ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

REPORT NO. 5

MAGNETICALLY SENSITIVE ELECTRICAL RESISTOR MATERIAL

October 1, 1954 to February 28, 1955

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Project 2136

DEPARTMENT OF THE ARMY
LABORATORY PROCUREMENT OFFICE
U.S. SIGNAL CORPS SUPPLY AGENCY
CONTRACT DA-36-039-SC-52601
FORT MONMOUTH, NEW JERSEY

March 1955

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ABSTRACT

Theory

Tables are given for the coefficients of the quadratic magneto resistance coefficients B in terms of the bracket symbols for the various crystal symmetries. The progress made so far, and what remains to be done, is surveyed (see Figure 1).

Experiment

Some improvements in our measuring equipment are mentioned. It was found that aging of Bi wires seems to produce a marked increase (~25 percent) in $\Delta\rho/\rho$. Measurements of $\Delta\rho/\rho$ for all directions of H were analyzed in terms of an ellipsoid which fitted the data very well, as predicted by theory. It was found that the maximum of $\Delta\rho/\rho$ lies in a direction almost (but in general, not exactly) normal to the wire.

OBJECTIVE

This project aims at developing the understanding of the magneto resistance effect (change of electrical resistivity in a magnetic field) by theoretical and experimental research, with the ultimate aim of developing materials with more favorable magneto resistance properties than are available at present.

A. INTRODUCTION

This is the fifth report on work done by L. P. Kao and E. Katz (theory) and by H. Patterson and W. Tantraporn (experiment) on the magneto resistance effect. The sections were integrated and edited by E. Katz.

At the time of writing of this report several phases of the work were in a stage which required further rounding off. Consequently, they will be reported on in the future. The present report then contains only a brief indication of the general direction.

In order to place the theoretical work of previous reports in proper perspective with the general plan followed, the section B.I was written.

B. THEORETICAL PART

I. PLAN OF WORK

From time to time it is useful to recall the general plan of the theoretical work and to determine which problems have been solved fully or partially and which ones are to be attacked next.

The aim of the theoretical work is to develop the theory of the conductivity $\sigma(\underline{H})$ of a wire* in a magnetic field $\underline{H}$. The work can be divided into one part which is phenomenological and another involving the theory of electrons. Each of these can be broken down into a number of steps, whose relations are surveyed in the bloc diagram shown in Figure 1.

1. Phenomenological Part of the Theory.—The purpose of the phenomenological theory is to interpret the measured change in resistance in terms of experimental parameters and of material constants. Most of the work presented in this and previous reports belongs to this part of the theory. It

*$\sigma(\underline{H})$ is defined as the measured ratio of the current density through the wire and the voltage gradient between the terminals.

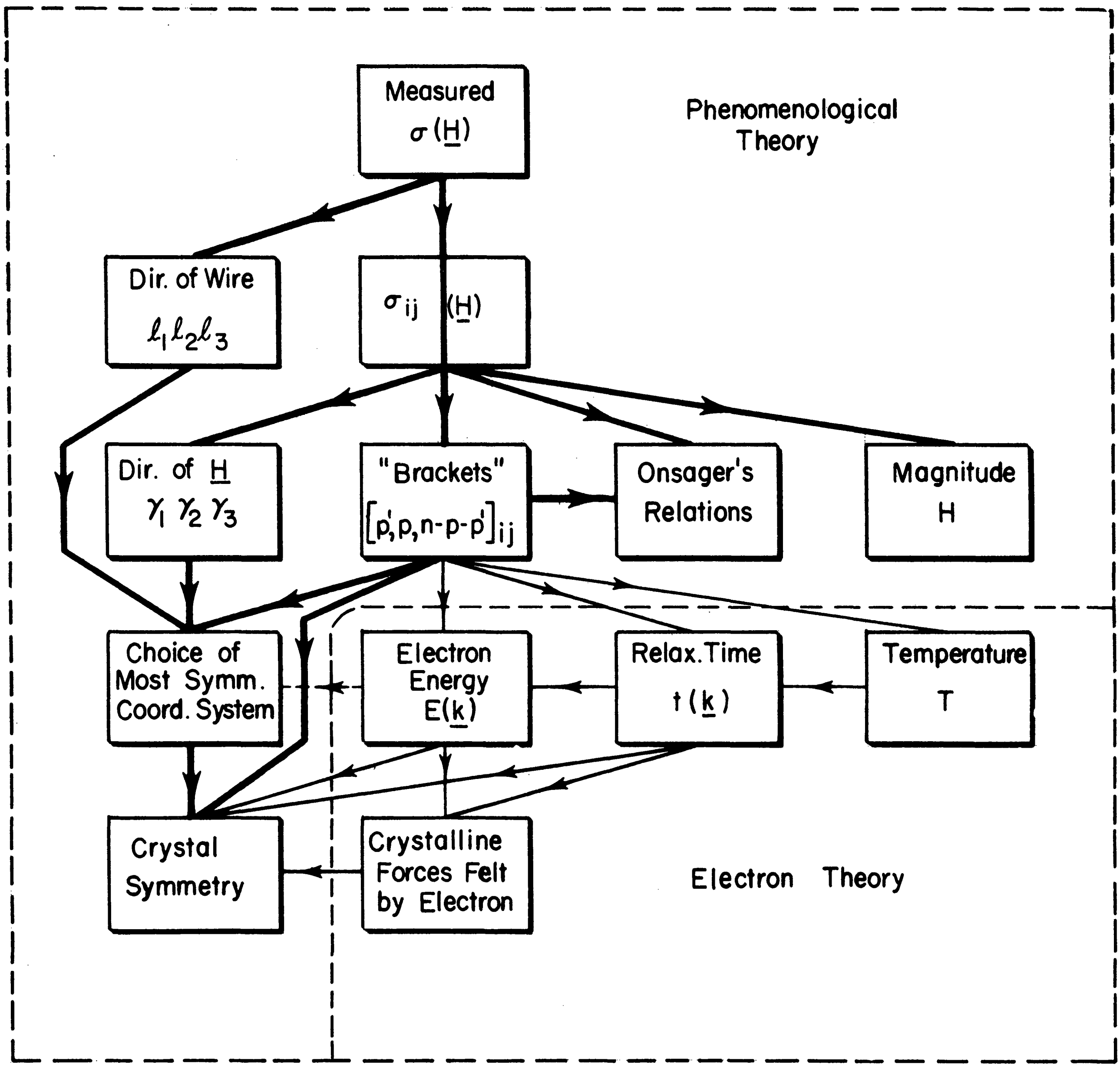

Figure 1. Bloc diagram of logical relations between various factors determining magneto resistance. Heavy lines represent relations whose analysis has essentially been given. Arrows show dependence of one bloc on another.

is concerned with the dependence of the magneto resistance on four sets of experimental parameters; namely, on

(1) the question to which of the 32 crystallographic point groups the sample belongs,

(2) the orientation of the wire sample with respect to the crystal axes,

(3) the orientation of the magnetic field with respect to the crystal axes, and

(4) the strength of the magnetic field.

In addition, the magneto resistance depends on material constants, called "brackets."

The present state of our work on these dependencies will now briefly be described.

