3B. WHERE ARE WE?
COMMENTS ON THE CONCEPT OF THE "CENTER OF POPULATION"

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1. Introduction

I was recently flabbergasted when I discovered how the Bureau of the Census calculates the location of the center of population of the United States following each decennial census. I found that:

> The center of population is the point at which an imaginary, flat, weightless, and rigid map of the United Sates would balance if weights of identical value were placed on it so that each weight represented the location of one person on April 1, 1980. Located at latitude 38 degrees, 8 minutes, 13 seconds north, and longitude 90 degrees, 34 minutes, 26 seconds west, ... The computation of the center of population in 1980 was based on the 1980 population counts and the 1970 centers of population for counties. County population centers have not been determined for 1980. The center is the point whose latitude (LAT) and longitude (LONG) satisfy the equations
>
> $$\text{LAT} = \frac{\Sigma w_i \times \text{lat}_i}{\Sigma w_i},$$
>
> $$\text{LONG} = \frac{\Sigma w_i \times \text{long}_i \times \text{Cos}(\text{lat}_i)}{\Sigma w_i \times \text{Cos}(\text{lat}_i)},$$
>
> where lat_i, long_i, w_i are the latitude, longitude, and population, respectively, of the counties. (U. S. Bureau of the Census, 1983, Appendix A, p. A-5.)

The statements were surprising for a number of reasons. First, as every good introductory physical geography text book points out, a flat map of the earth's curved surface is a *distorted* map. Though distortions can be reduced by a careful choice of projection, appropriately centered, some distortion always remains and cannot be eliminated. Of course, one may define something any way one wishes, but at least it should be stated which method of projection is used to create this "flat map." Second, the formulæ given suggest that east-west distances are measured (reasonably, though arbitrarily) along parallels of latitude, which are small circles. But distances are usually measured along great circles and this would produce different results. Third, the formulæ yield results that differ from results of other accepted definitions of the center of population. Fourth, the formulæ can produce some rather peculiar results. These will be discussed below.

The purpose of this paper is to discuss some of the difficulties in the concept of the "center of population" when applied to populations spread over enough of the earth's surface that its curvature is noticeable. A more satisfactory definition of the center of population is proposed.

2. Preliminary Remarks

When considering the characteristics of a large group of anything distributed over a region it is often useful to concentrate on only the most basic characteristics, the first three moments of the distribution: 1) the population, 2) the location, and 3) the spatial dispersion of the group. This paper concentrates on the second moment, the location. There are many ways location could be specified. Almost a century ago Hayford (1902) convincingly argued that the most appropriate measure of location of an area or population is a statistic called the average (arithmetic mean). Abler, *et al.* (1971) agree: "When we ask *where* questions about distributions we almost always desire an average location which represents the entire set." When averaging the location, each location is "weighted" according to the specific characteristic of interest and the result is the average location of the weighting character. The result is a "center of mass" if the weighting character is mass, a "center of area" (or geographic center) if the weighting character is area, a "center of population" if the weighting character is population, *etc.* Sviatlovsky and Eells (1937) have discussed in some detail the use and significance of the concept of the "center" in geographical regional analysis.

Locations can be described as vectors whose magnitudes and directions are taken as the distances and directions of the items whose center is to be calculated. Then the power and convenience of vector algebra can be used to calculate the center in one, two, three or even higher dimensional spaces. If the space is "flat" (Euclidean) then the process is quite straightforward, though tedious. Also, by using vectors in a Euclidean space to represent the distances and directions of individuals in a population, there exist several interesting and useful concepts. First, when the center of the coordinate system used is at the center of population then the vector sum of the distances of all the people is zero and the sum of the squares of the distances of all the individuals is at a minimum. The minimum sum of the distances squared when measured from the average is not an accident, but rather, the result of a fundamental mathematical relation between the two quantities. Sufficiently fundamental is this relation that Warntz and Neft (1960), for example, define the mean as the place from which the sum of squares of the distances to each member of the population is minimum. Second, when the center of the coordinate system used is *not* at the center of population then the vector sum of the distances of all the individuals provides the distance and direction of the center of population from the center of the coordinate system. I will use these characteristics later.

However, the surface of the earth is not "flat," but "curved," and though finite, is without a boundary. On such a surface one can get into difficulty with the concept of average location. Where, for example, is the "center of area" (geographic center) of the earth's entire surface? If one chooses to preserve an earth surface provincialism it is not clear how one can modify the "vector representation" of distance and direction for the locations of individuals in a population. Some criteria are needed.

