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Ann Arbor, MI
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1. PHOTO ESSAY

THE GREENING OF DETROIT, 1975-1992: PHYSICAL EFFECTS OF DECLINE

John D. Nystuen, The University of Michigan
Rhonda Ryznar, The University of Michigan
Thomas Wagner, Environmental Research Institute of Michigan

Figure 1. Landsat change image of Detroit showing changes in urban greenness from 1975 to 1992. Imagery and analysis are joint ventures between Environmental Research Institute of Michigan (ERIM) and The University of Michigan, College of Architecture and Urban Planning. Green areas show tracts with greenness increase; red areas with greenness decrease; black areas, no change. Change data derived from TM 1992 and MSS 1975 images of vegetation reflectance.

Figure 2. Ground truth, green area (reference to Figure 1). Areas of increased greenness are places where the social system is stressed. Houses are abandoned or destroyed. Much of the territory is vacant land. Sidewalks and alley surfaces are broken and overgrown with weeds. The overall effect is increased greenness over the time period. Indeed, we observed pheasants in overgrown parts of the central city. Much of the green part of the image in Figure 1, in the inner city of Detroit, are territories of this sort.

Figure 3. Ground truth, no change area--black (reference to Figure 1). In some neighborhoods in the central part of Detroit, the social structure is intact. The physical properties of these neighborhoods reflect this sustained social organization. This neighborhood has not changed much and shows as mostly black in the image (Figure 1). This is also true of cemeteries and parks where the vegetation cover has not changed over the time period.

Figure 4. Ground truth, red area (reference to Figure 1). In much of the outer edge of the city of Detroit, shown by the overlay of census tracts in Figure 1, the social structure of the neighborhoods remains intact but the physical changes indicate decline in vegetation cover. By field investigation, we noted open streets with an occasional single elm tree as shown in this figure. We attribute the decrease in greenness to the effects of the Dutch elm disease which in the decade of the 70s destroyed virtually the entire elm tree population. In 1975, at the time of the image, this process was in progress. By 1992, very few elm trees remained. In the early period, much of the street surface was shaded by these trees, whereas currently that is not the case, despite replanting of small trees.

Figure 5. Figure 5 shows a recently completed automobile plant in the City of Detroit. This location shows in Figure 1 as a red and black region with no green. The red resulted from conversion of low income neighborhoods, once green, to factory roof and parking lot surface. The black areas were pre-existing industrial areas that lacked vegetation and still have no vegetation.

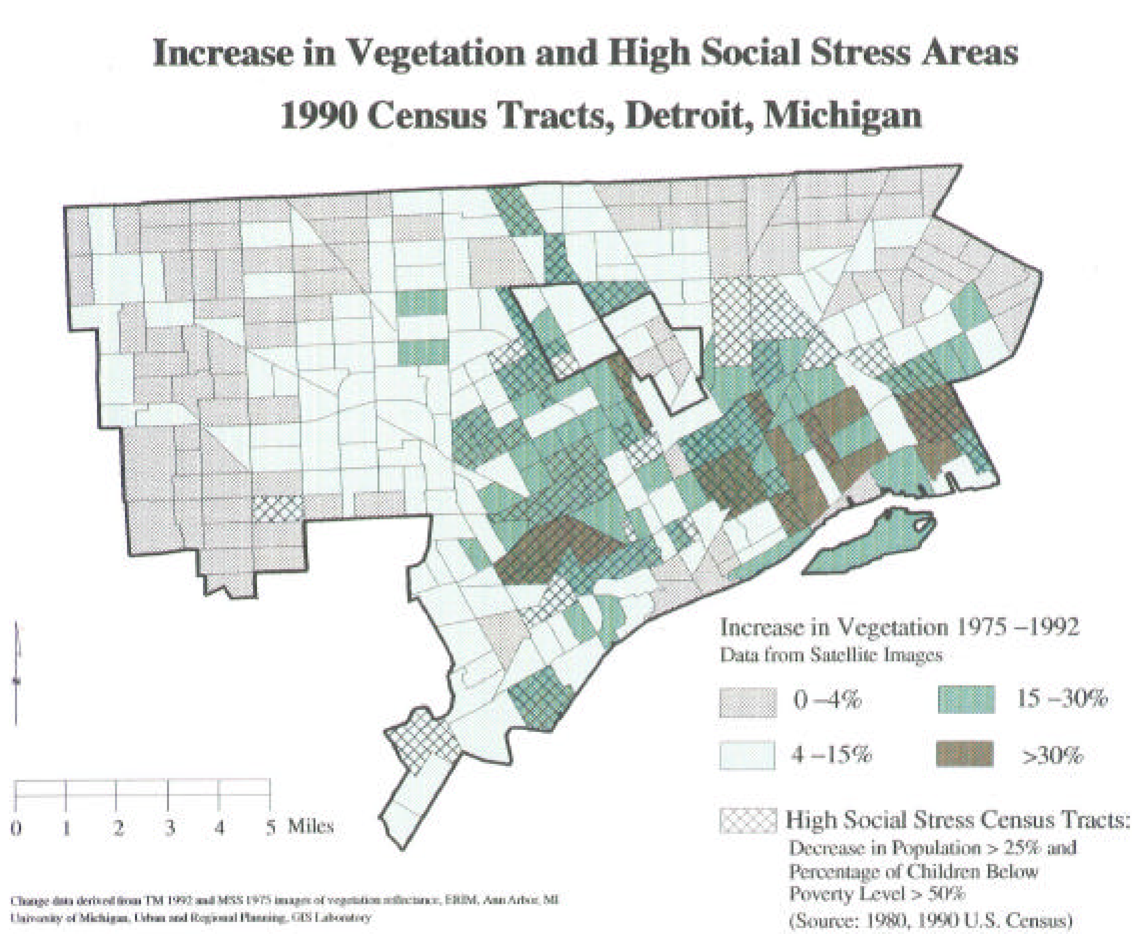

Figure 6. In Figure 6, census tracts with the largest increase in greenness are shown with the darkest green and brown tones. Superimposed on this colored pattern are census tracts of high social stress (cross-hatched). High social stress census tracts are defined as those tracts with decrease in population greater than 25% between 1980 and 1990 and with more than 50% of children below the poverty level. There is substantial correspondence between the two patterns.

