APPENDIX: CHAOS AND TRANSITIONS

Previous research showed how to use elements of mathematical chaos theory to suggest points of irreversibility in a semidesert soil science setting (Arlinghaus, Nystuen, and Woldenberg, 1992 a and b). The same sort of general strategy might work when coupled with transitions. This possibility is explored below, using Feigenbaum's graphical analysis.

Feigenbaum's graphical analysis--geometric feedback.

Feigenbaum's graphical analysis employs the line \( y = x \) as an alternate axis from which to insert inputs into a mathematical function. In Figure A.1, the function \( y = 4x(1-x) \) is graphed. Choose the input value \( x = 0.2 \) and find the \( y \) output that corresponds to it. Graphically, the \( y \)-output is the height of the vertical line from \( x = 0.2 \) to the curve. Use this output value as the next input or \( x \)-value. To do so, slide horizontally over to the line \( y = x \)--so that \( y \) becomes \( x \), or output becomes input. Now move vertically from this location to the curve. Continue this process indefinitely to produce an easy-to-understand picture of how (possibly) complex geometric feedback operates within a system (Feigenbaum, 1980; Hofstadter, 1981; Gleick, 1987; Devaney, 1989). The feedback is traced out as a pattern of directed lines bouncing back and forth between points on the curve and points on \( y = x \), much as singing might bounce back and forth from the crowd to the microphone in an auditorium. The pattern of lines is referred to as the "orbit" of the "seed" value (in this case, \( x = 0.2 \)). Different seed values give rise to different orbits.

Consider the seed value \( x = 0.4 \) in the graphical analysis of Figure A.1. Figure A.2 shows the feedback pattern created using this seed value. A dictionary defines chaos as "a state of utter confusion" (Webster's Seventh Collegiate). Hence, it is a state which defies the prediction of pattern. The decimal expressions of numbers such as \( \pi \) defy prediction of pattern; they exhibit a sort of arithmetic chaos. The feedback pattern in Figure A.2 is chaotic; Figure A.3 shows the detail of the pattern. Any seed value strictly between 0 and 1 will produce geometric chaos with respect to this parabola. The seed value of \( x = 0 \) is a fixed point; it is the first intersection of \( y = x \) with the curve--there is no room to bounce back and forth between curve and line.

When other curves are used, different geometric dynamics of feedback arise. Indeed, because it is possible to examine the pattern of feedback purely graphically, it is not even necessary to know the equation of a given curve in order to consider the geometric dynamics associated with it. It is better to know the equation, of course, for then one can interpolate or extrapolate; however, in situations based on accumulating real-world data, there will likely be no curve that fits them. One can of course choose to fit such data with a prescribed curve, either exactly (using cubic spline interpolation, for example) or approximately (using a logistic curve, for example), but then forecasts
Appendix: Population-Environment Dynamics

Economic dynamics in a reversible direction might help to guide decisions about a reduced expectation of policy to force the regulator to a threshold of instability-for example, in a scenario where thresholds are changed. For this reason, it's crucial to establish dynamics in the threshold parameter that have been described. Policies that would produce desirable results would follow curves which, if shaped properly, reflect the interactions in which some increase is desirable so long as it doesn't lead to environmental interaction in which some increase is desirable to its limit. If the output is reduced as demanded, it is desirable to its limit. If the input is reduced to its limit, then one would hope to be able to implement policy decisions for the threshold of reversibility. To the left of the boundary point, it might serve as a threshold of reversibility. Because there is symmetry around the reversible fixed point, it functions as a threshold fixed point. If the curve is a threshold fixed point, it functions as a threshold fixed point.

Consider the S-shaped curve in Figure A.1. There are three intersection points, one of which is a threshold fixed point. If the curve is a threshold fixed point, it functions as a threshold fixed point. If the curve is a threshold fixed point, it functions as a threshold fixed point.

Curve-Fitting decisions are limited by this initial decision. Graphical analysis offers a systematic and replicable strategy for looking at economic dynamics that is not necessarily based on initial sections in transition.
Use the output as the next input -- repeat the process.

Graph of $y = 4x(1-x)$.

Figure A.1
Figure A.2

Graphical Analysis, $y = 4x(1-x)$

Seed value, $x = 0.4$ — Chaotic Behavior
GRAPHICAL ANALYSIS, ENLARGEMENT

Orbit of $x = 0.4$ relative to $y = 4x(1-x)$
Directions for further research.

There are a number of conceptual issues that remain—what types of curves give what patterns of irreversibility? What is the relation of the inflection point to irreversibility? When repelliers and inflection points are linked, it may be that even more powerful conceptual tools emerge. The next stage, though, would be to use the ideas set forth here on a variety of real-world data. The back-and-forth nature of this sort of research strategy can produce a carefully-tailored fit of ideas leading, hopefully, to useful insights in policy decisions that can guide future interactions (measured as data output) between population and environment.