Chapter 6
Back to the Real World: Eratosthenes in Ann Arbor

eHighlights of this Chapter
Application of Eratosthenes sieve to parcel map of Ann Arbor
Virtual reality of downtown Ann Arbor: used in work on a maximum height in the downtown

Back to the Real World
The synthetic approach to central place geometry, in earlier chapters, has found application (by S. Arlinghaus and others as noted) in the usual and the unusual. The articles below have all appeared in *Solstice* and links to those articles are given below (*Solstice*, itself, is a Pirelli INTERNETional Award Semi-Finalist (top 80), 2001).

- 1990, June. "Beyond the Fractal, pages 17-35" suggests a transformation of the geometry presented in this work aligned with earlier work of Hagerstrand on the diffusion of an innovation. (Link is to scanned images of hardcopy printed from original TeX files sent as e-mail transmissions with instructions to the user to print out the file on a TeX enabled printer. See link in Bibliography for access to TeX code.)
- 1990, December. "Fractal Geometry of Infinite Pixel Sequences: "Super-definition" Resoultion? pages 98-103" offers an application of this geometry to the cathode ray tube, followed up on with queries from the business world. (Link is to scanned images of hardcopy printed from original TeX files sent as e-mail transmissions with instructions to the user to print out the file on a TeX enabled printer. See link in Bibliography for access to TeX code.)
- 1993, June. "Micro-cell Hex-nets" explores the use of this geometry in association with systems of relay towers on the geographic landscape. (Link is to scanned images of hardcopy printed from original TeX files sent as e-mail transmissions with instructions to the user to print out the file on a TeX enabled printer. See link in Bibliography for access to TeX code.)
- 1996, December. "Web Fractals: An Overview"
- 2002, December. "Spatial Synthesis: Research Program" with W. C. Arlinghaus.
- 2004, June. "Spatial Synthesis Sampler. Geometric Visualization of Hexagonal Hierarchies: Animation and Virtual Reality" with W. C. Arlinghaus, a Pirelli INTERNETional Award Semi-Finalist (top 80), 2003.
- 2004, December. "Spatial Synthesis: A 3D Atlas of Ann Arbor"
- 2005, June. "Spatial Synthesis. The Evidence of Cartographic Example: Centrality and Hierarchy." Serves as a lead-in to this book.

Communication of these ideas, an important part of real-world use of information, has appeared in a number of other locales, as well. A few links are given below to work by S. Arlinghaus (and others as noted).

- 1985. "Fractals Take A Central Place." *Geografiska Annaler, Series B*.
- 1986. "Steiner Transformations" in *Essays on Mathematical Geography*, Monograph 3, pages 55-70, Institute of Mathematical Geography.
- 1987. "Fractals: Constructions, Speculations, and Concepts" in *Essays on Mathematical Geography--II*, Monograph 5, pages 54-66, Institute of Mathematical Geography.
- 1989. "The Fractal Theory of Central Place Geometry: A Diophantine Analysis of Fractal Generators for Arbitrary Loschian Numbers" with W. C. Arlinghaus. *Geographical Analysis.*
- 1990 (January). "Geometry of boundary exchanges: Compression patterns for boundary dwellers" with John D. Nystuen. *The Geographical Review*, American Geographical Society of New York.
- 1991. "Table for Central Place Fractals" in *Essays on Mathematical Geography--III,* Monograph 14, pages 6-15, Institute of Mathematical Geography.
- 1991. "Tiling According to the Administrative Principle" in *Essays on Mathematical Geography--III*, Monograph 14, pages 16-25, Institute of Mathematical Geography.
- 1993. *Fractals in Geography* edited by Nina Siu-Ngan Lam and Lee De Cola, Chapter 10, "Central Place Fractals: Theoretical Geography in an Urban Setting." Prentice-Hall.
- 1993. "Electronic Geometry," The Geographical Review April, 1993. Vol. 83, No. 2, pp. 160-169.

Eratosthenes in Ann Arbor
Ann Arbor is a small city in southeastern Michigan. It is home to the main campus of The University of Michigan, a state university with over 35,000 students on the Ann Arbor campus. The student population composes about 1/3 of the population of the city. Much of the rest of the population works at the university in some capacity or in research industry, businesses, or institutions that locate near the campus. Most cities in the US have shapes that are topologically equivalent to a circle, in terms of paying taxes to the city: land parcels that lie within the city boundaries pays taxes to the city. There are, of course, cities that contain enclaves within their boundaries that are not part of the city itself: Detroit, for example, contains the small (in area) towns of Hamtramck and Highland Park totally within its borders. However, in the case of Ann Arbor and other small cities that contain large state universities, the city is more of an annulus (doughnut) in shape. A large hole, containing the university is cut out of the city: lands in this hole do NOT pay taxes to the city. Hence, a disproportionately large property tax burden is placed on owners of non-university parcels within the city.

For the city to offer a fine quality of life to all its residents (taxpayers and others) there need to be mechanisms to create continuing economic development within the city. One way to do this is to increase the stock of housing and space for commercial and other establishments in support of that housing. This path is all the more attractive in light of enduring interests in reducing "sprawl" and in preserving open space in the more rural surrounding lands. In a city with few remaining empty buildable lots, this approach seems to offer few alternatives, the most obvious of which is to increase the density of dwelling units within the city. When density increases are proposed in established residential neighborhoods there is often loud and long public objection from residents of those neighborhoods. There may also be serious environmental considerations, as well. Few residents, however, seem to object to increasing density in the downtown: many who already live there moved in with an acceptance of taller buildings and those who do not live there often do not care about the idea of increasing density in the downtown. What people do seem to care about, however, is what an increase in downtown residential density may mean to the character, appearance, and feeling of the downtown. To some, an 18 story building is a visual blight on the skyline that provokes negative comment everytime it is viewed; yet, others note that they have become accustomed to it and view it as an old, familiar friend. Building height can be a source of substantial dispute.

