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The purpose of Solstice is to promote interaction between geography and mathematics. Articles in which elements of one discipline are used to shed light on the other are particularly sought. Also welcome are original contributions that are purely geographical or purely mathematical. These may be prefaced (by editor or author) with commentary

suggesting directions that might lead toward the desired interactions. Individuals wishing to submit articles or other material should contact an editor, or send e-mail directly to sarhaus@umich.edu.

SOLSTICE ARCHIVES

Back issues of Solstice are available on the WebSite of the Institute of Mathematical Geography, http://www-personal.umich.edu/~sarhaus/image and on the GOPHER of the Arizona State University Department of Mathematics. Thanks to Bruce Long for taking the initiative in this matter. The connections to this GOPHER are available along a variety of routes through the Internet.

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TABLE OF CONTENTS, VOL. VII, No. 2.

1. WEB FRACTALS: AN OVERVIEW. Sandra Lach Arlinghaus.
2. PART II. ELEMENTS OF SPATIAL PLANNING: THEORY.
MERGING MAPS: NODE LABELING STRATEGIES
Sandra Lach Arlinghaus
3. INDEX TO SOLSTICE, VOLUME I (1990) TO THE PRESENT.

1.

WEB FRACTALS: AN OVERVIEW

Sandra Lach Arlinghaus
The University of Michigan

Documents written in HTML (Hyper-Text Markup Language) for WebSites not only offer the capability to place Home Pages on the World Wide Web, but also extend the dimension in which typesetting takes place. Broadly viewed, from a typesetting standpoint, HTML appears to be equivalent to a subset of commands of Plain TeX. One notable exception

arises, however, in the commands in HTML that enable the user to specify links to other documents, graphical or textual. A Home Page, or any other page or set of pages (paper or electronic), might be viewed as a one dimensional string of letters broken into words. Graphics offer an opportunity to extend the text into other dimensions. So too do the links from one web page to another. They offer an extraordinary capability to reach out within the text itself: an opportunity to "fill" a text-shed beyond the linear text-stream. Of course, one can mix and match links, graphics, text, and whatever is available on the web. Without loss of generality, interest might be confined to text because it is with text that the extension into extra dimensions appears straightforward.

Thus, it becomes of interest to ask how much of a text-shed is filled by the activity of creating links and how one might measure such filling of text-space. Concepts from fractal geometry and from chaos theory might be employed to consider these issues.

GLOBAL WEB VIEW

The global perspective employed in much of modern mathematics focuses not on the individual objects being studied but on the sets of transformations that enable one to move from one object, or type of object, to another. Thus, one learns group theory by studying the various morphisms that link groups, rather than by focusing on the tables that describe individual groups. Linkages expose structure.

Thus, one view of the Web would be to shrink to nodes all home pages and focus only on the links that exist between sites. The set of nodes is finite, but it is unbounded--one can imagine going beyond any upper bound on the number of sites, simply by adding one more. The set of links, too, is finite and unbounded. The morass of links defies good graphical description. Search engines try to make sense of the Web through various organizational schemes. One view of the pattern is as a cataloguerAs nightmare; another is as a chance to introduce order into seeming abstract chaos.

Yet another might be to engage in graphical analysis based on some sort of ordering scheme that permits the assessment of extent of space filled. The concept of attractors and repellers seems an important one to search engines. One such ordering scheme might be such as suggested below. Of course there are many others, too.

y equals x means link...both to and from me to you
y greater than x means link...me to you
y less than x means link...me from you

The algebraic, geometric, and topological structure of the web are

important to analyze: the space-filling ideas represented in fractal geometry suggest one style of approach. Any deep, careful mathematical analysis should, however, offer a systematic means to evaluate the current status of web content, and more important, provide a continuing framework in which to guide and understand web development. These comments offer mere hints at directions substantive work might take; hopefully, they offer encouragement to consider the web itself as a source of various styles of research opportunity.

Ann Arbor, MI
May, 1996

2.

PART II. ELEMENTS OF SPATIAL PLANNING: THEORY.

