PATENT POLICY, R&D AND ECONOMIC GROWTH

by

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DEDICATION

In memory of mom and grandma

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LIST OF ABBREVIATIONS

DGE	Dynamic General Equilibrium
FDA	Food and Drug Administration
GDP	Gross Domestic Product
NIHCM	National Institute for Health Care Management
PAC	Political Action Committee
PhRMA	Pharmaceutical Research and Manufacturers of America
R&D	Research & Development
SIC	Standard Industrial Classification
SIG	Special Interest Group
TFP	Total Factor Productivity
TRIPS	Trade-Related Aspects of Intellectual Property Rights
US	United States

CHAPTER I

Introduction

The literature on R&D-driven economic growth suggests that technological innovations result from entrepreneurial activities such as R&D investments. In the next two chapters of this dissertation, I develop tractable frameworks to characterize and quantitatively evaluate the effects of patent policy on R&D and economic growth. As Acemoglu (p. 1112, 2007) writes,

> "… we lack a framework similar to that used for the analysis of the effects of capital and labor income taxes and indirect taxes in public finance, which we could use to analyze the effects… of intellectual property right polices… on innovation and economic growth."

In Chapter IV, I analyze the political economy of patent policy.

What are the effects of blocking patents on R&D and consumption? To answer this question, in Chapter II, I develop a generalized quality-ladder growth model with overlapping intellectual property rights to quantify the inefficiency in the patent system. The analysis focuses on two policy variables: (a) patent breadth that determines the total profit received by a patent pool; and (b) the profit-sharing rule that determines the distribution of surplus between innovators. To quantify the inefficiency arising from blocking patents that are generated by these two policy variables, the model is calibrated to aggregate data of the US economy. Under parameter values that match key features of the US economy and show equilibrium R&D underinvestment, I find that eliminating blocking patents would lead to a conservatively estimated increase in R&D of 12% and long-run consumption of 4% per year. I also quantify the transition-dynamic effects of patent policy and show implications that are different from previous studies in important ways.

Is the patent length an effective policy instrument in stimulating R&D? To answer this question, in Chapter III, I develop a generalized variety-expanding growth model and calibrate the model to aggregate data of the US economy to quantify the effects of patent extension. At the empirical patent-value depreciation rates, extending the patent length beyond 20 years leads to a negligible increase in R&D despite equilibrium R&D underinvestment. On the other hand, shortening the patent length can lead to a significant reduction in R&D and consumption. The calibration exercise also suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D. Finally, I identify and analytically derive a dynamic distortion of patent policy on saving and investment in physical capital that has been neglected by previous studies, which consequently underestimate the distortionary effects of patent protection.

What are the welfare implications of pharmaceutical lobbying? Since the 80's, the pharmaceutical industry has benefited substantially from the strengthening of patent protection for brand-name drugs as a result of the industry's political influence. To analyze this phenomenon, in Chapter IV, I incorporate special interest politics into a quality-ladder growth model to consider the policymakers' tradeoff between the socially optimal patent length and campaign contributions. The welfare analysis suggests that the presence of a pharmaceutical lobby distorting patent protection is socially undesirable in a closed-economy setting but may improve global welfare in a multi-country setting, which features an additional efficiency tradeoff between monopolistic distortion and international free-riding on innovations. Finally, if the SIGs have asymmetric influences across countries, then the country, in which the government places a higher value on campaign contributions, would gain by less or even suffer a welfare loss.

CHAPTER II

Quantifying the Effects of Blocking Patents on R&D

> "Today, most basic and applied researchers are effectively standing on top of a huge pyramid… Of course, a pyramid can rise to far greater heights than could any one person... But what happens if, in order to scale the pyramid and place a new block on the top, a researcher must gain the permission of each person who previously placed a block in the pyramid, perhaps paying a royalty or tax to gain such permission? Would this system of intellectual property rights slow down the construction of the pyramid or limit its heights? … To complete the analogy, *blocking patents* play the role of the pyramid's building blocks." – Carl Shapiro (2001)

1. Introduction

What are the effects of blocking patents on R&D? In an environment with cumulative innovations, the scope of a patent (i.e. patent breadth) determines the level of patent protection for an invention against imitation and subsequent innovations. This latter form of patent protection, which is known as leading breadth in the literature, gives the patentholders property rights over future inventions, and the resulting overlapping intellectual property rights may dampen the incentives for R&D. This phenomenon is referred to as blocking patents.

The main contribution of this chapter is to develop an R&D-driven endogenous growth model to quantify this inefficiency in the patent system. To the best of my knowledge, this chapter is the first to perform a quantitative analysis on patent policy by calibrating a DGE model combining the following features: (a) overlapping intellectual property rights that are emphasized by the patent-design literature; (b) multiple R&D externalities that are commonly discussed in the growth literature; and (c) endogenous capital accumulation that leads to a dynamic distortionary effect of patent protection on saving and investment and transition-dynamic effects different from previous studies. As Acemoglu (p. 1112, 2007) writes,

> "… we lack a framework similar to that used for the analysis of the effects of capital and labor income taxes and indirect taxes in public finance, which we could use to analyze the effects… of intellectual property right polices… on innovation and economic growth."

The analysis focuses on two policy variables: (a) patent breadth that determines the total profit received by a patent pool; and (b) the profit-sharing rule that determines the distribution of surplus between innovators. In order to quantify the inefficiency arising from blocking patents and other externalities, the model is calibrated to aggregate data of the US economy.[1] I also show that the key equilibrium condition, which is used to identify the effects of blocking patents on R&D, can be derived analytically without relying on the entire structure of the DGE model. In particular, it can be derived from two conditions: (a) a zero-profit condition in the R&D sector; and (b) a no-arbitrage condition that determines the market value of patents. The DGE model serves the useful purpose of providing a structural derivation and interpretation on the effects of blocking patents.

The main result is the following. Blocking patents have a significant and negative effect on R&D, and eliminating them would lead to a minimum increase in R&D of 12%. This result has important policy implications because given previous empirical estimates on the social rate of return to R&D, the market economy underinvests in R&D relative to the social optimum. To understand this finding, the DGE framework has been made rich enough to be consistent with either R&D overinvestment or underinvestment by combining blocking patents with multiple R&D externalities. Whether the market economy overinvests or underinvests in R&D depends crucially on the degree of externalities in intratemporal duplication and intertemporal knowledge

[1] As a robustness check, the model is also calibrated to industry-level data of R&D-intensive industries.

spillovers, which in turn is calibrated from the balanced-growth condition between long-run TFP growth and R&D. The larger is the fraction of long-run TFP growth driven by R&D, the larger are the social benefits of R&D; as a result, the more likely it is for the market economy to underinvest in R&D. I use previous empirical estimates for the social rate of return to R&D to calibrate this fraction.

Furthermore, the effects of eliminating blocking patents on consumption in the long run and during the transition dynamics are considered. When blocking patents are eliminated, the balanced-growth level of consumption increases by a minimum of 4% per year. During the transition dynamics, the economy does not always experience a significant fall in consumption in response to the resource reallocation away from the production sector to the R&D sector. Over a wide range of parameters, upon eliminating blocking patents, consumption gradually rises towards the new balanced-growth path by reducing investment in physical capital and temporarily running down the capital stock. This finding contrasts with Kwan and Lai (2003), whose model does not feature capital accumulation and hence predicts consumption losses during the transition path.

Finally, I identify and analytically derive a *dynamic* distortionary effect of patent protection on saving and investment that has been neglected by previous studies on patent policy, which focus mostly on the *static* distortionary effect of markup pricing.[2] The dynamic distortion arises because the markup in the patent-protected industries creates a wedge between the marginal product of capital and the rental price. Proposition 2 derives the sufficient conditions

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[2] Laitner (1982) is the first study that identifies in an exogenous growth model with overlapping generations of households that the existence of an oligopolistic sector and its resulting pure profit as financial assets creates both the usual static distortion and an additional dynamic distortion on capital accumulation due to the crowding out of households' portfolio space. The current chapter extends this study to show that this dynamic distortion also plays an important role and through a different channel in an R&D-driven endogenous growth model in which both patents and physical capital are owned by households as financial assets.

under which: (a) the market equilibrium rate of investment in physical capital is below the socially optimal level; and (b) an increase in the markup reduces the equilibrium investment rate in physical capital. The numerical exercise also quantifies the discrepancy between the equilibrium capital investment rate and the socially optimal level and shows that eliminating blocking patents helps reducing this discrepancy.

1.1. Literature Review

This chapter relates to a number of studies on R&D underinvestment and provides through the elimination of blocking patents an effective method to mitigate the R&D-underinvestment problem suggested by Jones and Williams (1998) and (2000). Furthermore, the calibration exercise takes into consideration Comin's (2004) critique that long-run TFP growth may not be solely driven by R&D. The current chapter also complements the qualitative partial-equilibrium studies on leading breadth from the patent-design literature,[3] such as Green and Scotchmer (1995), O'Donoghue *et al* (1998) and Hopenhayn *et al* (2006), by providing a quantitative DGE analysis. O'Donoghue and Zweimuller (2004) is the first study that merges the patent-design and endogenous growth literatures to analyze the effects of patentability requirement, lagging and leading breadth on economic growth in a canonical quality-ladder growth model. However, their focus was not in quantifying the effects of blocking patents on R&D. In addition, the current chapter generalizes their model in a number of dimensions in order to perform a quantitative analysis on the transition dynamics. Other DGE analysis on patent policy includes Goh and Olivier (2002), Grossman and Lai (2004) and Li (2001).[4] These studies

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[3] The seminal work on optimal patent length is Nordhaus (1969). Some other recent studies on optimal patent design include Tandon (1982), Gilbert and Shapiro (1990), Klemperer (1990), O'Donoghue (1998), Hunt (1999) and Scotchmer (2004). Judd (1985) provides the first DGE analysis on optimal patent length.
[4] Goh and Olivier (2002) analyze the welfare effects of patent breadth in a two-sector variety-expanding growth model, and Grossman and Lai (2004) analyze the welfare effects of strengthening patent protection in developing countries as a result of the TRIPS agreement using a multi-country variety-expanding model. However, these studies do not analyze patent breadth in an environment with cumulative innovations. Li

are also qualitatively oriented and do not feature capital accumulation so that the dynamic distortionary effect of patent policy is absent.

In terms of quantitative analysis on patent policy, this chapter relates to Kwan and Lai (2003) and Chu (2007b). Kwan and Lai (2003) numerically evaluate the effects of extending the effective lifetime of patent in the variety-expanding model originating from Romer (1990) and find substantial welfare gains despite the temporary consumption losses during the transition path in their model. Chu (2007b) uses a generalized variety-expanding model and finds that whether or not extending the patent length would lead to a significant increase in R&D depends crucially on the patent-value depreciation rate. At the empirical range of patent-value depreciation rates estimated by previous studies, extending the patent length has only limited effects on R&D and thus social welfare. Therefore, Chu (2007b) and the current chapter together provide a comparison on the relative effectiveness of extending the patent length and eliminating blocking patents in mitigating the R&D-underinvestment problem. The crucial difference between these two policy instruments arises because extending the patent length increases future monopolistic profit while eliminating blocking patents raises current monopolistic profit for the inventors.

The rest of the chapter is organized as follows. Section 2 derives the equilibrium condition that is used to identify the effects of blocking patents. Section 3 describes the DGE model. Section 4 calibrates the model and presents the numerical results. The final section concludes. The proofs, derivations, tables and figures are relegated to the appendices.

(2001) analyzes the optimal policy mix of R&D subsidy and lagging breadth in a quality-ladder model with endogenous step size, but he does not consider leading breadth.

2. An Intuitive Derivation of the Market-Equilibrium Condition for R&D

In this section, I show that the steady-state equilibrium condition for R&D that is crucial for the calibration can be derived from: (a) a zero-profit condition in the R&D sector; and (b) a no-arbitrage condition that determines the market value of patents. Intuitively, given the level of R&D spending in the data, the private benefit of R&D can be inferred from the zero-profit condition. Then, given the private benefit of R&D, the amount of the monopolistic profit received by current inventors can be inferred from the no-arbitrage condition. Finally, the discrepancy between the amount of profit received by current inventors and the total amount of monopolistic profit is attributed to blocking patents, and the DGE model serves the useful purpose of providing a structural derivation and interpretation of this discrepancy.

The zero-profit condition in the R&D sector implies that

(1) $$\lambda V = R\,\&\,D\,.$$

V is the market value of a patented invention for its inventor, and λ is the Poisson arrival rate of innovations. $R\,\&\,D$ is the amount of R&D spending. The no-arbitrage condition implies that the market value of a patented invention is the expected present value of monopolistic profit received by the inventor such that

(2) $$V = \frac{\pi_{current\_inventor}}{r + \lambda - g_{\pi}},$$

where r is the real interest rate, and g_{π} is the growth rate of monopolistic profit. Because of blocking patents, an inventor may only capture a fraction of the monopolistic profit generated by her invention.

(3) $$\pi_{current\_inventor} = \nu\,\pi_{monopolistic},$$

where $\nu \in (0,1]$. At this point, the interpretation of ν is quite vague, and the DGE model provides a structural interpretation of ν as the backloading discount factor, which captures the effects of delayed reward due to profit-sharing in patent pools. Substituting (3) and (2) into (1) yields

$$R\,\&\,D = v\left(\frac{\lambda}{r+\lambda-g_{\pi}}\right)\pi_{monopolistic}\,. \tag{4}$$

Finally, the amount of monopolistic profit is given by

$$\pi_{monopolistic} = \left(\frac{\mu-1}{\mu}\right)Y\,. \tag{5}$$

In the case of industry-level data, μ is the industry markup and *Y* is the valued-added of R&D-intensive industries. Since empirical estimates for industry markup are known to be imprecise, I will perform the calibration using both industry-level data and aggregate data to ensure the robustness of the numerical results. Calibration based on aggregate data requires an additional assumption that economic profit in the economy is created by intellectual monopoly. Given this assumption, μ in (5) becomes the aggregate or average markup in the economy and *Y* in (5) becomes the value of GDP.

3. The Model

The model is a generalized version of Grossman and Helpman (1991) and Aghion and Howitt (1992). The final goods, which can be either consumed by households or invested in physical capital, are produced with a composite of differentiated intermediate goods. The intermediate goods are produced with labor and capital, and there are both competitive and monopolistic industries in the intermediate-goods sector. The relative price between the monopolistic and competitive goods leads to the usual *static* distortionary effect that reduces the output of final goods. The markup in the monopolistic industries drives a wedge between the marginal product of capital and the rental price; consequently, it leads to an additional *dynamic* distortionary effect that causes the market equilibrium rate of investment in physical capital to deviate from the social optimum. The R&D sector also uses both labor and capital as factor inputs.

To prevent the model from overestimating the social benefits of R&D and the extent of R&D underinvestment, the long-run TFP growth is assumed to be driven by R&D as well as an exogenous process as in Comin (2004). The class of first-generation R&D-driven endogenous growth models, such as Grossman and Helpman (1991) and Aghion and Howitt (1992), exhibits scale effects and is inconsistent with the empirical evidence in Jones (1995a).[5] In the present model, scale effects are eliminated by assuming decreasing *individual* R&D productivity as in Segerstrom (1998), which becomes a semi-endogenous growth model.[6]

The various components of the model are presented in Sections 3.1–3.7, and the decentralized equilibrium is defined in Section 3.8. Section 3.9 summarizes the laws of motion that characterize the transition dynamics, and Section 3.10 discusses the balanced-growth path. Section 3.11 derives the socially optimal allocations and the dynamic distortionary effect of patent protection.

3.1. Representative Household

The infinitely-lived representative household maximizes life-time utility that is a function of per-capita consumption c_t of the numeraire final goods and is assumed to have the iso-elastic form given by

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[5] See, e.g. Jones (1999) for an excellent theoretical analysis on scale effects.

[6] In a semi-endogenous growth model, the balanced-growth rate is determined by the exogenous labor-force growth rate. An increase in the share of R&D factor inputs raises the *level* of the balanced growth path while holding the balanced-growth rate constant. Since increasing R&D has no long-run growth effect in this model, the calibrated effects on consumption in the numerical exercises are likely to be more conservative than in other fully endogenous growth models.

(6)
$$U = \int_0^\infty e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt ,$$

where $\sigma \geq 1$ is the inverse of the elasticity of intertemporal substitution and ρ is the subjective discount rate. The household has $L_t = L_0 \exp(nt)$ members at time *t*. The population size at time 0 is normalized to one, and $n > 0$ is the exogenous population growth rate. To ensure that lifetime utility is bounded, it is assumed that $\rho > n$. The household maximizes (6) subject to a sequence of budget constraints given by

(7)
$$\dot{a}_t = a_t(r_t - n) + w_t - c_t .$$

Each member of the household inelastically supplies one unit of homogenous labor in each period to earn a real wage income w_t. a_t is the value of risk-free financial assets in the form of patents and physical capital owned by each household member, and r_t is the real rate of return on these assets. The familiar Euler equation derived from the intertemporal optimization is

(8)
$$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

3.2. Final Goods

This sector is characterized by perfect competition, and the producers take both the output price and input prices as given. The production function for the final goods Y_t is a Cobb-Douglas aggregator of a continuum of differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$ given by

(9)
$$Y_t = \exp\left(\int_0^1 \ln X_t(j) dj \right).$$ [7]

[7] To maintain the analytical tractability of the aggregate conditions, a Cobb-Douglas aggregator instead of the more general CES aggregator is adopted. With the CES aggregator, it becomes very difficult to derive the aggregate conditions when there are both competitive and monopolistic industries in the intermediate-goods sector. Furthermore, computation of the transition dynamics becomes possible under the Cobb-

The familiar aggregate price index is

$$P_t = \exp\left(\int_0^1 \ln P_t(j) dj \right) = 1, \tag{10}$$

and the demand curve for each variety of intermediate goods is

$$P_t(j) X_t(j) = Y_t . \tag{11}$$

3.3. Intermediate Goods

There is a continuum of industries producing the differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$. A fraction $\theta \in [0,1)$ of the industries is characterized by perfect competition because innovations in these industries are assumed to be non-patentable. Each of the remaining industries is dominated by a temporary industry leader, who owns the patent for the latest R&D-driven technology for production. Without loss of generality, the industries are ordered such that industries $j' \in [0,\theta)$ are competitive and industries $j \in [\theta,1]$ are monopolistic. The production function in each industry has constant returns to scale in labor and capital inputs and is given by

$$X_t(j) = z^{m_t(j)} Z_t K_{x,t}^{\alpha}(j) L_{x,t}^{1-\alpha}(j) \tag{12}$$

for $j \in [0,1]$. $K_{x,t}(j)$ and $L_{x,t}(j)$ are respectively the capital and labor inputs for producing intermediate-goods *j* at time *t*. $Z_t = Z_0 \exp(g_Z t)$ represents an exogenous process of productivity improvement that is common across all industries and is freely available to all producers. $z^{m_t(j)}$ is industry *j*'s level of R&D-driven technology, which is increasing over time through R&D investment and successful innovations. $z > 1$ is the exogenous step-size of a technological improvement arising from each innovation. $m_t(j)$, which is an integer, is the

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Douglas aggregator. Although the arrival rate of innovations varies along the transitional path, a tractable form for the law of motion for aggregate technology can still be derived under the Cobb-Douglas aggregator but not under the CES aggregator.

number of innovations that has occurred in industry *j* as of time *t*. The marginal cost of production in industry *j* is

$$MC_t(j) = \frac{1}{z^{m_t(j)} Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}, \tag{13}$$

where R_t is the rental price of capital. The optimal price for the leaders in the monopolistic industries is a constant markup $\mu(z,\eta)$ over the marginal cost of production given by

$$P_t(j) = \mu(z,\eta) MC_t(j) \tag{14}$$

for $j \in [\theta,1]$. The markup $\mu(z,\eta)$ is a function of the quality step size z and the level of patent breadth η (to be defined in Section 3.4). The competitive industries are characterized by competitive pricing so

$$P_t(j') = MC_t(j') \tag{15}$$

for $j' \in [0,\theta)$. The aggregate price level is

$$P_t = \tilde{\mu}(z,\eta,\theta) MC_t . \tag{16}$$

$\tilde{\mu}(z,\eta,\theta) \equiv \mu(z,\eta)^{1-\theta}$ is the aggregate markup in the economy. The aggregate marginal cost is

$$MC_t = \exp\left(\int_0^1 \ln MC_t(j) dj\right). \tag{17}$$

3.4. Patent Breadth

Before providing the underlying derivations, this section firstly presents the Bertrand equilibrium price and the amount of monopolistic profit generated by an invention under different levels of patent breadth, which is denoted by η.

$$P_t(j) = z^{\eta} MC_t(j), \tag{18}$$

(19) $$\pi_t(j) = (z^{\eta} - 1)MC_t(j)X_t(j),$$ [8]

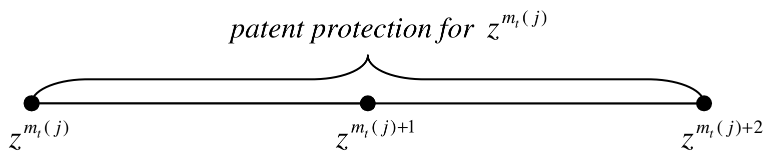
for $\eta \in \{1,2,3,...\}$ and $j \in [\theta,1]$. The expression for the equilibrium price is consistent with the seminal work of Gilbert and Shapiro's (1990) interpretation of "breadth as the ability of the patentee to raise price." A broader patent breadth corresponds to a larger η, and vice versa. Therefore, an increase in patent breadth *potentially* enhances the incentives for R&D by raising the amount of monopolistic profit generated by each invention but worsens the distortionary effects of markup pricing.

The patent-design literature has identified and analyzed two types of patent breadth in an environment with cumulative innovations: (a) lagging breadth; and (b) leading breadth. In a standard quality-ladder growth model, lagging breadth (i.e. patent protection against imitation) is assumed to be complete while leading breadth (i.e. patent protection against subsequent innovations) is assumed to be zero. The following analysis assumes complete lagging breadth and focuses on non-zero leading breadth, and the formulation originates from O'Donoghue and Zweimuller (2004).[9]

The level of patent breadth $\eta = \eta_{lag} + \eta_{lead}$ can be decomposed into lagging breadth denoted by $\eta_{lag} \in (0,1]$ and leading breadth denoted by $\eta_{lead} \in \{0,1,2,...\}$. In the following, complete lagging breadth is assumed such that $\eta = 1 + \eta_{lead}$. Nonzero leading breadth protects patentholders against subsequent innovations and gives the patentholders property rights over future inventions. For example, if $\eta_{lead} = 1$, then the most recent innovation infringes the patent of the second-most recent inventor. If $\eta_{lead} = 2$, then the most recent innovation infringes the patents of the second-most and the third-most recent inventors, etc. The following diagram

[8] Note that the inventor may only capture a fraction of this monopolistic profit because of blocking patents.
[9] See, e.g. Li (2001) for a discussion of incomplete lagging breadth.

illustrates the concept of nonzero leading breadth with an example in which the degree of leading breadth is two.



Therefore, nonzero leading breadth facilitates the new industry leader and the previous inventors, whose patents are infringed, to consolidate market power through licensing agreements or the formation of a patent pool resulting in a higher markup.[10] The Bertrand equilibrium price with leading breadth is

(20) $$P_t(j) = z^{1+\eta_{lead}} MC_t(j)$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$. Assumption 1 is *sufficient* to derive this equilibrium markup price.

Assumption 1: *An infringed patentholder cannot become the next industry leader while she is still covered by a licensing agreement in that industry.*[11]

Then, the amount of monopolistic profit captured by the licensing agreement at time *t* is

(21) $$\pi_t(j) = (z^{1+\eta_{lead}} - 1) MC_t(j) X_t(j)$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$.

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[10] See, e.g. Gallini (2002) and O'Donoghue and Zweimuller (2004), for a discussion on market-power consolidation through licensing agreements.

[11] The sufficiency of this assumption in determining the markup price is most easily understood with an example. Suppose leading breadth is one and lagging breadth is complete, the lower bound on the profit-maximizing markup is the square of z, which is the limit price from the collusion of the most recent and the second-most recent inventors against the third-most recent inventor, whose patent is not infringed upon by the most recent invention. In this example, the limit-pricing markup would be even larger if the third-most recent inventor happens to be the new industry leader. Continuing this reasoning, the markup could grow without bound; therefore, Assumption 1 is made to rule out this possibility. The empirical plausibility of this assumption is appealed to the existence of antitrust policy.

The share of profit obtained by each generation of patentholders in the patent pool depends on the profit-sharing rule (i.e. the terms in the licensing agreement). A stationary bargaining outcome is assumed to simplify the analysis.

Assumption 2: *The profit-sharing rule is symmetric across industries and is stationary. For each degree of leading breadth* $\eta_{lead} \in \{0,1,2,...\}$*, the profit-sharing rule is* $\Omega^{\eta_{lead}} = (\Omega_1,...,\Omega_\eta) \in [0,1]$*, where* Ω_i *is the share of profit received by the i-th most recent inventor, and* $\sum_{i=1}^{\eta} \Omega_i = 1$.

Although the shares of profit and licensing fees eventually received by the owner of an invention are constant overtime, the present value of profit is determined by the actual profit-sharing rule. The two extreme cases are: (a) *complete frontloading* $\Omega^{\eta_{lead}} = (1,0,...,0)$; and (b) *complete backloading* $\Omega^{\eta_{lead}} = (0,0,...,1)$. Complete frontloading maximizes the incentives on R&D provided by leading breadth by maximizing the present value of profit received by an inventor. The opposite effect of blocking patents arises when profit is backloaded, and complete backloading maximizes this damaging effect on the incentives for R&D. The law of motion for the market value of ownership for each generation of patentholders will be derived in Section 3.7.

3.5. Aggregation and Static Distortion

Define $A_t \equiv \exp\left(\int_0^1 m_t(j)dj \ln z\right)$ as the aggregate level of R&D-driven technology.

Also, define total labor and capital inputs for production as $K_{x,t} = \int_0^1 K_{x,t}(j)dj$ and $L_{x,t} = \int_0^1 L_{x,t}(j)dj$ respectively.

Lemma 1: *The aggregate production function for the final goods is*

(22) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha},$$

where $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* η *for* $\theta \in (0,1)$.

Proof: Refer to Appendix A.□

$\vartheta(\eta)$ captures the static distortionary effect of the markup z^{η}. Markup pricing in the monopolistic industries distorts production towards the competitive industries and reduces the output of the final goods. Also, $\vartheta(\eta)$ is initially decreasing in θ and subsequently increasing with $\vartheta(\eta) = 1$ for $\theta \in \{0,1\}$. Therefore, at least over a range of parameters, the static distortionary effect becomes increasingly severe as the fraction of competitive industries increases. The distortionary effect is not monotonic in θ because the relative-price distortion disappears when either all industries are monopolistic or competitive.

The market-clearing condition for the final goods is

(23) $$Y_t = C_t + I_t,$$

where $C_t = L_t c_t$ is the aggregate consumption and I_t is the investment in physical capital. The factor payments for the final goods are

(24) $$Y_t = w_t L_{x,t} + R_t K_{x,t} + \pi_t.$$

$\pi_t = \int_{\theta}^{1} \pi_t(j) dj$ is the total amount of monopolistic profit. Substituting (12) and (13) into (19) and then summing over all monopolistic industries yields

(25) $$\pi_t = (1-\theta)\left(\frac{z^{\eta} - 1}{z^{\eta}}\right) Y_t.$$

Therefore, the growth rate of monopolistic profit equals the growth rate of output. The factor payments for labor and capital inputs in the intermediate-goods sector are respectively

(26) $$w_t L_{x,t} = (1-\alpha)\left(\frac{z^{\eta}\theta + 1 - \theta}{z^{\eta}}\right)Y_t,$$

(27) $$R_t K_{x,t} = \alpha\left(\frac{z^{\eta}\theta + 1 - \theta}{z^{\eta}}\right)Y_t.$$

(27) shows that the markup drives a wedge between the marginal product of capital and the rental price. As will be shown below, this wedge creates a dynamic distortionary effect on the rate of investment in physical capital. Finally, the correct value of GDP should include R&D investment

(28) $$GDP_t = Y_t + w_t L_{r,t} + R_t K_{r,t}.$$ [12]

$L_{r,t}$ and $K_{r,t}$ are respectively the number of workers and the amount of capital for R&D.

3.6. Capital Accumulation

The market-clearing condition for physical capital is

(29) $$K_t = K_{x,t} + K_{r,t}.$$

K_t is the total amount of capital in the economy at time t. The law of motion for capital is

(30) $$\dot{K}_t = I_t - K_t\delta$$

δ is the rate of depreciation. The endogenous rate of investment in physical capital is

(31) $$i_t = (\dot{K}_t / K_t + \delta)K_t / Y_t$$

for all *t*. The no-arbitrage condition $r_t = R_t - \delta$ for the holding of capital and (27) imply that the capital-output ratio is

[12] In the national income account, private spending in R&D is treated as an expenditure on intermediate goods. Therefore, the values of investment and GDP in the data are I_t and Y_t respectively. The Bureau of Economic Analysis and the National Science Foundation's R&D satellite account provides preliminary estimates on the effects of including R&D as an intangible asset in the national income accounts.

(32) $$\frac{K_t}{Y_t} = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})(r_t + \delta)}.$$

$s_{K,t}$ is the endogenous share of capital in the R&D sector. Substituting (32) into (31) yields

(33) $$i_t = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})}\left(\frac{\dot{K}_t / K_t + \delta}{r_t + \delta}\right).$$

In the Romer model, (skilled) labor is the only factor input for R&D (i.e. $s_{K,t} = 0$); therefore, the distortionary effect of markup pricing on the *steady-state* rate of investment in physical capital is unambiguously negative (i.e. $\partial i / \partial \eta < 0$). In the current model, there is an opposing positive effect operating through the R&D share of capital. Intuitively, an increase in patent breadth potentially raises the private return on R&D and increases the R&D share of capital. Proposition 2 in Section 3.11 shows that the negative distortionary effect still dominates if the intermediate-goods sector is at least as capital intensive as the R&D sector.

3.7. R&D

$V_t(j)$ is the market value of the patent pool created by the most recent invention in industry $j \in [\theta,1]$ at time *t* and is determined by the following no-arbitrage condition

(34) $$r_t V_t(j) = \pi_t(j) + \dot{V}_t(j) - \lambda_t V_t(j).$$

The first terms in the right is the flow profit captured by the patent pool at time *t*. The second term is the capital gain due to the growth in the amount of monopolistic profit. The third term is the expected value of capital loss due to creative destruction, and λ_t is the Poisson arrival rate of the next invention that creates a new patent pool. However, the incentives for R&D depend on the market value of the shares in patent pools obtained by the next inventor. Denote $V_{i,t}(j)$ for $i \in \{1,...,\eta\}$ as the market value of ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$.

Proposition 1: $V_{i,t}(j)$ *for* $i \in \{1,2,...,\eta\}$ *and* $j \in [\theta,1]$ *is determined by the following law of motion*

(35) $$r_t V_{i,t}(j) = \Omega_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)),$$

where $V_{\eta+1,t}(j) = 0$. *The no-arbitrage condition for* $V_{1,t}(j)$ *can be re-expressed as*

(36) $$V_{1,t}(j) = \pi_t(j)\left(\sum_{k=1}^{\eta} \Omega_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)}\right)\right).$$

Proof: Refer to Appendix A.□

The intuition behind (35) is very similar to (34) with two differences. Firstly, the *i*-th most recent inventor in industry *j* only captures a share Ω_i of the flow profit. Secondly, when the next invention occurs, the *i*-th most recent inventor losses $V_{i,t}(j)$ but gains $V_{i,t+1}(j)$ as she becomes the *i*+1-th most recent inventor in the next patent pool. Once (35) has been derived for $i \in \{1,2,...,\eta\}$, (36) can be derived by recursive substitutions to obtain an expression for $V_{1,t}(j)$.

Assumption 3: *Innovation successes of the R&D entrepreneurs are randomly assigned to the industries in the intermediate-goods sector.*[13]

The expected present value of an invention obtained by the most recent inventor at time *t* is

(37) $$V_{1,t} = \int_{\theta}^{1} V_{1,t}(j)dj = (1-\theta)\left(\frac{z^{\eta}-1}{z^{\eta}}\right)Y_t\left(\sum_{k=1}^{\eta} \Omega_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}/V_{i,t}}\right)\right).$$[14]

[13] A reasonable implication of this assumption is that the equilibrium level of R&D is determined by the amount of monopolistic profits in the economy.

[14] Note that the second equality is obtained by firstly integrating over (35) and then by recursive substitutions.

The arrival rate of an innovation success for an R&D entrepreneur $h \in [0,1]$ is a function of labor input $L_{r,t}(h)$ and capital input $K_{r,t}(h)$ given by

(38) $$\lambda_t(h) = \overline{\varphi}_t K_{r,t}^{\beta}(h) L_{r,t}^{1-\beta}(h).$$

$\overline{\varphi}_t$ is a productivity parameter that the entrepreneurs take as given. The expected profit from R&D is

(39) $$E_t[\pi_{r,t}(h)] = V_{1,t}\lambda_t(h) - w_t L_{r,t}(h) - R_t K_{r,t}(h).$$

The first-order conditions are

(40) $$(1-\beta)V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta} = w_t,$$

(41) $$\beta V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta-1} = R_t.$$

To eliminate scale effects and capture various externalities, I follow Jones and Williams (2000) to assume that the *individual* R&D productivity parameter $\overline{\varphi}_t$ is given by

(42) $$\overline{\varphi}_t = \varphi(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma-1} / A_t^{1-\phi},$$

where $K_{r,t} = \int_0^1 K_{r,t}(h)dh$ and $L_{r,t} = \int_0^1 L_{r,t}(h)dh$. $\gamma \in (0,1]$ captures the negative externality in intratemporal duplication or the so-called "stepping-on-toes" effects, and $\phi \in (-\infty,1)$ captures the externality in intertemporal knowledge spillovers.[15] Given that the arrival of innovations follows a Poisson process, the law of motion for R&D-driven technology is given by

[15] This specification captures how semi-endogenous growth models eliminate scale effects as in Jones (1995b). $\phi \in (0,1)$ corresponds to the "standing-on-shoulder" effect, in which the *economy-wide* R&D productivity $A_q\overline{\varphi}$ increases as the level of R&D-driven technology increases (see the law of motion for R&D-driven technology). On the other hand, $\phi \in (-\infty,0)$ corresponds to the "fishing-out" effect, in which early technology is relatively easy to develop and $A_q\overline{\varphi}$ decreases as the level of R&D-driven technology increases.

(43) $$\dot{A}_t = A_t\lambda_t \ln z = A_t\overline{\varphi}_t K_{r,t}^{\beta} L_{r,t}^{1-\beta} \ln z = A_t^{\phi}(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}\varphi \ln z.^{16}$$

3.8. Decentralized Equilibrium

The analysis starts at $t = 0$. The equilibrium is a sequence of allocations $\{a_t, c_t, I_t, Y_t, X_t(j), K_{x,t}(j), L_{x,t}(j), K_{r,t}(h), L_{r,t}(h), K_t, L_t\}_{t=0}^{\infty}$ and a sequence of prices $\{w_t, r_t, R_t, P_t(j), V_{1,t}\}_{t=0}^{\infty}$ such that they are consistent with the initial conditions $\{K_0, L_0, Z_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also, in each period,

(a) the representative household chooses $\{a_t, c_t\}$ to maximize utility taking $\{w_t, r_t\}$ as given;

(b) the competitive firms in the final-goods sector choose $\{X_t(j)\}$ to maximize profit according to the production function taking $\{P_t(j)\}$ as given;

(c) each industry leader in the intermediate-goods sector chooses $\{P_t(j), K_{x,t}(j), L_{x,t}(j)\}$ to maximize profit according to the Bertrand price competition and the production function taking $\{R_t, w_t\}$ as given;

(d) the competitive firms in the intermediate-goods sector choose $\{K_{x,t}(j'), L_{x,t}(j')\}$ to maximize profit according to the production function taking $\{P_t(j'), R_t, w_t\}$ as given;

(e) each entrepreneur in the R&D sector chooses $\{K_{r,t}(h), L_{r,t}(h)\}$ to maximize profit according to the R&D production function taking $\{\overline{\varphi}_t, V_{1,t}, R_t, w_t\}$ as given;

(f) the market for the final-goods clears such that $Y_t = C_t + I_t$;

(g) the full employment of capital such that $K_t = K_{x,t} + K_{r,t}$; and

---

[16] This convenient expression is derived as $\ln A_t = \left(\int_0^1 m_t(j)dj\right)\ln z = \left(\int_0^t \lambda(\tau)d\tau\right)\ln z$; then, simple differentiation yields $\dot{A}_t / A_t = \lambda_t \ln z$.