I suggest that any reasonable definition of "center of population" should meet at least the following standards: (1) population distributions which are symmetric about some central point should have their center of population at this central point and (2) distances should be measured as true distances, either on the surface along great circles or in three dimensions along straight lines. It would also be desirable to have any definition satisfy additional restrictions: a) it should correspond to one's common understanding of "center of population," b) it should reduce to the usual definition of "center of population" when there is no curvature and c) it should be easily extended to nonspherical surfaces — for example, the International Ellipsoid or the geoid.

In the examples and discussions which follow, all angles and great circle arc lengths are given in degrees and decimal degrees. Azimuths of places from any point are measured from North toward East. Latitudes and longitudes are designated as North or South and East or West respectively. I have ignored the differences between the shape of the earth and a sphere, as does the Bureau of the Census, when calculating centers of population (U. S. Bureau of the Census, 1973). When calculating the 1980 center of population of the United States in the various examples, I have used the original published 1980 populations (U. S. Bureau of the Census, 1983) and the unpublished 1980 centers of population for the fifty states and the District of Columbia (see Appendix A). When calculating the 1910 and 1880 centers of population of the United States I have used the published populations and centers of population of the various states and the District of Columbia (U.S. Bureau of the Census, 1913 and 1914).

3. Census Bureau Center of Population Formulæ

Imagine a circumpolar population uniformly distributed along, say, the 70th parallel of latitude north (see Fig. 1). If longitude is measured from 180° W through 0° to 180° E then the Bureau of the Census formulæ put the center of population at 70° N on the Greenwich meridian. Yet, surely the center of this population is at the North Pole. The Bureau of the Census formulæ fail to meet the suggested standard.

This failure is not due to the choice of a circumpolar population. Even at mid-latitudes the formulæ fail to meet the suggested standard. Consider a second example, a collection of eight equally populated places located on a circle of 15 degrees of arc radius. Center the circle at 38° N and 90° W. Choose the position of the first place so that its azimuth from the center point is 15 degrees and each succeeding place has its azimuth 45 degrees greater than the preceding place (see Fig. 1). The latitude and longitude of each of the eight places can be calculated with spherical trigonometry. The results are shown in Table 1. When the Bureau of the Census formulæ are used to calculate the center of population of the odd numbered places, then of the even numbered places and finally for all eight places the results differ. Specifically, for the odd numbered places one finds:

$$\text{LAT} = 37.2351\,\text{N}, \qquad \text{LONG} = 90.0146\,\text{W}.$$

For the even numbered places one finds:

$$\text{LAT} = 37.2155\,\text{N}, \qquad \text{LONG} = 89.9855\,\text{W}.$$

While for all eight places one finds:

$$\text{LAT} = 37.2253\,\text{N}, \qquad \text{LONG} = 90.0000\,\text{W}.$$

Yet, surely the different symmetric distributions centered on the same point should have their center of population at the same place and surely that place (in this example) should be 38° N and 90° W ! Once more, the Bureau of the Census formulæ do not meet the suggested standard.

For a third example, consider the simplest possible case: two equally populated places equidistant and in opposite directions from a central place. Specifically, place the center at

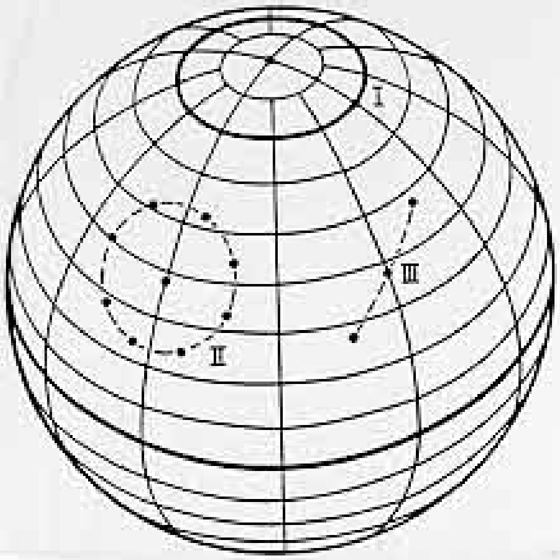

Figure 1. Example population centers and distributions used in Part 3 of the text. Figure available in hard copy only; content should be clear from text and from caption. A sphere is drawn containing parallels and meridians. Three figures are highlighted on this sphere. I: a circumpolar population distributed along the 70th parallel of latitude north. II. a symmetric population distribution centered at latitude 38° N and longitude 90° W. This population is spaced at eight locations around the perimeter of a circle. III: a simple symmetric population distribution centered at 38° N and longitude 30° W. This population is spaced at either end of a line segment centered at III. The precise locations of places in distributions II and III are listed in Tables 1 and 2, respectively.