(a) The measured conductivity $\sigma(\underline{H})$ depends on the six conductivity tensor components $\sigma_{ij}(\underline{H})$ and on the orientation of the wire with respect to the symmetry axes of the crystal. The orientation of the wire is specified by its direction cosines ℓ_1, ℓ_2, ℓ_3 with respect to an orthogonal set of coordinate axes, chosen most symmetrically with respect to the crystal symmetry axes. The dependence of $\sigma(\underline{H})$ on the $\sigma_{ij}(\underline{H})$ and on the ℓ_1 ℓ_2 ℓ_3 was given in crude form in the equations 24-27 of the second quarterly report (March 1954) and in more finished form in equations (3.3.6) and (3.3.7). These steps are completed.

(b) Each of the $\sigma_{ij}(\underline{H})$ depends on the magnetic field $\underline{H}$ and can be expanded in a power series in powers of the three components $H_i = \gamma_i H$ $(i = 1,2,3)$ along the coordinate axes:

$$\sigma_{ij}(\underline{H}) = \sum_{n=o}^{\infty} H^n \sum_{p=o}^{n} \sum_{p'=o}^{n-p} [p', p, n-p-p']_{ij} \gamma_1^{p'} \gamma_2^{p} \gamma_3^{n-p-p'} \qquad (3.3.8)$$

The coefficients of this expansion, which have been called "brackets" throughout the previous work, are sums of components of tensors of rank 2n+2 and are consequently subject to certain restrictions imposed by the symmetry of the crystal and by Onsager's relation*.

*In a previous report the bracket relation $[\,]_{ij} = (-)^n [\,]_{ij}$ was derived on the basis of the extended (electron) theory of Davis, A method was developed for finding these restrictions. The results are tabulated up to n=4 in Table VI of Report No. 4 (November 1954) for crystals belonging to any of the 32 possible symmetry groups known as crystallographic point groups.

From a phenomenological standpoint the brackets are the basic material constants.

While the basic work of step (b) has been completed, a number of details require further elaboration. For example, the general formula (4.6.6), which is the foundation for obtaining the symmetry restrictions for the brackets, can be stated in terms of a number of simple rules for certain crystal classes. In addition, formulas similar to (4.6.10), expressing the number of independent brackets as a function of n, can be obtained for crystal symmetries other than C'_{2h}. The presentation of these topics is deferred until they have been worked out more completely.

(c) The results of steps (a) and (b) can be synthesized. The conductivity $\sigma(\underline{H})$ can be expanded directly in a power series in terms of the three components of the magnetic field:

$$\sigma(\underline{H}) = \sum_{n=o}^{\infty} H^n \sum_{p=o}^{n} \sum_{p'=o}^{n-p} \sigma(p',p,n-p-p')\, \gamma_1^{p'} \gamma_2^{p} \gamma_3^{n-p-p'} \qquad (5.1.1)$$

In section 4 of Quarterly Report No. 3, it was proved that only even powers of H occur in the series (5.1.1). The results of steps (a) and (b) show that the coefficients $\sigma()$ depend only on the brackets and quadratically on the direction cosines $\ell_1\ \ell_2\ \ell_3$ of the

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by integrating the expressions for the brackets by parts. However, it is easy to show that this relation holds as a direct consequence of the Onsager relation $\sigma_{ij}(H) = \sigma_{ij}(-H)$ and hence can be placed in the phenomenological part of the theory. A consequence of this way of looking at the above bracket relation is that the relaxation time in the electron theory must be an even function of the wave vector $\underline{k}$.

Admittedly, the existence of a relaxation time is at present the weakest of all assumptions in the electron theory of all conduction phenomena including magneto resistance. Whatever improved formulation of the brackets in terms of electronic concepts will evolve in the future, the results derived in the phenomenological theory, using only symmetry and Onsager's relation, will remain valid.

wire. Their explicit form in terms of these quantities must be found.

The quantity measuring the magneto resistance effect is

$$\frac{\Delta\rho}{\rho} = -\frac{\Delta\sigma}{\sigma_0} .$$

It can likewise be expanded. Its expansion differs from (5.1.1) only by the omission of the H^0 term and through multiplication by $-1/\sigma_0$. Hence, step (c) completes the phenomenological part of the theory. It permits the expression of the magneto resistance in terms of the experimental parameters (1), (2), (3), and (4) and the material constants—the brackets—for which the effects of crystal symmetry have been taken into account.

The actual writing of the explicit form of the coefficients $\sigma_{()}$ in equation (5.1.1) is presently under study, and the first results, the explicit forms of the coefficients for n=2, are presented for various crystal symmetries in section 2 of this report. The corresponding magneto resistance has the form,

$$\frac{\Delta\rho}{\rho} = B_{k\ell}\, H_k\, H_\ell = B_{k\ell}\, \gamma_k\, \gamma_\ell\, H^2 * \qquad (5.1.2)$$

The phenomenological work done so far, and that planned for the future, gives insight of limited depth into the magneto resistance effect. It has the advantage of making use of very general principles only. Consequently, it is safe to state that its value will have a high degree of permanence.

2. Electron Theoretical Part.—The purpose of this part of the theory is to interpret the brackets in terms of electronic events and atomic constants, and vice versa. Mainly, two approaches suggest themselves.

(1) The theory of Davis-Wilson yields an interpretation as desired for a limited number of cases to which it can be applied. This theory, which is based on the Boltzmann equation, the existence of a "relaxation time," and a number of other assumptions and restrictions, can be generalized. In Quarterly Report No. 2 the assumptions and restrictions made by Davis were discussed. It was shown that five restrictions could be broadened. This

*The summation over k and ℓ is understood in the usual manner, so that the coefficient of $\gamma_1\, \gamma_2$, for example, is $B_{12} + B_{21} = 2\, B_{12}$.

led to equations (21) or (3.3.9), which expresses the brackets in terms of the derivatives of the electron energy E and of the relaxation time t with respect to the wave number vector $\underline{k}$.

At this point further progress becomes difficult because, in general, very little is known about the actual form of the functions $E(\underline{k})$ and $t(\underline{k})$ in any particular case, apart from their symmetry. The effects on the brackets of a number of choices for $E(\underline{k})$ and $t(\underline{k})$, representing one or two bands, are studied now.

In a number of special cases it was shown that a necessary and sufficient condition for vanishing magneto resistance could be found. This situation has been generalized and will be reported when it has reached the necessary completion.

Even attempts at an evaluation of the relative magnitudes of various brackets in general have, so far, been unsuccessful.

The usual interpretation of equation (3.3.9) is restricted to low temperatures. Preliminary attempts have been made at obtaining the general temperature dependence in more explicit form. All these matters will be reported if and when they reach a more rounded form.

(2) Another line of approach, less exact but more easily visualized than the generalization of the theory of Davis and Wilson, is now under study.

It is well known that an exact classical treatment of the magneto resistance effect leads to a prediction of zero magneto resistance. It is not difficult to show that this result is valid, even if the electron mass is taken as a tensor of second rank. Work is in progress to investigate the magneto resistance that would follow from a velocity dependent mass. If this approach yields a nonvanishing magneto resistance, much gain may be expected for the inverse problem: the drawing of inferences regarding the functions $E(\underline{k})$ and $t(\underline{k})$ from experimentally determined bracket values. This will be reported as soon as results warrant.