MERGING MAPS: NODE LABELING STRATEGIES

Sandra Lach Arlinghaus
The University of Michigan

Different inventories of roads produce different maps; two different maps can represent one set of actual roads. How is a planner to rectify this situation? To consider the problem abstractly, it is useful to characterize the road network as a graph, composed of nodes and edges joining nodes. Such style of consideration, as a graphical puzzle, dates from 1736 and the Konigsberg Bridge puzzle and from Hamilton's "Around the World" puzzle of 1859 (Harary, pp. 1, 4); as a planner's dilemma it has no doubt always been with us; and, as a cartographer's delight, it manifests itself over and over again in projection selection, a result of the one-point compactification theorem (visualized as stereographic projection) forcing us to recognize that the sphere can never be flattened out into the plane.

In Figure 1, two portions of road maps based loosely on the downtown Ann Arbor, Michigan, street map are shown superimposed--a conceptual view of a practical problem noted by John Nystuen, and implemented at the level of mapping by Andrea Frank and Jyothi Palathinkara (University of Michigan). As Nystuen et al. noted (unpublished communication), these nodes are, in some sense at least, fairly "close"--one can recognize, visually, what the correspondence is to be from one map to the other. The viewpoint adopted here is that as graph-theoretic trees they are homeomorphic (have the same structure or connection pattern).

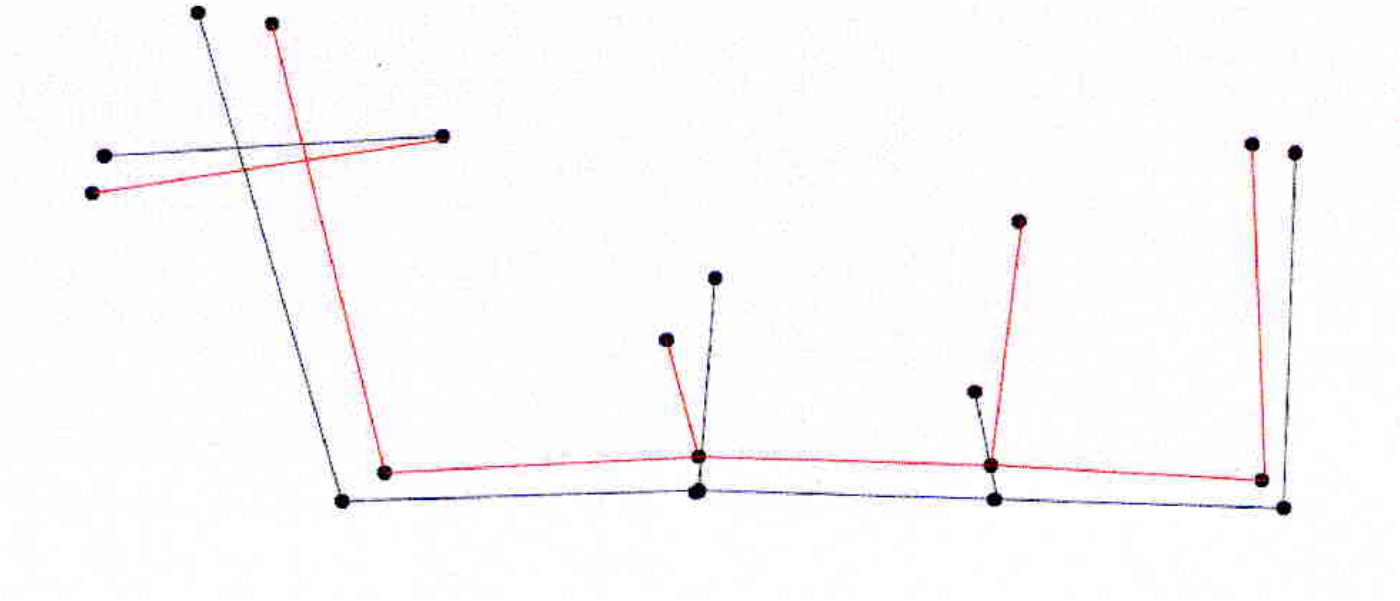

Figure 1. Two trees representing the same actual street pattern.

