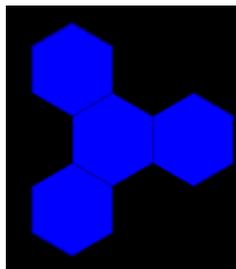


Spatial Synthesis: Volume I, Centrality and Hierarchy. Book 1.

Sandra Lach Arlinghaus and William C. Arlinghaus
University of Michigan and Lawrence Technological University

[Institute of Mathematical Geography](#), Ann Arbor, 2005

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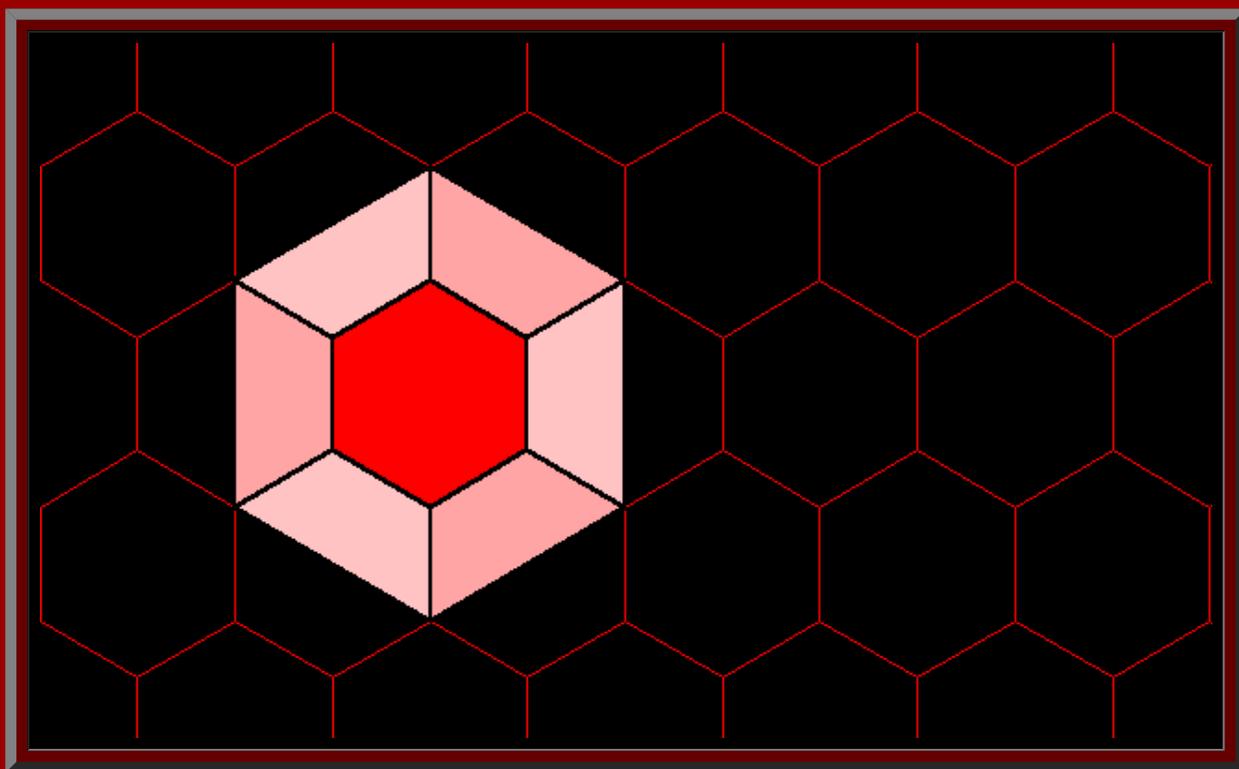
Spatial Synthesis: An eBook Series

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Volume I: Centrality and Hierarchy

Book 1: Regular Lattices, Geometry, and Number Theory.

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[Download Cosmo Player \(free\) Browser Plug-in for interactive virtual reality.](#)
[Link](#) showing more detail on how to use Cosmo Player.

Possible Future Volumes

Proposed topics for future volumes of Spatial Synthesis, as invited in

Solstice: An Electronic Journal of Geography and Mathematics,

Volume XIII, Number 2, Winter 2002

(<http://www.InstituteOfMathematicalGeography.org>).

