GROUP DECISION MAKING USING CARDINAL SOCIAL WELFARE FUNCTIONS

by

Ralph L. Keeney and Craig W. Kirkwood

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The Department of Industrial and Operations Engineering
The University of Michigan
Ann Arbor, Michigan 48104

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I. INTRODUCTION

Often the responsibility for making a decision falls on a group of individuals rather than a single person. In such cases, the group, whether it is the board of directors for a private corporation, the social committee for a college dormitory, or the voting public, must choose a method for selecting the alternative that is "best" in some sense.

The manner in which the group makes its decision can vary greatly in degree of formality. In some cases it may be possible for the members to discuss the alternatives and informally arrive at a concensus. However, in other situations, it may be desirable or necessary for the group to utilize a more formal decision making mechanism. In this paper, we suggest a method which explicitly addresses three complexities often present in group decision problems:

(1) The individual members have a different ranking of alternatives.

(2) There is uncertainty concerning the exact impact of each of the various alternatives.

(3) The individuals have different preference attitudes toward risks.

A cardinal social welfare function is developed which could be used for group decision making under uncertainty provided the assumptions made are justified for the problem at hand.

In the next section, the specific problem addressed in this paper is stated, and the ideas of utility theory that will be used in the development are briefly summarized. Section 3 reviews previous work on the problem. In the fourth section, a set of assumptions which is sufficient to imply that the cardinal social welfare function has either a multiplicative or additive form is stated. The interpretation of this result is also discussed in Section 4, and its limitations are considered in Section 5.

2. THE PROBLEM

Let $A = \{a_1, a_2, ..., a_n\}$ designate the set of alternatives available in a decision problem and $C = \{c_1, c_2, ..., c_m\}$ the set of possible consequences of these alternatives. At the time a decision must be made, there may be uncertainty about which consequence will result for each of the alternatives a_j. The problem is to choose the "best" alternative.

For individual decision makers, a theoretically sound method for making decisions in such situations has been developed. If the decision maker accepts the axioms of decision theory,* then he should choose the alternative which maximizes his expected utility,

$$E_j[u(c)] = \sum_c u(c)\ p_j(c), \tag{1}$$

*See, for example, von Neumann and Morgenstern [20], Pratt, Raiffa and Schlaifer [15], North [14], or Howard [8].

where u is the decision maker's cardinal utility function over the consequences c, and p_j is his subjective probability function for consequences given a_j is chosen. The probability function incorporates the decision maker's judgment about the uncertainties involved and the utility function encodes his preferences for possible consequences when there is uncertainty.

For the case of a group, there is no generally appealing normative theory for decision making under uncertainty applicable to all cases. The group Bayesian position, which is accepted in this paper, seems reasonable in certain situations.* This theory postulates that a group should be viewed as an entity whose decisions obey the axioms of utility theory. Hence, the group should select the alternative a_j which maximizes

$$E_j[W(c)] = \sum_c W(c)\; P_j(c) \;, \tag{2}$$

where P_j is the group's subjective probability function for the consequences of alternative a_j and W is the group's cardinal social welfare function.

The group Bayesians view a group as a " super individual" with its own subjective probability function and utility function. Given this approach, it is necessary to generate the P_j's and W in (2). One natural assumption to make about these is that the group preferences should depend only on the

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*A major objection to this approach is that decisions reached using it will not always be Pareto-optimal. Raiffa [16] has an interesting discussion of the advantages and disadvantages of the group Bayesian viewpoint.

preferences of the group members and the group judgments about the state of the world should depend only on the judgments of the group members. That is,

$$W(c) = w[u_1(c),\ u_2(c), \ldots,\ u_N(c)] \tag{3}$$

and

$$P_j(c) = P[p_j^1(c),\ p_j^2(c), \ldots,\ p_j^N(c)] \tag{4}$$

where u_k, $k=1,2,\ldots,N$, is the utility function of the kth group member for the possible consequences (N is the number of individuals in the group), p_j^k is his subjective probability function given alternative a_j is chosen, and w and p are unspecified functions.

