Chapter 1
Classical Central Place Hierarchies: A Real World View

Link to original file, with interactive mapplets:
CAUTION: This particular link to a file with Java applets may cause a browser crash depending on various configurations of hardware and software. No other link is known to have that possibility.

eHighlights of this Chapter
Animated central place hierarchies: global view and sequence of local views
Virtual reality representation of sample point hierarchies (requires Cosmo Player or Cortona)
The evidence of historical maps cast in the current technological setting

Circular regions surrounding lattice points

A scatter of points, spread evenly across the plane, may take on a variety of configurations. If one wishes to consider circular buffers around each point, then these buffers may overlap or be widely spaced. The space between buffers, in the background behind them, is interstitial space separating buffers. A natural issue to consider is to minimize interstitial space and provide some sort of maximal coverage of the plane by the buffers: to provide a "close packing" of the plane by circles.

Two close packings of the plane with circles of identical radius are shown in Figure 1.1: one with circles centered on a square lattice and the other with circles centered on a triangular lattice.

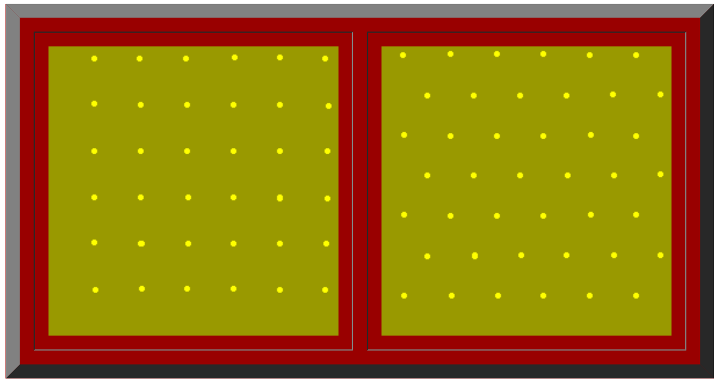
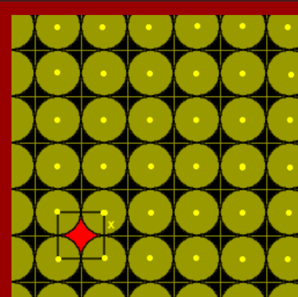


Figure 1.1 left, right. Animation shows square and triangular lattices in the plane and associated close packings with circles of identical radius. (Source of base image, no longer present at the cited url:http://www.psc.edu/~burkardt/src/graph_paper/graph_paper.html graph paper showing grid, triangular and hexagonal regions, John Burkardt, Pittsburgh Supercomputing Center, Carnegie Mellon University, University of Pittsburgh.).

It is a straightforward matter to calculate the interstitial space between circles associated with a square of side x in the square lattice and a corresponding parallelogram of side x in the triangular lattice. The radius of the circles in each figure is therefore x/2 and the area of the circles pi*x^2/4. Figure 1.2 shows a calculation of interstitial space: there is less interstitial area, between circles, within the parallelogram than there is within the square. Extrapolating this idea across the plane, one imagines naturally that the close packing of circles on the triangular lattice is denser than is the close packing of circles on the square lattice.

Side of length x, both square and parallelogram. Radius of any circle is x/2.	Area of any circle is pi*x^2/4
	Red Area = area of square with four one-quarter circular sectors subtracted out = area of square with one full circular area subtracted out. Area of square: x^2
	Red Area=area of parallelogram with two one-third circles and two one-sixth circular sectors

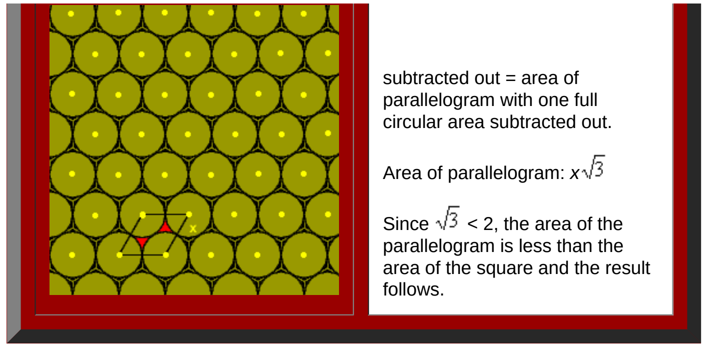


Figure 1.2. There is less interstitial area between circles within the parallelogram than there is between circles within the square. (Source of base image, no longer present at the cited url:http://www.psc.edu/~burkardt/src/graph_paper/graph_paper.html graph paper showing grid, triangular and hexagonal regions, John Burkardt, Pittsburgh Supercomputing Center, Carnegie Mellon University, University of Pittsburgh.).

This chain of reasoning indicates that the close packing of the plane by circles of identical radius is denser for a triangular lattice than it is for a square lattice. Gauss (1831) proved that, in fact, the densest lattice packing of the plane is the one based on the triangular lattice. In 1940, Fejes-Toth proved that that same packing is not only the densest lattice packing of the plane but is also the densest of all possible plane packings.

If one thinks, then, of the circles as if they were bubble-foam, the circles centered on a square grid pattern expand and collide to form a grid of squares (suggested in the top figure in Figure 1.2). The circles centered on a triangular grid pattern expand and collide to form a mesh of regular hexagons (suggested in the bottom figure in Figure 1.2). Because the close packing of circles centered on points spaced in a triangular grid is the densest possible close packing of the plane, the space-filling pattern of hexagonal cells associated with that packing also possesses unique character. If one were to enter a square or triangular grid into Geographic Information System software, and ask the software to calculate Thiessen polygons the grid of square cells would emerge from the square lattice and the grid of hexagonal cells would emerge from the triangular lattice. Thus, these cells can arise in a number of technical ways.

The theoretical issues surrounding tiling in the plane are complex; even deeper are those issues involving packings in three dimensional space. We confine our interest in that matter to simply illustrating Gauss's methods and stating Fejes-Toth's extension of it. The reader interested in probing this topic further is referred to the references. Interpretation of the simple triangular grid has range sufficient to fill this volume and more.

Basic assumptions

One classical interpretation of what dots on a lattice might represent is found in the geometry of "central place theory." Readers interested in the development of this theory, over time and with regard to a clear understanding of who did what and when it was done, are referred to the references. We simply present some of the ideas in order to focus on the geometry. Geometric central place models take the complex human process of urbanization and look at it in an abstract theoretical form in order to uncover principles which might endure despite changes over time, situation, cultural tradition, and all the various human elements that are truly the hallmarks of urbanization. Simplicity helps to reveal form: models are not representations of reality. They do, however, offer a way to look at some structural elements of complexity. Thus, consider dots on a triangular lattice as populated places (often, villages). Circles, expanding into hexagons, are areas that are tributary to the populated places. In the traditional formulation (described after Kolars and Nystuen) one considers four basic postulates (no one of which is "real" but each of which is simple):

- The backdrop of land supports uniform population density
- There is a maximum distance that residents can easily penetrate into the tributary area (often called "maximum range").
- There is slow, steady population growth
- Village residents who move, as a result of growth (or for other reasons), attempt to remain in close contact with their previous location (to maintain social or other networks).

Suppose, in a triangular lattice of villages, that one village adds to its retailing activities. After some time, growth occurs elsewhere. How might other villages compete to serve tributary areas: how will the larger, new villages share the tributary area? The answers lead to a surprising number of possible scenarios. Figure 1.3 shows the first in an infinite number of possibilities. Animated locations, for competing larger villages, are shown in Figure 1.3. The smallest villages are represented as small red dots; next nearest neighbors competing for intervening red dots are represented in blue; and, next nearest neighbors competing for intervening blue dots are represented in green. Of course, one is usually only willing to travel so far to go to a place only slightly larger, so the fact that the animated pattern could be extended to an infinite number of levels, beyond green, may not mirror the second postulate. Over time, however, one might suppose further growth and an entire hierarchy of populated places.

Figure 1.3. A triangular lattice of dots with animated locations for competing locations entering and vanishing from the picture.

Such a view of growth leads to another set of assumptions (after Christaller).

- The underlying market is spread evenly across an abstract geographic plane
- The underlying, discrete population is evenly spread
- There is a set of central place activities which reflects willingness to travel.

Within these added constraints, there are some embedded assumptions, as well. In particular, central place activities are viewed as themselves occupying no space: an important point in terms of retaining simplicity of form in order to look at structure. The problems of industrial location or of site plans for commercial activity are complex and important issues in themselves; however, they serve to obfuscate what is under consideration here. Hence, these complex activities are assumed to occupy mere points. Further, there is embedded the idea of a natural hierarchy of central place activities: that one might be willing to travel to get to a strip mall 5 miles away for routine shopping but for greater activities and service would be willing to travel 30 miles to a regional mall that offers all the goods and services of the smaller mall plus additional ones. These ideas are further elaborated in the following four points.

- The population of each location is a function of the number of goods and services it offers.
- The central place system is a closed system
- Each central place offers all goods and services offered by lower order places plus added ones: the number of central places in the system is therefore, in some sense, optimal
- Communication within the central place system is perfect.

