

UM-HSRI-78-58

MODELING THE IN-STOP TORQUE
PERFORMANCE OF HYDRAULICALLY-ACTUATED
TRUCK BRAKES

MVMA Project #1.34

T. Gillespie
L. Johnson
P. Fancher

Final Technical Report

December 1978

Highway Safety Research Institute
The University of Michigan
Ann Arbor, Michigan 48109

1. Report No. UM-HSRI-78-58		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle MODELING THE IN-STOP TORQUE PERFORMANCE OF HYDRAULICALLY-ACTUATED TRUCK BRAKES				5. Report Date December 1978	
				6. Performing Organization Code	
7. Author(s) T. Gillespie, L. Johnson, P. Fancher				8. Performing Organization Report No. UM-HSRI-78-58	
9. Performing Organization Name and Address Highway Safety Research Institute The University of Michigan Huron Parkway and Baxter Road Ann Arbor, Michigan 48109				10. Work Unit No. 361504	
				11. Contract or Grant No. MVMA Proj. #1.34	
12. Sponsoring Agency Name and Address Motor Vehicle Manufacturers Association 300 New Center Building Detroit, Michigan 48202				13. Type of Report and Period Covered Final 7/1/77-10/15/78	
				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract The characteristics of two truck rear-axle hydraulic brakes with different linings are determined from mobile dynamometer tests and are modeled by a regression analysis relating the brake effectiveness to actuation force, sliding velocity, and drum/lining interface temperature. A finite element heat transfer model is used to determine the interface temperature. A multi-nomial expression involving 27 regression coefficients for one brake and 36 regression coefficients for the other brake is found sufficient to characterize these brakes with high coefficients of determination. The regression model for one brake, developed from test data acquired on the constant-speed mobile dynamometer, is used to predict the earlier performance of that same brake on an inertia dynamometer. Though the agreement obtained generally demonstrates the capability of the model at predicting in-stop torque performance, errors due to effectiveness changes caused by work history are evident. Practical applications of the brake model for predicting FMVSS 105 stopping distance performance and other uses are discussed.					
17. Key Words brakes, hydraulic brakes, interface temperature, work history, brake modeling, processing brake data			18. Distribution Statement UNLIMITED		
19. Security Classif. (of this report) NONE		20. Security Classif. (of this page) NONE		21. No. of Pages 47	22. Price

TABLE OF CONTENTS

1. INTRODUCTION.	1
2. RESEARCH APPROACH	4
2.1 Brake Selection.	4
2.2 Brake Tests.	6
2.3 Data Analysis.	9
3. HYDRAULIC BRAKE EFFECTIVENESS FUNCTIONS	17
3.1 Typical Effectiveness Function	17
3.2 Evaluation of Modeling Errors.	25
3.3 Prediction of In-Stop Torque	28
4. APPLICATION OF RESULTS.	37
4.1 FMVSS 105 Braking Performance Predictions.	37
4.2 Wear Balancing	39
4.3 Long Grade Performance	40
5. SUMMARY AND RECOMMENDATIONS	41
REFERENCES.	43
APPENDIX A - Effectiveness Functions.	44

ACKNOWLEDGEMENTS

The Institute wishes to thank the Motor Vehicle Manufacturers Association and its supporting members for the sponsorship of this research program. In addition, the support of Mr. Peter Soltis and the Kelsey-Hayes Company is acknowledged for providing brake system hardware and the inertia dynamometer test data used in this project.

1.0 INTRODUCTION

An objective of the Motor Truck Braking and Handling Research program at HSRI (The Highway Safety Research Institute of The University of Michigan) is the development of a methodology to predict the braking performance of trucks and tractor-trailers. In the context of federal regulation of braking performance, this translates into a need to predict stopping distance with a high degree of accuracy.

Comprehensive digital computer simulation programs [1,2,3] have been developed as tools for investigating heavy truck braking and handling behavior. In these programs, various options have been available for representing brakes, ranging from the simplest representation by pressure-torque characteristics to more comprehensive models including the internal geometry of the brake shoes and mechanisms. Nevertheless, meaningful prediction of vehicle braking performance requires representation of the brakes in terms of their instantaneous torque output (effectiveness) as dependent on the instantaneous actuation effort, rotational speed, and internal temperature occurring as energy is absorbed by the brake. Because of the complex interdependence of various factors (especially the changing frictional characteristics of the brake linings), the available brake models are not suited to accurate representation over a broad range of operating conditions. Though brake fade can be specified in the models as a function of calculated brake temperature, no systematic method exists by which to determine the fade coefficient from available data. Rather, the brake is best represented by an empirical model derived from measured performance over a spectrum of conditions. This report presents methodology and techniques developed to describe brake performance as a function of actuation effort, sliding velocity, and lining/drum interface temperature in a way which allows calculation of the expected torque output through any arbitrary braking situation, the only limitation being that the operating conditions of

the brake are within the spectrum of conditions covered by the experimental tests from which the model is derived.

The material presented here is directed towards examining the performance of hydraulically-actuated heavy vehicle brakes, using techniques which have also been applied to the study of pneumatically-actuated brakes [4]. For air-braked vehicles, the existence of Federal Motor Vehicle Safety Standard 121 (FMVSS 121) [5] has led to careful examination and measurement of detailed information concerning time histories of brake performance during a stop in order to better understand (1) antilock system performance and (2) the influence of brake characteristics on the stopping distance obtained in emergency stops [6,7]. In studies of FMVSS 121 vehicles, the influences of in-stop fade and side-to-side variation in brake capability were found to be significant factors in determining wheels-unlocked stopping performance. These same factors are important for vehicles with hydraulically-actuated brakes (although the interaction of these factors with antilock system performance is not a current issue). As FMVSS 105 (the federal standard for vehicles with hydraulically-actuated brakes) is extended to medium and heavy vehicles, consideration of the variability in hydraulic brake performance and the influence of brake fade will become increasingly important in developing braking systems to meet governmental requirements.

The most costly and time-consuming activity associated with using computer simulations to predict vehicle performance is the acquisition of suitable parametric data for characterizing the vehicle and its components. Over the past several years, HSRI has employed various approaches for gathering brake data and representing the data in a form suitable for computer simulation. The process of developing an appropriate method for collecting and representing brake data has proven to be difficult because (1) brake characteristics change with work history, (2) there is a considerable variation from brake to brake of the same type, (3) brake adjustment can cause changes in brake performance (4) lining/drum interface temperatures (or average interface temperatures) are hard to

measure, and (5) the conventional methods for presenting results of inertia dynamometer tests do not contain all the information needed to predict stopping performance. The direct dependence of brake performance on the lining/drum interface temperature necessitates the inclusion of this parameter in the approach despite the added complexity of brake thermal modeling in order to achieve accurate brake representation. The approach taken is one of applying standard computer-based multiple regression methods to brake performance data and the significant independent variables. The method depends on having in-stop history data from dynamometer tests, as well as thermal modeling information on the brake, but in return for this effort provides algorithms for predicting the instantaneous torque output of a brake at arbitrary operating conditions.

The next section of this report describes the approach taken in terms of the brakes selected, the tests conducted, and the methods of data analysis. Section 3.0 presents the type of results obtained with an assessment of the error magnitudes associated with the model, and a comparison of the brake model in-stop torque predictions with the torque measured on an inertia dynamometer. Section 4.0 discusses practical applications of the developed methodology to several areas of need in brake system design.

2.0 RESEARCH APPROACH

In order to accomplish the hydraulic brake study described hereinafter, it was necessary to select appropriate brakes for study, test them on the Institute's Mobile Dynamometer, and analyze the brake performance data obtained.

2.1 Brake Selection

Guidance in selection of brakes for test was obtained through discussions with brake design engineers from two of the major truck manufacturers. For the program to be most beneficial, it was considered important to select brakes which are candidates for utilization in any anticipated FMVSS 105 regulation applicable to medium trucks. However, the selection of those brakes would be highly dependent on the severity of the stopping distance requirements imposed—potentially ranging from existing brake systems for moderate requirements to full disc brake systems for severe requirements. Nevertheless, the most critical and most likely candidate brake in FMVSS 105 systems would be of the 15 x 5 drum brake type currently used, with variations in lining type and cylinder size to tailor the brake to specific applications.

A cooperative arrangement was established with the Kelsey-Hayes Company whereby a popular 15 x 5 drum brake would be burnished and tested in a manner typical of FMVSS 105 conditions. The first brake selected, shown in Figure 1, was a 15 x 5 Twinplex drum brake, 9051 J lining, 1-1/2" wheel cylinders, with a Gunite 2603 drum. This brake is typically used with a 15,000-lb rated rear axle, and was tested by Kelsey-Hayes on an inertia dynamometer at 7,436-lb wheel load, 19.3" tire radius.

Consideration was given to testing a disc brake for comparison purposes, but the availability of the brake and adaptation hardware prevented its inclusion in the program. Instead, a second 15 x 5 drum brake with ABB 539 lining, typical of a 17,500-lb rated rear axle brake, was selected for test. The brake was obtained new,

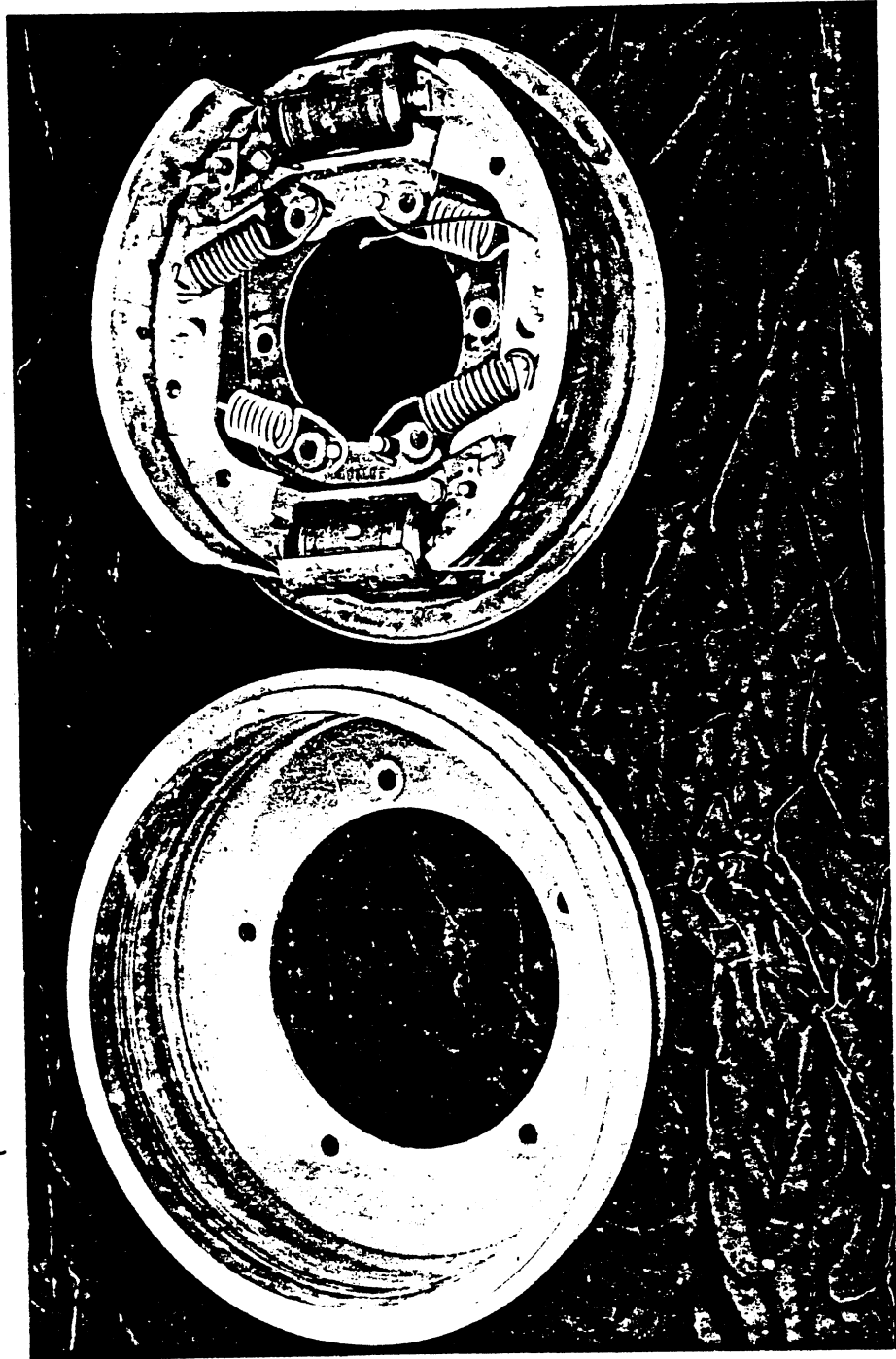


Figure 1. Photograph of Kelsey-Hayes 15 x 5 Twinplex Drum Brake.

tested at HSRI for first effectiveness (pre-burnish), burnished, and then tested for second effectiveness and a matrix of other conditions. -

2.2 Brake Tests

Brake tests at the Institute are carried out on the Mobile Truck Tire Dynamometer, as shown in Figure 2. The Dynamometer [8] consists of the straight-truck power unit on which an instrumentation van and auxiliary power systems are mounted. The truck tows a special trailer with a test wheel mounted on its centerline. The brake is mounted in a conventional fashion on the test wheel and is supported by a transducer for measuring the brake torque, longitudinal force and wheel load. Wheel load is provided by an air spring and is adjusted by the spring pressure.

