3. SHORT ARTICLE

GROUPS, GRAPHS, AND GOD

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Abstract

The fact that almost all graphs are rigid (have trivial automorphism groups) is exploited to argue probabilistically for the existence of God. This is presented in the context that applications of mathematics need not be limited to scientific ones.

Recently I was teaching some elementary graph theory to a class studying finite mathematics when, inevitably, someone asked the question, "But what is all this good for?" This question is posed often, and the answer rarely satisfies either the poser or the responder.

Usually the responder is a little annoyed at the question, for often a deeper look by the poser would have yielded some insight into the question. But also the responder is irritated on account of inability to give a satisfactory answer. Two obvious choices present themselves:

1. Most mathematicians find the process of discovery in mathematics rewarding in itself. An elegantly concocted proof of a pleasingly stated theorem gives a sense of satisfaction and a joy in the appreciation of beauty that makes real–world application unnecessary. But the questioner usually lacks the mathematical maturity necessary to appreciate this answer.

2. The most readily available sources of application are in the physical sciences, although there is an increasing use of mathematics in the social sciences. But often the mathematician lacks confidence in the extent of his knowledge of the appropriate science. This makes response somewhat tentative, and again the response fails to satisfy the questioner.

On this occasion, a third alternative presented itself. Being human, all people have some interest in philosophy, varying from formal study to informal discussion. What better place to find a meeting ground to answer the above question?

Definition 1 Let G be a finite graph. Then the automorphism group of G, Aut G, is the set of all edge– preserving 1–1 maps of the vertex set $V(G)$ onto itself, with composition the binary operation.

Informally, one can view the size of Aut G as a measure of the amount of symmetry that G possesses, the structure of Aut G as a measure of the way in which the symmetry occurs.

Definition 2 Let $g(n)$ be the number of n–point graphs which have non–identity automorphism group, $h(n)$ the number of n–point graphs. Define $f(n) = (g(n))/(h(n))$.

It is well–known [2, 3, 4, 6] that

$$\lim_{n \to \infty} f(n) = 0.$$

In other words, almost all graphs have identity automorphism group.

Viewed from a philosophical perspective, this says that the probability of symmetry existing in a complex world is virtually zero. Yet symmetry abounds in our own complex world. This provides plausibility for the view that the world did not evolve randomly, that some force shaped it; *i.e.*, it may be taken as a "proof" for the existence of God.

One might point out at this point that many other proofs for the existence of God rely on mathematical foundations. Causality depends on the belief that infinite regress through successive causes must eventually reach an infinite First Cause. Anselm's ontological argument involves the idea of being able to abstract the idea of perfection and then posit its existence. Pascal's view that one should behave as if God exists on the basis of expected value of reward if He does is surely a probabilistic view.

Since there is a whole first-order class of logical sentences about graphs [1] each of which is either almost always true or almost never true, further examples of this nature should be easy to find. Indeed, to close with one, observe that [3] almost every tree has non-trivial automorphisms. Thus even a random tree has some symmetry. This might lead one to question Joyce Kilmer's statement that "Only God can make a tree."

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