With these postulates and added constraints in mind, we examine the geometric configurations that are classically identified as central place hierarchies. These are Walter Christaller's view in looking for pattern from the broad to the particular; in interpreting these ideas, however, we present the characterization of August Lösch, in working from the small to the large, as a synthesis of conceptual material.

Marketing principle: *K*=3

Consider a central place point, *A*, in a triangular lattice (Figure 1.4). Unit hexagons (fundamental cells) surround each of the points in the lattice and represent the small tributary area of each village (red hexagons in Figure 1.5a). Growth at *A* has distinguished it from other villages in the system. It will now serve a tributary area larger than will the unit hexagon. There are six villages directly adjacent to *A*. The unit hexagons represent a partition of area based on even sharing of area between *A* and these six villages. When *A* expands its central place activities, others may also desire to do so as well. Figure 1.4 shows the locations for the next nearest competitors, such as A' in blue, to enter the system. Given that they, too, will share area evenly, a set of larger hexagons emerges (blue hexagons in Figure 1.5a). The competitors that enter are spaced at a distance, in terms of lattice points spaced one unit apart, of $\sqrt{3}$ units (Figure 1.4). The position of the competitors that enter the system in this scenario are as close as possible to *A*; expansion of goods and services at any of the six closest neighbors would constitute no change in basic pattern. One might imagine, therefore, that emphasis on distance minimization optimizes marketing capability--distance to market is at a minimum. Figure 1.5a shows the unit hexagons and the larger hexagons based on expansion of goods and services. To get a view of a large scatter of dots, arranged according to this idea, consider the linked virtual reality model. Use the Viewpoints to see an overview and a closeup; drive through it on your own.

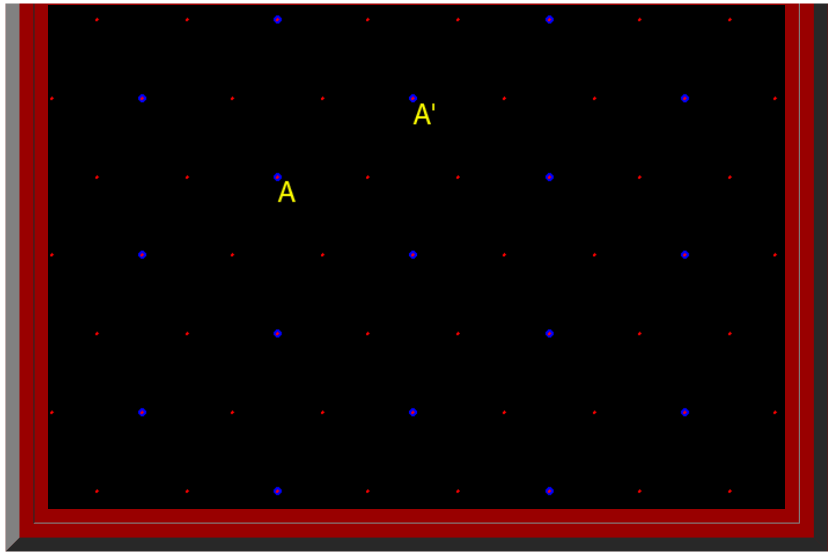


Figure 1.4. *K*=3: Marketing. Distance measurement between adjacent competing new centers, *A* and *A'*: in this case, competing centers, blue dots, are spaced $\sqrt{3}$ units apart, assuming a distance of one unit between adjacent red dots.

Thus, when competitors are chosen in this manner, the pattern of one layer of hexagons, in relation to another, has become known as a hierarchy arranged according to a "marketing principle" (Figure 1.5a). Notationally, it is captured by the square of the distance between competing centers: as a "*K*=3" hierarchy (Figure 1.4). Each large hexagon contains the equivalent of three smaller hexagons. One large hexagon = 1 small hexagon + six copies of 1/3 of a small hexagon = 3 small hexagons (Figure 1.5b, c, and d). Thus, the value *K*=3 is not only related to distance between competing centers but also to size of tributary areas generated by competition: as a constant of the hierarchy.

Figure 1.5a. *K*=3 hierarchy showing three layers of a nested hierarchy of hexagons of various sizes oriented with respect to one another according to the distance principle illustrated in Figure 1.4.

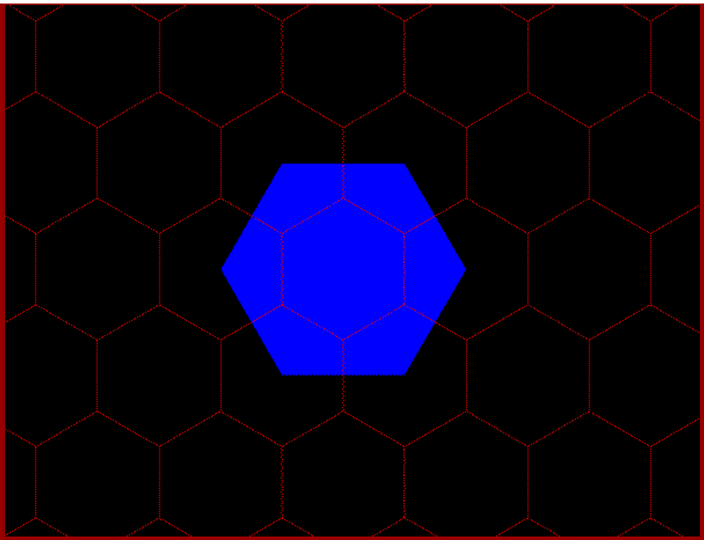


Figure 1.5b. Each blue hexagon contains the equivalent of three red hexagons: one entire red hexagon surrounded by six copies of 1/3 of a red hexagon.

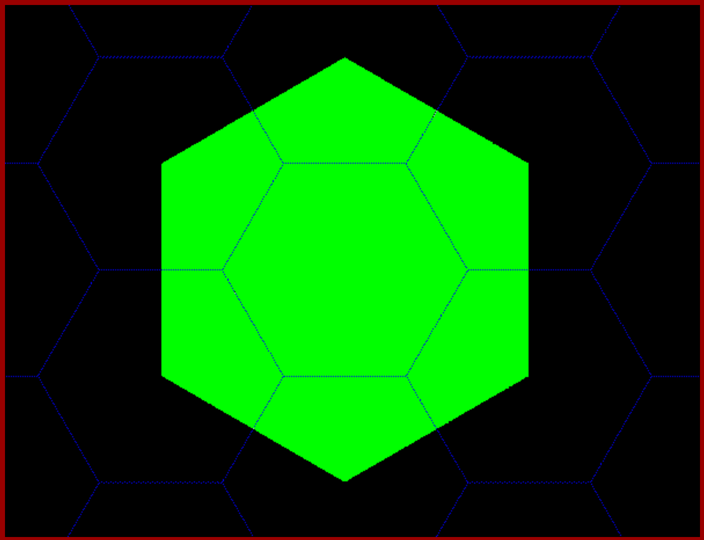


Figure 1.5c. Each green hexagon contains the equivalent of three blue hexagons: one entire blue hexagon surrounded by six copies of 1/3 of a blue hexagon.

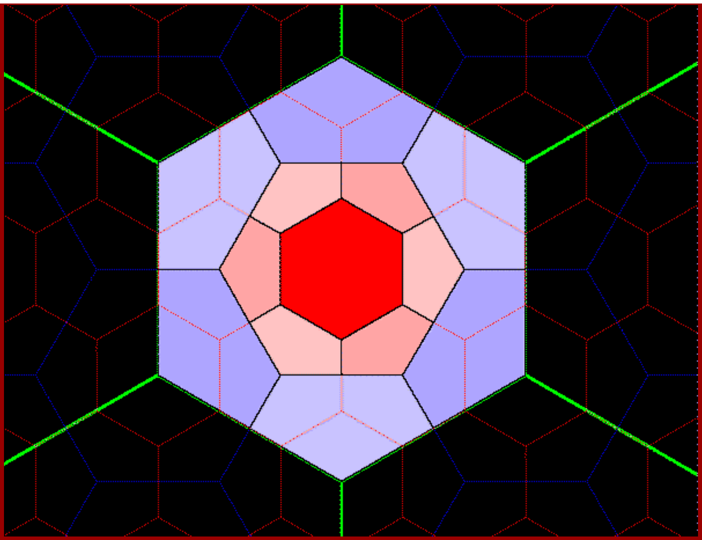


Figure 1.5d. The green hexagons contain the equivalent of 3^1 blue hexagons and 3^2 red hexagons.

Figure 1.5.
Back to Chapter 2 *K*=3 reference.

Transportation principle: *K*=4

Figure 1.6 shows the locations for the next nearest competitors, next beyond those from *K*=3, to enter the system. Given that they, too, will share area evenly, a set of even larger hexagons emerges. Figure 1.7a shows the unit hexagons and the larger hexagons based on expansion of goods and services. The competitors that enter are spaced at a distance, in terms of lattice points spaced one unit apart, of 2 units (Figure 1.6). The position of the competitors that enter the system in this scenario lie along radials that fan outward from *A* and pass along existing boundaries to tributary areas. One might imagine, therefore, that emphasis on market penetration, or transportation, is the focus here. To get a view of a large scatter of dots, arranged according to this idea, consider the linked virtual reality model. Use the Viewpoints to see an overview and a closeup; drive through it on your own.