If the planners who use these maps rely on a node labeling scheme, in which both maps are labeled from the top down, beginning in the upper left hand corner, proceeding horizontally and then in a serpentine pattern from top to bottom, the first node encountered is labeled with the numeral 1; the second node encountered is labeled with the numeral 2, and so forth (Figure 2). However, the labeling imposed on these two maps is not identical, as one would wish (to indicate their identity in topological form). Indeed, the label 3 on the red tree then corresponds to the label 4 on the black tree and vice-versa, as do labels 5 and 6; a situation Nystuen et al. noted and were able to uncover on a number of occasions on actual maps (Figure 2). Naturally, as they note, one would wish, in a situation such as this one, to have the numerals correspond in an appropriate manner.

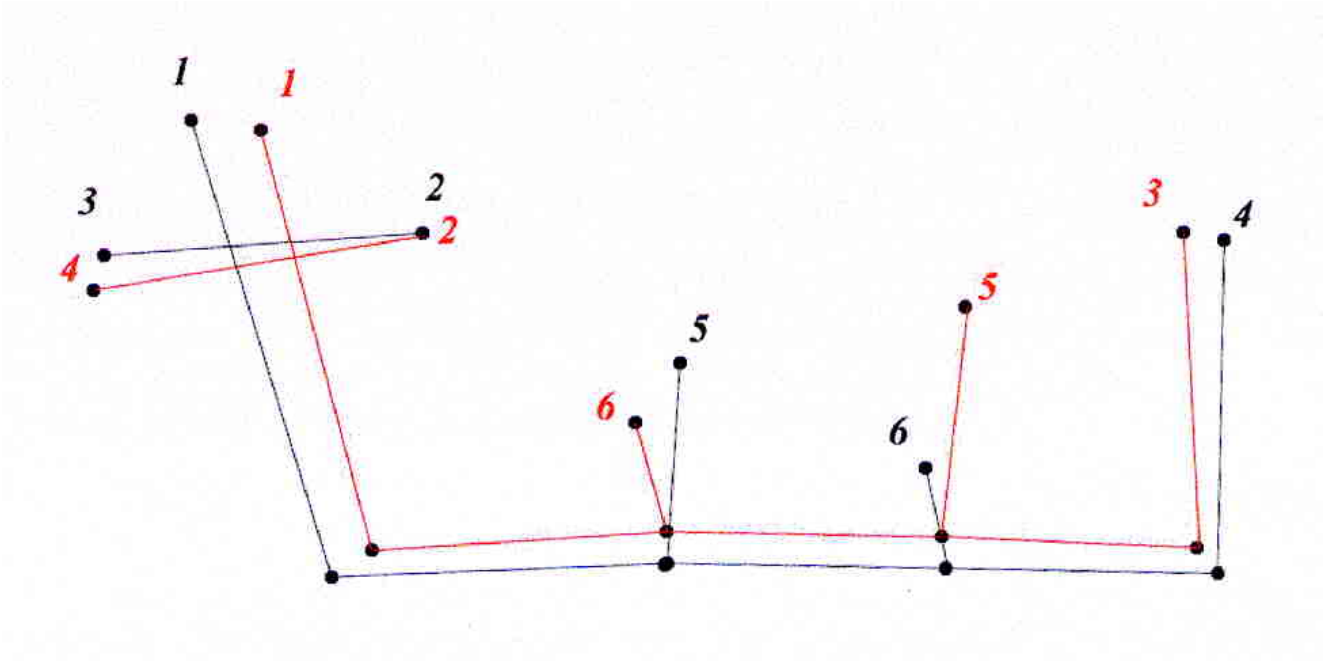

Figure 2. The red route and the black route: numbered nodes do not correspond as desired. The numbering scheme is a common one in which nodes are labeled from the top down in a left/right serpentine pattern (after Nystuen et al.).

Nystuen's observation came from a real-world example; it points, once again, to the appropriateness of considering different labeling schemes that will permit alignment of maps based on topology rather than solely on distance from the top of the screen. One strategy, that takes advantage of the structure of the underlying raster on which these maps were produced, proceeds as follows.

