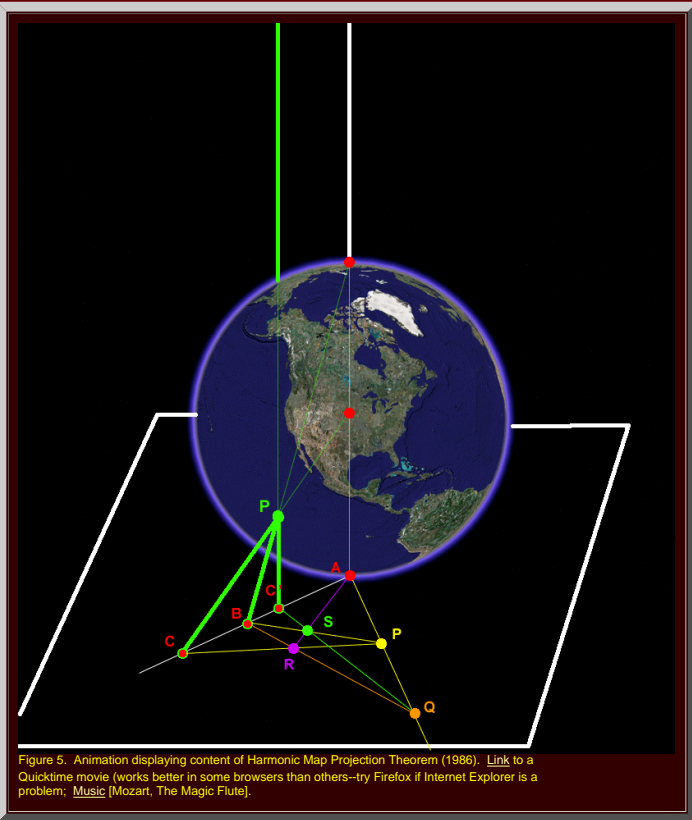


Figure 5. Animation displaying content of Harmonic Map Projection Theorem (1986). [Link to a Quicktime movie \(works better in some browsers than others--try Firefox if Internet Explorer is a problem; Music \[Mozart, The Magic Flute\].](#)

Naturally, it also follows that all the theorems of projective geometry that apply to harmonic conjugates also apply to all projections that satisfy the conditions of this theorem. Thus, for example, the entire set of perspective projections may be derived in the projective plane, alone, from the subset of projections with centers of projection contained within the sphere of projection. The unbounded problem of looking at an infinity of projection centers spread along an unbounded ray is thus converted to one of looking at an infinity of projection centers spread along a bounded line segment.

Further, the language of duality of projective geometry then applies to the geometry of all perspective map projections. At the broader level of identifying future research topics, this geo/metric/graphic unity of harmonic conjugacy and perspective projection suggests possible advantages in employing this highly symmetric geometry, that does not distinguish the ordinary from the infinite, to understand and to analyze other geographical problems that exhibit symmetry in underlying relations and that also embrace the concept of infinity.

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Mandelbrot, B. 1982. *The Fractal Geometry of Nature*. W. H. Freeman.

Mozart, W. A. Die Zauberkugel. <http://www.amazon.co.uk/exec/obidos/clipserve/B000001GX1002029/026-8133798-8086815>

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Yang, Q. H.; Snyder, J. P.; Tobler, W. R. 2000. *Map Projection Transformation: Principles and Applications*, Taylor & Francis.

Notes

The globe in Figures 1, 2, and 4 is derived from a screen capture from Google Earth, <http://earth.google.com/>

Figures 3a and 3b also appear in <http://www.YouTube.com>

[Construction of Harmonic Conjugates--1](#)

[Construction of Harmonic Conjugates--2](#)

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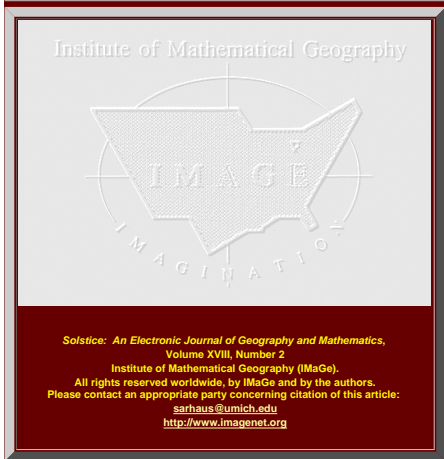
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THE ANIMATED PASCAL

Sandra Lach Arlinghaus

There are a number of elegant theorems about conics in the projective plane.
The animations below illustrate a few associated constructions.