There are at least two different types of decision problems where a group Bayesian analysis might be useful. In one case, a single person, or a small subgroup, would actually make the decision. However, this person might wish to incorporate the preferences of the entire group into his decision making process. That is, there would be a "benevolent dictator" -- someone (or a small group) who would have responsibility for the decision but who wished to take account of the views of others. In this case, it is reasonable to assume the dictator would specify the forms of w and p in (3) and (4).

The second type of group decision problem has the entire group collectively responsible for the decision. Here the group as a whole would have to decide on the forms of w and p.

Both types of group decision problem arise in practice. In this paper, an approach for specifying w is given which might be useful to either a benevolent dictator or a group acting collectively. The specification of p is a difficult and important problem which will not be addressed in this paper.*

3. PREVIOUS RELATED WORK

The problem of assessing social welfare functions has been investigated by numerous researchers.** However, most of this work has concerned ordinal social welfare functions. Perhaps the best known of this work is Arrow's [1]. He proved that in general there is no procedure for obtaining a group ordering of the various alternatives from the individual group member's ordinal rankings of alternatives that is consistent with five "reasonable" assumptions. Fleming [5,6] and Fishburn [4] have each postulated conditions which imply that the group's ordinal social welfare function can be written as the sum of ordinal utility functions of the group members.

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*Raiffa [16] discusses some of the difficulties in specifying p.

**Luce and Raiffa [12] provide a comprehensive survey of such work before 1957, and Kirkwood [11] provides a review that includes more recent work.

Nash [13] has considered group decisions under uncertainty.* However, his concern is finding an arbitrated solution <u>after</u> each individual's expected utility has been determined for each possible alternative. The group Bayesian approach is not directly concerned with the expected utilities of the individuals. It only seeks to determine the expected value for the group utility function.

Harsanyi [7] specifies a set of sufficient conditions for a cardinal social welfare function to be additive. In this case,

$$W(c) = \sum_{k=1}^{N} \lambda_k \, u_k(c) \ , \tag{5}$$

where the λ_k's are constants. These conditions are:

Condition 1: The social welfare function obeys the von Neumann-Morgenstern [20] axioms of utility theory.

Condition 2: The individual utility function u_k, $k=1,2,\ldots,N$, also obey the axioms of utility theory.

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*This work was generalized by Luce and Raiffa [12].

Condition 3: If two alternatives are indifferent from the standpoint of each individual, then they are indifferent for the group as a whole.*

In some instances, these three conditions seem appropriate. However, by use of examples, two implications of (5) which may not be reasonable in all situations can be shown. Suppose there are two individuals in the group with

$$W(c) = u_1(c) + u_2(c) \quad . \tag{6}$$

Now, consider three alternatives:

Alternative 1: $u_1 = 1$ and $u_2 = 0$ for certain.

Alternative II: there is a 50-50 chance of either $u_1 = 1$ and $u_2 = 0$ <u>or</u> $u_1 = 0$ and $u_2 = 1$.

Alternative III: there is a 50-50 chance of either $u_1 = 1$ and $u_2 = 1$ <u>or</u> $u_1 = 0$ and $u_2 = 0$.

Using (6), the group must be indifferent between Alternatives I, II, and III.

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*Condition 3 is similar in concept to the basic assumption used by Fishburn [3] to derive additive utility functions over multi-attribute consequences for individual decision makers.

Diamond [2] remarks that if equal utilities have the same meaning for each individual, then, because the group is indifferent between I and II, the decision making process seems to be unfair to individual 2. With Alternative I, he has no chance of receiving his preferred outcome (i.e., $u_2 = 1$) whereas he does have a chance with Alternative II. Thus, in order to give individual 2 a "fair shake," Diamond feels the group should prefer Alternative II.

Even in situations where all the group members get a "fair shake," Harsanyi's conditions may lead to undesirable results for the group. For example, with both Alternatives II and III, each individual has a 50-50 chance of obtaining his preferred consequence. However, in some situations, it seems reasonable that the group should prefer Alternative III to Alternative II. With III, either both individuals receive their preferred consequence or both receive their less preferred consequence. On the other hand, with Alternative II, there will always be an inequity in the consequence received--one individual will receive his preferred consequence while the other will receive his less preferred one. For many group decisions an attempt is made to avoid inequities within the group as much as possible. When this is the case, the above example indicates that equation (5) does not seem to be appropriate.