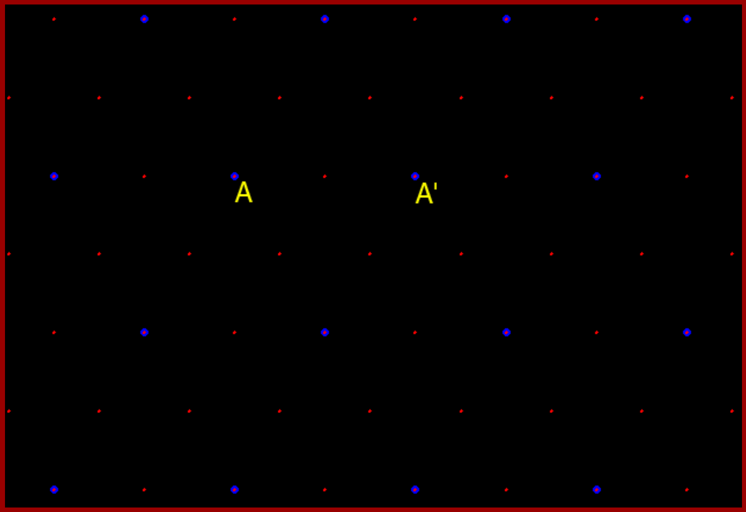


Figure 1.6. *K*=4: Marketing. Distance measurement between adjacent competing new centers, *A* and *A'* is 2 units, in this case (assuming a distance of 1 unit between adjacent red dots).

Thus, when competitors are chosen in this manner, the pattern of one layer of hexagons, in relation to another, has become known as a hierarchy arranged according to a "transportation principle" (Figure 1.7a). Notationally, it is captured by the square of the distance between competing centers: as a "K=4" hierarchy (Figure 1.6). Each large hexagon contains the equivalent of four smaller hexagons. One large hexagon = 1 small hexagon + six copies of 1/2 of a small hexagon = 4 small hexagons (Figure 1.7b, c, d). Thus, the value K=4 is not only related to distance between competing centers but also to size of tributary areas generated by competition--as a constant of the hierarchy.

Figure 1.7a. K= 4 hierarchy showing three layers of a nested hierarchy of hexagons of various sizes oriented with respect to one another according to the distance principle illustrated in Figure 1.6.

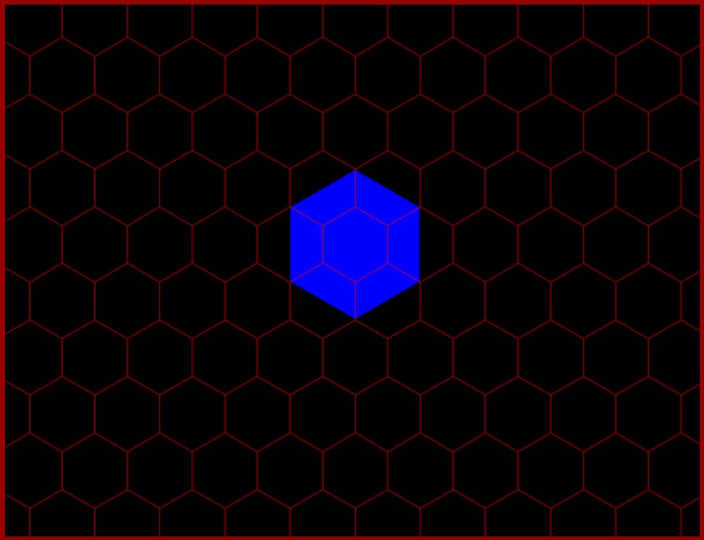


Figure 1.7b. Each blue hexagon contains the equivalent of four red hexagons: one entire red hexagon surrounded by six copies of 1/2 of a red hexagon.

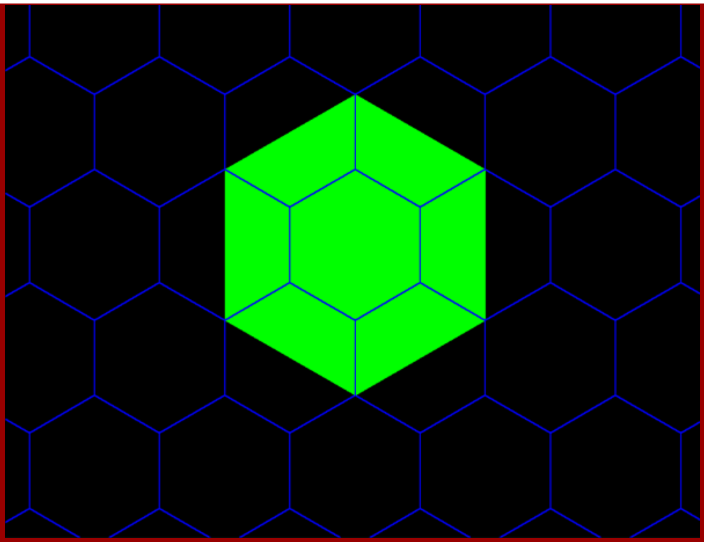


Figure 1.7c. Each green hexagon contains the equivalent of four blue hexagons: one entire blue hexagon surrounded by six copies of 1/2 of a blue hexagon.

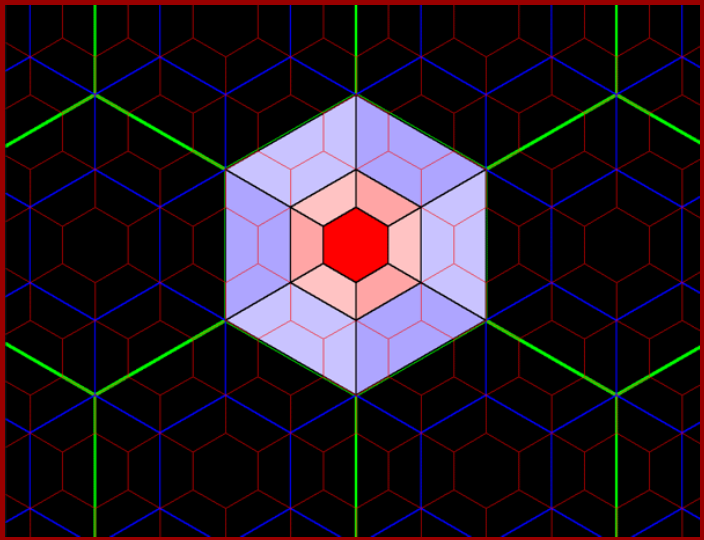


Figure 1.7d. The green hexagons contain the equivalent of 4^1 blue hexagons and 4^2 red hexagons.

Figure 1.7.

Back to Chapter 2 *K*=4 reference.

Administrative principle: *K*=7

Figure 1.8 shows the locations for the next nearest competitors, next beyond those from *K*=4, to enter the system. Given that they, too, will share area evenly, a set of even larger hexagons emerges. Figure 1.9a shows the unit hexagons and the larger hexagons based on expansion of goods and services. The competitors that enter are spaced at a distance, in terms of lattice points spaced one unit apart, of $\sqrt{7}$ units (Figure 1.8). The position of the competitors that enter the system in this scenario create larger hexagons whose boundaries pass through very few other populated places: hence, top-down control, or rule from the center is emphasized. One might imagine, therefore, an emphasis on administrative control here. To get a view of a large scatter of dots, arranged according to this idea, consider the linked virtual reality model. Use the Viewpoints to see an overview and a closeup; drive through it on your own.

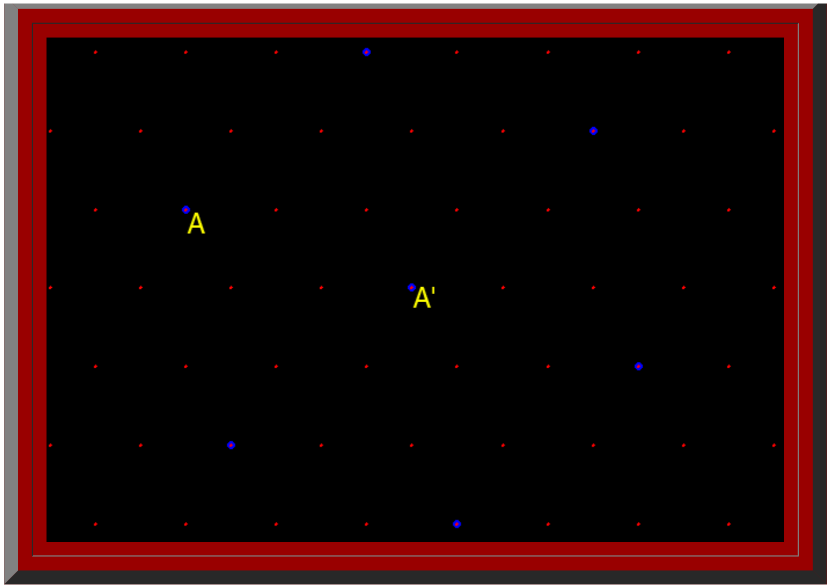


Figure 1.8. *K*=7: Marketing. Distance measurement between adjacent competing new centers, *A* and *A'* is $\sqrt{7}$ (assuming a distance of 1 unit between adjacent red dots).