Table 1. Locations of places, Example II

Place	N. Latitude	W. Longitude
1	52.3434	83.7050
2	44.1596	71.7938
3	32.8128	72.6948
4	24.7119	81.8100
5	23.4333	94.1868
6	29.5187	104.9264
7	40.3511	109.1501
8	50.4718	101.7316

38° N and 30° W and the two places 15 degrees of arc from the center to the northeast (Az = 45) and to the southwest (Az = 225). (See Fig. 1). The latitude and longitude of the two places can be calculated and are shown in Table 2. When the center of population is calculated for this very simple population distribution of two places using the Bureau of the Census formulæ the result is:

$$\text{LAT} = 37.2057\,\text{N}, \qquad \text{LONG} = 29.9626\,\text{W}.$$

Yet, surely the true center is at the central point: 38° N, 30° W ! And yet again, the Bureau of the Census formulæ do not meet the suggested standard.

Table 2. Locations of places, Example III

Place	N. Latitude	W. Longitude
9	47.6377	14.2401
10	26.7737	41.8289

What the Bureau of the Census formulæ calculate is not the latitude and longitude of the center of population, but rather, two different and separate statistics: 1) the average latitude of the population and 2) the longitude of the average east-west distance of the population *on a specific map projection.* This longitude is *neither* the average longitude *nor* the longitude of the center of population. The formulæ calculate the location of a place that differs from other common measures of the center, such as the median or mean location (as defined by Warntz and Neft, 1960, and used by Haggett, *et al.*, 1977). The result of the Bureau of the Census formulæ is the latitude and longitude of a place that cannot justifiably be named the "center of population" as the examples above clearly demonstrate.

For comparison with later examples and further discussion, I have calculated the 1980 center of population of the United States with the Bureau of the Census formulæ. The result is:

$$\text{LAT} = 38.1376\,\text{N}, \qquad \text{LONG} = 90.5737\,\text{W}.$$

This differs from the location published by the Bureau of the Census (latitude of 38°08′13″ or 38.1369 N and longitude of 90°34′26″ or 90.5739 W) by one to three seconds of arc. This very small difference results from my using the populations and centers of the fifty states and the District of Columbia rather than the much larger full list of populations and centers of all the individual counties or enumeration districts used by the Census Bureau. As the discussion and all the examples that follow are based on the same set of data this small difference is unimportant — the examples approximate and represent more extensive computations and their outcome well enough. The concerns in this paper are the methods used rather than the data on which they operate.

4. Census Bureau Center of Population Description

The Bureau of the Census description of the center of population does not give the map projection used. If the center of population (the "balance point" mentioned in the description) is calculated on a flat map constructed using various map projections the results vary. In order to demonstrate this I have calculated the 1980 centers of population (the balance point of the population distribution) for the U.S. using several different flat map projections. The projections used are all well known, having been developed in the 18th century, the 16th century and much earlier. For the projections chosen, descriptions given in numerous texts were sufficient for the derivation of the relevant formulæ for laying out the projections. Alternately, one may refer to detailed monographs, such as the one by Snyder (1987), for the appropriate formulæ. The results for each of the selected projections are listed in Table 3 and displayed in Fig. 2.

Table 3. The 1980 center of population of the United States when using the Bureau of the Census prose definition and various different map projections

No.	Projection and Comments	Center of Population N. Latitude	W. Longitude
1	Cylindrical Equal-Area	37.9818	90.4237
2	Equidistant Cylindrical (Plate Carrée)	38.1376	90.4237
3	Sinusoidal (centered at 0 W)	38.1376	90.2532
4	Sinusoidal (centered at 60 W)	38.1376	90.4655
5	Sinusoidal (centered at 120 W)	38.1376	90.6778
6	Sinusoidal (centered at 180 W)	38.1376	90.8901
7	Equatorial Mercator	38.2945	90.4237
8	Transverse Mercator (centered at 90 W)	39.2344	90.6732
9	Azimuthal Equal-Area (centered at 0N,0W)	39.2583	89.0197
10	Stereographic (centered at N. Pole)	39.7137	90.4888

Note that the calculated centers depend not only on the projection chosen but also on the center and the orientation selected for the projection. The results differ as little as they do from one another because the projections chosen leave the United States in those portions of the resulting maps where the distortions are not extreme. Indeed, using the Bureau of the Census descriptive definition of the center of population, I believe, that, given sufficient time and mischievousness, one could choose projections of various orientations and center that would place the center of population *any place one wished* !