Figure 1 shows an animated illustration of Pascal's Theorem:

PASCAL'S THEOREM [Coxeter, 1961]: If A, B, C, A', B', C' are six vertices of a hexagon inscribed in a conic, then the points of intersection of opposite sides of the hexagon, $L, M,$ and $N,$ are collinear (lying along line z).

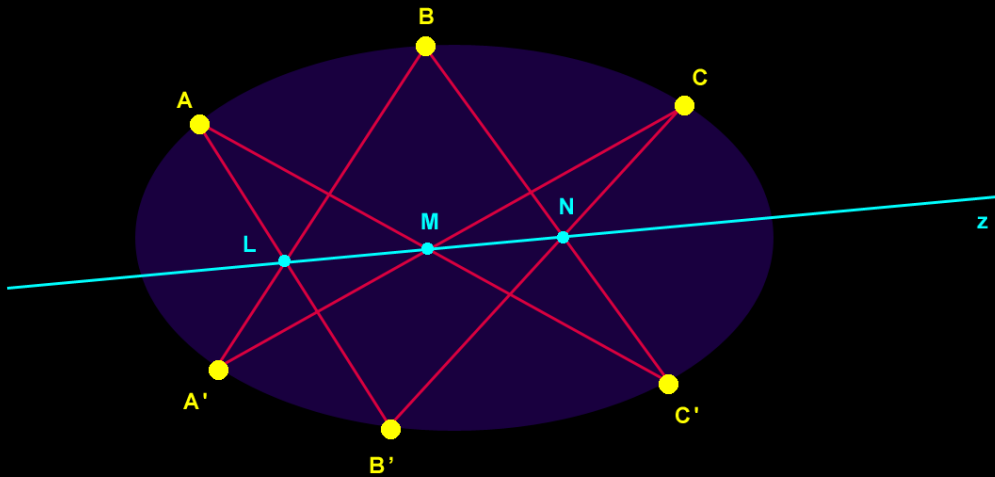


Figure 1. Pascal's Theorem.

Figure 2 shows an animated illustration of the dual of Pascal's Theorem:

BRIANCHON'S THEOREM [Coxeter, 1961]: If a, b, c, a', b', c' are six sides of a hexagon circumscribed about a conic, then the lines joining opposite vertices (the diagonals) of the hexagon, $l, m,$ and $n,$ are concurrent (at the point Z).

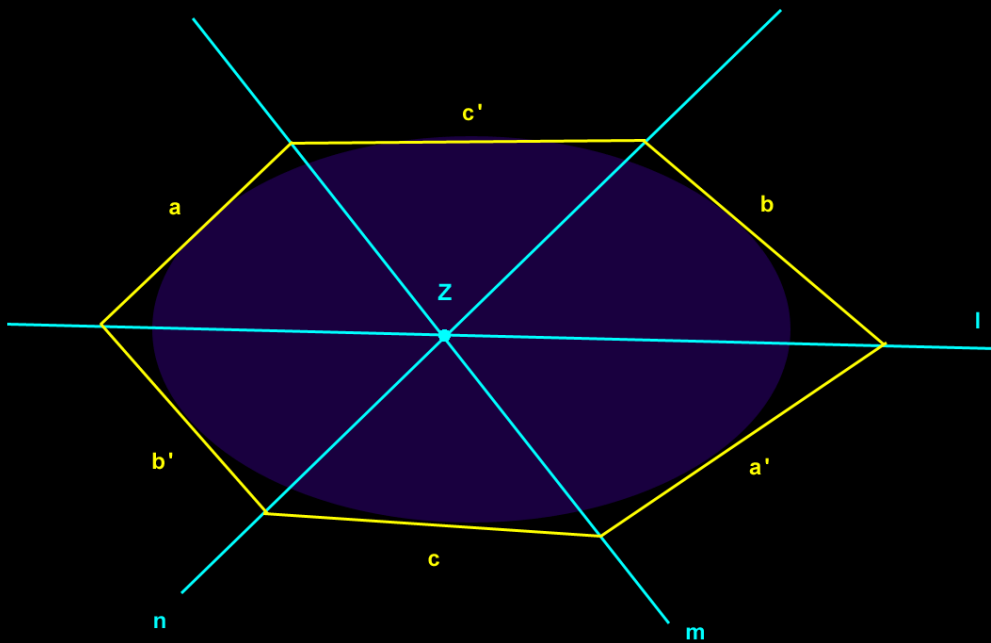


Figure 2. Brianchon's Theorem: dual of Pascal's Theorem.