Thus, when competitors are chosen in this manner, the pattern of one layer of hexagons, in relation to another, has become known as a hierarchy arranged according to an "administration principle" (Figure 1.9a). Notationally, it is captured by the square of the distance between competing centers: as a "*K*=7" hierarchy (Figure 1.8). Each large hexagon contains the equivalent of seven smaller hexagons. One large hexagon = 1 small hexagon + six copies of a small hexagon (underfit and overfit regions balance) = 7 small hexagons (Figure 1.9b, c, d). Thus, the value *K*=7 is not only related to distance between competing centers but also to size of tributary areas generated by competition--as a constant of the hierarchy.

Figure 1.9a. *K*= 7 hierarchy showing three layers of a nested hierarchy of hexagons of various sizes oriented with respect to one another according to the distance principle illustrated in Figure 1.8.

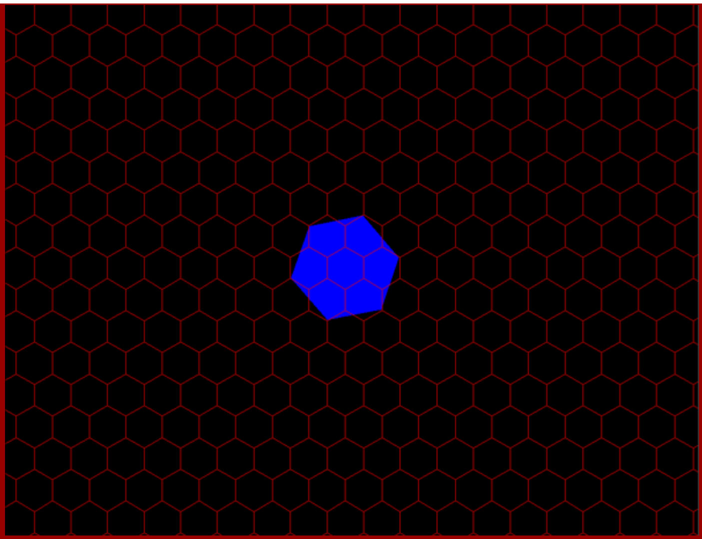


Figure 1.9b. Each blue hexagon contains the equivalent of seven red hexagons: one entire red hexagon surrounded by six copies equivalent to a single red hexagon. Each of the perimeter red hexagons is composed of 11/12 of a single red hexagonal cell plus 1/12 of an adjacent red cell: in an underfit/overfit pattern.

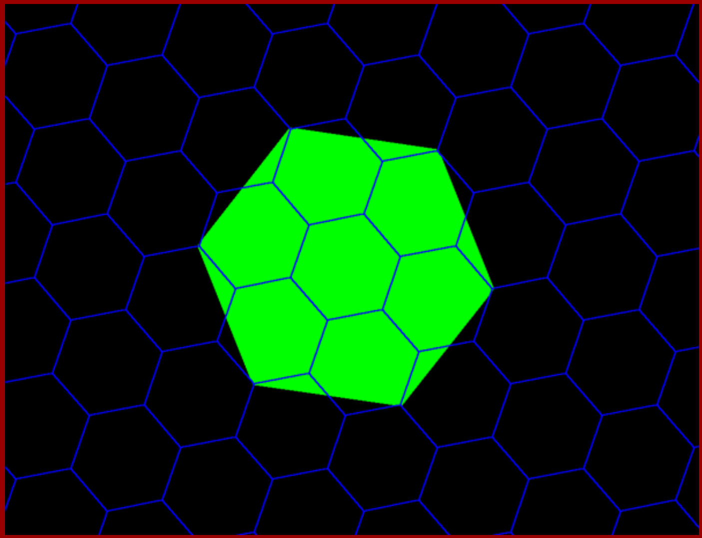


Figure 1.9c. Each green hexagon contains the equivalent of seven blue hexagons: one entire blue hexagon surrounded by six copies equivalent to a single blue hexagon. Each of the perimeter blue hexagons is composed of 11/12 of a single blue hexagonal cell plus 1/12 of an adjacent blue cell: in an underfit/overfit pattern.

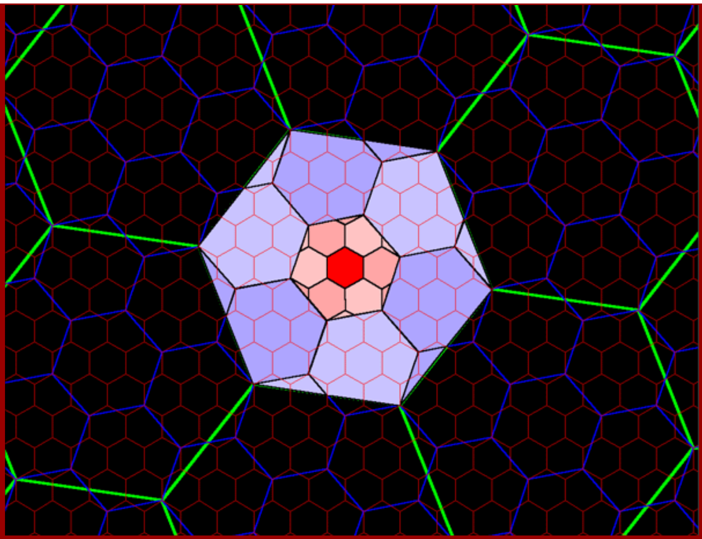


Figure 1.9d. The green hexagons contain the equivalent of 7^1 blue hexagons and 7^2 red hexagons.

Figure 1.9.
Back to Chapter 2 *K*=7 reference.

A tiled view of each hierarchy: mechanics of construction
The process of creating larger hexagons, as larger tributary areas representing expanded central place activities, can be carried out indefinitely. The set of figures above (1.5, 1.7, and 1.9) shows the general patterns that emerge and underscores, particularly, the importance of the constant of the hierarchy. Large hexagons in one layer contain the equivalent of K^1 hexagons of the next smallest size within them; they contain the equivalent of K^2 hexagons from the level two layers down in the hierarchy, and so forth. The *K* value is an invariant of each geometric hierarchy that uniquely characterizes it. The mathematical search for invariants as bench marks against which to view abstract structure is equivalent to the geographical search for bench marks in the field (physical or human) against which to view mapped, spatial structure.

Virtual reality central place skylines

The initial conditions from the real world may suggest building a hierarchy from small hamlets to large cities, in the style of Christaller, or they may suggest the reverse, in the style of Lösch (or any of an infinite number of possibilities in between--we confine ourselves to the extremes). Independent of such choice, however, the resulting geometry is identical: a pattern of hexagons of various sizes, dependent on spacing between competing centers at a given hierarchical level, is the result. To see the different initial patterns of dots, and to move them around on the screen, it is useful to capture the dot patterns above as virtual reality scenes. Animation of successive virtual reality scenes, as a sequence of screen captures of the scene from various vantage points, offers the reader a chance to examine closely the three different styles of initial dot scatter used to create the geometries of hexagons associated with *K*=3, *K*=4, and *K*=7.

The linked figures, Figures 1.L1 (derived from Figure 1.5), 1.L2 (derived from Figure 1.7), and 1.L3 (derived from Figure 1.9), offer a view of central place *K*=3, *K*=4, and *K*=7 landscapes as central place skylines in virtual reality. Links to each of these were given above, as they developed. Here, we simply accumulate the set in one set of links near each other for ease in making comparisons. Load CosmoPlayer and click on the links to drive through these landscapes and experience them as if the higher order central places represent taller urban structures, such as cell towers or other dominant skyline features. Figure 1.10 shows each of these virtual reality models, from the vantage point of "entry," as animated screen capture so that one might readily compare the three hierarchies. Figures 1.11 and 1.12 show similar animations, for purposes of comparison of pattern among *K*=3, *K*=4, and *K*=7, from the vantage point of "Overview" and "Close Up" respectively. Movies of these animations permit the reader to control the animation frame rate: Movie of 1.10, movie of 1.11, and movie of 1.12. (If the horizontal scroll bar does not show at the bottom of the linked image, expand your browser to show the full screen: in Microsoft Internet Explorer, for example, hit the F11 key.) In the next chapter, we use virtual reality to take a closer look at the geometry of these hierarchies; thus, it is worth the effort now to load the plug-in and to practice virtual navigation in a simple landscape.