5. Agreement between Description and Formulæ

As it happens, the description and the formulæ currently given by the Bureau of the Census agree for one map projection — a normal Sinusoidal (Sanson-Flamsteed) *with the central meridian of the map the same as the meridian of the center of population*. Indeed, the Bureau of the Census formula for the longitude of the center of population can be derived by answering the following question: What must the central meridian for a normal Sinusoidal projection be in order for the center of population (that is, the balance point of the population distribution) to lie on the central meridian?

But, why is the Sinusoidal projection (and associated formulæ) preferred? If this projection holds special appeal, why isn't the latitude of the center of population determined in a similar way? One could determine the longitude *and* latitude by answering the following question: What must the center for a Sinusoidal projection be in order for the center of population (the balance point of the population distribution) to lie at the center of the map projection? The result, for 1980, is:

$$\mathrm{LAT} = 39.1825\,\mathrm{N}, \qquad \mathrm{LONG} = 90.4934\,\mathrm{W}.$$

The Bureau of the Census first calculated the "center of population" of the United States following the census of 1870 (Walker, 1874). Then, as now, the concept of a balance point was stated as underlying the computation of the center of population. The description of the calculation method (the formulæ are not displayed) indicates the method was very similar to that currently used. East-west locations were taken as the distance from the 67th meridian

