

THE EARTH ISN'T FLAT. AND IT ISN'T ROUND EITHER!  
SOME SIGNIFICANT AND LITTLE KNOWN EFFECTS  
OF THE EARTH'S ELLIPSOIDAL SHAPE

Frank E. Barmore \*

Reprinted, with permission, from  
THE WISCONSIN GEOGRAPHER  
VOLUME 8, 1992  
pp. 1 - 8

**Abstract**

The small difference between the shape of the earth and a sphere is usually thought to be negligible except for work of very high accuracy such as geodesy. This is not the case. There are some examples where this small difference in shape makes an easily apparent difference in what is observed. This paper will comment on three problems and evaluate the impact of the non-spherical shape of the Earth on the result: 1) the qibla problem of Islamic geography, 2) the center of area (geographic center) and 3) the center of population.

**Introduction**

I have noticed that some common considerations in geography are often treated without due regard for the Earth's ellipsoidal shape. This is surprising. The Earth is not spherical (round). It is, rather, very nearly an ellipsoid of revolution with equatorial radii,  $a$  and  $b$ , of 6378.2 km. and polar radius,  $c$ , of 6356.6 km. — a difference of 21.6 km. This difference is significantly larger than the next largest pervasive topographic feature, the continent — ocean basin dichotomy of 5 km. Also, this shape, an ellipsoid of revolution, is not intrinsic to terrestrial planets. Venus is nearly spherical,  $a = b = c =$  ca. 6051.5 km. (Head, et al., 1981). Mars is reasonably well described as a tri-axial ellipsoid of  $a = 3399.2$  km,  $b = 3394.1$  km. and  $c = 3376.7$  km. (Mutch, et al., 1976).

This departure of the shape of the Earth from a sphere is often given as the flattening,

$$f = (a - c)/a = 0.0034 \text{ or } 0.34\%$$

or the eccentricity,  $e$ , where

$$e^2 = (a^2 - c^2)/a^2 = 0.0068.$$

The departure from a sphere also results in a difference between geocentric and geographic latitude of (at 45° latitude),

$$0.195^\circ = 0^\circ 11.7' = 0^\circ 11' 42''.$$

While these are small quantities, they are not insignificant. For comparison, consider the following difference or ratios of similar magnitude:

- a. one vacation day per year (which, in turn, is larger than the one day calendar adjustment every fourth or "leap" year),
- b. a watch which gains or loses five minutes per day,

- c. a two inch gap in a 50 foot brick wall.
- d. a 1/6 inch crack in a 48 inch table top.
- e. \$100 per \$30,000 of annual earnings.
- f. an angle of 1/3 of the apparent diameter of the sun or moon.

We routinely concern ourselves with such small differences in daily life. We expect and receive better accuracy from craftsmen. Differences in direction of this magnitude are easily seen.

Consistency would require that we be as concerned with equally small quantities in geography as we are in other circumstances. Therefore, all but the simplest considerations in geography should routinely take into account the Earth's ellipsoidal shape. Often this is not done. This paper will consider the impact of the Earth's non-spherical shape on the results in three cases: 1) the qibla problem of Islamic geography, 2) the computation of a geographic center (center of area) and 3) the computation of a center of population.

### The Qibla Problem

As I have previously commented (Barmore, 1985), a Koranic line which may be translated as "... wherever you are, turn your face towards it [the Holy Mosque — the Kaaba]" is often invoked to establish the correct orientation (the qibla) during the obligatory prayer (the salat), and hence the correct orientation for mosques. This requirement, in turn, is often considered as satisfied when a mosque is aligned with the direction of the Kaaba in Mecca. There is, in Islamic scientific literature, sufficient discussion of the direction of Mecca to indicate the usual definition of direction (King, 1979). The direction is that of the shortest arc of a great circle on a spherical Earth between the locality and Mecca. (But note that mediæval Islamic religious and legal scholars have often argued otherwise and, as a result, other orientation traditions have existed (King, 1972, 1982a, 1982b, and other work in preparation).) The direction is then specified by stating the azimuth of this arc of a great circle relative to the meridian.