Volume I; Centrality and Hierarchy (Books 2, 3, )

Volume II: Distance and Geodesic

Volume III: Adjacency and Connection

Volume IV: Minimax, Absolute/relative, and Density

Volume V: Scale, Orientation, and Dimension
Volume VI: Partition, Separation, and Diffusion
Volume VII: Transformation and Symmetry

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Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.
Sandra Lach Arlinghaus and William Charles Arlinghaus

ABOUT THE AUTHORS

(See their linked websites for more information.)

Sandra Lach Arlinghaus

- Ph.D. Theoretical Geography
- Pirelli INTERNETional Award Semi-Finalist (top 80) for submission in 2004, based on part of this book (appearing in *Solstice*).
- Pirelli INTERNETional Award Finalist (top 20 in the world of over 1200 invited entries), 2002 competition, for previous e-book, *Graph Theory and Geography: An Interactive View eBook*, John Wiley and Sons, 2002.
- Pirelli INTERNETional Award Semi-Finalist (top 80 in the world of over 1000 invited entries), 2001 competition, for *Solstice: An Electronic Journal of Geography and Mathematics*, Institute of Mathematical Geography, <http://www.InstituteOfMathematicalGeography.org> .
- Numerous publications, both classical and electronic, in a variety of disciplines (some are included in the attached e-Library).
- Biography appears in numerous Marquis publications, including *Who's Who in America* and *Who's Who in the World*.

William C. Arlinghaus

- Ph.D. Mathematics, Algebraic Graph Theory (book based on dissertation appears in the *Memoirs* Series of the American Mathematical Society)
- Pirelli INTERNETional Award Semi-Finalist (top 80) for submission in 2004, based on part of this book (appearing in *Solstice*).
- Pirelli INTERNETional Award Finalist (top 20) for previous e-book, *Graph Theory and Geography: An Interactive View eBook*, John Wiley and Sons, 2002.
- A variety of publications, mostly in graph theory.
- Biography appears in various biographical publications, including the ACBL *Encyclopedia of Bridge* and Marquis's *Who's Who in the Midwest*.

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Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.
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Acknowledgments

We live in exciting times and one of the reasons they are exciting is the great electronic resource called the Internet. It opens up remarkable possibilities involving "publication" in the broadest sense of the word: dissemination of information in creative, interesting, and even interactive ways. We have been exploring a few of the dimensions it has to offer in previous works and continue in this one, emphasizing the role of virtual reality in mathematical models. The Institute of Mathematical Geography (IMaGe) has been a leader in these efforts since 1985. Over the past 20 years, there have been more changes than we might ever have foreseen. The IMaGe website archives those efforts. Here, we note simply one previous work in a similar vein: a work by these two authors co-authored with the late Frank Harary. That work was entitled *Graph Theory and Geography: An Interactive View eBook*, published by John Wiley and Sons (Wiley-Interscience Series in Optimization and Discrete Mathematics) in April of 2003 as their first eBook publication. Professor Emeritus, Mitchell J. Rycus, Taubman College of Architecture and Urban Planning, The University of Michigan (Ann Arbor) notes of that one in a request for review sent by Wiley,

"I've had a chance to look at the Arlinghaus et al. book and I think it's just great! It is what an eBook is supposed to be--interactive, animated, easy to read and easily accessible, and of course, the context has to be as well written and as authoritative as any printed text. *Graph Theory [and Geography]* does indeed have all these elements, and more since it is both a math and geography text in one document. What a treat to have the opportunity to review it and (I hope) to keep it for reference for my work."

The current work draws partially from the success of the Wiley eBook as well as from the continuing experience with IMaGe and its electronic journal, *Solstice: An Electronic Journal of Geography and Mathematics* (founded, 1990). Articles appeared in *Solstice* as a means of testing book content. The comments below relate to these tests.

- Rycus:
"Wow! That was a good run--top 20 of *Pirelli* is certainly an honored position. I took a peek at your new work and it should have a better chance with the virtual machine and all. You guys are great!"
- Waldo Tobler, Professor Emeritus, Department of Geography, University of California, Santa Barbara, June 20, 2004.
"Sandra, This is a very impressive issue. I look forward to your and Bill's forthcoming book."
- Steve Quigley, Executive Editor, John Wiley and Sons, November 5, 2004.
"Sandra, ... you are the most forward thinking person about electronic delivery that I personally know. ... "

We hope that all readers of this document, and of those derivative ones to come, will also enjoy them and learn from them. This document would not have been possible without the kind help and wise counsel of numerous individuals. Klaus-Peter Beier, Adjunct Associate Professor of Engineering and Director of the 3D Laboratory of the Duderstadt Center, The University of Michigan, offered a marvelous course in Virtual Reality (Engineering 477) in which the first author had the privilege to participate in the Fall of 2003 and 2004. She was thus able, through Professor Beier's patience and fine lectures, to add virtual reality scenes to this book. The Graduate Student Instructors in the course offered clear instruction to help her to work through the laboratory exercises: Thana Chirapiwat (who had earlier been a student of hers in a GIS course), Bonnie Congxi Bao, and Jamie Cope. The staff of the 3D Laboratory of the Duderstadt Center was also of great help: Lars Schumann, Steffen Heise, Brett Lyons, Eric Maslowski, and Scott Hamm.