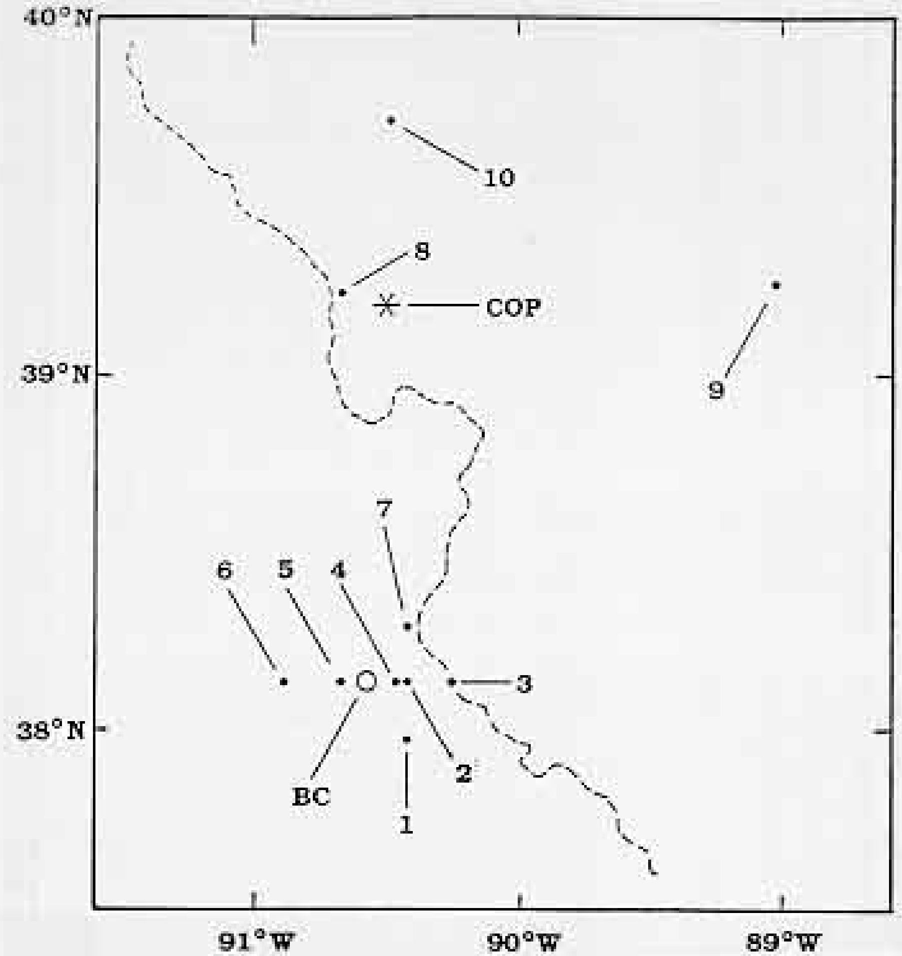

Figure 2. The "Centers of population" for 1980 for the United States calculated according to various definitions. The center shown by an asterisk (*) and labeled COP was determined by the method proposed in this paper which takes the curvature of the earth's surface into account in an appropriate manner. The place shown as an open circle (○) and labelled BC is that published by the Bureau of the Census as the location of the center of population. As discussed in the text, this location should not be called the center of population. Places shown as solid circles and numbered are those which result when the center of population is calculated on various map projections using the Bureau of the Census prose definition of the center. The prose definition does not specify which projection should be used. The numbers refer to the list of projections given in Table 3. The Illinois-Missouri boundary, shown dashed, was taken from The National Atlas (U. S. Geological Survey, 1970). This figure is available in hard copy only; its content should be clear from Table 3 when one also notes that the Census calculated "Center" lies in Missouri as do centers 1, 2, 4, 5, 6, and 7 (from Table 3); the COP Center lies in Illinois as do centers 8, 9, and 10 (from Table 3); Center 3 appears to lie on the border between Illinois and Missouri.

west, measured along parallels of latitude. North-south locations were taken as the latitude above the 24th parallel north. Thus, the calculations of the center of population for the 1879 census are equivalent to calculating the balance point on a Sinusoidal projection with its central meridian at 67° W. Since 1870, east-west distances were measured from other meridians, chosen to be near the estimated center of population. In 1910 for example, the 86th meridian was chosen (U. S. Bureau of the Census, 1913) and thus the calculations of the center of population for the 1910 census are equivalent to calculating the balance point on a Sinusoidal projection with its central meridian at 86° W.

6. Proposed Definition of the Center of Population

One could avoid some of the problems discussed above by avoiding the use of the "statistic," the average. There are other measures of the "center," such as the median. But as Hayford (1902) pointed out long ago, there are fundamental difficulties with the concept of the median for two dimensional distributions. There is no unique point that "divides" two dimensional distributions in half. Another possible measure is the "point of minimum aggregate travel" — the point for which the sum of all the distances to the various individuals would be minimum. But, as Eells (1930) demonstrates, this point has some peculiar characteristics. Also, as Court (1964) makes clear, the point of minimum aggregate travel is very difficult to find. There are now elegant, very powerful and very general techniques for solving such problems (Kirkpatrick, *et al.*, 1983). But, this is beside the point. The statistic that we want is one that reflects where people *are*, not where they might congregate with the least total travel. The appropriate statistic is the mean.

Whether calculating the mean location or the point of minimum aggregate travel, an arbitrary decision must be made: are the calculations to be done on the curved two dimensional surface of the globe (or appropriate flat map) or are they to be carried out in three dimensions? If the point of minimum aggregate travel were calculated in three dimensions the paths to be traveled and the resulting point would lie below the earth's surface. What value would there be in finding a point of least cumulative travel when the place to congregate and the paths to be traveled are inaccessible? I conclude that if one is interested in the point of minimum aggregate travel, it should be calculated on the earth's surface.

If the mean (average) location of a population is calculated in three dimensions, the resulting point is located below the surface. In the case of the United States in 1980, I find this mean location at:

$$\text{LAT} = 39.1823\,\text{N},$$

$$\text{LONG} = 90.3477\,W \text{ and Depth} = 0.0259 \times R$$

Where R is the radius of the sphere representing the earth. Taking $R = 6371$ km, the depth is about 165 km. But calculating the average location of simple surface distributions in three dimensions can yield some peculiar results. Consider three different equatorial population distributions, each consisting of four equally populated places with longitudes as follows:

Example IV: places at 50.00 W, 15.00 W, 15.00 E, 50.00 E.
Example V: places at 50.00 W, 15.00 W, 15.00 E, 130.00 E.
Example VI: places at 62.23 W, 60.00 W, 60.00 E, 62.23 E.

In all three examples the center of population (average location) ends up at the same place — on the equator at the Greenwich meridian — and differ only in their depth below the surface, if at all. (Examples V and VI have centers at the same depth.) But, we are largely confined to the surface of the earth and from this provincial point of view the center of population in example V should be far to the east of the centers in examples IV and VI. I find it unsatisfactory for populations of such different East-West distribution to have "centers" which differ only in depth or not at all. I conclude that average locations should be calculated *on* the earth's surface.

To insist that calculation of the average location or point of minimum aggregate travel must be done in three dimensions is no more (or less) reasonable than to insist that the only proper map projection is on a globe. I leave to others the task of championing the computations of two dimensional population distribution statistics in three dimensions. I believe it is legitimate to consider the population distribution a two dimensional distribution and display its characteristics on the two dimensional surface of a globe or appropriate flat map.