(h) the full employment of labors such that $L_t = L_{x,t} + L_{r,t}$.

3.9. Aggregate Equations of Motion

The transition dynamics of the decentralized equilibrium is characterized by the following differential equations. The capital stock is a predetermined variable and evolves according to

(44) $$\dot{K}_t = Y_t - C_t - K_t\delta .$$

R&D-driven technology is also a predetermined variable and evolves according to

(45) $$\dot{A}_t = A_t\lambda_t \ln z .$$

Consumption is a jump variable and evolves according to the Euler equation

(46) $$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

The market value of ownership in patent pools is also a jump variable and evolves according to

(47) $$\dot{V}_{i,t} = (r_t + \lambda_t)V_{i,t} - \lambda_t V_{i+1,t} - \Omega_i \pi_t$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$.

At the aggregate level, the generalized quality-ladder model is similar to Jones's (1995b) model, whose dynamic properties have been investigated by a number of recent studies. For example, Arnold (2006) analytically derives the uniqueness and local stability of the steady state with certain parameter restrictions. Steger (2005) and Trimborn *et al* (2007) numerically evaluate the transition dynamics of the model. In summary, to solve the model numerically, I firstly transform $\{K_t, A_t, c_t, V_{i,t}\}$ in the differential equations into its stationary form,[17] and then, compute the transition path from the old steady state to the new one using the relaxation algorithm developed by Trimborn *et al* (2007).

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[17] Refer to Appendix B for the details.

3.10. Balanced-Growth Path

Equating the first-order conditions (26) and (40) and imposing the balanced-growth condition on R&D-driven technology

(48) $$g_A = \overline{\varphi}_t L_{r,t}^{1-\beta} K_{r,t}^{\beta} \ln z$$

yield the steady-state R&D share of labor inputs given by

(49) $$\frac{s_L}{1-s_L} = \frac{1-\beta}{1-\alpha}\left(\frac{\lambda}{r+\lambda-g_Y}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+(1-\theta)}\nu(\Omega^{\eta_{lead}}),$$

where $\nu(\Omega^{\eta_{lead}}) \equiv \sum_{k=1}^{\eta}\Omega_k\left(\frac{\lambda}{r+\lambda-g_Y}\right)^{k-1} \in (0,1]$ is defined as the backloading discount factor. For example, in the case of complete frontloading, $\nu(\Omega^{\eta_{lead}}) = 1$. Similarly, solving (27), (41) and (48) yields the steady-state R&D share of capital inputs given by

(50) $$\frac{s_K}{1-s_K} = \frac{\beta}{\alpha}\left(\frac{\lambda}{r+\lambda-g_Y}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+(1-\theta)}\nu(\Omega^{\eta_{lead}}).$$

On the balanced-growth path, c_t increases at a constant rate g_c, so that the steady-state real interest rate is

(51) $$r = \rho + g_c\sigma.$$

The balanced-growth rate of R&D technology g_A is related to the labor-force growth rate such that

(52) $$g_A = \frac{(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma}}{A_t^{1-\phi}}\varphi \ln z = \left(\frac{\gamma\beta}{1-\phi}\right)g_K + \left(\frac{\gamma(1-\beta)}{1-\phi}\right)n.$$

Then, the steady-state rate of creative destruction is $\lambda = g_A / \ln z$. The balanced-growth rates of other variables are given as follows. Given that the steady-state investment rate is constant, the balanced-growth rate of per capita consumption is

(53) $$g_c = g_Y - n .$$

From the aggregate production function (22), the balanced-growth rates of output and capital are

(54) $$g_Y = g_K = n + (g_A + g_Z)/(1-\alpha) .$$

Using (52) and (54), the balanced-growth rate of R&D-driven technology is determined by the exogenous labor-force growth rate n and productivity growth rate g_Z given by

(55) $$g_A = \left(\frac{1-\phi}{\gamma} - \frac{\beta}{1-\alpha} \right)^{-1} \left(n + \frac{\beta}{1-\alpha} g_Z \right).$$

Long-run TFP growth denoted by $g_{TFP} \equiv g_A + g_Z$ is empirically observed. For a given g_{TFP}, a higher value of g_Z implies a lower value of g_A as well as a lower calibrated value for $\gamma/(1-\phi)$, which in turn implies that R&D have smaller social benefits and the socially optimal level of R&D spending is lower.

3.11. Socially Optimal Allocations and Dynamic Distortion

This section firstly characterizes the socially optimal allocations and then derives the dynamic distortion on capital accumulation. To derive the socially optimal rate of investment in physical capital and R&D shares of labor and capital, the social planner chooses i_t, $s_{L,t}$ and $s_{K,t}$ to maximize the representative household's lifetime utility given by $U = \int_0^\infty e^{-(\rho-n)t} \frac{((1-i_t)Y_t/L_t)^{1-\sigma}}{1-\sigma} dt$ subject to: (a) the aggregate production function given by $Y_t = A_t Z_t (1-s_{K,t})^\alpha (1-s_{L,t})^{1-\alpha} K_t^\alpha L_t^{1-\alpha}$; (b) the law of motion for capital given by $\dot{K}_t = i_t Y_t - K_t \delta$; and (c) the law of motion for R&D-driven technology given by $\dot{A}_t = A_t^\phi (s_{K,t})^{\beta\gamma} (s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi \ln z$. After deriving the first-order conditions, the social planner solves for i^*, s_L^* and s_K^* on the balanced-growth path.

Lemma 2: *The socially optimal steady-state rate of investment in physical capital is*

$$
\begin{aligned}
(56)\quad & i^* = \left(\alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta} \\
& \neq i = \frac{z^\eta \theta + 1 - \theta}{z^\eta} \left(\alpha + \beta \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}
\end{aligned},
$$

and the socially optimal steady-state R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

$$
\begin{aligned}
(57)\quad & \frac{s_L^*}{1 - s_L^*} = \frac{1-\beta}{1-\alpha} \left(\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \\
& \neq \frac{s_L}{1 - s_L} = \frac{1-\beta}{1-\alpha} \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})
\end{aligned},
$$

$$
\begin{aligned}
(58)\quad & \frac{s_K^*}{1 - s_K^*} = \frac{\beta}{\alpha} \left(\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \\
& \neq \frac{s_K}{1 - s_K} = \frac{\beta}{\alpha} \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})
\end{aligned}.
$$

Proof: Refer to Appendix A.□

(57) and (58) indicate the various sources of R&D externalities: (a) the negative externality in intratemporal duplication given by $\gamma \in (0,1]$; (b) the positive or negative externality in intertemporal knowledge spillovers given by $\phi \in (-\infty,1)$; (c) the static consumer-surplus appropriability problem given by $(1-\theta)(z^\eta - 1)/z^\eta \in (0,1]$, which is a positive externality; (d) the markup distortion in driving a wedge of $(z^\eta \theta + 1 - \theta)/z^\eta \geq 1$ between the factor payments for production inputs and their marginal products; (e) the positive externality of cumulative innovations together with the negative externality of creative destruction (i.e. the business-stealing effect) given by the difference between $g_A/(\rho - n + (\sigma-1)g_c + g_A)$ and $\lambda/(\rho - n + (\sigma - 1)g_c + \lambda)$; and (f) the negative effects of blocking patents on R&D through the

backloading discount factor $\nu(\Omega^{\eta_{lead}}) \in (0,1]$. Given the existence of positive and negative externalities, it requires a numerical calibration that will be performed in Section 4 to determine whether the market economy overinvests or underinvests in R&D.

If the market economy underinvests in R&D as also suggested by Jones and Williams (1998) and (2000), the government may want to increase patent breadth to reduce the extent of this market failure. However, the following proposition states that even holding the effects of blocking patents constant, an increase in η mitigates the problem of R&D underinvestment at the costs of worsening the dynamic distortionary effect on capital accumulation in addition to worsening the static distortionary effect.

Proposition 2a: *The decentralized equilibrium rate of capital investment i is below the socially optimal investment rate i^* if either there is underinvestment in R&D or labor is the only factor input for R&D.*

Proof: Refer to Appendix A.□

Proposition 2b: *Holding the backloading discount factor ν constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of capital investment i if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

Proof: Refer to Appendix A.□

A higher aggregate markup increases the wedge between the marginal product of capital and the rental price. This effect by itself reduces the equilibrium rate of investment in physical capital; however, there is an opposing effect from the R&D capital share. Proposition 2b shows that the intermediate-goods sector being more capital intensive than the R&D sector is a *sufficient* condition for the negative effect to dominate. As for Proposition 2a, the discrepancy between the equilibrium rate of investment in physical capital and the social optimum arises because of: (a) the aggregate markup; and (b) the discrepancy between the market equilibrium R&D capital share s_K and the socially optimal R&D capital share s_K^*. Since the equilibrium capital investment rate i is an increasing function of s_K, the underinvestment in R&D in the market equilibrium is sufficient for $i < i^*$. On the other hand, when there is overinvestment in R&D in the market equilibrium, whether i is below or above i^* depends on whether the effect of the aggregate markup or the effect of R&D overinvestment dominates. For the case in which labor is the only factor input for R&D (i.e. $s_K = 0$), only the effect of the aggregate markup is present.

4. Calibration

Using the framework developed above, this section provides a quantitative assessment on the effects of blocking patents. Figure II-1 shows that private spending on R&D in the US as a share of GDP has been rising sharply since the beginning of the 80's. Then, after a few years, the number of patents granted by the US Patent and Trademark Office also began to increase rapidly as shown in Figure II-2. Given the patent policy changes in the 80's, the structural parameters are calibrated using long-run aggregate data of the US's economy from 1953 to 1980 to examine the extent of R&D underinvestment and inefficiency arising blocking patents before these policy changes. The goal of this numerical exercise is to quantify the effects of eliminating blocking patents on R&D, consumption and capital investment. After calibrating the model using aggregate data, an alternative calibration based on industry-level data from R&D-intensive industries will be performed to ensure the robustness of the finding that the negative effect of blocking patents on R&D is significant.

4.1. Backloading Discount Factor

The first step is to calibrate the structural parameters and the steady-state value of the backloading discount factor ν. The average annual TFP growth rate g_{TFP} is 1.33%,[18] and the labor-force growth rate n is 1.94%.[19] The annual depreciation rate δ on physical capital and the household's discount rate are set to conventional values of 8% and 4% respectively. For the aggregate markup $\tilde{\mu} = \mu^{1-\theta}$, Laitner and Stolyarov (2004) estimate that the aggregate markup is about 1.1 (i.e. a 10% markup) in the data; on the other hand, Basu (1996) and Basu and Fernald (1997) estimate that the aggregate production function has constant return to scale and the aggregate profit share is 3%. These estimates imply that the aggregate markup is about 1.03.[20] To be conservative, I will set $\tilde{\mu}$ to the lower value at 1.03.[21] For a given $\tilde{\mu}$, each value of θ (the fraction of competitive industries in the intermediate-goods sector) corresponds to a unique value for the industry markup μ in monopolistic industries, and I will consider a wide range of values for $\theta \in \{0, 0.25, 0.5, 0.75\}$. A number of structural studies based on patent renewal models has estimated the arrival rate of innovations λ, and I will consider a reasonable range of values for $\lambda \in [0.04, 0.20]$.[22] For the capital intensity parameter in the R&D sector, I will set $\beta = \alpha$ as the benchmark case.[23]

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[18] Multifactor productivity for the private non-farm business sector is obtained from the Bureau of Labor Statistics.
[19] The data on the annual average size of the labor force is obtained from the Bureau of Labor Statistics.
[20] Cost minimization implies that the return to scale = the markup x (1 – profit share).
[21] As a robustness check, an aggregate markup of 1.1 implies that the negative effect of blocking patents on R&D is even more severe. Intuitively, a higher markup means increased profitability which must be offset by a stronger effect of blocking patents in order for the level of R&D in the data to be constant. In this case, eliminating blocking patents would lead to a more significant increase in R&D and consumption.
[22] For example, Lanjouw (1998) structurally estimate a patent renewal model using patent renewal data in a number of industries from Germany, and the estimated probability of obsolescence ranges 7% for computer patents to 12% for engine patents. Also, a conventional value for the rate of depreciation in patent value is about 15% (e.g. Pakes (1986)). On the other hand, Caballero and Jaffe (2002) estimate a mean rate of creative destruction of about 4%.
[23] I have considered different plausible values for $\beta \in \{0, \alpha, 2\alpha, 3\alpha\}$ as a sensitivity analysis. The extent of R&D underinvestment and the effects of eliminating blocking patent and increasing patent breadth on long-run consumption are robust to these parameter changes.

For the remaining parameters $\{\nu, \alpha, \sigma\}$, the model provides three steady-state conditions for the calibration: (a) R&D as a share of GDP; (b) labor share; and (c) capital investment rate.

(59) $$\frac{R\&D}{Y} = \left(\frac{\mu\theta + 1 - \theta}{\mu}\right)\left((1-\alpha)\frac{s_L}{1-s_L} + \alpha\frac{s_K}{1-s_K}\right),$$

(60) $$\frac{wL}{Y} = \frac{1-\alpha}{1-s_L}\left(\frac{\mu\theta + 1 - \theta}{\mu}\right),$$

(61) $$\frac{I}{Y} = \frac{\alpha(\mu\theta + 1 - \theta)}{\mu(1-s_K)}\left(\frac{n + g_{TFP}/(1-\alpha) + \delta}{\rho + \sigma g_{TFP}/(1-\alpha) + \delta}\right),$$

where $\frac{s_L}{1-s_L} = \frac{s_K}{1-s_K} = \left(\frac{\lambda}{\rho - n + (\sigma - 1)g_c + \lambda}\right)\frac{(\mu-1)(1-\theta)}{\mu\theta + 1 - \theta}\nu(\Omega^{\eta_{lead}})$ for the case of $\alpha = \beta$. The average private spending on R&D as a share of GDP is 0.0115,[24] and the labor share is set to a conventional value of 0.7. The average ratio of private investment to GDP is 0.203.[25]

Table II-1 presents the calibrated values for the structural parameters along with the real interest rate $r = \rho + \sigma g_{TFP}/(1-\alpha)$ and the industry markup $\mu = (1.03)^{1/(1-\theta)}$ for $\theta \in \{0, 0.25, 0.5, 0.75\}$ and $\lambda \in [0.04, 0.20]$. Table II-1 shows that for a given value of θ, the calibrated values for $\{\alpha, \sigma, r\}$ are invariant to different values of λ. The calibrated value for the elasticity of intertemporal substitution (i.e. $1/\sigma$) is about 0.42, which is closed to the empirical estimates from econometric studies.[26] The implied real interest rate is about 8.4%, which is

[24] The data is obtained from the National Science Foundation and the Bureau of Economic Analysis. R&D is net of federal spending, and GDP is net of government spending. The observations in the data series of R&D spending are missing for 1954 and 1955.

[25] This number is calculated using data obtained from the Bureau of Economic Analysis, and GDP is net of government spending.

[26] It is well-known that there is a discrepancy between the estimated elasticity of intertemporal substitution from dynamic macro models (closed to 1) and econometric studies. Guvenen (2006) shows that this discrepancy is due to the heterogeneity in households' preferences and wealth inequality. In short, the *average investor* has a high elasticity of intertemporal substitution while the *average consumer* has a much lower elasticity. Since my interest is on consumption, it is appropriate to calibrate the value of σ according to the average consumer.

slightly higher than the historical rate of return on the US's stock market, and this higher interest rate implies a lower optimal level of R&D spending and a higher steady-state value of the backloading discount factor. As a result, the model is less likely to overstate the extent of R&D underinvestment and the degree of inefficiency from blocking patents. Re-expressing (59) into (62) shows that ν decreases as λ increases.

(62) $$\nu = \frac{R\&D}{GDP} \bigg/ \frac{\lambda(1-\theta)(\mu-1)/\mu}{\rho - n + (\sigma-1)g_{TFP}/(1-\alpha)+\lambda}.$$

Furthermore, the fact that the calibrated values of $\nu \in [0.485, 0.892]$ are smaller than one suggests inefficiency from blocking patents in the economy. Therefore, eliminating blocking patents may be an effective method to stimulate R&D. After calibrating the externality parameters and computing the socially optimal level of R&D spending, the effects of eliminating blocking patents will be quantified.

4.2. Externality Parameters

The second step is to calibrate the values for the externality parameters γ (intratemporal duplication) and ϕ (intertemporal spillover). For each value of g_A, g_Z, n, α and β, the balanced-growth condition (55) determines a unique value for $\gamma/(1-\phi)$, which is sufficient to determine the effect of R&D on the balanced-growth level of consumption. However, holding $\gamma/(1-\phi)$ constant, a larger γ implies a faster rate of convergence to the balanced-growth path; therefore, it is important to consider different values of γ. As for the value of g_A, I will set $g_A = \xi g_{TFP}$ for $\xi \in [0, 1]$. The parameter ξ captures the fraction of long-run TFP growth that is driven by R&D, and the remaining fraction is driven by the exogenous process Z_t such that $g_Z = (1-\xi)g_{TFP}$. Table II-2 presents the calibrated values of ϕ for $\gamma \in [0.1, 1.0]$ and $\xi \in [0, 1]$. Table II-2 shows that the calibrated values for ϕ are very similar across different values of θ implying that the socially optimal level of R&D spending and the extent of R&D overinvestment or underinvestment are about the same across different values of θ.

To reduce the plausible parameter space of γ and ξ, I make use of the empirical estimates for the social rate of return to R&D. Following Jones and Williams' (1998) derivation, Appendix C shows that the net social rate of return $\tilde{r}$ can be expressed as

$$\tilde{r} = \frac{1+g_Y}{1+g_A}\left(1+g_A\left(\frac{\gamma}{s_r}+\phi\right)\right)-1. \tag{63}$$

With a lower bound of 0.30 for $\tilde{r}$, (63) pins down a lower bound for γ under each value of ξ. Table II-3 presents the implied social rate of return $\tilde{r}$ for $\gamma \in [0.1, 1.0]$ and $\xi \in [0, 1]$, and the values exceeding 0.30 are highlighted in bold.

4.3. Socially Optimal Level of R&D Spending

This section calculates the socially optimal level of R&D share $(1-\alpha)s_L^* /(1-s_L^*) + \alpha s_K^* /(1-s_K^*)$. Figure II-3 plots the socially optimal R&D shares for the range of values for γ and ξ that satisfies the lower bound of 0.30 for $\tilde{r}$. Figure II-3 shows that there was underinvestment in R&D prior to 1980 over the entire range of parameters. Since it is difficult to determine the empirical value of ξ, I will leave it to the readers to decide on their preferred values and continue to present results for this range of parameters.

4.4. Eliminating Blocking patents

Given the calibrated structural parameters, this section quantifies the effects of eliminating blocking patents on R&D and consumption. Table II-4 shows that eliminating blocking patents (i.e. setting $\nu = 1$) would lead to a substantial increase in the steady-state share of R&D by a minimum of 12% and a maximum of 106%. In the following, the effect of eliminating blocking patents is firstly expressed in terms of the percent change in the balanced-growth level of consumption per year. Along the balanced-growth path, per capita consumption increases at an exogenous rate g_c. Therefore, after dropping the exogenous growth path and some constant terms and solving for the balanced-growth path of R&D technology and steady-state capital-labor ratio, I derive the expression for the endogenous parts of long-run consumption as a function of the steady-state value of the backloading discount factor ν through the capital investment rate $i(\nu)$, and the R&D shares of capital and labor (where $s_r(\nu) = s_L(\nu) = s_K(\nu)$ because $\alpha = \beta$).

Lemma 3: For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

(64) $$c_0(\nu) = \left(i(\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\nu)) s_r(\nu)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}} (1-s_r(\nu))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}} \right).$$ [27]

Proof: Refer to Appendix A.□

Therefore, in the case of a change in ν, the percent change in long-run consumption can be decomposed into four terms.

(65) $$\Delta \ln c_0(\nu) = \left(\begin{array}{l} \left(\dfrac{\alpha(1-\phi+\gamma)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln i(\nu) + \Delta \ln(1-i(\nu)) + \\ \left(\dfrac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln s_r(\nu) + \left(\dfrac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln(1-s_r(\nu)) \end{array} \right).$$ [28]

Figure II-4 shows that eliminating blocking patents would lead to a substantial increase in long-run consumption by a minimum of 4% and a maximum of 67%. Also, a back-of-the-envelope calculation shows that the change in consumption mostly comes from

[27] The proof in Appendix I also derives the expression for the general case in which $\alpha \neq \beta$.
[28] Note that the coefficients are determined by $\gamma/(1-\phi)$ rather than the individual values of γ and ϕ.

$(\gamma/((1-\alpha)(1-\phi)-\alpha\gamma))\Delta \ln s_r(\nu)$; in other words, the other general-equilibrium effects only have secondary impacts on long-run consumption.

After examining the effect on long-run consumption, the next numerical exercise computes the entire growth path of consumption upon eliminating blocking patents. Figure II-5a compares the transition path (in blue) of log consumption per capita with its original balanced-growth path (in red) and its new balanced-growth path (in green) for the following parameters $\{\xi,\gamma,\lambda,\theta,\delta\} = \{0.55, 0.55, 0.1, 0, 0.08\}$ to illustrate the transition dynamics. Then, I will discuss the effects of changing these parameter values. Upon setting $\nu = \Omega_1 = 1$, consumption per capita gradually rises towards the new balanced growth path. Although factor inputs shift towards the R&D sector and the output of final goods drops as a result, the possibility of investing less and running down the capital stock enables consumption smoothing. To compare with previous studies, such as Kwan and Lai (2003), Figure II-5b presents the transition dynamics for $\{\xi,\gamma,\lambda,\theta,\delta\} = \{0.55, 0.55, 0.1, 0, 1\}$ as an approximation to a model with no capital accumulation. In this case, the result is consistent with Kwan and Lai (2003) that consumption falls in response to the strengthening of patent protection. In this case, consumption falls by about 2% on impact and only recovers to its original growth path after 4 years.

A sensitivity analysis has been performed for different values of ξ and γ. At a larger value of either ξ or γ, consumption increases by even more on impact. A larger ξ also implies a higher position of the new balanced-growth path. Holding ξ constant, a larger γ implies a faster rate of convergence. When both ξ and γ are smaller than 0.55, the household suffers small consumption losses during the initial phase of the transition path. For example, Figure II-5c presents the transition dynamics for $\{\xi,\gamma,\lambda,\theta,\delta\} =$ $\{0.3, 0.3, 0.1, 0, 0.08\}$. However, Figure II-

5d shows that when ξ is closed to one, γ could be as small as 0.3 without causing any short-run consumption losses. In summary, reallocating resources from the production sector to the R&D sector does not always lead to short-run consumption losses. Finally, at a larger value of λ, the calibrated value for ν becomes smaller (see Table II-1). This larger magnitude of the policy shock increases slightly the range of parameter values that corresponds to short-run consumption losses.

4.5. Dynamic Distortion

Proposition 2a derives the sufficient condition under which the market equilibrium rate of investment in physical capital is below the socially optimal level in (56). The following numerical exercise quantifies this wedge. Figure II-6 presents the socially optimal rates of capital investment along with the US's long-run ratio of private investment to GDP of 0.203, and the wedge is about 0.024 on average. The equilibrium rate of investment in physical capital in the long run is increasing in the R&D share of capital; therefore, eliminating blocking patents also increases the rate of capital investment. Table II-5 shows that upon eliminating blocking patents, the steady-state capital investment rate increases by 0.0017 on average and moves slightly toward the socially optimal level.

4.6. Robustness Check Based on Industry-Level Data

As mentioned in Section 2, the use of aggregate data relies on the assumption that economic profit in the economy is created by intellectual monopoly. In this section, I will perform a robustness check on the finding that the negative effect of blocking patents on R&D is significant by calibrating the following condition using industry-level data

$$\frac{R\&D_{industry}}{Y_{industry}} = \nu\left(\frac{\lambda}{r+\lambda-g_{\pi}}\right)\left(\frac{\mu-1}{\mu}\right). \tag{66}$$

This numerical exercise requires an estimate for the markup in R&D-intensive industries, and I will make use of the empirical estimates for industry-level returns to scale from Basu *et al* (2006). Assuming cost minimization and non-negative economic profit, the estimates for the returns to scale provide a lower bound for the industry markups.

Based on the data on R&D from the National Science Foundation, I choose four R&D-intensive industries that account for 67% of the private R&D spending in manufacturing from 1980 to 1997: chemical products (SIC 28); machinery (SIC 35); electrical equipment (SIC 36); and motor vehicles (SIC 371). I add up the industries' total R&D spending for each year and then divide this number by the value added of these industries. The annual average ratio of R&D over value added is 0.117. The real interest rate is set to 8.4% as before, and g_π is set to the long-run GDP growth rate of 3.4%. I will consider a range of values for $\lambda \in [0.04, 0.20]$, and the valued-added weighted average return to scale from Basu *et al* (2006) is 1.30 in these R&D-intensive industries. Then, I divide this number by the aggregate profit share of 0.03 from Basu and Fernald (1997) to obtain a conservative estimate of 1.34 for the average markup in these R&D-intensive industries.[29] Figure II-7 shows that the calibrated values for v are far below one unless the arrival rate of innovations λ is very small. Therefore, the data at both the aggregate and industry levels seems to suggest that blocking patents have a severe and negative effect on R&D.

5. Conclusion

This chapter has attempted to accomplish three objectives. Firstly, it develops a tractable framework to model the transition dynamics of an economy with overlapping intellectual

[29] This estimate is conservative because the share of economic profits in an R&D-intensive industry should be much higher than in the average industry because of the intellectual monopoly created by patent protection. For example, Comin (2004) argues that the average markup in patent-protected industries should be at least 1.5.

property rights and patent pools in a generalized quality-ladder growth model. Secondly, it identifies a dynamic distortionary effect of patent policy on capital accumulation that has been neglected by previous studies. Thirdly, it applies the model to the aggregate data of the US economy to quantify the extent of underinvestment in R&D and inefficiency arising from blocking patents. The numerical exercise suggests the following findings. If R&D investment is important for TFP growth, then the inefficiency created by blocking patents is a major reason for the underinvestment in R&D in the US. In addition, making the patent system more efficient can have substantial effects on R&D and consumption. From a policy perspective, the patent authority should mitigate the effects of blocking patents through the following policies: (a) compulsory licensing with an upper limit on the amount of licensing fees charged to subsequent inventors of more advanced technology; and (b) making patent-infringement cases in court favorable to subsequent inventors of more advanced technology.

Finally, the readers are advised to interpret the numerical results with some important caveats in mind. The first caveat is that although the quality-ladder growth model has been generalized as an attempt to capture more realistic features of the US economy, it is still an oversimplification of the real world. Furthermore, the finding of eliminating blocking patents having substantial and positive effects on R&D and consumption is based on the assumptions that the empirical estimates for the social rate of return to R&D and the data on private R&D spending are not incorrectly measured by an order of magnitude. The validity of these assumptions remains as an empirical question. Therefore, the numerical results should be viewed as illustrative at best.

CHAPTER III

Quantifying the Effects of Patent Length on R&D

1. Introduction

Is the patent length an effective policy instrument in stimulating R&D? The statutory term of patent in the US was 17 years from 1861 to 1995 and then extended to 20 years as a result of the TRIPS agreement. Suppose that there is underinvestment in R&D in the market economy as suggested by Jones and Williams (1998) and (2000), why hasn't the term of patent been lengthened to stimulate R&D?[30] Especially, Kwan and Lai (2003) show in a variety-expanding growth model that extending the effective lifetime of patents would lead to a substantial increase in R&D and consequently welfare gains.

This chapter attempts to provide an answer to the above questions by developing a generalized variety-expanding growth model and calibrating the model to the aggregate data of the US economy to analyze the effects of extending the patent length. It turns out that whether an extension in the patent length would lead to a significant increase in R&D depends crucially on whether the model is calibrated properly to match the empirical patent-value depreciation rate. The numerical exercise suggests that at the empirical range of patent-value depreciation rates, extending the patent length beyond 20 years leads to only a very small increase in R&D despite R&D underinvestment in the market economy. On the other hand, shortening the patent length

[30] The WTO's Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), initiated in the 1986-94 Uruguay Round, extends the statutory term of patent in the US from 17 years (counting from the issue date when a patent is granted) to 20 years (counting from the earliest claimed filing date) to conform with the international standard. Because of the difference in the starting date, the effective extension of patent length was much shorter than 3 years.

can lead to a significant reduction in R&D and consumption. In other words, the patent length loses its effectiveness in stimulating R&D at around 20 years. This chapter also makes use of the DGE framework to examine the fraction of TFP growth that is driven by R&D. The calibration exercise suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D. Finally, this chapter identifies and analytically derives a dynamic distortion of the patent length on saving and investment in physical capital that has been neglected by previous studies, which consequently underestimate the distortionary effects of patent protection. The dynamic distortion arises because when the patent length increases, the fraction of monopolistic industries goes up. The resulting higher aggregate markup causes the wedge between the marginal product of capital and the rental price to increase. As a result, the market equilibrium rate of investment in physical capital decreases and deviates further from the social optimum.

This chapter relates to a number of studies on R&D underinvestment. In a companion paper, Chu (2007a) numerically evaluates the effect of blocking patents on R&D in a generalized quality-ladder growth model with overlapping intellectual property rights, and he finds that eliminating blocking patents can be very effective in stimulating R&D. In performing a similar quantitative analysis, Chu (2007a) and the current chapter together provide a quantitative assessment on the relative effectiveness of extending the patent length and eliminating blocking patents in solving the R&D-underinvestment problem suggested by Jones and Williams (1998) and (2000). The crucial difference arises because extending the patent length affects future monopolistic profits while eliminating blocking patents affects current monopolistic profits. Furthermore, the calibration exercise takes into consideration Comin's (2004) critique. Comin (2004) argues two points: (a) Jones and Williams' (2000) finding of R&D underinvestment is based on the assumption in their calibration that the long-run TFP growth is solely driven by R&D; and (b) the level of R&D spending in the data may be optimal if R&D only drives a small fraction of the long-run TFP growth. The current chapter contributes to this debate by bringing in an additional moment that is the patent-value depreciation rate in order to calibrate the fraction of long-run TFP growth driven by R&D, and the details will be discussed in Section 2.9.

This chapter also complements the theoretical studies in the patent-design literature that is mostly based on a partial-equilibrium setting by providing a quantitative DGE analysis on patent policy. The seminal work on patent length is Nordhaus (1969), and he concludes that the optimal patent length should balance between the static distortionary effects of markup pricing and the gains from enhanced innovations. Gilbert and Shapiro (1990) argue that given the choices of patent length and patent breadth as policy instruments, the socially optimal policy combination is an infinite patent length and a minimum degree of patent breadth.[31] In a DGE setting, Judd (1985) also concludes that the optimal patent length is infinite.[32] On the other hand, Futagami and Iwaisako (2003) show that the optimal patent length may be finite when there is no underinvestment in R&D. However, the above studies do not feature endogenous capital accumulation so that the dynamic distortion on capital accumulation is absent.

In terms of quantitative analysis, the most closely related work is Kwan and Lai (2003), and they find substantial welfare gains from extending the effective lifetime of patent. There is an important reason for the contradicting results between Kwan and Lai (2003) and the current chapter. By using the same final-goods production function as in Romer (1990), Kwan and Lai (2003) necessarily restrict the size of the markup to the inverse of the capital share. This setup restricts the balanced-growth rate of monopolistic profits captured by each patent to equal the population growth rate that is nonnegative. Relaxing this parameter restriction indicates that at the

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[31] Some other studies on optimal patent design include Tandon (1982), Klemperer (1990), Green and Scotchmer (1995), O'Donoghue (1998), O'Donoghue, Schotchmer and Thisse (1998), Hunt (1999) and Schotchmer (2004).

[32] Some other DGE studies on patent policy include Li (2001), Goh and Olivier (2002), O'Donoghue and Zweimuller (2004) and Grossman and Lai (2004). However, all of these studies neglect the dynamic distortion on capital accumulation and are qualitatively oriented. Chu (2007a) provides a more detailed discussion on these studies.

empirical range of patent-value depreciation rates estimated by previous studies, the implied growth rates of the number of varieties (that are no longer the same as the TFP growth rate) are very high; consequently, the share of monopolistic profits captured by each patent declines sharply overtime rendering patent extension ineffective in stimulating R&D. In other words, the potentially rapid decline in the market share captured by each patent due to the introduction of new varieties enables the model to feature the empirically observed depreciation in the market value of patents. As a result, extending the patent length has limited effects on R&D.

Before closing the introduction, I briefly survey the empirical literature on estimating the market value of patents using patent renewal data to gather some information about the magnitude of the rate at which a patent's value declines overtime.[33] The pioneering study that estimates a deterministic patent renewal model is Pakes and Schankerman (1984), and they find that the market value of patents depreciates at a rate of 25% per year with a 95 percent confidence interval of 18%-36%. Schankerman and Pakes (1986) provide more recent data on a number of European countries, in which about half of all patents are not renewed within 10 years and only 10% of them are renewed until the end of the statutory term.[34] Pakes (1986) develops a stochastic renewal model to capture the effect of learning about a patent's value in the initial years, and he also finds high rates of depreciation ranging from 11.4% in Germany to 19.0% in the United Kingdom. Lanjouw (1998) uses a more general stochastic renewal model to estimate the value of patents in a number of industries. In addition to the rates of depreciation, her model also estimates the annual probability that a patent becomes obsolete (i.e. complete depreciation), and it ranges from 7% for computer patents to 12% for engine patents. Although the empirical estimates tend

[33] For a more detailed survey on early studies, see e.g. Griliches (1990).

[34] All the studies cited here are based on European data. In the US, patent maintenance fees were not initiated until 1982, and the fees are due 3.5 years ($900), 7.5 years ($2300) and 11.5 years ($3800) after a patent is granted, rather than annually as in some European countries.

to vary across studies, across countries and across industries, there seems to be suggestive evidence that the rates of depreciation and obsolescence are quite high for patents.

The rest of this chapter is organized as follows. Section 2 describes the model and derives the dynamic distortionary effect of the patent length. Section 3 calibrates the model to the data, and the final section concludes with some important caveats. All proofs are contained in Appendix E.

2. The Model

The variety-expanding model is a generalized version of Romer (1990). The basic framework is modified to introduce a finite patent length denoted by T for each invented variety of intermediate goods. The final goods are produced with labor and a composite of intermediate goods. The intermediate-goods industries are monopolistic for the producers owning a valid patent and become competitive once the patent expires. The relative price between the monopolistic and competitive goods leads to the usual *static* distortionary effect that reduces the output of final goods. The markup in the monopolistic industries drives a wedge between the marginal product of capital and the rental price; consequently, it leads to an additional *dynamic* distortionary effect that causes the market equilibrium rate of investment in physical capital to deviate from the social optimum. To prevent the model from overestimating the social benefits of R&D and hence the extent of R&D underinvestment, the long-run TFP growth is assumed to be driven by R&D as well as an exogenous process as in Comin (2004). In addition, this class of first-generation R&D-driven endogenous growth models exhibits scale effects and is inconsistent with the empirical evidence in Jones (1995a).[35] In the present model, scale effects are eliminated as in Jones (1995b). After eliminating scale effects, the resulting model becomes a semi-

[35] See Jones (1999) for an excellent theoretical analysis on scale effects.

endogenous growth model, in which the balanced-growth rate is proportional to the exogenous population growth rate.

The various components of the model are presented in Sections 2.1–2.8, and the competitive equilibrium is defined in Section 2.9. Section 2.10 derives the socially optimal allocations, and Section 2.11 derives the dynamic distortionary effect of the patent length on capital accumulation. The analysis focuses on the balanced-growth path.