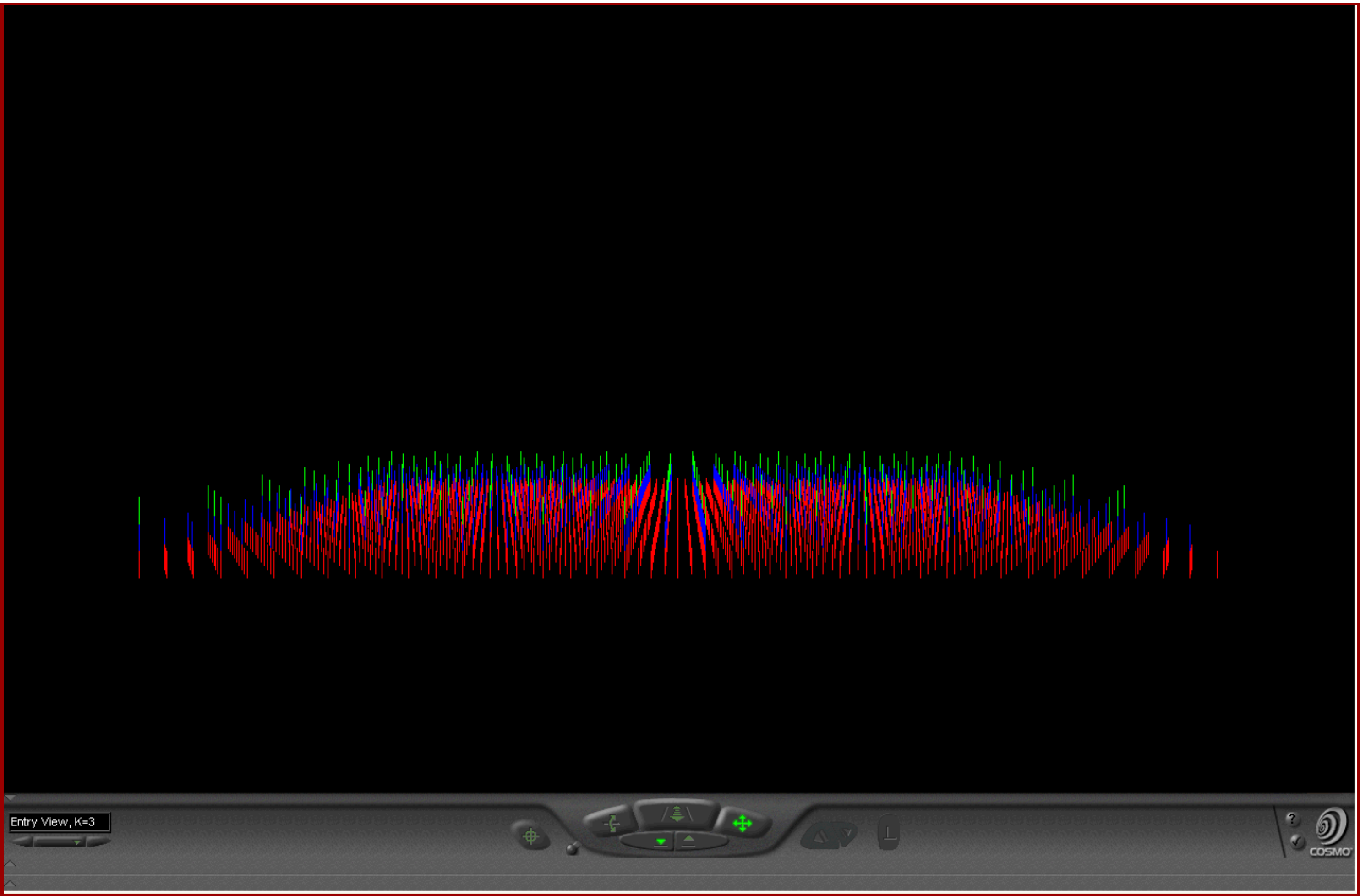


Figure 1.10. *K*=3, *K*=4, and *K*=7 hierarchies animated as viewed from the "entry" viewpoint of associated virtual reality models.

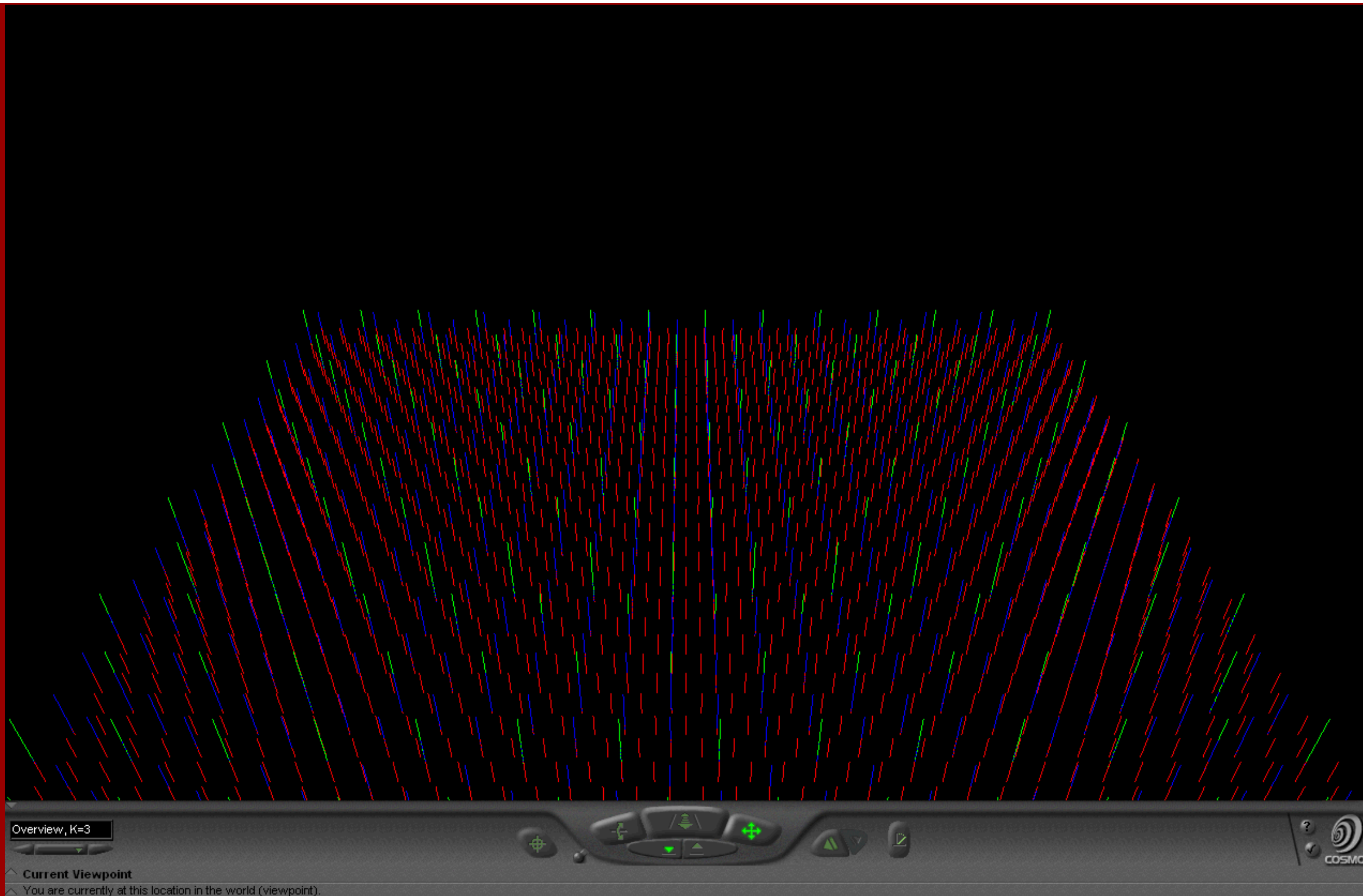


Figure 1.11. K=3, K=4, and K=7 hierarchies animated as viewed from the "overview" viewpoint of associated virtual reality models.