The mobile dynamometer brake tests are effectively constant-speed tests in contrast to inertia dynamometer tests which simulate a decelerating vehicle. In the research environment, the constant-speed test method has the advantage of reducing the number of variables during test.

Mobile dynamometer tests were conducted on both brakes. The first brake (15 x 5 with 9051 J lining) had been burnished and tested on the Kelsey-Hayes inertia dynamometer so that the only mobile dynamometer tests required were a matrix of conditions to characterize its performance. Table 1 summarizes the matrix of tests all conducted at an initial brake temperature of 220°F or less as measured by an SAE lining thermocouple. The first 32 tests were conducted at light braking levels to provide comparative data between the speed conditions of 20 and 40 mph. The 120 tests constituted the foundation of the matrix of conditions over which the brake was to be characterized. Periodically, tests were conducted at 3 mph to provide very low velocity conditions for determination of the brake's speed dependence. Throughout the tests, the brake application time was held to four seconds or less, if necessary, to keep the energy absorption during test to no more than one "g" equivalent.

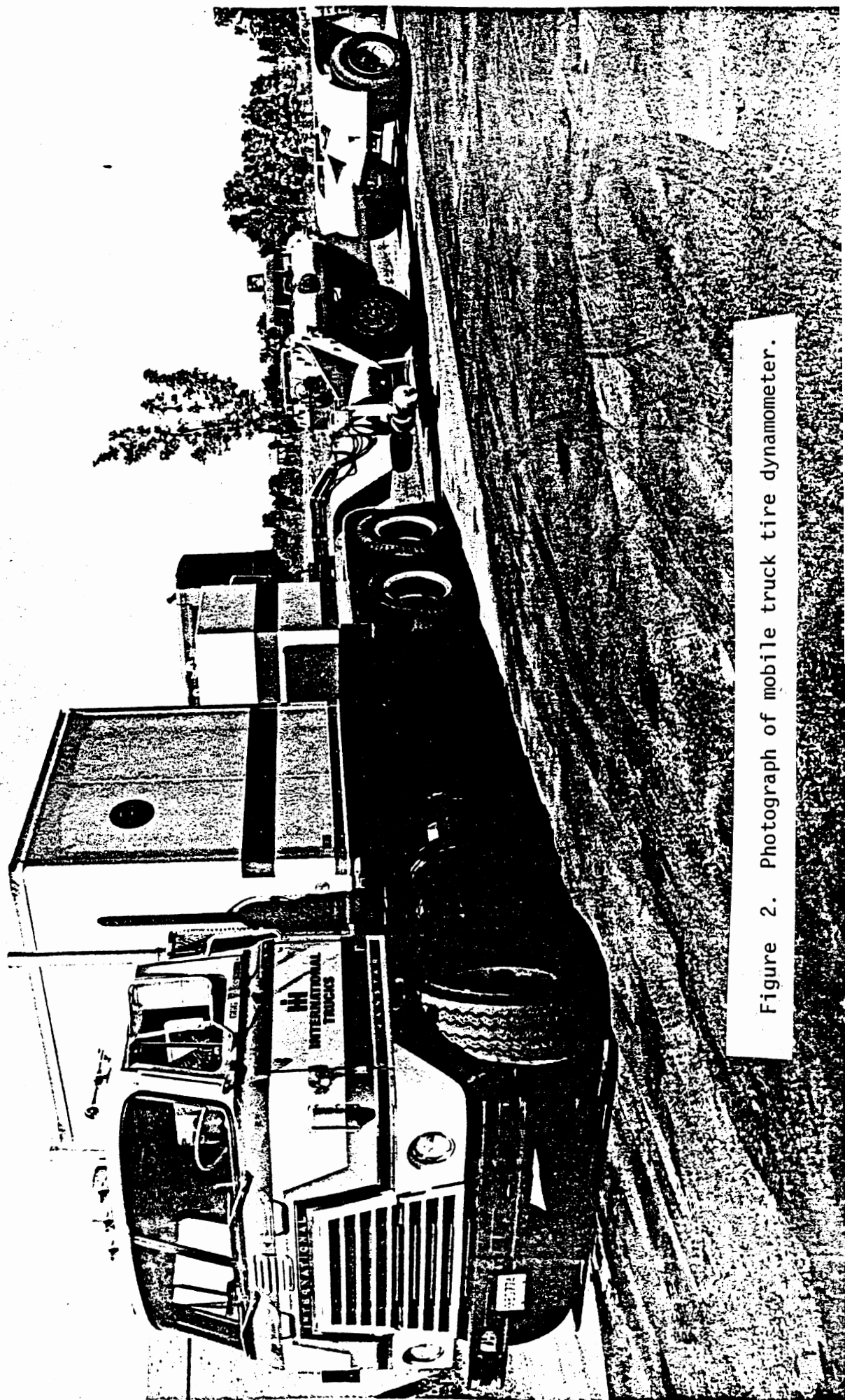


Figure 2. Photograph of mobile truck tire dynamometer.

Table 1. Summary of Test Matrix for the 15 x 5 Drum Brake, 9051 J Lining.

<u>Tests</u>	<u>Conditions</u>
32	Tests at 200 to 1000 psi; 20 and 40 mph
120	Tests at 200 to 1600 psi; 20, 40 and 60 mph
16	Tests at 200 to 1600 psi; 3 mph
71	Warm-up and control tests; 1000 psi at 40 mph

The second brake tested (15 x 5 with ABB 539 lining) was received unburnished. The brake was installed on the mobile dynamometer and was tested in a sequence simulating the FMVSS 105 test procedures, as summarized in Table 2. The effectiveness tests

Table 2. Summary of Test Matrix for the 15 x 5 Drum Brake, ABB 539 Lining.

<u>Tests</u>	<u>Conditions</u>
12	First effectiveness (8674 lb. equivalent load)
500	Burnish
12	Second effectiveness
16	Tests at 500 to 1500 psi; 15, 30, 45 and 60 mph; 150°F initial temperature
20	Tests at 500 to 1500 psi; 15, 30, 45 and 60 mph; 250°F initial temperature
20	Tests at 500 to 1500 psi; 15, 30, 45 and 60 mph; 350°F initial temperature

were performed at speed and torque conditions representing the average values that would be seen by the brake mounted on a vehicle at 8,674 lbs wheel load. The burnish was likewise performed at conditions representing such averages. After the second effectiveness, a matrix of conditions was tested covering a range of

actuation pressures, speeds and initial temperatures to provide a foundation of data for computer analysis and derivation of a brake model.

2.3 Data Analysis

2.3.1 Data Acquisition and Initial Processing. The following variables were recorded during each brake application:

- 1) brake torque
- 2) braking force at the tire-road interface
- 3) hydraulic line pressure
- 4) dynamometer speed
- 5) angular speed of the brake
- 6) lining temperature (SAE thermocouple)

In addition, the temperature of the rubbing surface of the drum was recorded by means of a thermocouple rubbing on the drum surface during the tests with the ABB 539 lining. A schematic diagram of this thermocouple is shown in Figure 3.

These data signals were recorded on an analog tape recorder. At the same time, selected data signals were monitored on a multi-channel pen-chart recorder so that the operator could observe the performance of the brake. Periodically throughout the testing, the operator placed calibration signals for each data channel on the tape recording.

At the conclusion of the tests on each brake, the analog tape recording was digitized on the Institute's AD-4 analog computer. The analog computer filters and amplifies each channel of data and passes them to analog-to-digital converters. The sampling rate of the converters was 10 msec for the 9051 J tests and 20 msec for the ABB 539 tests. The digitized data channels are then sent to a PDP-11 digital computer where they are further amplified and then written onto a digital magnetic tape. The calibration signals on the analog tape recording are also digitized so that the data channels can be scaled into physical units during processing of the digital tape.

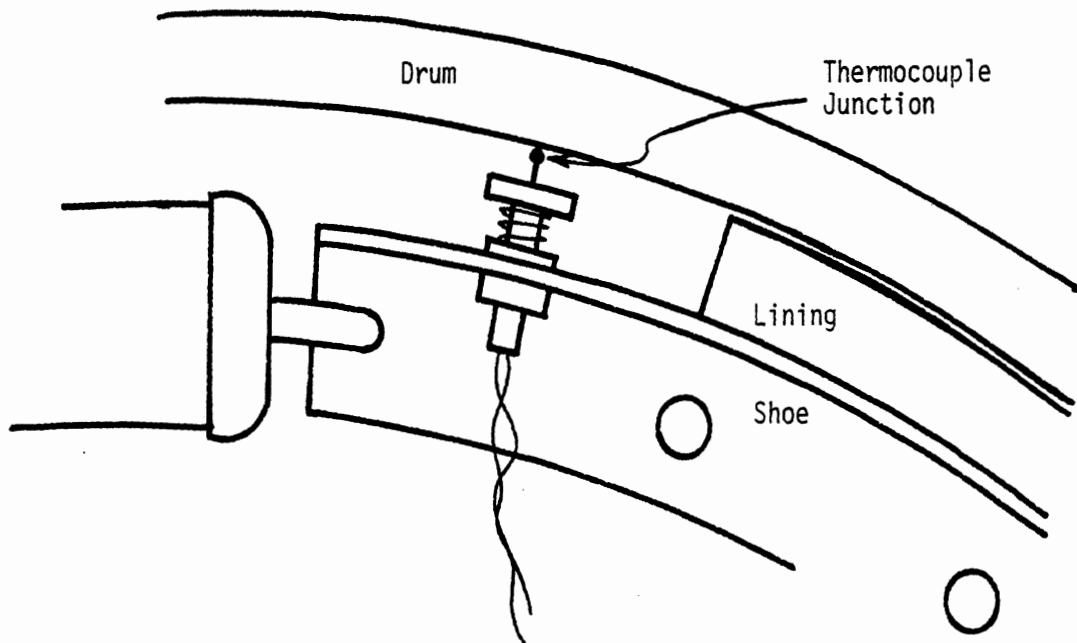


Figure 3. Illustration of drum surface thermocouple installation.

When the data is written onto the digital tape, each brake application is stored in a separate file. The digital tape is then edited so that the individual files are built into larger files containing related groups of brake applications, together with their relevant calibration files. Reformatting the tape in this manner results in better organization of the data for purposes of further processing. In addition, all extraneous and bad data is edited out of the tape at this time.

After the digitized tape is edited, it is processed by program TRANSMOBILE which converts the raw digitized data into physical units. This program creates two outputs. The first is a printed record for each brake application containing a tabularized record of the data at approximately 0.2-second intervals, together with some summary information. A sample of the output for an application is shown in Figure 4. It should be noted that the application time is the time from the onset of brake torque until the line pressure is turned off. Also, the initial values of torque and pressure are those at the time when the pressure has just reached a steady-state condition. The average values of torque, pressure, and rotational speed are computed from the time the initial torque and pressure are reached to the end of the application. In addition to the printed output, a magnetic tape is written, storing for future processing the detailed raw data from each brake application.

2.3.2 Data Reduction. The data records for each brake application, as compiled and corrected by the program TRANSMOBILE, require further reduction before performing the regression analysis. The digital data is processed by a program, DYNADRUM IIIA, to perform the following operations:

- 1) convert actuation pressure to shoe force
- 2) convert rotational speed to rubbing speed
- 3) calculate the instantaneous lining/drum interface temperature
- 4) calculate the instantaneous brake effectiveness.

KELSEY-HAYES 15 X 5 HYDRAULIC BRAKE 9051J LINING
 MVMA PROJECT #1.34 MOBILE DYNAMOMETER 5 APRIL, 1978
 SEQUENCE 27 - 200-1400 PSI, 57 MPH

RUN 170 1000 PSI 57 MPH

	TIME (SEC)	PRESSURE (PSI)	BRAKE TORQUE (K IN-LBS)	FX (LBS)	ROTATIONAL SPEED (RPS)	DYNAMOMETER SPEED (MPH)	LINING TEMPERATURE (DEG F)
1	0.0	72	2.0	140	7.22	55.6	158
2	0.20	1056	57.3	2840	6.89	55.3	169
3	0.41	1076	57.1	2755	6.98	55.8	159
4	0.61	1057	54.2	2710	6.82	55.1	167
5	0.81	1072	55.7	2790	6.82	54.7	158
6	1.02	1041	55.1	2770	6.85	55.7	171
7	1.22	1065	54.7	2770	6.80	55.0	171
8	1.43	1060	53.0	2750	6.75	54.6	172
9	1.63	1063	51.5	2525	6.79	54.3	178
10	1.83	1074	50.0	2525	6.84	54.5	182
11	2.04	1053	46.0	2345	6.79	53.9	185
12	2.24	1063	45.4	2345	6.87	53.8	191
13	2.44	1045	42.0	2140	6.73	53.8	194
14	2.65	1058	42.1	2125	6.73	54.1	201
15	2.85	1051	41.2	2180	6.70	53.7	209
16	3.05	1051	39.8	1995	6.76	53.8	218
17	3.26	1073	38.3	1825	6.74	53.1	222
18	3.46	1054	35.3	1810	6.67	53.5	227
19	3.66	1057	36.7	1960	6.75	53.7	237
20	3.79	1052	36.7	1900	6.73	53.8	243

APPLICATION TIME: 3.79 SEC AVERAGE TORQUE: 47.6 K IN-LBS

INITIAL TORQUE: 53.8 K IN-LBS AVERAGE PRESSURE: 1059 PSI

INITIAL PRESSURE: 1056 PSI AVERAGE ROTATIONAL SPEED: 6.84 RPS

(ABOVE VALUES OCCUR
 AT 0.052 SEC)

LINING TEMPERATURE
 INITIAL 168 DEG F
 MAXIMUM 245 DEG F
 RISE 77 DEG F

Figure 4. Sample output of TRANSMOBILE data reduction program.