2.1. Representative Household

There is a representative household whose lifetime utility is given by

$$U = \int_0^\infty e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt , \tag{1}$$

where $\sigma \geq 1$ is the inverse of the elasticity of intertemporal substitution and ρ is the exogenous subjective discount rate. The household has $L_t = e^{n.t}$ members at time *t*, and $n > 0$ is the constant exogenous population growth rate. ρ is assumed to be greater than n to ensure that utility is bounded. c_t is the per capita consumption of final goods (the numeraire). The household maximizes utility subject to a sequence of budget constraints given by

$$\dot{a}_t = (r_t - n)a_t + w_t - c_t \ . \tag{2}$$

Each member of the household inelastically supplies one unit of homogenous labor in each period to earn a wage income w_t. a_t is the amount of financial assets, which consist of physical capital and patents, owned by each member of the household, and r_t is the real rate of return on these financial assets. From the household's intertemporal optimization, the familiar Euler equation is

$$\dot{c}_t / c_t = (r_t - \rho) / \sigma \ . \tag{3}$$

Along the balanced growth path, c_t increases at a constant rate g_c. Therefore, the equilibrium real interest rate along the balanced-growth path is

(4) $$r = \rho + g_c \sigma .$$

2.2. Final Goods

The sector producing the final goods is characterized by perfect competition, and the producers take both the output price and input prices as given. In particular, the final-goods production function is

(5) $$Y_t = Z_t^{1-\alpha} L_{y,t}^{1-\alpha} \left(\int_0^{V_t} X_t^{\alpha\eta}(j) dj \right)^{1/\eta}$$

for $\eta \in (0, 1/\alpha)$. Y_t is the amount of final goods produced. $Z_t = Z_0 \exp(g_Z t)$ represents an exogenous process of productivity improvement that is freely available to all final-goods producers. $L_{y,t}$ is the number of production workers. $X_t(j)$ is the amount of intermediate goods of variety $j \in [0, V_t]$, in which V_t is the number of varieties that has been invented as of time *t*. The production function in (5) nests Romer (1990) as a special case with $\eta = 1$ and $Z_t = 1$ for all *t*. For $\eta = 1$, the monopoly markup μ is restricted to be $1/\alpha$ (i.e. roughly the inverse of the capital share); therefore, Jones and Williams (2000) propose a more realistic specification that allows η to differ from one so that the markup is given by $\mu = 1/(\alpha\eta)$ in order to relax the parameter restriction between the markup and the capital share.

The final-goods producers take $Z_t^{1-\alpha}$ as given. The current chapter includes this exogenous TFP process for two reasons: (a) to avoids the mistake in assuming that the long-run TFP growth in the data is solely driven by R&D; and (b) to relax the restriction between the patent-value depreciation rate and the long-run TFP growth rate denoted by g_{TFP}. This restriction will be discussed in details in Section 2.9. In short, by setting $g_z = 0$ (i.e. by assuming that the long-run TFP growth is solely driven by R&D), the balanced-growth rate of the number of varieties g_V is pinned down by the TFP growth rate g_{TFP}, the capital share α, and the markup $1/(\alpha\eta)$. Once g_V is determined, the patent-value depreciation rate is also uniquely pinned down, and the calibration results suggest that this implied patent-value depreciation rate differs substantially from the previous empirical estimates based on patent renewal data. Therefore, the current chapter adopts the specification in (5) that allows g_z to differ from zero in order to bring in the empirical estimates for the patent-value depreciation rate and perform a more realistic calibration.

Profit maximization yields the first-order conditions for the wage rate and the price of intermediate-goods $P_t(j)$ for $j \in [0, V_t]$ given by

(6) $$w_t = (1-\alpha) Y_t / L_{y,t},$$

(7) $$P_t(j) = \alpha Z_t^{1-\alpha} L_{y,t}^{1-\alpha} \left(\int_0^{V_t} X_t^{\alpha\eta}(j) dj \right)^{(1-\eta)/\eta} X_t^{\alpha\eta-1}(j) \ .$$

2.3. Intermediate Goods

There is a continuum of industries, indexed by $j \in [0, V_t]$, producing the differentiated intermediate goods $X_t(j)$. Once a variety has been invented, the production function in industry j is

(8) $$X_t(j) = K_{y,t}(j).$$

$K_{y,t}(j)$ is the amount of capital employed by industry j. The profit function facing the producer(s) of variety j is

(9) $$\pi_t(j) = P_t(j)X_t(j) - R_t K_{y,t}(j).$$

R_t is the rental price of capital. Denote the steady-state fraction of monopolistic industries by ω, which is endogenously determined by the patent length *T*. Without loss of generality, the industries are ordered such that industries $j \in [0, \omega V_t]$ are protected by patents and industries $j' \in (\omega V_t, V_t]$ are not protected by patents. Then, the first-order conditions are

(10) $$P_t(j) = R_t / \alpha\eta$$

for $j \in [0, \omega V_t]$, and

(11) $$P_t(j') = R_t$$

for $j' \in (\omega V_t, V_t]$.

2.4. Aggregate Production Function and Static Distortion

The total amount of capital employed by the intermediate-goods sector at time t is

(12) $$K_{y,t} = \int_0^{V_t} X_t(j)dj = V_t(\omega X_t(j) + (1-\omega)X_t(j')).$$

Lemma 1: *The aggregate production function for the final goods is*

(13) $$Y_t = \tilde{\omega} A_t^{1-\alpha} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha},$$

where A_t *is the level of R&D-driven TFP and is defined as*

(14) $$A_t^{1-\alpha} \equiv V_t^{(1-\alpha\eta)/\eta},$$

and $\tilde{\omega}$ *is defined as*

(15) $$\tilde{\varpi} \equiv \frac{(\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1-\omega)^{1/\eta}}{(\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1-\omega)^{\alpha}}.$$

$\tilde{\varpi}$ is strictly less than one for $\omega \in (0,1)$ and equals one for $\omega \in \{0,1\}$. In addition,

$$\partial\tilde{\varpi}/\partial\omega < 0 \text{ when } \omega < \overline{\varpi} \equiv \frac{1}{1-\alpha\eta}\left(\frac{1}{1-(\alpha\eta)^{1/(1-\alpha\eta)}} - \frac{\alpha\eta}{1-(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)}}\right) < 1.$$

Proof: See Appendix E.□

$\tilde{\varpi}$ captures the usual static distortionary effect of patent protection in creating a monopolistic markup in the patent-protected industries. In other words, the markup in the monopolistic industries distorts production towards the competitive industries and thus reduces the total output of the final goods. Increasing the fraction of monopolistic industries worsens this static distortionary effect when $\omega < \overline{\varpi}$. This static distortionary effect is not monotonic in the patent length because at an infinite patent length, all industries are monopolistic and the relative-price distortion disappears.

2.5. National Income Account Identities

The market-clearing condition for the final goods is

(16) $$Y_t = C_t + I_t.$$

$C_t = L_t c_t$ is aggregate consumption, and I_t is investment in physical capital. The correct value of GDP should include the amount of investment in R&D such that

(17) $$GDP_t = Y_t + w_t L_{r,t} + R_t K_{r,t}.$$ [36]

[36] In the national income account, private spending in R&D is treated as an expenditure on intermediate goods. Therefore, the values of capital investment and GDP in the data are I_t and Y_t respectively. The Bureau of Economic Analysis and the National Science Foundation's R&D satellite account provides preliminary estimates on the effects of including R&D as an intangible asset in the national income accounts.

$L_{r,t}$ and $K_{r,t}$ are respectively the number of workers and the amount of capital in the R&D sector that invents new varieties. The amount of monopolistic profits, the factor payments for production workers and capital in the intermediate-goods sector are given by

(18) $$w_t L_{y,t} = (1-\alpha)Y_t,$$

(19) $$R_t K_{y,t} = \alpha Y_t \hat{\omega},$$

(20) $$\pi_t V_t \omega = \alpha Y_t (1-\hat{\omega}),$$

where $\hat{\omega} \in [\alpha\eta, 1]$ is determined by the fraction of monopolistic industries ω and is defined as

(21) $$\hat{\omega} \equiv \frac{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega}.$$

A rise in the fraction of monopolistic industries ω leads to a decrease in $\hat{\omega}$ and consequently, increases the wedge between the marginal product of capital and the rental price. As will be shown below, this decrease in $\hat{\omega}$ also leads to a lower rate of investment in physical capital. Therefore, $\hat{\omega}$ captures the dynamic distortionary effect of the patent length on capital accumulation.

2.6. Capital Accumulation

The market-clearing condition for physical capital is

(22) $$K_t = K_{y,t} + K_{r,t}.$$

K_t is the total amount of capital available in the economy at time *t*. The law of motion for capital is

(23) $$\dot{K}_t = I_t - K_t \delta$$

δ is the rate of depreciation. Denote the balanced-growth rate of capital by g_K, the endogenous steady-state investment rate is

(24) $$i = (g_K + \delta) K_t / Y_t$$

for all *t*. The no-arbitrage condition $r_t = R_t - \delta$ implies that the steady-state capital-output ratio is

(25) $$\frac{K_t}{Y_t} = \frac{\alpha \hat{\omega}}{(1 - s_K)(\rho + g_c \sigma + \delta)}.$$

s_K is the endogenous steady-state R&D share of capital. Substituting (25) into (24) yields

(26) $$i = \frac{\alpha \hat{\omega}}{1 - s_K} \left(\frac{g_K + \delta}{\rho + g_c \sigma + \delta} \right).$$

In the Romer model, (skilled) labor is the only input for R&D (i.e. $s_K = 0$); therefore, the distortionary effect of patent length on the rate of investment in capital is unambiguously negative (i.e. $\partial i / \partial T < 0$). In the current model with $s_K \geq 0$, there is an opposing positive effect operating through s_K. Intuitively, an increase in the patent length raises the private return on R&D and hence the share of capital employed in the R&D sector. Proposition 1 in Section 2.11 shows that the negative effect still dominates.

2.7. R&D

The no-arbitrage value of a patent $P_{r,t}$ for a new variety invented at time *t* is the expected present value of the stream of monopolistic profits earned by an R&D entrepreneur until the patent expires given by

(27) $$P_{r,t} = \int_t^{t+T} e^{-r(\tau - t)} \pi_\tau d\tau = \Omega(T) \pi_t.$$

$\Omega(T) \equiv \dfrac{1 - e^{-(\rho + g_c \sigma - g_\pi) T}}{\rho + g_c \sigma - g_\pi}$ is the present-value discount factor after substituting in (4) for the steady-state real interest rate and defining g_π as the balanced-growth rate of monopolistic profits. When $g_\pi = 0$, the market value of a patent depreciates approximately at an annual rate of $1/T$.[37] For example, when the patent length is 20 years and $g_\pi = 0$, the market value of a patent depreciates at roughly 5% per year. However, the empirical estimates from the patent renewal data suggest that a reasonable range for the patent-value depreciation rates is between 15% and 25%; therefore, $g_\pi \in [-0.2,-0.1]$. In other words, an invention loses about 10% to 20% of its market share per year on average. The marginal effect of patent length on $\Omega(T)$ given by $\Omega'(T) = e^{-(\rho+g_c\sigma-g_\pi)T}$ is positive, and this marginal effect depends positively on the profit growth rate g_π. Therefore, a highly negative profit growth rate (i.e. a high patent-value depreciation rate) would render patent extension ineffective in raising the market value of patents.

The instantaneous probability $\lambda_t(k)$ of an innovation success for R&D entrepreneur $k \in [0,1]$ is

(28) $$\lambda_t(k) = \overline{\varphi}_t L_{r,t}^{1-\beta}(k) K_{r,t}^{\beta}(k),$$

where $\overline{\varphi}_t$ is the productivity parameter of R&D inputs that the entrepreneurs take as given. This specification nests the "knowledge-driven" specification in Romer (1990) as a special case with $\beta = 0$ and the "lab equipment" specification in Rivera-Batiz and Romer (1991) and Jones and Williams (2000) as a special case with $\beta = \alpha$. The R&D sector is characterized by constant returns to scale and perfect competition. The amount of expected profit of investing in R&D is

(29) $$\pi_{r,t}(k) = P_{r,t}\lambda_t(k) - w_t L_{r,t}(k) - R_t K_{r,t}(k).$$

The first-order conditions for R&D entrepreneur *k* are

(30) $$(1-\beta)P_{r,t}\overline{\varphi}_t(K_{r,t}(k)/L_{r,t}(k))^{\beta} = w_t,$$

(31) $$\beta P_{r,t}\overline{\varphi}_t(K_{r,t}(k)/L_{r,t}(k))^{\beta-1} = R_t.$$

[37] This approximation is exact when the interest rate is zero.

(30) and (31) together with (18) and (19) determine the resource allocation between production and R&D.

2.8. Law of Motion for the Number of Varieties

To eliminate scale effects and to ensure the existence of a balanced-growth path in the presence of population growth, I follow Jones and Williams (2000) to assume that the R&D productivity parameter $\overline{\varphi}_t$ is a function of V_t given by

$$\overline{\varphi}_t = \varphi V_t^{\phi}(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma-1}. \tag{32}$$

$\phi \in (-\infty,1)$ captures the externality in intertemporal knowledge spillovers,[38] and $\gamma \in (0,1]$ captures the negative externality in intratemporal duplication or the so-called "stepping-on-toes" effects. The law of motion for the number of varieties is

$$\dot{V}_t = \int_0^1 \lambda_t(k)dk = \overline{\varphi}_t K_{r,t}^{\beta} L_{r,t}^{1-\beta} = \varphi V_t^{\phi}(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}. \tag{33}$$

Along the balanced-growth path, $K_{r,t} = \int_0^1 K_{r,t}(k)dk$ increases at g_K, and $L_{r,t} = \int_0^1 L_{r,t}(k)dk$ increases at the exogenous population growth rate n. Therefore, the balanced-growth rate of V_t denoted by g_V is

$$g_V \equiv \frac{\dot{V}_t}{V_t} = \varphi \frac{(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}}{V_t^{1-\phi}} = \left(\frac{\gamma\beta}{1-\phi}\right) g_K + \left(\frac{\gamma(1-\beta)}{1-\phi}\right) n. \tag{34}$$

Finally, the steady-state fraction of patented varieties ω is given by

$$\omega \equiv \frac{\widetilde{V}_t}{V_t} = \frac{\dot{V}_t - \dot{V}_{t-T}}{\dot{V}_t} = 1 - e^{-g_V T}, \tag{35}$$

[38] As discussed in Jones (1995b), $\phi \in (0,1)$ corresponds to the "standing-on-shoulder" effect, in which R&D productivity increases as V_t increases, and $\phi \in (-\infty,0)$ refers to the "fishing-out" effect, in which R&D productivity decreases as V_t increases.

where $\widetilde{V}_t$ is the number of patented varieties at time *t* and $\dot{V}_t - \dot{V}_{t-T}$ is the net increase in the number of patented varieties at time *t*.

2.9. Decentralized Equilibrium and Balanced-Growth Path

The analysis starts at $t = 0$ when the economy has reached the balanced-growth path corresponding to the patent length *T*. The equilibrium is a sequence of allocations $\{c_t, a_t, X_t(j), Y_t, I_t, K_{y,t}, L_{y,t}, K_{r,t}, L_{r,t}, K_t, L_t\}_{t=0}^{\infty}$ and a sequence of prices $\{w_t, r_t, R_t, P_t(j), P_{r,t}\}_{t=0}^{\infty}$ that are consistent with initial conditions $\{K_0, L_0, Z_0, V_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also, in each period,

(a) the representative household chooses $\{c_t, a_t\}$ to maximize utility taking $\{w_t, r_t\}$ as given;

(b) the competitive firms in the final-goods sector choose $\{X_t(j), L_{y,t}\}$ to maximize profits according to the production function taking $\{P_t(j), w_t\}$ as given;

(c) the monopolistic firms $j \in [0, \omega V_t]$ in the intermediate-goods sector choose $\{P_t(j), K_{y,t}(j)\}$ to maximize profits according to the demand curve from the final-goods sector and the production function taking $\{R_t\}$ as given;

(d) the competitive firms $j' \in (\omega, 1]$ in the intermediate-goods sector choose $\{K_{y,t}(j')\}$ to maximize profits according to the production function taking $\{P_t(j'), R_t\}$ as given;

(e) the entrepreneurs $k \in [0,1]$ in the R&D sector choose $\{L_{r,t}(k), K_{r,t}(k)\}$ to maximize profits according to the production function taking $\{w_t, R_t, P_{r,t}, \overline{\varphi}_t\}$ as given;

(f) the market for the final goods clears such that $Y_t = C_t + I_t$;

(g) the full employment of capital such that $K_t = K_{y,t} + K_{r,t}$; and

(h) the full employment of labors such that $L_t = L_{y,t} + L_{r,t}$.

Equating the first-order conditions (18) and (30) and imposing the balanced-growth condition

(36) $$V_t = \frac{\overline{\varphi}_t K_{r,t}^{\beta} L_{r,t}^{1-\beta}}{g_V}$$

yield the steady-state shares of labor inputs given by

(37) $$\frac{s_L}{1-s_L} = \frac{1-\beta}{1-\alpha}\left(\frac{(1-e^{-(\rho+g_c\sigma-g_\pi)T})g_V}{\rho+g_c\sigma-g_\pi}\right)\frac{\alpha(1-\hat{\omega})}{\omega}.$$

Similarly, equating (19) and (31) and imposing (36) yield the steady-state shares of capital inputs as

(38) $$\frac{s_K}{1-s_K} = \frac{\beta}{\alpha}\left(\frac{(1-e^{-(\rho+g_c\sigma-g_\pi)T})g_V}{\rho+g_c\sigma-g_\pi}\right)\frac{\alpha(1-\hat{\omega})}{\omega}.$$

The balanced-growth rates of various variables are given as follows. From the aggregate production function in (13) and the steady-state investment rate in (26),

(39) $$g_Y = g_K = g_I = g_C = g_A + g_Z + n.$$

Therefore, $g_c = g_A + g_Z$. From the definition of R&D-driven TFP in (14),

(40) $$g_A = \frac{1}{1-\alpha}\left(\frac{1-\alpha\eta}{\eta}\right)g_V.$$

Finally, from (20), the balanced-growth rate of monopolistic profits is

(41) $$g_\pi = g_Y - g_V = g_A + g_Z + n - g_V.$$

Note that g_π is restricted to equal $g_Z + n > 0$ when $\eta = 1$ because $g_A = g_V$. However, when η is large (i.e. a low markup), g_V becomes large relative to g_A. Therefore, holding g_A

constant, an increase in η leads to a decrease in g_π. Eventually, g_π becomes negative for a low enough markup.

Denote the fraction of the long-run TFP driven by R&D by ξ such that $g_A = \xi g_{TFP} /(1-\alpha)$ and $g_Z = (1-\xi) g_{TFP} /(1-\alpha)$. If $\xi = 1$, then the value of g_π is pinned down by g_A, α, η and n according to (40) and (41). The resulting implied value of g_π could be seriously biased due to the misspecification of the model. Therefore, I allow ξ to differ from one and calibrate this parameter from the data. Firstly, I make use of the previous empirical estimates for the patent-value depreciation rate to determines g_π. Note that once g_π is given, (41) pins down a unique value for g_V for given values of g_{TFP}, α and n. Then, given g_V, (40) pins down a unique value for g_A for given values of α and η.

Finally, from (34) and using (39) and (40), the balanced-growth condition that determines the externality parameters γ and ϕ is given by

(42) $$g_V = \left(\frac{1-\phi}{\gamma} - \frac{\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \right)^{-1} (\beta g_Z + n).$$

2.10. Socially Optimal Allocations

To derive the socially optimal rate of capital investment i^* and R&D shares of labor s_L^* and capital s_K^*, the social planner maximizes

(43) $$U = \int_0^\infty e^{-(\rho-n)t} \frac{((1-i_t)Y_t / L_t)^{1-\sigma}}{1-\sigma} dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(44) $$Y_t = V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} (1-s_{K,t})^{\alpha} (1-s_{L,t})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of i_t given by

(45) $$\dot{K}_t = i_t Y_t - K_t \delta;$$

and (c) the law of motion for the number of varieties expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(46) $$\dot{V}_t = V_t^{\phi} (s_{K,t})^{\beta\gamma} (s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi.$$

After deriving the first-order conditions, the social planner solves for i^*, s_L^* and s_K^* on the balanced-growth path.

Lemma 2: *The socially optimal rate of capital investment i^* is*

(47) $$i^* = \left(\alpha + \beta \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma-1) g_c + (1-\phi) g_V} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta},$$

and the socially optimal R&D shares of labor s_L^* and capital s_K^* are respectively

(48) $$\begin{aligned} \frac{s_L^*}{1-s_L^*} &= \frac{1-\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma-1) g_c + (1-\phi) g_V} \\ \neq \frac{s_L}{1-s_L} &= \frac{1-\beta}{1-\alpha} \left(\frac{\alpha(1-\hat{\omega})}{\omega} \right) \frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T}) g_V}{\rho - n + (\sigma-1) g_c + g_V} \end{aligned},$$

(49) $$\begin{aligned} \frac{s_K^*}{1-s_K^*} &= \frac{\beta}{\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma-1) g_c + (1-\phi) g_V} \\ \neq \frac{s_K}{1-s_K} &= \frac{\beta}{\alpha} \left(\frac{\alpha(1-\hat{\omega})}{\omega} \right) \frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T}) g_V}{\rho - n + (\sigma-1) g_c + g_V} \end{aligned}.$$

Proof: See Appendix E.□

(48) and (49) indicate the various R&D externalities: (a) the negative externality in intratemporal duplication $\gamma \in (0,1]$; (b) the positive or negative externality in intertemporal knowledge spillovers $\phi \in (-\infty, 1)$; (c) the positive externality from the dynamic surplus-appropriability

problem due to a finite patent length given by $(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})<1$; and finally, (d) the positive externality from the static surplus-appropriability problem given by $(1-\alpha\eta)/\eta>\alpha(1-\hat{\omega})/\omega$ for all *T*. Given the existence of positive and negative externalities, it requires a careful calibration that will be performed in Section 3 to determine whether the market economy over- or under-invests in R&D.

2.11. Dynamic Distortion

If the market economy underinvests in R&D, the government may want to increase the patent length to reduce the extent of this market failure. However, an increase in *T* would worsen the dynamic distortionary effect on capital accumulation. Therefore, the government needs to trade off the gains from the increase in R&D and the losses caused by the dynamic distortion and potentially the static distortion. Proposition 1 provides the condition under which the markup-pricing distortion moves the market equilibrium rate of capital investment i away from the social optimum i^*.

Proposition 1: *The decentralized equilibrium capital investment rate* i *is below the socially optimal investment rate* i^* *if either there is underinvestment in R&D or labor is the only factor input for R&D. In addition, an increase in the patent length always reduces the equilibrium capital investment rate* i.

Proof: See Appendix E.□

The second part of the proposition is quite intuitive. When the patent length increases, the fraction of monopolistic industries rises. The resulting higher aggregate markup drives a bigger wedge between the marginal product of capital and the rental price. Therefore, the market equilibrium rate of investment in physical capital decreases. As for the first part of the proposition, the discrepancy between the market equilibrium rate of investment in physical capital and the social optimum arises because of: (a) the aggregate markup; and (b) the discrepancy between the market equilibrium R&D capital share s_K and the socially optimal R&D capital share s_K^*. Since the market equilibrium capital investment rate i is an increasing function of s_K, the underinvestment in R&D in the market equilibrium is sufficient for $i < i^*$. On the other hand, when there is overinvestment in R&D in the market equilibrium, whether i is below or above i^* depends on whether the effect of the aggregate markup or the effect of R&D overinvestment dominates. For the case in which labor is the only factor input for R&D, $s_K = 0$; therefore, only the effect of the aggregate markup is present.

3. Calibration

This section firstly calibrates the structural and externality parameters using long-run aggregate data of the US economy and then computes the changes in R&D and consumption from varying the patent length. After that, the dynamic distortionary effects are also examined.

3.1. Structural Parameters

The statutory patent length *T* in the US is 20 years, and the average annual labor-force growth rate n is 1.66%.[39] The annual discount rate ρ and the annual rate of depreciation δ for physical capital are set to conventional values of 0.04 and 0.08 respectively. β is set equal to α corresponding to the lab-equipment specification in Rivera-Batiz and Romer (1991) and Jones and Williams (2000).[40] Once the above parameters are determined, the model provides five

[39] This number is calculated using data between 1956 and 2006 from the Bureau of Labor Statistics.
[40] I have considered different plausible values for $\beta \in \{0, \alpha, 2\alpha, 3\alpha\}$ as a sensitivity analysis, and the results are robust to these parameter changes.

steady-state conditions (summarized in (50)-(54) below) to match the following five moments in the data and to determine the remaining five structural parameters $\{\sigma, \alpha, \eta, g_V, \xi\}$. The ratio of private investment to GDP is 20.21%,[41] and the labor share of total income is set to a conventional value of 0.7. The ratio of private spending on R&D to GDP is 1.49%.[42], and the average annual TFP growth rate $g_{TFP} = (1-\alpha)(g_A + g_Z)$ is 1.02%.[43] As discussed in Section 2.7, the empirical estimates based on the patent renewal data suggest that a reasonable range for the patent-value depreciation rates is between 15% and 25%; therefore, $g_\pi \in [-0.2, -0.1]$.

(50) $$\frac{I}{Y} = \frac{\alpha\hat{\omega}}{1-s_K}\left(\frac{n+g_c+\delta}{\rho+g_c\sigma+\delta}\right),$$

(51) $$\frac{wL}{Y} = \frac{1-\alpha}{1-s_L},$$

(52) $$\frac{wL_r + RK_r}{Y} = (1-\alpha)\frac{s_L}{1-s_L} + \hat{\omega}\alpha\frac{s_K}{1-s_K},$$

(53) $$g_{TFP} = \frac{1}{\xi}\left(\frac{1-\alpha\eta}{\eta}\right)g_V.$$

(54) $$g_\pi = g_{TFP}/(1-\alpha) + n - g_V.$$

For $\alpha = \beta$, the R&D share of labor and capital is

$$\frac{s_r}{1-s_r} = \left(\frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})g_V}{\rho-n+(\sigma-1)g_c+g_V}\right)\frac{\alpha(1-\hat{\omega})}{\omega}.$$

Table III-1 lists the calibrated structural parameters along with the implied markup $\mu = 1/(\alpha\eta)$ and the implied real interest rate $r = \rho + g_A\sigma$. The implied markup is within the

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[41] This number is calculated using data between 1956 and 2006 from the Bureau of Economic Analysis, and GDP is net of government spending.
[42] This number is calculated using data between 1956 and 2004 from the Bureau of Economic Analysis and the National Science Foundation. R&D is net of federal spending, and GDP is net of government spending.
[43] Multifactor productivity for private non-farm business sector from the Bureau of Labor Statistics is available from 1956 to 2002.

empirically plausible range. For example, Laitner and Stolyarov's (2004) estimated markup is 1.09-1.11, and Basu and Fernald (1997) estimate that the aggregate profit share in the US is about 3%.[44] Also, the implied real interest rate is closed to the historical rate of return in the US stock market. The calibrated values for ξ suggest that roughly 35% to 45% of the long-run TFP growth in the US is driven by R&D.

3.2. Externality Parameters

For each set of the calibrated parameters, the balanced-growth condition in (42) determines a unique value for $\gamma/(1-\phi)$, which is sufficient to determine the effect of R&D on the balanced-growth level of consumption. However, holding $\gamma/(1-\phi)$ constant, a larger γ implies a faster rate of convergence to the balanced-growth path; therefore, it is important to consider different values for $\gamma \in [0.1, 1.0]$ that is the parameter for the negative externality in intratemporal duplication. The calibrated values for ϕ that is the parameter for the externality in intertemporal knowledge spillovers are listed in Table III-2, and the positive values suggest positive knowledge spillovers (i.e. the standing-on-shoulder effect).

3.3. Socially Optimal R&D

This section computes the socially optimal level of R&D share $(1-\alpha)s_L^*/(1-s_L^*)+\alpha s_K^*/(1-s_K^*)$. Figure III-1 shows that there is underinvestment in R&D unless γ is very small. To reduce the plausible parameter space for γ, I make use of the empirical estimates for the social rate of return to R&D. Following Jones and Williams' (1998) derivation, Appendix F shows that the net social rate of return $\tilde{r}$ can be expressed as

[44] Assuming cost minimization, the return to scale = markup x (1 - the profit share). Basu and Fernald's (1997) estimates also suggest that "a typical industry has roughly constant returns to scale." (p. 250)

$$\tilde{r} = \frac{1+g_Y}{1+g_V}\left(1+g_V\left(\frac{1-\alpha\eta}{\eta}\frac{\gamma}{s_r}+\phi\right)\right)-1. \tag{55}$$

Holding other things constant, $\tilde{r}$ is increasing in γ. Table III-3 shows that for the range of values for $\gamma \leq 0.2$ that exhibits R&D underinvestment, the implied social rates of return $\tilde{r}$ are less than 8%, which are far below the empirical estimates summarized in Jones and Williams (1998). Since the empirical estimates for the social rate of return to R&D vary across studies, I will leave it to the readers to decide on their preferred values. For the relevant range of γ, there is underinvestment in R&D. This finding of R&D underinvestment is due to the calibration result that a non-negligible fraction of long-run TFP growth is driven by R&D. In other words, if the calibrated values for ξ were smaller, then the socially optimal levels of R&D spending would be lower. This is because a lower calibrated value for ξ implies a lower calibrated value for $\gamma/(1-\phi)$, which in turn implies that R&D would have a smaller effect on consumption.

3.4. Patent Extension

Given the above finding of R&D underinvestment, a natural question to ask is whether extending the patent length can effectively mitigate this problem. Figure III-2 shows that the magnitude of the increases in R&D from extending the patent length beyond 20 years depends on the patent-value depreciation rate. At a high patent-value depreciation rate, the stimulating effect of the patent length on R&D is almost negligible. At a low patent-value depreciation rate, extending the patent length from 20 to 50 years increases R&D slightly by 0.2 percentage points, and the resulting level of R&D is still far below the social optimum. Therefore, this numerical exercise suggests that patent extension is not an effective method in stimulating R&D confirming the intuition discussed in Section 2.7. On the other hand, shortening the patent length can reduce R&D spending significantly.

The next exercise computes the percentage changes in long-run consumption when the patent length varies from 20 years.

Lemma 3: For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

$$c(T) = \begin{pmatrix} \tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} i(T)^{\frac{\alpha\eta(1-\phi)+\alpha\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} (1-i(T)) \\ s_r(T)^{\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} (1-s_r(T))^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}} \end{pmatrix}. \tag{56}$$

Proof: See Appendix E.□

Figure III-3 plots the percentage changes in long-run consumption. Given the small increases in R&D from patent extension, the positive effect on long-run consumption is no more than 3% at the lower bound of the patent-value depreciation rates and is as small as 0.61% at the upper bound. A back-of-the-envelope calculation shows that the increase in consumption mostly comes from $\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}\Delta \ln s_r(T)$ that is the direct effect of increasing R&D spending on consumption. In other words, the other general-equilibrium and distortionary effects only have secondary impacts.

3.5. Dynamic Distortion

Proposition 1 derives the sufficient condition under which the market equilibrium rate of investment in physical capital is below the socially optimal level in (47). The next numerical exercise quantifies this discrepancy. Figure III-4 presents the socially optimal rates of investment in physical capital along with the US's long-run investment rate, and the difference is about 1.7% on average. The equilibrium rate of investment in physical capital is decreasing in the aggregate markup; therefore, extending the patent length decreases the capital investment rate and causes it to deviate further from the social optimum. Figure III-5 presents the equilibrium rates of capital investment at different patent length. Extending the patent length from 20 to 50 years would cause the steady-state equilibrium rate of investment in physical capital to decrease slightly by at most 0.24%.

4. Conclusion

This chapter provides a quantitative framework to evaluate the effects of extending the patent length. At the empirical range of patent-value depreciation rates, extending the patent length beyond 20 years leads to only a very small increase in R&D. Therefore, the policy implication is that the patent length is not an effective policy instrument in solving the R&D-underinvestment problem. Although the analysis focuses on the balanced-growth path, taking into consideration the transition dynamics should not alter this policy implication. This is because if the long-run effects on consumption are so small, accounting for the potential short-run consumption losses would make the overall welfare gains even more negligible. The calibration exercise also suggests that about 35% to 45% of the long-run TFP growth in the US is driven by R&D and extending the patent length would worsen the dynamic distortionary effect slightly by reducing the steady-state rate of investment in physical capital by at most 0.24%.

The readers are advised to interpret the numerical results with some important caveats in mind. Although the variety-expanding growth model has been generalized to capture more realistic features of the US economy, it is still an oversimplification of the real world. Furthermore, the finding of R&D underinvestment is based on the assumptions that the empirical estimates for the social rate of return to R&D and the data on private R&D spending are not incorrectly measured by an order of magnitude. The validity of these assumptions remains as an empirical question. Therefore, the numerical results should be viewed as illustrative at best.

CHAPTER IV

Special Interest Politics and Intellectual Property Rights

> "What's true of the eight-hundred-pound gorilla is true of the colossus that is the pharmaceutical industry. It is used to doing pretty much what it wants to do." – Marcia Angell (2005, p. 3)

1. Introduction

Since the 80's, the pharmaceutical industry has benefited substantially from a series of laws enacted by the Congress, such as the Bayh Dole Act in 1980,[45] the Hatch-Waxman Act in 1984,[46] the Prescription Drug User Fee Act of 1992,[47] and the Food and Drug Administration Modernization Act of 1997.[48] These policies result in an extension of the commercial lifetime of patents for brand-name drugs.[49] So, why did the government implement all these policy changes?

At the first glance, the answer could be quite obvious that there must be incentives for the policymakers to do so. But, the real puzzle is about what exactly these incentives are. It may be

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[45] The Bayh-Dole Act essentially allows universities and other non-profit institutions to patent discoveries from publicly funded medical research and then to grant exclusive licenses to drug companies. See, e.g. Jaffe (2000), Eisenberg (2001) and Angell (2005, chapter 1).

[46] The Hatch-Waxman Act enables drug companies to extend monopoly rights by restoring up to five years of lost patent time on premarket testing and the FDA approval process. See, e.g. Eisenberg (2001) and Gallini (2002). Also, as a result of this Act, "…if a brand-name company sues a generic company for patent infringement, FDA approval of the generic drug will automatically be delayed for [up to] thirty months… In effect, the FDA will add thirty months to the brand-name drug's exclusivity." Angell (2005, p. 179-180)

[47] The Prescription Drug User Fee Act of 1992 authorizes the FDA to collect user fees from the pharmaceutical industry and at the same time requires the FDA to significantly reduce its approval time for drugs. As a result, the FDA approval time decreases from 2.6 years to 1.4 years, which in turn increases the commercial lifetime of patents for drugs. See, e.g. NIHCM (2000).

[48] The Food and Drug Administration Modernization Act of 1997 extends the monopoly rights for a drug by six months if it is tested in children. "The result is that drug companies now test their blockbusters, including drugs to treat primarily adult diseases like high blood pressure, in children, just because the extra protection is so lucrative." Angell (2005, p. 182)

[49] Refer to Table 2 for details. The commercial lifetime of patent for a drug refers to the number of years remaining in the drug's patent term after clinical testing and FDA's approval of the drug.

the case that the government has the incentives to act as a social planner and to maximize social welfare as we, economists, usually like to assume. However, the reality is neither so simple nor ideal. Douglass North (2005, p. 67) describes the incentives of the government as,

> "The Government is not a disinterested party in the economy. By the very nature of the political process..., the government has strong incentives to behave opportunistically to maximize the rents of those with access to the government decision-making process... [I]t means that the government will cartelize economic activity in favor of politically influential parties. In rare cases the government designs and enforces a set of rules of the game that encourage productive activity."

The $200-billion industry not only has access to the government's decision-making process,[50] but it is indeed so politically influential that "PhRMA, this lobby, has a death grip on Congress", in the words of Senator Richard J. Durbin.[51] This political influence potentially comes from the impressive amount of the industry's lobbying expenditures and campaign contributions. For example, the industry's total expenditure on lobbying from 1998 to 2006 was $1,087 million, and total campaign contributions were $139 million during the election cycles from 1990 to 2006.[52] In fact, given the nature of the industry, it is easy to understand that it is in the drug companies' best interest to have access to the policymakers, who can easily return favors at low political costs. For a blockbuster (a drug that has sales of over a billion dollars a year), an extension of the patent's effective lifetime for a few years could be extremely profitable given the usually negligible marginal cost of production for drugs.