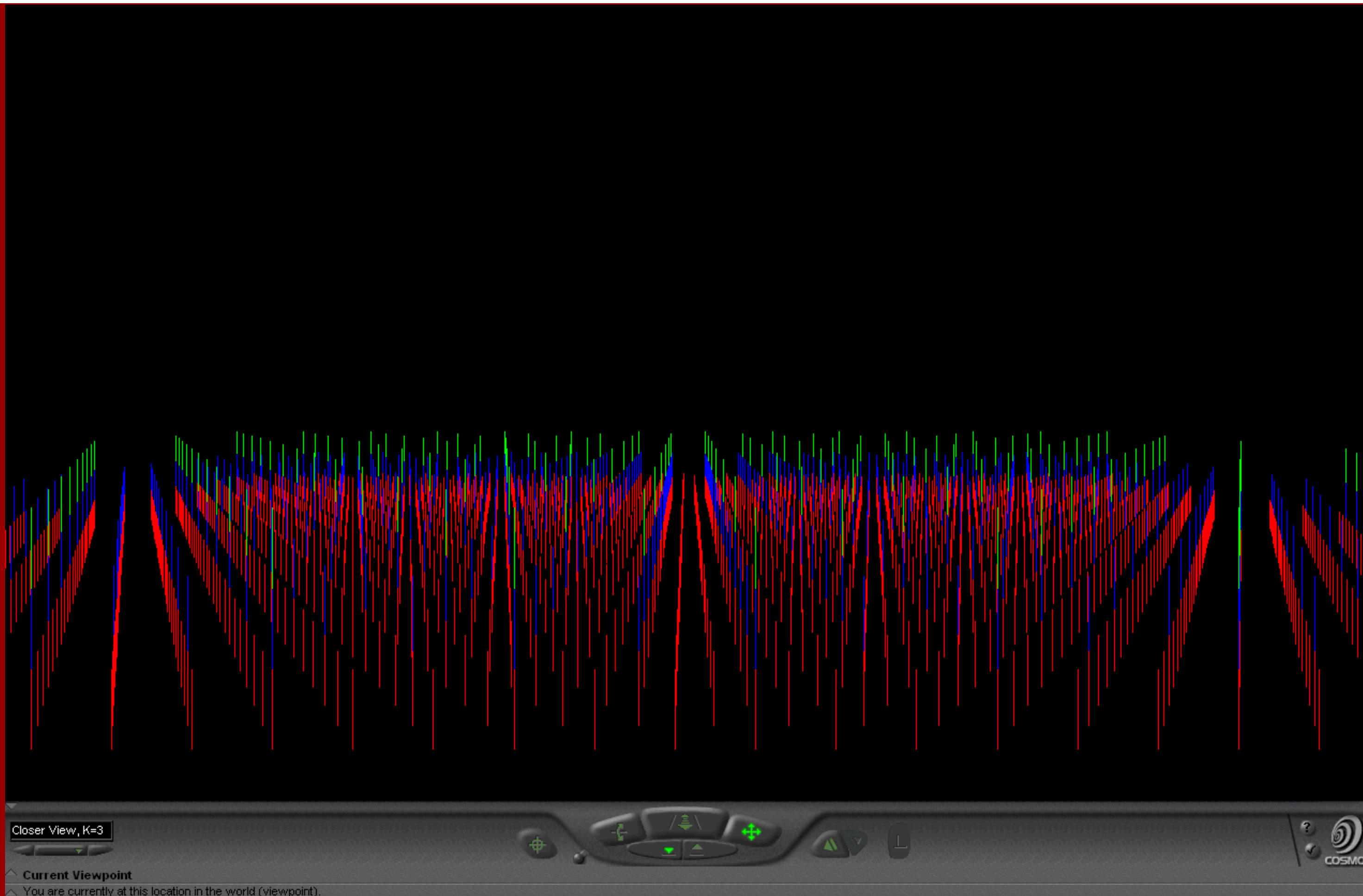


Figure 1.12. *K*=3, *K*=4, and *K*=7 hierarchies animated as viewed from the "close up" viewpoint of associated virtual reality models.

Mapplets

Another method, that is available only in current technology, for looking simultaneously at connection patterns between multiple layers of central place networks involves capturing them as Java (TM) Applets: as "Mapplets." This process also suggests some sort of stability of the geometric structure: the mapplet reflects stability in structural form. Shake the mapplet; it settles down to the expected hierarchy. Pull the mapplet; still it settles down to the underlying hierarchical form. One might speculate about real-world forces that pull on mathematical form: the form endures.

Figure 1.13. *K*=3 Mapplet

Figure 1.14.*K*=4 Mapplet

Figure 1.15. *K*=7 Mapplet

Mapplets focus on connection patterns between successive hierarchical layers and, when *K* values are loaded as distances between hierarchies, they also suggest some elusive form of structural stability of geometric form. Animated maps of the central place geometry of the plane, coupled with mapplets showing animated hierarchical pattern alone, suggest another sort of three dimensional view of central place geometry.

Central place hierarchies as Thiessen polygons

One difficulty with considering central place landscapes rests in their visualization. Slight errors in placement of points get magnified in overlay alignments. To create a meshed hierarchy in which overlays are aligned is a drafting task of substantial proportions, when done by hand. Geographic Information System software, however, offers an easy and accurate method of constructing central place landscapes at almost any level of complexity (up to the limits of hardware and software capability). Figures 1.4-1.9 were created using ArcView GIS (v. 3.2, ESRI). The method for creating GIS-generated central place landscapes employed the following steps:

- obtain as a base map a triangular lattice shape file; such a file may be created in ArcView using EdTools extension to precisely translate a point.

ensure that each record in the underlying database has a unique code entered in "number" format (using the "add record number" feature of Animal Movement extension, if need be).

- if desired, create in a separate layer, a bounded region to serve as limits within which to calculate the landscape--a rectangle, for example. One way to create such a region is to calculate the minimum convex polygon (convex hull) of the distribution of red dots using Home Range extension.
- load Spatial Analyst extension (ESRI) to ArcView and calculate Thiessen polygons using the Analysis|Assign Proximity command; choose the rectangle layer as the region within which to calculate the Thiessen polygons. Alternately, employ the same strategy using Home Range extension and calculate Dirichlet regions.
- The result will appear as a set of small hexagons surrounding the dots, as in the red layers in Figures 1.4-1.9.
- Repeat the procedure on other triangular lattices, with broader spacing of lattice points as in the blue and green points above, derived from the base lattice. The result will produce landscapes such as those in Figures 1.4-1.9 depending on how the broader spacing pattern is selected.

This strategy is simple to execute and straightforward to discover. Its importance, however, lies in its ease of use and its accuracy. Large environments can be created almost as easily as small ones and historical central place studies can be captured electronically. Indeed, central place studies done by hand can be checked and planning documents from the past can be recast as central place geometries.

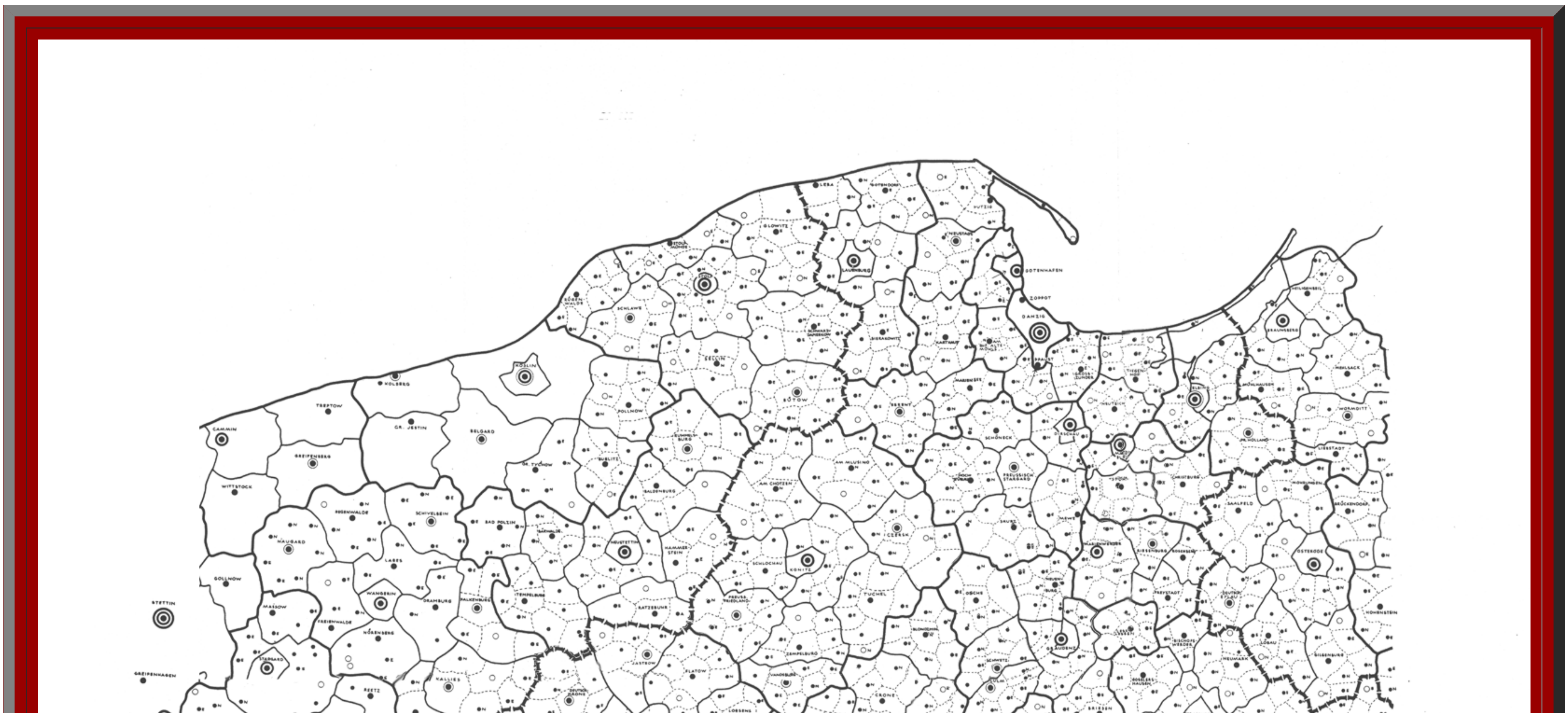
Christaller and settlements in Central Europe

The map in Figure 1.16 shows Walter Christaller's sense of regional settlement planning in Eastern Europe. Harris and Berry, note in "Walter Christaller: An Appreciation," that central place theory served as the basis for settlement planning in Eastern Europe:

> "...Christaller's involvement in the tumultuous social movements that affected Germany profoundly influenced his life. Like many young Germans, he became aligned with left-wing causes during the 1920s, and later his work was used as the basis of Nazi settlement planning in Eastern Europe. As a result this gentle and kindly man was shunned politically by both East and West after World War II, even as he was being accorded belated recognition for his pioneering contributions to theoretical geography."