Calculation of the shoe actuation force from the actuation pressure is a straightforward process based on the brake cylinder size. The return spring force, readily determined from the experimental tests, is subtracted in the process. Likewise, the rubbing speed is determined from the rotational speed by multiplication by the drum radius.

The instantaneous lining/drum interface temperature is a parameter calculated from a finite element heat transfer model of the brake drum [4]. The drum is treated as a two-dimensional structure divided into elements, as shown in Figure 5. At the beginning of each brake application, the drum is considered to be at a uniform temperature equivalent to the experimentally measured initial temperature. The model calculates the time-dependent heat flow through each element of the drum when subject to the heat flux input at the surface equivalent to the time history of energy dissipation by the brake (torque times speed). Ninety-five percent of the dissipated energy is assumed to flow into the drum, with the remaining five percent going into the lining. Heat flow through the mounting flange and cooling at the outside of the drum is negligible for these short-duration brake applications. The temperature in each element at each point in time is calculated from numerical integration of the conductivity equation. The physical dimensions of the drum model are shown in Figure 5, and the thermal conductivity and diffusivity constants used in the calculations were 30 BTU/hr/ft/°F, and 0.43 ft²/hr, respectively. A more extensive discussion of the interface temperature modeling is provided in Reference [4].

Finally, the brake effectiveness parameter representing the torque output per unit of input force is calculated for each instant of time. The processed data is printed out as illustrated in the example of Figure 6, and is placed on a digital tape file for regression analysis.

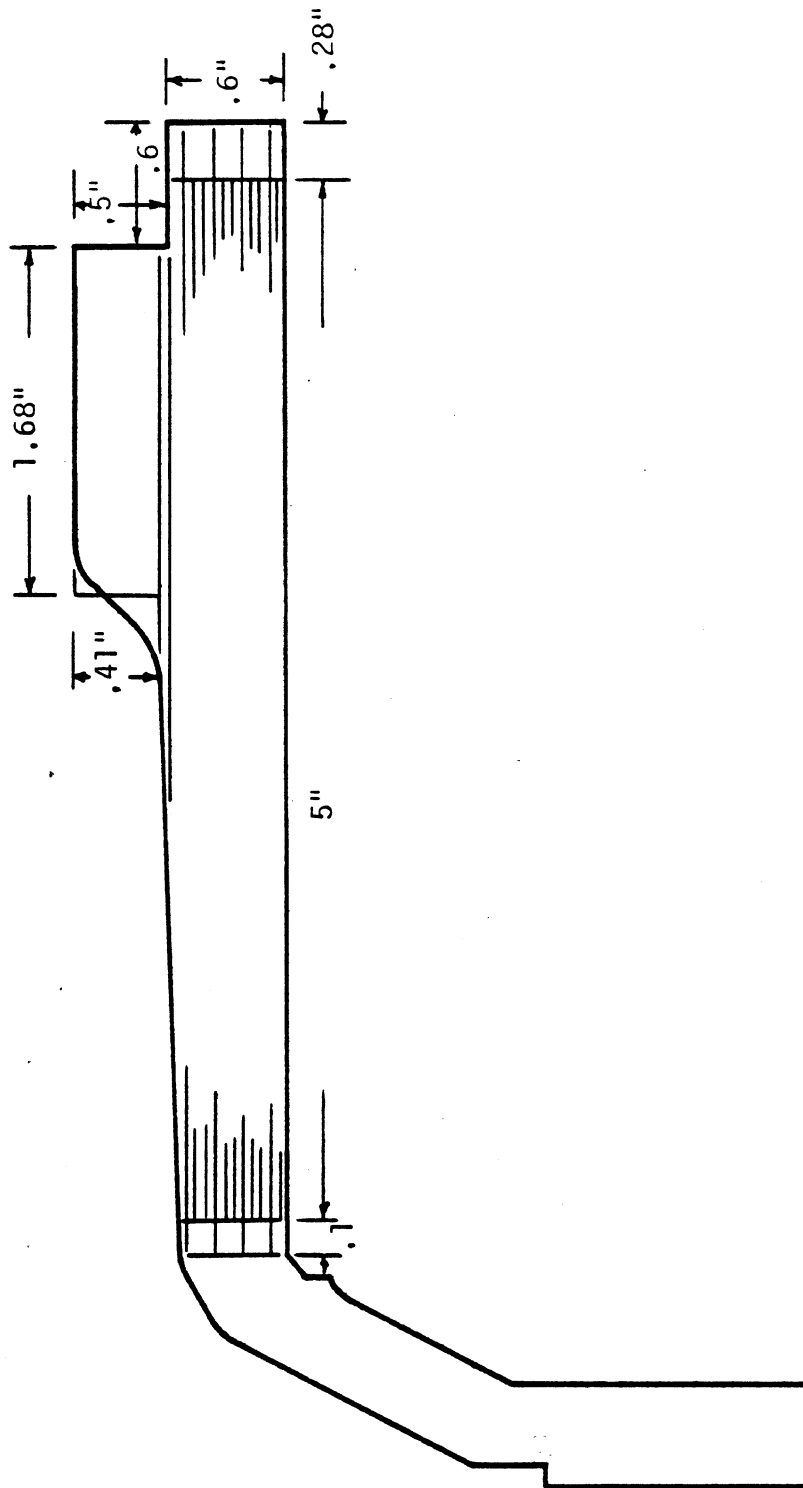


Figure 5. Finite element heat transfer model of the brake drum.

KELSEY-HAYES 15 X 5 HYDRAULIC BRAKE 9051J LINING
 MVMA PROJECT #1.34 MOBILE DYNAMOMETER 5 APRIL, 1978
 SEQUENCE 27 - 200-1400 PSI, 57 MPH

RUN 170 1000 PSI 57 MPH

TIME (SEC)	PRESSURE (PSI)	ROTATIONAL SPEED (RPS)	BRAKE TORQUE (K IN-LBS)	APPLICATION FORCE (LBS)	RUBBING SPEED (FPS)	SURFACE TEMP (DEG F)	EFFECTIVE PRESS (IN-LBS/IN)
1	72	7.22	2.0	-27	28.4	168	---
2	1057	6.91	57.5	1714	27.1	243	33.6
3	1055	6.89	57.0	1709	27.1	306	33.3
4	1071	7.00	56.9	1738	27.5	383	32.8
5	1059	6.82	54.5	1717	26.8	435	31.7
6	1072	6.81	55.8	1739	26.8	478	32.1
7	1045	6.84	55.3	1692	26.9	517	32.7
8	1066	6.81	54.9	1728	26.8	551	31.8
9	1056	6.77	52.9	1712	26.6	576	30.9
10	1065	6.80	51.4	1727	26.7	597	29.8
11	1066	6.80	49.7	1729	26.7	615	28.7
12	1054	6.79	47.2	1707	26.7	632	27.6
13	1061	6.81	45.9	1720	26.7	644	26.7
14	1052	6.75	42.9	1704	26.5	654	25.2
15	1063	6.77	42.6	1724	26.6	663	24.7
16	1048	6.74	40.4	1698	26.4	668	23.8
17	1058	6.75	39.6	1715	26.5	676	23.1
18	1057	6.73	38.4	1712	26.4	682	22.4
19	1052	6.72	37.5	1704	26.4	689	22.0
20	1057	6.70	37.3	1713	26.3	693	21.8
21	1052	6.73	36.7	1704	26.4	698	21.5

Figure 6. Sample output from DYNADROM IIIA data reduction program.

2.3.3 Regression Analysis of the Effectiveness Function. In its processed form, the digital data contains multiple data sets consisting of the effectiveness, actuation force, sliding velocity, and interface temperature parameters organized into files for individual brake applications. The challenge is then to derive a mathematical model that can express the effectiveness as a function of the three other variables with sufficient flexibility that it can reasonably match all the data sets.

The approach taken here was to fit the data by a multi-variable regression analysis in which the effectiveness can be expressed in the form:

$$e = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} a_{ijk} \theta^{i-1} v^{j-1} F^{k-1} . \quad (1)$$

where

θ = interface temperature

v = sliding velocity

F = actuation force

The effectiveness function is obtained by a conventional computer library statistical program which generates a least squares curve fit to the processed data [4]. The function is then represented by a set of coefficients for the above equation. In addition, the coefficient of determination and standard error can be used to assess the goodness of the curve-fitting process.

3.0 HYDRAULIC BRAKE EFFECTIVENESS FUNCTIONS

The experimental data obtained from the hydraulic brake tests were analyzed by the methods described in the preceding sections to investigate the nature of the effectiveness function, degree of accuracy with which the model could be matched to the brake characteristics over all conditions, and the ability of the model to predict torque performance under arbitrary test conditions.

3.1 Typical Effectiveness Functions

The effectiveness functions take the form shown in Figures 6, 7, and 8. The effectiveness is plotted as a function of calculated interface temperature for different sliding velocities, with each graph for a different actuation force level. Figures 6 and 7 are both for the brake with 9051J lining based on tests near the beginning (sequences 7-10) and near the end (sequences 23-37) of the test program. For this brake, the effectiveness falls typically in the range of 30-40 in-lb/lb with strong temperature dependence, especially above 500°F. The brake data shows a peculiarly high temperature dependence at the very low (5 fps) speed. At other speeds, the performance seems more consistent with effectiveness curves that tend to overlap each other. A comparison of the plots in Figures 6 and 7 shows a systematic increase in the effectiveness during the latter test sequence indicating changing brake characteristics, a point which will be discussed in detail later.

Figure 8 shows the effectiveness determined for the brake with ABB 539 lining. It may be characterized by an overall higher effectiveness level, greater speed dependence (the plots for different speeds are separated), and more temperature sensitivity. Since the two brakes were mechanically similar, the difference between the plots of Figures 6 and 7 versus Figure 8 are primarily a reflection of differences in lining coefficient. In general, the

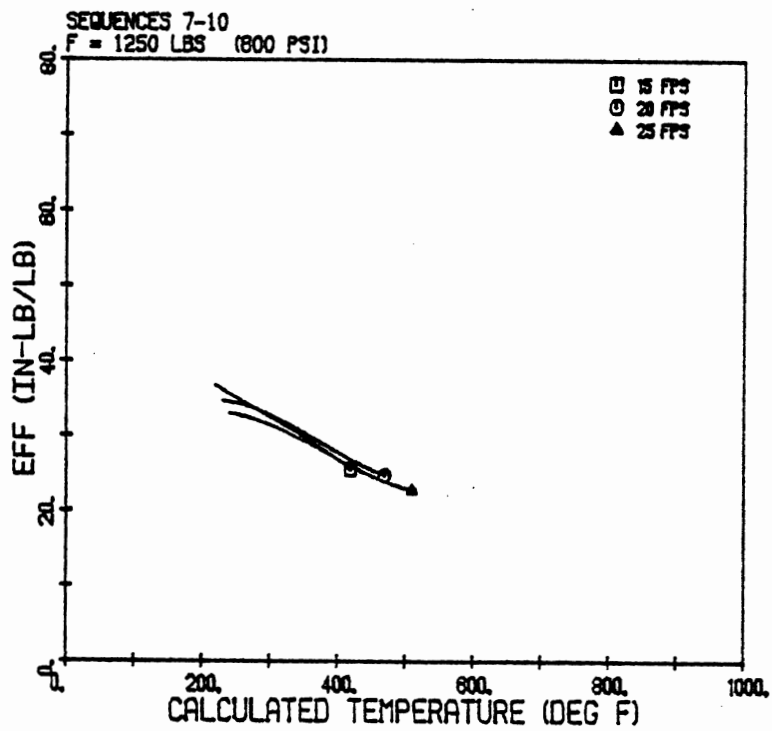
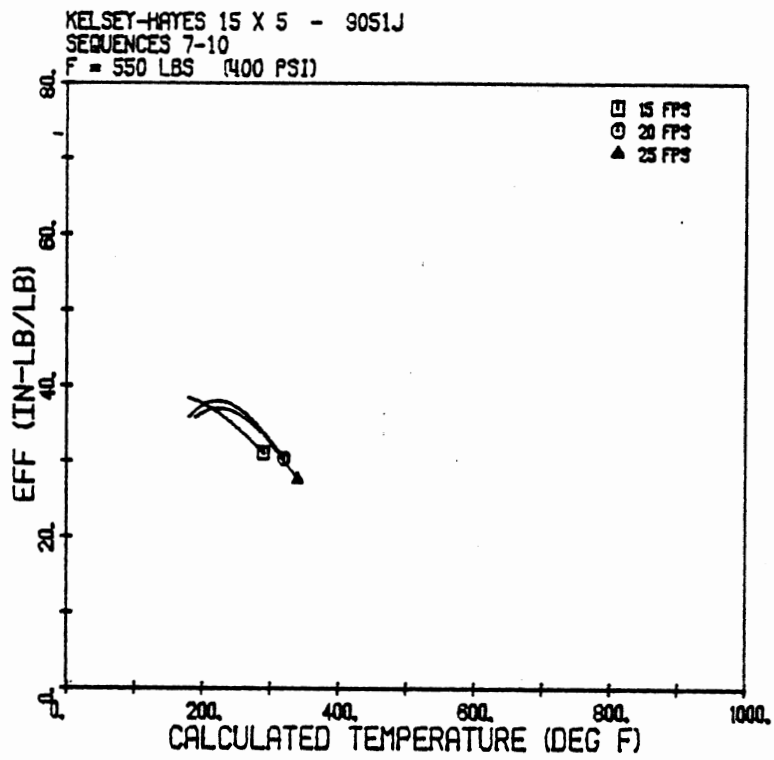


Figure 6a, b. Effectiveness plots for brake with 9051J lining - 400 psi and 800 psi, test sequence 7-10.