They also thank Merle Johnson of the City of Ann Arbor ITS Department and Chandra Hurd Gochanour of

the City of Ann Arbor Planning Department: the former for his generosity in sharing aerials and maps from City of Ann Arbor files and the latter for sharing her expertise in analyzing city data and in checking selected files related to material on this site. Donald T. Uchman, Coordinator of Space Graphics, Space Information and Planning, Plant Extension--AEC, The University of Michigan, kindly shared official University map files on locations and sizes of buildings.

This work has been in progress for over 15 years and is an ongoing venture. Numerous colleagues have been helpful, often in ways that might be considered indirect, but generally which feed into an important infrastructure in understanding on which we build our work. At the risk of missing some, we thank particularly, Frank Harary (1921-2005), John D. Nystuen, Kenneth H. Rosen, Matthew Naud, Steve Quigley, Susanne Steitz, Penelope Kaiserlian, Sylvia Hecimovich, Douglas Kelbaugh, Fred Beal, Susan Pollay, Karen Hart, Paul Mohai, Mitchell J. Rycus, Waldo R. Tobler, Michael Batty, Michael J. Woldenberg, John F. Kolars, Marvin Mikesell, Chauncy D. Harris (1914-2003), and Marco Farinelli.

The authors wish to thank members of their respective university communities who were supportive, in various ways, of this project. S. Arlinghaus thanks the Dean of the School of Natural Resources and Environment of The University of Michigan, Rosina Bierbaum, for her consistent appointment of this author to her annual post. This author hopes that a strong win-win situation is created when scholars publish materials in association with such appointments. W. Arlinghaus wrote the chapters on elementary number theory and on binary quadratic forms while on a semester sabbatical from the Mathematics Department at Lawrence Technological University. He thanks those who made that sabbatical possible, particularly his Department Chair, David Bindschadler. Also, both authors wish to thank Donald F. Lach (1917-2000), Bernadotte E. Schmitt Professor of History, University of Chicago, and the staff of the Joseph Regenstein Library of the University of Chicago who helped procure German maps and texts from 1941.

Errors that remain are, of course, ours alone.

Sandra Lach Arlinghaus
William Charles Arlinghaus

Ann Arbor, Michigan
June 21, 2005.

Software used in the course of making this book was made by the following firms:

- Environmental Systems Research Institute, ArcView and ArcGIS software
- MacroMedia, Dreamweaver, MX.
- Adobe, Photoshop and ImageReady, 7.0.
- Sun Microsystems
- Microsoft
- Alta 4 (Germany), ImageMapper
- Discreet, 3D Studio Max
- Animal Movement (Alaska Biological Station)

Hardware used centered on the equipment in the "Usability Lab" of the Duderstadt Center (with two flat screen monitors working together for a wide view) and on a Hewlett-Packard Pavilion f1903, with an Intel Pentium 4 Processor, 512 MB PC3200 DDR SDRAM memory, Intel Graphics Media Accelerator, 200GB hard drive and a 19 inch flat screen monitor set to highest resolution.

Acknowledgment

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Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.
Sandra Lach Arlinghaus and William Charles Arlinghaus

Dedication

To our Parents

Alma S. Lach
and to the memory of
Donald F. Lach

To the memories of:
Francis A. Arlinghaus
Blanche S. Arlinghaus

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Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.

Sandra Lach Arlinghaus and William Charles Arlinghaus

Kioskland: A Strategy for Linking Hierarchical Levels of Virtual Reality Maps*

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Adjunct Professor of Mathematical Geography and Population-Environment Dynamics
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with input from**:
Klaus-Peter Beier, Matthew Naud, John D. Nystuen
Taejung Kwon, Adrien Lazzaro, Paul Oppenheim, Aaron Rosenbaum (Fall 2003)
Nikolai Nolan, Rasika Ramesh, Itzhak Shani (Fall 2004)

Technical note:

The images linked to this article require the reader to install Cosmo Player or Cortona or another browser plug-in designed for viewing Virtual Reality files on the internet.