Figure 3 shows an animated illustration of the converse of Pascal's Theorem used to create a specific, important construction:

BRAIKENRIDGE-MACLAURIN CONSTRUCTION [Coxeter, 1961]: Construction of a conic through 5 given points based on:

The point C' lies on the conic.

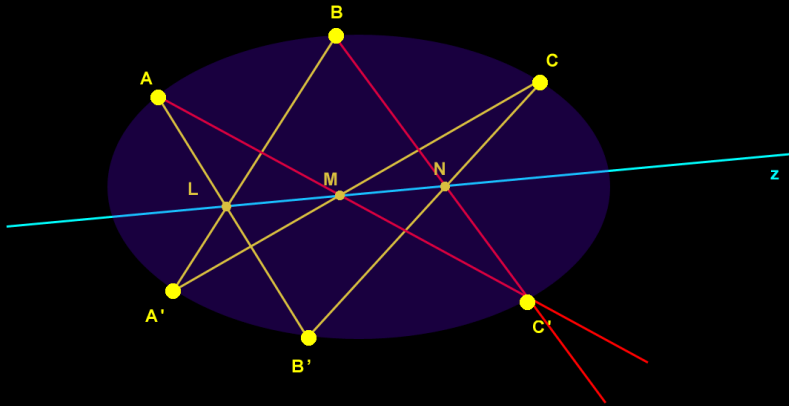


Figure 3a. This construction is the converse of Pascal's Theorem. Choose line z through L , leading to point C' on the conic through the 5 given points, A, B, C, A', B' .

This point C' also lies on the conic.

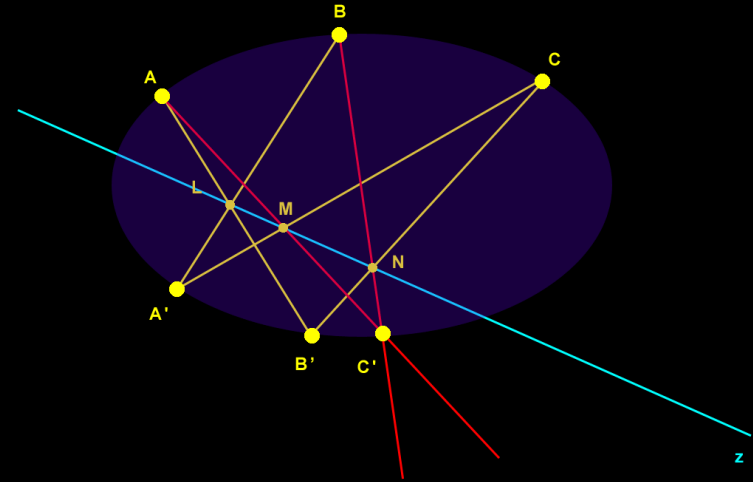


Figure 3b. This construction is independent of the choice of z . Choose a different line z through N . It, too, will produce a point C' in a location different from that of Figure 3a but the new C' will also lie on the conic through the 5 given points!

Part of the motivation for creating these animations lies in offering helpful graphics to illuminate notation. Another part of it is to build a foundation on which to continue ongoing research linking non-Euclidean geometries and the compression of (geo)graphics [Arlinghaus and Batty, 2005] -- within the digital world but perhaps not beyond the "limits" of the Escher series! [Escher, Circle Limit series; kmz file [link](#)].

