Dead Time, Pileup, and Accurate Gamma-Ray Spectrometry

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The accuracy of gamma-ray spectrometric measurements is ultimately limited by the precision of Poisson counting statistics. With careful attention to detail, all other sources of error in the ratio of activities of two sources of a radionuclide in small samples can be made insignificant, even when the statistical limit is well below one percent. An important source of error comes from the finite time required by the counting electronics to detect and process pulses. Dead-time losses (mostly in the analog-digital converter) are usually compensated very well by the pulse-height analyzer, but pileup losses (mostly in the amplifier) may not be. Errors of 10% or more may easily result. Several methods are available for detecting and correcting rate-related losses. These methods are sufficiently reliable and well understood that a decaying source can be measured with acceptably small errors even at count rates as high as tens of thousands per second.

Introduction

We define accuracy in measurement to be the absence of bias. An accurate measurement in gamma-ray spectrometry is indistinguishable from the truth within the bounds of precision set by the statistics of counting. In the following discussion we concentrate on detecting and removing biases which depend on counting rate. We ignore such issues as reproducible counting geometry, self-absorption of gamma rays in the sample, efficiency calibration, cascade summing, and bias in peak integration and other computational issues, all of which must be taken into account in accurate work and which are treated in the literature in standard works.

Despite all the opportunities for bias, radioactivity measurements in practice can be performed so as to make all these errors smaller than the precision of counting statistics, even when that limit is much less than 1%. Since in metrology “knowledge increases…not by the direct perception of truth but by a relentless bias toward the perception of error,” a desirable approach toward accuracy is to search for inaccuracy, devise correction procedures, and validate these procedures under conditions more severe than those encountered in routine practice. The present paper discusses losses due to the finite response time of the gamma-ray spectrometer system.

For additional information on these topics, we refer the reader to earlier review articles and standard texts.

Magnitude of rate-related losses

Significant events may be lost in all parts of the gamma-ray spectrometer system: the germanium detector crystal, preamplifier, amplifier, analog-digital converter (ADC), and multichannel analyzer (MCA) or computer. As an example, if we aim to measure a source of $^{199}$Au with a counting precision of $\sigma = 0.1\%$, then Poisson statistics requires that more than $10^6$ counts be accumulated in the 412-keV gamma-ray peak. If we wish to complete the measurement in an hour’s time, and if the peak/total ratio of the detector is 0.2, we must acquire more than 1400 cps. Even at this modest rate, losses of several percent may occur because of the finite response time of the system. In many applications, the gamma-ray spectrometer is expected (and usually assumed) to respond linearly over a dynamic range from background up to tens of thousands of counts per second. In practice, if the dead-time indicator on the ADC shows 10% then a nonlinearity bias of the order of 10% should be anticipated even if the dead-time correction circuit is working properly.

Ten years ago an intercomparison was organized by the International Atomic Energy Agency to test the adequacy of dead time and pileup corrections. The task of the participants was the simplest metrology: to determine the activity, relative to a reference source counted at 1000 cps, of four radionuclides in each of four radioactive sources ranging up to 15 times the
activity of the reference. Ninety-eight sets of results were submitted by laboratories in 24 countries. The results were sobering: the median error for the most active source was 6%, and the maximum error was 36%.

Mechanisms of rate-related losses
Counts are lost because of the finite response time of the successive components in a counting system: the detector, preamplifier, amplifier, ADC, and MCA or computer. The slowest unbuffered units are the ADC and the amplifier, followed by the detector itself. Most of the charge in the detector is collected in less than 100 ns (large detectors are slower), and shaped in the amplifier during the next 1-10 \( \mu s \). ADC dead time is relatively less important than it was a decade ago. For example, a modern successive-approximation ADC in our laboratories requires 6 \( \mu s \) to digitize a pulse in the middle of an 8192-channel spectrum, compared with 60 \( \mu s \) for the Wilkinson device which it replaced; the ADC and amplifier dead times are thus comparable.