II. THE EXPLICIT FORM OF $B_{k\ell}$

Finding the explicit form of the coefficients $B_{k\ell}$ in equation (5.1.2) is straightforward but lengthy. The results differ for the various crystal classes, since the relations between the brackets differ. In Table I the expressions for $B_{k\ell}$ are listed for the classes C_{4h}, C_{6h}, D_{2h}, D_{3i}, D_{4h}, D_{6h}, T_h, and O_h. The expressions for the three classes of lowest symmetry (S_2, C_{2h}, S_6) are very lengthy and contain practically no actual materials of interest for magneto resistance work. Consequently, the expressions for C_{2h} and S_6 have been omitted from the tables, although we have them on file. The case S_2 was of prohibitive length and has not been calculated.

With these eleven classes possessing a center of symmetry, all 32 crystallographic point groups are covered, as was discussed in section 7 of the third quarterly report (July, 1954).

TABLE I

THE EXPLICIT FORM OF $B_{k\ell}$ FOR THE CLASSES

$\underline{C_{4h}, C_{6h}, D_{4h}, D_{6h}}$ (terms with * are zero).

$$D = [000]_{11}\left\{[000]_{33}+\ell_3^2([000]_{11}-[000]_{33})\right\}$$

$$\begin{aligned}-B_{11}D = {} & \ell_1^2([200]_{11}[000]_{33}+[100]_{31}^2{}^*) \\ & +\ell_2^2([200]_{22}[000]_{33}+[100]_{23}^2) \\ & +\ell_3^2([200]_{33}[000]_{11}+[100]_{23}^2+[100]_{31}^2{}^*)[000]_{11}/[000]_{33} \\ & +2\ell_1\ell_2([200]_{12}[000]_{33}-[100]_{23}[100]_{31})^*\end{aligned}$$

$$\begin{aligned}-B_{22}D = {} & \ell_1^2([200]_{22}[000]_{33}+[100]_{23}^2) \\ & +\ell_2^2([200]_{11}[000]_{33}+[100]_{31}^2{}^*) \\ & +\ell_3^2([200]_{33}[000]_{11}+[100]_{23}^2+[100]_{31}^2{}^*)[000]_{11}/[000]_{33} \\ & -2\ell_1\ell_2[200]_{12}[000]_{33}-[100]_{23}[100]_{31})^*\end{aligned}$$

$$\begin{aligned}-B_{33}D = {} & [000]_{33}([002]_{11}+[001]_{12}^2/[000]_{11}) \\ & -\ell_3^2([002]_{11}[000]_{33}-[002]_{33}[000]_{11}^2/[000]_{33}+[001]_{12}^2[000]_{33}/[000]_{11})\end{aligned}$$

$$\begin{aligned}-(B_{23}+B_{23})D = {} & 2\ell_2\ell_3([011]_{23}[000]_{11}-[100]_{23}[001]_{12}) \\ & +2\ell_1\ell_3([011]_{31}[000]_{11}+[100]_{31}[001]_{12})^*\end{aligned}$$

$$\begin{aligned}-(B_{31}+B_{13})D = {} & 2\ell_2\ell_3(-[011]_{31}[000]_{11}-[100]_{31}[001]_{12})^* \\ & +2\ell_1\ell_3([011]_{23}[000]_{11}-[100]_{23}[001]_{12})\end{aligned}$$

$$\left.\begin{aligned}-(B_{12}+B_{21})D = {} & (\ell^2-\ell_2^2)([110]_{11}[000]_{33}+2[100]_{23}[100]_{31})^* \\ & +2\ell_1\ell_2([110]_{12}[000]_{33}-[100]_{23}^2+[100]_{31}^2{}^*)\end{aligned}\right\} C_{4h}, D_{4h}$$

$$\left.\begin{aligned}-(B_{12}+B_{21})D = {} & 2(\ell_1^2-\ell_2^2)(-[200]_{12}[000]_{33}+[100]_{23}[100]_{31})^* \\ & +2\ell_1\ell_2\left\{([200]_{11}-[200]_{22})[000]_{33}-[100]_{23}^2+[100]_{31}^2{}^*\right\}\end{aligned}\right\} C_{6h}, D_{6h}.$$

TABLE I, Continued

THE EXPLICIT FORM OF $B_{k\ell}$ FOR THE CLASS

$\underline{D_{2h}}$

$$D = \ell_1^2[000]_{22}[000]_{33} + \ell_2^2[000]_{33}[000]_{11} + \ell_3^2[000]_{11}[000]_{22}$$

$$\begin{aligned}-B_{11}D = {} & \ell_1^2[200]_{11}[000]_{22}[000]_{33}/[000]_{11} \\ & +\ell_2^2([200]_{22}[000]_{33} + [100]_{23}^2)[000]_{11}/[000]_{22} \\ & +\ell_3^2([200]_{33}[000]_{22} + [100]_{23}^2)[000]_{11}/[000]_{33}\end{aligned}$$

$$\begin{aligned}-B_{22}D = {} & \ell_1^2([020]_{11}[000]_{33} + [010]_{31}^2)[000]_{22}/[000]_{11} \\ & +\ell_2^2[020]_{22}[000]_{11}[000]_{33}/[000]_{22} \\ & +\ell_3^2([020]_{33}[000]_{11} + [010]_{31}^2)[000]_{22}/[000]_{33}\end{aligned}$$

$$\begin{aligned}-B_{33}D = {} & \ell_1^2([002]_{11}[000]_{22} + [001]_{12}^2)[000]_{33}/[000]_{11} \\ & +\ell_2^2([002]_{22}[000]_{11} + [001]_{12}^2)[000]_{33}/[000]_{22} \\ & +\ell_3^2[002]_{33}[000]_{11}[000]_{22}/[000]_{33}\end{aligned}$$

$$-(B_{23}+B_{32})D = 2\ell_2\ell_3([011]_{23}[000]_{11} - [010]_{31}[001]_{12})$$

$$-(B_{31}+B_{13})D = 2\ell_3\ell_1([101]_{31}[000]_{22} - [100]_{23}[001]_{12})$$

$$-(B_{12}+B_{21})D = 2\ell_1\ell_2([110]_{12}[000]_{33} - [100]_{23}[010]_{31})$$