The problem of where to locate tall buildings, with sensitivity to existing building types on adjacent and nearby lots, is a difficult one. In Ann Arbor, building height is currently limited by "floor area ratio" (FAR). The FAR is calculated as the ratio of floor area in a building divided by parcel area, times 100. If a given parcel has an FAR of 100 assigned to it, then a building footprint built lot line to lot line may have a height of 1 story. If a parcel has an FAR of 200 assigned to it, then a building footprint built lot line to lot line may have a height of 2 stories. Similarly, an FAR of 300, assigned to a parcel, yields a building of height 3 stories covering the entire parcel. Thus, on a parcel with an FAR of 300, one might, instead, build a building on half of the lot area but of height six stories, or on a third of the lot area but of height 9 stories. On the same parcel, a 30 story building could be built only if its footprint covered one-tenth of the land area of the parcel.

The FAR provides a height limit based on the size of foundation needed to support a tall building. It also offers subtle encouragement for preserving some amount of open space and visual variation in the region to which it applies. The drawback is that a tall building may get built with no regard to the broader context of how a new building will fit in with existing buildings on the surrounding parcels. A possible side effect of using FAR (alone) to limit height is that it might encourage parcel amalgamation by large developers, thereby driving out desired local small business owners.

The issue of creating greater height in the downtown is a complex one. In 2003, Mayor John Hieftje and the Ann Arbor City Council established, in conjunction with the Downtown Development Authority, DDA, (Susan Pollay, Executive Director) a Taskforce to study increasing residential land use in the downtown. Some of the materials studied by that group was created by the first author, in collaboration with others (Fred J. Beal, President Beal Construction, and Douglas S. Kelbaugh, Dean Taubman College of Architecture and Urban Planning, The University of Michigan). The material that follows highlights the use of concepts from this book that were used in that context. It, and associated materials were used in public hearings and by City Council in passing the recommendations of that Taskforce. A more complete story of the policy and 3D modeling aspects is available elsewhere (link to Arlinghaus, Beal, and Kelbaugh).

A Prime Parcel Sieve.
The FAR is assigned by zoning type. In the downtown, there are currently parcels assigned to each of 22 different zoning categories (AG, C1, C1A, C1AR, C2A, C2AR, C2B, C2BR, C3, M1, M1A, M2, O, P, PL, PUD, R1D, R2A, R2B, R4B, R4C, R4D). Roughly speaking, any category beginning with C is a commercial category; M is for manufacturing; R is for residential. The AG category is for agricultural zoning, O is for office, P (except for PUD) is for Public Land (as for the University of Michigan), and PUD is for Planned Unit Development.

Enumeration of parcels via geometric partition

In Figure 6.1, the animated map shows the downtown area of the City of Ann Arbor parcel map colored as a thematic map by zoning category: the broad PL zoning is part of the central campus of the University of Michigan. The curved line near the left side of the map. representing the Ann Arbor Railroad corridor, has most of the manufacturing parcels adjacent to it. Separate categories enter the picture in sequence, arranged according to alphabetical ordering of zoning category. The coloring scheme is exhaustive: every parcel is covered. It is also mutually exclusive: no parcel has more than one color. Thus, the zoning classification serves as a geometric partition of the parcels. It offers a way to enumerate all the parcels: to order them in some logical fashion and to assign some quality to them as well, in much the way that we assign both cardinal and ordinal properties to numbers.

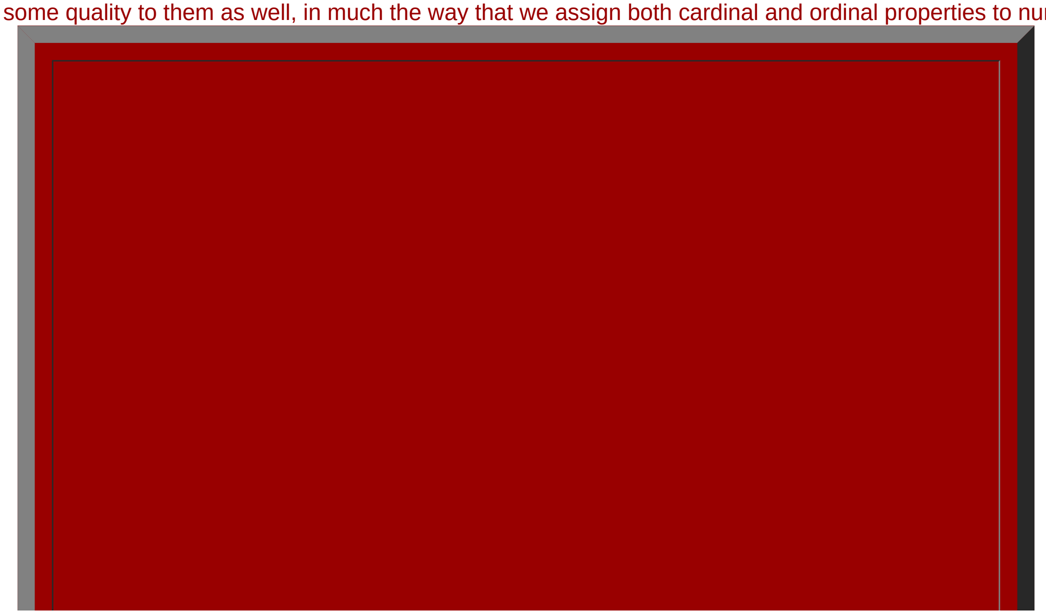

Figure 6.1. Zoning animation of 22 zoning categories: AG, C1, C1A, C1AR, C2A, C2AR, C2B, C2BR, C3, M1, M1A, M2, O, P, PL, PUD, R1D, R2A, R2B, R4B, R4C, R4D. Zones enter the animation in alphabetical succession. Attached labels are added in the final frame.

The casting out of parcels

With an enumeration of all parcels in hand, groups of parcels will be removed in accordance with various ideas. The goal is to select targets of opportunity for taller projects. The mechanism for such selection is similar to the mechanism for the selection of prime numbers employed by Eratosthenes (link back to Eratosthenes's Sieve). In the case of the downtown, parcels meeting certain criteria are cast out from further consideration as prime parcels much as numbers meeting certain criteria (multiples of 2, multiples of 3, multiples of 5, and so forth) were cast out from further consideration as prime numbers. What remains, are "primes": prime numbers or prime parcels. These prime parcels will then be viewed as possible targets of opportunity. Local considerations of various sorts might then be superimposed on them, such as design standards, spacing of tall buildings, or whatever constraints are within the legal code of the city.