The purpose of this chapter is to provide a theoretical analysis on this phenomenon by taking as a premise the hypothesis that campaign contributions are for the purpose of buying

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[50] The figure of $200 billion refers only to how much Americans spent on prescription drugs in 2002, and it does not include the other large expenses on drugs administered in hospitals, nursing homes, or doctors' offices. Combining all these expenses, the industry's revenue is roughly $400 billions in 2002. See, e.g. Angell (2005, p. 4-5).

[51] Robert Pear, "Drug Companies Increase Spending on Efforts to Lobby Congress and Governments" *New York Times*, June 1, 2003.

[52] These two figures are quoted in nominal terms. Refer to Table 1 for the detailed breakdown in real terms.

legislative influence. In particular, it incorporates special interest politics into a quality-ladder growth model to analyze the political-economic tradeoff facing the policymakers, who have to balance between the optimal level of patent protection to maximize social welfare on one hand and the interests of the SIG in exchange for campaign contributions on the other.[53] It is analytically shown that the government placing a higher value on campaign contributions increases: (a) the patent length; (b) the amount of monopolistic profits; (c) the market value of patents; and (d) R&D investments. These results are consistent with the data in Section 2. Furthermore, this chapter derives a unique level of the government's bargaining power above which the amount of campaign contributions increases with respect to the weight that the government places on campaign contributions.

In terms of welfare implications, it is not surprising and perhaps trivial that the presence of a SIG lobbying the government to distort the level of patent protection is socially undesirable in a closed-economy setting. However, in a symmetric multi-country setting, the presence of a SIG lobbying its own government may improve social welfare in each country because the level of patent protection in the multi-country Nash equilibrium without lobbying is suboptimally low due to the positive externality of international spillovers in innovation suggested by previous studies, such as Grossman and Lai (2004) and Lai and Qiu (2003). Therefore, the multi-country political equilibrium features an additional efficiency tradeoff between monopolistic distortion and the detrimental effects of international free-riding on innovations, and the finding of potential welfare gains arises from the SIGs increasing the level of patent protection from a globally suboptimal level and acting as an externality-correcting device. In addition, the pharmaceutical lobby may even improve the welfare of the consumers, who do not own any patent, when the

[53] This setup is motivated by Grossman and Helpman's (2002, p. 5) insight that "[p]oliticians value contributions for their potential usefulness in financing campaigns, but many also wish to enact policies that benefit the public in order to improve their popularity among the well informed or to fulfill their sense of social responsibility."

degree of international free-riding on innovations is severe enough. Finally, this chapter shows that if the SIGs have asymmetric influences across countries, then the country, in which the government places a higher value on campaign contributions, would gain by less or even suffer a welfare loss.

This chapter relates to four strands of literature: (a) political economy of trade policy, (b) political economy and economic growth; (c) the welfare effects of lobbying; and (d) international patent protection. Feenstra (2004, chapter 9) provides a comprehensive discussion on the political economy of trade policy. An elegant formulation of special interest politics and trade policy is provided by Grossman and Helpman (1994), who later develop a set of modeling tools on special interest politics in Grossman and Helpman (2001), from which this chapter borrows to analyze patent policy. Drazen (2000, chapter 11) and Persson and Tabellini (2000, chapter 14) provide a comprehensive discussion on political economy and economic growth. The literature has so far focused on the relations between economic growth and a number of issues, such as income inequality through redistributive policies, political institutions through property rights, and the adoption of new technology through the distribution of technological-vintage-specific human capital. This chapter provides a theoretical analysis on economic growth and the political economy of patent policy on one hand and analyzes the welfare effects of lobbying on the other. Recent studies on the welfare effects of lobbying, such as Besley and Coate (2001) and Coate and Morris (1999), analyze the negative effects of lobbying on social welfare arising from coordination failures between multiple SIGs, the excessive entry of citizens into running for office, and the implementation of Pareto dominated policies due to the fear of lobbying-driven hysteresis in policy decisions.

In the literature on international patent protection, Grossman and Lai (2004) and Lai and Qiu (2003) analyze the welfare effects of strengthening patent protection in developing countries as a result of the TRIPS agreement using a multi-country variety-expanding growth model. In contrast, the current chapter develops a multi-country quality-ladder growth model to analyze international patent protection and the political-economy aspects of patent policy. Dinopoulous *et al* (2005) analyze international patent protection using a North-South quality-ladder model. In their model, the level of patent protection is an exogenous parameter, and they are interested in the effects of patent protection on long-run growth and technology transfer; on the other hand, in the current chapter, the level of patent protection is a policy instrument chosen by the government to maximize its objective function. Furthermore, the current chapter incorporates special interest politics into the growth model to analyze the welfare implications of a pharmaceutical lobby. Lai (2005) extends the variety-expanding model in Grossman and Lai (2004) to analyze the effects of political contributions on the level of patent protection and shows insufficient patent protection prior to the TRIPS agreement. The current chapter complements and extends Lai (2005) by taking insufficient patent protection as a starting point and by investigating whether the increasing political influence of the pharmaceutical lobby has improved social welfare. In addition, the current chapter considers the welfare implications under both symmetric and asymmetric SIGs.

Before closing the introduction, I briefly discuss two common critiques against the hypothesis of campaign contributions as political investments. The first one is the small amount of campaign contributions relative to the potential financial returns at stake. Helpman and Persson (2001) provide an interesting model to show that a small amount of contributions does not necessarily imply that it has no effects on policy outcomes. In their model, lobbying has important influences on policy outcomes and the equilibrium amount of contributions is as small as zero because of the competition between legislators. The other common critique is that individuals are the main source of money in US campaigns. Based on this observation, Ansolabehere *et al* (2003) argue that campaign contributions should be considered as consumption goods rather than political investments. However, this argument does not rule out the possibility that SIGs contribute in order to influence the policymakers on certain policies. Ansolabehere *et al* (2003) write, "[a]lthough aggregate campaign expenditures primarily reflect consumption, it may be that a subset of donors, mainly corporate and industry PACs, behave as if they expected favors in return. These contributors may in fact receive a reasonable rate of return – say 20 percent…"

The rest of this chapter is organized as follows. Section 2 provides some stylized facts about the pharmaceutical industry. Section 3 describes the model and presents the theoretical results. The final section concludes. Some of the proofs are contained in Appendix H.

2. Some Stylized Facts about the Pharmaceutical Industry

Table IV-1 indicates that the industry spends a large amount of money on lobbying each year and on campaign contributions during each election cycle. Another feature of the data is that lobbying expenditures in real terms have been rising rapidly at an average annual growth rate of 9.96%. Proposition 2 suggests that this rising trend may be interpreted as an indication for the increasing importance of lobbying and campaign contributions that motivate the comparative static exercise in Section 3.

Table IV-2 presents the details of the extension in the commercial lifetime of patent and market exclusivity for drugs as a result of the policy changes during the 80's and 90's. Note that the effects captured in Table IV-2 underestimates the true extension of monopoly rights for brand-name drugs because they do not take into account the fact that the companies could strategically utilize the market exclusivity provided by the Hatch-Waxman Act to protect a new generation of slightly-modified products.[54] To summarize, the data presented so far is consistent with the hypothesis that the pharmaceutical industry has successfully exercised its political power, which comes from lobbying and campaign contributions, to influence the policymakers in order to strengthen patent protection for brand-name drugs. The next question is whether these policy changes have led to handsome financial returns for the industry.

Indeed, they have. As Angell (2005, p. 3) writes, "[t]he watershed year was 1980. Before then, it was a good business, but afterward, it was a stupendous one." Table IV-4 presents some financial time series for the top 10 pharmaceutical companies in Fortune 500 as an example. Each number in the table represents the annual average of the combined figures of the companies; for example, the first row presents the annual averages of the combined sales of the companies in the past decades. The data indicates that both sales and pretax income have been rising at a faster rate than the real GDP. Also, as a result of the stronger patent protection, the market value of pharmaceutical patents should have increased as well. A rough proxy for the value of patents owned by the companies is the value of intangible assets,[55] which has also risen substantially.

Motivated by the data presented above, the theoretical model in Section 3 attempts to achieve the following analytical results to confirm the basic intuition of the model. An increase in the value that the government places on campaign contributions in its objective function should lead to an extension in the patent length for brand-name drugs, and consequently an increase in the amount of monopolistic profits generated by the industry and the market value of patents. In addition, the model predicts that R&D investments on pharmaceuticals would increase. Table IV-

[54] From NIHCM (2000), "…if a manufacturer timed the introduction of a new use, such as a more convenient dosing form, to coincide with the expiration of the 'mother' drug's patent, it could have shielded the 13.9 to 15.4 years franchise of the drug for an additional three years, for as long as 17 to 18 years overall."
[55] This proxy is a rough approximation at best for two reasons: (a) it includes other items, such as trade secrets, trademarks, and goodwill; and (b) the accounting valuation methods may not accurately reflect the market value.

4 shows that this is also true in the data. Private R&D spending on drugs in real terms has been increasing at an average annual growth rate of 7.28% since the 80's.

3. A Quality-Ladder Growth Model with Special Interest Politics

The quality-ladder growth model originates from Aghion and Howitt (1992) and Grossman and Helpman (1991), in which scale effects are eliminated by assuming decreasing *individual* R&D productivity as in Segerstrom (1998).[56] The basic framework is modified to introduce: (a) patent protection in the form of patent length; and (b) an organized SIG representing the owners of pharmaceutical patents. The model is further modified to include homogenous goods and quality-enhancing goods (i.e. pharmaceuticals).

Sections 3.1 – 3.4 firstly describe the different components of the model in a closed-economy setting, and then the decentralized equilibrium is defined in Section 3.5. After characterizing the socially optimal patent length, lobbying is incorporated into the model by assuming that the government values campaign contributions in addition to social welfare. As a result, the SIG is able to distort the level of patent protection to benefit itself. The comparative static result shows that an increase in the importance of campaign contributions in the government's objective function increases: (a) the patent length; (b) the amount of monopolistic profits; (c) the market value of patents; and (d) R&D investments. Finally, the different welfare implications of lobbying are derived under the closed-economy and multi-country settings with symmetric and asymmetric SIGs.

[56] See, e.g. Jones (1995a) and Jones (1999) for a discussion of scales effects in R&D-driven endogenous growth models. After eliminating scale effects, the resulting model becomes the so-called semi-endogenous growth model, in which the growth rate along the balanced-growth path is proportional to the labor-force growth rate. An increase in the share of R&D workers raises the *level* of technology while holding its balanced-growth rate constant.

3.1. Households

There is a continuum of households indexed by $i \in [0,1]$, and their lifetime utility function is

$$U(i) = \int_0^{\infty} e^{-(\rho - n)t} (c_{h,t}(i) + \beta \ln c_{q,t}(i)) dt \,. \quad (1)$$

[57]

$c_{h,t}(i)$ is the per capita consumption of homogenous goods, and it is chosen as the numeraire. β is a preference parameter reflecting the importance of the per capita consumption of quality-enhancing goods, denoted by $c_{q,t}(i)$. Each household has $L_t = e^{n.t}$ members at time *t*, and $n > 0$ is the exogenous population growth rate. ρ is the subjective discount rate. To ensure that utility is bounded,

$$\rho > n \,. \quad \text{(a1)}$$

Each household maximizes utility subject to a sequence of budget constraints

$$\dot{a}_t(i) = (r_t - n)a_t(i) + w_t - c_{h,t}(i) - P_{q,t}c_{q,t}(i) \,. \quad (2)$$

Each member of a household inelastically supplies one unit of homogenous labor in each period to earn a wage income w_t. r_t is the real rate of return on the financial assets, which represent the market value of patents in equilibrium. Each household $i \in [0,1]$ belongs to either of two types $\in \{I, II\}$. Type-I household $i \in [0, \theta]$ and Type-II household $i \in (\theta,1]$ differ in their investment opportunity sets.

Assumption 1: *Type-I households have access to the financial market while Type-II households are restricted from participating in the financial market such that* $a_t(II) = 0$.[58]

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[57] The quasi-linear preference enables me to derive the patent length in the political equilibrium with a minimal assumption of efficient bargaining between the government and the SIG.

[58] In this setting, the identity of the two types of households is exogenously imposed. In fact, this identity can be endogenously derived by assuming an exogenous difference in the discount rates across the two

This setup leads to a conflict of interest between *the patent owners* (i.e. Type-I households) who benefit from an increase in the rate of return on R&D investments and *the consumers* (i.e. Type-II households) who don't, and this conflict of interest gives rise to the importance of political economics.

From Type-I households' intertemporal optimization, the Euler equation derived from the quasi-linear preference equates the rate of return on financial assets to the subjective discount rate such that

(3) $$r_t = \rho .$$

The intratemporal optimality condition determines the amount of spending on the quality-enhancing goods in each period to be

(4) $$P_{q,t} c_{q,t}(i) = \beta .$$

for $i \in \{I, II\}$. Since both types of households consume the same amount of quality-enhancing goods, notation simplifies to $c_{q,t} \equiv c_{q,t}(i)$. Aggregate consumption of pharmaceuticals $C_{q,t} = L_t c_{q,t}$ is a standard Cobb-Douglas aggregator of a continuum of differentiated quality-enhancing intermediate goods $j \in [0,1]$ (i.e. differentiated drugs). The production function is given by

(5) $$Y_{q,t} = \exp\left(\int_0^1 \ln Y_{q,t}(j) dj \right),$$

and the familiar aggregate price index is

(6) $$P_{q,t} = \exp\left(\int_0^1 \ln P_{q,t}(j) dj \right).$$

The first-order condition for differentiated drug $j \in [0,1]$ is

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types of households along with a borrowing constraint. Then, the households who have a lower discount rate would be the owners of financial assets in equilibrium.

(7) $$P_{q,t}(j)Y_{q,t}(j) = P_{q,t}C_{q,t}\,.$$

3.2. Homogenous Goods

There exists a larger number of competitive firms producing the homogenous goods $Y_{h,t}$. The production function has constant returns to scale in labor input $L_{h,t}$ given by

(8) $$Y_{h,t} = \alpha L_{h,t}\,.$$

The marginal cost of production is

(9) $$MC_{h,t} = w_t / \alpha\,.$$

Since the homogenous goods are chosen to be the numeraire and this sector is characterized by marginal-cost pricing, the marginal cost is also one. Therefore, the real wage is given by $w_t = \alpha$, and the market-clearing condition is

(10) $$Y_{h,t} = C_{h,t} = L_t(\theta c_{h,t}(I) + (1-\theta)c_{h,t}(II))\,,$$

where $C_{h,t}$ is the total consumption of homogenous goods.

3.3. The Pharmaceutical Industry

Within the pharmaceutical industry, there is a continuum of sub-industries on the unit interval producing differentiated drugs. Each sub-industry $j \in [0, \omega_t]$ is dominated by a temporary monopolistic leader, who holds a valid patent on the latest technology, and each sub-industry $j \in (\omega_t, 1]$ is characterized by perfect competition because the most recent patent has expired. ω_t is endogenously determined by the patent length chosen by the government. The following formulation of patent length is originally developed by Grossman and Lai (2004) for a variety-expanding model and subsequently modified by Dinopoulos *et al* (2005) for a quality-ladder model. In standard quality-ladder models, a patent's effective lifetime ends when a higher level of technology is invented in the same industry. As in Dinopoulos *et al* (2005), I assume that a monopolistic leader can maintain its monopoly position until the end of the statutory patent life, denoted by T . This assumption is especially relevant for the pharmaceutical industry, in which the effective patent life for drugs usually coincides with the statutory patent life.[59]

Assumption 2: *Entrepreneurs can only invest in R&D to develop a better drug after the latest patent in the sub-industry has expired.*

When a patent expires, the monopolistic sub-industry temporarily becomes a generic sub-industry characterized by marginal-cost pricing until the next innovation occurs. Therefore, extending the statutory patent life has two opposing effects on social welfare. On one hand, it enhances the incentives for R&D by raising the present value of the stream of profits earned by a monopolistic leader. On the other hand, it increases the dead weight loss by increasing the fraction of monopolistic sub-industries due to a longer period of patent protection. At the constrained social optimum, the government balances these opposing effects to maximize social welfare.

The production function in sub-industry *j* is

(11) $$Y_{q,t}(j) = z^{m_t(j)} L_{q,t}(j) .$$

$L_{q,t}(j)$ is the number of workers hired by sub-industry *j* at time *t*. $z^{m_t(j)}$ is the monopolistic leader's marginal product of labor, which is increasing over time due to technological improvement. $z > 1$ is the exogenous step-size of productivity improvement arising from each

[59] In this case, the statutory patent life refers to the patent length granted by the US Patent and Trademark Office as well as the length of exclusive marketing rights granted by the FDA.

innovation. $m_t(j)$, which is an integer, is the number of innovations that has occurred in sub-industry *j* as of time *t*. The marginal cost of production is

$$MC_{q,t}(j) = w_t / z^{m_t(j)} . \tag{12}$$

For a monopolistic sub-industry $j \in [0, \omega_t]$, the price-setting behavior is governed by the closest rival's marginal cost of production $w_t / z^{m_t(j)-1}$. The familiar Bertrand-equilibrium price is

$$P_{q,t}(j) = zMC_{q,t}(j) = z\alpha / z^{m_t(j)} . \tag{13}$$

Thus, the monopolistic leader charges a markup of z over the marginal cost, and the equilibrium profit is

$$\pi_{q,t}(j) = (z-1)MC_{q,t}(j)Y_{q,t}(j) = (z-1)\alpha L_{q,t}(j) . \tag{14}$$

In equilibrium, $L_{q,t}(j)$ is the same across $j \in [0, \omega_t]$ because of the Cobb-Douglas specification in (5), so that $\pi_{q,t}(j)$ is also the same across $j \in [0, \omega_t]$.

Substituting (11) into (5), the aggregate production function for pharmaceuticals becomes

$$Y_{q,t} = A_t L^e_{q,t} . \tag{15}$$

$L^e_{q,t} = \exp\left(\int_0^1 \ln L_{q,t}(j) dj \right)$ is an index of *effective* labor inputs,[60] and $A_t = \exp\left(\int_0^1 m_t(j) dj \ln z \right)$ is the aggregate level of technology for pharmaceuticals. Without loss of generality, the sub-industries are ordered such that *j* is monopolistic for $j \in [0, \omega_t]$ and j' is generic for $j' \in (\omega_t, 1]$. Substituting (13) into (6), the aggregate price index for pharmaceuticals becomes

$$P_{q,t} = z^{\omega_t} \alpha / A_t . \tag{16}$$

[60] This index is different from the *actual* number of workers in the pharmaceutical industry because the number of workers varies across sub-industries in the present model as a result of the presence of both monopolistic and generic sub-industries.

Substituting (16) into (4) and multiplying by L_t, aggregate consumption of pharmaceuticals becomes

$$C_{q,t} = (A_t / z^{\omega_t})(\beta L_t / \alpha). \tag{17}$$

$C_{q,t}$ decreases in ω_t reflecting the markup-pricing distortion and increases over time due to A_t reflecting technological improvements. Using (15) and (17), the index of effective labor inputs becomes $L^e_{q,t} = \beta L_t /(z^{\omega_t}\alpha)$. The first-order condition from (7) implies that the ratio of relative labor inputs in the monopolistic and generic sub-industries is $L_{q,t}(j')/L_{q,t}(j) = z$. Therefore, the index of effective labor inputs can be re-expressed as $L^e_{q,t} = z^{1-\omega_t} L_{q,t}(j)$, and the number of workers in monopolistic sub-industry *j* is $L_{q,t}(j) = \beta L_t /(z\alpha)$. Substituting this condition into (14) yields

$$\pi_{q,t}(j) = \left(\frac{z-1}{z}\right)\beta L_t, \tag{18}$$

which is the same across all monopolistic sub-industries and increases over time as the total population grows. The total amount of monopolistic profit is

$$\pi_{q,t} = \int_0^{\omega_t} \pi_{q,t}(j)dj = \omega_t\left(\frac{z-1}{z}\right)\beta L_t. \tag{19}$$

Finally, the *actual* number of workers employed in the pharmaceutical industry is

$$L_{q,t} = \omega_t L_{q,t}(j) + (1-\omega_t) L_{q,t}(j') = \left(\frac{\omega_t + (1-\omega_t)z}{z}\right)\frac{\beta L_t}{\alpha}, \tag{20}$$

which increases over time at the population growth rate.

3.4. R&D

The no-arbitrage value of a pharmaceutical patent V_t for a new successful innovation at time t is the present value of the stream of profits earned by the R&D entrepreneur who becomes the next industry leader until the patent expires. From (19),

$$V_t = \int_{\tau=t}^{t+T} e^{-\rho(\tau-t)} \pi_{q,\tau}(j')d\tau = \frac{\Omega(T)}{\rho-n}\left(\frac{z-1}{z}\right)\beta L_t \,, \tag{21}$$

where $\Omega(T) \equiv 1-\exp(-(\rho-n)T)$ has the properties of $\Omega(0)=0$, $\Omega(\infty)=1$, and $\Omega'(T)>0$.

The Poisson arrival rate $\lambda_t(k)$ of a successful innovation for an R&D entrepreneur $k \in [0,1]$ is a function of labor inputs given by

$$\lambda_t(k) = \overline{\varphi}_t L_{r,t}(k) \,, \tag{22}$$

where $\overline{\varphi}_t$ is the *individual* R&D productivity parameter that the entrepreneur takes as given. The R&D sector is characterized by constant returns to scale and perfect competition. The expected profit earned by entrepreneur *k* is

$$\pi_{r,t}(k) = V_t \lambda_t(k) - w_t L_{r,t}(k) \,. \tag{23}$$

The first-order condition is

$$V_t \overline{\varphi}_t = \alpha \,, \tag{24}$$

which is the key condition that determines the number workers allocated to the R&D sector.

To eliminate scale effects, the *individual* R&D productivity parameter $\overline{\varphi}_t$ at time t is assumed to be decreasing in the level of technology A_t such that

$$\overline{\varphi}_t = \frac{\varphi L_{r,t}^{\gamma-1}}{A_t^{1-\phi}} \,, \tag{25}$$

where $L_{r,t} = \int_0^1 L_{r,t}(k)dk$. $\phi \in (-\infty,1)$ captures the externality of intertemporal knowledge spillovers,[61] and $\gamma \in (0,1]$ captures the intratemporal negative congestion externality or the so-called "stepping-on-toes" effects. The law of motion for pharmaceutical technology is

(26) $$\dot{A}_t = A_t \lambda_t \ln z = A_t \overline{\varphi}_t L_{r,t} \ln z = A_t^{\phi} L_{r,t}^{\gamma} \varphi \ln z.$$ [62]

3.5. Decentralized Equilibrium

The equilibrium is a sequence of prices $\{w_t, r_t, V_t, P_{q,t}(j), MC_{q,t}(j)\}_{t=0}^{\infty}$ and a sequence of allocations $\{a_t(I), c_{h,t}(i), c_{q,t}(i), Y_{q,t}(j), L_{q,t}(j), L_{r,t}(k), L_{h,t}\}_{t=0}^{\infty}$ that are consistent with the initial conditions $\{L_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also,

(a) each Type-I household chooses $\{a_t(I), c_{h,t}(I), c_{q,t}(I)\}$ and supplies labor to maximize utility taking $\{w_t, r_t, P_{q,t}(j)\}$ as given;

(b) each Type-II household chooses $\{c_{h,t}(II), c_{q,t}(II)\}$ and supplies labor to maximize utility taking $\{w_t, P_{q,t}(j)\}$ as given;

(c) competitive firms in the homogenous-goods sector chooses $\{L_{h,t}\}$ to maximize profits according to the production function taking $\{w_t\}$ as given;

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[61] As discussed in Jones (1995b), $\phi \in (0,1)$ corresponds to the "standing-on-shoulder" effect, in which *aggregate* R&D productivity $A_q\overline{\varphi}$ increases as the level of technology increases (see the law of motion for technology). On the other hand, $\phi \in (-\infty,0)$ corresponds to the "fishing-out" effect, in which early technology is relatively easy to develop and $A_q\overline{\varphi}$ decreases as the level of technology increases.

[62] This expression is derived as $\ln A_t = \left(\int_0^1 \ln z^{m_t(j)} dj\right) = \left(\int_0^1 m_t(j)dj\right) \ln z = \left(\int_0^t \lambda(\tau)d\tau\right) \ln z$; then, differentiation with respect to time yields $\dot{A}_t / A_t = \lambda_t \ln z$.

(d) each monopolistic leader in pharmaceutical sub-industry $j \in [0, \omega_t]$ chooses $\{P_{q,t}(j), L_{q,t}(j)\}$ to maximize profits according to the Bertrand price competition and its production function taking $\{MC_{q,t}(j), Y_{q,t}(j)\}$ as given;

(e) competitive firms in pharmaceutical sub-industry $j' \in (\omega_t, 1]$ choose $\{L_{q,t}(j')\}$ to maximize profits according to the production function taking $\{P_{q,t}(j'), MC_{q,t}(j'), Y_{q,t}(j')\}$ as given;

(f) each entrepreneur $k \in [0,1]$ in the R&D sector chooses $\{L_{r,t}(k)\}$ to maximize profits according to the production function taking $\{w_t, V_t, \overline{\varphi}_t\}$ as given;

(g) The market for the homogenous goods clears such that $Y_{h,t} = C_{h,t}$;

(h) The market for pharmaceuticals clears such that $Y_{q,t} = C_{q,t}$; and

(i) The labor market clears such that $L_{h,t} + L_{q,t} + L_{r,t} = L_t$.

3.6. Balanced-Growth Path

Along the balanced-growth path, $L_{r,t}$ increases at the population growth rate (to be shown below). Therefore, the balanced-growth rate of technology for pharmaceuticals denoted by g must be proportional to the population growth rate such that

(27) $$g \equiv \frac{\dot{A}_t}{A_t} = \frac{L_{r,t}^{\gamma}}{A_t^{1-\phi}} \varphi \ln z = \frac{\gamma}{1-\phi} n .$$

Then, the steady-state Poisson arrival rate of innovations is $\lambda = g / \ln z$. The fraction of monopolistic sub-industries at time *t* is the sum of all inventions from the last *T* periods

(28) $$\omega_t = \int_0^T \lambda_{t-\tau} d\tau .$$

Therefore, the steady-state value of ω is simply $T\lambda$. To ensure that $\omega \in (0,1]$, an upper bound given by

(a2) $$T \leq 1/\lambda = \frac{(1-\phi)\ln z}{n\gamma}$$

is imposed on the patent length.

Solving the first-order condition (24) from the R&D sector and imposing the balanced-growth condition yield the steady-state share of workers in the R&D sector given by

(29) $$s_r = \lambda \frac{\Omega(T)}{\rho - n}\left(\frac{z-1}{z}\right)\frac{\beta}{\alpha}.$$

Substituting (20) and (29) into $s_h + s_q + s_r = 1$ closes the model by solving for the steady-state share of workers in the homogenous-goods sector given by

(30) $$s_h = 1 - \frac{\beta}{\alpha}\left(\frac{T\lambda + (1-T\lambda)z}{z} + \lambda \frac{\Omega(T)}{\rho - n}\left(\frac{z-1}{z}\right)\right).$$

The per capita supply of homogenous goods is

(31) $$\alpha s_h = \alpha - \beta\left(\frac{T\lambda + (1-T\lambda)z}{z} + \lambda \frac{\Omega(T)}{\rho - n}\left(\frac{z-1}{z}\right)\right).$$

From the budget constraints, the steady-state per capita consumption of homogenous goods is

(32) $$c_h(I) = w - P_q c_q + (\rho - n)a(I) = \alpha - \beta + \frac{\lambda}{\theta}\left(T - \frac{\Omega(T)}{\rho - n}\right)\left(\frac{z-1}{z}\right)\beta,$$

(33) $$c_h(II) = w - P_q c_q = \alpha - \beta.$$

Therefore, the following parameter restriction is sufficient to ensure an interior solution for both types of households

(a3) $$\alpha > \beta.$$

$(\rho - n)a(I)$ is the amount of dividends received by each member of Type-I households, and the total value of financial assets owned by Type-I households equals the market value of all existing patents. From (21), the market value of a pharmaceutical patent that has a remaining life time of τ years is

$$V_t(\tau) = \frac{\Omega(\tau)}{\rho - n}\left(\frac{z-1}{z}\right)\beta L_t . \tag{34}$$

Therefore, the market value of all existing patents in the monopolistic industries is

$$\frac{\omega}{T}\int_{\tau=0}^{T} V_t(\tau)d\tau = \lambda\left(\int_{\tau=0}^{T}\Omega(\tau)d\tau\right)\left(\frac{z-1}{z}\right)\frac{\beta L_t}{\rho - n}, \tag{35}$$

where $\left(\int_{\tau=0}^{T}\Omega(\tau)d\tau\right) = \left(T - \frac{\Omega(T)}{\rho - n}\right)$.

To see the conflict of interests across the two types of households, differentiating $c_h(I)$ and $c_h(II)$ with respect to T yields

$$\frac{\partial c_h(I)}{\partial T} = \frac{\lambda}{\theta}\left(1 - \frac{\Omega'(T)}{\rho - n}\right)\left(\frac{z-1}{z}\right)\beta > 0, \tag{36}$$

$$\frac{\partial c_h(II)}{\partial T} = 0. \tag{37}$$

Note that $\Omega'(T) = \exp(-(\rho - n)T)(\rho - n)$. An increase in T has three effects on the welfare of each household. (36) and (37) indicate the asymmetric effect coming from the increase in the value of patents that benefits only the patent owners. The two symmetric effects as shown in (17) are the negative effect of markup-pricing distortion and the positive effect of a higher level of pharmaceutical technology.

3.7. Desired Patent Lengths for patent owners and consumers

Given the equilibrium conditions, the lifetime utility for each type of households along the balanced growth path can be derived by rewriting (1) into

$$U(i,T)=\frac{c_h(i,T)+\beta\ln c_{q,0}(T)}{\rho-n}+\int_0^\infty e^{-(\rho-n)t}(g\,t)dt\,, \tag{38}$$[63]

for $i\in\{I,II\}$. $c_h(I,T)$ is a function of T because an increase in patent length raises the value of dividends that increases the consumption of the patent owners. From (17),

$$\ln c_{q,0}(T)=\ln A_0(T)-T\lambda\ln z+\ln(\beta/\alpha)\,. \tag{39}$$

$T\lambda\ln z$ is a function of T because an increase in T worsens the markup-pricing distortion by increasing the fraction of monopolistic sub-industries. $\ln A_0(T)$ is a function of T because an increase in T enhances the incentives for R&D, which in turn raises the *level* of technology for pharmaceuticals. From (27),

$$A_0(T)=\left(\frac{\varphi\ln z}{g}\right)^{1/(1-\phi)}L_{r,0}^{\gamma/(1-\phi)}(T)\,. \tag{40}$$

$\int_0^\infty e^{-(\rho_i-n)t}(g\,t)dt$ represents the balanced growth path of A_t and is independent of T because of the semi-endogenous growth formulation. Substituting (29), (39) and (40) into (38) and dropping some constant terms and the exogenous growth path, the lifetime utility can be further simplified to

$$U(i,T)=(\rho-n)^{-1}\left(c_h(i,T)+\beta\left(\frac{\gamma}{1-\phi}\right)\ln\Omega(T)-\beta T\lambda\ln z\right) \tag{41}$$

for $i\in\{I,II\}$. Therefore, the first-order condition that characterizes the desired patent length is

[63] The conclusion briefly discusses the generality of a steady-state welfare analysis that ignores transition dynamics.

(42) $$\frac{\partial U(i,T)}{\partial T} = (\rho - n)^{-1}\left(\frac{\partial c_h(i,T)}{\partial T} + \beta\left(\frac{\gamma}{1-\phi}\right)\frac{\Omega'(T)}{\Omega(T)} - \beta\lambda \ln z\right) = 0$$

for $i \in \{I, II\}$. Denoted the solution to (42) by T_i^* for $i \in \{I, II\}$.

Lemma 1: $T_I^* > T_{II}^*$.

Proof: Note that $\frac{\partial c_h(I,T)}{\partial T} > \frac{\partial c_h(II,T)}{\partial T} = 0$ from (36) and (37).□

Intuitively, the patent owners have a longer desired patent length than the consumers because of the asymmetric ownership of patents as financial assets whose market value increases in patent length. This setup leads to a conflict of interest across the two types of households and gives rise to the potential role of the pharmaceutical lobby influencing the government's policy choice through campaign contributions.

3.8. Optimal Patent Length

A benevolent government chooses the patent length to maximize social welfare subject to the equilibrium conditions. Social welfare is defined as

(43) $$W(T) \equiv \int_0^1 U(i,T)di = \theta U(I,T) + (1-\theta)U(II,T).$$

The first-order condition that characterizes the optimal patent length is

(44) $$\frac{\partial W(T)}{\partial T} = \theta\frac{\partial U(I,T)}{\partial T} + (1-\theta)\frac{\partial U(II,T)}{\partial T} = 0.$$

Denote the solution to (44) by T^*. The second-order condition at the social optimum is

(45) $$\frac{\partial^2 W(T^*)}{\partial T^2} = -\Omega''(T^*)\frac{\beta}{\rho - n}\left(\frac{\gamma}{1-\phi}\right)\left(\frac{n}{\rho - n}\left(\frac{z-1}{z\ln z}\right) - \frac{1}{\Omega(T^*)^2}\right) = 0.$$

The sign of (45) is given by

(46) $$sign\left(\frac{\partial^2 W(T^*)}{\partial T^2}\right) = sign\left(\frac{n}{\rho - n}\left(\frac{z-1}{z\ln z}\right) - \frac{1}{\Omega(T^*)^2}\right).$$

To ensure that the second-order condition is satisfied for a maximum, the following assumption is imposed on the parameter values

(a4) $$\left(\frac{z\ln z}{z-1}\right) > \frac{n}{\rho - n}.$$

This assumption rules out the possibility that an increase in T would increase $c_h(I,T)$ by so much that T^* is unbounded. Comparing with (42), (44) is simply a weighted average of the first-order conditions for the two types of households. The larger is θ, the closer T^* is to T_I^*, and vice versa. Lemma 2 summarizes these findings.

Lemma 2: $T^* \in (T_{II}^*, T_I^*)$ *and* $\partial T^* / \partial \theta > 0$ *for* $\theta \in (0,1)$.

Proof: Note (42) and (44).□

3.9. Campaign Contributions

This subsection employs the modeling tools in Grossman and Helpman (2001, chapter 7) to model campaign contributions for legislative influence. In addition to social welfare, the government is assumed to value campaign contributions, denoted by C_0, and its objective function is a weighted average of $W(T)$ and C_0 given by

(47) $$\widetilde{W}(T, C_0) \equiv (1-\varsigma)W(T) + \varsigma C_0.$$ [64]

$\varsigma \in (0,1]$ is the weight that the government places on campaign contributions. The pharmaceutical lobby and the government are assumed to engage in efficient bargaining and be able to commit to the bargaining outcome on patent length and campaign contributions. The

[64] Grossman and Helpman (2001, chapter 10) and (2002, chapter 2) show that this objective function represents a proper reduced form of a model with electoral competition.

quasi-linear preference enables me to derive the political-equilibrium patent length without imposing further assumptions on the bargaining process.[65] The efficient bargaining outcome on the patent length can be derived from the constrained maximization problem $\underset{T,C_0}{Max}\tilde{W}(T,C_0)$ subject to $\theta U(I,T)-C_0 \geq \hat{U}$ for some value of $\hat{U}$. It is in the government's best interest to choose the highest possible C_0 to make this inequality constraint binding. Therefore, the constraint can be re-expressed as

(48) $$C_0 = \theta U(I,T) - \hat{U} .$$

After substituting (48) into (47) and dropping the constant term, the constrained maximization becomes

(49) $$\underset{T}{Max}\tilde{W}(T) = \theta U(I,T) + (1-\varsigma)(1-\theta)U(II,T) .$$

In other words, when the government values campaign contributions, the pharmaceutical lobby is able to alter the relative weight that the government places on the households' welfare. In particular, a higher value that the government places on campaign contributions leads to a larger relative weight that the government places on Type-I households' welfare. The first-order condition that characterizes the political-equilibrium patent length, denoted by $T^{SIG}(\varsigma)$, is

(50) $$\frac{\partial \tilde{W}(T)}{\partial T} = \theta \frac{\partial U(I,T)}{\partial T} + (1-\varsigma)(1-\theta)\frac{\partial U(II,T)}{\partial T} = 0 .$$

Comparing (44) and (50), $T^{SIG}(\varsigma) > T^*$ for $\varsigma \in (0,1]$ and $\partial T^{SIG}(\varsigma)/\partial\varsigma > 0$. As the government places a larger weight on campaign contributions, it chooses a longer patent length. As a result, the amount of monopolistic profits generated by the pharmaceutical industry and the value of pharmaceutical patents also increase. This leads to an increase in R&D investments,

[65] Although the political-equilibrium patent length can be determined, the amount of campaign contributions cannot be determined unless further assumptions are imposed on the bargaining process. See Proposition 2.

reflected by an increase in the fraction of workers in the R&D sector. Proposition 1 summarizes these results.