4. MORE GENERAL CARDINAL SOCIAL WELFARE FUNCTIONS

In this section forms of the social welfare function W which are more general than the additive case, but still consistent with (3) are presented. Harsanyi's conditions 1 and 2 are kept, but two weaker conditions are substituted for his Condition 3. Since these conditions are the basis for deriving the results below, they will be carefully discussed below. The conditions are:

Condition A. For any individual k, $k=1,2,\ldots,N$, if all the other N-1 members of the group are indifferent among all possible consequences, then the preferences of the group for any lotteries over these consequences will be the preferences of individual k.

Condition B. For any two individuals k and h, $k=1,2,\ldots,N$, $h \neq k$, if all the other N-2 members of the group are indifferent among all possible consequences, then the preferences of the group for any lotteries over these consequence will be governed only by the preferences of individuals k and h.

These two conditions seem fairly plausible. They say, in essence, that if only one or two people in a group care what happens then those one or two people should decide what to do. Notice that this does not mean that all but one or two people will be indifferent among the consequences in the actual decision problem being faced. Rather, the conditions merely specify what the group decision mechanism must do if these extreme cases ever arise.

This approach to specifying conditions in analogous to that taken by Harsanyi in specifying his Condition 3. He doesn't say that there will be two alternatives that all group members are indifferent between in any particular group decision problem. He merely states that if this situation ever arises, then the group must be indifferent between the two alternatives.

Imposing any of these conditions may be viewed as placing restrictions on how the social welfare function should behave in extremely simple cases where it is fairly clear what should happen, and then seeing how these restrict its behavior in the more complicated cases that are likely to arise in practice.

Mathematically, Condition A may be stated as follows: Suppose that $u_h(c) = u_h^o$, $h \neq k$, for all c where the u_h^o's are constants. Then if $p_1(c)$ and $p_2(c)$ designate probability mass functions over the consequences for lotteries 1 and 2, it must be true that if

$$\sum_c u_k(c)\ p_1(c) \geq \sum_c u_k(c)\ p_2(c) \tag{7}$$

then

$$\sum W(c)\ p_1(c) \geq \sum W(c)\ p_2(c) \tag{8}$$

regardless of the form of $p_1(c)$ and $p_2(c)$. (Of course, this doesn't mean that (7) will hold for <u>any</u> specified $p_1(c)$ and $p_2(c)$. It merely means that if (7) holds, then (8) must hold irrespective of what $p_1(c)$ and $p_2(c)$ are.)

The statement above must hold, of course, because expected utility is the choice criterion used by individual k, and expected social welfare is the choice criterion used by the group. By Condition A, these must agree in the special case specified.

In utility theory terms, both $u_k(c)$ and $W(c)$ are cardinal utility functions. Thus, if $W(c)$ is to make the same decisions as $u_k(c)$, von Neumann and Morgenstern have proved [20, pp. 627-28] that $W(c)$ must be a positive linear transformation of $u_k(c)$. That is,

$$W(c) = \alpha_k + \beta_k u_k(c), \; k = 1,2,\ldots,N, \tag{9}$$

where α_k and β_k are unrestricted constants except that $\beta_k > 0$. But in view of equation (3), this may be rewritten

$$w[u_{\bar{k}}^o; u_k(c)] = \alpha_k + \beta_k u_k(c), \; k = 1,2,\ldots,N, \tag{10}$$

where

$$u_{\bar{k}}^o \equiv (u_1^o, u_2^o, \ldots, u_{k-1}^o, u_{k+1}^o, \ldots, u_N^o)$$

so that

$$(u_{\bar{k}}^o; u_k) = (u_1^o, u_2^o, \ldots, u_{k-1}^o, u_k, u_{k+1}^o, \ldots, u_N^o).$$