Use a recent version of Internet Explorer as the browser (or an early version of Netscape, series 4.7x) or other browser that supports virtual reality.

These images are also best viewed on a current computer with at least 512 MB of RAM (more is better!) with the screen resolution set as high as possible.

The electronic pages of this journal have, for the past few years, contained numerous examples of virtual reality maps: from planning for extra residential units in downtown [Ann Arbor](#) to tracking rugged voyages of [Lewis and Clark](#). Many maps cover large areas of terrain; they are global in scale. Virtual reality, however, often is best executed in the small, at a local scale (Beier (lectures in [Engineering 477](#), The University of Michigan), [Crispen](#)). The virtual maps tend to become large in file size quickly, causing the maps not to load properly. One problem is that software that easily creates virtual maps may not also optimize file size. That persistent problem of the virtual modeler can be partially addressed by importing Virtual Reality Modeling Language (vrmf) files exported from Geographic Information System software (GIS software) into a software package that executes polygon reduction of the vrmf code. Beyond such reduction, however, there remains the geographer's dilemma of scale transformation and a need to map both globally and locally. A [classical](#) way to execute such transformation is to arrange the spatial information in layers of a nested hierarchy and use a well-defined transformation to move from one level of that hierarchy to another. In the case of virtual reality maps (VR-maps), one puts individual maps in separate layers, separating maps before they become too heavy to run smoothly, or adequately, on current computing equipment (the creator, of course, needs to decide the target audience and the computing environment in which its members are likely to function).

Earlier work on creating a 3D Atlas of Ann Arbor has produced thousands of separate virtual maps of the downtown ([inventory](#) of previous work; click on the "Archive" button). Some are maps that show the current stock of buildings. Others are maps that suggest future buildout scenarios based on concepts provided by leading architectural and construction experts. The images below show screen capture of separate virtual reality models. The image on this offers a simple solution to link the different hyperlinked images, using kiosks that transform the user, via the internet, from one hierarchical level to another.

File Navigation Notes

- Click [here](#) to download a copy of Cosmo Player to use as a plug-in for your browser in order to view virtual reality files.
- The screen captures of virtual reality models in this article are taken from browsers with Cosmo Player installed.
- View the virtual reality models in your browser:
 - To use pre-defined viewpoints, and have controlled navigation of the model, pull down the "[viewpoints](#)" menu in the lower lefthand corner.
 - When using the driving controls to navigate the model freely, adjust driving speed using the "[preferences](#)" menu in the lower righthand corner.
- Other plug-ins for viewing virtual reality files in browsers are available for free download. Cortona is one such option. Go to Google, Yahoo, or other to search for them if the links on the page for Cosmo Player, above, fail to work.

This [linked](#) image shows a screen capture of an associated [virtual reality](#) model of downtown Ann Arbor with buildings represented as simple boxes extruded from digitized building footprints. Over several years, such images, of both actual and projected cityscape, served as one set of basic materials for planning efforts for city officials involved in various aspects of municipal planning. Visualizing alternative outcomes associated with different zoning types and land use patterns was helpful in understanding how decisions might be reached. Maps and decisions often go hand in hand.

These maps and box-models were useful in a general way: for looking at patterns of vacant land, possible infill sites, general skyline resulting from proposed infill, relation of topography to the built environment, and as a general inventory of the existing downtown scene by building height, zoning type, and special designation (such as historic district or floodplain designation or State-owned property, as are University of Michigan buildings). Many buildings, however, are not boxes, and all buildings have texture. In the municipal arena of

professional planning, supplementary architectural drawings and models are therefore critical (until the day arrives that virtual models are easy and inexpensive to make at all levels of detail). Thus, one wishes for virtual maps that show more detail: not only for planning purposes but for others as well. An important other use centers on [emergency response](#): of [firefighters](#) (using virtual training of firefighters in their downtime in the firehouse, for example), of [police officers](#), of [environmental management personnel](#), and of a whole host of [first responders](#) ([link](#) to the 3D Laboratory, Duderstadt Center, The University of Michigan).