Animated maps are useful for showing change; static maps are useful when one wishes to take a longer look at pattern without regard to change in pattern. Thus, Figure 6.2 shows the final frame of the animated map in Figure 6.1 along with a layer showing the boundary of the Downtown Development Authority (DDA) in yellow, the railline in black/yellow, the floodway (channel) outline of Allen's Creek in blue, and the floodplain outline of Allen's Creek in cyan. We begin the process of casting out parcels according to some replicable logic (not as 'multiples' but according to some logical scheme). (In Ann Arbor, the "downtown" generally refers to the DDA: a state-enabled authority that can capture increases in taxable value to pay for improvements within the defined boundaries.)

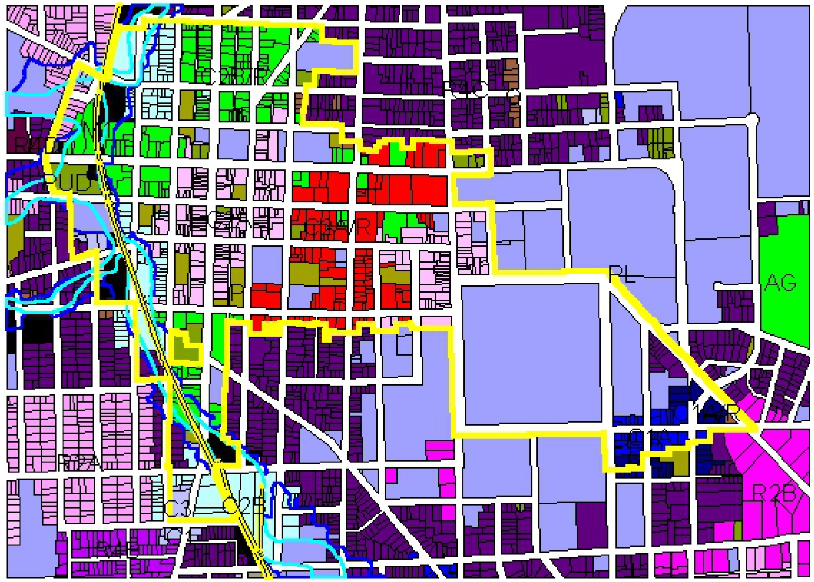

Figure 6.2. Zoning categories, DDA boundary, railline, floodway and floodplain.

CASTING OUT:

- **All zoning categories that cannot support height in excess of FAR are cast out.**
  Consider only zoning categories, as the downtown "core," of C1A, C1AR, C2A, C2AR, C2BR as those categories that might contain parcels that are targets of opportunity for building structures of height greater than that permitted by FAR. Thus, all parcels of zoning type other than those 5 are cast out from further consideration. This process opens up quite a bit of land within the DDA as possibilities for tall buildings (shown in white in Figure 6.3). As in the prime number seive, the first step removes a substantial set, parcels in color, from further consideration. Subsequent steps remove smaller subsets from further consideration.

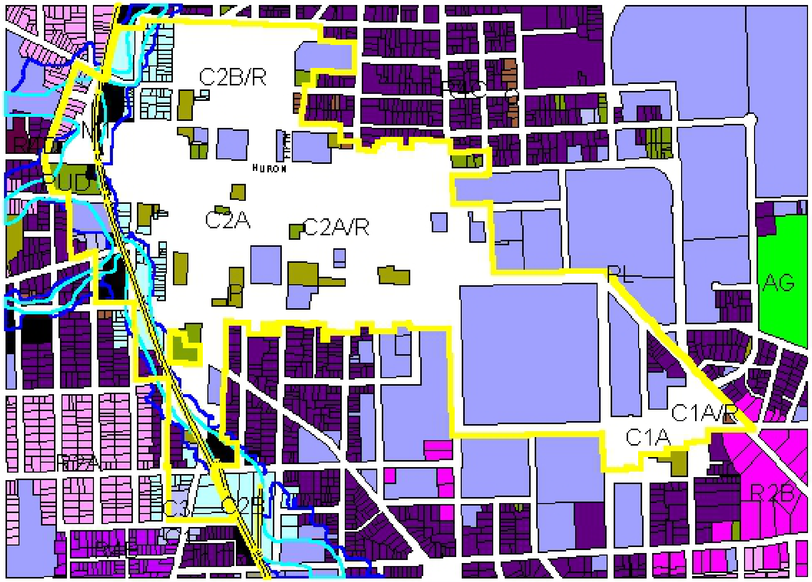

Figure 6.3. All zoning categories other than C1A, C1AR, C2A, C2AR, C2BR zoning categories are cast out from further consideration. Those 5 zoning categories cover the lands represented in white (as in open for further consideration). The parcels assigned a color other than white have been removed from consideration.

- **All historic district parcels are cast out.**

  When parcels containing historic district designation (and often relatively short buildings) are superimposed on the pattern in Figure 6.3, further limitation of targets of opportunity is the result (in Figure 6.4, historic district parcels are red). In this step, all historic district parcels are cast out, further limiting the number of parcels eligible to be considered for "prime" parcel status (Figure 6.4). The white area, targets of opportunity for tall buildings within the DDA, remains large but not as large as in Figure 6.3. Successive casting out of other parcels continues in the steps to follow.

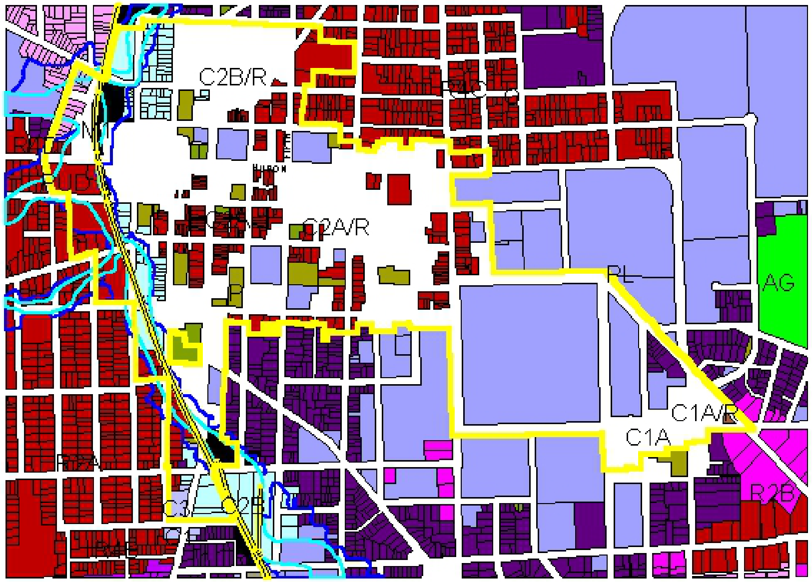

Figure 6.4. This map is identical to the map in Figure 6.3 with historic district parcels superimposed in red. The historic district designation further limits the targets of opportunity.