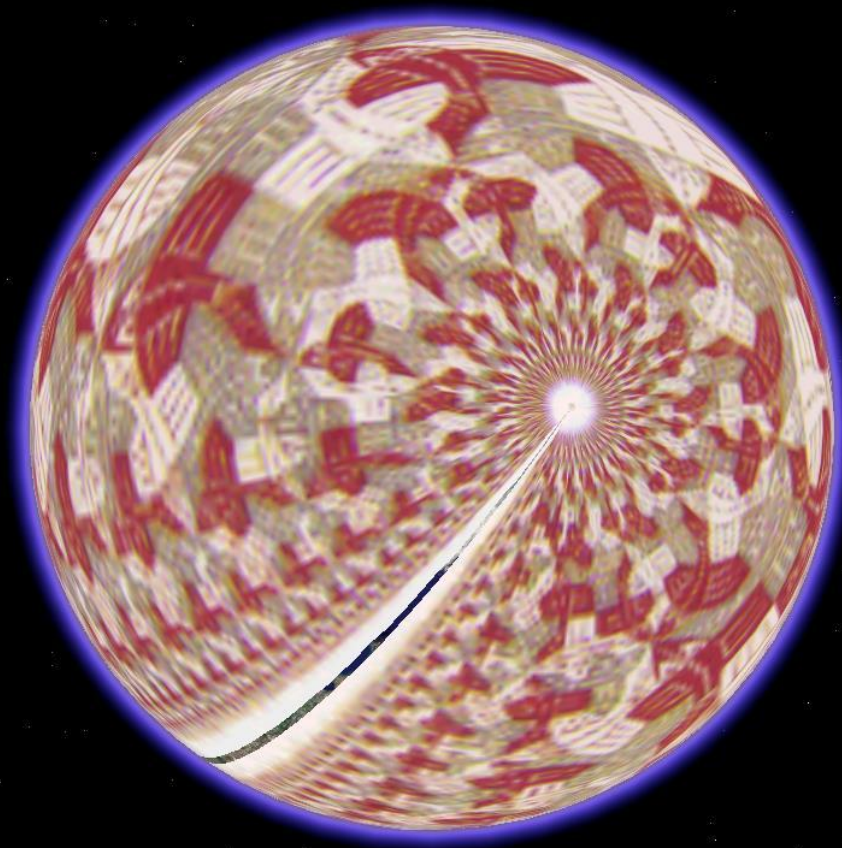


Image NASA
Image © 2007 TerraMetrics

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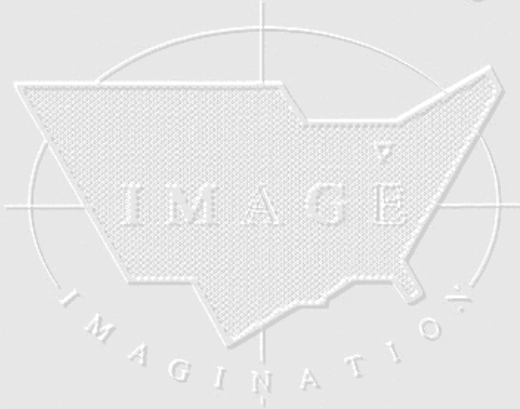
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- Arlinghaus, S. L. and Batty, M. "Zipf's Hyperboloid?" *Solstice: An Electronic Journal of Geography and Mathematics*, Vol. XVII, No. 1. Institute of Mathematical Geography, <http://www.imagenet.org/>
- Coxeter, H. S. M. 1961. *Introduction to Geometry*. New York: John Wiley & Sons. 252-254.
- Escher, M. C. 1956. [http://en.wikipedia.org/wiki/M. C. Escher](http://en.wikipedia.org/wiki/M._C._Escher)

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DESARGUES'S TWO-TRIANGLE THEOREM
Sandra Lach Arlinghaus

Often the introduction of added perspective from another dimension sheds light on pattern. So it is with the following theorem from Projective Geometry which becomes easy to visualize in three dimensions. Google Earth will assist in creating graphics for such visualization.

DESARGUES'S THEOREM [Coxeter, 1961]: If two triangles, A, B, C , and A', B', C' are perspective from point O ($AA', BB',$ and CC' are concurrent), then the intersections of corresponding triangle sides, $L=AB':A'B, M=AC':A'C,$ and $N=BC':B'C,$ are collinear.

Figure 1 shows an animated illustration of Desargues's Theorem in the plane, only.