The introduction of photographic textures on files causes rapid enlargement of file size. The map in this [linked](#) image shows a screen capture of one view of a [virtual reality](#) file in which textures have been applied to a number of buildings in the downtown area. The file is about at the limit of what a typical desktop modern machine running 512 MB of RAM can run in a reasonable (not optimal) time (anywhere from 1 to 5 minutes to load the model depending on various local conditions such as amount of free hard drive space, fragmentation of the hard drive, internet connection speed, age of machine, and so forth). In this file, there are textures only on selected buildings. There are no street or sidewalk textures and there is no street "furniture" nor are there any other related accessories such as traffic lights, lane markers, or cars.

The extra detail that comes at high cost in terms of file size is quite attractive for many applications. Another way to introduce extra detail is to put links on the virtual buildings. Thus, in the last [linked](#) virtual reality model, one can click on selected buildings to look inside: not at an image of the interior but to internet links to the interior via a firm's own webpage and so forth. Try [219 S. Main Street](#): look at the bottom of Cosmo Player to see building addresses as the mouse is moved across virtual buildings with active links to html pages. The corresponding html page that comes up when the building is clicked on in the virtual reality model is [linked](#) here. The background color of the html page is significant. All buildings in the same physical, four-sided, block have the same color background. Buildings in adjacent blocks (with adjacency defined as being "across the street from") have a different color.*** When environmental managers choose to insert, in the html pages, detail about environmental hazards known to be in particular buildings (for private municipal use only), then this model provides even more information valued by these managers, and other first responders, in times of disaster. One advantage to this approach using html files linked to virtual buildings is the small file size of the linked web page; another is that no special knowledge, beyond simple html, is required to maintain the model.

Finally, one may wish to introduce even more detail with street textures, benches, trees, and so forth. All of this detail quickly increases file size. Over the course of two years, student groups worked for three months to produce more detailed files. In the Fall of 2003, in Professor Klaus-Peter Beier's Engineering 477 course (The University of Michigan), with Faculty Advisors Arlinghaus, Naud, and Nystuen, students Kwon, Lazzaro, Oppenheim, and Rosenblum produced a detailed model of the intersection of Main and Liberty Streets that Beier edited substantially (look both at the [screen capture](#) and at the [virtual reality](#) model). In the Fall of 2004, again in Engineering 477, students Nolan, Ramesh, and Shani, produced a detailed model of Main and Huron Streets that Beier edited substantially (look both at the [screen capture](#) and at the [virtual reality](#) model). Both of these models were edited by the author in terms of cleaning of photographic images (removing the foreground where it remained), adjusting building shape and height to be correct and to be compatible with materials from the City of Ann Arbor and The University of Michigan, and adjusting building location so that the detailed models would fit correctly and seamlessly with the City of Ann Arbor files and with The University of Michigan files. These adjustments involved the use of ArcView GIS software from ESRI, Adobe PhotoShop, Windows notepad as a vrmf editor, and 3D Studio Max from Discreet. The adjustments were executed in the Usability Lab of the Duderstadt Center of The University of Michigan using computing equipment configured with a double flat-screen monitor for wide-angle viewing, and also on a home desktop with a large single flat screen monitor.

While each model has its own uses, an obvious desire is to be able to move from one to another without returning to a page and links: to move directly from one level of the hierarchy to another and stay within the virtual world. One way to execute such movement is to insert markers, "kiosks," in files to link from one level to another. Each virtual kiosk has an internet link on it: the virtual kiosk serves as a center of information based on the virtual matter of the internet much as its real-world kiosk counterpart serves as a center of information based on printed matter. Thus, the virtual kiosks, when loaded with correct information regarding Uniform Resource Locators (URLs), serve as well-defined transformations from one hierarchical level of virtual map to another. The table below shows how to use this single file. The rows represent layers in a hierarchy: moving across the rows is a lateral move within the same hierarchy or virtual image. The columns represent movement from one virtual image to another; in this case, using kiosks loaded with appropriate URLs to execute that movement.

Enter the Virtual World [here](#)

The links to images below show the path through that World

Entry to file.
The viewpoints menu suggests the lateral move (blue highlight).

[Image](#)

The direct link to this world is given in the table heading, above.

Lateral move to kiosk, at Main and Liberty, to next level. Click on the kiosk, in the virtual reality file, to go to next image, below.