Landsat Data Analysis by ERIM

Two Landsat data sets: MSS data from May 10, 1975 and TM data from May 16, 1992 were employed in the analysis. A restoration resampling algorithm rectified the images to the same spatial reference frame (the State Plane Coordinate system). A principal component procedure was employed to create a "greenness" vector from signals returned by several spectral bands available from the satellites' instruments. The "greenness" vector has been identified with intensity of vegetation. The resampling and rectification allows for the creation of a change image measuring the difference in "greenness" per pixel (at 25 meter change image was then divided into three classes: increased greenness (colored green), no change in greenness (colored black), and decrease in greenness (colored red) to create the image shown here. Detroit census tracts (1990) were superimposed on this image and counts of pixels by each class by census tracts could then be compared with census data on socioeconomic variables. A map of percent increase in vegetation was created by classifying census tracts by the proportion of green pixels to total pixels contained in each. The remote sensing analysis was carried out by the Environmental Research Laboratory of Michigan (ERIM), Ann Arbor, Michigan. Comparisons to socioeconomic data were done by the Urban and Regional Planning GIS Laboratory, University of Michigan.

ALGEBRAIC ASPECTS OF RATIOS

Sandra Lach Arlinghaus
Institute of Mathematical Geography;
The University of Michigan;
Community Systems Foundation

To Douglas R. McManis, Editor of the *Geographical Review*,
whose cleverness with words has helped this author tame many a distorted thought!
Congratulations on your unparalleled service to the American Geographical Society
(May 30, 1996).

Article to appear in *Solstice: An Electronic Journal of Geography and Mathematics,*
Vol. VI, No. 1, June, 1996 (http://www-personal.umich.edu/~sarhaus/image)

A particularly exciting feature of the Internet permits documents stored on the World Wide Web to incorporate graphics, as well as text, on the same page. It is generally easy to cut and paste text from a word processor into the online file in which the home page is being created (into the html file). It is almost as easy to upload an image of a scanned photo (for example, saved in gif or jpg format) from a computer in one's home to that same online file (or to one linked to it). Indeed, the whole technical process is so easy, once one has a small set of keys that open merely a few doors, that documents can be created in advance of even simple knowledge about enduring concepts.

Thus, one might anticipate a host of web sites that do not take account of simple cartographic principles or of drafting lessons, let alone elements of written linguistic or mathematical style. From a more positive standpoint, however, one might see that the creation of web documents can serve as a favorable opportunity to cast new light on traditional material. Practice can motivate theory: in mathematics, geography, linguistics, or elsewhere.

Consider the simple task of creating web stationery incorporating an established logo. The American Geographical Society (New York) has given permission to experiment with its relatively symmetric logo that includes a sphere (Bird, M. L., 1996). When the logo is scanned at a relatively low resolution that even older or cheaper scanners can produce, and loaded as a scanned image onto a Web page, the result appears as it does below (Figure 1). The shape of the globe appears correct to the eye, but the image is too large to be viewed on one moderately sized computer screen. Clearly, this situation is unsatisfactory for general use: a large image requires not only too much visual space, but also too much computer space, and too much time to come up on the screen. The obvious answer is to scale the image down in size.

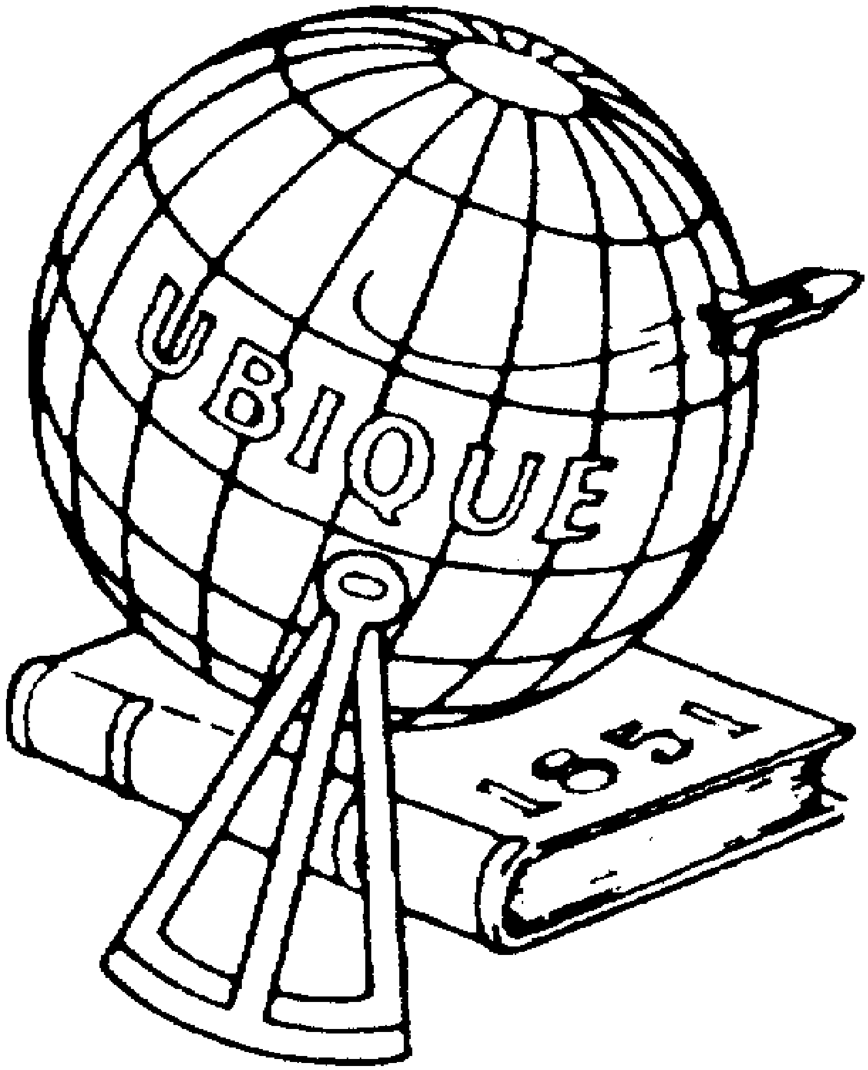

Figure 1.
Logo of the American Geographical Society, height 922 pixels, width 810 pixels.

With a rectangular image such scaling is of course a simple matter: divide the width in half (for example). To retain the same shape of image as the original, also divide the height in half (Figure 2). More generally, to retain shape when altering the scale of a two-dimensional rectangular shape, preserve the aspect ratio--the ratio of the horizontal dimension of an image to the vertical dimension of an image. Varying the aspect ratio of an image causes distortion, as in Figure 3, in which the width is divided by 2 and the height is divided by 3.

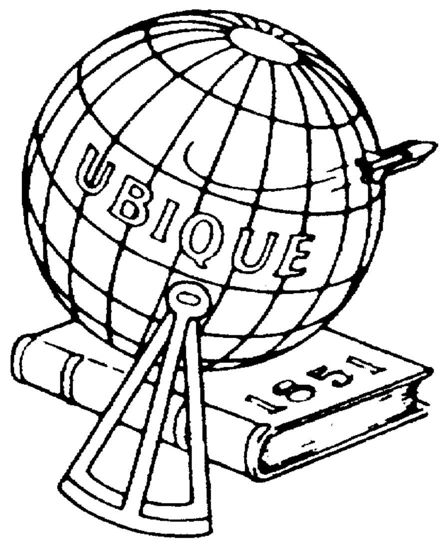

Figure 2.
Logo of the American Geographical Society, height of Figure 1 divided by 2 and width divided by 2. Correctly scaled image.