- **Floodplain parcels are cast out.**

  All parcels whose centroids lie within the floodplain of Allen Creek are cast out as possible prime parcels. Parcels that intersect the floodplain, but whose centroid does not lie in the floodplain remain. The reason to use the centroid analysis is that public benefit might come from having a building erected on the part of the parcel not in the floodplain and saving the part of the parcel in the floodplain as part of a greenway or other broad environmental amenity.
- **All parcels containing "steep slopes" are cast out.**

  Many of these are also floodplain parcels, as above. Elsewhere, the definition of "steep slope" is problematic. Generally, the DDA is fairly flat, away from the creek valley.
- **Church parcels are cast out.**

  Parcels containing churches, as listed in published official records, are cast out.

All possible prime parcels, as targets for opportunity to support buildings of height in excess of the FAR, are those that pass through this sieve: those that are not cast out based on zoning, environmental, and cultural factors above.

Zooming in

To take a closer look at the area, zoom in on a portion of it. One idea is to use the street hierarchy to select general target areas: wider streets support taller nearby structures. Thus, the zoom-in of Figure 6.5 is on Huron Street, the widest street in the DDA. Insert an aerial photo behind the map: the "white" area of earlier maps has been replaced with aerial so that one can see directly the current content of all potential prime parcels. With the aerial replacing the white zone, one can see what is currently located on each parcel. Thus, identification of immediate targets is easy: note the surface parking lots (with clearly underutilized existing infrastructure). Parcels already containing buildings might, or might not, become targets.

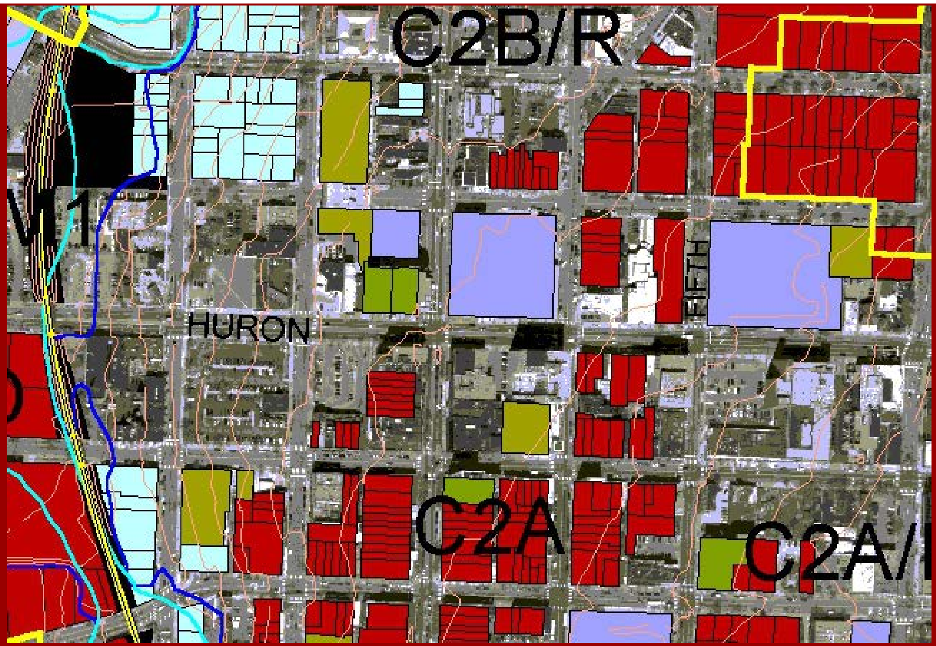

Figure 6.5. Zoom in--the aerial replaces the white zone in Figures 6.3 and 6.4. The locations of existing buildings and surface parking lots become evident.

The map in Figure 6.5 shows each parcel from lot line to lot line. Building footprints were digitized from aerials provided by the City. Associated databases gave estimates of building height. Interactive maps offered a constructive way to link a variety of two dimensional sets of information (as in Maps 6.1, 6.2, and 6.3).

- **I-Map 6.1:** Click here for a link to an interactive map showing building footprints and height (on mouse-over) as well as building address and street names (on mouse-over). Parcel boundaries are shown on the underlying aerial and on the green Downtown Development Authority (DDA) area. The Allen Creek floodway (underground) and flood plain are shown, shaded, respectively in blue and turquoise. Click on a building or a street to see associated entries in the underlying database.
- **I-Map 6.2:** Click here for a link to an interactive aerial showing parcel boundaries, zoning, building height (on mouse-over), and street name. DDA outline, only, is shown in light yellow so the user may zoom in to get a closer view of the aerial within the DDA (up to 800% enlargement--can see cars clearly). The Allen Creek floodway (underground) and flood plain are shown, outlined, respectively in blue and light blue; again, because the shading is removed, the viewer may look at the content of the floodway/floodplain in greater detail than above. Click on a building or a street to see associated entries in the underlying database.
- **I-Map 6.3:** Click here for a link to an interactive aerial showing zoning boundaries in the downtown, zoning type (on mouse-over), building height (in the "zoneht" record of the database), and street name. Click on a building or a street to see associated entries in the underlying database.

Designing the Vertical City: the Use of Virtual Reality Models

Once the parcels that could serve as targets of opportunity for tall buildings were agreed upon by the Taskforce, the prime parcels, a series of several hundred virtual reality models were developed to visualize alternatives for the DDA. The reader interested in the detail of the planning process, local policy matters, and the mechanics involved in model construction are referred to the following materials in the references. The focus here is on the use of a concept identical to Erastosthenes prime number seive to sift out parcels elegible for further consideration, as above, and the subsequent application of virtual reality to visualize complex geometric situations, as in the case of Christaller nets, in order to facilitate scientific communication and eventual policy decisions by official municipal bodies.