The first author contacted Harris directly, in writing and on the telephone, to find out more about the source of that statement. The linked letter shows one part of his fascinating answer. He also offers citations to appropriate documents to read. The map in Figure 1.16 is based directly on material from one of these sources.

In it, larger cities are spaced farther apart than are smaller ones. Towns and villages fill interstitial space. Surely it must have taken Christaller much time and effort to construct this map. We have scanned it from a map in the 1941 reference and pieced together the scanned images. The scanning strategy was a multistep process because the map was quite large. First, 8.5 by 14 inch photocopies were made of the map in the book. It took 8 of those to capture the map, with overlap. Then, each piece was scanned at 600dpi on a flatbed scanner. The goal was to reassemble the scanned pieces on a canvas in Adobe Photoshop. File size of the 8 pieces prohibited such strategy and caused file oversize errors. Thus, each piece was reduced in size from 6900 pixels wide to 2000 pixels wide (preserving the aspect ratio). Then, each of these smaller images (smaller in physical size but not in resolution) was saved as a transparent .gif. The images with transparent backgrounds were then brought into Adobe Photoshop and assembled as a set of layers; use of transparency was important because it was only then that we could see how to fit the pieces together. To see an even larger version of this map (on which place names can be read easily), click here; the attached file is over 5 MB in size: scroll both horizontally and vertically to see the linked map.



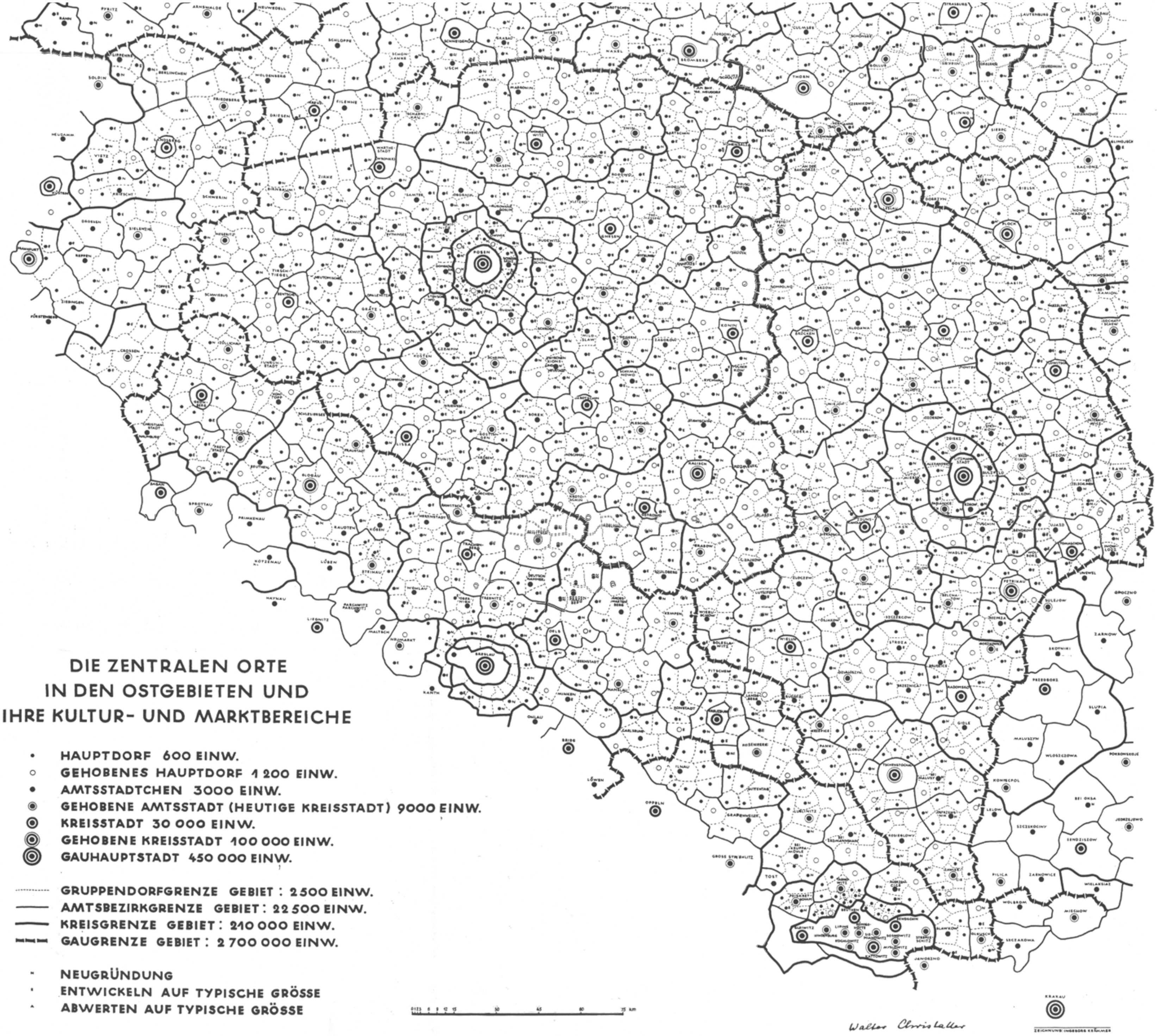

DIE ZENTRALEN ORTE
IN DEN OSTGEBIETEN UND
IHRE KULTUR- UND MARKTBEREICHE
HAUPTDORF 600 EINW.
GEHOBENES HAUPTDORF 1 200 EINW.
AMTSSTADTCHEN 3000 EINW.
GEHOBENE AMTSSTADT (HEUTIGE KREISSTADT) 9000 EINW.
KREISSTADT 30 000 EINW.
GEHOBENE KREISSTADT 100 000 EINW.
GAUHAUPTSTADT 450 000 EINW.
GRUPPENDORFGRENZE GEBIET : 2 500 EINW.
AMTSBEZIRKGRENZE GEBIET : 22 500 EINW.
KREISGRENZE GEBIET : 210 000 EINW.
GAUGRENZE GEBIET : 2 700 000 EINW.
NEUGRÜNDUNG
ENTWICKELN AUF TYPISCHE GRÖSSE
ABWERTEN AUF TYPISCHE GRÖSSE
Walter Christaller

Figure 1.16. Walter Christaller's view of central places in Eastern Germany, 1941. Click on the map above to see a larger image.
(Source: Struktur und Gestaltung der Zentralen Orte des Deutschen Ostens, Gemeinschaftswerk im Auftrage der Reichsarbeitsgemeinschaft fur Raumforschung, Teil 1, Dr. Walter Christaller, Die Zentralen Orte in den Ostgebieten und ihre Kultur- und Marktbereiche, K. F. Koehler Verlag, Leipzig, 1941).

Digitize the original map.

One question that arises is, does the map drawn by Christaller, showing a plan for settlement in Eastern Europe, actually conform to a central place theory pattern as might be generated today? Preston (1992) notes that Christaller used an administrative hierarchy, as well as a mixed hierarchy, in creating plans for the east, mixing marketing, transportation, and administration/separation principles. A map of the sort above is complex. Current technology permits creation of geometric consideration of historical evidence. It is a straightforward manner to digitize the dots set down by Christaller in different layers in a GIS according to assigned hierarchical level. Use of Thiessen polygons to create pattern is also simple to execute. Thus, when the image of Figure 1.16 was brought into Geographic Information System software, we separated the layers of dots as Christaller had suggested in the legend. The sizes of the colored dots in Figure 1.17 show an animated view of the seven levels of urban areas entering the system: from largest to smallest.

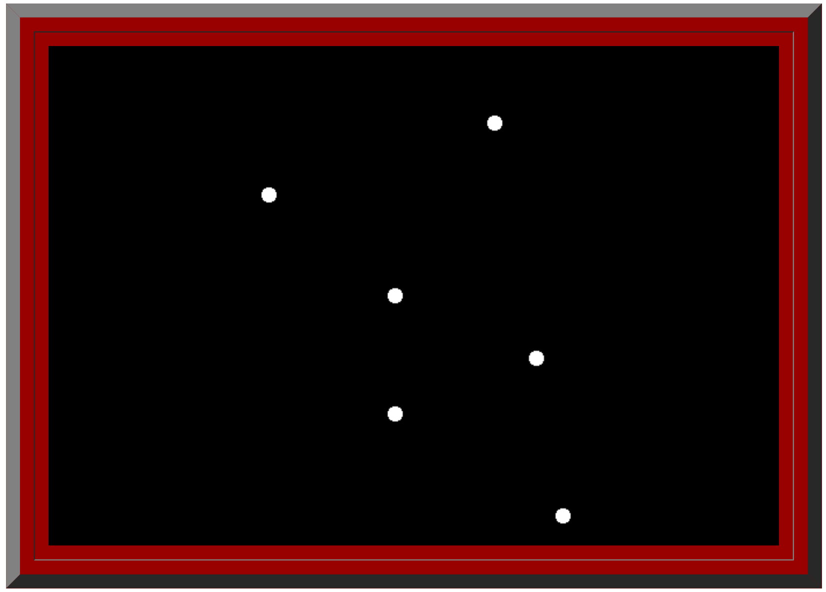


Figure 1.17. The central places of Figure 1.16 entered into a Geographic Information System.