Given the geographic coordinates of a locality and of Mecca the azimuth of Mecca is easily calculated with spherical trigonometry, provided a spherical Earth is assumed. Tables of such information, both historical and contemporary, exist in great number. These tables, as well as numerous individual calculations in the literature discussing the many facets of Islamic culture, often give their results to the nearest minute of arc (or even the nearest second of arc). The implication is that the results are correct to the same level of accuracy. But the Earth is not spherical. The Earth is ellipsoidal in shape. If qibla azimuths are calculated assuming a spherical Earth, they do not represent the real case with an accuracy approaching a minute of arc. In every case I have examined, the calculations were done as if the Earth were a sphere. In order to illustrate the errors that result, I have calculated the simple azimuth as well as the geodetic azimuth of the Kaaba in Mecca for a number of places. (The simple azimuth is calculated on a sphere while the geodetic azimuth more closely represents the correct case (See Appendix A).) The qibla error,

$$QE = az(S) - AZ(E),$$

is the amount that must be subtracted from the incorrect but more easily calculated simple azimuth,  $az(S)$ , in order to obtain the more accurate geodetic azimuth,  $AZ(E)$ , calculated on the ellipsoid representing the Earth. The locations of various places were taken from *The*

*Times Atlas of the World* (1990). The location of the Kaaba in Mecca was taken from a large scale map of Mecca (1970). The result, for Clarke's (1866) Ellipsoid, is displayed in Table 1 for selected localities and shown for the world in Figure 1.

Table 1

The error in the qibla azimuth for various places when calculated on a sphere. The results are given in decimal degrees and in minutes of arc. A tabulated value of the qibla error,  $QE = az(S) - AZ(E)$ , is the amount that must be subtracted from the incorrect but more easily calculated simple azimuth,  $az(S)$ , in order to obtain the more accurate geodetic azimuth,  $AZ(E)$ , calculated on Clarke's (1866) Ellipsoid representing the earth.

Name	Place		Qibla		Qibla Error	
	Lat(N-)	Long(E+)	$az(S)$	$AZ(E)$	degree	min.arc
Baghdad	33.3333	44.4333	200.0637	200.1583	-.0946	-5.67
Cairo	30.0500	31.2500	136.2137	136.0561	.1576	9.46
Chicago	41.8333	-87.7500	48.5875	48.5209	.0666	3.99
Cordoba	37.8833	-4.7667	100.3041	100.1910	.1131	6.78
Damascus	33.5000	36.3167	164.7021	164.6278	.0743	4.46
Istanbul	41.0333	28.9500	151.5875	151.4770	.1105	6.63
Jakarta	-6.1333	106.7500	295.1509	294.9765	.1744	10.46
Jidda	21.5000	39.1667	97.2106	97.1680	.0426	2.55
Kabul	34.5167	69.2000	250.8290	250.9551	-.1261	-7.56
Khartoum	15.5500	32.5333	48.5631	48.7363	-.1732	-10.39
Marrakech	31.8167	-8.0000	91.5913	91.5139	.0774	4.65
Medina	24.5000	39.5833	175.8084	175.7846	.0239	1.43
Mombasa	-4.0667	39.6667	0.3437	0.3460	-.0024	-0.14
Riyadh	24.6500	46.7667	244.5752	244.7080	-.1328	-7.97
Tashkent	41.2667	69.2167	240.3127	240.4550	-.1423	-8.54
Tehran	35.6667	51.4333	218.5599	218.7030	-.1431	-8.59
Tombouctou	16.8167	-2.9833	76.4880	76.5301	-.0421	-2.53
Trabzon	41.0000	39.7167	179.6976	179.6962	.0013	0.08

When these results are considered it is clear that qibla errors on the order of 0.1 degrees ( $0^{\circ} 06'$ ) will result when azimuths are calculated assuming a spherical earth. Not only is this true for qibla azimuths, but it is also true for azimuths calculated for any other purpose. Clearly, azimuths calculated assuming a spherical earth will not, in general, be accurate to a tenth of a degree and should not be given in a way that implies such accuracy.