TABLE I, Continued

THE EXPLICIT FORM OF $B_{k\ell}$ FOR THE CLASS

$\underline{D_{3i}}$

$$D = [000]_{11}\left\{[000]_{33} + \ell_3^2([000]_{11} - [000]_{33})\right\}$$

$$\begin{aligned}-B_{11}D = {} & \ell_1^2[200]_{11}[000]_{33} \\ & +\ell_2^2([200]_{22}[000]_{33} + [100]_{23}^2) \\ & +\ell_3^2([200]_{33}[000]_{11} + [100]_{23}^2)[000]_{11}/[000]_{33} \\ & +2\ell_2\ell_3[200]_{23}[000]_{11}\end{aligned}$$

$$\begin{aligned}-B_{22}D = {} & \ell_1^2([200]_{22}[000]_{33} + [100]_{23}^2) \\ & +\ell_2^2[200]_{11}[000]_{33} \\ & +\ell_3^2([200]_{33}[000]_{11} + [100]_{23}^2)[000]_{11}/[000]_{33}\end{aligned}$$

$$\begin{aligned}-B_{33}D = {} & [000]_{33}([002]_{11} + [001]_{12}^2/[000]_{11}) \\ & -\ell_3^2([002]_{11}[000]_{33} - [002]_{33}[000]_{11}^2/[000]_{33} + [001]_{12}^2[000]_{33}/[000]_{11})\end{aligned}$$

$$\begin{aligned}-(B_{23}+B_{32})D = {} & (\ell_1^2-\ell_2^2)\ [011]_{11}[000]_{33} \\ & +2\ell_2\ell_3([011]_{23}[000]_{11} - [100]_{23}[001]_{12})\end{aligned}$$

$$\begin{aligned}-(B_{31}+B_{13})D = {} & 2\ell_3\ell_1([011]_{23}[000]_{11} - [100]_{23}[001]_{12}) \\ & +2\ell_1\ell_2[011]_{11}[000]_{33}\end{aligned}$$

$$\begin{aligned}-(B_{12}+B_{21})D = {} & 4\ell_3\ell_1[200]_{23}[000]_{11} \\ & +2\ell_1\ell_2\left\{([200]_{11} - [200]_{22})[000]_{33} - [100]_{23}^2\right\}\end{aligned}$$

TABLE I, Concluded

THE EXPLICIT FORM OF $B_{k\ell}$ FOR THE CLASSES:

<u>T_h, O_h</u>

For O_h take $[020]_{11} = [002]_{11}$ (see * marks)

$$D = [000]_{11}^2$$

$$\begin{aligned}-B_{11}D = {} & \ell_1^2[200]_{11}[000]_{11} \\ & +\ell_2^2([002]_{11}[000]_{11} + [100]_{23}^2) \\ & +\ell_3^2([020]_{11}^*[000]_{11} + [100]_{23}^2)\end{aligned}$$

$$\begin{aligned}-B_{22}D = {} & \ell_1^2([020]_{11}^*[000]_{11} + [100]_{23}^2) \\ & +\ell_2^2[200]_{11}[000]_{11} \\ & +\ell_3^2([002]_{11}[000]_{11} + [100]_{23}^2)\end{aligned}$$

$$\begin{aligned}-B_{33}D = {} & \ell_1^2([002]_{11}[000]_{11} + [100]_{23}^2) \\ & +\ell_2^2([020]_{11}^*[000]_{11} + [100]_{23}^2) \\ & +\ell_3^2[200]_{11}[000]_{11}\end{aligned}$$

$$-(B_{23}+B_{32})D = 2\ell_2\ell_3([011]_{23}[000]_{11} - [100]_{23}^2)$$

$$-(B_{31}+B_{13})D = 2\ell_3\ell_1([011]_{23}[000]_{11} - [100]_{23}^2)$$

$$-(B_{12}+B_{21})D = 2\ell_1\ell_2([011]_{23}[000]_{11} - [100]_{23}^2)$$

III. CONCLUSIONS AND OUTLOOK

The table yields the following conclusions.

1. For O_h.—Experiments on various samples of different cut (different $\ell_1\ \ell_2\ \ell_3$) and various directions of H (longitudinal and transverse is in general not enough) permit at best the determination of the four combination of constants:

$$[000]_{11},\ [200]_{11},\ ([000]_{11}[002]_{11} + [100]_{23}^2),\ ([002]_{11} + [011]_{23}).$$

In addition, there is one check relation between the constants experimentally determined. This may be seen by comparing equation (5.1.2) with Table I for O_h. It follows that the identity

$$\frac{B_{11} - B_{22}}{B_{22} - B_{33}} \equiv \frac{(B_{13}/B_{23})^2 - 1}{1 - (B_{13}/B_{12})^2} \qquad (5.1.3)$$

must be satisfied. By measuring with H at various angles the direction cosines $\ell_1\ \ell_2\ \ell_3$ of a sample can be determined in this case, if one of them is known.

By measuring the Hall effect in one direction normal to the wire, it can be shown that $[100]_{23}$ is determined and hence $[022]_{11}$ and $[011]_{23}$ can then also be evaluated separately. Thus for a complete determination of all five basic constants of O_h for $n = 0, 1, 2$, it is necessary and sufficient to measure the magneto resistance and Hall effects.

2. For T_h.—Experiments on various samples of different cut and various directions of H permit at best the determination of the five combination of constants:

$$[000]_{11}, [200]_{11}, ([000]_{11}[002]_{11}+[100]_{23}^2),\ ([002]_{11}-[020]_{11}), ([002]_{11}+[011]_{23}).$$

Again the determination, in addition, of the Hall effect yields $[100]_{23}$ and this is sufficient to break up all the above combinations and determine the six basic constants for this case. The materials with diamond structure belong in this class.

3. For Other Classes.—The corresponding analysis remains to be completed in the future.

Explicit expressions for the various classes, adapted for ~~use~~ in situations arising from experiment, must still be evolved. Similar work, for $n > 2$, restricted to the cubic case, must be carried out in order to permit the interpretation of deviations from equation (5.1.2) in terms of basic constants of higher order.

C. EXPERIMENTAL PART

I. EQUIPMENT

A larger magnet was assembled; it has two coils of more than 3,000 turns each and thus is capable of giving higher fields with much smaller energizing current than what is required in the old magnet reported earlier. The accurate calibration of this magnet is being planned. The approximate calibration gives, for $I < 1.0$ amp,

$$H = 2450 \, I \quad \pm 5 \text{ percent.}$$

At higher current the field is not a linear function of I, but tapers off and reaches a value of approximately 9,000 gauss at 8 amp. For $I < 2$ amp, the magnet can be energized continuously, while for higher current care must be taken to adjust a rheostat to compensate for the dropping of the current due to heating of the coils. Also, the period of use is shorter for higher current. At present this magnet is being used as the source of a constant, low field for the study of the MR effect for the H^2 term.

To speed up the measurement, a Leeds and Northrup pen-recorder has been connected to the measuring circuit parallel to the galvanometer. The Cardan suspension holder reported earlier is turned by a synchronous motor, and the signal from the sample is fed to the DC amplifier of the recorder. Curves similar to those in Figure 2 of the last report can thus be obtained in only a few minutes. The response time of the recorder is small and the position of the maximum of $\Delta\rho/\rho$ can be determined, which is important in the analysis of the ellipsoid of magneto resistance to be given in this report. An attempt is being made to put the MR effect signal on the oscilloscope screen for the study of symmetry from the polar diagram to be obtained.

II. MATERIALS

The Bi samples used are the same as those reported in the last quarterly report, plus some newly made from the same Fitzpatrick Bi. Bismuth is now being purchased from American Smelting and Refining Company, South Plainfield, N. J. The metal is 99.99 percent pure.