[Image](#)

	<p>More detailed view of scene at Main and Liberty. Note the kiosk in the street. Click on it, in the virtual reality file, to go to the next image below, showing even more detail of a region more local than that shown here.</p> <p>Image Panorama Direct link to this world</p>	<p>The viewpoints menu suggests the lateral move (blue highlight).</p> <p>Image</p> <p>In the various views at this hierarchical level, the streets outlined in white lie within the Downtown Development Authority (DDA); those outlined in yellow do not.</p>	<p>More detailed view of scene at Main and Huron. Note the kiosk in the street. Click on it, in the virtual reality file, to go to the next image below, showing even more detail of a region more local than that shown here.</p> <p>Image Panorama</p>
	<p>Most detailed view of Main and Liberty; note the variety of viewpoints available. Try them. Click on the "artifacts" switch to see trees and other detail.</p> <p>Image Panorama: entry Image Panorama: intersection Direct link to this world</p>		<p>Most detailed view of Main and Huron; note the variety of viewpoints available. Try them.</p> <p>Image Panorama Direct link to this world</p>

Follow the path suggested in the table above to become familiar with navigation through the various layers of the model. Use viewpoints or drive freely within layers. One entry point to the virtual world offers access to a wide range of virtual files. An important aspect of such transformation is thus that one can have many files together in a single file no heavier to view than the single heaviest layer of the hierarchy. It would not have been possible, using merging techniques or some such equivalent strategy, to create the entire set as a single file. When kiosks (or some conceptually equivalent mechanism for jumping between hierarchical levels) are employed, it is easy to imagine emergency managers training first responders to fly through a small file of the entire city with chunky buildings in order to learn the general city layout. When each street corner or other strategic location in this general model is marked with a kiosk then with one click of the kiosk a detailed model pops up, while additional clicks on individual buildings in this detailed view reveal the interiors of buildings as html files. This sort of arrangement has been created for the City of Ann Arbor (and given to them). Creation of the model has taken several years and involved the use of a wide variety of resources, both human and digital. Maintenance, however, can be achieved using a simple text editor to alter the underlying code in either the html files or in the vrm files (part of the code for one building is [attached](#)). Thus, highly computer-literate City employees, armed with digital cameras and good computing resources can maintain the model and insert changes and additions to suit a variety of needs and interests: the process is in place. The arrangement of spatial information in a hierarchy is an efficient arrangement not only in the classical worlds of pen and paper but also in the contemporary world of digital models as well as in the practical implementation of such hierarchical arrangements in the real world.

*

The author thanks Merle Johnson of the City of Ann Arbor ITS Department and Chandra Hurd of the City of Ann Arbor Planning Department: the former for his generosity in sharing aeriels and maps from City of Ann Arbor files and the latter for sharing her expertise in analyzing city data and in checking selected files related to material on this site. She also thanks Donald T. Uchman, Coordinator of Space Graphics, Space Information and Planning, Plant Extension--AEC, The University of Michigan, for his willingness to share official University map files on locations and sizes of buildings.

Prof. Klaus-Peter Beier, Ph.D., Director of the 3D Laboratory of the Duderstadt Center of The University of Michigan has offered extraordinary insight and support on various phases of this project. The author is indebted to his graciousness in sharing computing as well as intellectual resources. She is also extremely appreciative of the assistance and wise advice from staff members in the 3D Laboratory of the Duderstadt Center, Scott Hamm, Steffan Heise, Brett Lyons, Eric Maslowski, and Lars Schumann, and also to Graduate Student Instructors in Engineering 477, Thana Chirapiwat, Jamie Cope, and Bonnie Congxi Bao.

The name "Kioskland" derives from a paper by that name by the author (1976, Department of Geography, The University of Michigan, unpublished).

** Matthew Naud, M. S. and M. P. P., is Environmental Coordinator for the City of Ann Arbor and a principal member of the Emergency Management team. John D. Nystuen, Ph.D. is Professor Emeritus of Geography and Urban Planning, The University of Michigan.

In the Fall of 2003, Taejung Kwon was a Ph.D. Candidate in the Taubman College of Architecture and Urban Planning, The University of Michigan, Adrien Lazzaro was a Master's degree Candidate in the School of Information, The University of Michigan, and Paul Oppenheim and Aaron Rosenbaum were undergraduates (Seniors) majoring

in computer science.

In the Fall of 2004, Nikolai Nolan was an undergraduate majoring in General Studies, Rasika Ramesh was a beginning graduate student in the School of Information, and Itzhak Shani was an undergraduate (Senior) majoring in Engineering.

***[The Four Color Theorem](#) is used in color assignment; thus, in a grid street pattern, two colors are both necessary and sufficient to distinguish all blocks (animated Four Color map [link](#)). No more than four colors are ever required (viewing the block geometry as part of a plane or a sphere). Thus, background color of the linked html page also gives subtle information as to location.

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