CANTOR PIXEL-LABELING ALGORITHM

1. Designate as a fundamental screen unit the smallest dot size to be used to represent a node; without loss of generality it can be assumed to be a "pixel."

2. In the manner that has come to be conventional with cathode

ray tubes, use the top of the screen as the positive x-axis and the left side of the screen as the positive y-axis (with origin at the upper left-hand corner of the screen).

3. Assign to each pixel on the CRT an ordered pair of Cartesian coordinates: (1,2) is a pixel (an indivisible area) located one unit to the right of the left edge of the screen and two units down from the top.

4. Assign to each node in one map the pixel coordinates of the node as follows. If the pixel coordinates are (x,y), label the node with the rational number y/x.

This sort of strategy differs from what may be a computer default labeling scheme. Each possible node location has a unique label that is a single rational number indicating pixel location referred both to raster position and to location in relation to other pixels. The serpentine labeling scheme is still preserved even though its form is different, and perhaps, not evident. Cantor's enumeration technique shows that there is a one-to-one correspondence between the set of natural numbers and the set of ordered pairs of natural numbers. These two sets have the same cardinal number, aleph null (Hahn, pp. 1595-1596). Cantor's serpentine pattern through a square lattice converts the two-dimensional array into a one dimensional stream clearly equivalent to a number line (Figure 3).

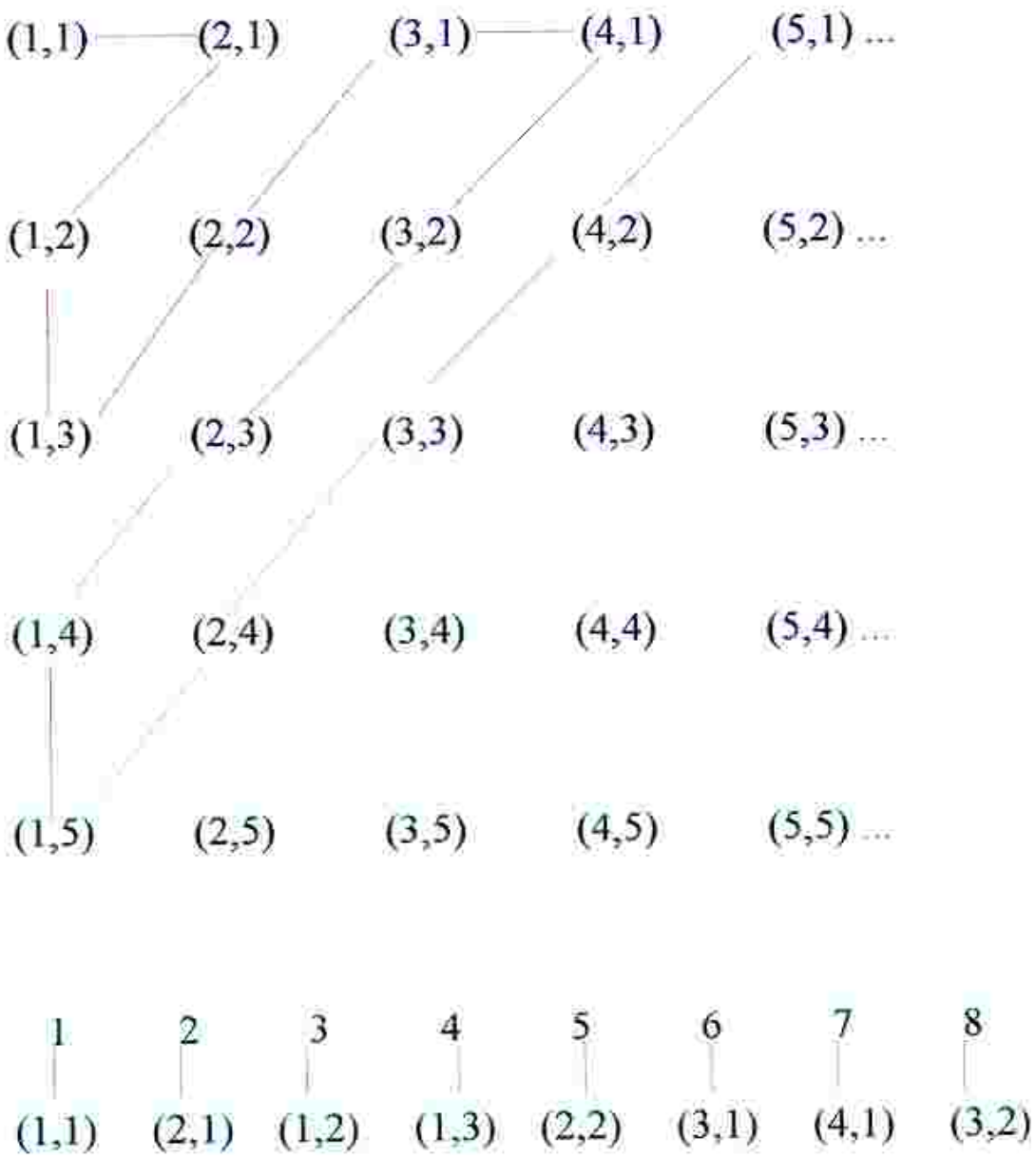
(1,1) (2,1) (3,1) (4,1) (5,1) ...
(1,2) (2,2) (3,2) (4,2) (5,2) ...
(1,3) (2,3) (3,3) (4,3) (5,3) ...
(1,4) (2,4) (3,4) (4,4) (5,4) ...
(1,5) (2,5) (3,5) (4,5) (5,5) ...
1 2 3 4 5 6 7 8
(1,1) (2,1) (1,2) (1,3) (2,2) (3,1) (4,1) (3,2)