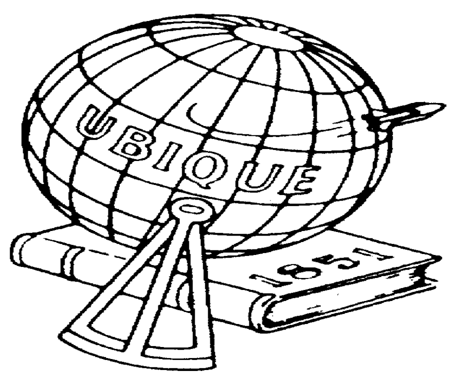

Figure 3.
Logo of the American Geographical Society, height of Figure 1 divided by 3 and width divided by 2. Distorted image.

PIXEL ALGEBRA

In the case of the AGS logo, the scanned image in Figure 1 (a .gif file) measured 810 pixels in width and 922 pixels in height. Pixels are the fundamental units in which these images are measured; fractions of pixels are not permitted. The algebra of pixels is an integer algebra that can be viewed to operate on a bounded, finite set of pixels (the dimensions of the cathode ray tube). In the image of the AGS logo, both dimensions are even numbers and so clearly both are divisible by 2. The aspect ratio, 810/922, can be preserved by taking 1/2 of each dimension; 810/922 = (405*2)/(461*2) = 405/461. Are both numerator and denominator divisible by 3?

In times when "flashy" applications relied more on elegant abstractions, the distributive law was often a source of the unusual or the "cute."

DISTRIBUTIVE LAW

For integers in a domain of two operations, + and *:

Suppose that a, b, and c are integers. Then a*(b+c)=a*b + a*c.

One application of the distributive law yields the "trick" that to determine whether or not a number is divisible by three, add the digits which form it; repeat the process until a single digit is reached. If that single digit is divisible by 3, then the entire number is divisible by 3. In the case of the AGS logo dimensions 810 becomes 8+1+0=9 which is divisible by 3 so that 810 is also divisible by 3. The number 922 becomes 9+2+2=13 which becomes 1+3=4 which is not divisible by 3 so that 922 is not divisible by 3. Thus, 1/3 would not be a scaling factor for the scanned AGS logo that would preserve the aspect ratio: 3 is not a factor of both the numerator (810) and the denominator (922) of the aspect ratio of 810/922.

To see why this procedure is an application of the distributive law, work through 810 as an example, and note that numbers such as 9, 99, 999 are always divisible by 3 (conversation with W. C. Arlinghaus):

810 = 8*100 + 1*10 + 0*1 --- first use of distributive law

= 8*(99+1) + 1*(9+1) + 0*1 --- partitioning of powers of ten into convenient summands

= 8*99 + 8*1 + 1*9 + 1*1 + 0*1 --- second use of distributive law

= 8*99 + 1*9 + 8*1 + 1*1 + 0*1 --- commutative law of addition (a+b=b+a)

notice that the sum 8*99 + 1*9 is divisible by 3 because 9 and 99 are divisible by 3:

8*99 + 1*9 = (8*33 + 1*3)*3--third use of the distributive law, so now,

810 = (8*33 + 1*3)*3 + (8*1 + 1*1 + 0*1) and for the number 810 to be divisible by 3 (fourth use of the distributive law) all that is thus required is for the right summand in parentheses, 8*1 + 1*1 + 0*1 to be divisible by 3. Hence, the rule of three, for determining whether or not a given integer is divisible by 3 becomes clear.

A corresponding rule of nine is not difficult to understand, as are numerous other shortcuts for determining divisibility criteria. Clearly, one need test candidate divisors only up to the square root of the number in question. However, when one is faced with an image on the screen, it would be nice not to have taken the trouble (however little) of finding that 810 is divisible by 3, only to find that 922 is not. A far better approach is to rewrite each number using some systematic procedure and then compare a pair of expressions to determine, all at once, which numbers are divisors of BOTH 810 and 922. For this purpose, the Fundamental Theorem of Arithmetic is critical.

FUNDAMENTAL THEOREM OF ARITHMETIC

Any positive integer can be expressed uniquely as a product of powers of prime numbers (numbers with no integral divisors other than themselves and one).

Thus, in the AGS logo example,

810=2*405=2*5*81=2*3*3*3*3*5 (superscripts avoided by repetitive multiplication) and,

922=2*461.

The number 810 was easy to reduce to its unique factorization into powers of primes; one might not know whether or not 461 is a prime number or whether further reduction is required to achieve the prime power factorization of 922.

To this end, the Sieve of Eratosthenes (the same Librarian at Alexandria who measured the circumference of the Earth) works well. To use the sieve, simply test the prime numbers less than the square root of the number in question. The square root of 461 is about 21.47. So, the only primes that can possibly be factors are: 2, 3, 5, 7, 11, 13, 17, and 19. Clearly, 2, 3, and 5 are not factors of 461. A minute or two with a calculator shows that 7, 11, 13, 17, and 19 are also not factors of 461. Notice that these calculations need only be made once--when one has the unique factorization all divisors of both numbers are known from looking at the two factorizations, together. Thus:

810 = 2*3*3*3*3*5

922 = 2 * 461

and 2 is the only factor common to both numbers.

PRESERVATION OF THE ASPECT RATIO

Because the only factor the two numbers 810 and 922 have in common is 2 it follows that the only scaling factor that can be used, THAT WILL PRESERVE THE ASPECT RATIO, is 1/2. Had 1/3 or some other ratio been employed, a distorted view of the AGS logo would have been the result. Figure 4 shows the result of using a scaling factor of 1/10, so that the image is 81 pixels wide and 92 pixels high (rounded off from 92.2 pixels). The image looks good, but in fact the aspect ratio was not preserved. Level of sensitivity to image distortion will vary with the individual; it is important to have absolute techniques that guarantee correct answers. Once one knows them, then one can choose when, and when not, to violate them.

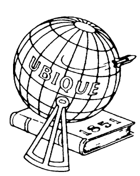

Figure 4.

Logo of the American Geographical Society, height and width divided by 10, aspect ratio not preserved. Distortion present but not evident.