However, there is no reason that α_k and β_k might not differ depending on the value of $u_{\bar{k}}^o$. Thus, it is more accurate to write (10) as

$$w[u_{\bar{k}}^o; u_k(c)] = \alpha_k(u_{\bar{k}}^o) + \beta_k(u_{\bar{k}}^o)u_k(c), \qquad k = 1,2,\ldots,N, \tag{11}$$

where α_k and $\beta_k > 0$ are unspecified functions of $u_{\bar{k}}^o$. The relation shown in (11) must be true for <u>any</u> value of $u_{\bar{k}}^o$ since Condition A holds regardless of what the specific utilities are of the individuals who are indifferent among the consequences.

Similarly, Condition B means that if $u_\ell(c) = u_\ell^o$, $\ell \neq h$ or k, for all c then if

$$\sum_c U_{kh}[u_k(c),\ u_h(c)]p_1(c) \geq \sum_c U_{kh}[u_k(c),\ u_h(c)]p_2(c) \tag{12}$$

it must be true that

$$\sum_c W(c)p_1(c) \geq \sum_c W(c)p_2(c) \tag{13}$$

where $u_{\overline{kh}}^o \equiv (u_1^o,\ldots,u_{k-1}^o,u_{k+1}^o,\ldots,u_{k-1}^o,u_{h+1}^o,\ldots,u_N^o)$

so that $(u_{\overline{kh}}^o; u_k, u_h) = (u_1^o,\ldots,u_{k-1}^o,u_k,u_{k+1}^o,\ldots,u_{h-1}^o,u_h,$ $u_{h+1}^o,\ldots,u_N^o)$. (Notice that $U_{kh}[u_k(c),u_h(c)]$ is the social welfare function that individuals k and h use to make decisions together. The form of this has not been specified and there is no need to do so to obtain the results below.) This follows by exactly the same reasoning as that which led to (11). Also, by the same reasoning that led to (11), it must be true that

$$w[u_{\overline{kh}}^{o};u_k(c),u_h(c)] = \alpha_{kh}(u_{\overline{kh}}^{o}) + \beta_{kh}(u_{\overline{kh}}^{o})U_{kh}[u_k(c),u_h(c)], \quad (14)$$

$$k = 1,2,\ldots,N; h \neq k,$$

where α_{kh} and $\beta_{kh} > 0$ are unspecified functions.

The equations (11) and (14) are central to the proofs of Theorems 1 and 2 below. The proofs proceed by realizing that these equations are formally identical to equations that arise in multiattribute utility theory. In particular, if $u_1,u_2,\ldots,u_N$ are viewed formally as attributes of a multiattribute utility function $w(u_1,u_2,\ldots,u_N)$ then Equation (11) is equivalent to saying that any attribute u_k is utility independent of the other attributes, and that its conditional utility function is linear in u_k.

Of course, the problem here is not a multiattribute utility problem. However, any results in multiattribute utility theory that depend on utility independence can be applied here because the mathematical equation used in applying the utility independence condition is identical to Equation (11).

Similarly, Equation (14) is identical to that obtained in multiattribute utility theory if any two attributes u_k and u_h are jointly utility independent of the other attributes. Thus, any results in multiattribute utility theory that depend on this fact can be applied here since the equation is the same.

Keeping these comments in mind, the following two theorems are true.

Theorem 1. Given conditions 1, 2, and A hold, then

$$W(c) = \sum_{k=1}^{N} \lambda_k u_k(c) + \sum_{\substack{k=1 \\ j>k}}^{N} \lambda_{kj} u_k(c) u_j(c) + \cdots + \lambda_{1,2,\ldots,N} u_1(c) u_2(c) \cdots u_N(c) \tag{15}$$

where W and the u_k's are scaled from zero to one, the λ's are constants, and $0 < \lambda_k < 1$ for all k.

Proof. As noted above, Condition A can be interpreted as a utility independence condition. When this is done, Theorem 1 follows directly from a proof given by Keeney [9, Theorem 4] in the context of developing multiattribute utility functions for individual decision makers so the proof will not be given here. Because W satisfies the axioms of utility theory it is unique up to a positive linear transformation.