All multichannel analyzers compensate for ADC dead time by gating the livetime clock off during the conversion; compensation can be essentially perfect. Pulse-pileup (also called random summing) losses, which may or may not be compensated, can be an equally important source of inaccuracy. The accidental arrival of a signal from two unrelated gamma rays at the amplifier within a time interval short compared to the shaping time results in a composite pulse whose amplitude is the sum of the two events. As a result two counts are lost from the low-energy portion of the spectrum and one is added at high energy. For accurate spectrometry, each such random-sum pulse should be excluded from the spectrum (pileup rejection), and the system dead time should be adjusted to compensate for the time the system is busy inspecting and rejecting this event (pileup live-time correction).

Figure 1 shows the influence of pileup on the shape of the spectrum, and the improvement gained by the use of a pileup rejector signal from the amplifier to veto questionable events.

Theory of count rate losses: software corrections

Case 1: Pileup only
Pileup in the amplifier is the classical case of extending dead time. That is, the arrival of a second pulse on the tail of the first increases the time that the counting system is busy. Since the ADC live time is well compensated by the circuitry, we may approximate any additional losses as if pileup alone is the dominant effect, or equivalently the losses have the same mathematical dependence on count rate. This assumption will be examined and justified in what follows. If the counting rate is \( I_0 \) into the amplifier, then the output rate \( I \) is

\[
I = I_0 e^{-k \cdot I_0}
\]

where \( k \) is a constant. Wyttenbach expressed this in a particularly simple way. Using the ADC dead time as a measure of the input rate \( I_0 \), he gave an expression for the pileup factor which can be rewritten as

\[
f_p \equiv \frac{I_0}{I} = e^{P \cdot DT/LT}
\]

where:

- \( LT = \) live time of the measurement interval, s
- \( DT = \) macroscopic dead time = \( CT - LT \), s
- \( CT = \) clock time, s
- \( P = \) pileup constant, of order unity.

The pileup constant \( P \) is conveniently measured by an extension of the classical two-source method of determining detector dead times. A reference source such as \( ^{60}\text{Co} \) of moderate activity is counted several times in the presence of large, varying amounts of another source which does not contain this radionuclide. The strong source may be \( ^{137}\text{Cs} \) or

\[
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\]

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(better) a mixed source of the matrix to be analyzed. For this
calibration the range of dead times should bracket that expected for samples.

Taking logarithms of equation (2) and rearranging, we have for each measurement of the photopeak counting rate \( I \) of the reference source

\[
\ln I = \ln I_0 - P \frac{DT}{LT}
\]

which is a linear function of the two unknowns \( \ln I_0 \) (the peak counting rate of the reference source at zero dead time) and the pileup constant \( P \). For three or more observations the equations may be solved by weighted least squares to determine \( P \) and its uncertainty. This formulation is used for pileup correction in a number of commercial software packages.

Since pileup is a consequence of the finite amplifier resolving time, the pileup correction should become less important as the shaping time constant \( \tau \) of the amplifier is reduced. As can be seen in Figure 2, this is observed experimentally. For each value of \( \tau \), the dependence of \( f_P \) on dead time is approximately exponential (strictly, an exponential in \( DT/LT \), while fractional dead time is conventionally \( DT/CT \)). Wytenbach’s derivation implies that the pileup constant \( P \) should be proportional to \( \tau \). This is a good approximation as long as \( \tau \) is small compared with the ADC digitization time, as shown in Figure 3. Clearly, the smaller the time constant the smaller the pileup correction and the smaller its uncertainty. There is a minimum value of \( \tau \), determined by the time required for charge collection from the volume of the detector, below which the resolution may not be acceptable. For the detector used in these measurements the peaks were broadened below \( \tau = 2 \) µs, but even for the shortest \( \tau \) the model of equation (2) describes the data well.