Reduction of the AGS logo by 1/2, producing an image 1/4 of the original size, may still not be desirable. One can use the unique factorization to build what is desired. The value of 810 has a number of factors; thus, one might choose to rescan the image, holding the width at 810 pixels and shaving just a bit off the height (there appears to be room to do so in the background without touching the line drawing of the globe). Now, to create the possibility of various scaling factors, consider a tiny sliver removed to create a height of 920 = 2*2*2*5*23; there is an extra factor of 5 that 920 has in common with 810 that 922 did not, so all of 1/2, 1/5, and 1/10 (one over 2*5) are scaling factors that will preserve the aspect ratio. However, one might wish still more possible scaling factors; if a slightly larger sliver can be removed, so that the height of the scanned image is 900 pixels, with 900 = 2*2*3*3*5*5, then 810 and 900 have common prime power factors of 2, 3, 3*3, and 5, so that the set of scaling factors has been substantially expanded to include all of:

1/2, 1/3, 1/9, 1/5,	combination of four things one at a time--4!/(1!*3!)
1/6, 1/18, 1/10; 1/27, 1/15; 1/45	combination of four things two at a time--4!/(2!*2!)
1/54, 1/30, 1/90, 1/135	combination of four things three at a time--4!/(3!*1!)
1/270	combination of four things four at a time--4!/(4!*0!).

This set of values for the height, in pixels, should offer enough choices in various size ranges for the stationery to have a logo of reasonable size on it.

IMAGE SECURITY

In creating images, it is useful to have an aspect ratio with numerator and denominator with many common factors; greater flexibility in changing the image is a consequence. However, there may be situations, particularly on the Internet where downloading is just a right-click of the mouse away, in which one wishes to inhibit easy rescaling of an image. When that is the case, unique factorization via the Fundamental Theorem of Arithmetic again yields an answer: choose to scan the image so that the numerator and denominator of the aspect ratio are relatively prime--that is, so that numerator and denominator have no factors in common other than 1. In the case of the AGS logo, altering the aspect ratio from the original of 810/922 to 810/923 would serve the purpose: 923 is not divisible by any of 2, 3, or 5. Anyone downloading the AGS logo could not resize it without distortion: distortion, for once, becomes the mapmaker’s friend.

FINALE

The abstract lessons learned in Modern Algebra are as important in the electronic world as they are elsewhere; indeed, the current technical realm therefore offers a refreshing host of new example to motivate theory, as suggested in this simple scaling example that drew on a variety of algebraic concepts including: the distributive law, the commutative law of addition, the Fundamental Theorem of Arithmetic and the needed associated concept of prime number, facts involving square roots and divisors, the Sieve of Eratosthenes, basic material on permutations and combinations together with the convention that 0!=1, and the idea of relatively prime numbers. The web is a rich source of example when one brings a rich source of abstract liberal arts training to it.

CITATIONS

Bird, M. L. Executive Director, American Geographical Society, May, 1996.

REFERENCES

For the reader wishing to learn more about modern algebra--not easy reading for most, but well worth looking at to gain some appreciation for the vastness of this field.

Birkhoff, G. and Mac Lane, S. A Survey of Modern Algebra. Macmillan,
Herstein, I., N. Topics in Algebra.
Mac Lane, S. and Birkhoff, G. Algebra.

3. EDUCATION GRAPHIC ESSAY

US ROUTE 12 BUFFER

Daniel Jacobs
The University of Michigan

A child's turf is an ordered space in which the child plots data points at important spots. I will use maps generated on a geographical information system (Atlas GIS for Windows, v. 3.0) as one set of tools to broaden student perspective. The student population will be mainly 9-10 year old students from urban and suburban settings. I hope to initially probe three areas of student perspectives:

a. Their sense of space immediately around them in their daily lives, including their school, home, block, city, county, and state;
b. Their sense of the order of the physical world around them, including cardinal directions, directions of major roads and highways, positions of buildings, cities, and states along U.S. 12 (base map in Figure 1);
c. Their knowledge of the people, buildings, and stories behind their known turf (sample of a map set shown in Figure 2).

I am most interested in finding where different student maps end, where their Unknown begins, and what they think is beyond.