I suggest that the appropriate definition of the center of population is one similar to the descriptive definition given by the Bureau of the Census but with one addition. Specifically, the center of population is the point at which an imaginary, flat, weightless, and rigid map of the United States *constructed by a specific method of projection* would balance if weights of identical value were placed on it so that each weight represented the location of one person .... It only remains for one to choose the specific type of projection. It is distance and direction which are central to any calculation of the center of any population distribution. Therefore, I suggest that the only map projection (on which to find the balance point of the population distribution) is one where distances and directions of the individuals in the population are undistorted. If one chooses to measure the distances and directions from the center of an Azimuthal Equidistant map, or on the surface of a globe, they will be undistorted. Then one can create vectors whose magnitudes and directions are the true distances and directions of the various populated places. A vector sum can be done and the result is an *estimate* of the distance and direction of the center of population. It is only an estimate because, although a map is flat, the earth's surface is not. However, it is a very good estimate, *and the closer the map projection's center is to the center of population the better is the estimate.* Whether one chooses to carry out the computation on an Azimuthal Equidistant map or on the surface of a sphere is immaterial, since the process is algebraically identical and the results are numerically identical.

Because the calculating of the center only produces an estimate, the procedure must be an iterative one, with the center of the projection in each iteration being the estimate of the center of population from the previous iteration. But the estimate is an excellent estimate, so the process converges rapidly. The iteration continues until the center of population is as close to the center of the map as one wishes. When calculating the U. S. center of population in this manner I find:

Latitude of the 1980 U. S. center of population = 39.1980 N.
Longitude of the 1980 U. S. center of population=90.4978 W.

This point lies about 125 km from the center given by the Bureau of the Census and is in Greene County, Illinois, about 14 km southwest by south of Carrollton, the county seat (see Figs. 2 and 3).

The iterative calculation is not the computational nightmare that one might imagine (see Appendix B). Even when choosing the initial starting point at latitude zero and longitude zero or at latitude 20° S and longitude 20° E, the process rapidly converges to within 0.000001 degrees of the answer in four or five iterations. But one knows that the U.S. center of population is not in or near Africa — there is no point in beginning the computations there. As the approximate center of population can be guessed, only two or three iterations are necessary to calculate the center of population to ample accuracy.

I have also tested this procedure on the three example distributions used in Part 3 above and find that in all three cases the process rapidly converges on the expected central point.

Therefore, I suggest that the proper definition of the center of population of the United States (or for any population distributed over a substantial portion of the earth's surface) is:

> The center of population is the point at which an imaginary, flat, weightless, and rigid map of the United States would balance if weights of identical value were placed on it so that each weight represented the location of one person on a specific date. The map in question is an azimuthal equidistant map whose center is at the center of population which must be calculated by successive approximation.

This suggested definition of the center of population has the following advantages: (1) The center of populations symmetric about some central point is at that central point. (2) The true distance of each place is used in the computation. (Thus, this definition satisfies the two suggested standards given in Part 2 above.) (3) The suggested definition of the center of population also satisfies two of the three additional restrictions desired and stated in Part 2 above: (a) it corresponds to one's common understanding of center of population in that it does find the balance point of a distribution — though one must be very specific about how the distribution is displayed, and (b) mathematically there is a correspondence to the usual definition of the center in the sense of the average — the vector sum of the "distances" is zero when measured from the center and the sum of the squares of the "distances" is minimum when the distances are measured from the center. In addition, when the center of the map is not at the center of population the vector sum of the "distances" points approximately to the center of population. Finally, our definition would reduce to the usual mathematical definition when there is no curvature.

It is not clear that the definition suggested can be extended to non-spherical curved surfaces and thus satisfy the additional desired restriction (c) mentioned in Part II above. I believe it would work for the center of population of the United States on an ellipsoid of revolution but there could be difficulties for non-spherical surfaces in general — the shortest distance between two points may not be uniquely defined and one may end up with several centers of population, all equally legitimate.

A widely known use of the decennial centers of population determined by the Bureau of the Census is their display on a map of the United States depicting the historic westward shift of the population. In addition to this westward shift, these centers have slowly moved south since the turn of the century. By 1980, the center determined by the Bureau of the Census was more than a degree of latitude (*ca.* 110 km) south of where it was located in 1790. In contrast, the center of population calculated by the proposed method has followed a different path, diverging from the other path, and in 1980 was located at about the same latitude as the 1790 center. (The smaller the east-west dispersion of the population, the smaller will be the difference between the center of population calculated by the proposed method and the center the Bureau of the Census calculates. Thus, one would expect the locations calculated by either method to be about the same in 1790, before extensive westward national expansion occurred.)