When Condition B is also added a stronger result can be proven.

Theorem 2. Given that conditions 1, 2, A, and B hold and the group has at least three members, then

$$W(c) = \sum_{k=1} \lambda_k u_k(c) + \lambda \sum_{\substack{k=1 \\ j>k}} \lambda_k \lambda_j u_k(c) u_j(c) + \tag{16}$$

$$+ \lambda^{N-1} \lambda_1 \lambda_2 \cdots \lambda_N u_1(c) u_2(c) \cdots u_N(c),$$

where again W and the u_k's are scaled from zero to one, the various λ's are constants, $0<\lambda_k<1$ for all k and $\lambda> -1$.

Proof. With the multiattribute utility theory interpretation of Equations (11) and (14), this result also follows directly from a theorem in multiattribute utility theory [10, Theorem 1].

As must be the case because of the requisite assumptions, (16) is a special case of (15). Furthermore, notice that special cases of both (15) and (16) result in the additive social welfare function (5). In particular, this results from (16) when $\lambda=0$. If λ is not zero, one can multiply both sides of (16) by λ and add one to obtain

$$\lambda W(c) + 1 = \prod_{k=1}^{N} [\lambda \lambda_k u_k(c) + 1] \tag{17}$$

which is a multiplicative social welfare function.

Now, using Theorem 1, reconsider Alternatives I, II, and III introduced in the previous section. For this situation of two individuals, equation (15) is

$$W(c) = \lambda_1 u_1(c) + \lambda_2 u_2(c) + \lambda_{12} u_1(c) u_2(c), \tag{18}$$

where

$$\lambda_1 + \lambda_2 + \lambda_{12} = 1 \tag{19}$$

since W, u_1, and u_2 must all be scaled from zero to one. Suppose, for example, that $\lambda_1 = \lambda_2 = 0.4$ so $\lambda_{12} = 0.2$ and (18) becomes

$$W(c) = 0.4u_1(c) + 0.4u_2(c) + 0.2u_1(c)u_2(c). \tag{20}$$

Then the social welfares for the three Alternatives I, II, and III are calculated as 0.4, 0.4, and 0.5 respectively. Thus, since I and II are still indifferent, Diamond's objection to Harsanyi's result is not met by (15).

However, Alternative III is preferred to Alternative II. Because the constant λ_{12} is positive, a concern for the equity of the distribution of utilities consequences the group members has been introduced that was not possible with the additive form.

Comments on Assessing the λ's

Theorem 1 and 2 are theoretically interesting. However, if one wishes to actually apply them the various scaling constants (that is, the λ's) must be assessed. The approach needed to do this would depend on the type of decision problem being considered. If the problem is that of the "benevolent dictator" mentioned earlier, then W is his or her utility function so the λ's are assessed by this person. The assessment involves addressing questions of trading off utility to one individual against utility to another individual. This involves simultaneously trying to consider (i) the inherent value of different measured utilities to each individual (inter-personal comparison of utility) and (ii) the relative weight that the dictator wishes to give to the preferences of different individuals.*

If the group as a whole is to have the final responsibility for the decision, the problem of assessing the λ's becomes more difficult. The two considerations raised above must be addressed by the entire group. The scaling constants need to be evaluated by an arbitration scheme that will hopefully lead to a set of constants with which

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*This problem is considered in Kirkwood [11]. Sen [17] notes that in some cases useful results can be obtained without completely specifying the inter-personal comparison of utility.

each member of the group agrees. Theil [18] and van den Bogaard and Versluis [19] have considered this problem for the weighted additive social welfare function. Their procedures might be generalizable to the more general forms of equations (15) and (16). Clearly, this is an area where more research is needed.

5. LIMITATIONS OF THE MULTIPLICATIVE FORM

Conditions A and B are reasonable for many situations. In particular, when the entire group has structured W, the members should realize that the social welfare function is based on a "fair" method of making decisions. Thus, if events occur which yield favorable consequences for some individuals in the group and unfavorable consequences for the others, they will realize that this did not result from an unfair social welfare function. However, when a benevolent dictator is making the decisions, the situation is different. In this case, the individuals receiving unfavorable consequences may not have been involved in structuring the social welfare function. Thus, they may not interpret the outcome as due to chance, but rather as due to unfairness on the part of the overall decision maker.