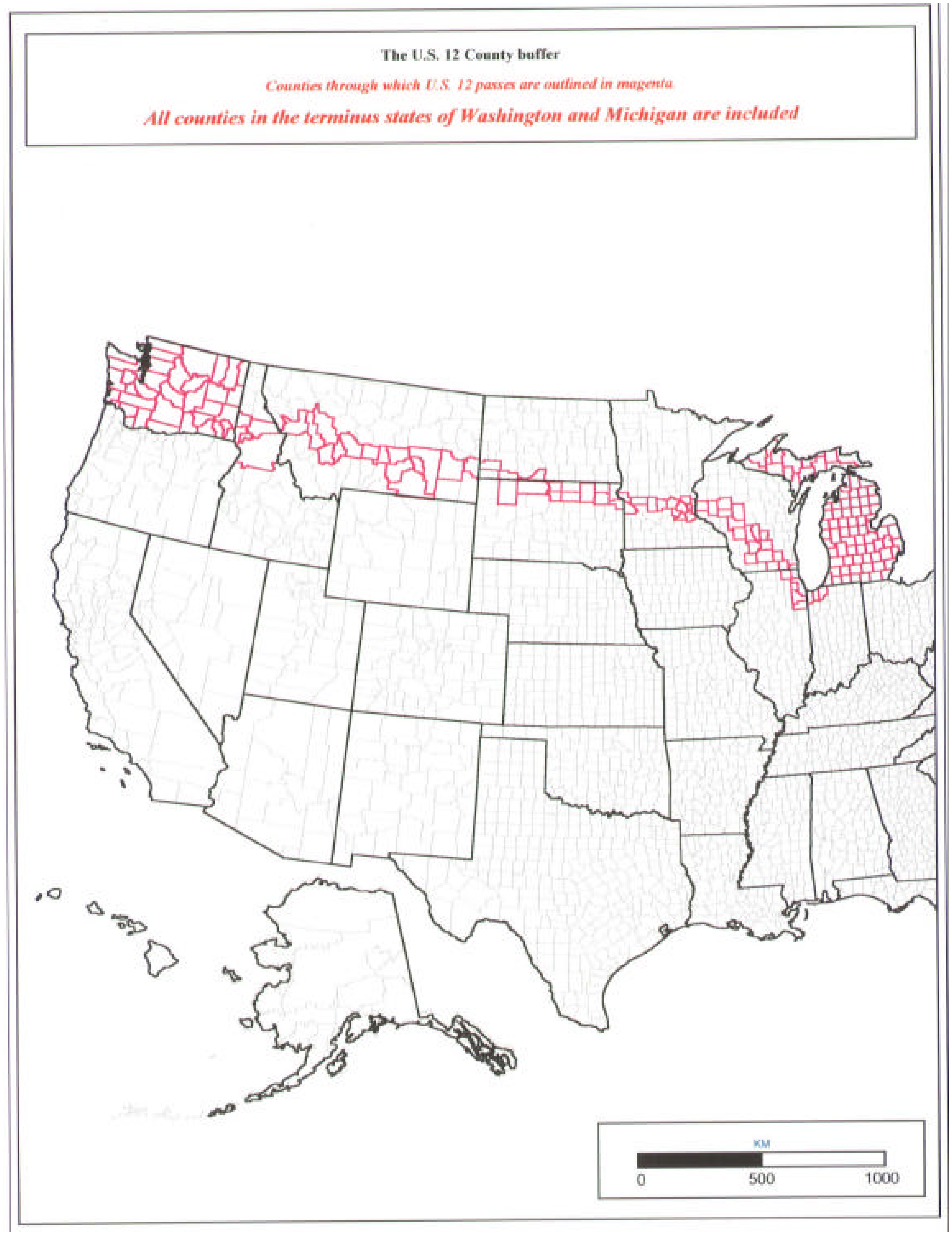

Figure 1. U.S. 12 county buffer.

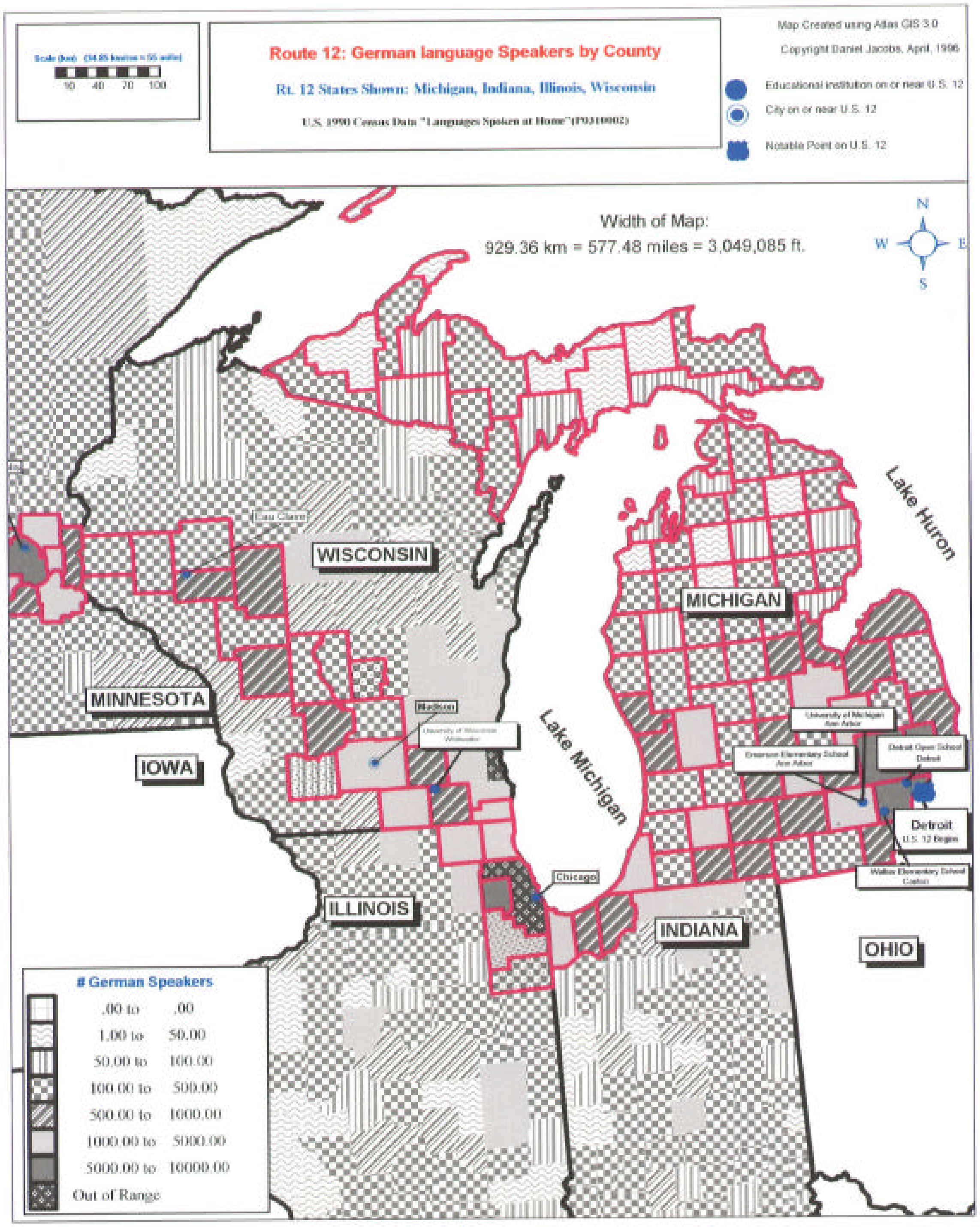

Figure 2. Number of German speakers by county; U.S. 12 buffer outlined in magenta.

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3.

WEB FRACTALS

Sandra Lach Arlinghaus
The University of Michigan

Documents written in HTML (Hyper-Text Markup Language) for WebSites not only offer the capability to place Home Pages on the World Wide Web, but also extend the dimension in which typesetting takes place. Broadly viewed, from a typesetting standpoint, HTML appears to be equivalent to a subset of commands of Plain TeX. One notable exception arises, however, in the commands in HTML that enable the user to specify links to other documents, graphical or textual. A Home Page, or any other page or set of pages (paper or electronic), might be viewed as a one dimensional string of letters broken into words. Graphics offer an opportunity to extend the text into other dimensions. So too do the links from one web page to another. They offer an extraordinary capability to reach out within the text itself: an opportunity to “fill” a text-shed beyond the linear text-stream. And of course, one can mix and match links, graphics, text, and whatever is available on the web. Without loss of generality, interest might be confined to text because it is with text that the extension into extra dimensions appears straightforward.

Thus, it becomes of interest to ask how much of a text-shed is filled by the activity of creating links and how one might measure such filling of text-space. Concepts from fractal geometry and from chaos theory will be employed to consider these issues.

GLOBAL WEB VIEW

The global perspective employed in much of modern mathematics focuses not on the individual objects being studied but on the sets of transformations that enable one to move from one object, or type of object, to another. Thus, one learns group theory by studying the various morphisms that link groups, rather than by focusing on the tables that describe individual groups. Linkages expose structure.

Thus, one view of the Web would be to shrink to nodes all home pages and focus only on the links that exist between sites. The set of nodes is finite, but it is unbounded--one can imagine going beyond any upper bound on the number of sites, simply by adding one more. The set of links, too, is finite and unbounded. The morass of links defies good graphical description. Search engines try to make sense of the Web through various organizational schemes. One view of the pattern is as a cataloguer’s nightmare; another is as a chance to introduce order into seeming abstract chaos.