Proposition 1: *An increase in* ς *leads to an increase in: (a)* T^{SIG} *; (b)* π_q *; (c)* V *; and (d)* s_r *.*

Proof: For (a), note (50). Then, recall from (28) that $\partial\omega/\partial T = \lambda > 0$. For (b), note (19). For (c), note (21). For (d), note (29).□

Determining the impact of ς on the equilibrium amount of campaign contributions requires an additional assumption on the bargaining outcome between the government and the pharmaceutical lobby.

Assumption 3: *The government and the pharmaceutical lobby maximizes the total surplus, and the constant share of total surplus captured by the government is* $\sigma \in [0,1]$.

Proposition 2: *There exists a* $\hat{\sigma} \in (0,1)$ *such that if and only if* $\sigma \geq \hat{\sigma}$, $\partial C_0 / \partial\varsigma \geq 0$ *for* $\varsigma \in (0,1)$.

Proof: See Appendix H.□

Corollary 1a: *The government having the first-mover advantage to make a take-or-leave-it offer to the SIG is equivalent to* $\sigma = 1$. *In this case,* $C_0 = \theta(U(I, T^{SIG}) - U(I, T^*))$ *and* $\partial C_0 / \partial\varsigma > 0$ *for* $\varsigma \in (0,1)$.

Proof: See Appendix H.□

Corollary 1b: *The SIG having the first-mover advantage to make a take-or-leave-it offer to the government is equivalent to* $\sigma = 0$. *In this case,* $C_0 = (W(T^*) - W(T^{SIG}))(1-\varsigma)/\varsigma$ *and* $\partial C_0 / \partial\varsigma < 0$ *for* $\varsigma \in (0,1)$.

Proof: See Appendix H.□

Intuitively, whether the amount of campaign contributions increases or decreases in response to a larger ς depends on the relative bargaining power between the government and the SIG. When the government has all the bargaining power (i.e. $\sigma = 1$), it is able to extract all the surplus from the SIG, and hence, $\partial C_0 / \partial\varsigma > 0$. Similarly, when the SIG has all the bargaining power (i.e. $\sigma = 0$), it contributes the smallest amount possible in order to make the government indifferent between T^{SIG} and T^*.

3.10. Welfare Analysis in Closed vs. Open Economy

Grossman and Lai (2004) and Lai and Qiu (2003) show that the Nash-equilibrium level of patent protection in a multi-country setting is suboptimally low because each country has the incentive to free ride on other countries' innovations. Therefore, when all countries act according to their best response functions in choosing the level of patent protection, the Nash-equilibrium patent length is much shorter than the global optimum. This is a realistic description of the real world because innovations occur worldwide. At least, innovations come from multiple industrial countries. Therefore, the closed-economy setting is now extended to a multi-country setting in order to compare the different welfare implications. The theoretical result is that the pharmaceutical lobbies may improve social welfare in the multi-country setting by increasing the patent length from a globally suboptimal level.

There are N symmetric countries. For simplicity, transportation costs are assumed to be zero, and there is no international lending and borrowing as commonly assumed in the literature to pin down a unique trade pattern. When an R&D entrepreneur has an innovation success, she obtains a patent in each country. Therefore, in addition to the stream of monopolistic profits from the domestic economy, she is also entitled to the profits from abroad. At the aggregate level, these transfers of monopolistic profits for patent services are balanced by an equal value of trade in homogenous goods. The symmetry assumption implies that each country owns an equal fraction of patented innovations in the world.

The two equilibrium concepts for the open-economy model are as follows.

1. Nash Equilibrium. *In the absence of lobbying, the benevolent government in each country chooses the patent length to maximize the welfare of its own citizens taking the level of patent protection abroad as given. The resulting first-order condition for each country* $s \in \{1,...,N\}$ *is a social best response function given by*

$$\frac{\partial W^s(T^1,...,T^N)}{\partial T^s} = \theta \frac{\partial U^s(I,T^1,...,T^N)}{\partial T^s} + (1-\theta)\frac{\partial U^s(II,T^1,...,T^N)}{\partial T^s} = 0 \tag{51}$$

(i.e. the multi-country analog of (44)), and the Nash equilibrium is the solution of the social best response functions of all countries.

2. Political Equilibrium. *In the presence of lobbying, the SIG in each country is assumed to lobby its own government taking the action of the SIGs in other countries as given. The resulting first-order condition for each country* $s \in \{1,...,N\}$ *is a SIG best response function given by*

$$\frac{\partial \widetilde{W}^s(T^1,...,T^N)}{\partial T^s} = \theta \frac{\partial U^s(I,T^1,...,T^N)}{\partial T^s} + (1-\varsigma)(1-\theta)\frac{\partial U^s(II,T^1,...,T^N)}{\partial T^s} = 0 \tag{52}$$

(i.e. the multi-country analog of (50)), and the political equilibrium is the solution of the SIG best response functions of all countries.

The followings sketch out the key equations of the multi-country model. The no-arbitrage value of a patent is the present value of the stream of monopolistic profits from all countries $s \in \{1,...,N\}$.

$$V_t = \sum_{s=1}^{N}\left(\int_{\tau=t}^{t+T^s} e^{-\rho(\tau-t)}\pi_{q,\tau}^{s}(j')d\tau\right) = \left(\sum_{s=1}^{N}\Omega(T^s)\right)\left(\frac{z-1}{z}\right)\frac{\beta L_t}{\rho-n} \tag{53}$$

The statutory patent length in country *s* is denoted by T^s. In the Nash equilibrium, T^s is the same across countries because of symmetry. The law of motion for technology is

$$\dot{A}_t = A_t\overline{\varphi}_t\left(\sum_{s=1}^{N}L_{r,t}^{s}\right)\ln z\,. \tag{54}$$

The first implicit assumption behind (54) is that each country has the immediate access to the technological innovations from abroad, and this is a reasonable assumption because the inventors have the incentive to patent their innovations internationally. The second assumption is that technological innovations across countries are perfect substitutes because of the identical households' preferences across countries, and this is likely to be a reasonable description of the demand for pharmaceuticals in industrial countries. The first-order condition from the R&D sector becomes

$$\frac{L_{r,t}^{s}}{L_t} = \lambda\left(\sum_{s=1}^{N}\frac{\Omega(T^s)}{N}\right)\left(\frac{z-1}{z}\right)\frac{\beta/\alpha}{\rho-n}\,. \tag{55}$$

Using the balanced-growth path condition, the multi-country analog of (40) is

$$A_0(T^1,...,T^N) = \left(\frac{\varphi\ln z}{g}\right)^{1/(1-\phi)}\left(\sum_{s=1}^{N}L_{r,0}^{s}(T^1,...,T^N)\right)^{\gamma/(1-\phi)}. \tag{56}$$

(55) and (56) reveal one of the two key differences between the closed-economy model and the multi-country model in which a unilateral increase in T^s leads to a much smaller percentage increase in $\sum_{s=1}^{N} L_{r,0}^s(T^1,...,T^N)$ and consequently a much smaller percentage increase in $A_0(T^1,...,T^N)$. The market value of all existing patents in the global economy at time *t* is

$$(57) \qquad \sum_{s=1}^{N}\left(\frac{\omega(T^s)}{T^s}\int_{\tau=0}^{T^s}\Omega(\tau)d\tau\right)\left(\frac{z-1}{z}\right)\frac{\beta L_t}{\rho-n}.$$

Therefore, the amount of dividends received by each member of Type-I households in country *s* is

$$(58) \qquad (\rho-n)a^s(I)=\frac{\lambda}{N\theta}\sum_{s=1}^{N}\left(T^s-\frac{\Omega(T^s)}{\rho-n}\right)\left(\frac{z-1}{z}\right)\beta.$$

The derivative of $c_h^s(I,T^1,...,T^N)$ with respect to T^s is given by

$$(59) \qquad \frac{\partial c_h^s(I,T^1,...,T^N)}{\partial T^s}=\frac{\lambda}{N\theta}\left(1-\frac{\Omega'(T^s)}{\rho-n}\right)\left(\frac{z-1}{z}\right)\beta,$$

which reveals the other key difference that an unilateral increase in T^s has a smaller impact on domestic consumption because a large fraction of the increase in monopolistic profits is accrued to foreigners.

The social best response function (51) of country $s \in \{1,...,N\}$ becomes

$$(60) \qquad \theta\frac{\partial c_h^s(I,T^1,...,T^N)}{\partial T^s}+\beta\left(\frac{\gamma}{1-\phi}\left(\Omega'(T^s)/\sum_{s=1}^{N}\Omega(T^s)\right)-\lambda\ln z\right)=0.$$

By imposing symmetry, the condition that characterizes the symmetric Nash-equilibrium patent length T^{NE} simplifies to

$$(61) \qquad \frac{\Omega'(T^{NE})}{\Omega(T^{NE})}=n\left(N-\Omega(T^{NE})\left(\frac{z-1}{z\ln z}\right)\right).$$

Lemma 3: *The Nash-equilibrium patent length is decreasing in N and is strictly below the symmetric globally optimal patent length when the number of countries is at least two.*

Proof: See Appendix H.□

In the case of special interest politics, the SIG best response function (52) of country $s \in \{1,...,N\}$ becomes

$$\theta \frac{\partial c_h^s(I, T^1,...,T^N)}{\partial T^s} + (1-\varsigma(1-\theta))\beta\left(\frac{\gamma}{1-\phi}\left(\Omega'(T^s) / \sum_{s=1}^{N} \Omega(T^s)\right) - \lambda \ln z\right) = 0. \tag{62}$$

The condition that characterizes the symmetric political-equilibrium patent length T^{SIG} simplifies to

$$\frac{\Omega'(T^{SIG})}{\Omega(T^{SIG})} = n\left(N - \Omega(T^{SIG})\left(\frac{z-1}{z \ln z}\right)\frac{1}{1-\varsigma(1-\theta)}\right). \tag{63}$$

Proposition 3: *When* $N = 1$, $W(T^{SIG}) < W(T^*)$ *for* $\varsigma \in (0,1]$. *When* $N \geq 2$, *there exists a* $\hat{\varsigma}$ *such that for* $\varsigma \in (0, \hat{\varsigma}]$, $W^s(T^{SIG}) > W^s(T^{NE})$, *and* $\hat{\varsigma}$ *is increasing in* N. *Furthermore, for any given* $\tilde{\varsigma} \in (0,1)$, *there exists a* $\tilde{N}$ *such that if* $N \geq \tilde{N}$, $U^s(II, T^{SIG}) > U^s(II, T^{NE})$.

Proof: See Appendix H.□

In a closed-economy setting, the pharmaceutical lobby distorts the level of patent protection from the social optimum and thus leads to lower social welfare. However, in a multi-country Nash equilibrium, the level of patent protection is suboptimally low; therefore, the pharmaceutical lobbies may indeed improve global welfare by increasing the patent length from a suboptimal level. This is especially likely when their political influence is not overly substantial. Figure IV-1 illustrates this intuition in a two-country setting. As the number of countries increases, the degree of international free-riding on innovations becomes more severe; as a result, the threshold on the level of the SIGs' political influence that corresponds to welfare improvement increases. Furthermore, even the welfare of the consumers (i.e. Type-II households) may be improved when the degree of international free-riding on innovations is severe enough.

Proposition 3 shows that in a multi-country setting, the presence of a symmetric SIG may improve the social welfare of each country. However, if the SIGs have asymmetric influences across countries, then the country, in which the government places a higher value on campaign contributions, would gain by less or even suffer a welfare loss compared to the symmetric Nash equilibrium. Proposition 4 proves this statement in a two-country setting.

Proposition 4: *Suppose that at the symmetric political equilibrium with* $\varsigma^1 = \varsigma^2 = \hat{\varsigma} > 0$*, both countries are better off compared to the symmetric Nash equilibrium. Then, there must exist a* $\breve{\varsigma} \in (0, \hat{\varsigma})$ *such that when* $\varsigma^1 = \hat{\varsigma}$ *and* $\varsigma^2 \in [0, \breve{\varsigma}]$*, country 1 is worse off compared to the symmetric Nash equilibrium.*

Proof: See Appendix H.□

Intuitively, if the SIG in country 1 is more politically influential than the SIG in country 2, then the patent length in country 1 would be longer than in country 2 in the political equilibrium. In this case, country 1 bears a disproportional share of the distortionary effect, and the resulting welfare loss may outweigh the gain from the increase in R&D.

4. Conclusion

Since the 80's, the pharmaceutical industry has benefited substantially from a series of policy changes that have strengthened the patent protection for brand-name drugs as a result of the industry's political influence, which potentially comes from lobbying and campaign contributions. This chapter incorporates special interest politics into a quality-ladder growth model to analyze the policymakers' tradeoff between the socially optimal patent length and campaign contributions. The welfare analysis suggests that the presence of a pharmaceutical lobby distorting the level of patent protection is socially undesirable in a closed-economy setting. However, in a multi-country setting, the presence of a symmetric SIG may improve the social welfare of each country. If the SIGs have asymmetric influences across countries, then the country that has a more politically influential SIG would gain by less or even suffer a welfare loss. It remains as an empirical question as to whether the pharmaceutical lobby in the US is more or less politically influential than its foreign counterparts.

Before closing the chapter, I briefly discuss the generality of the steady-state welfare analysis. The transition dynamics is omitted for analytical tractability; however, the theoretical predictions should be robust for two reasons. Firstly, the government may want to maximize social welfare that includes the transition dynamics. So long as there is a positive externality in patent protection, the Nash-equilibrium patent length is globally suboptimal. Thus, an increase in patent length due to political influences may still improve social welfare. Secondly, the resource reallocation from production to R&D as a result of increasing patent protection does not necessarily lead to short-run consumption losses.[66] In this case, improving steady-state welfare would be sufficient to improve social welfare.

[66] For example, Chu (2007a) shows that this result holds true over a range of parameters in a model with capital accumulation.

CHAPTER V

Conclusion

In the long run, economic growth is driven by technological innovations, which partly result from profit-seeking entrepreneurial activities such as investments in R&D. In this dissertation, I develop quantitative frameworks to analyze the effects of patent length and blocking patents on R&D in developed countries, such as the US. I find that eliminating blocking patents is more effective in stimulating R&D than extending the patent length. A potential topic for future research is to develop a suitable quantitative framework to analyze technology transfer from developed countries to less-developed economies that could lead to important policy implications on closing the world-income gap.

The political economy of patent policy is also an area that deserves further investigation, and the analysis may provide surprising insights. For example, in Chapter IV, I show that the presence of a pharmaceutical SIG lobbying the government, who would otherwise act as a social planner, to distort the level of patent protection may improve global welfare in a multi-country setting. Another potential topic for future research is to investigate the implications on economic growth and social welfare from modifying standard economic assumptions based on well-established empirical evidence on political behaviors.

APPENDICES

Appendix A: Proofs for Chapter II

Lemma 1: *The aggregate production function for the final goods is*

$$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}, \tag{A1}$$

where $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* η *for* $\theta \in (0,1)$.

Proof: Recall that the production function for the final goods is given by

$$Y_t = \exp\left(\int_0^1 \ln X_t(j) dj \right). \tag{A2}$$

After substituting $X_t(j)$ for $j \in [0,1]$ into (A2) and assuming cost minimization in the intermediate-goods sector, the aggregate production function becomes

$$Y_t = A_t Z_t \left(\frac{K_{x,t}}{L_{x,t}} \right)^{\alpha} L_{x,t}^{e}, \tag{A3}$$

where $L_{x,t}^{e}$ is defined as

$$L_{x,t}^{e} \equiv \exp\left(\int_0^1 \ln L_{x,t}(j) dj \right) \neq \left(\int_0^1 L_{x,t}(j) dj \right) = L_{x,t} \tag{A4}$$

because of the competitive industries. Define $\vartheta(\eta)$ as the ratio of $L_{x,t}^{e}$ and $L_{x,t}$, which is given by

$$\vartheta(\eta) \equiv \frac{L_{x,t}^{e}}{L_{x,t}} = \frac{(z^{\eta})^{\theta}}{z^{\eta}\theta + 1 - \theta} \in (0,1) \tag{A5}$$

for $\theta \in (0,1)$. $\vartheta(\eta)$ represents the static distortionary effect of markup pricing, and it enters the aggregate production function as

$$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}. \tag{A6}$$

Finally, simple differentiation shows that for $\theta \in (0,1)$, (A7) $$\frac{\partial \vartheta(\eta)}{\partial \eta} = -\frac{\theta(1-\theta)(z^{\eta}-1)\ln z}{z^{\eta}\theta + 1 - \theta}\vartheta(\eta) < 0 . \square$$

Proposition 1: *$V_{i,t}(j)$ for $i \in \{1,2,...,\eta\}$ and $j \in [\theta,1]$ is determined by the following law of motion*

(A8) $$r_t V_{i,t}(j) = \Omega_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)),$$

where $V_{\eta+1,t}(j) = 0$. The no-arbitrage condition for $V_{1,t}(j)$ can be re-expressed as

(A9) $$V_{1,t}(j) = \pi_t(j)\left(\sum_{k=1}^{\eta} \Omega_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)}\right)\right).$$

Proof: The expected present value of the ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$ is

(A10) $$V_{i,t}(j) = \int_t^{\infty}\left(\int_t^{s} \Omega_i \pi_x(j)\exp\left(-\int_t^{x} r_v dv\right)dx\right)f(s)ds + \int_t^{\infty} V_{i+1,s}(j)\exp\left(-\int_t^{s} r_v dv\right)f(s)ds,$$

where $f(s) = \lambda_s \exp\left(-\int_t^{s} \lambda_x dx\right)$ is the density function of *s* that is a random variable representing the time when the next innovation occurs and follows the Erlang distribution. The first term in $V_{i,t}(j)$ is the expected present value of monopolistic profit captured by the *i*-th most recent inventor in the current patent pool. The second term in $V_{i,t}(j)$ is the expected present value of the ownership in patent pools when the *i*-th most recent inventor becomes the *i*+1-th most recent inventor. Note that $V_{\eta+1,t}(j) = 0$ because the $\eta+1$-th most recent inventor is no longer in any patent pool. In order to derive (A8), I differentiate (A10) with respect to *t*. To simplify notations, I firstly define a new function such that (A10) becomes

(A11)
$$V_{i,t}(j)=\int_t^{\infty} g(t,s)ds\,,$$

where $g(t,s)=\left(\int_t^s \Omega_i\pi_x(j)\exp\left(-\int_t^x r_v dv\right)dx+V_{i+1,s}(j)\exp\left(-\int_t^s r_v dv\right)\right)f(s)$. Then, using the formula for differentiation under the integral sign,

(A12)
$$\dot{V}_{i,t}(j)\equiv\frac{\partial V_{i,t}(j)}{\partial t}=-g(t,t)+\int_t^{\infty}\frac{\partial g(t,s)}{\partial t}ds\,,$$

where $g(t,t)=\lambda_t V_{i+1,s}(j)$, and

(A13)
$$\frac{\partial g(t,s)}{\partial t}=\left((\lambda_t+r_t)\left(\int_t^s \Omega_i\pi_x(j)\exp\left(-\int_t^x r_v dv\right)dx+V_{i+1,s}(j)\exp\left(-\int_t^s r_v dv\right)\right)-\Omega_i\pi_t(j)\right)f(s)\,.$$

After a few steps of mathematical manipulation, (A12) becomes

(A14)
$$\dot{V}_{i,t}(j)=-\lambda_t V_{i+1,s}(j)+(\lambda_t+r_t)V_{i,t}(j)-\Omega_i\pi_t(j)\int_t^{\infty}f(s)ds\,.$$

Finally, after setting $\int_t^{\infty}f(s)ds=1$, (A14) becomes (A8).

Upon deriving (A8), each $V_{i,t}(j)$ for $i\in\{1,2,...,\eta\}$ can be rewritten as

(A15)
$$V_{i,t}(j)=\frac{\Omega_i\pi_t(j)+\lambda_t V_{i+1,t}(j)}{r_t+\lambda_t-\dot{V}_{i,t}(j)/V_{i,t}(j)}\,,$$

where $V_{\eta+1,t}(j)=0$. Recursive substitutions show that $V_{1,t}(j)$ can be re-expressed as (A9).□

Lemma 2: *The socially optimal steady-state rate of investment in physical capital is*

(A16)
$$i^*=\left(\alpha+\beta\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right)\frac{g_K+\delta}{\rho+g_c\sigma+\delta}\,,$$

and the socially optimal steady-state R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

(A17) $$\frac{s_L^*}{1-s_L^*}=\frac{1-\beta}{1-\alpha}\left(\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right),$$

(A18) $$\frac{s_K^*}{1-s_K^*}=\frac{\beta}{\alpha}\left(\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right).$$

Proof: To derive the socially optimal rate of capital investment and R&D shares of labor and capital, the social planner chooses i_t, $s_{L,t}$ and $s_{K,t}$ to maximize

(A19) $$U=\int_0^\infty e^{-(\rho-n)t}\,\frac{((1-i_t)Y_t/L_t)^{1-\sigma}}{1-\sigma}\,dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(A20) $$Y_t=A_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of i_t given by

(A21) $$\dot{K}_t=i_tY_t-K_t\delta;$$

and (c) the law of motion for R&D technology expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(A22) $$\dot{A}_t=A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi\ln z.$$

The current-value Hamiltonian H is

(A23) $$\begin{aligned}H=&(1-\sigma)^{-1}\left(\frac{(1-i_t)A_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}}{L_t}\right)^{1-\sigma}\\&+\upsilon_{K,t}(i_tA_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}-K_t\delta)\\&+\upsilon_{A,t}A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi\ln z\end{aligned}.$$

The first-order conditions are

(A24) $$H_i = -\frac{1}{(1-i_t)}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}Y_t = 0,$$

(A25) $$H_{s_L} = -\left(\frac{1-\alpha}{1-s_{L,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{1-\alpha}{1-s_{L,t}}\right)i_t Y_t + \upsilon_{A,t}\left(\frac{(1-\beta)\gamma}{s_{L,t}}\right)\dot{A}_t = 0,$$

(A26) $$H_{s_K} = -\left(\frac{\alpha}{1-s_{K,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{\alpha}{1-s_{K,t}}\right)i_t Y_t + \upsilon_{A,t}\left(\frac{\beta\gamma}{s_{K,t}}\right)\dot{A}_t = 0,$$

(A27) $$H_K = \frac{\alpha}{K_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\alpha\frac{i_t Y_t}{K_t} - \delta\right) + \upsilon_{A,t}\left(\beta\gamma\frac{\dot{A}_t}{K_t}\right) = (\rho - n)\upsilon_{K,t} - \dot{\upsilon}_{K,t},$$

(A28) $$H_A = \frac{1}{A_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\frac{i_t Y_t}{A_t}\right) + \upsilon_{A,t}\left(\phi\frac{\dot{A}_t}{A_t}\right) = (\rho - n)\upsilon_{A,t} - \dot{\upsilon}_{A,t}.$$

Note that the first-order conditions with respect to the co-state variables $\upsilon_{K,t}$ and $\upsilon_{A,t}$ yield the law of motions for capital and R&D technology. Then, imposing the balanced-growth conditions yields

(A29) $$H_i : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} = (1-i)\upsilon_{K,t}Y_t,$$

(A30) $$H_{s_L} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_L}{s_L}\right) = \frac{1-\alpha}{1-\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t\right),$$

(A31) $$H_{s_K} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_K}{s_K}\right) = \frac{\alpha}{\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_K i Y_t\right),$$

(A32) $$H_K : \alpha\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t\right) + \beta\gamma g_A A_t \upsilon_{A,t} = (\rho + g_c\sigma + \delta)K_t \upsilon_{K,t},$$

(A33) $$H_A : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t = (\rho - n + (\sigma - 1)g_c + (1-\phi)g_A)A_t \upsilon_{A,t}.$$

Finally, solving (A29)-(A33) yields (A16)-(A18).□

Proposition 2a: *The decentralized equilibrium rate of capital investment* i *is below the socially optimal investment rate* i^* *if either there is underinvestment in R&D or labor is the only factor input for R&D.*

Proof: The socially optimal capital investment rate i^* is

(A34) $$i^* = \left(\alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

The market equilibrium rate of capital investment i is

(A35) $$i = \frac{z^\eta \theta + 1 - \theta}{z^\eta} \left(\alpha + \beta \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

Therefore, either $\beta = 0$ or the underinvestment in R&D such that

$\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A} > \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})$ is *sufficient*

for $i^* > i$ because $(z^\eta \theta + 1 - \theta)/z^\eta < 1$ for $\theta \in [0,1)$.□

Proposition 2b: *Holding the backloading discount factor* ν *constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of capital investment* i *if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

Proof: Differentiating i with respect to η yields

(A36) $$\frac{\partial i}{\partial \eta} = -\frac{(1-\theta)\ln z}{z^\eta} \left(\alpha - \beta \frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda} \nu\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

Since $\rho > n$ and $\sigma \geq 1$, $\alpha \geq \beta$ is a *sufficient* condition for $\partial i / \partial \eta < 0$.□

Lemma 3: For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

(A37) $$c_0(\nu) = \left(i(\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\nu)) s_r(\nu)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}} (1-s_r(\nu))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}} \right).$$

Proof: The following derivation applies to the more general case in which α and β can be different. Without loss of generality, time is re-normalized such that time 0 is the first-period in which the economy reaches the balanced-growth path. The balanced-growth path of per capita consumption (in log) can be written as

(A38) $$\ln c_t = \ln c_0 + g_c t,$$

where $g_c t$ represents the exogenous growth path of consumption because of the semi-endogenous growth formulation. The balanced-growth level of per capital consumption at time 0 is

(A39) $$c_0 = \tilde{\vartheta}(1-i)(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0 Z_0 \left(\frac{K_0}{L_0} \right)^{\alpha},$$

where Z_0 is normalized to one. The capital-labor ratio K_0 / L_0 and the level of R&D-driven technology A_0 at time 0 are respectively

(A40) $$\left(\frac{K_0}{L_0} \right)^{\alpha} = \left(\frac{\tilde{\vartheta} i \,(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0}{g_K + \delta} \right)^{\alpha/(1-\alpha)},$$

(A41) $$A_0 = \left(s_K^{\beta} s_L^{1-\beta} \left(\frac{K_0}{L_0} \right)^{\beta} \right)^{\gamma/(1-\phi)} \left(\frac{\varphi \ln z}{g_A} \right)^{1/(1-\phi)}.$$

After dropping the exogenous growth path and some constant terms, the expression for the endogenous parts of per capita consumption on the balanced-growth path that depends on $\tilde{\vartheta}(\eta)$, $i(\eta,\nu)$, $s_K(\eta,\nu)$ and $s_L(\eta,\nu)$ is

$$c_0(\eta,\nu) = \left(\begin{array}{l} \tilde{\vartheta}(\eta)^{1/(1-\alpha)} i(\eta,\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\eta,\nu)) \\ s_K(\eta,\nu)^{\frac{\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-s_K(\eta,\nu))^{\frac{\alpha(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \\ s_L(\eta,\nu)^{\frac{(1-\beta)\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-s_L(\eta,\nu))^{\frac{(1-\alpha)(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \end{array} \right). \tag{A42}$$

Finally, after setting $\alpha = \beta$ and dropping $\tilde{\vartheta}(\eta)$, (A42) becomes (A37).□

Appendix B: Transition Dynamics for Chapter II

This appendix provides the details of transforming the variables in equations (44) – (47) into their stationary forms for the purpose of computing the transition dynamics numerically. To simplify the analysis, the transformation is performed for the special case of $\alpha = \beta$. The Euler equation is

(B1) $$\dot{c}_t = c_t (r_t - \rho)/\sigma .$$

Define a stationary variable $\tilde{c}_t \equiv c_t /(A_t Z_t)^{1/(1-\alpha)}$, and its resulting law of motion is

(B2) $$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\sigma}(r_t - \rho) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for physical capital is

(B3) $$\dot{K}_t = Y_t - C_t - K_t \delta .$$

Define a stationary variable $k_t \equiv K_t /(L_t (A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(B4) $$\frac{\dot{k}_t}{k_t} = (1 - s_{r,t})\vartheta(\eta) k_t^{\alpha-1} - \frac{\tilde{c}_t}{k_t} - (\delta + n) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for R&D-driven technology is

(B5) $$\dot{A}_t = A_t \lambda_t \ln z .$$

Define a stationary variable $a_t \equiv k_t^{\alpha\gamma} A_t^{\alpha\gamma/(1-\alpha)-(1-\phi)} Z_t^{\alpha\gamma/(1-\alpha)} L_t^{\gamma} \varphi$, and its resulting law of motion is

(B6) $$\frac{\dot{a}_t}{a_t} = \alpha\gamma(1 - s_{r,t})\vartheta(\eta) k_t^{\alpha-1} - \alpha\gamma \frac{\tilde{c}_t}{k_t} - (1-\phi)\lambda_t \ln z + (n\gamma - \alpha\gamma(\delta + n)) .$$

The law of motion for the market value of ownership in patent pools is given by

(B7) $$\dot{V}_{i,t} = (r_t + \lambda_t) V_{i,t} - \lambda_t V_{i+1,t} - \Omega_i \pi_t$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$. Define a stationary variable $\tilde{v}_{i,t} \equiv V_{i,t} /(L_t (A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(B8) $$\frac{\dot{\tilde{v}}_{i,t}}{\tilde{v}_{i,t}} \equiv (r_t + \lambda_t) - \lambda_t \frac{\tilde{v}_{i+1,t}}{\tilde{v}_{i,t}} - (1 - s_{r,t})\vartheta(\eta)\Omega_i(1-\theta)\left(\frac{\mu - 1}{\mu}\right)\frac{k_t^{\alpha}}{\tilde{v}_{i,t}} - n - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z)$$

for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1,t} = 0$. To close this system of differential equations, the endogenous variables $\{r_t, s_{r,t}, \lambda_t\}$ are also expressed in terms of the four newly defined stationary variables. The interest rate is

(B9) $$r_t = \alpha\vartheta(\eta)k_t^{\alpha-1}(\mu\theta + 1 - \theta)/\mu - \delta .$$

From the first-order condition of the R&D sector, the share of factor inputs in R&D is

(B10) $$s_{r,t} = \frac{1}{k_t^{\alpha/(1-\gamma)}}\left(\frac{a_t\tilde{v}_t}{\vartheta(\eta)}\right)^{1/(1-\gamma)}\left(\frac{\mu}{\mu\theta + 1 - \theta}\right)^{1/(1-\gamma)} .$$

From the law of motion of R&D-driven technology, the Poisson arrival rate of innovations is

(B11) $$\lambda_t = s_{r,t}^{\gamma} a_t .$$

Finally, the steady-state values of the variables are

(B12) $$\lambda = g_A / \ln z ,$$

(B13) $$\frac{s_r}{1 - s_r} = \frac{\lambda v(\mu - 1)(1-\theta)/(\mu\theta + (1-\theta))}{\rho - n + (\sigma - 1)g_c + \lambda},$$

(B14) $$a = \lambda / s_r^{\gamma} ,$$

(B15) $$k = \left(\frac{\alpha\vartheta(\eta)(\mu\theta + 1 - \theta)}{\mu(\delta + \rho + \sigma(g_A + g_Z)/(1-\alpha))}\right)^{1/(1-\alpha)} ,$$

(B16) $$\tilde{c} = (1 - s_r)\vartheta(\eta)k^{\alpha} - k\left(\delta + n + \frac{g_A + g_Z}{1-\alpha}\right),$$

(B17) $$\tilde{v}_i = \frac{k^{\alpha}(1 - s_r)\vartheta(\eta)\Omega_i(1-\theta)(\mu - 1) + \lambda\,\tilde{v}_{i+1}}{\alpha\vartheta(\eta)k^{\alpha-1}(\mu\theta + 1 - \theta) + \mu\left(\frac{1}{\ln z} - \frac{1}{1-\alpha}\right)g_A - \mu\left(\frac{g_Z}{1-\alpha} + \delta + n\right)}$$

for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1} = 0$. Note that upon eliminating blocking patents, the backloading discount factor ν and the share of profit captured by the most recent inventor Ω_1 become one.