The set of buildings that currently exist within the DDA was used as a base on which to build alternative scenarios of what might be done. Figure 6.6a shows a screen-capture of a virtual reality model built by extruding building footprints according to height records from the City of Ann Arbor Planning Department (Figure 6.6b is the virtual reality model; try driving around in it).

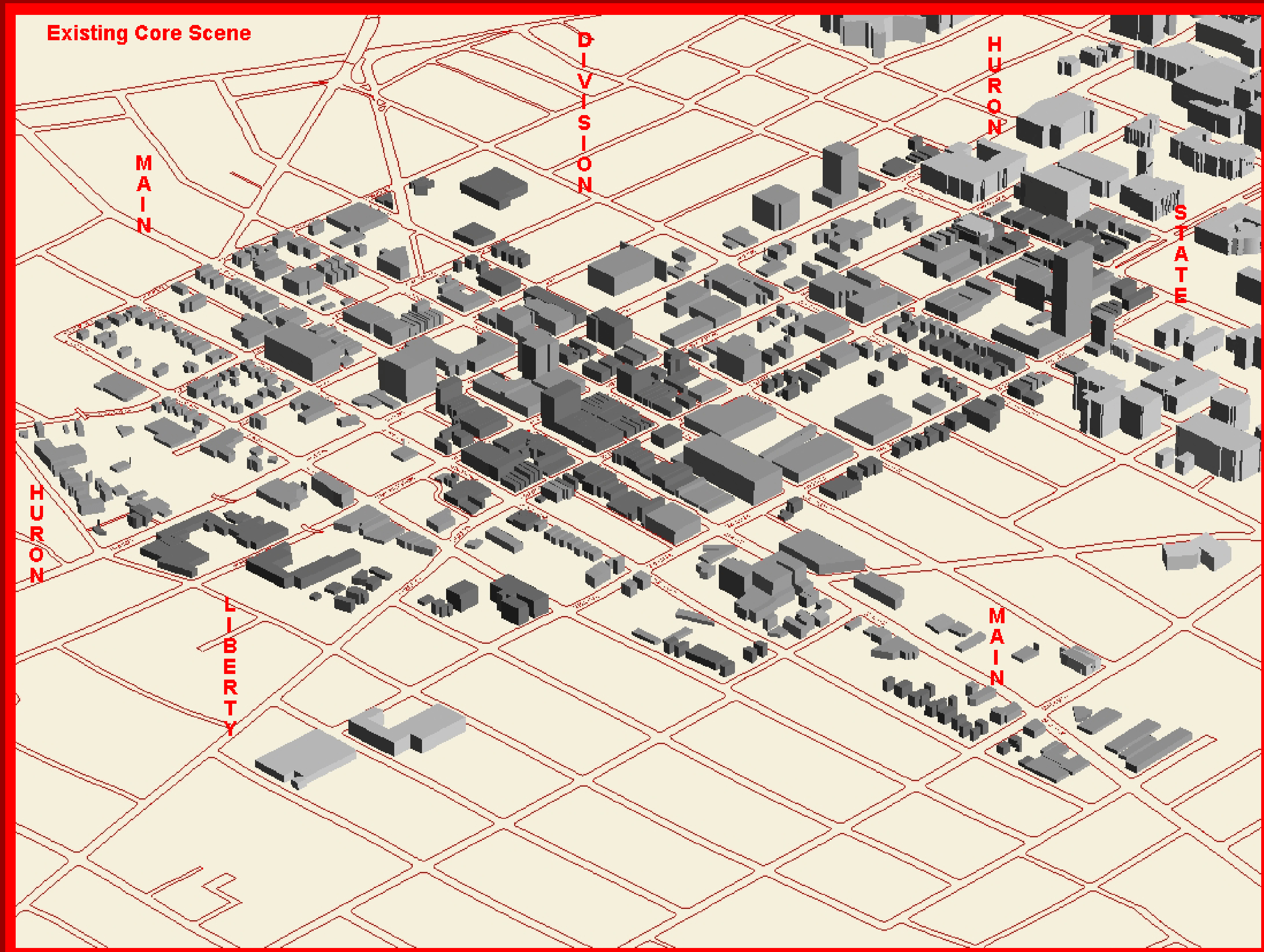

Figure 6.6a. Downtown Ann Arbor buildings extruded from footprints digitized from aerial.
Figure 6.6b: link to virtual reality model from which Figure 6.6a was taken.

Figures 6.7a and b, 6.8a and b suggest how that model was used: colored buildings are added, only on prime parcels, according to numbers of residential units to be added and to guidelines suggested by Taskforce members (selected for expertise in construction, urban planning, architecture, and so forth). Different building colors respond to different particular suggestions. This set of four models represents merely one possible scenario; input from numerous sources was important in creating a variety of alternatives for policy makers to consider.