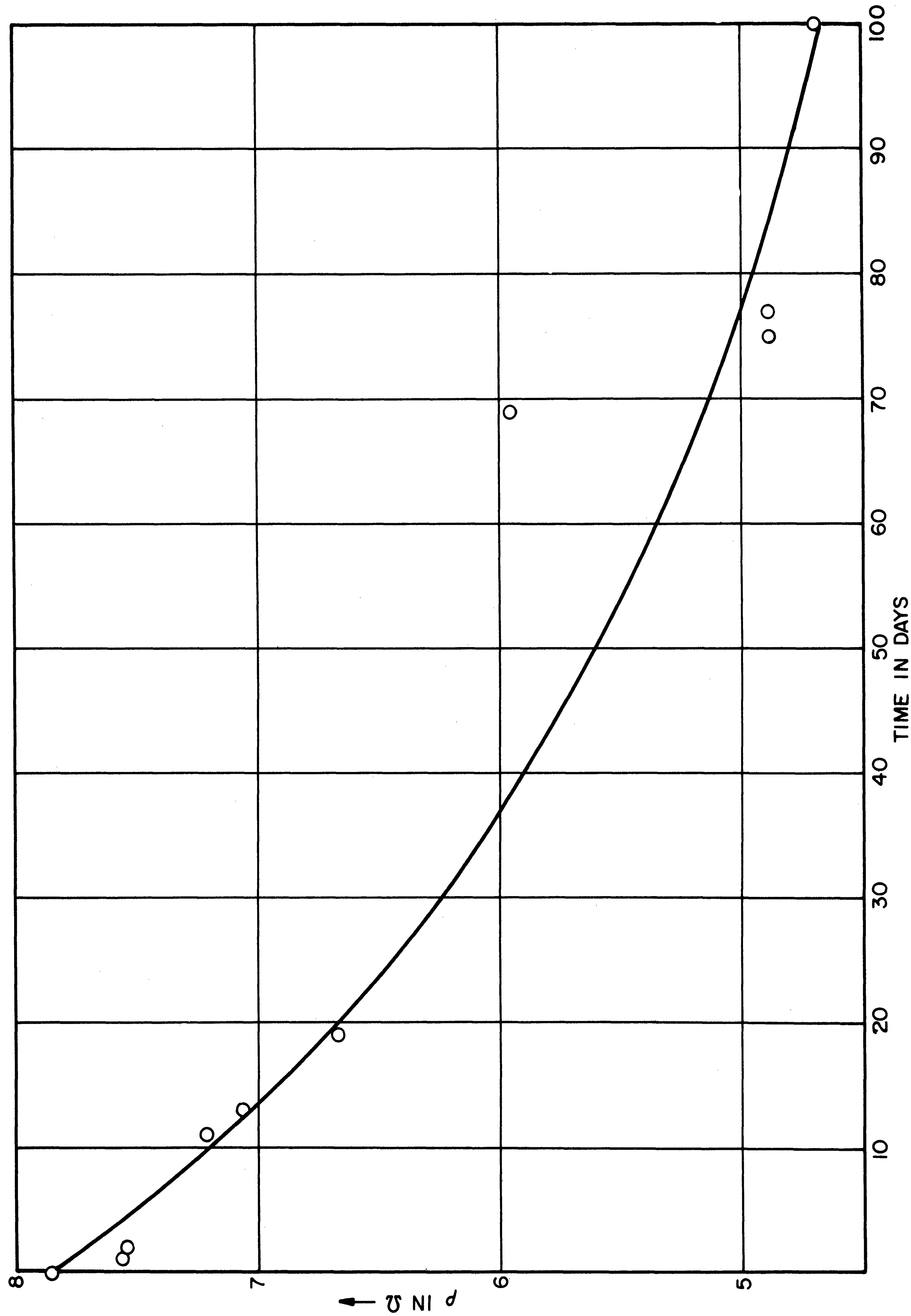

Figure 2. Aging effect: Decrease in resistance as function of time (data in Table II).

III. MEASUREMENTS

Not much measurement has been done since the closing of the last report. Time was spent in reassembling the equipment, testing and calibrating (approximately) the magnet, testing and improving the input impedance of the Northrup recorder for best result, making more wire samples, negotiating for the method of growing bismuth crystal, and determination of the crystal axes, since the etching of Bi with nitral failed.

1. Determination of Crystal Axes.—The nitral etchant is used at various mixture ratios between alcohol and nitric acid by volume. (Concentration: HNO_3 5n., alcohol (ethyl) 100%). The failure is believed to be due to the smallness of the area to be etched (0.003 mm^2 approximately). For a large rod of Bi (diam. $\sim$ 2 mm) still in glass tubing, 72 hours of etching with 5n. HNO_3 resulted in a tilting pyramid on the exposed end of the rod. The sides of the pyramid are not smooth, however. A similar result has not been obtained with any other rod. It has also been found that the etching method is unlikely to lead to the accurate determination of the crystal axes, so the etching method is, temporarily at least, abandoned, and determination of orientation will have to be done by x-ray methods.

2. Coating Bi on Glass.—Some efforts were made to achieve better control of coating conditions.

3. Aging Effect of the Samples.—The resistance of the Bi samples (in glass tubings) mounted inside the lucite capsules with gallium well contacts seems to be decreasing monotonically with time. An example is given in Table II and Figure 2. Note that the measured values of ρ are dependent on the measuring current i. The room temperature was not recorded but was fairly constant. The decrease in resistance is fastest when the sample is new, and may change considerably in a matter of hours. Hence, the aging cannot be due to seasonal weather conditions. Possibly the gallium may have something to do with this change in resistance. It may be of interest to note that the value of $\Delta\rho/\rho$ increases as the resistance decreases, and is about inversely proportional. If one assumes that some gallium may diffuse over the surface of the rods, causing a parallel resistance, then it is not clear why $\Delta\rho$ remains approximately constant when ρ changes by a factor of almost two, unless the gallium has a much larger magneto resistance than bismuth. Possibly this question will be explained when the x-ray work gives results.

TABLE II

AGING EFFECT OF SAMPLE RESISTANCE

Date	ρ Ω	Measuring Current i (ma)	$\left(\frac{\Delta\rho}{\rho}\right)_{max}$ for same. H = 800 gauss
July 9, 1954	7.85	40	
July 10, 1954	7.56	20	
July 11, 1954	7.54	20	63.3×10^{-4}
July 20, 1954	7.21	20	
July 22, 1954	7.06	20	
July 28, 1954	6.67	20	
Sept. 16, 1954	5.95	20	
Sept. 22, 1954	4.88	15, 10, 5	
Sept. 24, 1954	4.88	5	
Oct. 17, 1954	4.74	20	80.5×10^{-4}

IV. ELLIPSOID OF MAGNETO RESISTANCE

For the lower field range (H < 1000 gauss) the magneto resistance expression is of the form

$$B = \frac{\Delta\rho}{\rho H^2} = B_{11}\gamma_1^2 + B_{22}\gamma_2^2 + B_{33}\gamma_3^2 + 2B_{12}\gamma_1\gamma_2 + 2B_{23}\gamma_2\gamma_3 + 2B_{31}\gamma_3\gamma_1 \quad (5.7.1)$$

where the γ's are the direction cosines of the magnetic field vector with respect to the crystal axes. The transverse magneto resistance (with H in the plane normal to the wire) has two maxima as can be seen in the expression for this transverse case:

$$B = C_1 + C_2 \sin 2\phi \qquad (5.7.2)$$

If $B\left(\frac{\vec{H}}{H}\right)$ is plotted for all directions of $\vec{H}$ with respect to the wire, keeping

$|H|$ constant, the resulting diagram is similar to that shown in Figure 3. If $\vec{B}'$ is plotted, where

$$\vec{B}' = \frac{\vec{H}}{|H|} \cdot \frac{1}{\sqrt{B}} \tag{5.7.3}$$

for all directions, an ellipsoid would result. This is obvious by rearranging the equation (5.7.1) above in terms of B'. The diagonalized form of the ellipsoid is:

$$\frac{\left(\frac{\gamma_1'}{\sqrt{B}}\right)^2}{\left(\frac{1}{\sqrt{B_1}}\right)^2} + \frac{\left(\frac{\gamma_2'}{\sqrt{B}}\right)^2}{\left(\frac{1}{\sqrt{B_2}}\right)^2} + \frac{\left(\frac{\gamma_3'}{\sqrt{B}}\right)^2}{\left(\frac{1}{\sqrt{B_3}}\right)^2} = 1 \tag{5.7.4}$$

where the gamma-primes refer to the principal axes of the ellipsoid, and $\frac{1}{\sqrt{B_1}}, \frac{1}{\sqrt{B_2}}, \frac{1}{\sqrt{B_3}}$ are the lengths of the principal axes, as in Figure 4.