Figure 3. Assignment scheme for illustrating the cardinality of the set of rational numbers (after Hahn, p. 1595).

This strategy was the basis for Georg Cantor's (perhaps counter-intuitive) proof that the cardinal number of the set of rational numbers is aleph null. The set of ordered pairs contains the set of rationals--for, when ordered pairs are converted to rationals, as in the algorithm above, some are equivalent in value although not in visual form--1/2 and 2/4 are equivalent in value but do not look the same. However, the rational numbers contain the natural numbers as a subset. Thus, the cardinal number of the rationals is sandwiched between two sets each of whose cardinal number is aleph null--consequently, the set of rational numbers must also have cardinal number aleph null. Thus, the set of numbers y/x exactly covers the screen.

REPETITION OF THE CANTOR ALGORITHM

Suppose that the Cantor Algorithm has been applied once to a map: to the red tree in Figure 2, for example. Each node of the red tree will have a distinct, unique rational number label based on its position relative to the top and the side of the screen and in relation to other nodes. In a different layer, label the black street map in the same manner. It appears that at most one node on these two maps will have the same label; indeed, using rational number labeling, two nodes have the same label if and only if they are represented by the same pixel location in both maps. What is desired is to identify (match up or glue together) a node from one map with a node from the other map both of which represent the same physical location.

MERGING ALGORITHM

The following algorithm offers a strategy for making such an identification. It can be executed by buffering nodes, in an iterative fashion, and identifying two nodes at the first instance a node from one map falls into a node-buffer in another map.

1. Draw a buffer of one pixel width around each node in the red map. Assign identical labels to any pair of nodes on red and black maps both of which lie within or on the buffer.

2. Draw a buffer of one pixel width around each buffered node created in step 1. Assign identical labels to any pair of nodes on the

red and black maps both of which lie within or on this newly drawn ring buffer region.

3. Repeat the process until one pair of buffer zones comes into contact with each other. Then stop the process.

If this Merging Algorithm provides a complete identification of nodes (with no extra nodes lying in the region outside the buffered zone), with identical connection patterns in both, then the two maps are said to be pairwise "well-matched." There is no difficulty in merging the two maps from different sources.

TRANSLATION OF THE MERGING ALGORITHM

Maps may fail to be pairwise well-matched for a variety of reasons. The following list suggests some reasons for such failure, but it may not be exhaustive.