In order to show the increasing divergence of the two paths I have calculated the centers of population for 1910 and 1880 using the proposed method. The results are shown in Fig. 3 and labeled COP. Also shown (and labeled BC) are the locations determined and published by the Bureau of the Census for the same years. One can see that the average latitude of the population (which is what the Bureau of the Census calculates) has moved farther south than has the center of population.

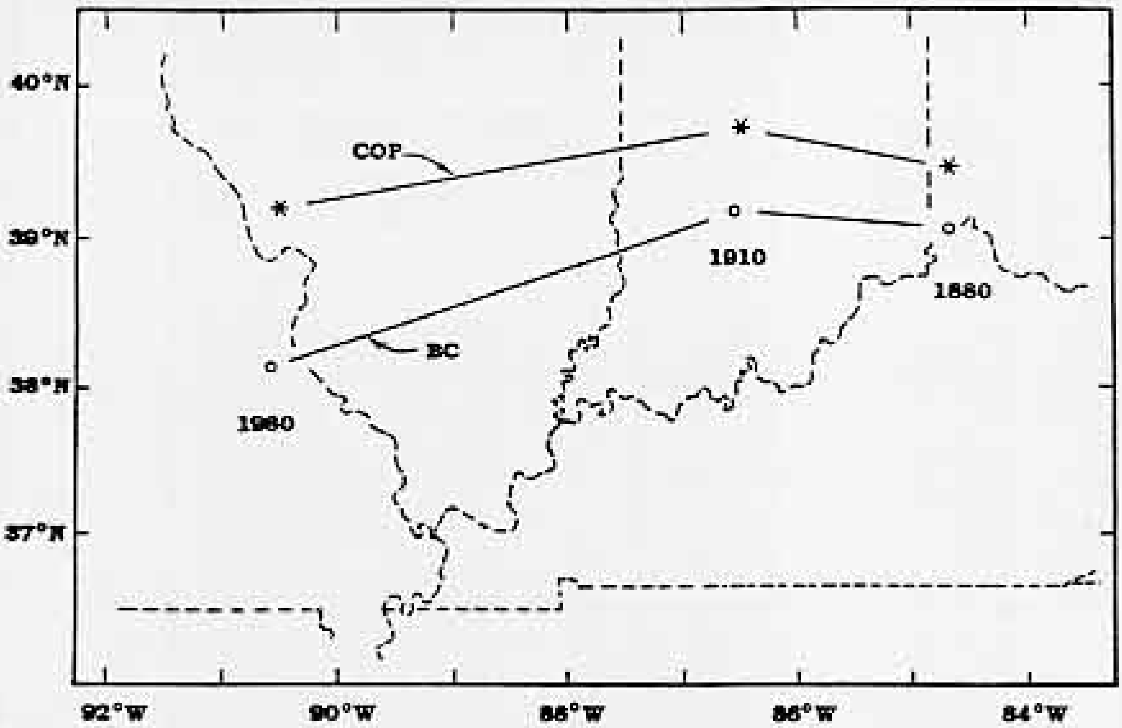

Figure 3. "Centers of population" for 1880, 1910 and 1980 for the United States calculated according to various definitions. Centers shown by asterisks (*) and labeled COP were determined by the method proposed in this paper which takes the curvature of the earth's surface into account in an appropriate manner. Places shown as open circles (o) and labeled BC are those published by the Bureau of the Census as the location of the center of population. As discussed in the text, these locations should not be called the centers of population. State boundaries, shown dashed, were taken from *The National Atlas* (U. S. Geological Survey, 1970). This map is available in hard copy only; it does not transmit electronically. Its content should be clear from the combination of the text and this caption.

7. Summary

For more than a century the Bureau of the Census has been calculating and displaying on maps a place designated as the "center of population" of the United States. The method used in this computation is equivalent to calculating the average location of the population on a Sinusoidal map projection. As indicated in the previous discussion, such a method does not adequately take into account the curvature of the earth's surface. As a result, what the Bureau of the Census calculates should not be called the "center of population." It is, rather, the location of a point that has the population's average latitude and the population's average distance (measured east-west along parallels of latitude) from an arbitrarily chosen meridian.

A different method of calculating the center of population has been proposed in this paper. Like the Bureau of the Census method of calculation, the proposed method is based on the concept of the balance point of the population distribution and thus corresponds to one's common understanding of the center. In contrast to the Bureau of the Census method, the proposed method takes into account the curvature of the earth's surface and map projection distortions in an appropriate manner and is based on measuring distances along great circles.