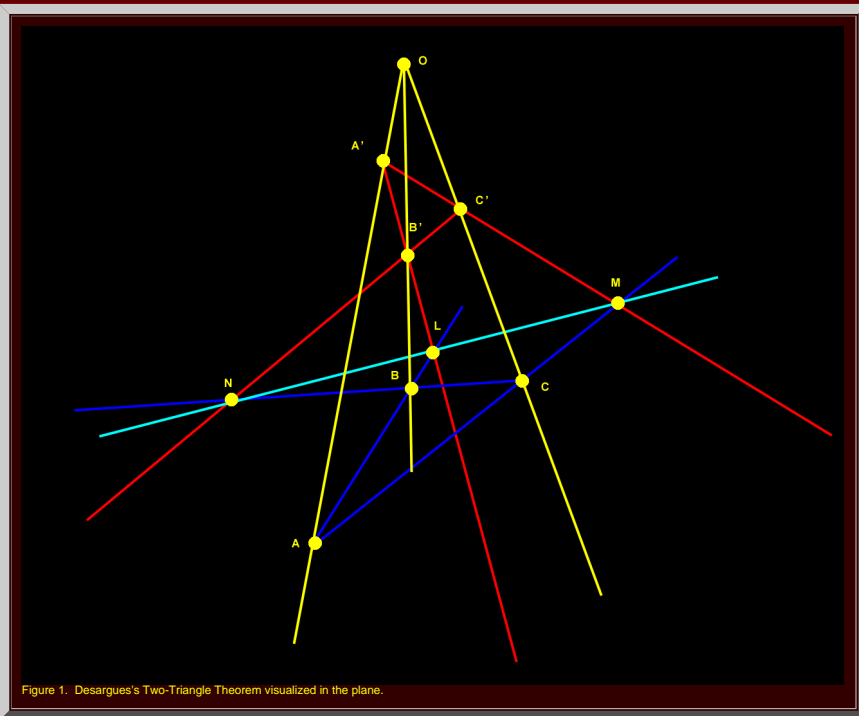
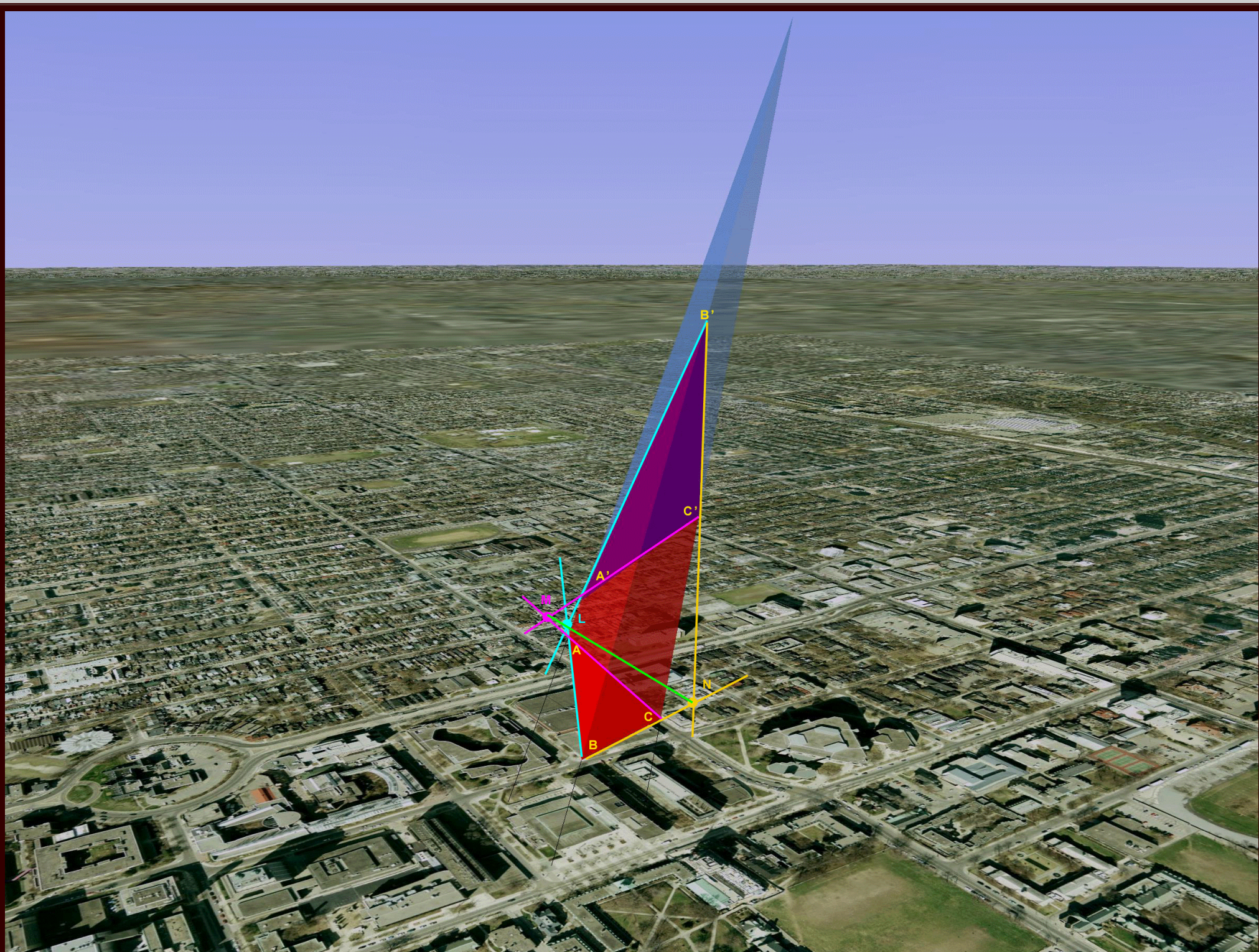


Figure 1. Desargues's Two-Triangle Theorem visualized in the plane.