1. The numbers of nodes in the red and black maps are different from each other.
2. The number of nodes in the two maps is the same, but the number of edges is different.
3. The number of nodes and edges is the same in both maps but the topology, the pattern of connection, is different.

4. Some or all of the three factors above may hold, in whole or in part, but the offset of pattern is so large that it falls outside the would-be buffer zone and gets visually confused with other pattern.

Nystuen notes in some of his examples that some parts of each map in the pair are shifted; the structure is recognizable from one to the other, but offset some significant amount. One way to deal with situations of these sorts is to permit translation, locally, of parts of the graph in one map. The Merging Algorithm will run in a satisfactory manner, up to a point. Beyond this point, nodes fall outside buffer zones. In such situations, use the diameter of the buffer zone (number of maximum number of pixels that came about in the use of the Merging Algorithm) as a translation scaling number. Add this number to each coordinate (in the local region) in one map. Now run the Merging Algorithm. Repeat the procedure as needed.

DIRECTIONS FOR FURTHER WORK

This issue of merging maps appears rich from an abstract

viewpoint. Some of the other directions one might consider involve use of Hasse's Algorithm applied to graphs, development of error detection and correction criteria, use of sum-graphs or a similar strategy in which the topology of the graph guides the labeling pattern, and any of a host of theorems related to these topics.

Ann Arbor, MI
April, 1996.

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4.

INDEX TO VOLUMES I (1990) TO VOL. VI

VOL. VII, NUMBER 1.

TABLE OF CONTENTS

1. PHOTO ESSAY
THE GREENING OF DETROIT, 1975-1992:
PHYSICAL EFFECTS OF DECLINE
John D. Nystuen, Rhonda Ryznar, Thomas Wagner
2. ALGEBRAIC ASPECTS OF RATIOS
Sandra Lach Arlinghaus
3. EDUCATION GRAPHIC ESSAY
U.S. ROUTE 12 BUFFER
Daniel Jacobs
4. INDEX TO VOLUMES I (1990) TO V (1995).

Vol. VI, No. 2, December, 1995. TABLE OF CONTENTS

Elements of Spatial Planning: Theory--Part I.
Sandra L. Arlinghaus
MapBank: An Atlas of On-line Base Maps
Sandra L. Arlinghaus
International Society of Spatial Sciences

Volume VI, Number 1, June, 1995.

Fifth Anniversary of Solstice
New format for Solstice and new Technical Editor
Richard Wallace. Motor Vehicle Transport and Global Climate Change:
Policy Scenarios.

Expository Article. Discrete Mathematics and Counting Derangements in Blind Wine Tastings.
Sandra L. Arlinghaus, William C. Arlinghaus, John D. Nystuen

Volume V, No. 2, Winter, 1994.
Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary: The Paris Metro: Is its Graph Planar?
Planar graphs; The Paris Metro; Planarity and the Metro; Significance of lack of planarity.
Sandra Lach Arlinghaus: Interruption!
Classical interruption in mapping; Abstract variants on interruption and mapping; The utility of considering various mapping surfaces--GIS; Future directions.
Reprint of Michael F. Dacey: Imperfections in the Uniform Plane. Forewords by John D. Nystuen.
Original (1964) Nystuen Foreword; Current (1994) Nystuen Foreword; The Christaller spatial model; A model of the imperfect plane; The disturbance effect; Uniform random disturbance; Definition of the basic model; Point to point order distances; Locus to point order distances; Summary description of pattern; Comparison of map pattern; Theoretical model; Point to point order distances; Locus to point order distances; Summary description of pattern; Comparison of map pattern; Theoretical order distances; Analysis of the pattern of urban places in Iowa; Almost periodic disturbance model; Lattice parameters; Disturbance variables; Scale variables; Comparison of M(2) and Iowa; Evaluation; Tables.
Sandra L. Arlinghaus: Construction Zone: The Brakenridge-MacLaurin Construction.
William D. Drake: Population Environment Dynamics: Course and Monograph--descriptive material.