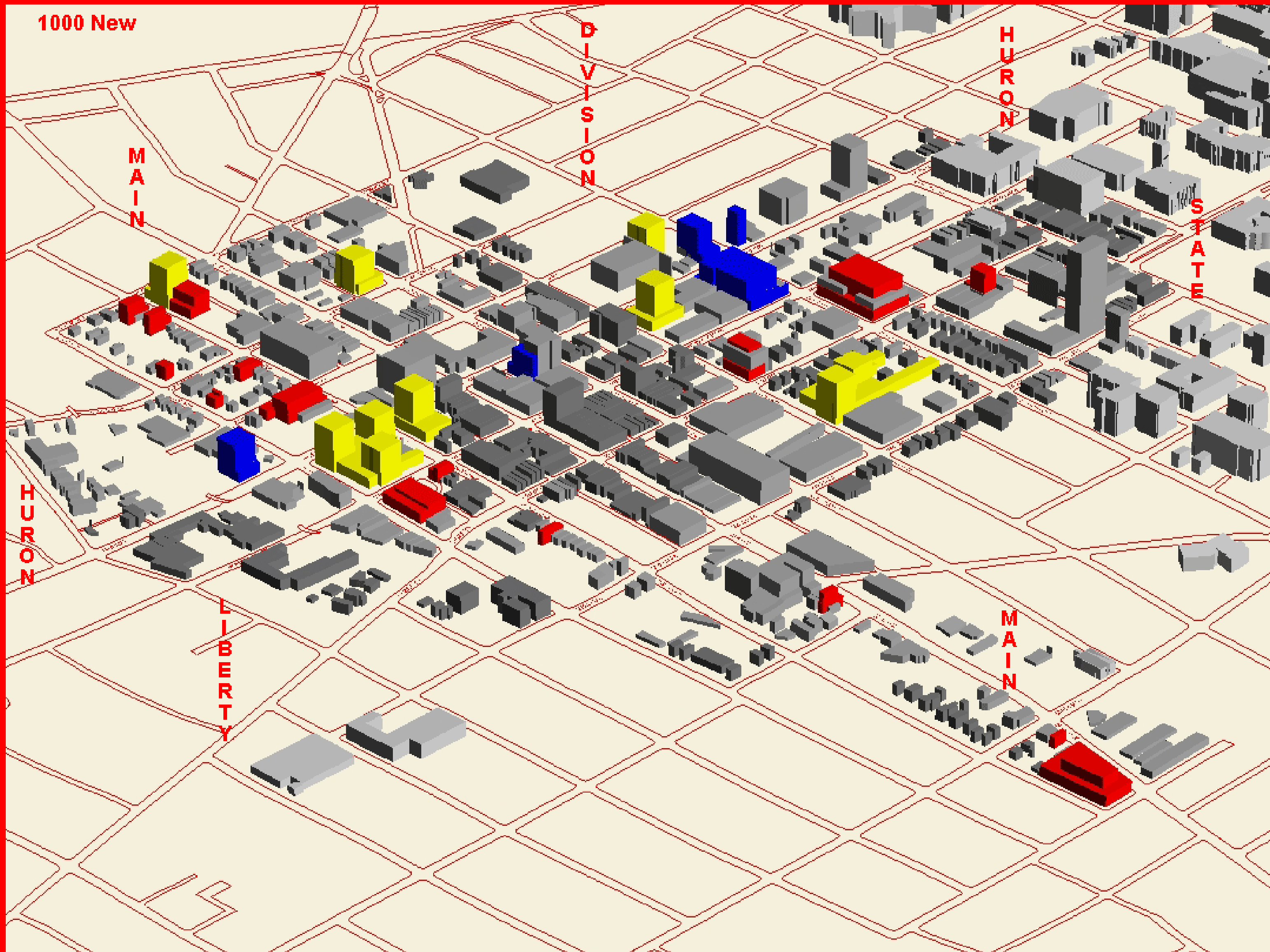

Figure 6.7a. Downtown Ann Arbor buildings extruded from footprints digitized from aerial. Gray buildings are existing buildings. Yellow buildings are built on empty lots or on surface parking lots. Blue buildings are built along Huron Street (possibly as replacements for, or on top of, existing buildings). Red buildings are built with upper story setbacks on a subset of streets identified by the Taskforce. The number of new buildings is sufficient to support 1000 new residential units. All non-gray buildings are built on prime parcels.
Figure 6.7b: link to virtual reality model from which Figure 6.7a was taken.

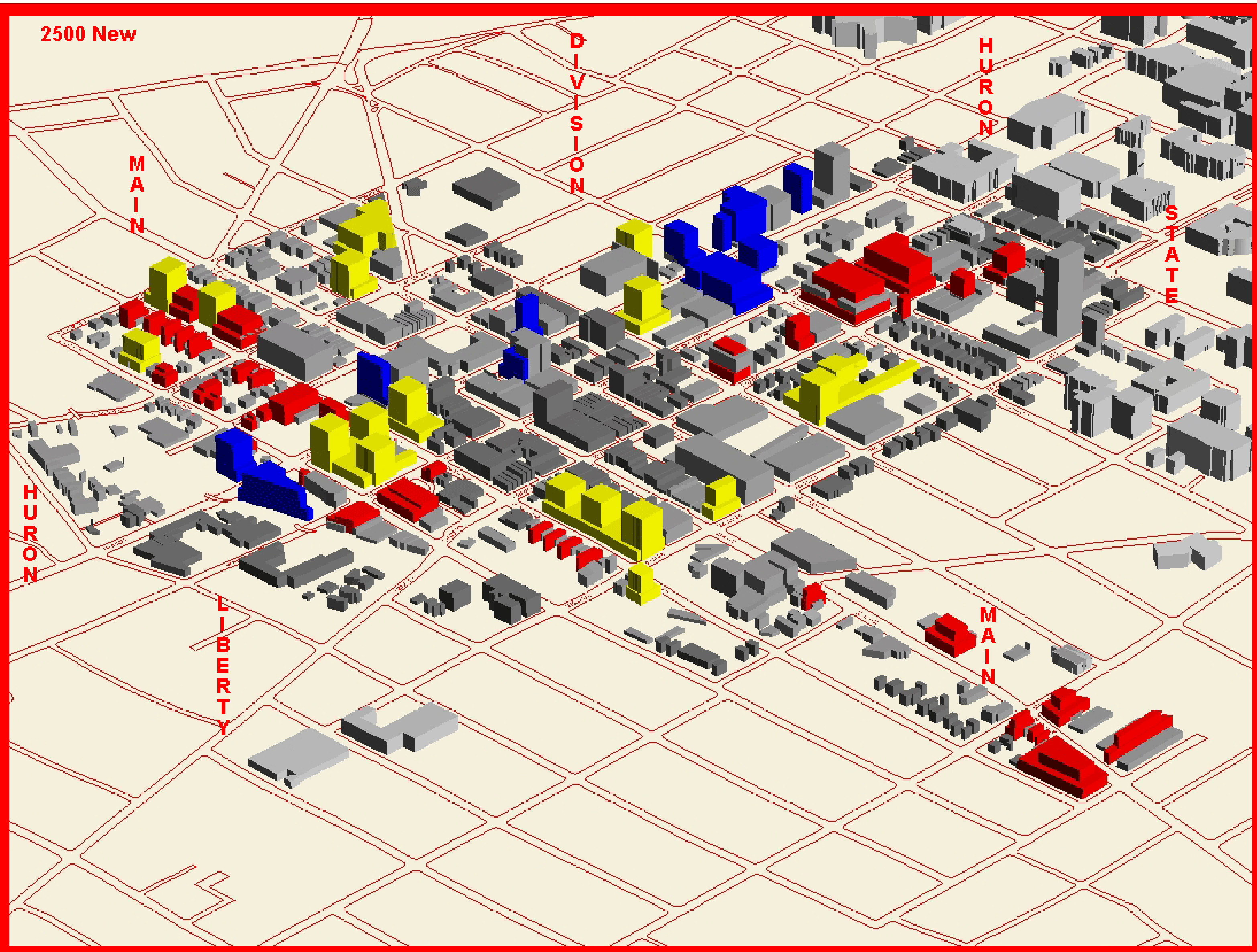

Figure 6.8a. Downtown Ann Arbor buildings extruded from footprints digitized from aerial. Gray buildings are existing buildings. Yellow buildings are built on empty lots or on surface parking lots. Blue buildings are built along Huron Street (possibly as replacements for, or on top of, existing buildings). Red buildings are built with upper story setbacks on a subset of streets identified by the Taskforce. The number of new buildings is sufficient to support 2500 new residential units. All non-gray buildings are built on prime parcels.
Figure 6.8b: link to virtual reality model from which Figure 6.8a was taken.