Yet another might be to engage in graphical analysis based on some sort of ordering scheme that permits the assessment of extent of space filled. The concept of attractors and repellers seems an important one to search engines. One such ordering scheme might be such as suggested below. Of course there are many others, too.

$y=x$ means link…both to and from me to you
$y>x$ means link…me to you
$y<x$ means link…me from you

The algebraic, geometric, and topological structure of the web are important to analyze: the space-filling ideas represented in fractal geometry suggest one style of approach. Any deep, careful mathematical analysis should, however, offer a systematic means to evaluate the current status of web content, and more important, provide a continuing framework in which to guide and understand web development. These comments offer mere hints at directions substantive work might take; hopefully, they offer encouragement to consider the web itself as a source of various styles of research opportunity.

Ann Arbor, MI
May, 1996.

2.

PART II. ELEMENTS OF SPATIAL PLANNING: THEORY.

MERGING MAPS: NODE LABELING STRATEGIES

Sandra Lach Arlinghaus
The University of Michigan

Different inventories of roads produce different maps; two different maps can represent one set of actual roads. How is a planner to rectify this situation? To consider the problem abstractly, it is useful to characterize the road network as a graph, composed of nodes and edges joining nodes. Such style of consideration, as a graphical puzzle, dates from 1736 and the Konigsberg Bridge puzzle and from Hamilton's "Around the World" puzzle of 1859 (Harary, pp. 1, 4); as a planner's dilemma it has no doubt always been with us; and, as a cartographer's delight, it manifests itself over and over again in projection selection, a result of the one-point compactification theorem (visualized as stereographic projection) forcing us to recognize that the sphere can never be flattened out into the plane.

In Figure 1, two portions of road maps based loosely on the downtown Ann Arbor, Michigan, street map are shown superimposed—a conceptual view of a practical problem noted by John Nystuen, and implemented at the level of mapping by Andrea Frank and Jyothi Palathinkara (University of Michigan). As Nystuen et al. Noted (unpublished communication), these nodes are, in some sense at least, fairly "close"--one can recognize, visually, what the correspondence is to be from one map to the other. The viewpoint adopted here is that as graph-theoretic trees they are homeomorphic (have the same structure or connection pattern).

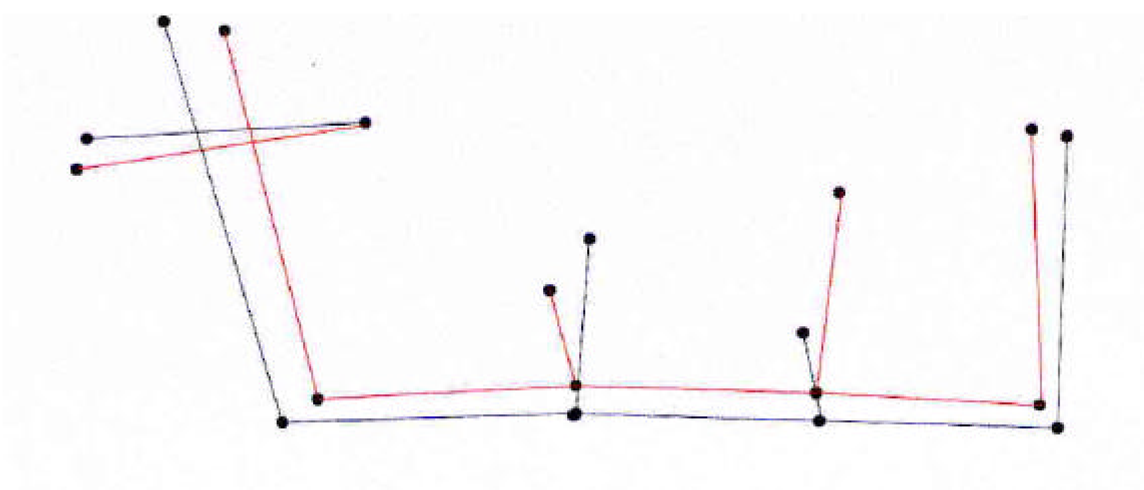

Figure 1. Two trees representing the same actual street pattern.

If the planners who use these maps rely on a node labeling scheme, in which both maps are labeled from the top down, beginning in the upper left hand corner, proceeding horizontally and then in a serpentine pattern from top to bottom, the first node encountered is labeled with the numeral 1; the second node encountered is labeled with the numeral 2, and so forth (Figure 2). However, the labeling imposed on these two maps is not identical, as one would wish (to indicate their identity in topological form). Indeed, the label 3 on the red tree then corresponds to the label 4 on the black tree and vice-versa, as do labels 5 and 6; a situation Nystuen et al. noted and were able to uncover on a number of occasions on actual maps (Figure 2). Naturally, as they note, one would wish, in a situation such as this one, to have the numerals correspond in an appropriate manner.

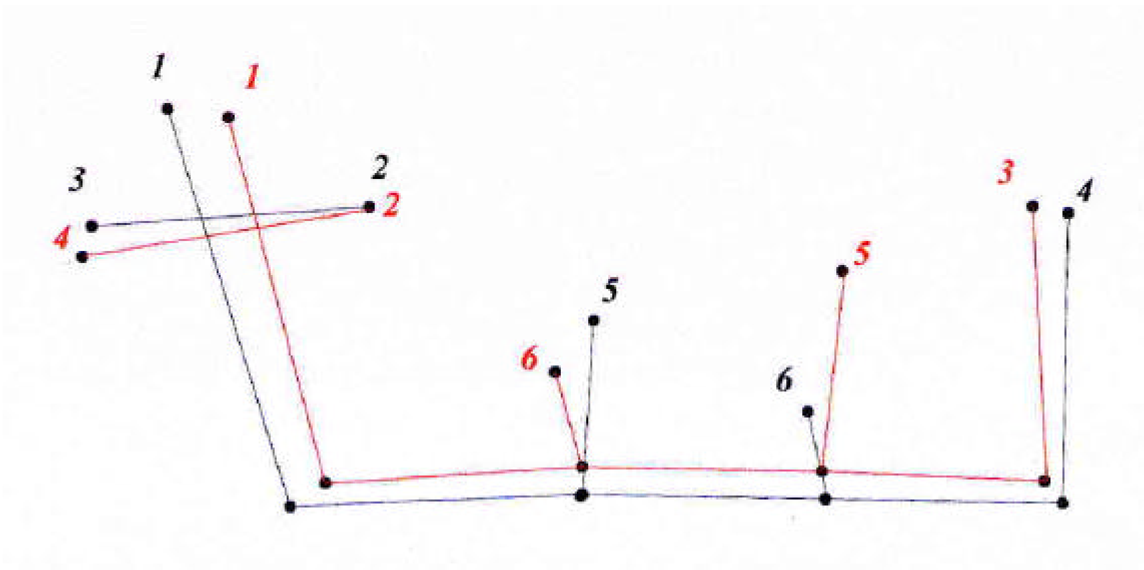

Figure 2. The red route and the black route: numbered nodes do not correspond as desired. The numbering scheme is a common one in which nodes are labeled from the top down in a left/right serpentine pattern (after Nystuen et al.).