Appendix C: The Social Rate of Return to R&D for Chapter II

Jones and Williams (1998) define the social rate of return as the sum of the additional output produced and the reduction in R&D that is made possible by reallocating one unit of output from consumption to R&D in the current period and then reducing R&D in the next period to leave the subsequent path of technology unchanged. To conform to their notations, I rewrite the law of motion for R&D technology as

(C1) $$\dot{A}_t = G(A_t, R_t) \equiv A_t^{\phi} R_t^{\gamma} \varphi \ln z \,,$$

where $R_t \equiv K_{r,t}^{\alpha} L_{r,t}^{1-\alpha}$. The aggregate production function is rewritten as

(C2) $$Y_t = F(A_t, X_t) \equiv \vartheta(\eta) A_t Z_t X_t \,,$$

where $X_t \equiv K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}$. Using the above definition, Jones and Williams (1998) show that the gross social return is

(C3) $$1 + \tilde{r} = \left(\frac{\partial G}{\partial R}\right)_t \left(\frac{\partial F}{\partial A}\right)_{t+1} + \frac{(\partial G / \partial R)_t}{(\partial G / \partial R)_{t+1}} \left(1 + \left(\frac{\partial G}{\partial A}\right)_{t+1}\right).$$

After imposing the balanced-growth conditions, the net social return becomes

(C4) $$\tilde{r} = \frac{1 + g_Y}{1 + g_A} \left(1 + g_A \left(\frac{\gamma}{s_r} + \phi\right)\right) - 1 \,.$$

Appendix D: Tables and Figures for Chapter II

Table II-1a: Structural Parameters for θ = 0						Table II-1b: Structural Parameters for θ = 0.25					
λ	ν	α	σ	r	μ	λ	ν	α	σ	r	μ
0.04	0.847	0.287	2.359	0.084	1.030	0.04	0.852	0.288	2.363	0.084	1.040
0.06	0.696	0.287	2.359	0.084	1.030	0.06	0.700	0.288	2.363	0.084	1.040
0.08	0.620	0.287	2.359	0.084	1.030	0.08	0.624	0.288	2.363	0.084	1.040
0.10	0.575	0.287	2.359	0.084	1.030	0.10	0.578	0.288	2.363	0.084	1.040
0.12	0.545	0.287	2.359	0.084	1.030	0.12	0.548	0.288	2.363	0.084	1.040
0.14	0.523	0.287	2.359	0.084	1.030	0.14	0.526	0.288	2.363	0.084	1.040
0.16	0.507	0.287	2.359	0.084	1.030	0.16	0.510	0.288	2.363	0.084	1.040
0.18	0.495	0.287	2.359	0.084	1.030	0.18	0.497	0.288	2.363	0.084	1.040
0.20	0.485	0.287	2.359	0.084	1.030	0.20	0.487	0.288	2.363	0.084	1.040
Table II-1c: Structural Parameters for θ = 0.5						**Table II-1d: Structural Parameters for θ = 0.75**					
λ	ν	α	σ	r	μ	λ	ν	α	σ	r	μ
0.04	0.862	0.288	2.372	0.084	1.061	0.04	0.892	0.288	2.397	0.085	1.126
0.06	0.708	0.288	2.372	0.084	1.061	0.06	0.732	0.288	2.397	0.085	1.126
0.08	0.631	0.288	2.372	0.084	1.061	0.08	0.652	0.288	2.397	0.085	1.126
0.10	0.585	0.288	2.372	0.084	1.061	0.10	0.604	0.288	2.397	0.085	1.126
0.12	0.554	0.288	2.372	0.084	1.061	0.12	0.572	0.288	2.397	0.085	1.126
0.14	0.532	0.288	2.372	0.084	1.061	0.14	0.549	0.288	2.397	0.085	1.126
0.16	0.515	0.288	2.372	0.084	1.061	0.16	0.532	0.288	2.397	0.085	1.126
0.18	0.503	0.288	2.372	0.084	1.061	0.18	0.519	0.288	2.397	0.085	1.126
0.20	0.492	0.288	2.372	0.084	1.061	0.20	0.508	0.288	2.397	0.085	1.126

Table II-2a: Calibrated Values of ϕ for θ = 0										
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.81	0.63	0.44	0.25	0.07	-0.12	-0.31	-0.49	-0.68	-0.86
0.90	0.79	0.59	0.38	0.17	-0.04	-0.24	-0.45	-0.66	-0.86	-1.07
0.80	0.77	0.53	0.30	0.07	-0.17	-0.40	-0.63	-0.86	-1.10	-1.33
0.70	0.73	0.47	0.20	-0.07	-0.33	-0.60	-0.86	-1.13	-1.40	-1.66
0.60	0.69	0.38	0.07	-0.24	-0.55	-0.86	-1.18	-1.49	-1.80	-2.11
0.50	0.63	0.25	-0.12	-0.49	-0.86	-1.24	-1.61	-1.98	-2.36	-2.73
0.40	0.53	0.07	-0.40	-0.86	-1.33	-1.80	-2.26	-2.73	-3.19	-3.66
0.30	0.38	-0.24	-0.86	-1.49	-2.11	-2.73	-3.35	-3.97	-4.59	-5.21
0.20	0.07	-0.86	-1.80	-2.73	-3.66	-4.59	-5.53	-6.46	-7.39	-8.32
0.10	-0.86	-2.73	-4.59	-6.46	-8.32	-10.19	-12.05	-13.92	-15.78	-17.64
0.00	-∞	-∞	-∞	-∞	-∞	-∞	-∞	-∞	-∞	-∞

Table II-2b: Calibrated Values of ϕ for $\theta = 0.25$										
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.81	0.63	0.44	0.25	0.07	-0.12	-0.31	-0.49	-0.68	-0.86
0.90	0.79	0.59	0.38	0.17	-0.04	-0.24	-0.45	-0.66	-0.86	-1.07
0.80	0.77	0.53	0.30	0.07	-0.17	-0.40	-0.63	-0.86	-1.10	-1.33
0.70	0.73	0.47	0.20	-0.07	-0.33	-0.60	-0.86	-1.13	-1.40	-1.66
0.60	0.69	0.38	0.07	-0.24	-0.55	-0.86	-1.18	-1.49	-1.80	-2.11
0.50	0.63	0.25	-0.12	-0.49	-0.86	-1.24	-1.61	-1.98	-2.36	-2.73
0.40	0.53	0.07	-0.40	-0.86	-1.33	-1.80	-2.26	-2.73	-3.20	-3.66
0.30	0.38	-0.24	-0.86	-1.49	-2.11	-2.73	-3.35	-3.97	-4.59	-5.22
0.20	0.07	-0.86	-1.80	-2.73	-3.66	-4.59	-5.53	-6.46	-7.39	-8.32
0.10	-0.86	-2.73	-4.59	-6.46	-8.32	-10.19	-12.05	-13.92	-15.78	-17.65
0.00	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

Table II-2c: Calibrated Values of ϕ for $\theta = 0.5$										
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.81	0.63	0.44	0.25	0.07	-0.12	-0.31	-0.49	-0.68	-0.87
0.90	0.79	0.59	0.38	0.17	-0.04	-0.24	-0.45	-0.66	-0.87	-1.07
0.80	0.77	0.53	0.30	0.07	-0.17	-0.40	-0.63	-0.87	-1.10	-1.33
0.70	0.73	0.47	0.20	-0.07	-0.33	-0.60	-0.87	-1.13	-1.40	-1.66
0.60	0.69	0.38	0.07	-0.24	-0.55	-0.87	-1.18	-1.49	-1.80	-2.11
0.50	0.63	0.25	-0.12	-0.49	-0.87	-1.24	-1.61	-1.98	-2.36	-2.73
0.40	0.53	0.07	-0.40	-0.87	-1.33	-1.80	-2.26	-2.73	-3.20	-3.66
0.30	0.38	-0.24	-0.87	-1.49	-2.11	-2.73	-3.35	-3.97	-4.60	-5.22
0.20	0.07	-0.87	-1.80	-2.73	-3.66	-4.60	-5.53	-6.46	-7.39	-8.33
0.10	-0.87	-2.73	-4.60	-6.46	-8.33	-10.19	-12.06	-13.92	-15.79	-17.65
0.00	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

Table II-2d: Calibrated Values of ϕ for $\theta = 0.75$										
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.81	0.63	0.44	0.25	0.07	-0.12	-0.31	-0.49	-0.68	-0.87
0.90	0.79	0.59	0.38	0.17	-0.04	-0.24	-0.45	-0.66	-0.87	-1.07
0.80	0.77	0.53	0.30	0.07	-0.17	-0.40	-0.63	-0.87	-1.10	-1.33
0.70	0.73	0.47	0.20	-0.07	-0.33	-0.60	-0.87	-1.13	-1.40	-1.67
0.60	0.69	0.38	0.07	-0.24	-0.56	-0.87	-1.18	-1.49	-1.80	-2.11
0.50	0.63	0.25	-0.12	-0.49	-0.87	-1.24	-1.61	-1.99	-2.36	-2.73
0.40	0.53	0.07	-0.40	-0.87	-1.33	-1.80	-2.27	-2.73	-3.20	-3.67
0.30	0.38	-0.24	-0.87	-1.49	-2.11	-2.73	-3.35	-3.98	-4.60	-5.22
0.20	0.07	-0.87	-1.80	-2.73	-3.67	-4.60	-5.53	-6.46	-7.40	-8.33
0.10	-0.87	-2.73	-4.60	-6.46	-8.33	-10.20	-12.06	-13.93	-15.80	-17.66
0.00	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

Table II-3a: The Implied Social Rates of Return for θ = 0

ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.15	0.27	**0.38**	**0.49**	**0.61**	**0.72**	**0.83**	**0.95**	**1.06**	**1.18**
0.90	0.14	0.24	**0.34**	**0.45**	**0.55**	**0.65**	**0.75**	**0.86**	**0.96**	**1.06**
0.80	0.13	0.22	**0.31**	**0.40**	**0.49**	**0.58**	**0.67**	**0.76**	**0.86**	**0.95**
0.70	0.12	0.20	0.28	**0.35**	**0.43**	**0.51**	**0.59**	**0.67**	**0.75**	**0.83**
0.60	0.11	0.17	0.24	**0.31**	**0.38**	**0.44**	**0.51**	**0.58**	**0.65**	**0.71**
0.50	0.09	0.15	0.21	0.26	**0.32**	**0.37**	**0.43**	**0.49**	**0.54**	**0.60**
0.40	0.08	0.13	0.17	0.22	0.26	**0.30**	**0.35**	**0.39**	**0.44**	**0.48**
0.30	0.07	0.10	0.14	0.17	0.20	0.23	0.27	**0.30**	**0.33**	**0.36**
0.20	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.21	0.23	0.25
0.10	0.05	0.06	0.07	0.07	0.08	0.09	0.10	0.11	0.12	0.13
0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

Table II-3b: The Implied Social Rates of Return for θ = 0.25

ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.15	0.27	**0.38**	**0.49**	**0.61**	**0.72**	**0.83**	**0.95**	**1.06**	**1.18**
0.90	0.14	0.24	**0.35**	**0.45**	**0.55**	**0.65**	**0.75**	**0.86**	**0.96**	**1.06**
0.80	0.13	0.22	**0.31**	**0.40**	**0.49**	**0.58**	**0.67**	**0.76**	**0.86**	**0.95**
0.70	0.12	0.20	0.28	**0.35**	**0.43**	**0.51**	**0.59**	**0.67**	**0.75**	**0.83**
0.60	0.11	0.17	0.24	**0.31**	**0.38**	**0.44**	**0.51**	**0.58**	**0.65**	**0.71**
0.50	0.09	0.15	0.21	0.26	**0.32**	**0.37**	**0.43**	**0.49**	**0.54**	**0.60**
0.40	0.08	0.13	0.17	0.22	0.26	**0.30**	**0.35**	**0.39**	**0.44**	**0.48**
0.30	0.07	0.10	0.14	0.17	0.20	0.23	0.27	**0.30**	**0.33**	**0.36**
0.20	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.21	0.23	0.25
0.10	0.05	0.06	0.07	0.07	0.08	0.09	0.10	0.11	0.12	0.13
0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

Table II-3c: The Implied Social Rates of Return for θ = 0.5

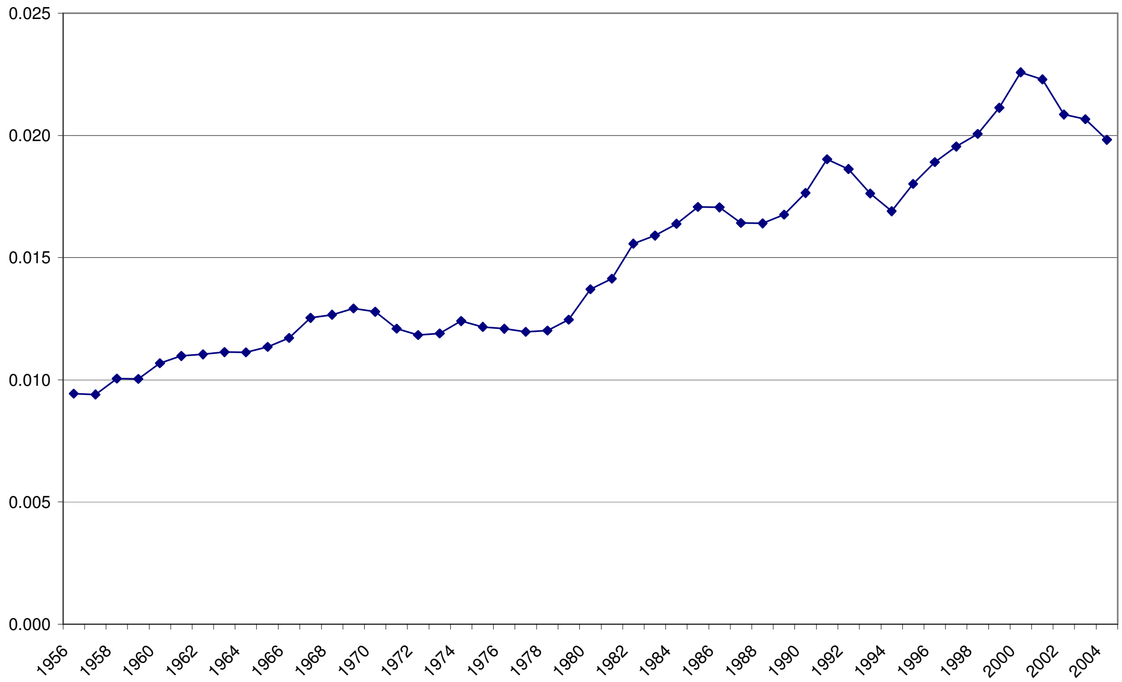
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.15	0.27	**0.38**	**0.49**	**0.61**	**0.72**	**0.84**	**0.95**	**1.06**	**1.18**
0.90	0.14	0.24	**0.35**	**0.45**	**0.55**	**0.65**	**0.75**	**0.86**	**0.96**	**1.06**
0.80	0.13	0.22	**0.31**	**0.40**	**0.49**	**0.58**	**0.67**	**0.76**	**0.86**	**0.95**
0.70	0.12	0.20	0.28	**0.36**	**0.43**	**0.51**	**0.59**	**0.67**	**0.75**	**0.83**
0.60	0.11	0.17	0.24	**0.31**	**0.38**	**0.44**	**0.51**	**0.58**	**0.65**	**0.71**
0.50	0.09	0.15	0.21	0.26	**0.32**	**0.37**	**0.43**	**0.49**	**0.54**	**0.60**
0.40	0.08	0.13	0.17	0.22	0.26	**0.30**	**0.35**	**0.39**	**0.44**	**0.48**
0.30	0.07	0.10	0.14	0.17	0.20	0.23	0.27	**0.30**	**0.33**	**0.36**
0.20	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.21	0.23	0.25
0.10	0.05	0.06	0.07	0.07	0.08	0.09	0.10	0.11	0.12	0.13
0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

Table II-3d: The Implied Social Rates of Return for θ = 0.75										
ξ / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.15	0.27	**0.38**	**0.49**	**0.61**	**0.72**	**0.84**	**0.95**	**1.06**	**1.18**
0.90	0.14	0.24	**0.35**	**0.45**	**0.55**	**0.65**	**0.76**	**0.86**	**0.96**	**1.06**
0.80	0.13	0.22	**0.31**	**0.40**	**0.49**	**0.58**	**0.67**	**0.77**	**0.86**	**0.95**
0.70	0.12	0.20	0.28	**0.36**	**0.43**	**0.51**	**0.59**	**0.67**	**0.75**	**0.83**
0.60	0.11	0.17	0.24	**0.31**	**0.38**	**0.44**	**0.51**	**0.58**	**0.65**	**0.72**
0.50	0.09	0.15	0.21	0.26	**0.32**	**0.37**	**0.43**	**0.49**	**0.54**	**0.60**
0.40	0.08	0.13	0.17	0.22	0.26	**0.30**	**0.35**	**0.39**	**0.44**	**0.48**
0.30	0.07	0.10	0.14	0.17	0.20	0.23	0.27	**0.30**	**0.33**	**0.37**
0.20	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.21	0.23	0.25
0.10	0.05	0.06	0.07	0.07	0.08	0.09	0.10	0.11	0.12	0.13
0.00	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

Table II-4: R&D Shares without Blocking Patents									
θ / λ	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
0.00	0.0136	0.0165	0.0185	0.0200	0.0211	0.0219	0.0226	0.0232	0.0237
0.25	0.0135	0.0164	0.0184	0.0199	0.0210	0.0218	0.0225	0.0231	0.0236
0.50	0.0133	0.0162	0.0182	0.0196	0.0207	0.0216	0.0223	0.0228	0.0233
0.75	0.0129	0.0157	0.0176	0.0190	0.0201	0.0209	0.0216	0.0221	0.0226

Table II-5: Capital Investment Rates without Blocking Patents									
θ / λ	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
0.00	0.204	0.204	0.205	0.205	0.205	0.205	0.206	0.206	0.206
0.25	0.204	0.204	0.205	0.205	0.205	0.205	0.206	0.206	0.206
0.50	0.204	0.204	0.205	0.205	0.205	0.205	0.205	0.206	0.206
0.75	0.204	0.204	0.204	0.205	0.205	0.205	0.205	0.205	0.206

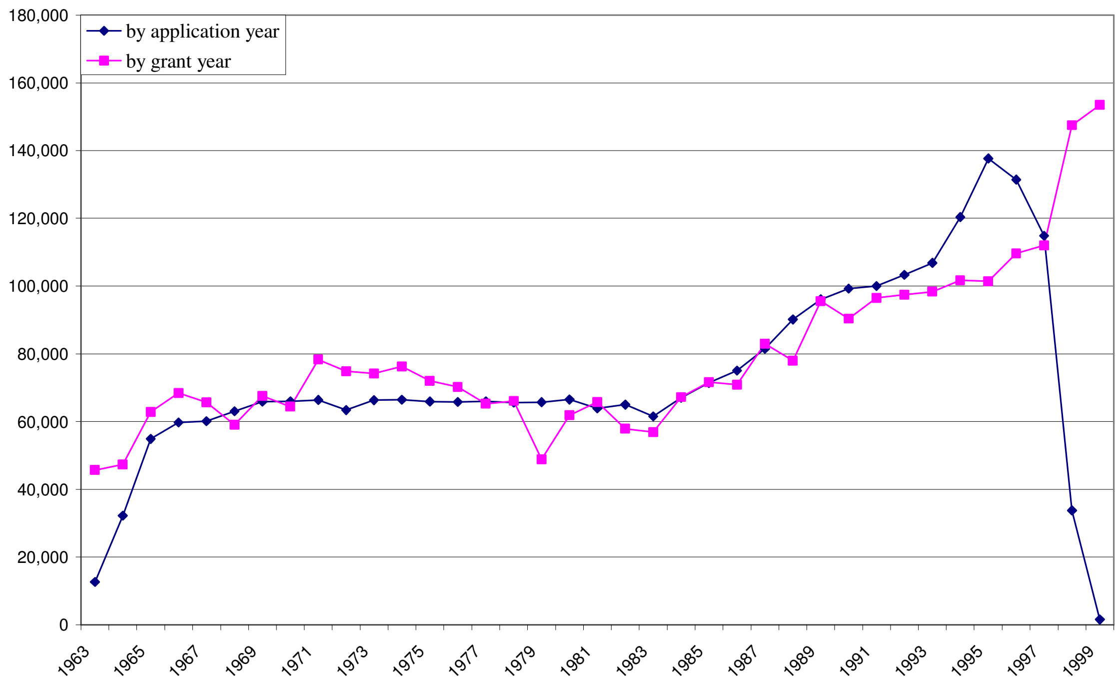
Figure II-1: Private Spending on R&D as a Share of GDP



Data Sources: (a) Bureau of Economic Analysis: National Income and Product Accounts Tables; and (b) National Science Foundation: Division of Science Resources Statistics.

Footnote: R&D is net of federal spending, and GDP is net of government spending.

Figure II-2: Number of Patents Granted



Data Source: Hall, Jaffe and Trajtenberg (2002): The NBER Patent Citation Data File.

Figure II-3a: Socially Optimal R&D Shares for θ = 0

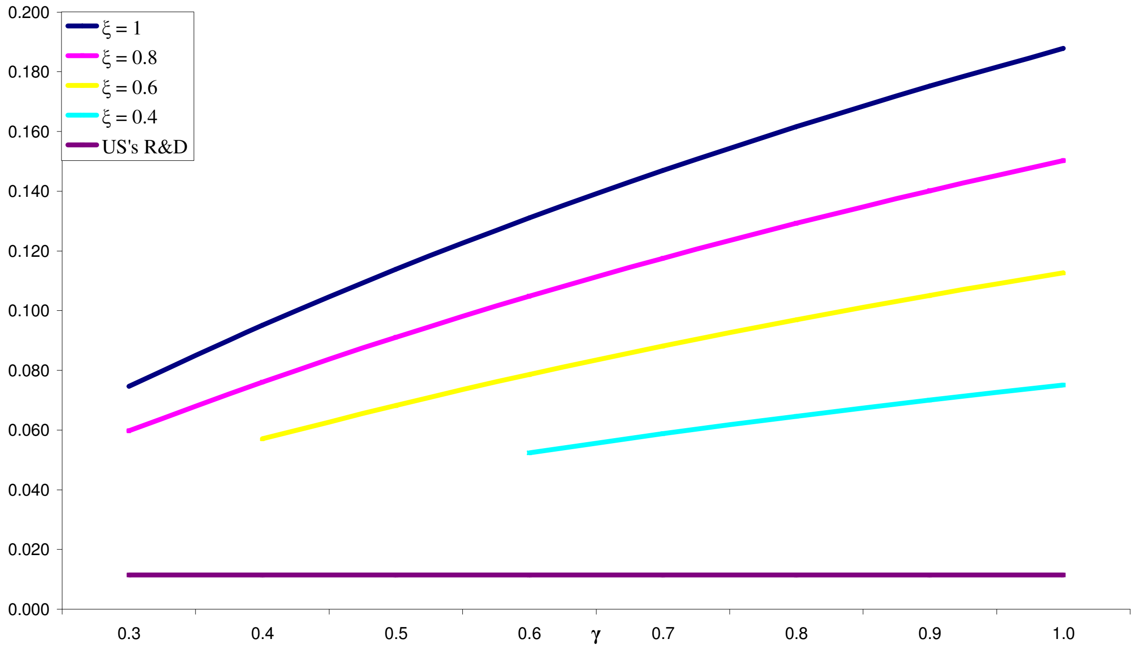


Figure II-3b: Socially Optimal R&D Shares for θ = 0.25

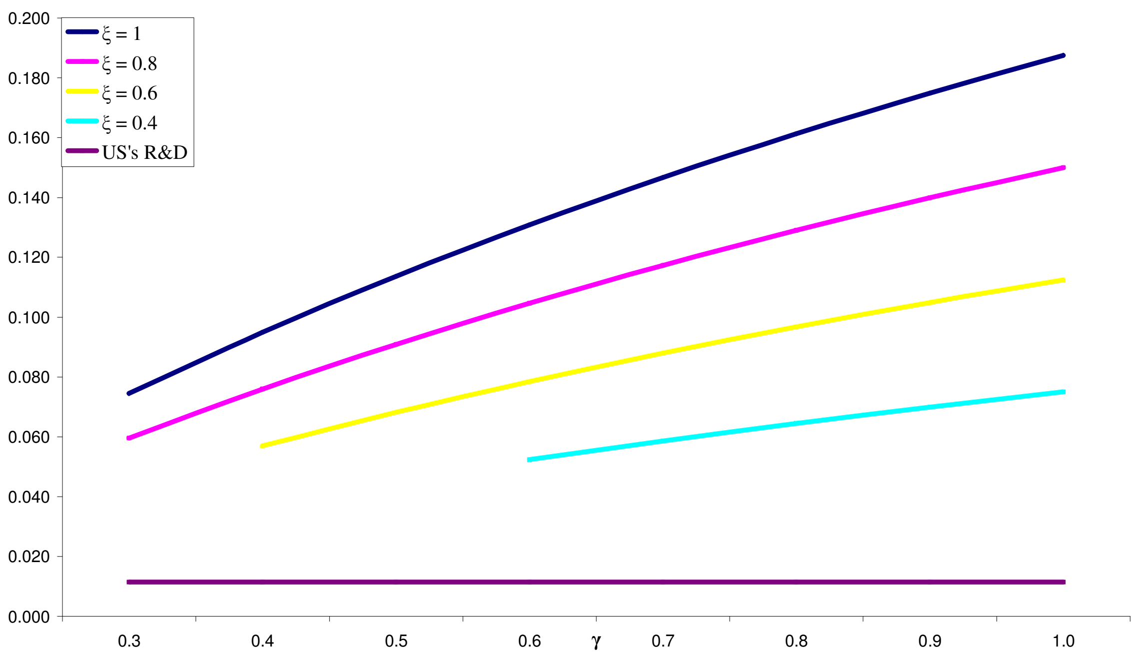


Figure II-3c: Socially Optimal R&D Shares for θ = 0.5

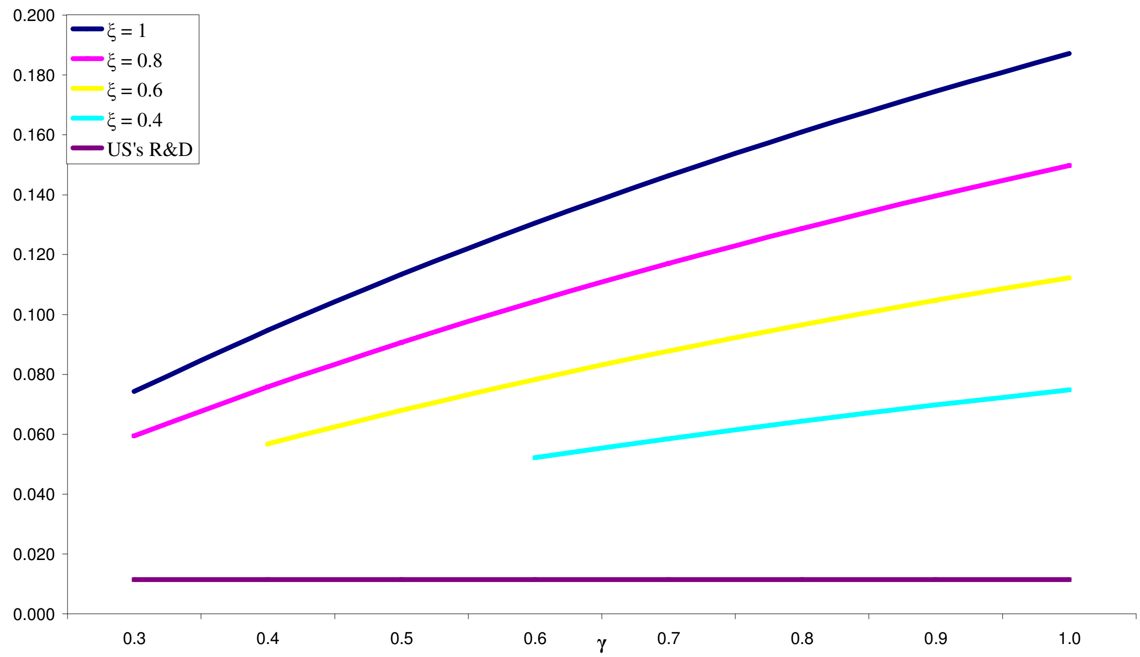


Figure II-3d: Socially Optimal R&D Shares for θ = 0.75

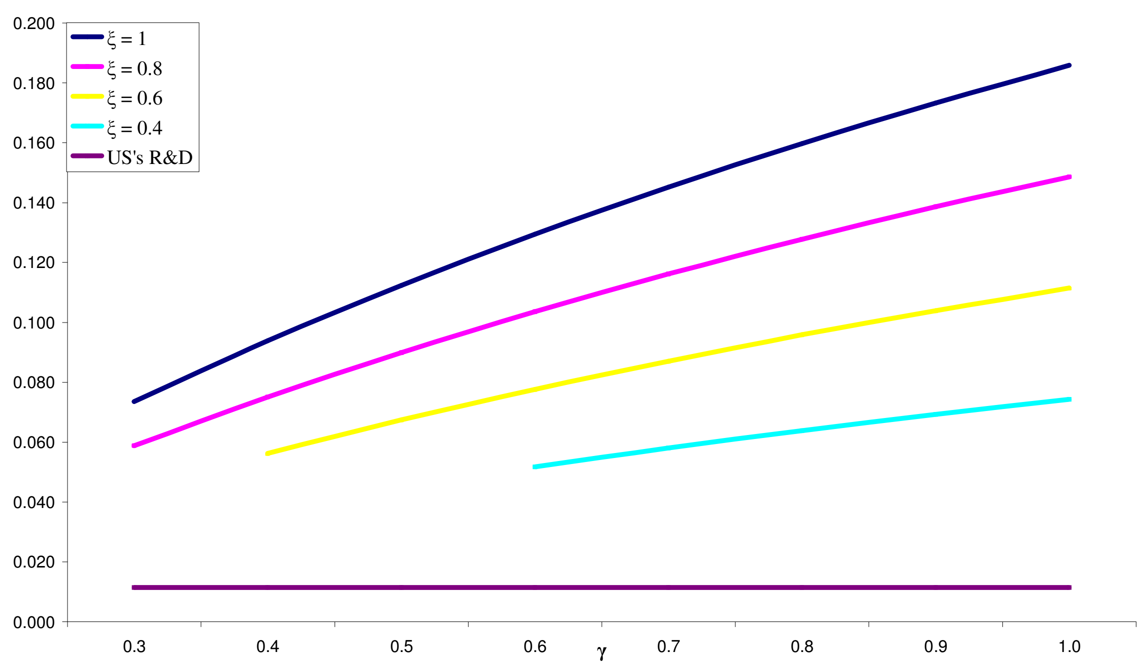


Figure II-4a: Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0

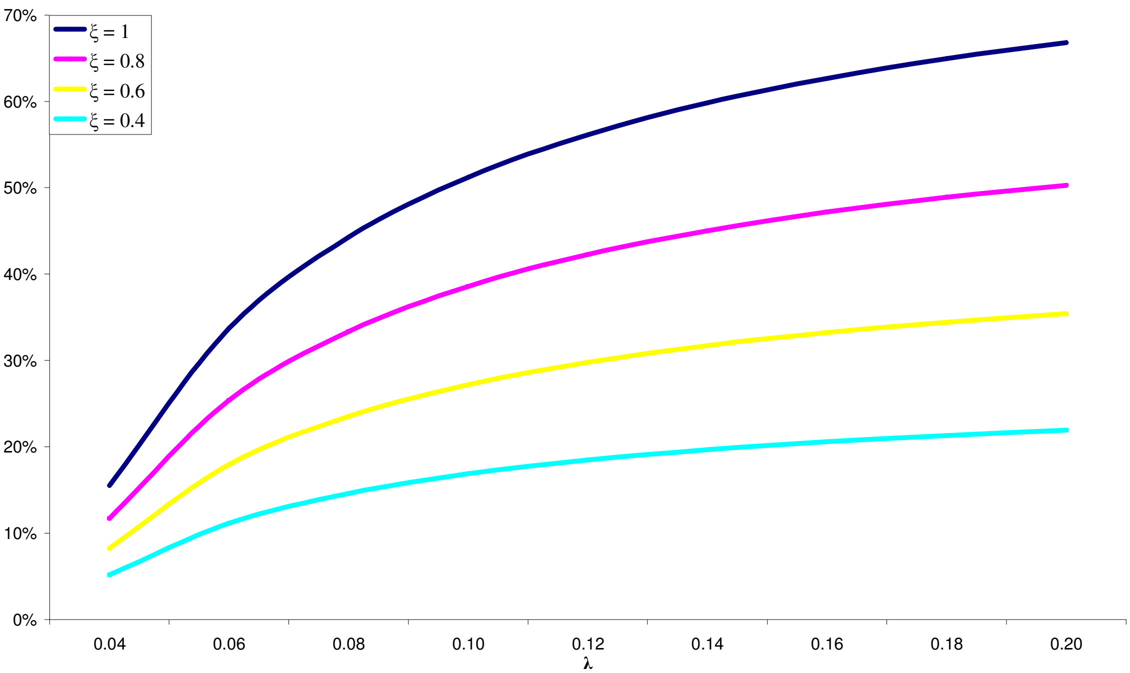


Figure II-4b: Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.25

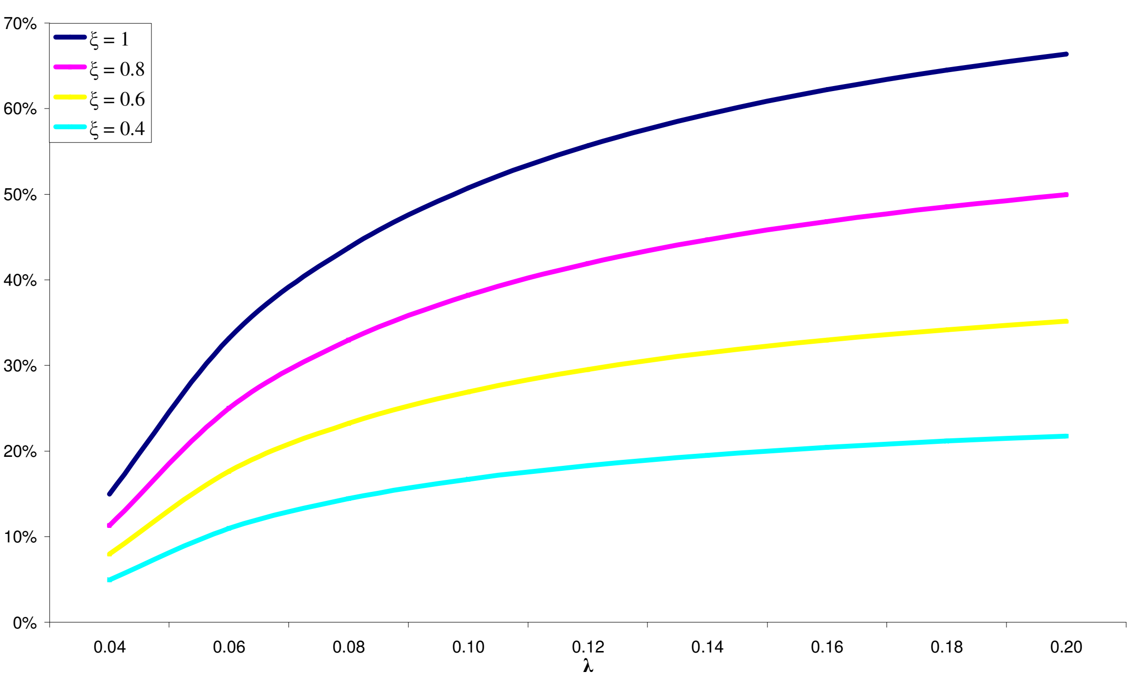


Figure II-4c: Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.5

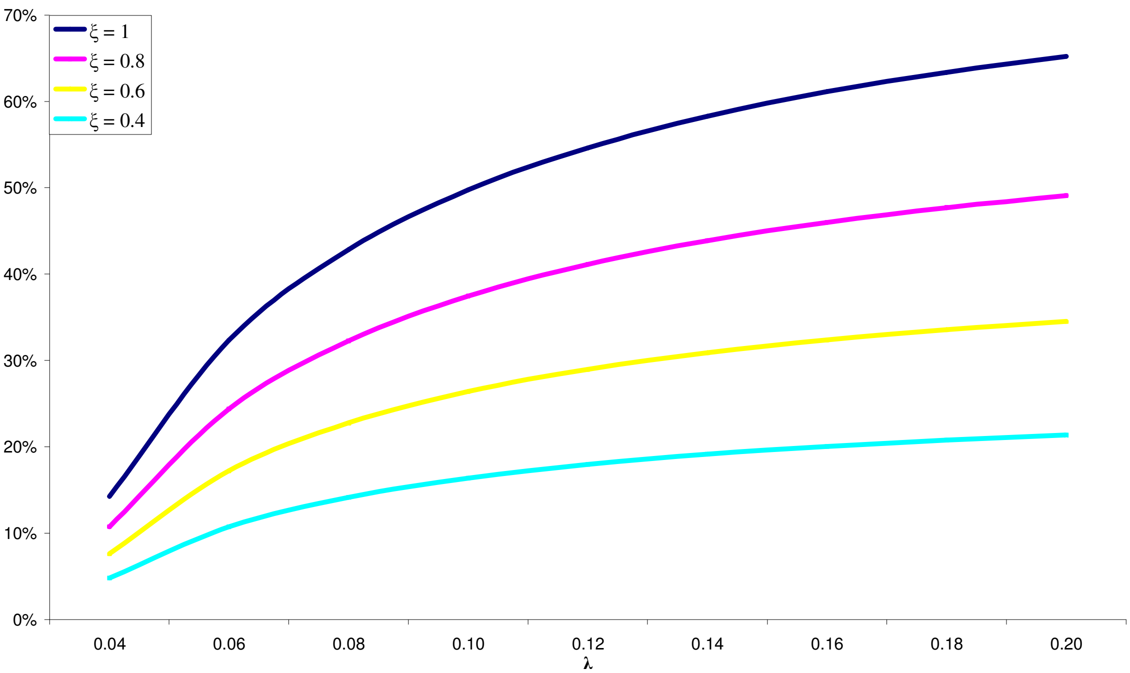


Figure II-4d: Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.75

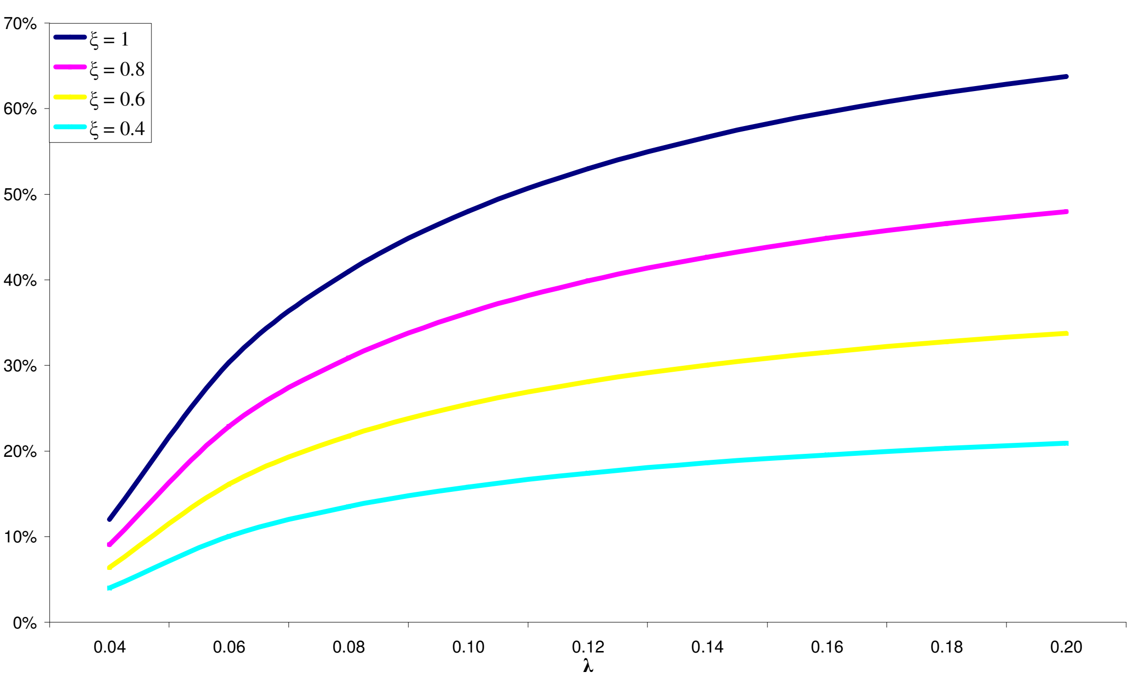


Figure II-5a: Transition Dynamics of Consumption for ξ = γ = 0.55 with Partial Capital Depreciation