Volume V, No. 1, Summer, 1994.
Virginia Ainslie and Jack Licate: Getting Infrastructure Built. Cleveland infrastructure team shares secrets of success;
What difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; How can local communities translate these successes to their own settings?
Frank E. Barmore: Center Here; Center There; Center, Center Everywhere.
Abstract; Introduction; Definition of geographic center; Geographic

center of a curved surface; Geographic center of Wisconsin; Geographic center of the conterminous U.S.; Geographic center of the U.S.; Summary and recommendations; Appendix A: Calculation of Wisconsin's geographic center; Appendix B: Calculation of the geographical center of the conterminous U.S.; References.

Barton R. Burkhalter: Equal-Area Venn Diagrams of Two Circles: Their Use with Real-World Data

General problem; Definition of the two-circle problem; Analytic strategy; Derivation of B% and AB% as a function of r(B) and d(AB).

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary, John D. Nystuen. Los Angeles, 1994 -- A Spatial Scientific Study.

Los Angeles, 1994; Policy implications; References; Tables and complicated figures.

Volume IV, No. 2, Winter, 1993.

William D. Drake, S. Pak, I. Tarwotjo, Muhilal, J. Gorstein, R. Tilden. Villages in Transition: Elevated Risk of Micronutrient Deficiency.

Abstract; Moving from traditional to modern village life: risks during transtion; Testing for elevated risks in transition villages; Testing for risk overlap within the health sector; Conclusions and policy implications

Volume IV, No. 1, Summer, 1993.

Sandra L. Arlinghaus and Richard H. Zander: Electronic Journals: Observations Based on Actual Trials, 1987-Present.

Abstract; Content issues; Production issues; Archival issues; References

John D. Nystuen: Wilderness As Place.

Visual paradoxes; Wilderness defined; Conflict or synthesis; Wilderness as place; Suggested readings; Sources; Visual illusion authors.

Frank E. Barmore: The Earth Isn't Flat. And It Isn't Round Either:

Some Significant and Little Known Effects of the Earth's Ellipsoidal Shape.

Abstract; Introduction; The Qibla problem; The geographic center; The center of population; Appendix; References.

Sandra L. Arlinghaus: Micro-cell Hex-nets?

Introduction; Lattices: Microcell hex-nets; References

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary:

Sum Graphs and Geographic Information.

Abstract; Sum graphs; Sum graph unification: construction; Cartographic application of sum graph unification; Sum graph

unification: theory; Logarithmic sum graphs; Reversed sum graphs; Augmented reversed logarithmic sum graphs; Cartographic application of ARL sum graphs; Summary.

Volume III, No. 2, Winter, 1992.

Frank Harary: What Are Mathematical Models and What Should They Be? What are they?

Two worlds: abstract and empirical; Two worlds: two levels; Two levels: derivation and selection; Research schema; Sketches of discovery; What should they be?

Frank E. Barmore: Where Are We? Comments on the Concept of Center of Population.

Introduction; Preliminary remarks; Census Bureau center of population formulae; Census Bureau center of population description; Agreement between description and formulae; Proposed definition of the center of population; Summary; Appendix A; Appendix B; References.

Sandra L. Arlinghaus and John D. Nystuen: The Pelt of the Earth: An Essay on Reactive Diffusion.

Pattern formation: global views; Pattern formation: local views; References cited; Literature of apparent related interest.

Volume III, No. 1, Summer, 1992.

Harry L. Stern: Computing Areas of Regions with Discretely Defined Boundaries.

Introduction; General formulation; The plane; The sphere; Numerical examples and remarks; Appendix--Fortran program.

Sandra L. Arlinghaus, John D. Nystuen, Michael J. Woldenberg: The Quadratic World of Kinematic Waves.

Volume II, No. 2, Winter, 1991.

Reprint of Saunders Mac Lane: Proof, Truth, and Confusion, The Nora and Edward Ryerson Lecture at The University of Chicago in 1982.

The fit of ideas; Truth and proof; Ideas and theorems; Sets and functions; Confusion via surveys; Cost-benefit and regression; Projection, extrapolation, and risk; Fuzzy sets and fuzzy thoughts; Compromise is confusing.

Robert F. Austin: Digital Maps and Data Bases: Aesthetics versus accuracy.