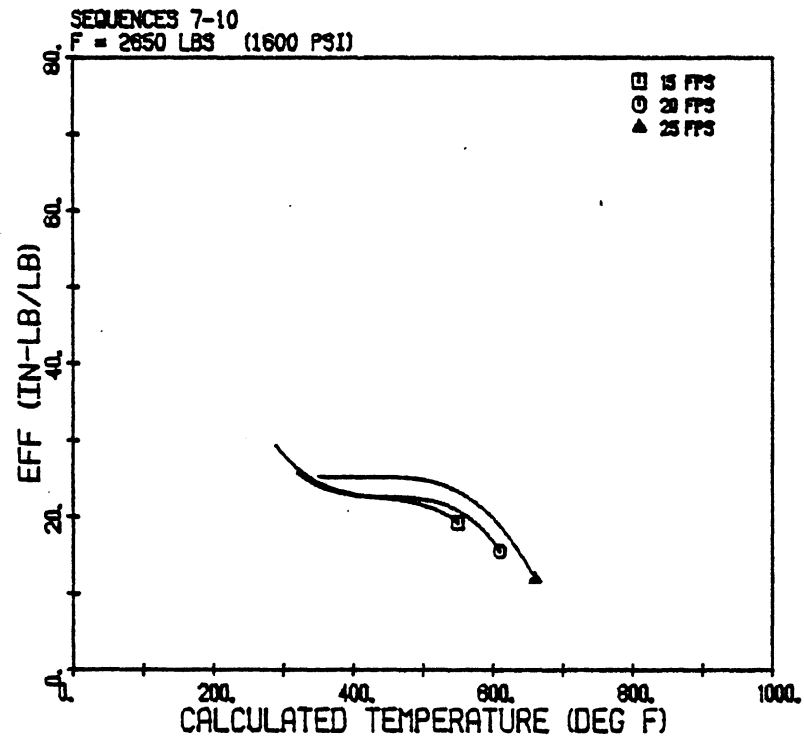
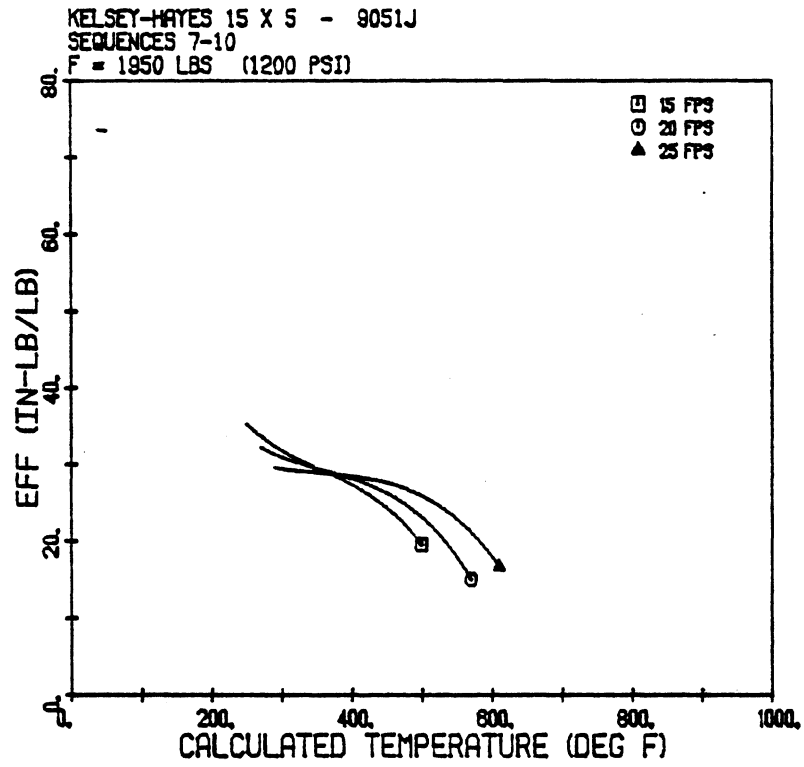


Figure 6c, d. Effectiveness plots for brake with 9051J lining - 1200 psi and 1600 psi, test sequence 7-10.

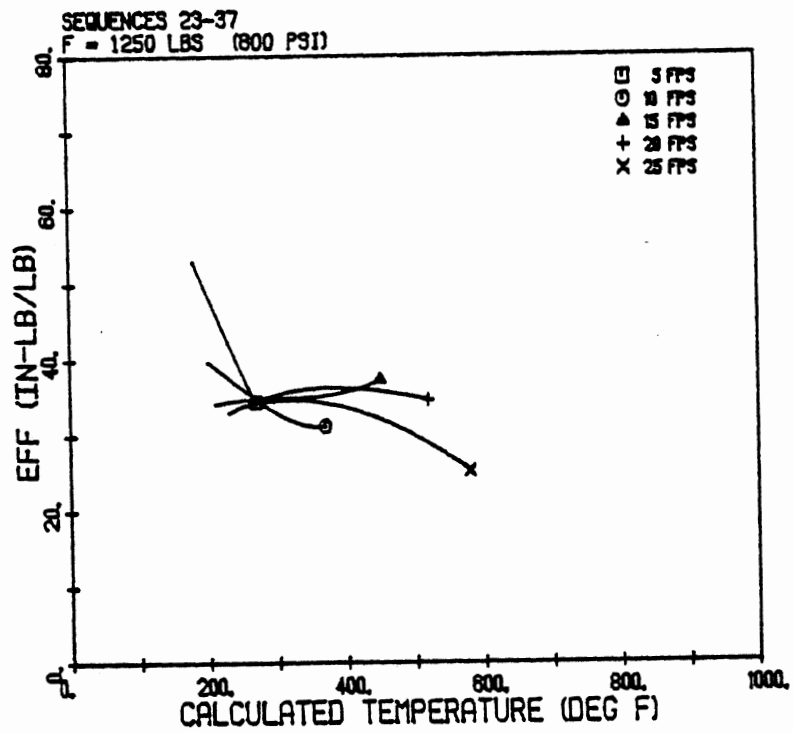
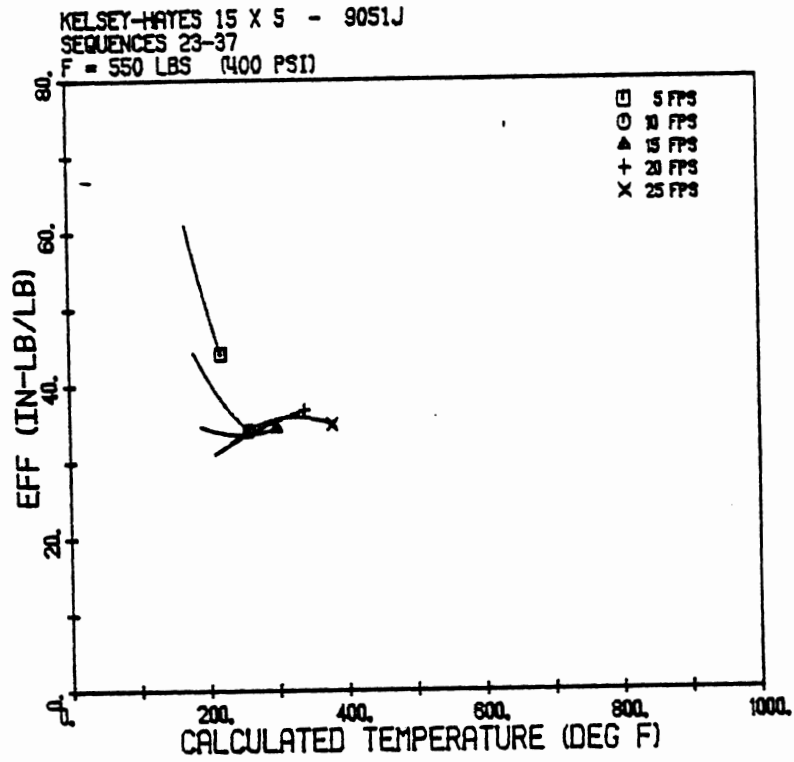


Figure 7a, b. Effectiveness plots for brake with 9051J lining - 400 psi and 800 psi, test sequence 23-37.

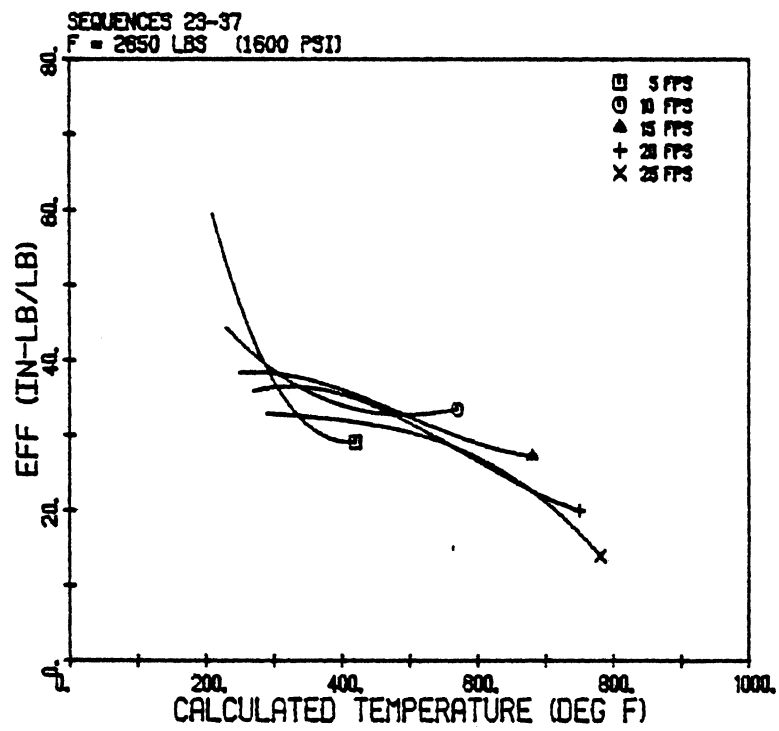
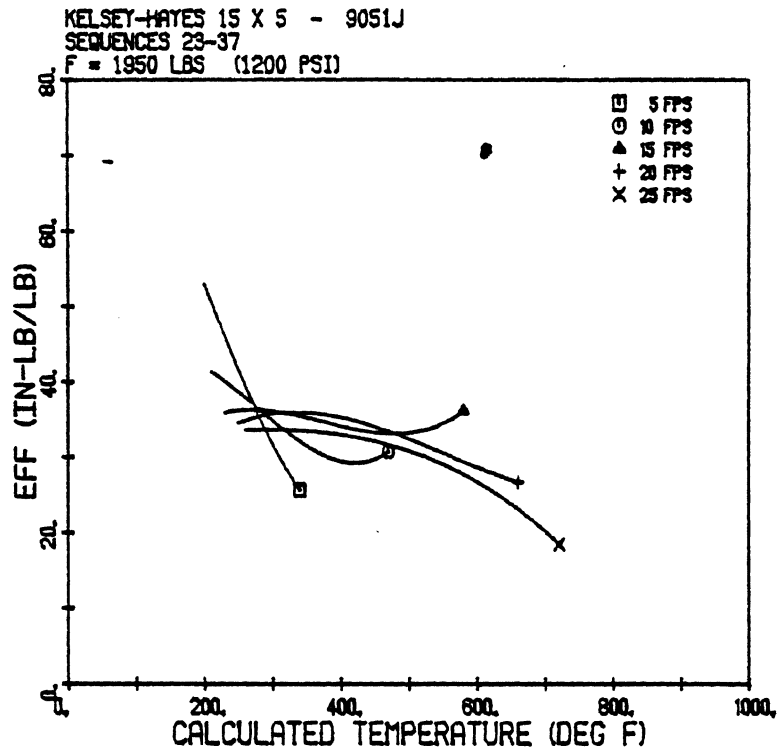


Figure 7c, d. Effectiveness plots for brake with 9051J lining - 1200 psi and 1600 psi, test sequence 23-37.