When calculated as proposed, the center of population's location differs substantially from that calculated by the Bureau of the Census. Not only is this true for 1980, but also for other census years, and the greater the east-west dispersion of the population, the greater will be the difference.

8. Appendix A

The unpublished 1980 population centers of the fifty states and the District of Columbia used in the various examples were obtained from the Bureau of the Census. As they are unpublished, a complete list of the center latitudes and longitudes that were used in the computations discussed in this paper is supplied below.

This is the data used as the representative example data set in "Where are we? Comments on the concept of the 'center of population' " by Frank E. Barmore, published in *The Wisconsin Geographer*, Vol. 7, pp. 40-50, (1991), a publication of the Wisconsin Geographical Society.

The table below shows the original state populations and also, State Centers of Population supplied by the Bureau of the Census. The first coordinate for the center of population is measured in degrees of longitude west of the prime meridian; the second coordinate is measured in degrees of latitude north of the equator.

Place	Population 1980	Center of population 1980	Center of population 1980
Alabama	3,893,888	86.7750	32.9923
Alaska	401,851	148.4964	61.3650
Arizona	2,718,215	111.7186	33.3245
Arkansas	2,286,435	92.4340	34.9718
California	23,667,902	119.4380	35.4746
Colorado	2,889,964	105.1809	39.4868
Connecticut	3,107,576	72.8760	41.4906
Delaware	594,338	75.5636	39.4450
D. C.	638,333	77.0088	38.9074
Florida	9,746,324	81.6735	27.7948
Georgia	5,463,105	83.8100	33.1866
Hawaii	964,691	157.6129	21.2009
Idaho	943,935	114.9358	44.2072
Illinois	11,426,518	88.4070	41.2073
Indiana	5,490,224	86.2835	40.1759
Iowa	2,913,808	93.0582	41.9858
Kansas	2,363,679	96.6379	38.4544
Kentucky	3,660,777	85.2228	37.7918
Louisiana	4,205,900	91.4656	30.7177
Maine	1,124,660	69.6408	44.4125
Maryland	4,216,975	76.7904	39.1598
Massachusetts	5,737,037	71.3844	42.2792
Michigan	9,262,078	84.1083	42.8410
Minnesota	4,075,970	93.6489	45.2543
Mississippi	2,520,638	89.6224	32.5778
Missouri	4,916,686	92.0799	38.4815
Montana	786,690	110.9159	46.8610
Nebraska	1,569,825	97.5697	41.1991
Nevada	800,493	116.7563	37.5535

New Hampshire	920,610	71.4735	43.1783
New Jersey	7,364,823	74.4172	40.4640
New Mexico	1,302,894	106.2391	34.6202
New York	17,558,072	74.7181	41.5458
North Carolina	5,881,766	79.6756	35.5676
North Dakota	652,717	99.5101	47.4277
Ohio	10,797,630	82.7006	40.5199
Oklahoma	3,025,290	96.8876	35.5880
Oregon	2,633,105	122.5648	44.6942
Pennsylvania	11,863,895	77.2024	40.4699
Rhode Island	947,154	71.4419	41.7595
South Carolina	3,121,820	81.0355	34.0472
South Dakota	690,768	99.0563	44.1116
Tennessee	4,591,120	86.4217	35.7793
Texas	14,229,191	97.4571	30.9925
Utah	1,461,037	111.8261	40.5165
Vermont	511,456	72.8055	44.0566
Virginia	5,346,818	78.0021	37.6381
Washington	4,132,156	121.5325	47.3363
West Virginia	1,949,644	80.9407	38.7202
Wisconsin	4,705,767	88.9756	43.7192
Wyoming	469,557	106.9348	42.6568

9. Appendix B

All the computations reported here were done on an Apple IIgs computer using the spread sheet in AppleWorks 3.0. Computation time for the problems varied. When calculating the center of population using the proposed method, each iteration took about 85 seconds. Computations using a more detailed list of populated places would take longer in direct proportion to the number of places used. Computers and software with enormously greater speed and capability are widely available. There is no computational reason for not using the proposed method.

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NOTE: The original article contains several printing errors which might cause misunderstanding. These were corrected in the reprints of the article. The *Solstice* copy was prepared from the corrected reprint. The errors in question in the original include: i. the longitudes of places in Examples IV, V and VI were incorrect and ii. near the end of part 6 of the text a date of 1790 was incorrectly given as 1970. Also, the location, about 14 km southwest by south of Carrollton, was incorrectly given as about 7 km southeast of Carrollton.