Introduction; Basic issues; Map production; Digital maps; Computerized data bases; User community.

Volume II, No. 1, Summer, 1991.

Sandra L. Arlinghaus, David Barr, John D. Nystuen:

The Spatial Shadow: Light and Dark -- Whole and Part.

This account of some of the projects of sculptor David Barr attempts to place them in a formal systematic, spatial setting based on the postulates of the science of space of William Kingdon Clifford (reprinted in Solstice, Vol. I, No. 1.).

Sandra L. Arlinghaus: Construction Zone--The Logistic Curve. Educational feature--Lectures on Spatial Theory.

Volume I, No. 2, Winter, 1990.

John D. Nystuen: A City of Strangers: Spatial Aspects of Alienation in the Detroit Metropolitan Region.

This paper examines the urban shift from "people space" to "machine space" (see R. Horvath, Geographical Review, April, 1974) in the Detroit metropolitan regions of 1974. As with Clifford's Postulates, reprinted in the last issue of Solstice, note the timely quality of many of the observations.

Sandra Lach Arlinghaus: Scale and Dimension: Their Logical Harmony.

Linkage between scale and dimension is made using the Fallacy of Division and the Fallacy of Composition in a fractal setting.

Sandra Lach Arlinghaus: Parallels Between Parallels.

The earth's sun introduces a symmetry in the perception of its trajectory in the sky that naturally partitions the earth's surface into zones of affine and hyperbolic geometry. The affine zones, with single geometric parallels, are located north and south of the geographic parallels. The hyperbolic zone, with multiple geometric parallels, is located between the geographic tropical parallels. Evidence of this geometric partition is suggested in the geographic environment--in the design of houses and of gameboards.

Sandra L. Arlinghaus, William C. Arlinghaus, and John D. Nystuen: The Hedetniemi Matrix Sum: A Real-world Application.

In a recent paper, we presented an algorithm for finding the shortest distance between any two nodes in a network of n nodes when given only distances between adjacent nodes (Arlinghaus, Arlinghaus, Nystuen, Geographical Analysis, 1990). In that previous research, we applied the algorithm to the generalized road network graph surrounding San Francisco Bay. Here, we examine consequent changes in matrix entries when the underlying adjacency pattern of the road network was altered by the 1989 earthquake that closed the San Francisco--Oakland Bay Bridge.

Sandra Lach Arlinghaus: Fractal Geometry of Infinite Pixel Sequences: "Super-definition" Resolution?

Comparison of space-filling qualities of square and hexagonal pixels.

Sandra Lach Arlinghaus: Construction Zone--Feigenbaum's number; a triangular coordinatiztion of the Euclidean plane; A three-axis coordinatization of the plane.

Volume I, No. 1, Summer, 1990.

Reprint of William Kingdon Clifford: Postulates of the Science of Space.

This reprint of a portion of Clifford's lectures to the Royal Institution in the 1870s suggests many geographic topics of concern in the last half of the twentieth century. Look for connections to boundary issues, to scale problems, to self-similarity and fractals, and to non-Euclidean geometries (from those based on denial of Euclid's parallel postulate to those based on a sort of mechanical `polishing'). What else did, or might, this classic essay foreshadow?

Sandra Lach Arlinghaus: Beyond the Fractal.

The fractal notion of self-similarity is useful for characterizing change in scale; the reason fractals are effective in the geometry of central place theory is because that geometry is hierarchical in nature. Thus, a natural place to look for other connections of this sort is to other geographical concepts that are also hierarchical. Within this fractal context, this article examines the case of spatial diffusion.

When the idea of diffusion is extended to see "adopters" of an innovation as "attractors" of new adopters, a Julia set is introduced as a possible axis against which to measure one class of geographic phenomena. Beyond the fractal context, fractal concepts, such as "compression" and "space-filling" are considered in a broader graph-theoretic setting.

William C. Arlinghaus: Groups, Graphs, and God.

Sandra L. Arlinghaus: Theorem Museum--Desargues's Two Triangle Theorem from projective geometry.

Construction Zone--centrally symmetric hexagons.