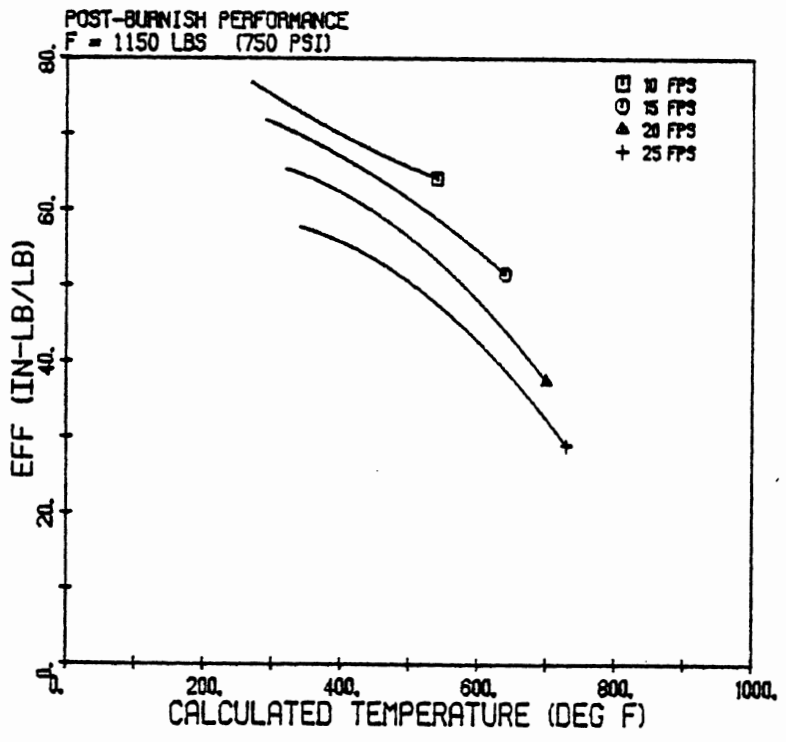
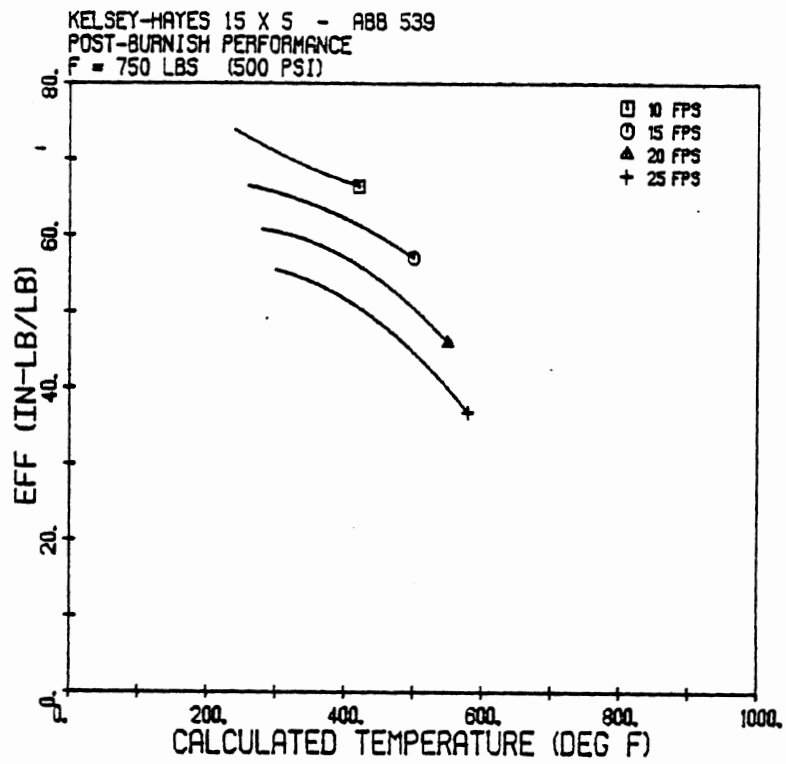


Figure 8a, b. Effectiveness plots for brake with ABB 539 lining - 500 psi and 750 psi.

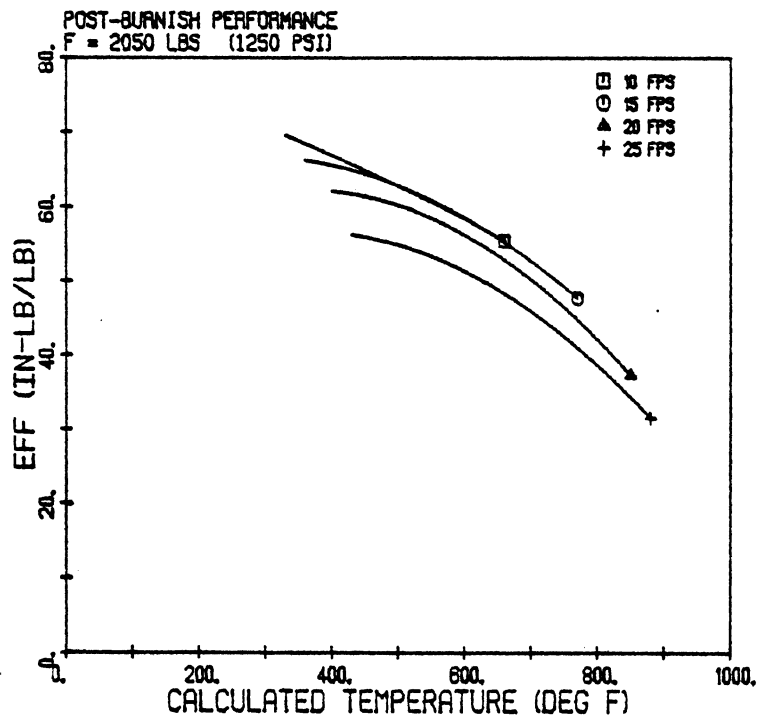
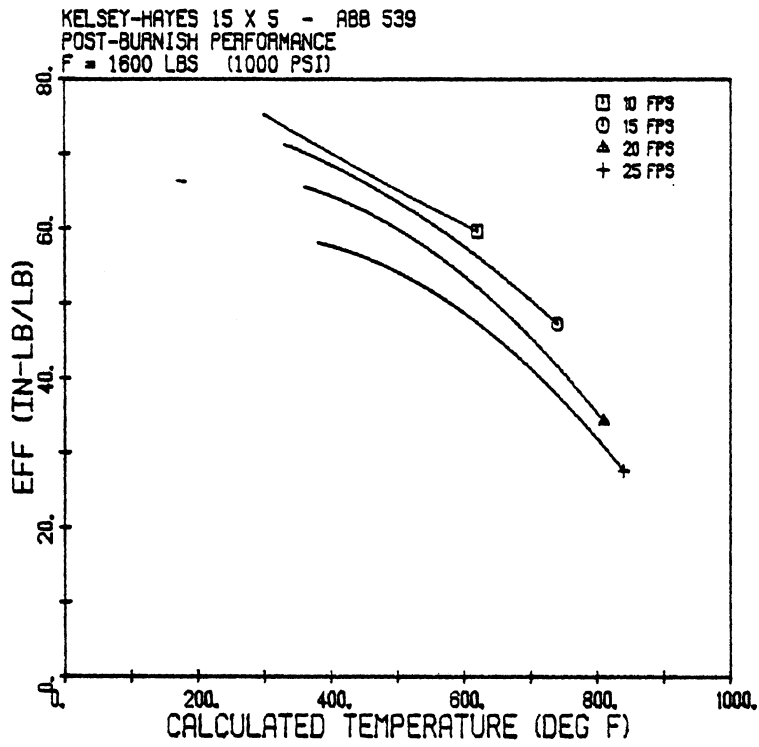


Figure 8c, d. Effectiveness plots for brake with ABB 539 lining - 1000 psi and 1250 psi.

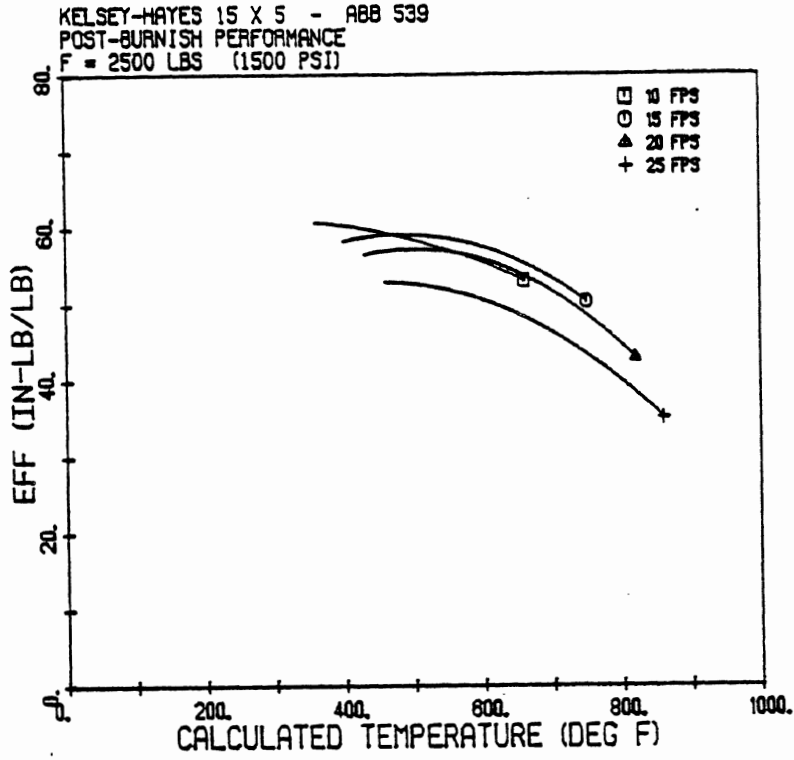


Figure 8e. Effectiveness plot for brake with ABB 539 lining - 1500 psi.

characteristics exhibited by the effectiveness function may be viewed as closely parallel to the characteristics of lining coefficient.

The effectiveness functions shown in these figures are actually plots of Equation (1) using the coefficients, a_{ijk} , obtained in the regression analysis. Appendix A contains summary results of the regression analysis.

3.2 Evaluation of Modeling Errors

The regression analysis to obtain the effectiveness function is, in effect, an effort to fit a multi-nomial expression to a set of experimental data. A practical concern in such an exercise is to obtain a measure of the error associated with the fit, both to assess the quality of the experimental data and to judge the adequacy of the multi-nomial expression used. In a regression analysis, the quality of fit can be judged by two statistics routinely calculated—the coefficient of determination and the standard error. The first is a measure of the fraction (ranging from 0 to 1) of the variation in the data that is accommodated by the multi-nomial expression. Random variation in the experimental data and systematic changes in brake behavior (without a matching variable in the regression analysis) both reduce the coefficient of determination. Coefficient of determination values greater than 0.9 are generally considered indicative of a good fit in regression analysis. The standard error is analogous to the standard deviation of the differences between the experimental and predicted data points. Once a good fit to the data is obtained (indicated by a high coefficient of determination), the standard error provides a measure of the random variation in the experimental data.

The adequacy of the regression model was evaluated by examining the coefficient of determination obtained when the number of terms in the multi-nomial was varied and when various sets of experimental data covering extended periods of testing were combined. The number of terms in the multi-nomial is the product of the upper limits on the summations in Equation (1).

For the hydraulic brakes tests, an expression consisting of 27 (for the ABB 539 lining) to 36 terms (9051J lining) proved optimal in the sense that fewer terms were insufficient to conform to the variational effects, reducing the coefficient of determination, and a greater number of terms did not improve the coefficient of determination significantly. The optimal number of terms in the regression models tended to result in coefficients of determination in the range of 0.92 to 0.95.

Brakes characteristically experience systematic changes in performance with use. The implication is that work history is a variable influencing the effectiveness. Though work history was not of direct interest in this modeling study, its effect had to be considered in the development of the study because the systematic variations would influence the fit of the regression model. Ultimately, it would be a logical extension of this study to develop a characteristic variable representing work history and include it as an independent variable in the regression analysis. A broader discussion of this subject is included in Reference [4].

Nevertheless, in this project work history was only considered as a potentially confounding effect on quality of fit. The effect becomes evident in the coefficient of determination obtained when sets of experimental data are combined into one analysis. The 15 x 5 brake with 9051J lining was put through over 200 tests, including three duplicate sets of applications covering 200-1600 psi and 20, 40, and 60 mph. The first set (sequence 7-10, Fig. 6) was clearly distinguished from the last two sets (sequence 23-37, Fig. 7) by a reduction in the coefficient of determination when all three were combined. On the other hand, the last two were combined on the justification that the coefficient of determination for the combination was not significantly different from that obtained from each when individually analyzed. Thus, the indication is that the brake, received burnished and tested on an inertia dynamometer, continued to change in effectiveness with testing on the mobile dynamometer through approximately one-half of the test program, finally stabilizing during the last half of the program.

Recognizing that the brake with the 9051J lining must be characterized at different points by the separate effectiveness functions, it is appropriate to examine the standard error obtained to assess the error in the predictions that may be expected because of the random variability of the brake. The standard error may be considered as a measure of the difference between predicted and observed performance at any arbitrarily selected point in a test. Because the brake torque does not vary randomly at each instant of time throughout a test (i.e., it is not high one instant and low the next), the variations may not necessarily average out over an application. Hence, the stop-to-stop torque variability of a brake is likely to be in proportion to the standard error, and this statistic can be expected to reflect the overall variability of the brake.

Table 3 summarizes the standard errors obtained with the two tested brakes. When normalized by the mid-range value of the

Table 3. Standard Errors Obtained in the Regression Analysis.

<u>Brake Lining</u>	<u>Standard Error</u>	<u>Mid-Range Effectiveness</u>	<u>Percent</u>
9051J (sequence 7-10)	1.72	25	7
9051J (sequence 23-37)	2.79	37	7
ABB 539	4.81	53	9

effectiveness, the standard errors are equivalent to 7-9 percent. These values are typical of the range of variability commonly ascribed to brakes. A practical interpretation of this statistic is to say that the brake torque will be repeatable within 14% (9051J lining) to 18% (ABB 539 lining) for 95% of brake applications (i.e., two standard errors). Likewise, the higher variability of the more effective ABB 539 lining is an expected trend.