It would not be appropriate to criticize historical works concerning the qibla problem for lacking such accuracy. However, knowledge of the ellipsoidal shape of the Earth is now widely known — clear descriptions are to be found in many texts on physical geography. I wish to

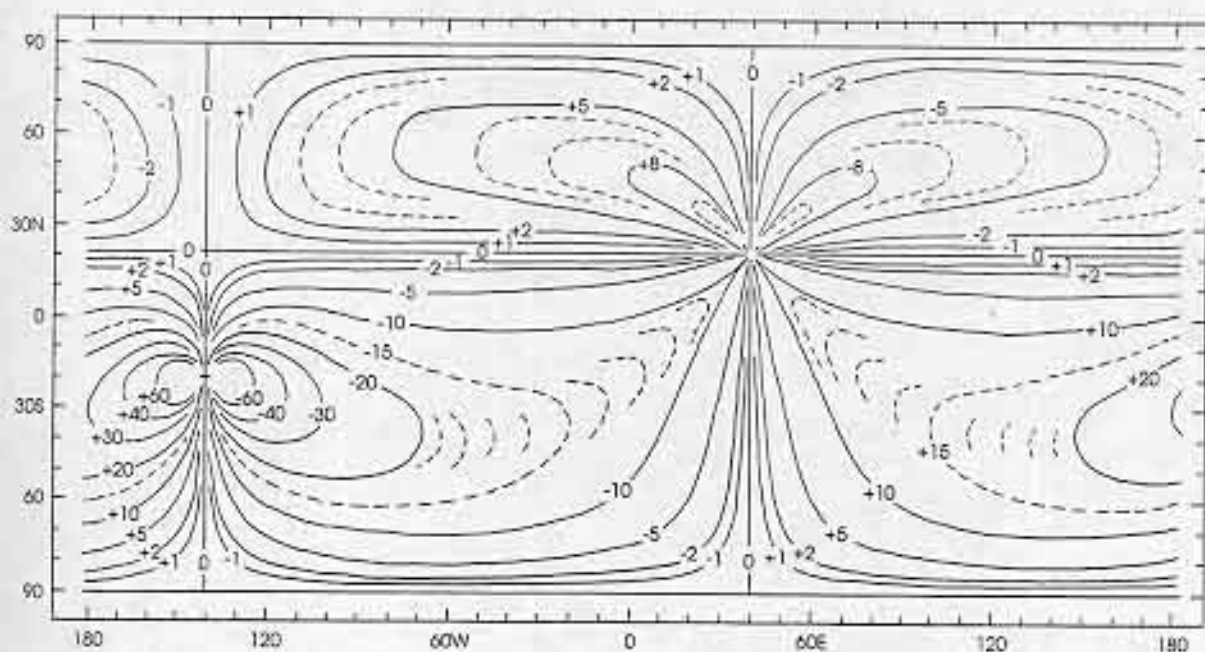


Figure 1. The error in the qibla azimuth for various places when calculated on a sphere. The results are given in minutes of arc. The plotted value of the qibla error,  $QE = az(S) - AZ(E)$ , is the amount that must be subtracted from the incorrect but more easily calculated simple azimuth,  $az(S)$ , in order to obtain the more accurate geodetic azimuth,  $AZ(E)$ , calculated on Clarke's (1866) Ellipsoid representing the Earth. The variations are complex near Mecca, located at 21.4 degrees N., 39.8 degrees E., and at the antipodes of Mecca. Note the non-uniform contour intervals, the incomplete contours in regions of high contour line density and some intermediate contour fragments, shown dashed.

raise two questions: 1) Is there an instance in recent or contemporary works concerning the "qibla problem" where the problem has been considered with due regard for the ellipsoidal (non-spherical) shape of the Earth? 2) Would Islamic legal, religious or geographic scholars have any interest in this small but noticeable correction to a traditional solution of the "qibla problem"?

### The Geographic Center

There exists, in north central Wisconsin, less than 3/4 kilometer to the north and west of the very small community Poniowski, a monument with the following text:

The location of Poniowski near this unique geographic point has given it sufficient fame to be mentioned in newspaper articles, some tourist literature and even celebrated in song (Berryman, 1989).

If the Earth were spherical or much more nearly so, then the statements on the marker would be true enough. But, as a result of the Earth's ellipsoidal shape: a) the place marked is not halfway between the Equator and the pole, b) the place marked is well removed from the "center" and c) the halfway point and the center are well separated from one another.