Both the legend of Figure 1.16 and the animated map of Figure 1.17 suggest that all layers in this hierarchy are similar in nature. A closer look, however, at Figure 1.16 might see the first five layers as those with central place activities and see layers six and seven (the most rural) as working together to form a single, final layer that fills in between central places. No sixth or seventh layer places have links to subordinate places. Figure 1.18 shows those two most rural layers represented as a single layer as a scatter of yellow dots. These yellow dots are then analyzed using Thiessen polygons to create a cellular mesh that appears quite local in nature.

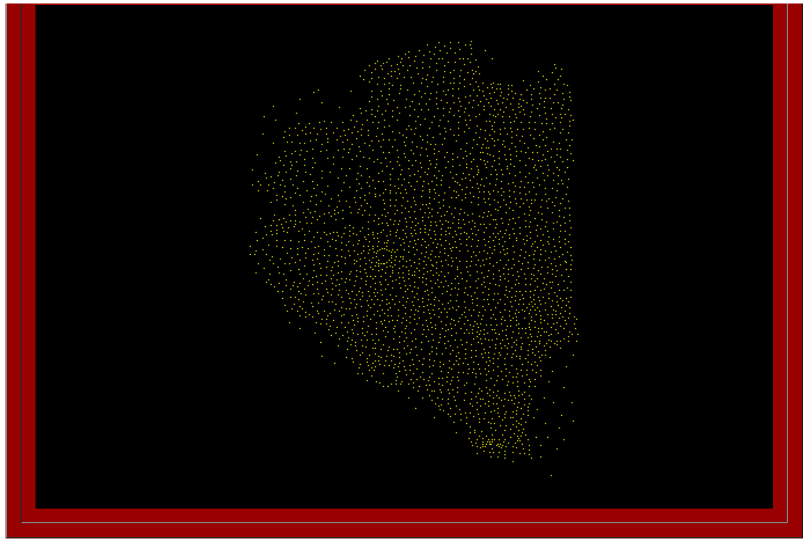


Figure 1.18. Thiessen polygons form a cellular mesh based on the two most rural layers of the hierarchy from map 1.16.

Figure 1.19 shows a sample overlay of the local cellular mesh on the original Christaller map.

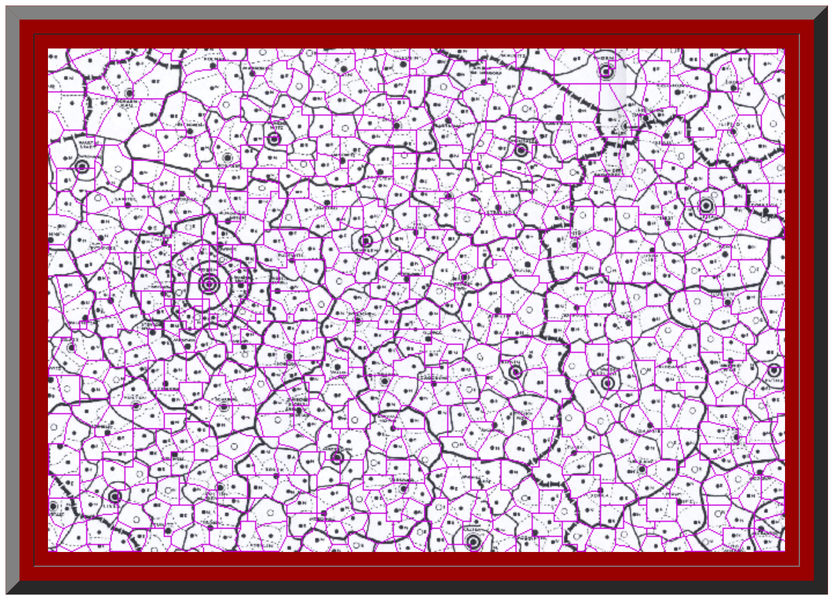


Figure 1.19. Overlay showing cellular mesh superimposed on Christaller map. Posen is the large city located near the middle/left.

Next, we constructed, using the GIS, Thiessen central place networks for each of the first five layers of the hierarchy identified by Christaller. Figure 1.20 shows these animated layers embedded in the convex hull of the map in Figure 1.16.

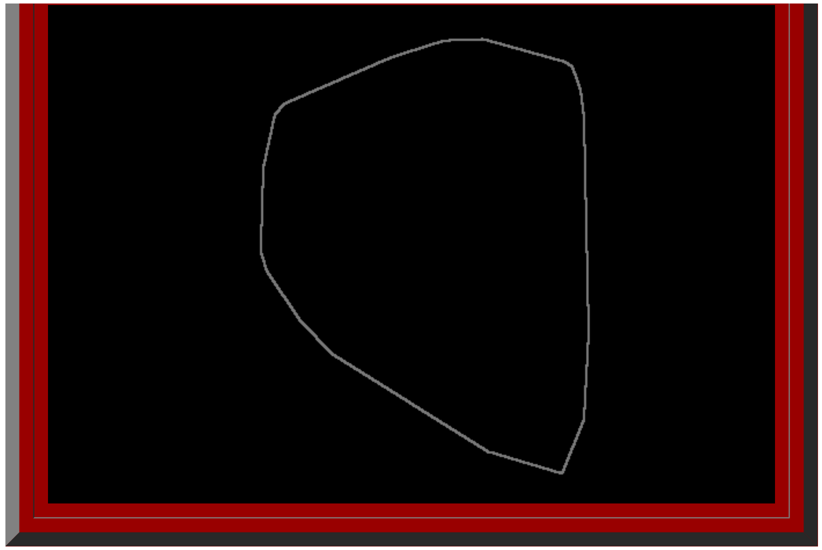
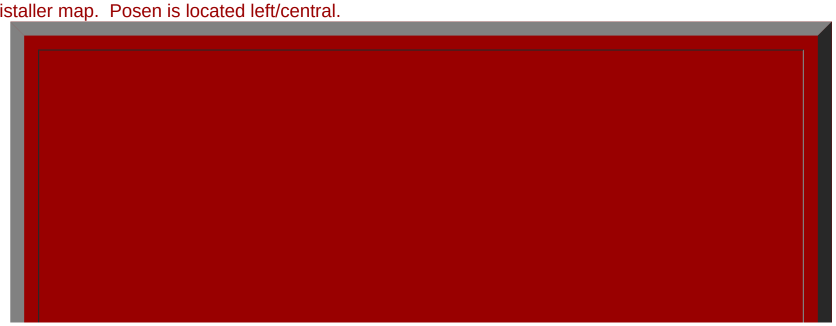


Figure 1.20. Thiessen-generated mesh for each of the five layers of largest urban places in the legend of the Christaller map.

Figure 1.21 shows a closeup comparing each of these layers to the actual map. Note the underfit/overfit pattern at the most global level (as with fractal generators for *K*=7).

Figure 1.21. Each of the layers of Figure 1.20 overlain on the Christaller map. Posen is located left/central.



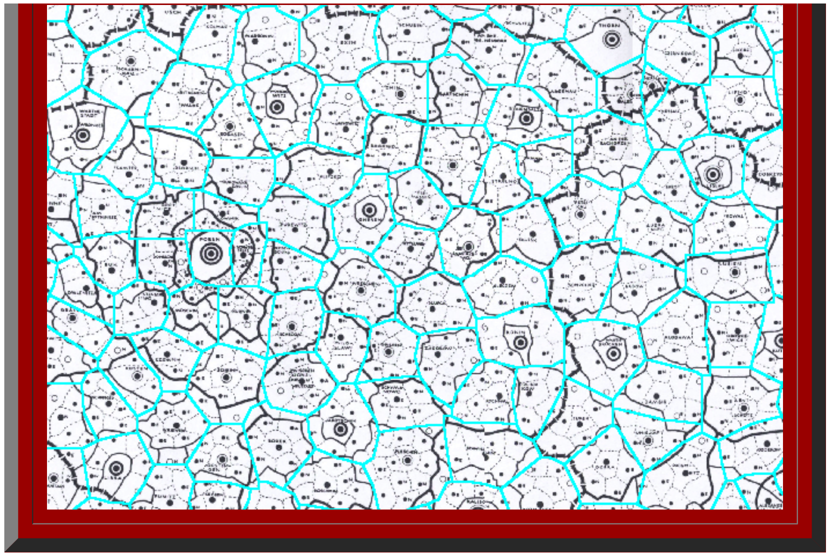


Figure 1.22. Isolated frame from the animation in Figure 1.21, showing the correspondence between the cyan Thiessen-generated polygons and the underlying Christaller map. When the place names are linked, via the hierarchy shown in Christaller's map, as a Mapplet (Figure 1.23), the hierarchical pattern becomes clear in yet another visual manner.

Figure 1.23.

Clearly one can recreate maps from the past using techniques of the present. The strategy of aligning existing work with new technique will be carried forward into the next chapter. Establishing such alignment is interesting; projecting theorems and concepts on which it is based, into future research, is the real reason for making such alignments: to see that new procedures provide adequate results on known cases so that they serve as a procedural benchmark to extend into new theoretical and practical realms.

Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.
Sandra Lach Arlinghaus and William Charles Arlinghaus