An estimate of the uncertainty in the pileup correction factor \( f_P \) is obtained by conventional error propagation. The uncertainty in \( P \) is obtained as part of the calibration process just described. Multichannel analyzers record the elapsed live times (LT) and clock times (CT) with a fixed time resolution \( \delta \) (typically 0.01 or 1 s) so at least one of these times and their difference \( DT \) has an uncertainty of \( \pm \delta \). Treating this uncertainty as a formal standard deviation, and assuming that LT is known exactly, we have the relative uncertainty in the pileup factor

\[
\frac{\sigma_{f_P}}{f_P} = \left( \frac{DT}{LT} \sigma_P \right)^2 + \left( \frac{P}{LT} \delta \right)^2 \right)^{\frac{1}{2}}
\]

The advent of successive-approximation ADCs, with a constant digitization time for any pulse height, has removed the small dependence on spectrum shape observed with Wilkinson ADCs. With this pileup correction, the accuracy in measuring relative counting rates can be as good as a few hundredths of a percent.

**Case 2: Pileup, dead time, and decay**

We treat in detail the case of a single radionuclide with decay constant \( \lambda \) giving a time-dependent input count rate \( I_0(t) \), with
an extending dead time $\alpha$ in the detector-amplifier combination, followed by a non-extending dead time $\Theta$ in the ADC to deliver an output count rate $I'$ to the memory.\textsuperscript{12,17} The model is given schematically in Figure 4. Here $I_0$, $I$, and $I'$ are the instantaneous event rates into the detector, out of the amplifier, and out of the ADC, respectively. The relations among these rates are given by\textsuperscript{10,13}

\[
I_0(t) = I_0(0)e^{-\lambda t} \quad \text{[decay]} \quad (5)
\]

\[
I(t) = I_0(t)e^{-\alpha t} I_d(t) \quad \text{[extending : pileup]} \quad (6)
\]

\[
I'(t) = \frac{I(t)}{1 + \Theta I(t)} \quad \text{[non-extending : ADC]} \quad (7)
\]

The last equation cannot be strictly correct, since its derivation presupposes a Poisson interval distribution,\textsuperscript{7,18} whereas the output pulses from the amplifier cannot have spacing shorter than $\alpha$. Experiment will show how important this assumption is.

The number of counts $C$ recorded in clock time $CT$ beginning at decay time $t_1$ is

\[
C = \int_{t_1}^{t_1+CT} I(t)dt = \int_{t_1}^{t_1+CT} I_0(0)e^{-\lambda t_1} e^{-\alpha t I_d(t)} e^{-\lambda t} dt \quad (8)
\]

We assume that the pulse pileup and ADC dead-time effects are approximately separable; that is, the effect of pileup on dead time is small. This is valid for some value of $\alpha$ if decay is moderate during the counting interval. This assumption gives

\[
C \equiv e^{-\alpha I_d(0)\exp(-\lambda t_1)} \int_{t_1}^{t_1+CT} I_0(0)e^{\lambda t} e^{\alpha I_d(t)} dt \quad (9)
\]

The integral can now be carried out and the experimental quantities grouped together to give

\[
A(t_1) = \frac{\lambda I_0}{1 - e^{-\lambda \cdot DT}} \left[ e^{\frac{\lambda}{\lambda \cdot DT} - 1} \right] = I_0(0)e^{\alpha I_d(0)\exp(-\lambda t_1)} \quad (10)
\]

where as before the macroscopic dead time $DT = CT \cdot LT$. Thus the slope of a plot of $\ln A(t_1)$ vs $\exp(-\lambda t_1)$ will yield the pileup parameter $\alpha$. This model has been tested\textsuperscript{17} with a decaying source of $^{64}$Cu, and shown (Figure 5) to be valid to better than 0.3\% over at least six half-lives.