The magnitudes and directions of the three principal axes can be found experimentally. Note that equation (5.7.4) is, for $|H|$ = constant,

$$B = B_1\gamma_1'^2 + B_2\gamma_2'^2 + B_3\gamma_3'^2 \quad . \tag{5.7.5}$$

Let $B_3^{-1/2}$ be the minimum axis of the ellipsoid of equation (5.7.4), and therefore B_3 is the absolute maximum of B. Let $B_2^{-1/2}$ be the maximum axis of the ellipsoid, and therefore B_2 is the absolute minimum of all B. Let $B_1^{-1/2}$ be the intermediate axis. Now consider the general $\vec{B}$ diagram in Figure 3. Let OW be the wire, with a fixed but arbitrarily assigned direction. The transverse effect is represented by a vector in the OPQ plane. Let OP be the maximum transverse effect. In describing the experimental results two stages of interpretation must be clearly distinguished.

In the earlier measurements it was found that the absolute maximum B_3 of all $\vec{B}$ lies so close to OP that for all practical purposes it could be taken to coincide with OP. This approximation constitutes the first stage of interpretation, the second stage taking account of the difference between B_3 and OP. Discussion of some results from the first standpoint follows.

The absolute minimum B_2 must then lie in the plane normal to OP and it was found to make an angle ϕ_0 with the wire. The direction of the intermediate axis B_1 follows then normal to B_2 and B_3. In this direction B_1 was measured. Measurements of B were then taken for a set of ϕ-values covering the range $0 - 2\pi$, for one value of ψ (see Figure 3). The same was done for another value of ψ, etc., covering a range of $0 - \pi/2$ for ψ. Thus only one half (two opposite quarters) of the ellipsoid was measured.