To illustrate this point, consider two different situations where all but one group member is indifferent between the consequences. In one case all the members that are indifferent between alternatives are impacted in a manner they individually feel to be very undesirable and in the other case all such individuals feel they are very desirably impacted. According to Condition A, the group social welfare function should count only the preferences of the individual who is not indifferent in both cases. However, if everyone else's utility is low, then, because of pressures among group members, the benevolent dictator may prefer consequences where that individual also receives an undesirable consequence rather than a desirable one. On the other hand, the situation may be reversed if all the other individuals have a high utility.

If such distinctions are important, then Condition A is violated so Theorems 1 and 2 do not hold. Similar arguments can be made against Condition B.

Another limitation of the multiplicative form is illustrated using the following:

Condition C. If all the members of a group have the same utility function over the consequences, then the group social welfare function should always yield the same decisions as the common utility function. This leads to

Theorem 3. Given that Conditions 1, 2, A and C hold, then the social welfare function must be the additive form (5).

Proof. Because Conditions 1, 2, and A hold, the social welfare function $W(c)$ is given by (15). However, since Condition C holds, then for all k, $u_k(c) = u(c)$ for some function u and (15) simplifies to

$$W(c) = u(c)\sum_{k=1}^{N}\lambda_k + [u(c)]^2\sum_{\substack{k=1\\ j>k}}^{N}\lambda_{kj} \tag{21}$$

$$+ \dots + [u(c)]^N \lambda_{1,2,\dots,N}$$

But, from Condition C, using the previously quoted result of von Neumann and Morgenstern,

$$W(c) = \alpha + \beta u(c), \tag{22}$$

where α and $\beta > 0$ are unspecified constants. Equations (21) and (22) are compatible only if $W(c)$ is the additive form (5).

Theorem 3 is slightly disturbing since Condition C appears to be reasonable and yet, when taken in conjunction with some other reasonable assumptions, a form of the social welfare function is implied to which objections were raised earlier in this paper.

There are situations where Condition C may not be as reasonable as it appears on the surface. For example, suppose the Board of Directors of a company is considering various business ventures with monetary payoffs to the company being the only consequences of importance to any of the board members. Then, even if all the members have identical risk averse utility functions for money accruing to the company, it might be reasonable for the group cardinal social welfare function to be less risk averse than the individual functions. This may result from the fact that the responsibility for the decision is being shared by all the group members, as opposed to falling entirely on one individual. In the latter case, if a very undesirable consequence happened to result from the decision, the individual might "feel terible" about the situation he had "caused." However, that individual board member might not have felt as bad if the original decision had been made by the entire group. The moral and emotional responsibility for the decision is in some sense divided among the members, whereas the entire monetary consequences accrue to the group as a whole.

In general, whenever all the individuals have the same utility function, each of them will be impacted by any consequence in the same manner, and hence, because "misery loves company" and the individuals are "all in the same boat together," the undesirable consequences will not seem so bad. Therefore, the group may be willing to act less risk averse.

6. CONCLUSIONS

This paper has addressed the problem of constructing a cardinal social welfare function for a group that obeys certain "reasonable" sets of conditions. The results are representation theorems providing for the functional form of a social welfare function which uses as arguments the utility functions of individual members of the group. In addition, for any particular decision, the group must assess certain scaling constants to completely specify the social welfare function . This requires the inter-personal comparison of utility, which is an ever present problem in social choice situations.

The results in the paper show that mathematical analogies can be drawn between certain problems in social welfare theory and in multiattribute utility theory. These allow the representation theorems in the paper to be easily proved.

Kirkwood [11] has used the ideas of this paper in structuring three diverse group decision problems--one involving preferences of users of a time-share computer system, one the preferences of voters, and the third, the preferences of families being relocated due to urban highway construction.

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