Virtual reality models served to pique conversations and help Taskforce members to visualize alternatives. Eventually, they saw a need to compare one virtual reality model to another. For that purpose, "viewpoints" were inserted into the VR models (find the "viewpoints" pull down in Figures 6.6b, 6.7b, 6.8b, so that different models could each be viewed from exactly the same vantage point (guided by the computer). While the insertion of viewpoints was helpful, direct comparison still required retention of a great deal of visual information in one's memory. Animation of the models of the DDA, as in Figure 6.9 offered a way to look at multiple global alternatives.

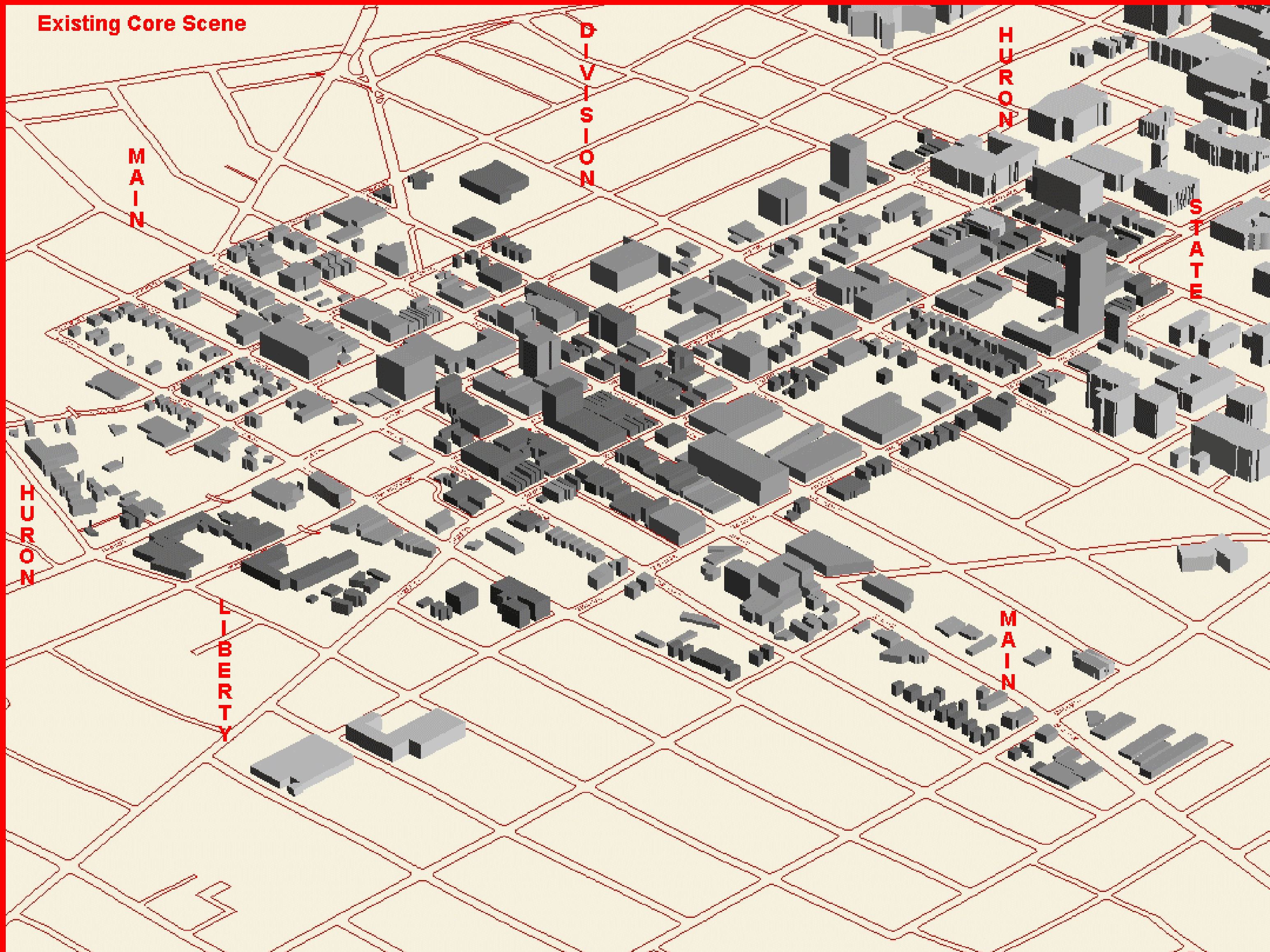

Figure 6.9. Animation of various alternative screen captures from multiple VR models permits consistent visual comparison of models from one vantage point.

Thus, the solution for local views was also to take screen captures from the same viewpoint of different VR models and stack them in an animation. The 3D VR models above were built on May 27, 2004. Earlier models were shown to the public on April 27; public reaction was used to create a responsive model on May 3. The May 3 model was used in conjunction with the April 27 model as a stacked animation built from the taller April 27 model fading into the shorter response model. The results were presented to the Taskforce, May 10. as in Figures 6.10, 6.11, 6.12, and others. These exemplify the idea of using animation to make clear visual comparisons by taking screen captures from VR models from the same viewpoint: the VR models might all be from the same time, as in Figure 6.9, or they might be from different times as well, as in Figures 6.10, 6.11, and 6.12.

Figure 6.10. Animation of various alternative screen captures from multiple VR models permits consistent visual comparison of models from one vantage point.