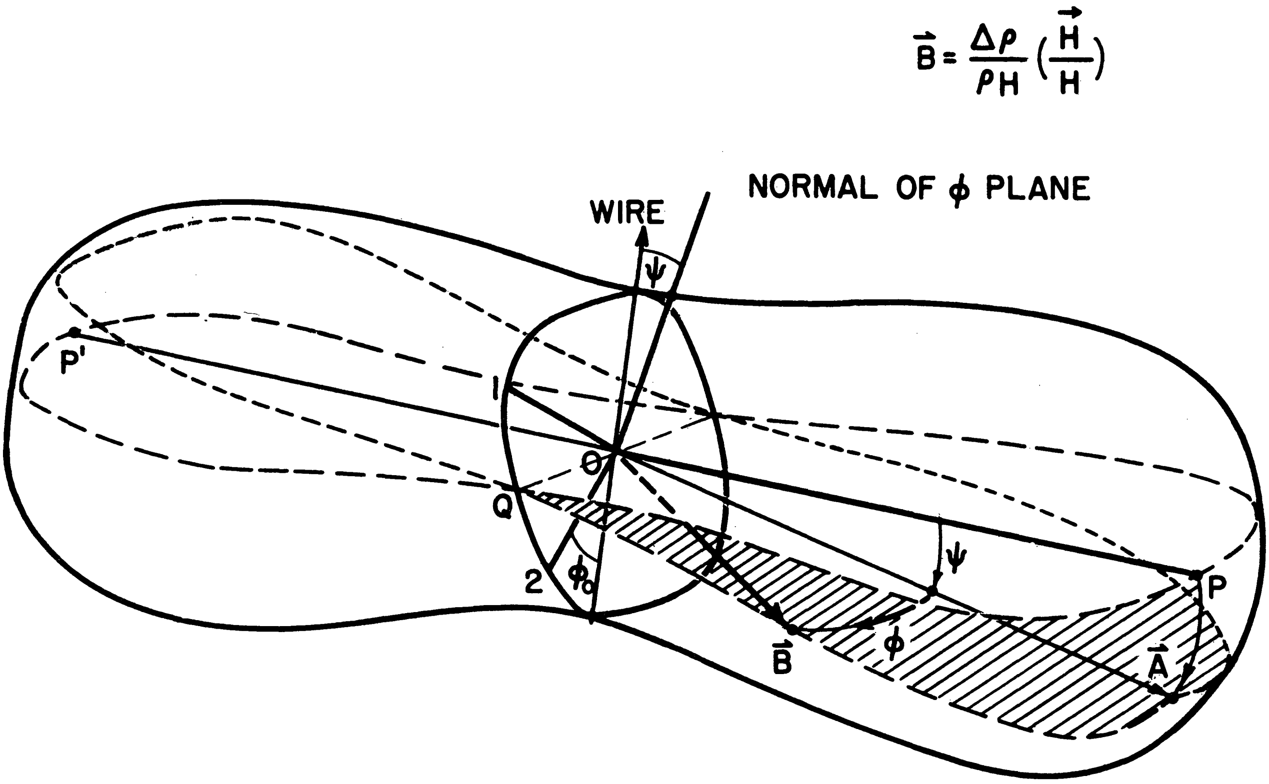

Figure 3. General diagram of Δρ/ρ

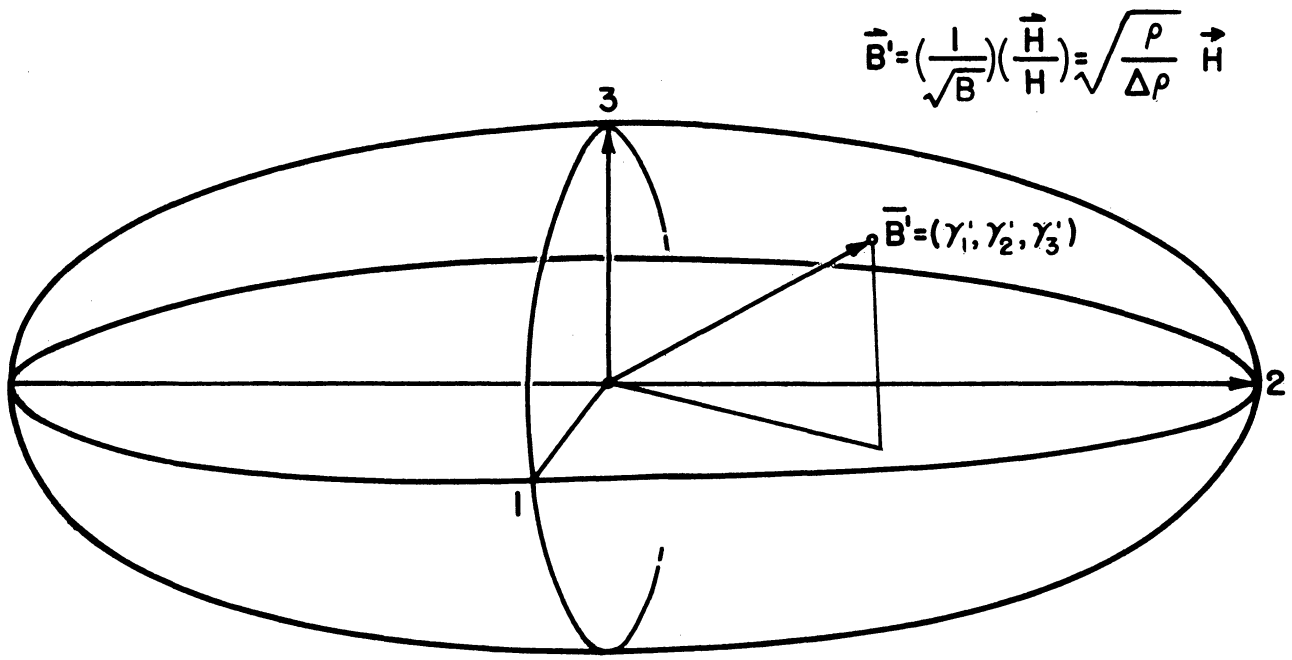

Figure 4. The magneto resistance ellipsoid

In order to check the measured values of B against the ellipsoid formula 5.7.5, B-values were calculated for all measuring positions ϕ, ψ, using the values B_1, B_2, B_3 as determined above and their directions; the required values of γ_1', γ_2', γ_3' were found for each pair of ϕ, ψ according to

$$\begin{aligned}
\gamma_1' &= \sin\phi\cos\phi_o - \cos\phi\sin\psi\sin\phi_o \\
\gamma_2' &= \sin\phi\sin\phi_o + \cos\phi\sin\psi\cos\phi_o \\
\gamma_3' &= \cos\phi\cos\psi
\end{aligned}$$

The agreement between the values of B thus calculated from B_1, B_2, B_3, and those observed was very satisfactory for all samples. The conclusion may be drawn, therefore, that the ellipsoid theory is in agreement with the experimental findings.

As a first example, consider the sample for which the results of measurement were given in Figure 3 of the previous report. This sample is of special interest because the curves of constant ψ all have their maxima and minima at the same value of $\phi = 90°$ and the curve for one value of ψ (in this case $\psi = 20°$) is a straight line. This is just what would be expected when the B_2 axis (direction of $\Delta\rho/\rho$ max) lies along the wire, i.e. $\phi_o = 0$, and consequently the intermediate B_1 axis lies along QQ. It is well known that a triaxial ellipsoid has two symmetrical circular cross sections passing through the intermediate axis B_1. One of these yields the observed straight line. This situation will arise only if the wire coincides with a binary axis or with the bisector between two binary axes, while if the wire coincides with the trigonal axis the resulting ellipsoid must be of revolution about the wire (i.e. the straight line occurs for $\psi = 0°$). The experimental determination of axis directions by means of x-rays for these samples will follow later.

As another example, consider the wire for which the results of measurement were given in Figure 2 of the previous report. In Figure 5 the difference between the measured $\Delta\rho/\rho$ and that calculated for the principal axes as specified has been charted in a polar diagram. Circles around W and radii from W represent lines of constant latitude and longitude on a sphere with pole at W. Circles through QQ are lines of constant ψ as marked, and the orthogonal trajectories to these (also circles) are lines of constant ϕ. At every point where a measurement was taken, an integer is found, representing the difference in percent of measured and calculated values, rounded off to the nearest whole percent. No least square fit of principal axes was attempted and thus a slightly more accurate fit may be possible. However, with the present choice of axes, there seems to be very little systematic deviation, and the errors as marked can easily be accounted for as experimental errors due to setting and reading the angles. The distribution of the errors is as follows.

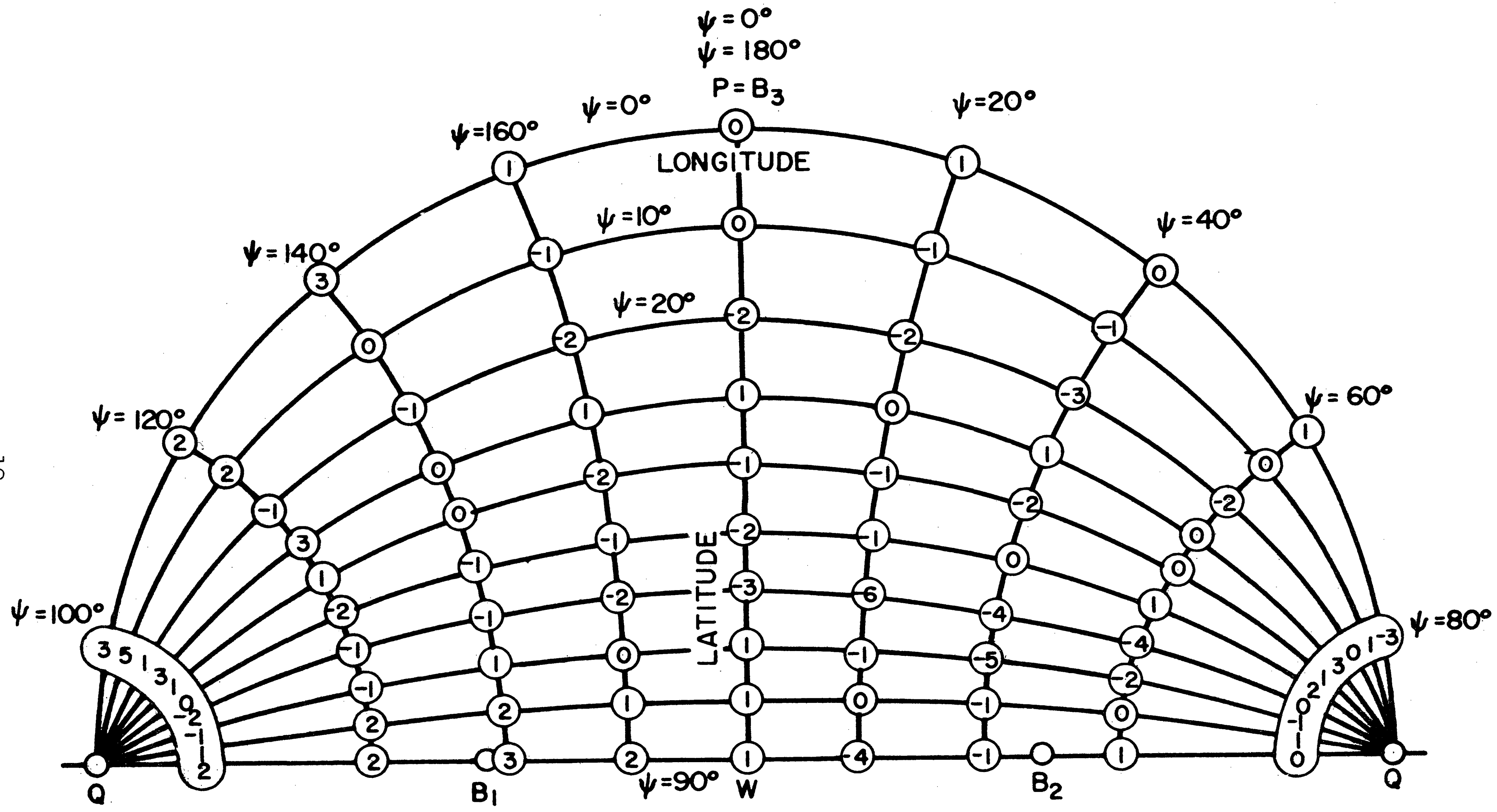

Figure 5. Percent deviation of measured $\Delta\rho/\rho$ from calculated values. (Sterographic projection of spherical coordinates).