Once the value of $\alpha$ has been determined for a particular nuclide and counting system, the pileup correction factor $f_p$ by which to multiply the observed decay-corrected counting rate $A_0$ may be determined. That is

\[
A_{0,\text{true}} = f_p \cdot A_{0,\text{obs}} \quad (11)
\]

We recognize that for the general case for which $I_0(0)$ is not known, the pileup factor is given by

\[
f_p = e^{\alpha I_d(0)\exp(-\lambda t_1)} \quad (12)
\]

The solution to this equation\textsuperscript{12} is, by iteration,

\[
f_p = \exp(A(t_1)e^{-\lambda t_1}f_p \exp(A(t_1)e^{-\lambda t_1}f_p(...))) \quad (13)
\]

which quickly converges when solved numerically.

A stringent test of the model was the assay of the former National Bureau of Standards (now the National Institute of Standards and Technology) Standard Reference Material l633a Fly Ash by assaying aluminum via 2.240-min $^{28}$Al, the major radioactivity present for many minutes after a short reactor irradiation of this material.\textsuperscript{20} In this work 0.2\% precision was attained, and from the assay of single-crystal $\text{Al}_2\text{O}_3$ an accuracy of 0.3\% was inferred, even though the pileup correction in a tuned counting system was as high as 46\%.

**Hardware solutions**

The time-honored method for counting loss corrections is to include a pulser with a known repetition rate in the spectrum.\textsuperscript{21,23} Eight of the best ten laboratories in the International Atomic Energy Agency (IAEA) intercomparison used the pulser method. The correction factor for dead time and pileup is simply the ratio of the pulser rate with and without the sample. The precision of the correction is determined by the number of events missing from the periodic pulser peak.\textsuperscript{24} A minor perturbation on the correction arises from the fact that pulses from a periodic pulser cannot pile up with each other as detector pulses can.\textsuperscript{6,25,26} A random pulser may be used, but the precision is not as good as with a periodic pulser at the same rate. Another difficulty is that typical Ge detector pulses from the preamplifier rise rapidly ($\tau \approx 10$ ns) and fall slowly (usually $\tau = 50$ $\mu$s). Since most inexpensive commercial pulsers rise slower and decay faster, the pole-zero cancellation in the preamplifier and main amplifier cannot be made correct for both kinds of pulses. The resulting mismatch may impair the detector’s resolution by several tenths of a keV. Research-grade pulsers can be purchased which do not greatly degrade the detector signal; special pulsers optimized for gamma-ray spectrometry have been constructed.\textsuperscript{23,27} Excellent frequency stability can be obtained by slaving the pulser to a reference frequency generator, which can be obtained as stable as one part per million per day for a few hundred dollars.

Active mixtures of rapidly decaying sources cannot be meas-
ured quantitatively even with a pulser, unless the dead time as a function of time is known\textsuperscript{28,29} or the pulser rate is made proportional to the input counting rate.\textsuperscript{22,30,31} In another approach, the dead time is stabilized at a constant value,\textsuperscript{32,33} which makes the efficiency independent of the counting rate up to a certain limit by decreasing the throughput.

As mentioned earlier, some commercial amplifiers produce logic signals to reject piled-up pulses, thereby removing spurious peaks and continuum from the spectrum (Figure 1). Since this cleanup action requires time, a correction signal is sent from the amplifier to the ADC clock to extend the live time appropriately. Considerable improvement is achieved in spectral quality and quantitative accuracy of the counting rate. However, because a wide bandwidth is needed to detect closely spaced pulse pairs, the pileup detection circuit may be sensitive to high-frequency noise, for example from a nearby switching power supply, and thus overcompensate by increasing the system dead time too much. A good test is to perform a software pileup calibration as described in equation (3): if $P$ is near zero then the rejection and correction logic is operating properly.