3.3 Prediction of In-Stop Torque

The ultimate purpose of the brake effectiveness model is for prediction of the torque output during a brake application. The output is, of course, directly dependent on the time histories of temperature, actuation force, and velocity, thus making the result specific to the particular test situation, whether it be on-vehicle or on a brake dynamometer.

Tests were made to compare the torque predictions of the effectiveness model obtained from the regression analysis of mobile dynamometer data against the measured performance on the inertia dynamometer. Such data were available for the brake with the 9051J lining. Torque and speed traces for tests on the inertia dynamometer were obtained from Kelsey-Hayes along with their Test Report No. 77-37 documenting the dynamometer test results for this brake. A computer simulation model of an inertia dynamometer was prepared and programmed with the equivalent test wheel load of 7436 lb and a 19.3-inch rolling radius. Torque input to the inertia dynamometer simulation was obtained at each instant of time from the effectiveness function according to the equation:

$$\text{Torque} = e(\theta, v, F) \cdot F \quad (2)$$

where

e = effectiveness function

θ = interface temperature

v = sliding velocity

F = actuation force

The interface temperature at each instant of time was determined from the initial temperature and the time-dependent changes calculated in the finite element heat transfer model described earlier.

The effectiveness function used was obtained from the first full set of tests (sequence 7-10) on the Mobile Dynamometer.

(See Appendix A for values of the 36 coefficients for this effectiveness function.) Figures 9-14 show the comparison of torque traces for two actuation levels over three speeds. In these plots the "measured" refers to the inertia dynamometer test trace while the "simulated" means results from the inertia dynamometer simulation model. The best agreement is obtained at the low braking levels with errors of increasing magnitude at the high speed, high actuation force levels. The differences seen here are reasonable when viewed in light of the earlier observation that the brake effectiveness was increasing from test to test in the first portion of the mobile dynamometer tests on this brake. In the 20-mph (800 and 1600 psi) and the 40-mph (800 psi) tests, the rubbing speed traces are in very close agreement despite some apparent differences in the torque-time plots. However, to achieve such close agreement in speed, the torque values must also be in close agreement. Hence, it must be concluded that much of the difference between the measured and simulated torque values is a result of zero and calibration errors on the inertia dynamometer strip chart, or due to errors in reading the torque values off the strip chart.

In the 60-mph comparisons, the differences due to the suspected increasing brake effectiveness show up. In both 60-mph tests, the simulated dynamometer results indicate higher torques and higher deceleration levels than those measured earlier on the inertial dynamometer, leading to the conclusion that the brake effectiveness at these conditions had increased by the time of testing on the mobile dynamometer.

These comparisons vividly illustrate the phenomena of brake effectiveness changes with work history. Although the brake model itself appears quite capable of assimilating data from any source (inertia dynamometer, constant-speed mobile dynamometer, or even on-vehicle tests) to produce a characterization of the brake capable of predicting its output under other circumstances, that capability is limited when brake effectiveness changes occur. The changes occur with work history, indicating that the brake's characteristics

KELSEY-HAYES 15 X 5 - 9051J
 POST-BURNISH EFFECTIVENESS
 STOP 4 20 MPH 800 PSI 200 DEG F

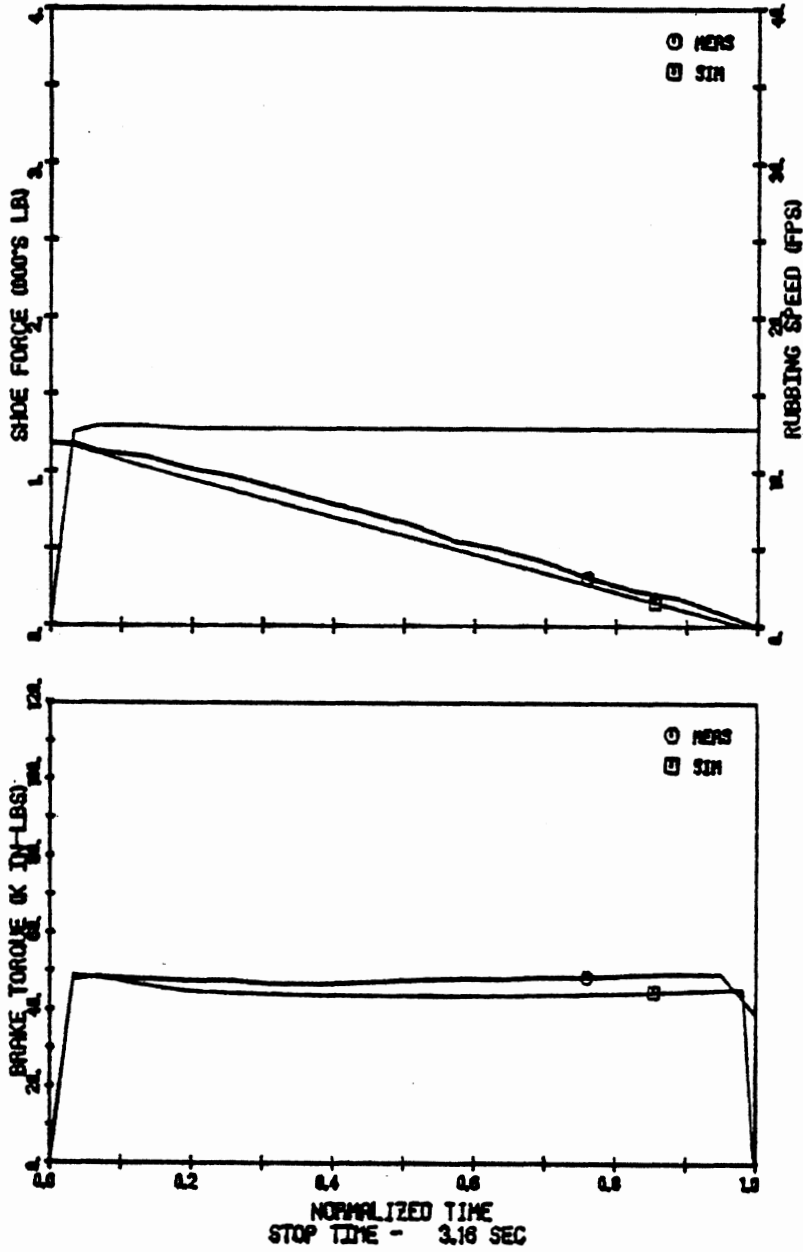


Figure 9. Comparison of the measured and simulated brake performance on an inertia dynamometer - 800 psi, 20 mph.

KELSEY-HAYES 15 X 5 - 8051J
 POST-BURNISH EFFECTIVENESS
 STOP 8 20 MPH 1600 PSI 200 DEG F

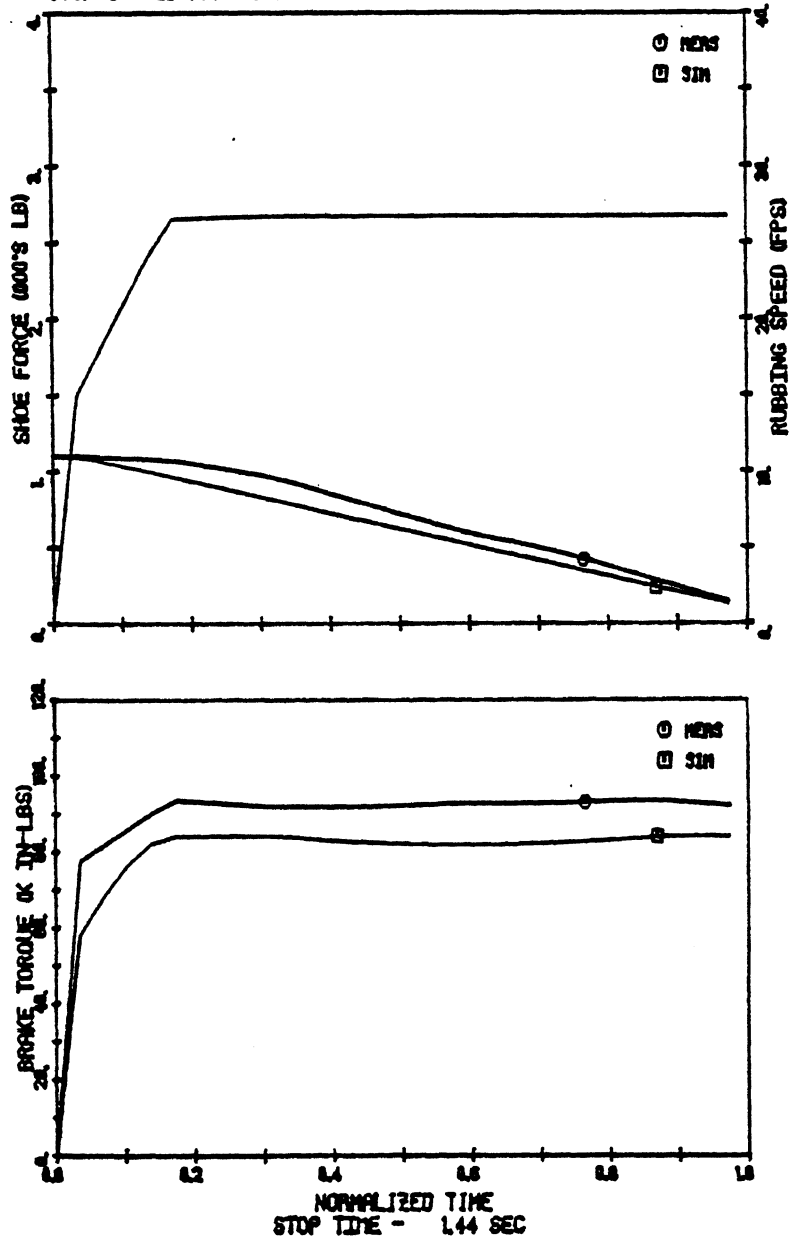


Figure 10. Comparison of the measured and simulated brake performance on an inertia dynamometer - 1600 psi, 20 mph.

KELSET-HAYES 15 X 5 - 9051J
 POST-BURNISH-EFFECTIVENESS
 STOP 13 40 MPH 800 PSI 200 DEG F

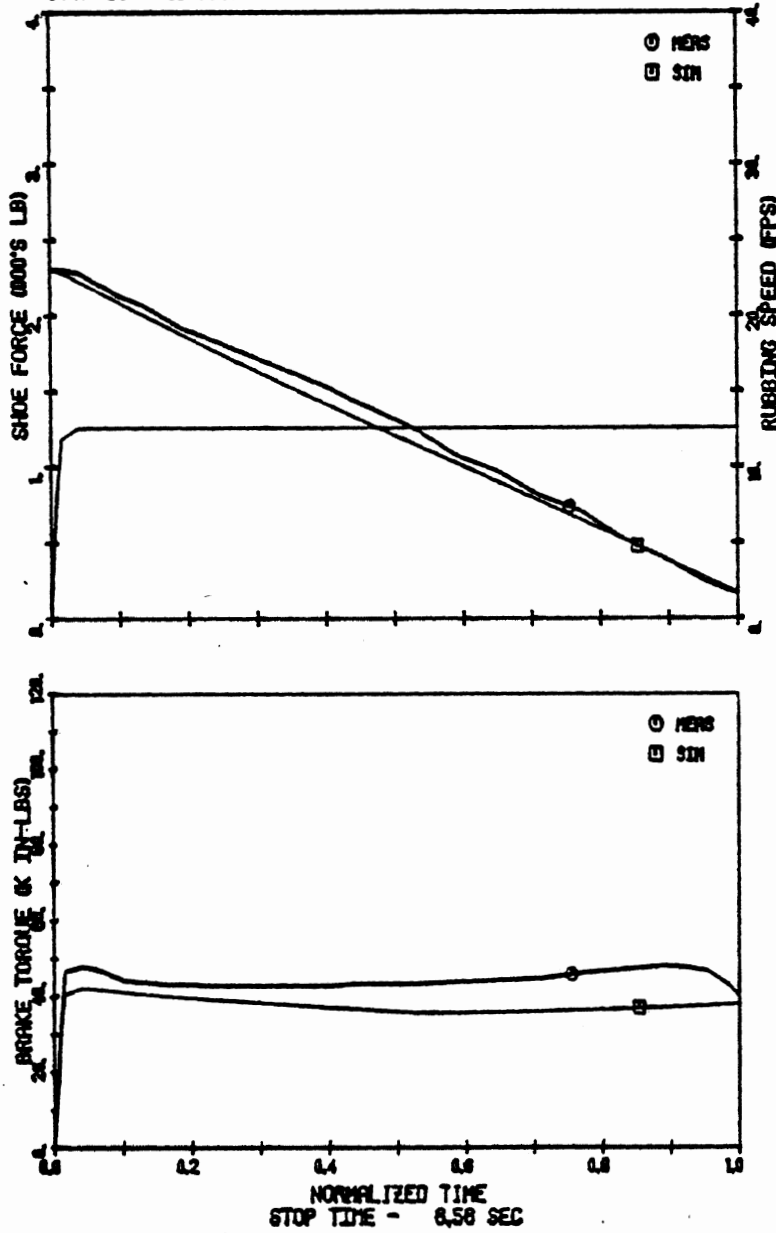


Figure 11. Comparison of the measured and simulated brake performance on an inertia dynamometer - 800 psi, 40 mph.