### GEOLOGICAL MARKER

*This marker in Section 14, in the Town of Rietbrock, Marathon County is the exact center of the northern half of the Western Hemisphere. It is here that the 90th meridian of longitude (sic) bisects the 45 parallel of latitude, meaning it is exactly halfway between the North Pole and the Equator, and is a quarter of the way around the earth from Greenwich, England.*

### MARATHON COUNTY PARK COMMISSION

---

---

(Note, however, the Earth's ellipsoidal shape notwithstanding, the monument does mark the place, 90 W longitude, 45 N latitude, well enough.) The monument's failure in marking the halfway point and the center is substantial and each failure will be discussed in turn.

#### *Halfway Point:*

Because of the ellipsoidal shape of the Earth, the length (measured on the surface) of a degree of geographic (that is, geodetic) latitude varies with latitude. As a result, the point that is equidistant from the pole and the equator is not simply the midpoint in latitude. Using Clarke's (1866) ellipsoid and the various relationships in the geometry of an ellipsoid (Bomford, 1971, Appendix A) it is a straightforward calculus problem to find the equidistant point. It is at the geographic latitude  $45.1447 = 45^{\circ}08'41''$  (see Appendix B). The place with this latitude is about 16 km. from the one marked and sufficiently far from Poniatowski as to place it well into the next county to the north, Lincoln County.

#### *Center:*

The concept of the geographic center (center of area) for a curved surface is not as straightforward as when the area is flat. What is usually meant by the center is the average (or mean) location. The location coordinates used (latitude and longitude) are curvilinear rather than rectangular. Because of this, one may not average the latitude and longitude of the elements of area that make up the whole in order to find the center (average location) of the whole area. In order to make this point more clear, consider Figure 2. Shown shaded is the northwest quadrant of the Earth. On a sphere, this area shows a great deal of symmetry about the point at latitude  $45^{\circ}$  N., longitude  $90^{\circ}$  W. Surely the center of this quadrant on the surface of a sphere is at this central point. But, if one calculates the average latitude of the various area elements that make up the northwest quadrant on the surface of a sphere, the result is 32.7042 degrees or  $32^{\circ}42'15''$  N. Surely the center is not there. (Other statistics are no better when applied to latitude alone — the median latitude is  $30^{\circ}$  N. and the modal latitude is  $0^{\circ}$ .) What must be averaged is the location, **not** the coordinates of the location. Phrased differently, the latitude of the center of area is different from the average latitude of the same area.

Any satisfactory method of finding the center must take into account the curved surface of the Earth in a suitable way. One method is to calculate the center by assuming that the quantities spread over the two dimensional surface of a sphere are distributed in a three-dimensional Euclidean space (as indeed they are). One early geographical use (the earliest I have noted) of this "three-dimensional" method for finding centers of population (or area) on the surface of a sphere was derived by I. D. Mendeleev and used by his father, D. I.