Figure 2 shows an animated illustration of Desargues's Theorem in the plane, with background derived from Google Earth.



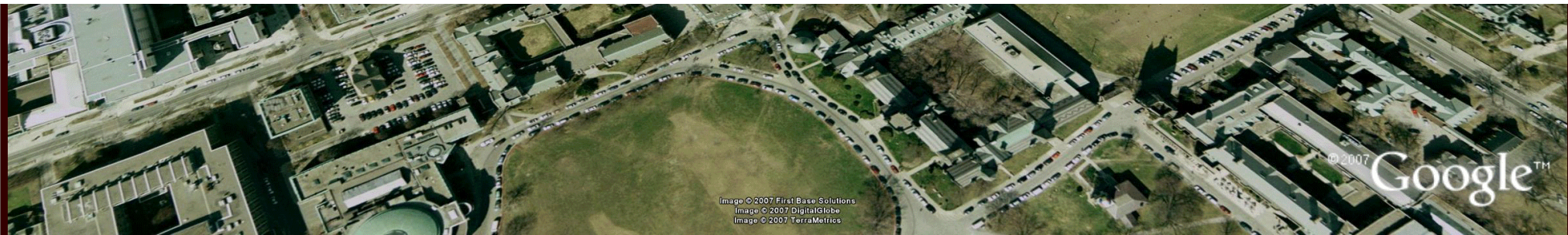


Figure 2. Desargues's Two-Triangle "Tower" centered on the Sidney Smith Building, home of the Mathematics Department of the University of Toronto and the academic home of Professor H. S. M. Coxeter for many years.

Figure 3 is a .kmz file which must be viewed in Google Earth. It permits the reader to fly around the Leaning "Tower" of Desargues and examine positions of triangle vertices. Download the kmz file [here](#).

To visualize the theorem in the plane while flying through the .kmz file, tilt the configuration so that it appears to flatten out in the plane. When triangle sides are "parallel" in the Euclidean sense the theorem still holds. The lines forming the sides intersect at points at infinity, which, in the projective plane, are no different from any others. The proof of this theorem is often presented in three, rather than in two, dimensions.

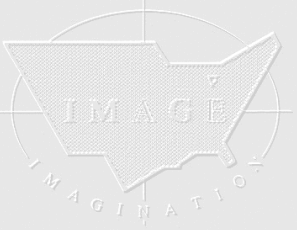
References:

Coxeter, H. S. M. 1961. *Introduction to Geometry*. New York: John Wiley & Sons. 252-254.
 Google Earth.
 Google SketchUp Pro, version 6.
 Moulton, F. R. "A simple non-Desarguesian plane geometry," *Transactions of the American Mathematical Society*, Vol. 3, No. 2 (Apr., 1902), pp. 192-195

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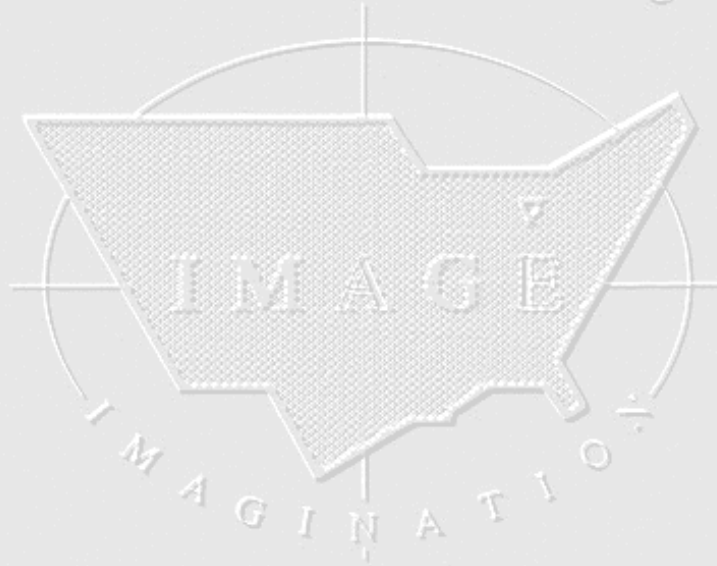
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