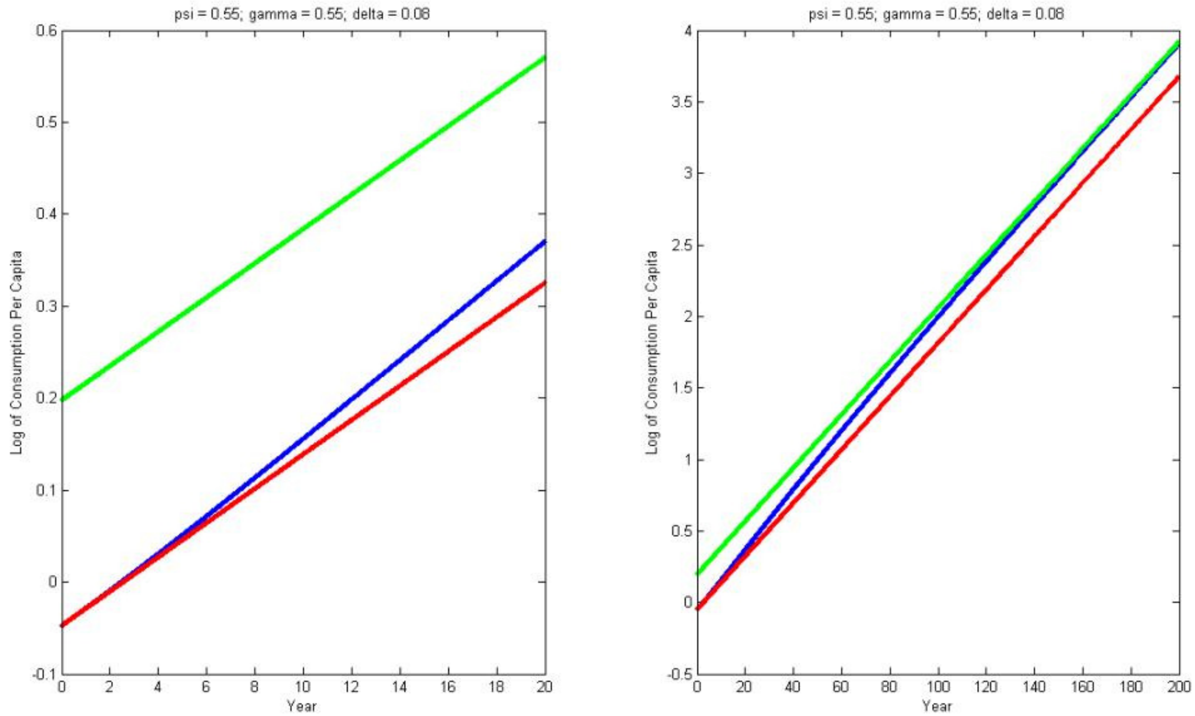


Figure II-5b: Transition Dynamics of Consumption for ξ = γ = 0.55 with Complete Depreciation

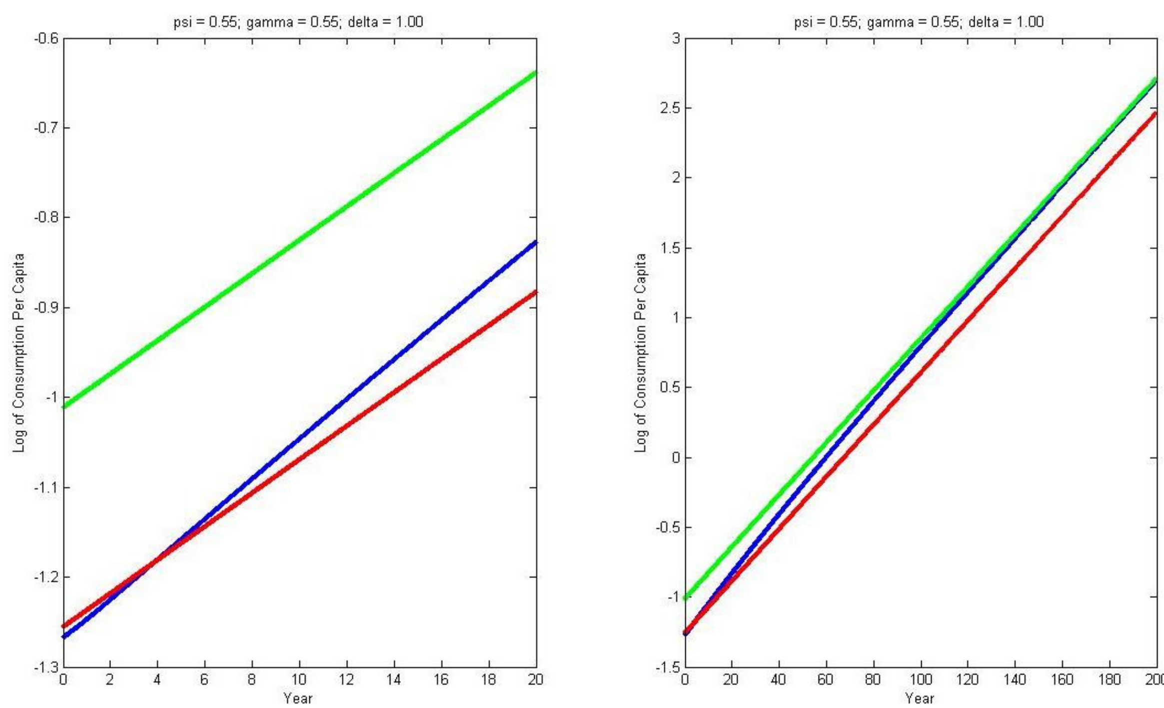


Figure II-5c: Transition Dynamics of Consumption for $\xi = \gamma = 0.3$ with Partial Depreciation

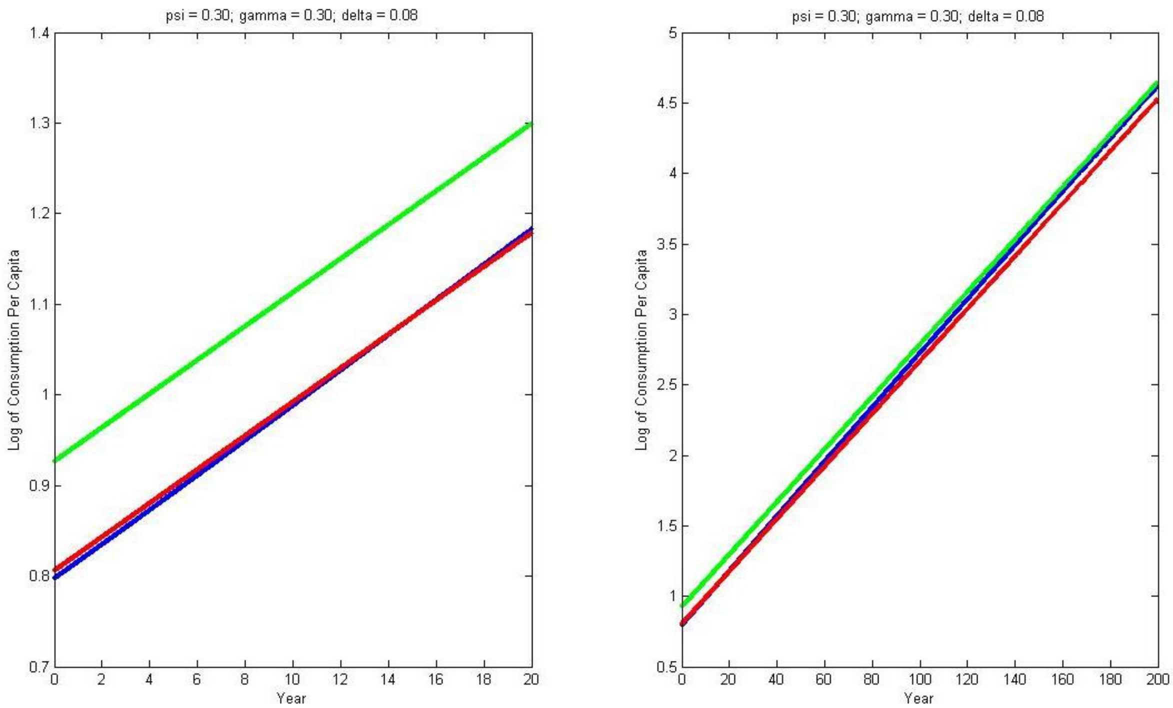


Figure II-5d: Transition Dynamics of Consumption for $\xi = 0.95$ and $\gamma = 0.3$ with Partial Depreciation

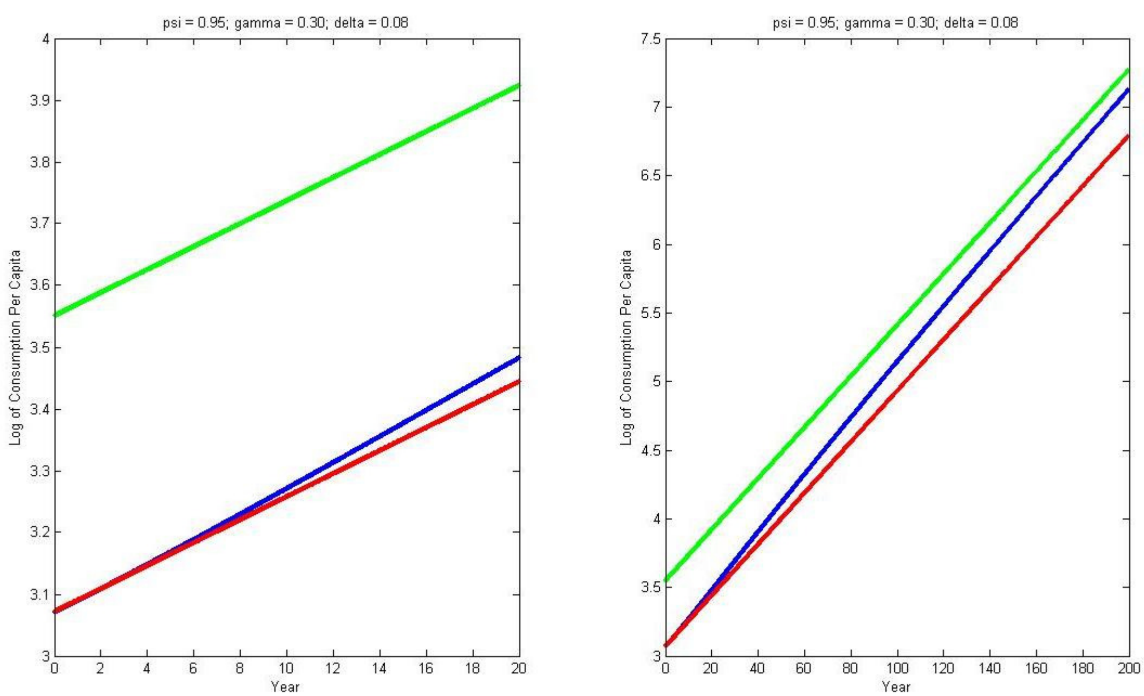


Figure II-6a: Socially Optimal Rates of Capital Investment for θ = 0

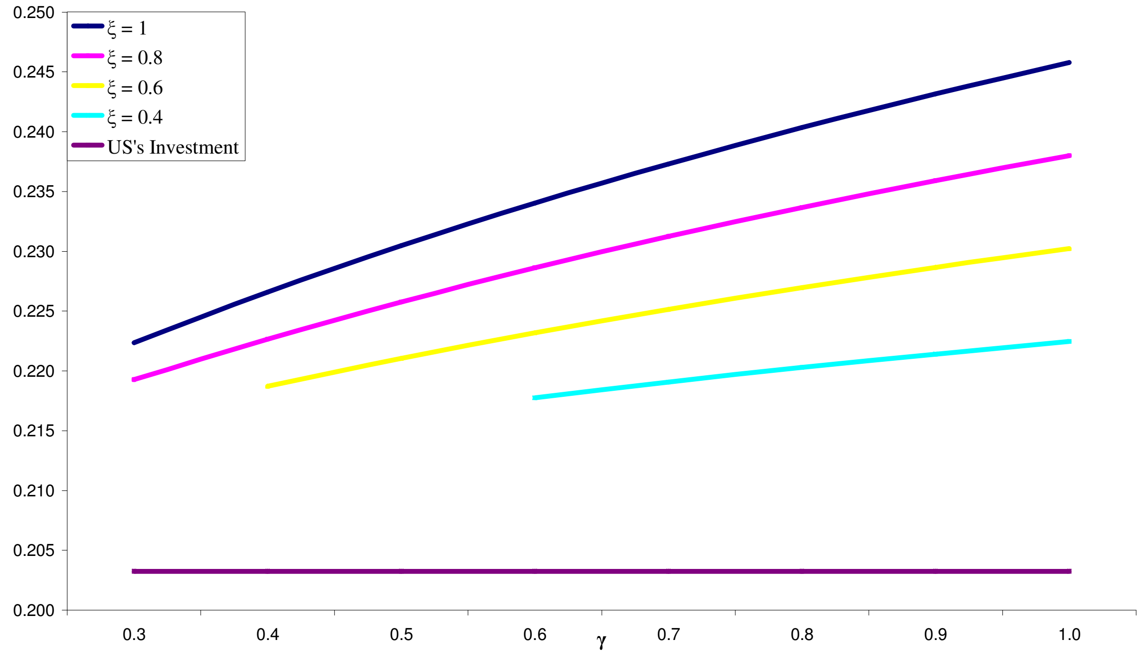


Figure II-6b: Socially Optimal Rates of Capital Investment for θ = 0.25

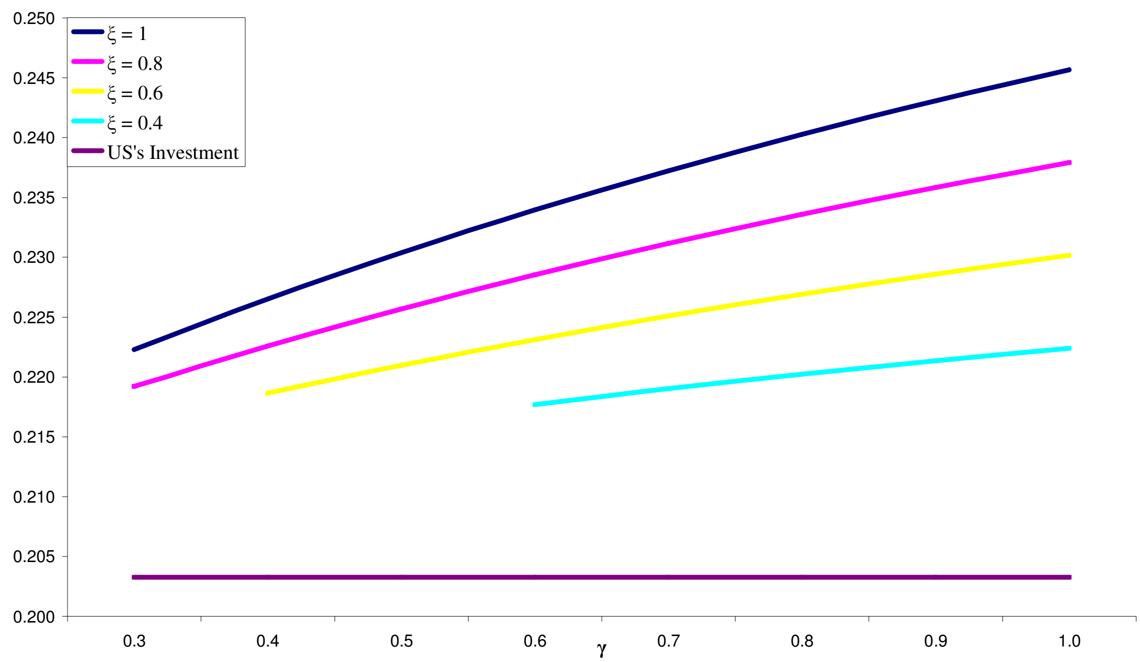


Figure II-6c: Socially Optimal Rates of Capital Investment for θ = 0.5

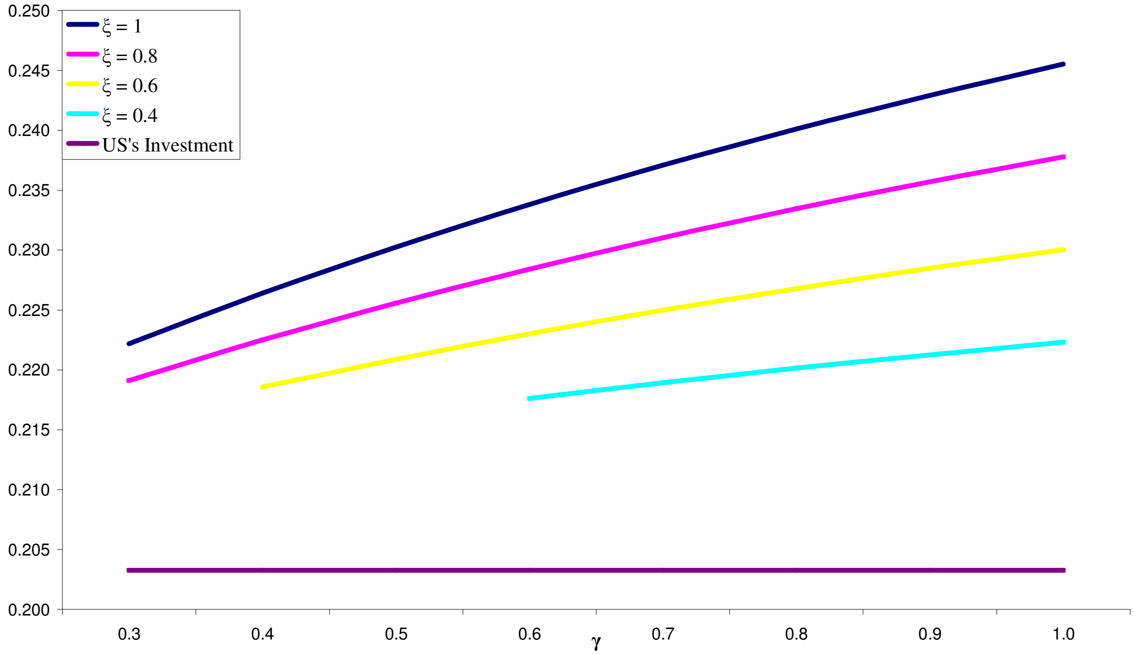


Figure II-6d: Socially Optimal Rates of Capital Investment for θ = 0.75

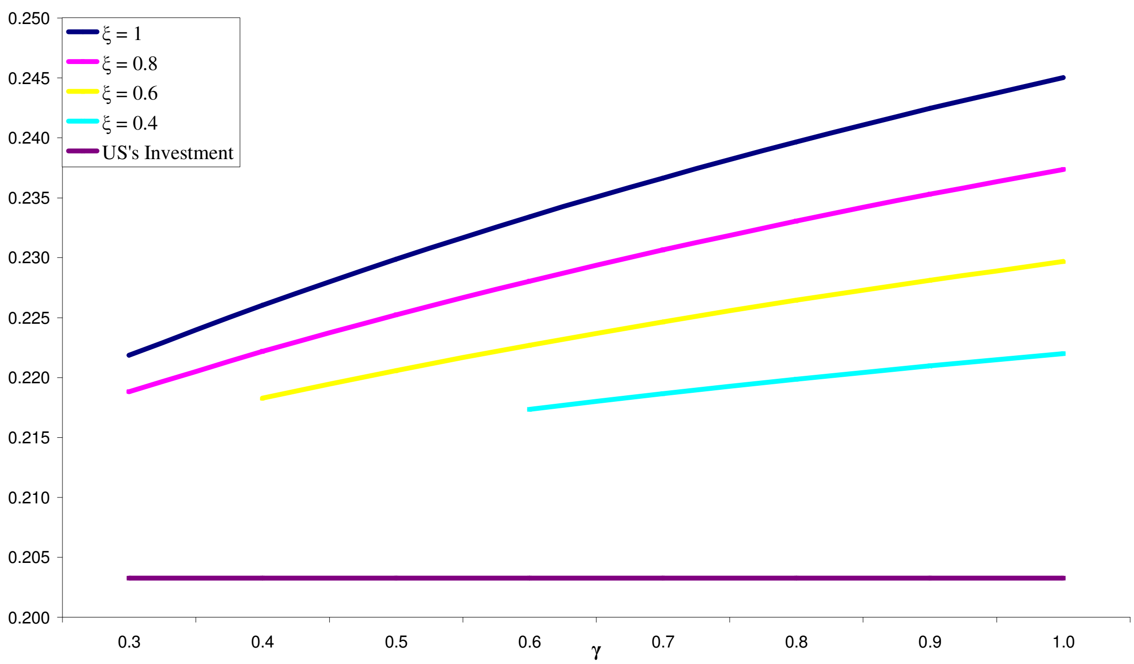
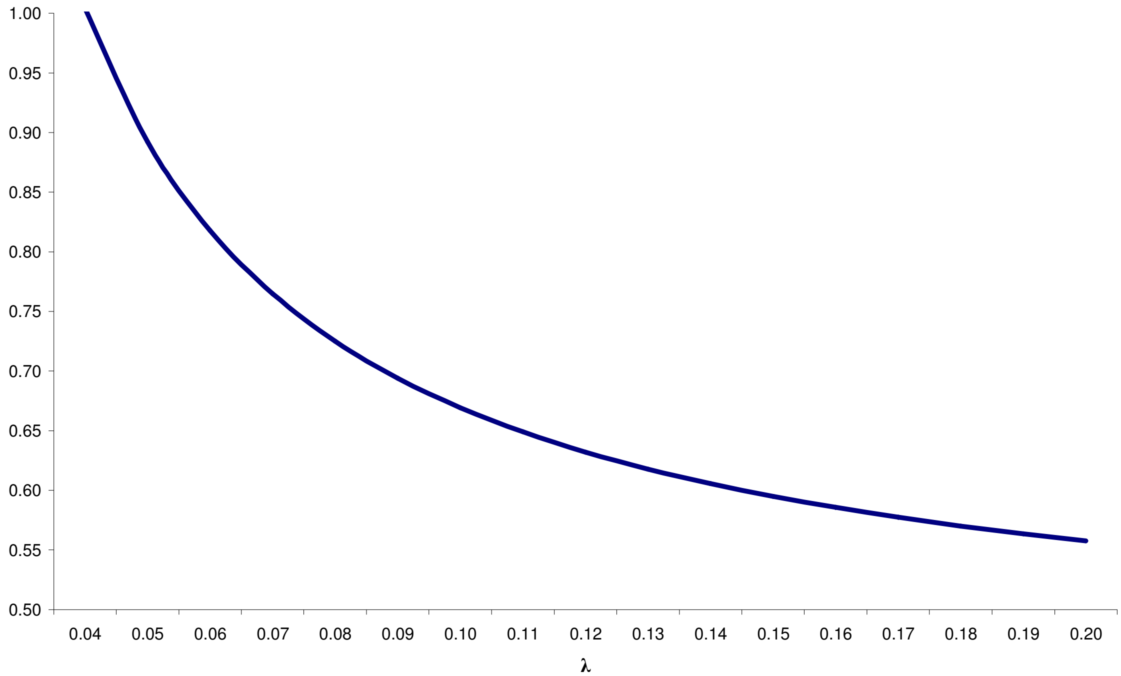


Figure II-7: Calibrated Values for the Backloading Discount Factor Based on Industry-Level Data



Appendix E: Proofs for Chapter III

Lemma 1: *The aggregate production function for the final goods is*

$$Y_t = \tilde{\omega} A_t^{1-\alpha} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha}, \tag{E1}$$

where A_t *is the level of R&D-driven TFP and is defined as*

$$A_t^{1-\alpha} \equiv V_t^{(1-\alpha\eta)/\eta}, \tag{E2}$$

and $\tilde{\omega}$ *is defined as*

$$\tilde{\omega} \equiv \frac{(\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega)^{1/\eta}}{(\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega)^{\alpha}}. \tag{E3}$$

$\tilde{\omega}$ *is strictly less than one for* $\omega \in (0,1)$ *and equals one for* $\omega \in \{0,1\}$. *In addition,*

$$\partial\tilde{\omega}/\partial\omega < 0 \text{ when } \omega < \overline{\omega} \equiv \frac{1}{1-\alpha\eta}\left(\frac{1}{1-(\alpha\eta)^{1/(1-\alpha\eta)}} - \frac{\alpha\eta}{1-(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)}}\right) < 1.$$

Proof: (7), (10) and (11) imply that $X_t(j) = (\alpha\eta)^{1/(1-\alpha\eta)} X_t(j')$. Substituting this condition into (12) and then the resulting condition into (5) yields the aggregate production function. For $\omega \in \{0,1\}$, $\tilde{\omega}$ equals one. Simple differentiation yields

$$\frac{\partial \ln\tilde{\omega}}{\partial\omega} = \frac{1}{\eta}\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega} - \alpha\frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}. \tag{E4}$$

At $\omega = \overline{\omega}$, $\partial\ln\tilde{\omega}/\partial\omega = 0$ and

$$\left.\frac{\partial^2 \ln\tilde{\omega}}{\partial\omega^2}\right|_{\omega=\overline{\omega}} = -\left(\frac{1}{\eta}\left(\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega}\right)^2 - \alpha\left(\frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}\right)^2\right) > 0. \square \tag{E5}$$

Lemma 2: *The socially optimal rate of capital investment* i^* *is*

$$i^* = \left(\alpha + \beta \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}, \tag{E6}$$

and the socially optimal R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

$$\frac{s_L^*}{1-s_L^*} = \frac{1-\beta}{1-\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V}, \tag{E7}$$

$$\frac{s_K^*}{1-s_K^*} = \frac{\beta}{\alpha} \left(\frac{1-\alpha\eta}{\eta} \right) \frac{\gamma g_V}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_V}. \tag{E8}$$

Proof: To derive the socially optimal rate of capital investment and R&D shares of labor and capital, the social planner chooses i_t, $s_{L,t}$ and $s_{K,t}$ to maximize

$$U = \int_0^\infty e^{-(\rho - n)t} \frac{((1-i_t) Y_t / L_t)^{1-\sigma}}{1-\sigma} dt \tag{E9}$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

$$Y_t = V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} (1-s_{K,t})^{\alpha} (1-s_{L,t})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}; \tag{E10}$$

(b) the law of motion for capital expressed in terms of i_t given by

$$\dot{K}_t = i_t Y_t - K_t \delta; \tag{E11}$$

and (c) the law of motion for R&D technology expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

$$\dot{V}_t = V_t^{\phi} (s_{K,t})^{\beta\gamma} (s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi. \tag{E12}$$

The current-value Hamiltonian H is

$$H = (1-\sigma)^{-1}\left(\frac{(1-i_t)V_t^{(1-\alpha\eta)/\eta}Z_t^{1-\alpha}(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}}{L_t}\right)^{1-\sigma}$$

(E13) $$+\upsilon_{K,t}(i_tV_t^{(1-\alpha\eta)/\eta}Z_t^{1-\alpha}(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}-K_t\delta)$$

$$+\upsilon_{V,t}V_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi \quad .$$

The first-order conditions are

(E14) $$H_i = -\frac{1}{(1-i_t)}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}Y_t = 0,$$

(E15) $$H_{s_L} = -\left(\frac{1-\alpha}{1-s_{L,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{1-\alpha}{1-s_{L,t}}\right)i_tY_t + \upsilon_{V,t}\left(\frac{(1-\beta)\gamma}{s_{L,t}}\right)\dot{V}_t = 0,$$

(E16) $$H_{s_K} = -\left(\frac{\alpha}{1-s_{K,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{\alpha}{1-s_{K,t}}\right)i_tY_t + \upsilon_{V,t}\left(\frac{\beta\gamma}{s_{K,t}}\right)\dot{V}_t = 0,$$

(E17) $$H_K = \frac{\alpha}{K_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\alpha\frac{i_tY_t}{K_t}-\delta\right) + \upsilon_{V,t}\left(\beta\gamma\frac{\dot{V}_t}{K_t}\right) = (\rho-n)\upsilon_{K,t} - \dot{\upsilon}_{K,t},$$

(E18) $$H_V = \frac{1-\alpha\eta}{V_t\eta}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\frac{(1-\alpha\eta)i_tY_t}{V_t\eta}\right) + \upsilon_{V,t}\left(\phi\frac{\dot{V}_t}{V_t}\right) = (\rho-n)\upsilon_{V,t} - \dot{\upsilon}_{V,t}.$$

Note that the first-order conditions with respect to the co-state variables $\upsilon_{K,t}$ and $\upsilon_{V,t}$ yield the laws of motion for capital and the number of varieties. Then, imposing the balanced-growth conditions yields

(E19) $$H_i : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} = (1-i)\upsilon_{K,t}Y_t,$$

(E20) $$H_{s_L} : \gamma g_V V_t \upsilon_{V,t}\left(\frac{1-s_L}{s_L}\right) = \frac{1-\alpha}{1-\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}iY_t\right),$$

(E21) $$H_{s_K} : \gamma g_V V_t \upsilon_{V,t}\left(\frac{1-s_K}{s_K}\right) = \frac{\alpha}{\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_K iY_t\right),$$

(E22) $$H_K : \alpha\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right) + \beta\gamma g_V V_t \upsilon_{V,t} = (\rho + g_c\sigma + \delta)K_t \upsilon_{K,t},$$

(E23) $$H_V : \frac{1-\alpha\eta}{\eta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right) = (\rho - n + (\sigma-1)g_c + (1-\phi)g_V)V_t \upsilon_{V,t}.$$

Finally, solving (E19)-(E23) yields (E6)-(E8).□

Proposition 1: *The decentralized equilibrium capital investment rate* i *is below the socially optimal investment rate* i^* *if either there is underinvestment in R&D or labor is the only factor input for R&D. In addition, an increase in the patent length always reduces the equilibrium capital investment rate* i.

Proof for $i < i^*$**:** The socially optimal investment rate i^* is

$$i^* = \left(\alpha + \beta\left(\frac{1-\alpha\eta}{\eta}\right)\frac{\gamma g_V}{\rho - n + (\sigma-1)g_c + (1-\phi)g_V}\right)\frac{g_K + \delta}{\rho + g_c\sigma + \delta}.$$

The market equilibrium rate of investment i is

$$i = \hat{\omega}\left(\alpha + \beta\left(\frac{\alpha(1-\hat{\omega})}{\omega}\right)\frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})g_V}{\rho - n + (\sigma-1)g_c + g_V}\right)\frac{g_K + \delta}{\rho + g_c\sigma + \delta}.$$

Thus, either (i) labor is the only factor input for R&D such that $\beta = 0$, or (ii) there is underinvestment in R&D such that

$\left(\frac{1-\alpha\eta}{\eta}\right)\frac{\gamma g_V}{\rho - n + (\sigma-1)g_c + (1-\phi)g_V} > \left(\frac{\alpha(1-\hat{\omega})}{\omega}\right)\frac{(1-e^{-(\rho-n+(\sigma-1)g_c+g_V)T})g_V}{\rho - n + (\sigma-1)g_c + g_V}$ is a *sufficient* condition for $i < i^*$ because $\hat{\omega} < 1$.□

Proof for $\partial i / \partial T < 0$**:** Recall that $\hat{\omega}(T)$ is a function of *T* and is given by

$$\hat{\omega}(T) = \frac{\omega(T)(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega(T)}{\omega(T)(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega(T)},$$

where $\omega(T) = (1 - e^{-g_V T})$. Differentiating $\hat{\omega}$ with respect to *T* yields

$$\frac{\partial \hat{\omega}(T)}{\partial T} = -\hat{\omega}(T)\left(\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(T)((\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1) + 1} - \frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(T)((\alpha\eta)^{1/(1-\alpha\eta)} - 1) + 1}\right)\frac{\partial \omega(T)}{\partial T} < 0$$

for $T \in (0, \infty)$ because $\frac{(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{\alpha\eta/(1-\alpha\eta)} + 1 - \omega} > \frac{(\alpha\eta)^{1/(1-\alpha\eta)} - 1}{\omega(\alpha\eta)^{1/(1-\alpha\eta)} + 1 - \omega}$. Recall that the equilibrium rate of capital investment is $i(T) = \frac{\alpha\hat{\omega}(T)}{1 - s_K(T)}\left(\frac{g_K + \delta}{\rho + g_c\sigma + \delta}\right)$. Differentiating $i(T)$ with respect to *T* yields

$$\frac{\partial i(T)}{\partial T} = i(T)\left(\frac{1}{\hat{\omega}(T)}\frac{\partial \hat{\omega}(T)}{\partial T} + \frac{1}{1 - s_K(T)}\frac{\partial s_K(T)}{\partial T}\right),$$

where

$$\frac{\partial s_K(T)}{\partial T} = s_K(T)(1 - s_K(T))\left(\frac{(\rho - n + (\sigma-1)g_c + g_V)e^{-(\rho-n+(\sigma-1)g_c+g_V)T}}{1 - e^{-(\rho-n+(\sigma-1)g_c+g_V)T}} - \frac{g_V e^{-g_V T}}{1 - e^{-g_V T}} - \frac{\partial \hat{\omega}(T)/\partial T}{1 - \hat{\omega}(T)}\right)$$

. Finally,

$$\frac{\partial i(T)}{\partial T} = i(T)\left(\begin{array}{l}\left(\frac{1 - \hat{\omega}(T)(1 - s_K(T))}{\hat{\omega}(T)(1 - \hat{\omega}(T))}\right)\frac{\partial \hat{\omega}(T)}{\partial T} \\ + s_K(T)\left(\frac{(\rho - n + (\sigma-1)g_c + g_V)e^{-(\rho-n+(\sigma-1)g_c+g_V)T}}{1 - e^{-(\rho-n+(\sigma-1)g_c+g_V)T}} - \frac{g_V e^{-g_V T}}{1 - e^{-g_V T}}\right)\end{array}\right) < 0$$

because $\frac{(\rho - n + (\sigma-1)g_c + g_V)e^{-(\rho-n+(\sigma-1)g_c+g_V)T}}{1 - e^{-(\rho-n+(\sigma-1)g_c+g_V)T}} < \frac{g_V e^{-g_V T}}{1 - e^{-g_V T}}$ for $T \in (0, \infty)$.□

Lemma 3: *For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is*

(E24)
$$c(T)=\begin{pmatrix}\tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}}\, i(T)^{\frac{\alpha\eta(1-\phi)+\alpha\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}}(1-i(T))\\ s_r(T)^{\frac{\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}}(1-s_r(T))^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\alpha\gamma(1-\alpha\eta)}}\end{pmatrix}.$$

Proof: The following derivation applies to the more general case in which α and β can be different. Along the balanced growth path, per capita consumption is

(E25)
$$c_t=(1-i)\left(\frac{Y_t}{L_t}\right)=\tilde{\omega}(1-i)(1-s_L)\left(\frac{K_{y,t}}{L_{y,t}}\right)^{\alpha}A_t^{1-\alpha}Z_t^{1-\alpha}.$$

The capital-labor ratio $K_{y,t}/L_{y,t}$ in the intermediate-goods sector is

(E26)
$$\frac{K_{y,t}}{L_{y,t}}=A_tZ_t\left(\frac{i\tilde{\omega}(1-s_K)}{g_A+n+\delta}\right)^{1/(1-\alpha)}.$$