KELSEY-HAYES 15 X 5 - 9051J
 POST-BURNISH EFFECTIVENESS
 STOP 17 40 MPH 1600 PSI 200 DEG F

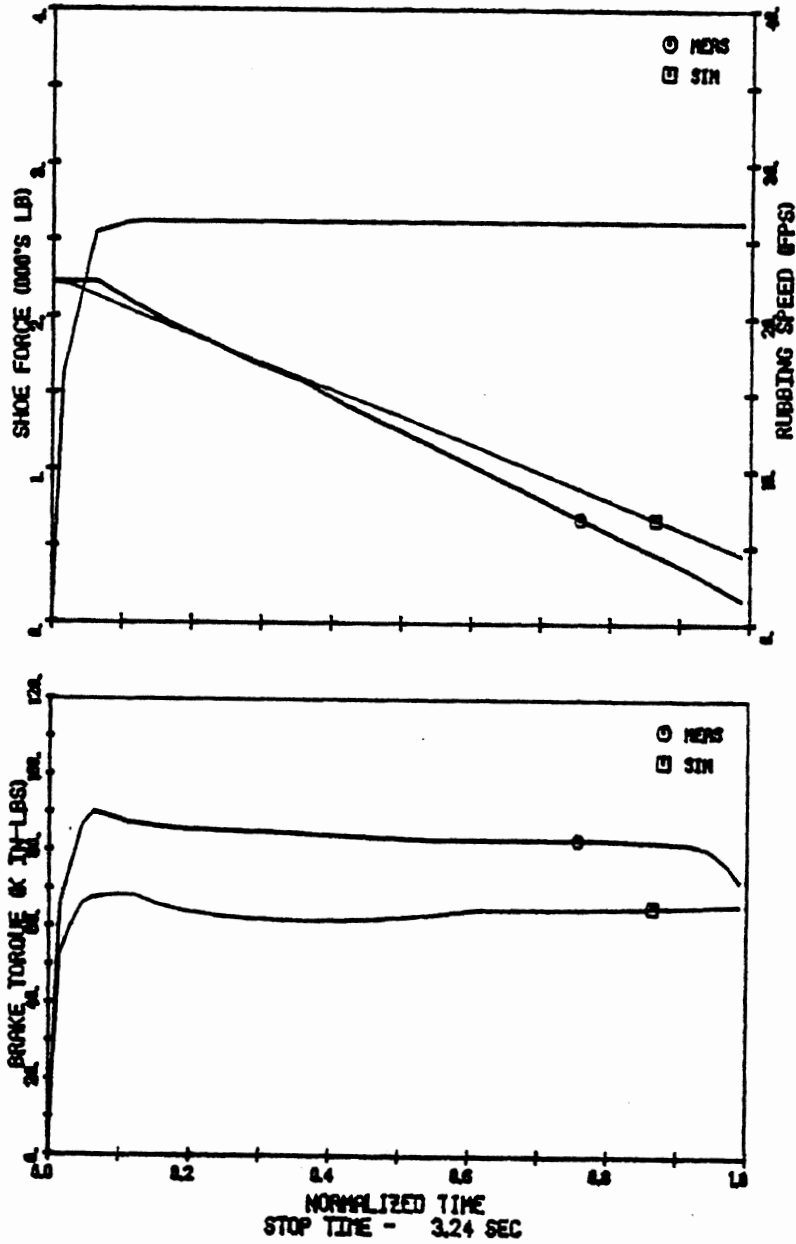


Figure 12. Comparison of the measured and simulated brake performance on an inertia dynamometer - 1600 psi, 40 mph.

KELSEY-HAYES 16 X 5 - 9051J
 POST-BURNISH EFFECTIVENESS
 STOP 17 60 MPH 800 PSI 200 DEG F

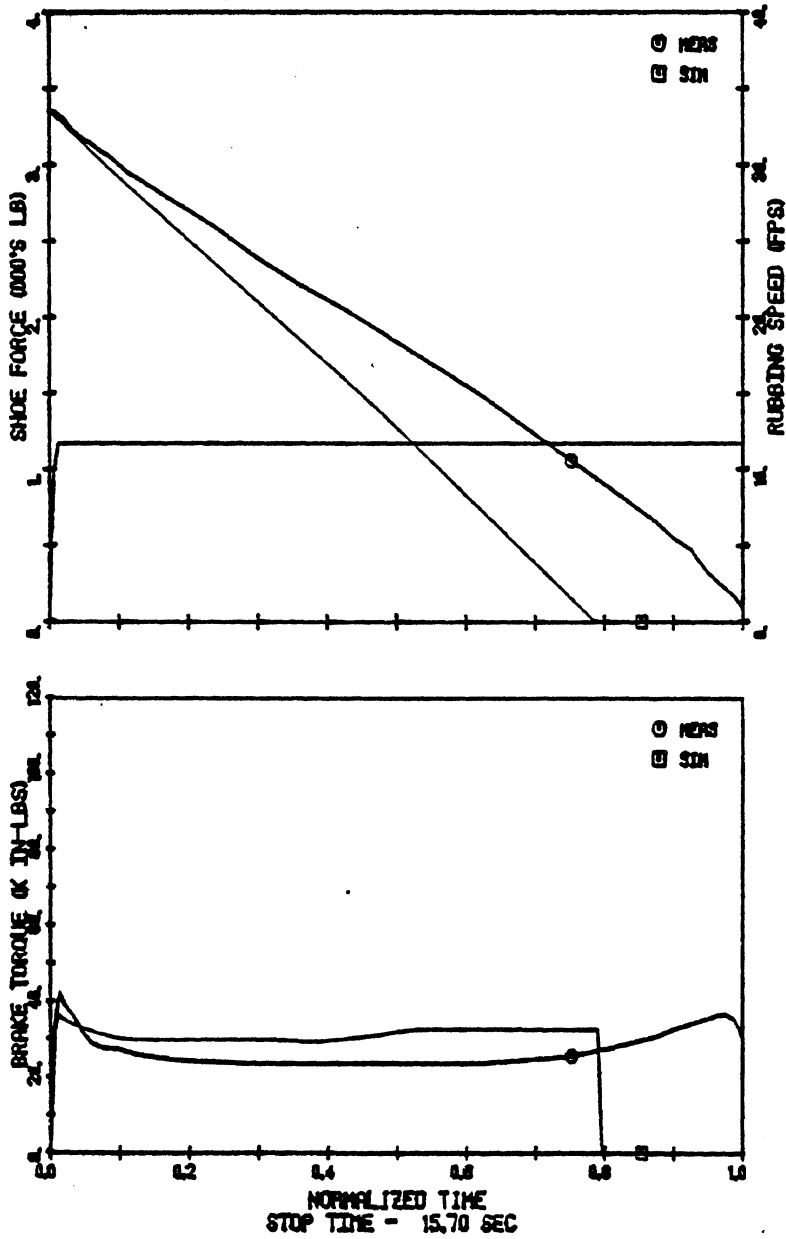


Figure 13. Comparison of the measured and simulated brake performance on an inertia dynamometer - 800 psi, 60 mph.

KELSEY-HAYES 15 X 5 - 9051J
 POST-BURNISH EFFECTIVENESS
 STOP 26 60 MPH 1600 PSI 200 DEG F

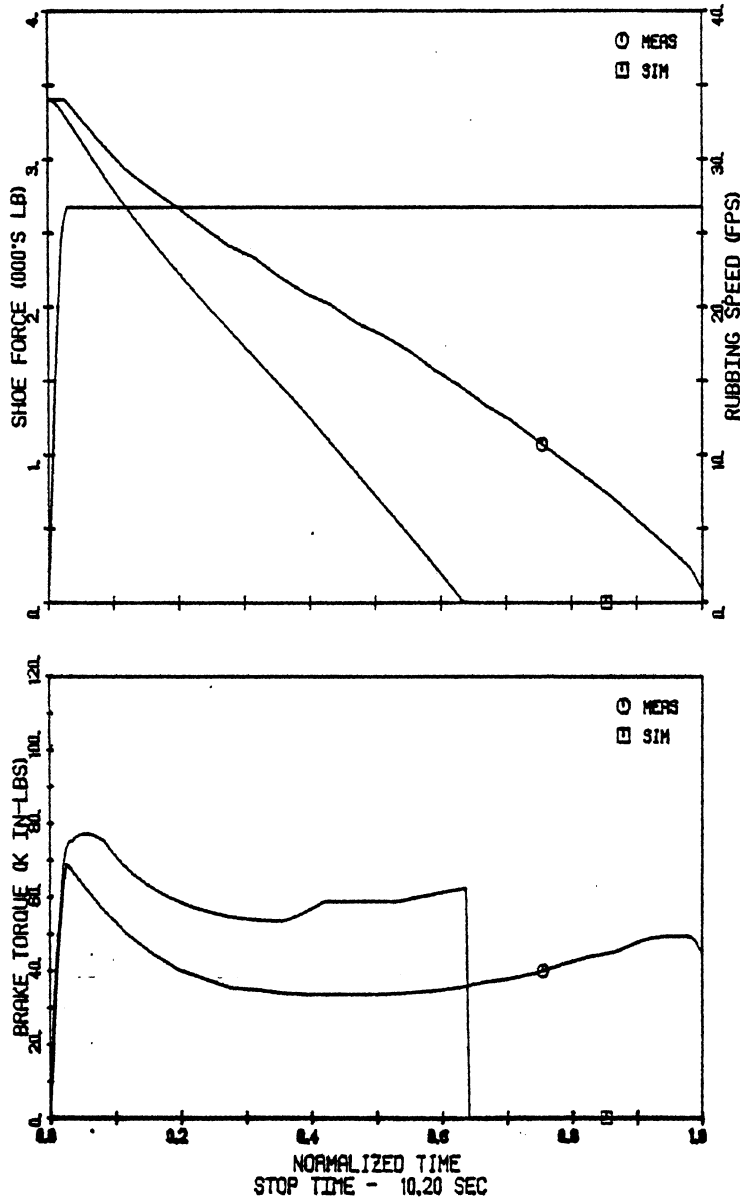


Figure 14. Comparison of the measured and simulated brake performance on an inertia dynamometer - 1600 psi, 60 mph.

are not consistent but change with each application, whereas the brake regression model as developed here is intended to characterize a consistent brake under arbitrary stopping conditions. In order to also characterize the work history effects as well, one or more work history variables would have to be added to the regression analysis. At this stage of understanding in brake technology, the work history variables are unknown but would likely include factors related to the chemistry and reaction rates of the brake linings.

4.0 APPLICATION OF RESULTS

The primary intent of this research program has been to apply available technological methods to the problem of predicting performance of hydraulically-actuated brakes in order to assess the potential for developing improved braking performance models needed for regulatory compliance. The exercise has proved most instructive in ways relating directly to FMVSS 105-type compliance issues, but has also generated new ideas relating to other areas of applicability.

4.1 FMVSS 105 Braking Performance Predictions

In the event an FMVSS 105 braking performance regulation is promulgated for commercial trucks, the manufacturing industry is faced with the requirement of developing methodology for assuring compliance for the broad spectrum of vehicles represented. Because of the design and operating variables associated with vehicles over 10,000-lb GVW (i.e., different axle rating combinations, tire sizes, wheelbases, etc.), a number of brake system components and combinations must be developed. For any moderately challenging performance requirement, accuracy in performance prediction can often minimize the complexity of an overall brake release with significant savings in development costs. In addition, the resort to costly alternatives such as antilock systems can most likely be avoided with the development of accurate predictive methods.

In contrast to FMVSS 121, FMVSS 105 braking performance depends only on the peak tire traction limits on dry surfaces and avoids the complex issues of tire traction under high slip conditions and on low coefficient (wet) surfaces. Hence, characterization of the tire traction limits for FMVSS 105 are rather straightforward, allowing accurate modeling in this respect. Hence, the primary challenge in development of accurate predictive methodology for FMVSS 105 performance lies in the modeling of instantaneous brake torque so that the stopping distance for specific brake combinations can be calculated. With accurate brake torque models,

computer routines can readily be developed to labor through the matrix of vehicle combinations offered by a manufacturer, in order to define the acceptable limits of brake proportioning appropriate to each. Once reduced to this level, the design of a brake release to meet FMVSS 105 performance requirements becomes only the rational process of selecting the necessary proportioning schemes. In the event the FMVSS 105 performance requirements are so stringent that a given front/rear brake combination cannot meet the requirements with a single proportioning scheme over the range of conditions reflected by the first through fourth effectiveness tests, such a computer routine can easily identify the nature of the compliance problems and where performance improvements are needed.

The logical question arising at this point is this—is the regression fit brake effectiveness model used in this research sufficiently accurate to meet these needs? Obviously, the final assessment of the required accuracy depends on the stringency of the FMVSS 105 performance requirements eventually imposed. Nevertheless, for a typical drum brake, it is anticipated that the instantaneous torque performance (at a given point in the brake's work history) can be predicted throughout any arbitrary braking test with a nominal accuracy of 15%, about 95% of the time. This approach represents a substantial step forward when compared with the more traditional measures of peak and average torque because these measures are defined only for certain pressure, load, and initial speed conditions and estimates of on-vehicle torque characteristics can only be inferred by interpolation from these conditions.