Nystuen's observation came from a real-world example; it points, once again, to the appropriateness of considering different labeling schemes that will permit alignment of maps based on topology rather than solely on distance from the top of the screen. One strategy, that takes advantage of the structure of the underlying raster on which these maps were produced, proceeds as follows.

CANTOR PIXEL-LABELING ALGORITHM

1. Designate as a fundamental screen unit the smallest dot size to be used to represent a node; without loss of generality it can be assumed to be a "pixel."

2. In the manner that has come to be conventional with cathode ray tubes, use the top of the screen as the positive x-axis and the left side of the screen as the positive y-axis (with origin at the upper left-hand corner of the screen).

3. Assign to each pixel on the CRT an ordered pair of Cartesian coordinates: (1,2) is a pixel (an indivisible area) located one unit to the right of the left edge of the screen and two units down from the top.

4. Assign to each node in one map the pixel coordinates of the node as follows. If the pixel coordinates are (x,y), label the node with the rational number y/x.

This sort of strategy differs from what may be a computer default labeling scheme. Each possible node location has a unique label that is a single rational number indicating pixel location referred both to raster position and to location in relation to other pixels. The serpentine labeling scheme is still preserved even though its form is different, and perhaps, not evident. Cantor's enumeration technique shows that there is a one-to-one correspondence between the set of natural numbers and the set of ordered pairs of natural numbers. These two sets have the same cardinal number, aleph null (Hahn, pp. 1595-1596). Cantor's serpentine pattern through a square lattice converts the two-dimensional array into a one dimensional stream clearly equivalent to a number line (Figure 3).

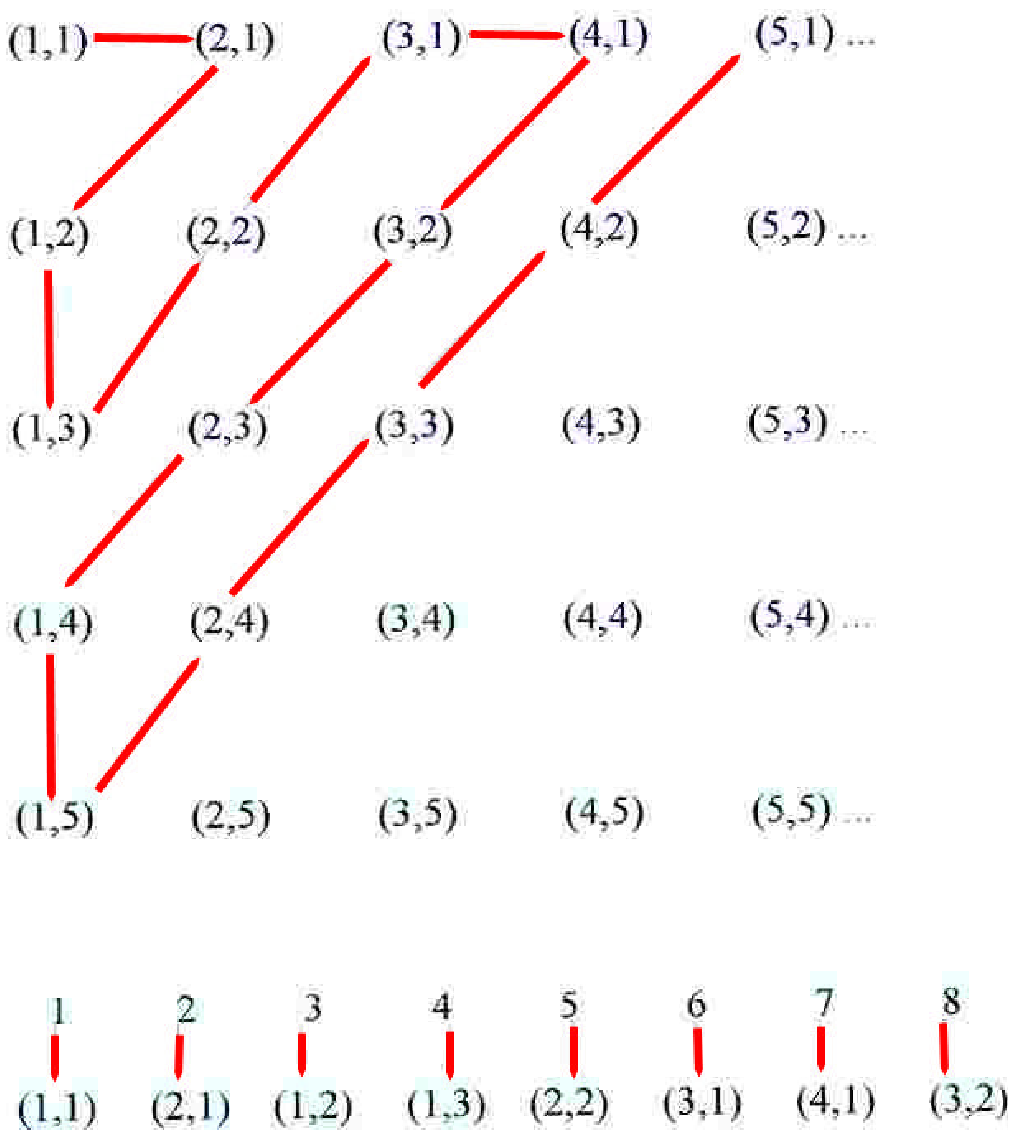

Figure 3. Assignment scheme for illustrating the cardinality of the set of rational numbers (after Hahn, p. 1595).

This strategy was the basis for Georg Cantor's (perhaps counter-intuitive) proof that the cardinal number of the set of rational numbers is aleph null. The set of ordered pairs contains the set of rationals--for, when ordered pairs are converted to rationals, as in the algorithm above, some are equivalent in value although not in visual form--1/2 and 2/4 are equivalent in value but do not look the same. However, the rational numbers contain the natural numbers as a subset. Thus, the cardinal number of the rationals is sandwiched between two sets each of whose cardinal number is aleph null--consequently, the set of rational numbers must also have cardinal number aleph null. Thus, the set of numbers y/x exactly covers the screen.