The balanced-growth path of V_t from (46) is

(E27)
$$V_t=\left(A_t^{\beta\gamma}Z_t^{\beta\gamma}(s_K)^{\beta\gamma}(s_L)^{(1-\beta)\gamma}\left(\frac{1-s_L}{1-s_K}\right)^{\beta\gamma}\left(\frac{i\tilde{\omega}(1-s_K)}{g_A+n+\delta}\right)^{\beta\gamma/(1-\alpha)}\frac{\varphi}{g_V}L_t^{\gamma}\right)^{1/(1-\phi)}.$$

Therefore, the balanced-growth path of A_t from (E27) and (14) is

(E28)
$$A_t=\left(\frac{(i\tilde{\omega})^{\beta\gamma/(1-\alpha)}(s_K)^{\beta\gamma}(s_L)^{(1-\beta)\gamma}(1-s_L)^{\beta\gamma}(1-s_K)^{\beta\gamma\alpha/(1-\alpha)}}{(g_A+n+\delta)^{\beta\gamma/(1-\alpha)}}\frac{\varphi}{g_V}L_t^{\gamma}Z_t^{\beta\gamma}\right)^{\frac{1-\alpha\eta}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}.$$

Finally, substituting (E26) and (E28) into (E25) and dropping the exogenous terms yield the expression for the balanced-growth level of per capita consumption corresponding to the patent length *T* given by

(E29)
$$c(T)=\begin{pmatrix} \tilde{\omega}(T)^{\frac{\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}\, i(T)^{\frac{\alpha\eta(1-\phi)+\beta\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-i(T)) \\ s_K(T)^{\frac{\beta\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-s_K(T))^{\frac{\alpha\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}} \\ s_L(T)^{\frac{(1-\beta)\gamma(1-\alpha\eta)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}}(1-s_L(T))^{\frac{(1-\alpha)\eta(1-\phi)}{\eta(1-\phi)(1-\alpha)-\beta\gamma(1-\alpha\eta)}} \end{pmatrix}.$$

Finally, after setting $\alpha=\beta$, (E29) becomes (E24).□

Appendix F: The Social Rate of Return to R&D for Chapter III

Jones and Williams (1998) define the social rate of return as the sum of the additional output produced and the reduction in R&D that is made possible by reallocating one unit of output from consumption to R&D in the current period and then reducing R&D in the next period to leave the subsequent path of technology unchanged. I rewrite the law of motion for R&D technology as

(F1) $$\dot{V}_t = G(V_t, R_t) \equiv \varphi V_t^{\phi} R_t^{\gamma},$$

where $R_t \equiv K_{r,t}^{\alpha} L_{r,t}^{1-\alpha}$. The aggregate production function is rewritten as

(F2) $$Y_t = F(V_t, Z_t, L_{y,t}, K_{y,t}) \equiv \tilde{\omega} V_t^{(1-\alpha\eta)/\eta} Z_t^{1-\alpha} L_{y,t}^{1-\alpha} K_{y,t}^{\alpha}.$$

Using the above definition, Jones and Williams (1998) show that the gross social return is

(F3) $$1+\tilde{r} = \left(\frac{\partial G}{\partial R}\right)_t \left(\frac{\partial F}{\partial V}\right)_{t+1} + \frac{(\partial G/\partial R)_t}{(\partial G/\partial R)_{t+1}} \left(1+\left(\frac{\partial G}{\partial V}\right)_{t+1}\right).$$

After imposing the balanced-growth conditions, the net social return becomes

(F4) $$\tilde{r} = \frac{1+g_Y}{1+g_V}\left(1+g_V\left(\frac{1-\alpha\eta}{\eta}\frac{\gamma}{s_r}+\phi\right)\right)-1.$$

Appendix G: Tables and Figures for Chapter III

Table III-1: Calibrated Structural Parameters

g_π	σ	α	η	ξ	g_V	μ	r
-0.10	2.872	0.310	2.970	0.338	0.131	1.085	0.082
-0.15	2.907	0.310	3.008	0.391	0.181	1.071	0.083
-0.20	2.933	0.310	3.024	0.458	0.231	1.065	0.083

Table III-2: The Implied Values for ϕ

g_π / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.10	0.984	0.968	0.952	0.935	0.919	0.903	0.887	0.871	0.855	0.839
-0.15	0.988	0.977	0.965	0.953	0.942	0.930	0.918	0.907	0.895	0.883
-0.20	0.991	0.982	0.973	0.963	0.954	0.945	0.936	0.927	0.918	0.908

Table III-3: The Implied Social Rates of Return to R&D

g_π / γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.10	5.0%	7.0%	8.9%	10.8%	12.7%	14.6%	16.5%	18.4%	20.3%	22.2%
-0.15	5.3%	7.4%	9.6%	11.7%	13.9%	16.0%	18.1%	20.3%	22.4%	24.6%
-0.20	5.6%	8.0%	10.5%	12.9%	15.3%	17.8%	20.2%	22.7%	25.1%	27.5%

Figure III-1: Socially Optimal R&D Shares

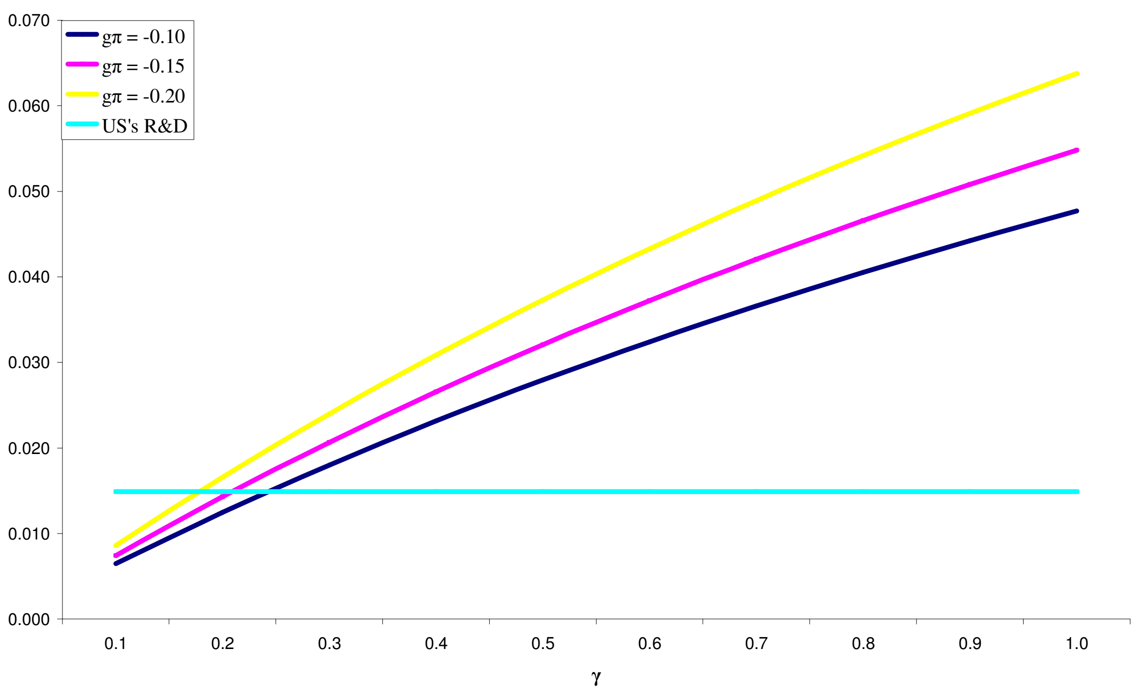


Figure III-2: R&D Shares at Different Patent Length

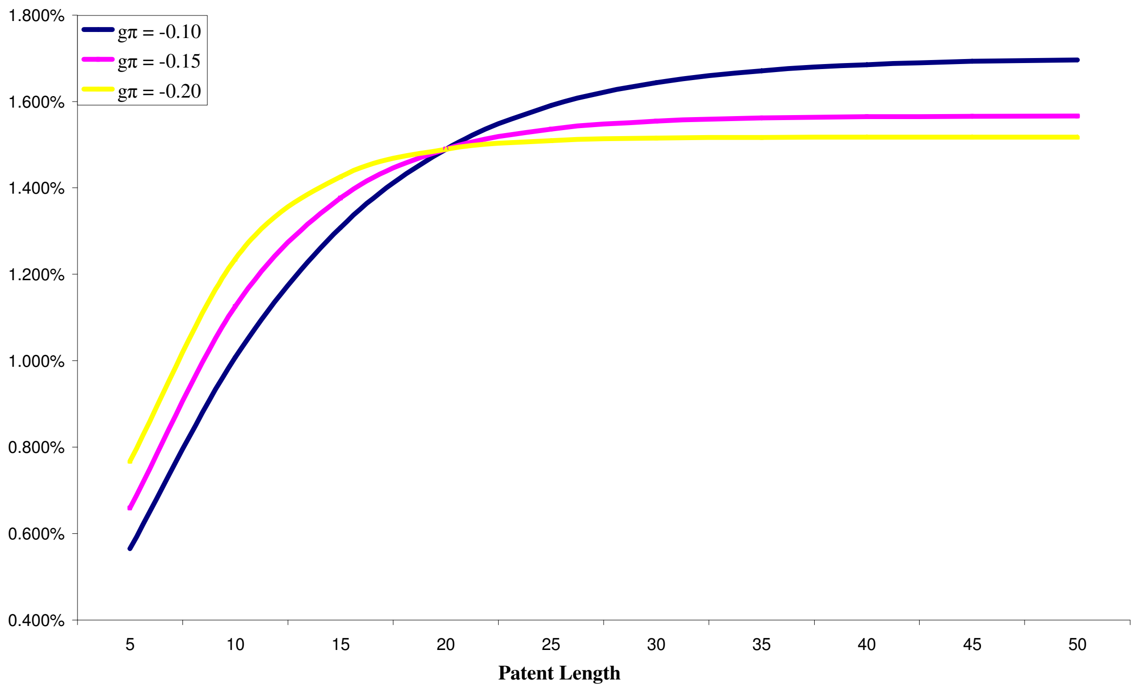


Figure III-3: Percentage Changes in Long-Run Consumption

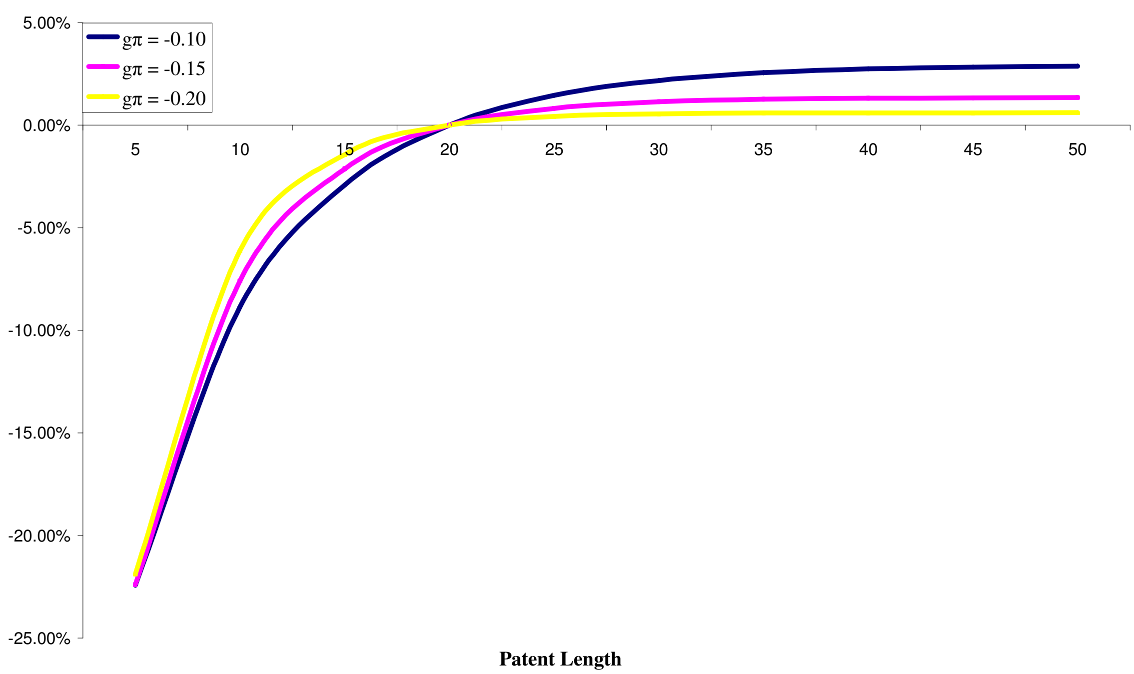


Figure III-4: Socially Optimal Rates of Investment in Physical Capital

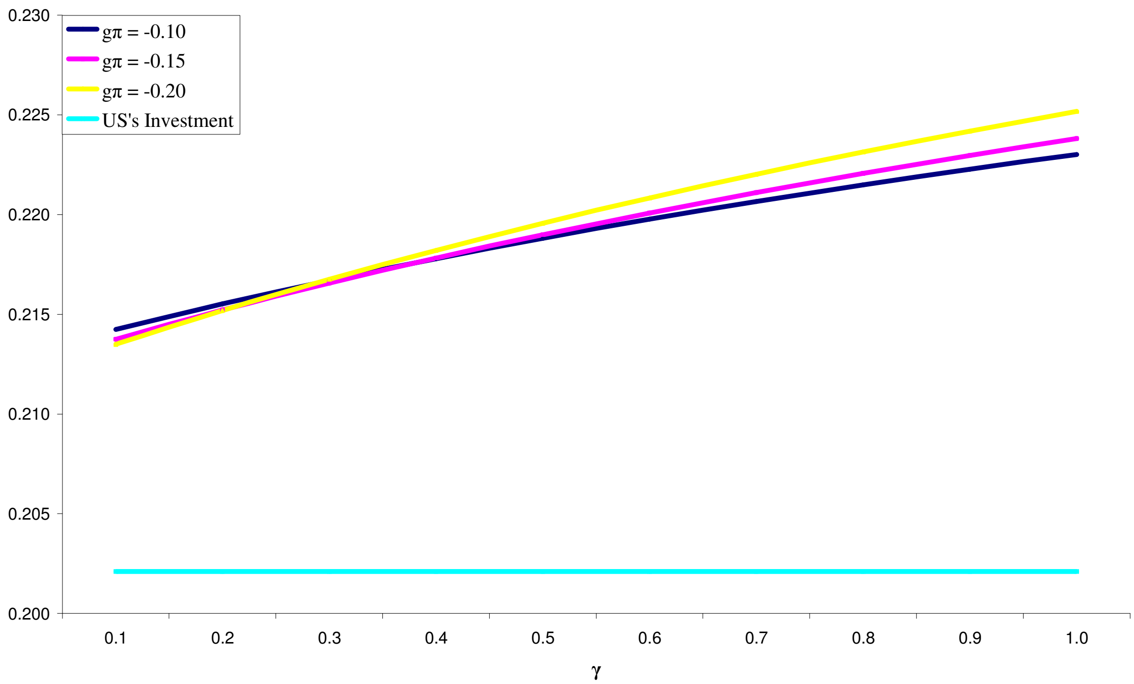
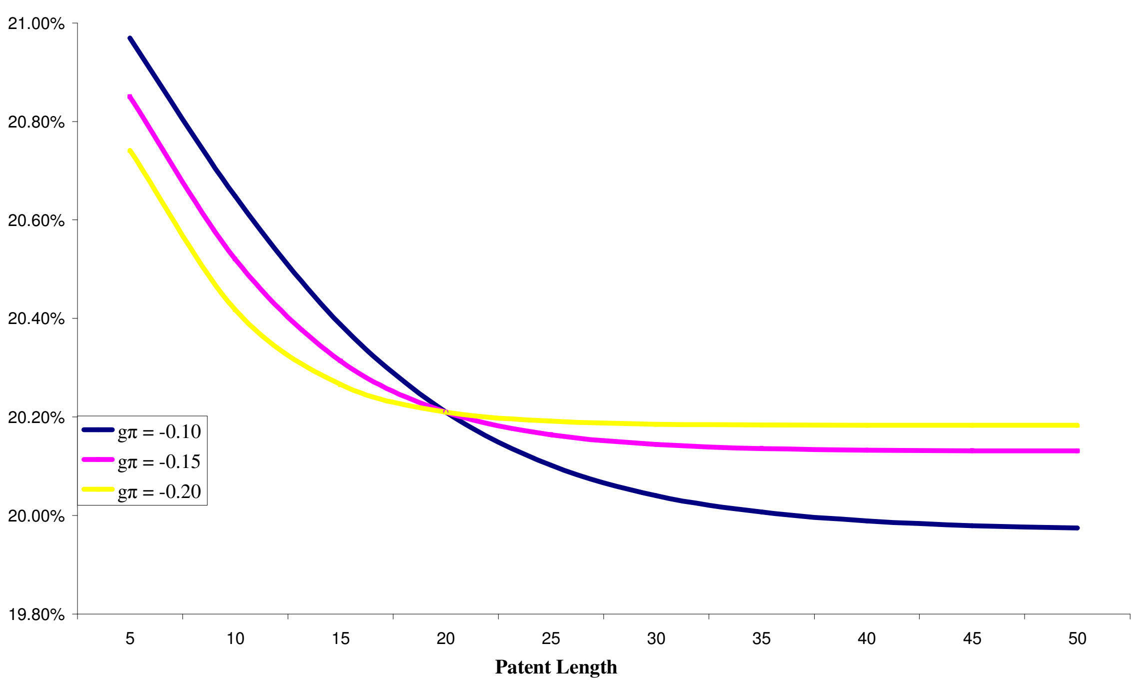


Figure III-5: Equilibrium Rates of Investment in Physical Capital at Different Patent Length



Appendix H: Proofs for Chapter IV

Proposition 2: *There exists a* $\hat{\sigma} \in (0,1)$ *such that if and only if* $\sigma \geq \hat{\sigma}$, $\partial C_0 / \partial \varsigma \geq 0$ *for* $\varsigma \in (0,1)$.

Proof: The welfare of the government is firstly rescaled so that one dollar for the government has the same utility weight as one dollar for the SIG. Dividing (47) by ς,

(H1) $$\widetilde{W}(T, C_0) / \varsigma = W(T)(1-\varsigma)/\varsigma + C_0 .$$

The total surplus of the government and the SIG at T^{SIG} is

(H2) $$TS = [W(T^{SIG}) - W(T^*)](1-\varsigma)/\varsigma + \theta[U(I, T^{SIG}) - U(I, T^*)] .$$

Differentiating (H2) with respect to T^{SIG} yields

(H3) $$\frac{\partial TS}{\partial T^{SIG}} = \left(\frac{1-\varsigma}{\varsigma}\right)\frac{\partial W(T^{SIG})}{\partial T^{SIG}} + \theta \frac{\partial U(I, T^{SIG})}{\partial T^{SIG}} = 0 .$$

Multiplying (H3) by ς and substituting (44) into (H3) yields the same first-order condition as (50). From Assumption 3, the amount of surplus captured by the government is σTS such that

(H4) $$[W(T^{SIG}) - W(T^*)](1-\varsigma)/\varsigma + C_0 = \sigma TS .$$

Rearranging some terms, (H4) becomes

(H5) $$C_0 = (\sigma - 1)[W(T^{SIG}) - W(T^*)](1-\varsigma)/\varsigma + \sigma\theta[U(I, T^{SIG}) - U(I, T^*)] .$$

Differentiating (H5) with respect to ς yields

(H6) $$\frac{\partial C_0}{\partial \varsigma} = \frac{\sigma - 1}{\varsigma}\left([W(T^*) - W(T^{SIG})]\frac{1}{\varsigma} + \frac{\partial W(T^{SIG})}{\partial \varsigma}(1-\varsigma)\right) + \sigma\theta \frac{\partial U(I, T^{SIG})}{\partial \varsigma} .$$

Note that $W(T^*) - W(T^{SIG}) > 0$, $\partial W(T^{SIG}) / \partial \varsigma < 0$, and $\partial U(I, T^{SIG}) / \partial \varsigma > 0$. The next step is to show that there exists a unique $\hat{\sigma} \in (0,1)$ such that $\partial C_0 / \partial \varsigma \big|_{\sigma = \hat{\sigma}} = 0$. I will show this in three steps: (i) $\partial^2 C_0 / \partial \varsigma \partial \sigma > 0$; (ii) $\partial C_0 / \partial \varsigma \big|_{\sigma = 0} < 0$; and (iii) $\partial C_0 / \partial \varsigma \big|_{\sigma = 1} > 0$.

(H7) $$\frac{\partial C_0}{\partial \varsigma \partial \sigma} = \frac{1}{\varsigma}\left([W(T^*) - W(T^{SIG})]\frac{1}{\varsigma} + \frac{\partial W(T^{SIG})}{\partial \varsigma}(1-\varsigma) \right) + \theta \frac{\partial U(I, T^{SIG})}{\partial \varsigma} > 0 .$$

Note that $[W(T^*) - W(T^{SIG})]/\varsigma + (1-\varsigma)\partial W(T^{SIG})/\partial \varsigma > 0$ because $1/\varsigma > (1-\varsigma)$ for $\varsigma \in (0,1)$ and $[W(T^*) - W(T^{SIG})] > -\partial W(T^{SIG})/\partial \varsigma$.

(H8) $$\left.\frac{\partial C_0}{\partial \varsigma}\right|_{\sigma=0} = -\frac{1}{\varsigma}\left([W(T^*) - W(T^{SIG})]\frac{1}{\varsigma} + \frac{\partial W(T^{SIG})}{\partial \varsigma}(1-\varsigma) \right) < 0 .$$

(H9) $$\left.\frac{\partial C_0}{\partial \varsigma}\right|_{\sigma=1} = \theta \frac{\partial U(I, T^{SIG})}{\partial \varsigma} > 0 .$$

Thus, there must exist a $\hat{\sigma} \in (0,1)$ such that if and only if $\sigma \geq \hat{\sigma}$, $\partial C_0 / \partial \varsigma \geq 0$ for $\varsigma \in (0,1)$.□

Corollary 1a: *The government having the first-mover advantage to make a take-or-leave-it offer to the SIG is equivalent to* $\sigma = 1$. *In this case,* $C_0 = \theta(U(I, T^{SIG}) - U(I, T^*))$ *and* $\partial C_0 / \partial \varsigma > 0$ *for* $\varsigma \in (0,1)$.

Proof: At the social optimum T^*, the welfare of the SIG is

(H10) $$\theta U(I, T^*) .$$

In the political equilibrium $\{T^{SIG}, C_0\}$, the welfare of the SIG is

(H11) $$\theta U(I, T^{SIG}) - C_0 .$$

Therefore, the maximum amount that the SIG is willing to pay as campaign contributions is the amount for which it is indifferent between $\{T, C_0\} = \{T^{SIG}, \theta(U(I, T^{SIG}) - U(I, T^*))\}$ and $\{T, C_0\} = \{T^*, 0\}$. Therefore, the participation constraint is

(H12) $$C_0 \leq \theta(U(I, T^{SIG}) - U(I, T^*)) .$$

Since the government has the first-mover advantage, it would make an offer to the SIG such that the participation constraint is binding. Note that setting $\sigma = 1$ in (H5) yields

(H13) $$C_0 = \theta[U(I, T^{SIG}) - U(I, T^*)].$$

Therefore, $\sigma = 1$ is equivalent to the case in which the government has the first-mover advantage to make a take-or-leave-it offer to the SIG. In this case, $\partial C_0 / \partial \varsigma > 0$ for $\varsigma \in (0,1)$ as shown in (H9).□

Corollary 1b: *The SIG having the first-mover advantage to make a take-or-leave-it offer to the government is equivalent to* $\sigma = 0$. *In this case,* $C_0 = (W(T^*) - W(T^{SIG}))(1-\varsigma)/\varsigma$ *and* $\partial C_0 / \partial \varsigma < 0$ *for* $\varsigma \in (0,1)$.

Proof: At the social optimum T^*, the welfare of the government is

(H14) $$W(T^*)(1-\varsigma)/\varsigma.$$

In the political equilibrium $\{T^{SIG}, C_0\}$, the welfare of the government is

(H15) $$W(T^{SIG})(1-\varsigma)/\varsigma + C_0.$$

Therefore, the minimum amount that the government is willing to accept as campaign contributions to implement T^{SIG} is the amount for which it is indifferent between $\{T, C_0\} = \{T^*, 0\}$ and $\{T, C_0\} = \{T^{SIG}, [W(T^*) - W(T^{SIG})](1-\varsigma)/\varsigma\}$. Therefore, the participation constraint is

(H16) $$C_0 \geq [W(T^*) - W(T^{SIG})](1-\varsigma)/\varsigma.$$

Since the SIG has the first-mover advantage, it would make an offer to the government such that the participation constraint is binding. Note that setting $\sigma = 0$ in (H5) yields

(H17) $$C_0 = [W(T^*) - W(T^{SIG})](1-\varsigma)/\varsigma.$$

Therefore, $\sigma = 0$ is equivalent to the case in which the SIG has the first-mover advantage to make a take-or-leave-it offer to the government. In this case, $\partial C_0 / \partial \varsigma < 0$ for $\varsigma \in (0,1)$ as shown in (H8).□

Lemma 3: *The Nash-equilibrium patent length is decreasing in N and is strictly below the symmetric globally optimal patent length when the number of countries is at least two.*

Proof: Recall that T^{NE} is characterized by

(H18) $$\frac{\Omega'(T^{NE})}{\Omega(T^{NE})} = n\left(N - \Omega(T^{NE})\left(\frac{z-1}{z \ln z}\right)\right).$$

Taking the total differentials of (H18), $sign\left(\frac{dT^{NE}}{dN}\right) = sign\left(\frac{n}{\rho - n}\left(\frac{z-1}{z \ln z}\right) - \frac{1}{\Omega(T^{NE})^2}\right)$,

which is negative by (a4). The symmetric globally optimal patent length, denoted by T^{GO}, is characterized by

(H19) $$\frac{\Omega'(T^{GO})}{\Omega(T^{GO})} = n\left(1 - \Omega(T^{GO})\left(\frac{z-1}{z \ln z}\right)\right).$$

Therefore, for $N \geq 2$, $T^{NE} < T^{GO}$.□

Proposition 3: *When* $N = 1$, $W(T^{SIG}) < W(T^*)$ *for* $\varsigma \in (0,1]$. *When* $N \geq 2$, *there exists a* $\hat{\varsigma}$ *such that for* $\varsigma \in (0, \hat{\varsigma}]$, $W^s(T^{SIG}) > W^s(T^{NE})$, *and* $\hat{\varsigma}$ *is increasing in* N. *Furthermore, for any given* $\tilde{\varsigma} \in (0,1)$, *there exists a* $\tilde{N}$ *such that if* $N \geq \tilde{N}$, $U^s(II, T^{SIG}) > U^s(II, T^{NE})$.

Proof: Recall that the symmetric SIG patent length T^{SIG} is characterized by

(H20) $$\frac{\Omega'(T^{SIG})}{\Omega(T^{SIG})} = n\left(N - \Omega(T^{SIG})\left(\frac{z-1}{z\ln z}\right)\frac{1}{1-\varsigma(1-\theta)}\right).$$

Equating (H19) and (H20) yields

(H21) $$(N-1) = \frac{\varsigma(1-\theta)}{1-\varsigma(1-\theta)}\Omega(T^{GO})\left(\frac{z-1}{z\ln z}\right),$$

where T^{GO} is determined by (H19) and is independent of N. Denote the value of ς that solves (H21) by $\hat{\varsigma}$. When $N = 1$, $\hat{\varsigma}$ must equal zero for (H21) to hold. When $N \geq 2$, $\hat{\varsigma} > 0$ and $\partial\hat{\varsigma}/\partial N > 0$. Note that when $\varsigma > \hat{\varsigma}$, it is not necessarily true that $W^s(T^{SIG}) < W^s(T^{NE})$. In this case, it simply involves the comparison of two globally suboptimal levels of patent protection T^{NE} and $T^{SIG}(\varsigma)$.

To show that even Type-II households may benefit from the pharmaceutical lobbies, (42) implies that the symmetric desired patent length for Type-II households is given by

(H22) $$\frac{\Omega'(T_{II}^*)}{\Omega(T_{II}^*)} = n.$$

Equating (H22) and (H18) yields

(H23) $$(N-1) = \Omega(T^{NE})\left(\frac{z-1}{z\ln z}\right).$$

The left-hand side is increasing in *N* while the right-hand side is decreasing is decreasing in *N*. Therefore, there exists a unique *N* for which the Nash-equilibrium patent length coincides with the symmetric desired patent length of Type-II households. When the number of countries exceeds this threshold, even Type-II households would find the Nash-equilibrium patent length too short. Finally, equating (H22) and (H20) yields

(H24) $$(N-1) = \frac{1}{1-\varsigma(1-\theta)}\Omega(T_{II}^*)\left(\frac{z-1}{z\ln z}\right).$$

Denote the value of ς that solves (H24) by $\tilde{\varsigma}$. For any given $\tilde{\varsigma} \in (0,1)$, there exists a $\tilde{N}$ such that (H24) holds. In this case, T^{SIG} coincides with T_{II}^{*}.

Proposition 4: *Suppose that at the symmetric political equilibrium with* $\varsigma^1 = \varsigma^2 = \hat{\varsigma} > 0$*, both countries are better off compared to the symmetric Nash equilibrium. Then, there must exist a* $\breve{\varsigma} \in (0, \hat{\varsigma})$ *such that when* $\varsigma^1 = \hat{\varsigma}$ *and* $\varsigma^2 \in [0, \breve{\varsigma}]$*, country 1 is worse off compared to the symmetric Nash equilibrium.*

Proof: The SIG best response function for country $s \in \{1,2\}$ is

$$\frac{\lambda\Omega(T^s)}{2}\left(\frac{z-1}{z}\right)\beta + (1-\varsigma^s(1-\theta))\beta\left(\frac{\gamma}{1-\phi}\left(\frac{\Omega'(T^s)}{\Omega(T^1)+\Omega(T^2)}\right) - \lambda \ln z\right) = 0. \tag{H25}$$

The best response function shows that T^1 and T^2 are strategic substitutes and $\partial T^s / \partial \varsigma^s > 0$. Therefore, the political-equilibrium pairs of patent length are $T^1\left(\underset{+}{\varsigma^1}, \underset{-}{\varsigma^2}\right)$ and $T^2\left(\underset{-}{\varsigma^1}, \underset{+}{\varsigma^2}\right)$.

Suppose that at the symmetric political equilibrium (T^{SIG}, T^{SIG}) with $\varsigma^1 = \varsigma^2 = \hat{\varsigma} > 0$, both countries are better off compared to the symmetric Nash equilibrium (T^{NE}, T^{NE}). We know from Proposition 3 that such $\hat{\varsigma}$ always exists. At $\varsigma^1 = \hat{\varsigma}$ and $\varsigma^2 = 0$, $T^1 > T^{SIG}$ and $T^2 < T^{NE}$. Therefore, there must exist a $\breve{\varsigma} \in (0, \hat{\varsigma})$ such that when $\varsigma^1 = \hat{\varsigma}$ and $\varsigma^2 = \breve{\varsigma}$, $T^2 = T^{NE}$. At this point, the social best response of country 1 is to set $T^1 = T^{NE}$, but it is setting $T^1 > T^{SIG} > T^{NE}$. Therefore, it must be worse off compared to the symmetric Nash equilibrium. Finally, if country 1 is worse off at $\varsigma^2 = \breve{\varsigma}$, it must also be worse off for $\varsigma^2 \in [0, \breve{\varsigma}]$. □

Appendix I: Tables and Figures for Chapter IV

Table IV-1: Lobbying Expenditures and Campaign Contributions by the Pharmaceutical Industry

(in real 2000 US$ million)

Year	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Lobbying expenditures	72.08	88.58	102.14	97.53	125.26	120.73	128.64	139.62	148.16
Election cycle	**1990**	**1992**	**1994**	**1996**	**1998**	**2000**	**2002**	**2004**	**2006**
Campaign contributions	3.97	9.17	8.54	14.67	13.65	26.69	28.30	16.48	16.64

Data source: opensecrets.org (http://www.opensecrets.org)

Footnotes:

(a) Campaign contributions include direct and soft-money contributions. Soft-money contributions to the national parties were not publicly disclosed until the 1991-92 election cycle.

(b) Soft-money contributions were banned by the Bipartisan Campaign Finance Reform Act following the 2002 elections.

Table IV-2: Extension in Commercial Lifetime of Patents and Market Exclusivity for Drugs

	Number of Years Added
Hatch-Waxman	+2.3 years
PDUFA	+2.1 years
URAA	+1 year
FDAMA	+0.5 year

Source: NIHCM (2000)

Footnotes:

(a) Hatch-Waxman: The Drug Competition and Patent Term Restoration Act of 1984

(b) PDUFA: The Prescription Drug User Fee Act of 1992

(c) URAA: The Uruguay Round Agreements Act of 1994

(d) FDAMA: The Food and Drug Administration Modernization Act of 1997

Table IV-3: Financial Figures of the Top 10 Pharmaceutical Companies in Fortune 500

(in real 2000 US$ million)

Year	**1960-69**	**1970-79**	**1980-89**	**1990-99**	**2000-05**
Sales	13,714	31,878	49,433	100,886	180,901
Pretax Income	2,713	6,051	9,732	23,376	42,000
Intangible Assets	582	1,028	2,978	22,956	65,474

Data source: COMPUSTAT North America

Footnotes:

(a) According to CNNMoney.com, the Top 10 companies in 2006 are: Abbott Laboratories, Amgen, Bristol-Myers Squibb, Eli Lilly, Forest Laboratories, Johnson & Johnson, Merck, Pfizer, Schering-Plough, and Wyeth.

(b) The figures do not include the data for Amgen because it does not have a complete time series for the above years.

(c) For J&J, Eli Lilly, Pfizer, and Schering-Plough, intangible assets include both intangible assets and other assets because intangible assets were included in other assets for some years.

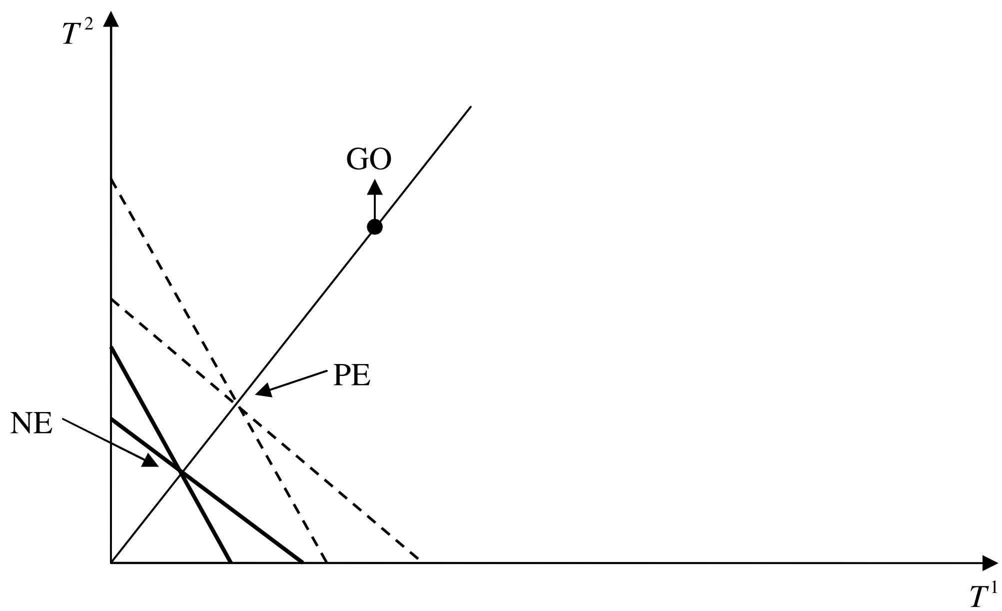
Table IV-4: Private R&D Spending on Phamaceuticals and Medicines

(in real 2000 US$ million)

1981	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**
3,494	3,946	4,445	4,897	4,997	5,135	5,595	6,473	7,015	7,251	8,225
1992	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
9,182	10,332	10,664	11,078	10,410	12,143	13,025	12,502	12,854	9,899	13,614

Data source: National Science Foundation - Division of Science Resources Statistics
Footnote: R&D is net of Federal spending.

Figure IV-1: Nash Equilibrium vs. Political Equilibrium



1. NE refers to the Nash-equilibrium patent length, and the bold lines are the social best response functions of country 1 and country 2.
2. PE refers to the political-equilibrium patent length, and the dotted lines are the SIG best response functions of country 1 and country 2.
3. GO refers to the symmetric globally optimal patent length.

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