4. THE PELT OF THE EARTH: AN ESSAY ON REACTIVE DIFFUSION

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Reactive diffusion (see the many references to Murray) is an idea that draws on the concept that boundary shape can influence the spatial pattern of the developing forms and processes interior to that boundary. A second idea is involved. Once a natural diffusive process has been at work, there is reaction to it, altering the shape of the underlying diffusion. Reactive diffusion is thus a dynamic process that is, to some extent, self-adjusting to change.

This sort of idea is one that has met with many expressions in the past — in the biological as well as in the geographical landscape (Arlinghaus, Nystuen, and Woldenberg, 1992). Boundary shape can determine how matter and energy travel within a closed system. Standing waves can be created in this manner, be they standing waves of translation of pigment on animal coats, producing striped animals; or standing waves of oscillation of water, producing seiches as water-stripes in reaction to lake depth and coastline shape of the containing vessel (e.g., Lake Michigan or Lake Geneva). One might even be tempted to speculate on a possible role for seiche-like stripes in the "parting of the Red Sea."

Xu, Vest, and Murray (1983) created mock animal outlines on laminar plates shaped like two-dimensional pelts — as "maps" of three-dimensional animals; when small adjustments in the outlines were made, vibrational patterns formed in a surface dust placed within these outlines created various spotted and striped patterns as a reaction to the boundary shape. Indeed, a circular drum head boundary offers one way for the roll of the drum wave of noise to interact with the boundary; using a fractal boundary for the drum head can produce a vastly different pattern of resonating pockets of drum roll (Science, 1991). The continuing work of Batty and Longley (1985 and later) in using fractal concepts to track the pattern of the urban fringe might also be (but has not yet been, to our knowledge) cast in the framework of reactive diffusion. Three-dimensional solids, covered with a coat of spots, might also have their spot patterns determined by some underlying vibrational process that causes the substance of the spots on the surface to react with the three-dimensional volume over which the surface is stretched. Thus, the calico cat and the earth might have a great deal in common when land masses, driven by tectonic rather than by biological rhythms, are seen as the calico spots on the pelt of the earth.

For example, the burn pattern created by random lightning strikes in a forest, and the reaction of firefighters to these strikes, displays a clear case of reactive diffusion and pattern formation on the earth. For, in the absence of firefighters, the random strikes start fires which coalesce to form an advancing front that may ultimately burn the entire region. When firefighters enter the scene, they work to confine the random strikes; the fire may leap the barriers they create, and when it does, the firefighters talk about it and react by moving to control the new hotspot. Ultimately, the spread of communication among firefighters, in response to the leapfrogging character of forest fires, produces a forest spotted with burnt dark patches. The reaction of firefighters to the diffusion of information about the location of fires produces characteristic, and predictable, patterns on the earth.

1. Pattern Formation: Global Views

Nystuen noted (1966) that "spatial processes depend upon the shape of the partitions created by their boundary patterns. If the boundary shape is changed the process itself is changed. In fact, the very existence of the process may depend on the boundary shape." The biologist Joseph Birdsell noted that coastline shape has affected genetic diversity in the Australian aborigine population (1950); migration patterns forced away from the concave-up portions of the northern coastline were dispersed, while those forced away from the concave-down portions were focused. With dispersal of hunting and gathering came genetic complexity; with focusing came genetic inbreeding. Arlinghaus (1977; 1986) drew on ideas from Birdsell and Nystuen in using boundary shape of a limited access arterial to suggest where new pockets of population concentration and dispersion will appear relative to the concavity of the arterial. In all of these, as with reactive diffusion, there is an adjustment of process (geographical or biological) to boundary, with implications for the spatial organization of associated human activity coming as a reaction to that adjustment.

Indeed, even in mediæval guilds, retail services clustered in pockets across the geographical landscape, as "stripes" or "spots" of commercial activity, in reaction to the diffusion of information as to type of service available (Vance, 1980). Similar urban patterns are evident in modern developing countries; and this context thus suggests, very generally, an interesting human dimension in exploring global urban change (Drake, 1993; Meadows *et al.* 1992).

In a classical urban context, one might imagine Harris and Ullman's "multiple nuclei" model recast within the replicable theoretical framework of reactive diffusion. As diffusion causes change surrounding and within the nuclei, there is a reaction, and the nuclei shift, or new nuclei spring up. The spatial evidence of reactive diffusion might be substituted for the historical evidence on which Harris and Ullman (1945) based their observational model, pulling the multiple nuclei model more in line with the earlier spatial models of Burgess and Hoyt. The multiple nuclei pattern appears as a reaction to incompatible land uses; it arises from an alternative resistance to residential and commercial land uses in which further employment centers leapfrog over existing urban neighborhoods, leading to extensive additional urban growth.