Westphal,\textsuperscript{34} building on earlier work,\textsuperscript{35,36} perfected a method for correcting counting losses by storing compensating pulses in the MCA on a time scale comparable to the inter-pulse interval. This technique is called loss-free counting (LFC). The fraction of the counting time that the system is busy and unable to receive an incoming pulse, either because of preamplifier reset, amplifier pulse shaping, pileup rejection, or ADC digitization time, is directly measured on a sub-millisecond time scale, most successfully by the use of a virtual pulser. Based on this instantaneous probability, a number greater than or equal to one is added to the memory address computed by the ADC. Because the probability that a count at time $t$ will be stored in channel $N$ is the same as the probability at a very short time later $t + \delta$, the shape of the accumulated spectrum at any time in the counting interval is indistinguishable from the spectrum that would have been acquired with an infinitely fast counting system. The live time is equal to the clock time, so that the kinetic equations for a set of several decaying radionuclides can be solved without additional information. In addition to its use in measuring time-varying count rates, the LFC technology extends the usable dynamic range of the counting system to rates well over 50 kHz. The fundamental correctness of what at first sight seems like something for nothing has been verified by Monte Carlo calculations\textsuperscript{37} and by experiment.\textsuperscript{36} The number of counts in each corrected channel, however, no longer follows a Poisson distribution ($\sigma < \sqrt{n}$), so that for proper error analysis two spectra must be recorded simultaneously: LFC-corrected for peak areas and uncompensated for statistical error estimation. Such equipment and the appropriate software are commercially available.

A particularly powerful means of analyzing signals, which is beginning to be applied to nuclear spectroscopy,\textsuperscript{38} is to use digital methods as early as possible in the signal processing chain. Consider the output signal from a transistor-reset preamplifier, the voltage-time behavior of which resembles a staircase. All the information about the charge deposited by the gamma ray in the detector, and thus the energy of the gamma ray, is contained in the height of the step. The best possible measurement of that height is obtained by observing the shape of the step for the entire period after the end of the pulse

Figure 5  Experimental test of the model of Figure 4 and equation (10), using a decaying source of $^{64}$Cu. The model fits the data within the experimental uncertainty (1$\sigma$ error bars range from 0.06% to 0.25%).

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preceding the pulse of interest, and comparing it with the step preceding the next pulse after the one of interest. No analog amplifier can analyze randomly spaced signals so well; only by continuously digitizing the signal can one choose the pre-pulse baseline measurement time optimally. Furthermore, the quality of the spectrum can be improved by inspecting the leading-edge shape of the event; for instance, a multiple step implies pulse pileup. Rejection of slow-rising pulses has been shown to improve the low-energy background and peak/Compton ratio as much as fivefold. This approach to signal analysis requires fast digital processors and large data storage devices, as a simple calculation shows. If we want to acquire data at a modest 10,000 cps, then digitizing the signal with 8192-channel resolution at an average sampling frequency of 100 points per event would require a digitizing time of less than 1 µs (equivalent to an 8-GHz Wilkinson ADC) and generate 1.5 Mbytes of data per second, which for sustained operation must be processed as rapidly as it is generated. Because of recent advances in computer technology, such data rates are now becoming manageable.

Results
The adequacy of the dead-time and pileup corrections by the methods described above has been demonstrated. In the IAEA intercomparison the best results were obtained with the pulser method. In this exercise at NIST, both the modified Wyttenbach software correction (equation 2) and a commercial pileup rejection—live-time correction module (Canberra 1468) independently produced 0.3% precision and accuracy. In a nuclear safeguards application of gamma-ray spectrometry, the modified Wyttenbach pileup correction (equation 2) was used to obtain an accuracy of order 0.02-0.03% in assaying 235U even though the pileup correction was as high as 30%. Even with a rapidly decaying source an accuracy of 0.3% has been reached with the a formalism, as mentioned above. Loss-free counting has been proven in Vienna, and is being applied successfully elsewhere. These measures have been successfully applied to the analysis of reference materials and other samples in routine work at NIST.

Summary
Data losses in gamma-ray spectrometry due to high counting rates can be substantial. Hardware and software methods are available, however, to eliminate or correct these losses with excellent accuracy.

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Note
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References

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