TABLE III

DISTRIBUTION OF ERRORS BETWEEN CALCULATED AND MEASURED
$\Delta\rho/\rho$ AMONG 90 MEASURED POINTS ON 1/4 OF THE ELLIPSOID

Error %	<-3	-3	-2	-1	0	1	2	3	>3
Number of Cases	5	2	11	18	19	19	8	7	1

The conclusion to be drawn from these results is, that in the future it is sufficient to determine the principal axes of the ellipsoid by seeking its absolute maximum and minimum; no further measurements in directions other than the axes are required since the ellipsoid describes them adequately, provided H lies in the low range where higher terms than the quadratic ones are negligible.

The fact that the absolute maximum of $\vec{B}$ was always found perpendicular to the wire in a direction OP was now scrutinized, first from a theoretical standpoint, and the prediction confirmed experimentally.

It is easy to prove that B_{max} must indeed be exactly transverse to the wire for a large number of wire orientations: If the wire makes an angle α with k_3 (the trigonal axis), and the normal to the plane through k_3 and the wire makes an angle β with a binary axis, then B_{max} lies exactly transverse to the wire if either $\alpha = m\ 90°$ $(m = 0, 1)$ or $\beta = n\ 60°$ $(n = 0, 1, 2, 3, 4, 5)$. For other angles α, β it is likely that the deviation from exact transversality will be relatively small. However, it can be shown also that B_{max} <u>cannot</u> be exactly transverse for <u>all</u> α, β orientations unless a number of brackets vanish which is very implausible since there is no reason, on grounds of symmetry.

Consequently a series of measurements was undertaken with a sample of the same general type as the one discussed last (see Figure 2 of the previous report), with ϕ constant and ψ ranging from 0 to π instead of 0 to $\pi/2$ as before. Indeed the maximum was found at $4 \pm 0.5°$ off the normal plane to the wire (see Figure 6). This result sheds an interesting light on why historically the transverse and longitudinal effects have been studied primarily, although from the standpoint of symmetry this is not justified.

The connections between the principal axes of the ellipsoid and the crystal axes were not given in the above analysis. They will be investigated later when the axes are known accurately from x-ray work.

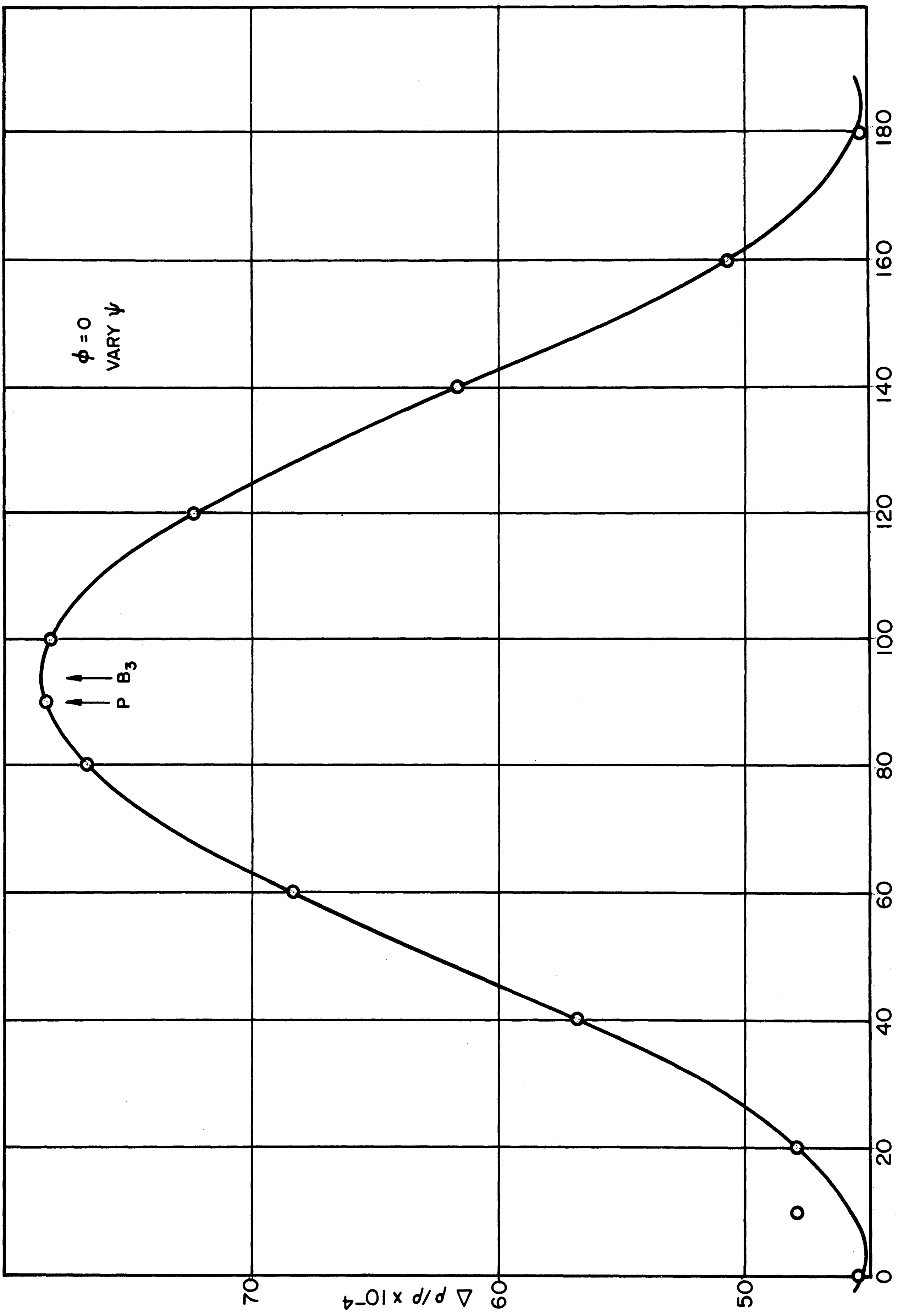

Figure 6. Measured magneto resistance as a function of ψ in the plane ϕ = 0 (plane of P, the point of maximum transverse MR) showing maximum B_3 off 90°.

V. PLANS FOR THE IMMEDIATE FUTURE

Our aim is to standardize a procedure for determining the brackets from combined measurements of several x-rayed samples whose $\Delta\rho/\rho$ are measured.