Within the Detroit metropolitan region, for example, the complex changing nature of the local political scene coupled with the increasing crime rates associated with downtown Detroit, often encapsulated quite simply in the minds of many Detroiters by the closing of the downtown Detroit Hudson's store, led to the consequent reaction of many businesses to move to the suburbs. Thus, suburban Southfield became an early hub of urban reaction in the Detroit metropolitan region — here, a new nucleus emerged. Efforts to restore the prominence of the downtown on the Detroit River are typified by the Renaissance Center — here, the old nucleus shifts toward the River banks.

Indeed, the characterization of the collapse of the central city in terms of the failure of the downtown headquarters of Hudson's department store may not be a strictly simple-minded view. Like the stars, the life of the city may take different paths — at one time a center may be a viable unit, and at another time the relative size and density of the urban area may cause inner city collapse. In a central place context, in which the "threshold" of a firm refers to the minimum number of sales which allows the firm to succeed and give an adequate return to its owners, the situation with Hudson's was simply a matter that the buying population at the center was too small to meet the threshold number. Related central place terminology involves the notions of the maximum range of a good and the minimum range of a good. The maximum range is the absolute limit on the demand of a good — beyond this limit, transportation costs reduce demand for the good to zero. The minimum range is the distance over which the firm must ship its goods to include the threshold populations. A logical consequence, all else being equal, is that the minimum range of a good is less in a densely settled region than it is in a sparsely settled one.

Thus, the common sense notion of "how can a big store like Hudson's fail in downtown Detroit?" can be translated as follows. Migration of the affluent population to the suburbs reduced the number of potential customers in the center. The minimum range therefore needed to be extended outward from downtown in order to include the threshold number of customers. But, suburban Hudson stores were already in place and also worked to attract those customers that the downtown branch now required to succeed. The three large sub-urban stores competed with the downtown store for these customers, won them over, and the downtown store failed. Stability in competition (Hotelling, 1929) was restored when the "empty" center was divided among the peripheral competitors, in a sort of central place (two-dimensional) Hotelling model. This sort of geometric view is a minimalist approach — a best-case scenario; when additional social (and other) issues are superimposed, accelera-tion along the path to collapse is more likely. When one next considers that this pattern will repeat on the periphery of these suburban stores and within the maximum ranges of the various goods, a sort of leapfrogging of circular/hexagonal trade areas occurs and sug-gests, once again, a conceptual context of reactive diffusion as an alternative, and addition to traditional spatial analysis.

Unlike earlier models of urban ecologists (Burgess, 1925; Hoyt, 1939), this sort of urban view of the world is not a generalization of a particular example — that is why it is important to see reactive diffusion cast in the geographical as well as the biological (or other) realms. The pattern of clusters of urban activity on a regional part of the earth's surface is one that is produced in reaction to the diffusion of urban process.

2. Pattern Formation: Local Views

Some current urban research strives to develop indices that offer an easy means for replication of experiments and that are sensitive to the role of boundary. Thus, Morrill (1991) proposed an index of segregation, modified by boundary considerations, to quantify urban spatial segregation. Wong (1992) modifies Morrill's indices by arguing that the length of the boundary separating adjacent urban areal units, as well as the shape of these adjacent units, is significant in determining segregation. Indices such as these, that already are sensitive to some boundary considerations, may offer one means to tighten the focus of application of the concept of reactive diffusion in various specific urban situations.

Often reactions to incompatible urban land uses are circumscribed by the boundary of the system of local jurisprudence. When these reactions fit reasonably well within the laws, competing commercial and residential land uses are in relative harmony. Laws, such as the apocryphal "it is illegal to tie an alligator to a parking meter" suggest a reaction to an unusual situation. When that reaction is passed as law, it diffuses to the population of the surrounding area and may disturb the sensitive balance between incompatible land uses.

Perhaps the most difficult situation of this sort is in establishing rules (legal, ethical, or otherwise) to position locally unwanted land uses ("lulus"). Human laws permit or forbid institutional boundaries that can influence how process works. Typically, a lulu, such as an adult bookstore or a toxic waste site, causes a strong local reaction around this "hotspot." This reaction is confined and suppressed by municipal authorities using the local legal system as their "hose" or "barrier" to confine the effects of the unwanted activity. As with the forest fire example, the lulu leapfrogs, and yet another hotspot of locally unwanted activity occurs.

Reactive diffusion offers an attractive conceptual context in which to examine pattern formation on the pelt of the earth: from local scenarios that mimic the forest fire example to global scenarios that examine entire closed and bounded surfaces. Beyond this essay, the next step is to use this context in specific urban or physical settings.

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