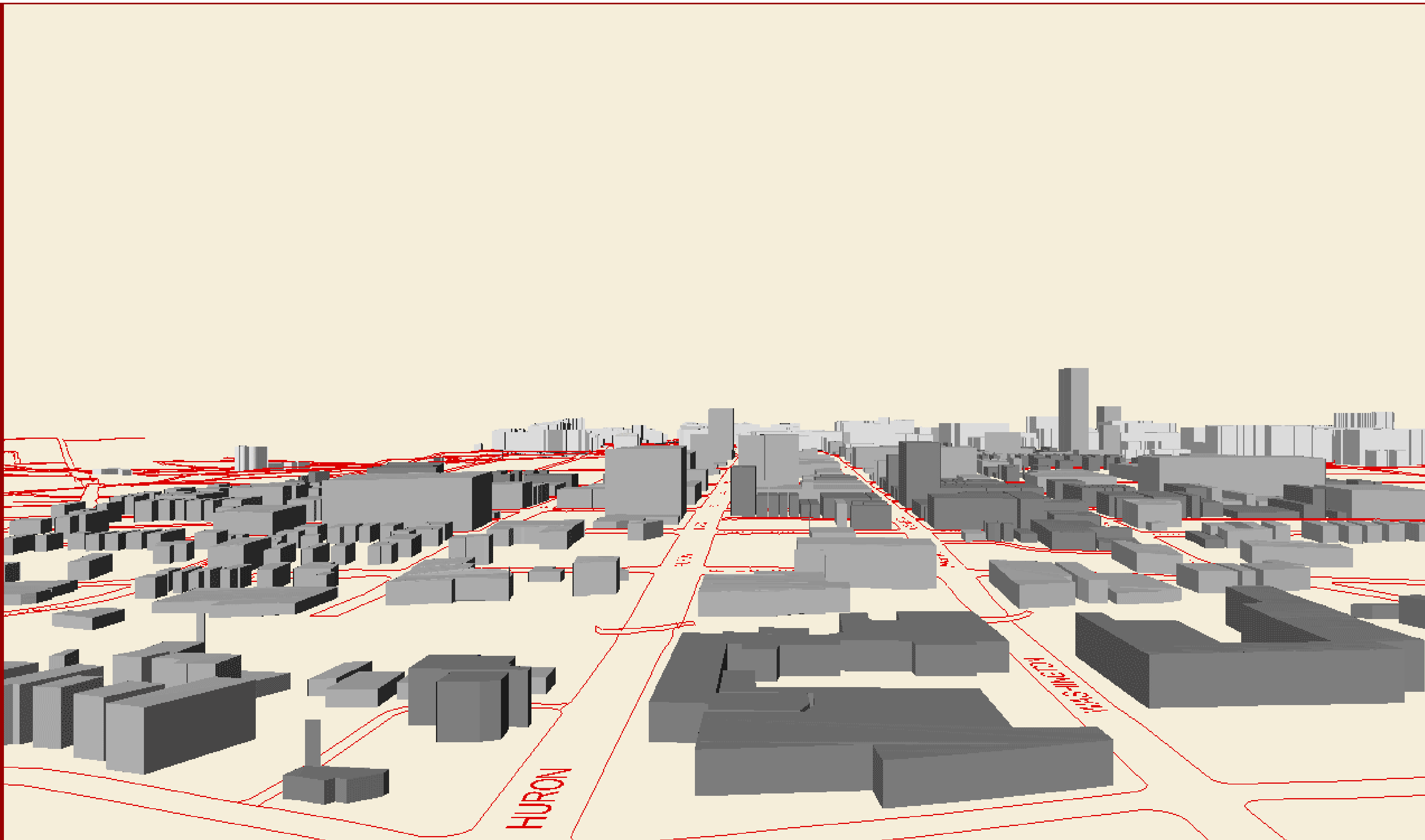

Figure 6.11. Animation of various alternative screen captures from multiple VR models permits consistent visual comparison of models from one vantage point.

Figure 6.12. Animation of various alternative screen captures from multiple VR models permits consistent visual comparison of models from one vantage point.

These models, and results from hundreds of others, were presented to the Taskforce in 11 different meetings in the spring of 2004. They were presented in a public hearing to the citizens of Ann Arbor on April 27, 2004 as chronicled in a front page story in The Ann Arbor News with a picture of a model on the front page. They were presented to the Mayor and City Council on June 7, 2004. A previous article in Solstice describes this activity in detail. On July 19, 2004, the resolution derived in part from these models was approved by City Council with a request for a special session in September subsequent to the opportunity for City Council to analyze the mass of materials. From sieve to virtual reality, the strategy employed in one local study condensed time across millennia.

*The author, S. Arlinghaus, acknowledges productive meetings on the Ann Arbor study with, and assistance from, the following individuals and groups (during the time work was done):

- her colleagues on the City of Ann Arbor Planning Commission (Sandra Arlinghaus (Chair), Kevin McDonald (Vice-Chair), Scott Wade (Secretary), Braxton Blake, Jean Carlberg, Kristen Gibbs, Christopher Graham, William Hanson, and Steve Thorp);
- the Ordinance Revisions Committee of that Commission (Hanson, Chair; Carlberg, Arlinghaus, Blake);
- the City of Ann Arbor Planning Department staff (Karen Hart, Planning Director; Wendy Rampson, Coy Vaughn, Donna Johnson, Jeff Kahan, Chandra Hurd, Alexis Marcarello, Christopher Cheng, and Matthew Kowalski);

Merle Johnson, City of Ann Arbor, Information Technology Services;
- Heather Edwards, Historic District Preservation Coordinator, City of Ann Arbor;
- Matthew Naud, Environmental Coordination Services Director and Emergency Management Director, City of Ann Arbor
- John D. Nystuen, Professor Emeritus, Taubman College of Architecture and Urban Planning, The University of Michigan
- Peter Beier, Professor of Engineering and Director, 3D Laboratory, Media Union, The University of Michigan.
- Fred Beal, Beal Construction, Ann Arbor, MI
- Douglas Kelbaugh, Dean, College of Architecture and Urban Planning, The University of Michigan
- Mayor's Task Force on Height in the Downtown
- DDA Executive Director, Susan Pollay
- the Mayor of Ann Arbor, His Honor, John Hieftje

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Spatial Synthesis: Centrality and Hierarchy, Volume I, Book 1.
Sandra Lach Arlinghaus and William Charles Arlinghaus