However, as indicated in the earlier sections, the major problem in achieving this accuracy derives from the difficulty in following the brake characteristics as the work history develops. The problem is in no way unique to this approach, but, in fact, is a manifestation of the chronic problem of characterizing brake performance through some period of use, whether on a dynamometer or on a vehicle. The approach of regression modeling simply gives a

precise quantitative method of describing the changes that occur with brake use, thus serving the functions of:

- 1) Providing a precise method for describing brake characteristics at any point in FMVSS 105 effectiveness testing.
- 2) Providing a quantitative method for judging the overall variability of a brake on a stop-to-stop basis and throughout its work history.

While this second function may not be common practice in the industry at this time, it may very well prove an essential consideration in brake selection under FMVSS 105 regulation.

It is clear that further research is required to understand the nature of changes in brake torque characteristics with work history. Though a given brake type may exhibit reasonably predictable performance in the "green" condition, the burnish and subsequent changes in performance may be expected to vary with vehicle application because of the different companion brakes and/or proportioning ratios used. Hence, the research should focus on discovery of factors in the work history that control the changing characteristics so that the long-term changes in effectiveness can be predicted just as the short-term (in-stop) changes are predicted in this research by the regression model.

4.2 Wear Balancing

One criterion of brake system design is achievement of reasonable wear balance between the front and rear axles. For hydraulically-braked trucks this aspect of design can be complicated by the numerous front and rear axle brake combinations offered, and by the potential number of proportioning schemes necessary should FMVSS 105 regulations be imposed. Though brake wear balance is a function of many variables, many of which must be empirically determined, temperature balance between axles is recognized as one of the important variables. Though absolute prediction of brake

temperatures in any duty cycle is yet a difficult task, the brake torque regression model offers opportunities to analytically predict temperature balance as a tool in achieving acceptable wear balance throughout a spectrum of vehicles. Through its capability to model brake effectiveness as a function of temperature, it can be used to obtain a first-order estimate of braking effort and hence brake temperatures in selected duty cycles, thereby reducing the need for experimental development in this aspect of brake system design.

4.3 Long Grade Performance

The performance of a truck braking system in negotiating long grades in the roadway is similarly dependent on front/rear axle temperature balance and the fade characteristics of the individual brakes. Though performance prediction on long grades necessarily requires modeling of brake cooling effects, the regression effectiveness model provides a foundation on which long grade performance models can be based. As with the wear balance problem above, the model offers new opportunities to trade off less costly predictive engineering methods for more costly experimental development effort.

5.0 SUMMARY AND RECOMMENDATIONS

The objective of this study of truck hydraulic brakes was to develop tools and methodology for the analytical prediction of braking performance. Past experience with characterizing the performance of brakes has demonstrated that the torque at any instant in time is a complex function of many variables. The method selected for modeling a brake was by use of a multi-nomial expression involving the suspected variables of influence, fitted to the brake performance data by regression analysis. The method proved capable of modeling the performance of the two tested brakes (exclusive of work history effects) with an expression involving actuation force, sliding velocity, and drum/lining interface temperature. As a result, the brake's performance could be described over a broad range of representative operating conditions by a set of empirically-determined coefficients.

This approach to describing a brake's performance has several disadvantages:

- 1) The characterization of a brake by its regression coefficients is not as straightforward as the commonly used methods of quoting torque levels.
- 2) The methods of determining the coefficients require digital processing of dynamometer test data.
- 3) The importance of drum/lining interface temperature requires the use of either a heat transfer model or improved experimental techniques for measuring interface temperature.

Despite these disadvantages, this method of characterizing brake performance has the significant advantage of being able to predict instantaneous torque in any arbitrary stopping situation (assuming experimental test data covered a comparable range of operation). These same predictions cannot be made validly, using

dynamometer torque data as it is currently reported. Yet the same dynamometer tests are a sufficient basis for regression analysis to obtain a multi-nomial model. Though the approach requires the additional effort of providing descriptive input data for the heat transfer model, this aspect of the approach simply reflects the recognized importance of temperature on brake performance and would be essential to any analytical approach to this problem.

It is recommended that this approach to brake characterization be given serious consideration by brake manufacturers and users as a means to improve brake system design technology by expanding the range of conditions over which system performance is evaluated in the design process. This recommendation is most appropriate in the truck manufacturing industry where many front/rear brake combinations must be designed to be compatible.

Though this study has shown that work history can alter a brake's performance characteristics, reducing the accuracy of torque predictions, those effects should be amenable to characterization in this model; and it is recommended that research on this aspect of brake performance be encouraged.

REFERENCES

1. Murphy, R.W. Bernard, J.E., and Winkler, C.B. A Computer-Based Mathematical Method for Predicting the Braking Performance of Trucks and Tractor-Trailers. Phase I Report, Motor Truck Braking and Handling Performance Study, Highway Safety Research Institute, Univ. of Michigan, September 15, 1972.
2. Bernard, J.E., Winkler, C.B., and Fancher, P.S. A Computer-Based Mathematical Method for Predicting the Directional Response of Trucks and Tractor-Trailers. Phase II Technical Report, Motor Truck Braking and Handling Performance Study, Highway Safety Research Institute, Univ. of Michigan, Report #UM-HSRI-PF-73-1, June 1, 1973.
3. Winkler, C.B., et al. Predicting the Braking Performance of Trucks and Tractor-Trailers. Phase III Technical Report, Truck and Tractor-Trailer Braking and Handling Project, Highway Safety Research Institute, Univ. of Michigan, Report #UM-HSRI-76-26-1, June 1976.
4. Johnson, L.K., Fancher, P.S., and Gillespie, T.D. An Empirical Model for the Prediction of the Torque Output of Commercial Vehicle Air Brakes. MVMA Project #1.36, Highway Safety Research Institute, Univ. of Michigan, Report #UM-HSRI-78-53, December 1978.
5. Federal Motor Vehicle Safety Standard No. 121, Air Brake Systems: 49CFR 571.121
6. MacAdam, C.C. and Fancher, P.S. Survey of Antilock System Properties. MVMA Project #1.37, Highway Safety Research Institute, Univ. of Michigan, Report #UM-HSRI-78-47, October 1978.
7. Fancher, P.S. "Pitching and Bouncing Dynamics Excited During Antilock Braking of a Heavy Truck." 5th VSD-2nd IUTAM Symposium on Dynamics of Vehicles on Roads and Tracks, Vienna, September 1977.

APPENDIX A
EFFECTIVENESS FUNCTIONS

This appendix contains detailed presentations of the effectiveness functions obtained for each brake in the form:

$$e = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} a_{ijk} \theta^{i-1} V^{j-1} F^{k-1}$$

where

e = effectiveness (in-lb/lb)

θ = interface temperature ($^{\circ}F$)

V = sliding velocity (fps)

F = actuation force (lb)

BRAKE: Kelsey-Hayes 15 x 5 Twinplex Brake, 1 1/2" wheel
cylinders, 9051J lining, Gunitite 2603 drum, sequence 7-10

EFFECTIVENESS:

		$n_k = 1$			
		$n_i = 1$	2	3	4
$n_j = 1$	1	956.21	-10144×10^{-3}	33680×10^{-6}	-35546×10^{-9}
	2	-96.761	1096.6×10^{-3}	-3734.2×10^{-6}	3997.9×10^{-9}
	3	2.1898	-25.28×10^{-3}	87.565×10^{-6}	-95.318×10^{-9}

		$n_k = 2$			
		$n_i = 1$	2	3	4
$n_j = 1$	1	-383.74×10^{-3}	5929.6×10^{-6}	-22738×10^{-9}	25655×10^{-12}
	2	40.344×10^{-3}	-660.16×10^{-6}	2634.6×10^{-9}	-3051.3×10^{-12}
	3	$-.83707 \times 10^{-3}$	14.899×10^{-6}	-61.907×10^{-9}	73.807×10^{-12}

		$n_k = 3$			
		$n_i = 1$	2	3	4
$n_j = 1$	1	-14.927×10^{-6}	-616.37×10^{-9}	3451.4×10^{-12}	-4439×10^{-15}
	2	4.0839×10^{-6}	56.454×10^{-9}	-387.94×10^{-12}	533.81×10^{-15}
	3	$-.15774 \times 10^{-6}$	$-.96038 \times 10^{-9}$	8.7226×10^{-12}	-12.856×10^{-15}

BOUNDS:

Actuation Force: $300 \leq F \leq 2690$ lb

Sliding Speed: $11.4 \leq V \leq 28.8$ fps

Temperature:

Lower $\theta = 174.32 - 1.1741V + 10.852 \times 10^{-3}F + 2.5781 \times 10^{-3}VF$

Upper $\theta = 126.97 + 2.7508V - .038253V^2 - 83.2 \times 10^{-3}F$
 $+ 31.743 \times 10^{-3}VF - .58756 \times 10^{-3}V^2F$
 $+ 58.121 \times 10^{-6}F^2 - 9.8611 \times 10^{-6}VF^2$
 $+ .20583 \times 10^{-6}V^2F^2$

BRAKE: Kelsey-Hayes 15 x 5 Twinplex Brake, 1 1/2" wheel
cylinders, 9051J lining, Gunitite 2603 drum, sequence 23-37

EFFECTIVENESS:

$$n_k = 1$$

$n_j = 1$	$n_i = 1$	2	3	4
1	382,53	$-2409,6 \times 10^{-3}$	$4377,5 \times 10^{-6}$	$-770,78 \times 10^{-9}$
2	-21,795	$128,57 \times 10^{-3}$	$-157,17 \times 10^{-6}$	$-95,084 \times 10^{-9}$
3	.18413	$-.32124 \times 10^{-3}$	$-2,8355 \times 10^{-6}$	$6,3278 \times 10^{-9}$

$$n_k = 2$$

$n_j = 1$	$n_i = 1$	2	3	4
1	$-395,24 \times 10^{-3}$	$3591,3 \times 10^{-6}$	-10127×10^{-9}	$8015,8 \times 10^{-12}$
2	$22,359 \times 10^{-3}$	$-209,53 \times 10^{-6}$	$618,32 \times 10^{-9}$	$-511,51 \times 10^{-12}$
3	$-.16132 \times 10^{-3}$	$-1,8710 \times 10^{-6}$	$-6,7816 \times 10^{-9}$	$6,4678 \times 10^{-12}$

$$n_k = 3$$

$n_j = 1$	$n_i = 1$	2	3	4
1	$143,71 \times 10^{-6}$	$-1293,3 \times 10^{-9}$	$3738,1 \times 10^{-12}$	$-3225,8 \times 10^{-15}$
2	$-10,411 \times 10^{-6}$	$94,755 \times 10^{-9}$	$-278,84 \times 10^{-12}$	$247,16 \times 10^{-15}$
3	$.16812 \times 10^{-6}$	$-1,5804 \times 10^{-9}$	$4,8434 \times 10^{-12}$	$4,5004 \times 10^{-15}$

BOUNDS:

Actuation Force: $320 \leq F \leq 2750$ lb

Sliding Speed: $2,2 \leq V \leq 27,2$ fps

Temperature:

$$\text{Lower } \theta = 153,53 + 1,8102V + 13,634 \times 10^{-3}F + -.75961 \times 10^{-3}VF$$

$$\begin{aligned} \text{Upper } \theta = & 203,45 - 2,9043V + .047848V^2 - 52,64 \times 10^{-3}F \\ & + 21,149 \times 10^{-3}VF - .074507V^2F + 22,331 \times 10^{-6}F^2 \\ & - 1,4046 \times 10^{-6}VF^2 - .098195 \times 10^{-6}V^2F^2 \end{aligned}$$

BRAKE: Kelsey-Hayes 15 x 5 Twinplex Brake, 1 1/2" wheel cylinder
 ABB 539 lining, Gunitite 2603 drum

EFFECTIVENESS:

$$n_k = 1$$

	$n_i = 1$	2	3
$n_j = 1$	266.84	-904.90×10^{-3}	1165.2×10^{-6}
2	-28.078	113.96×10^{-3}	-138.88×10^{-6}
3	0.73471	-2.8801×10^{-3}	3.2529×10^{-6}

$$n_k = 2$$

	$n_i = 1$	2	3
$n_j = 1$	-163.47×10^{-3}	713.39×10^{-6}	-916.09×10^{-9}
2	31.658×10^{-3}	-109.37×10^{-6}	116.97×10^{-9}
3	-0.93553×10^{-3}	3.0355×10^{-6}	-2.9145×10^{-9}

$$n_k = 3$$

	$n_i = 1$	2	3
$n_j = 1$	49.139×10^{-6}	-196.86×10^{-9}	218.18×10^{-12}
2	-10.498×10^{-6}	33.828×10^{-9}	-30.950×10^{-12}
3	0.31550×10^{-6}	-0.97147×10^{-9}	0.81245×10^{-12}

BOUNDS:

Actuation Force: $750 \leq F \leq 2450$ lbs

Sliding Speed: $6.0 \leq V \leq 29.0$ fps

Temperature:

$$\text{Lower } \theta = 158.35 + 2.8003V + 55.787 \times 10^{-3}F + 1.6101 \times 10^{-3}VF$$

$$\begin{aligned} \text{Upper } \theta = & 134.16 - 4.9104V + 0.24447V^2 + 39.345 \times 10^{-3}F \\ & + 59.806 \times 10^{-3}VF - 1.3786 \times 10^{-3}V^2F \\ & + 23.072 \times 10^{-6}F^2 - 17.757 \times 10^{-6}VF^2 \\ & + 0.42010 \times 10^{-6}V^2F^2 \end{aligned}$$