REPETITION OF THE CANTOR ALGORITHM

Suppose that the Cantor Algorithm has been applied once to a map: to the red tree in Figure 2, for example. Each node of the red tree will have a distinct, unique rational number label based on its position relative to the top and the side of the screen and in relation to other nodes. In a different layer, label the black street map in the same manner. It appears that at most one node on these two maps will have the same label; indeed, using rational number labeling, two nodes have the same label if and only if they are represented by the same pixel location in both maps. What is desired is to identify (match up or glue together) a node from one map with a node from the other map both of which represent the same physical location.

MERGING ALGORITHM

The following algorithm offers a strategy for making such an identification. It can be executed by buffering nodes, in an iterative fashion, and identifying two nodes at the first instance a node from one map falls into a node-buffer in another map.

1. Draw a buffer of one pixel width around each node in the red map. Assign identical labels to any pair of nodes on red and black maps both of which lie within or on the buffer.

2. Draw a buffer of one pixel width around each buffered node created in step 1. Assign identical labels to any pair of nodes on the red and black maps both of which lie within or on this newly drawn ring buffer region.

3. Repeat the process until one pair of buffer zones comes into contact with each other. Then stop the process.

If this Merging Algorithm provides a complete identification of nodes (with no extra nodes lying in the region outside the buffered zone), with identical connection patterns in both, then the two maps are said to be pairwise "well-matched." There is no difficulty in merging the two maps from different sources.

TRANSLATION OF THE MERGING ALGORITHM

Maps may fail to be pairwise well-matched for a variety of reasons. The following list suggests some reasons for such failure, but it may not be exhaustive.

1. The numbers of nodes in the red and black maps are different from each other.
2. The number of nodes in the two maps is the same, but the number of edges is different.
3. The number of nodes and edges is the same in both maps but the topology, the pattern of connection, is different.

4. Some or all of the three factors above may hold, in whole or in part, but the offset of pattern is so large that it falls outside the would-be buffer zone and gets visually confused with other pattern.

Nystuen notes in some of his examples that some parts of each map in the pair are shifted; the structure is recognizable from one to the other, but offset some significant amount. One way to deal with situations of these sorts is to permit translation, locally, of parts of the graph in one map. The Merging Algorithm will run in a satisfactory manner, up to a point. Beyond this point, nodes fall outside buffer zones. In such situations, use the diameter of the buffer zone (number of maximum number of pixels that came about in the use of the Merging Algorithm) as a translation scaling number. Add this number to each coordinate (in the local region) in one map. Now run the Merging Algorithm. Repeat the procedure as needed.

DIRECTIONS FOR FURTHER WORK

This issue of merging maps appears rich from an abstract viewpoint. Some of the other directions one might consider involve use of Hasse's Algorithm applied to graphs, development of error detection and correction criteria, use of sum-graphs or a similar strategy in which the topology of the graph guides the labeling pattern, and any of a host of theorems related to these topics.

Ann Arbor, MI
April, 1996.

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Sandra L. Arlinghaus, David Barr, John D. Nystuen:
The Spatial Shadow: Light and Dark -- Whole and Part.

This account of some of the projects of sculptor David Barr attempts to place them in a formal systematic, spatial setting based on the postulates of the science of space of William Kingdon Clifford (reprinted in Solstice, Vol. I, No. 1.).

Sandra L. Arlinghaus: Construction Zone--The Logistic Curve.

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John D. Nystuen: A City of Strangers: Spatial Aspects of Alienation in the Detroit Metropolitan Region.

This paper examines the urban shift from "people space" to "machine space" (see R. Horvath, Geographical Review, April, 1974) in the Detroit metropolitan regions of 1974.

As with Clifford's Postulates, reprinted in the last issue of Solstice, note the timely quality of many of the observations.

Sandra Lach Arlinghaus: Scale and Dimension: Their Logical Harmony.

Linkage between scale and dimension is made using the Fallacy of Division and the Fallacy of Composition in a fractal setting.

Sandra Lach Arlinghaus: Parallels Between Parallels.

The earth's sun introduces a symmetry in the perception of its trajectory in the sky that naturally partitions the earth's surface into zones of affine and hyperbolic geometry. The affine zones, with single geometric parallels, are located north and south of the geographic parallels. The hyperbolic zone, with multiple geometric parallels, is located between the geographic tropical parallels. Evidence of this geometric partition is suggested in the geographic environment--in the design of houses and of gameboards.

Sandra L. Arlinghaus, William C. Arlinghaus, and John D. Nystuen: The Hedetniemi Matrix Sum: A Real-world Application.

In a recent paper, we presented an algorithm for finding the shortest distance between any two nodes in a network of n nodes when given only distances between adjacent nodes (Arlinghaus, Arlinghaus, Nystuen, Geographical Analysis, 1990). In that previous research, we applied the algorithm to the generalized road network graph surrounding San Francisco Bay. Here, we examine consequent changes in matrix entries when the underlying adjacency pattern of the road network was altered by the 1989 earthquake that closed the San Francisco--Oakland Bay Bridge.

Sandra Lach Arlinghaus: Fractal Geometry of Infinite Pixel Sequences: "Super-definition" Resolution?

Comparison of space-filling qualities of square and hexagonal pixels.

Sandra Lach Arlinghaus: Construction Zone--Feigenbaum's number; a triangular coordinatiztion of the Euclidean plane; A three-axis coordinatization of the plane.

Volume I, No. 1, Summer, 1990.

Reprint of William Kingdon Clifford: Postulates of the Science of Space.

This reprint of a portion of Clifford's lectures to the Royal Institution in the 1870s suggests many geographic topics of concern in the last half of the twentieth century. Look for connections to boundary issues, to scale problems, to self-similarity and fractals, and to non-Euclidean geometries (from those based on denial of Euclid's parallel postulate to those based on a sort of mechanical `polishing'). What else did, or might, this classic essay foreshadow?

Sandra Lach Arlinghaus: Beyond the Fractal.

The fractal notion of self-similarity is useful for characterizing change in scale; the reason fractals are effective in the geometry of central place theory is because that geometry is hierarchical in nature. Thus, a natural place to look for other connections of this sort is to other geographical concepts that are also hierarchical. Within this fractal context, this article examines the case of spatial diffusion.

When the idea of diffusion is extended to see "adopters" of an innovation as "attractors" of new adopters, a Julia set is introduced as a possible axis against which to measure one class of geographic phenomena. Beyond the fractal context, fractal concepts, such as "compression" and "space-filling" are considered in a broader graph-theoretic setting.

William C. Arlinghaus: Groups, Graphs, and God.

Sandra L. Arlinghaus: Theorem Museum--Desargues's Two Triangle Theorem from projective geometry.

Construction Zone--centrally symmetric hexagons.

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