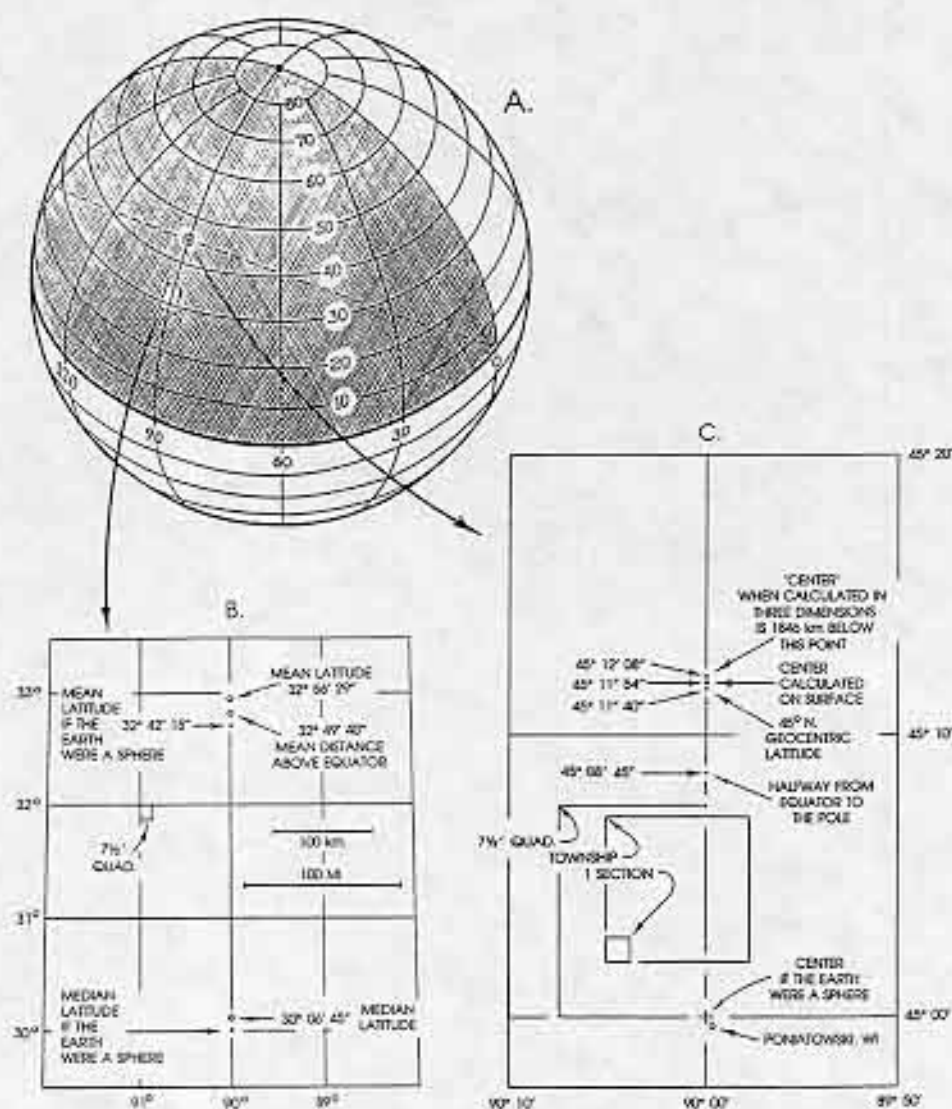


Figure 2. The geographic center (center of area) of the northwest quadrant of the Earth (or the upper left quadrant of a sphere or an ellipsoid) and other statistics. A) An oblique view of the Earth showing the northwest quadrant. B) The region of the northwest quadrant near the median and mean latitudes of the quadrant on the 90th meridian. C) The region of the northwest quadrant near the geographic center. The center was determined by the preferred method (Barmore, 1991); that is, calculated with the computations and the result restricted to the surface. The ellipsoid is Clarke's (1866) ellipsoid.

Mendelev (1907 and earlier) to find the centers of area and population of Russia. Such a method is easily extended to calculating the center of area or population on the surface of an ellipsoid.

I believe an alternative method is preferable — a method that restricts the computations and the results to the surface of a sphere. We are largely confined to the Earth's surface and it is appropriate to adopt this provincial viewpoint when determining the center of

population or geographic center. This is discussed elsewhere in some detail (Barmore, 1991). Whichever of the two methods is used (computations in the earth in three dimensions or computations on the surface in two dimensions) the geographic center (center of area) of the northwest quadrant of a spherical Earth is at  $90^\circ$  W. longitude and  $45^\circ$  N. latitude.

But the Earth is not spherical. The Earth is ellipsoidal in shape. When these computations are done for an ellipsoid, one finds that the geographic center is far removed from  $45^\circ$  N. latitude (though it remains on the 90th meridian). I have used both methods to calculate the geographic center of the northwest quadrant for Clarke's (1866) ellipsoid using the ellipsoidal geometry found in Bomford (1977) and find the center is about 22 km. to the north, well into the next county, Lincoln County, at about  $45^\circ 12'$  N. latitude. In addition to being far above the 45th parallel and far removed from Poniowski, Wisconsin, the center is also far removed from the point midway between the equator and the pole (see Figure 2). Though the monument marks the intersection of the 45th parallel of latitude with the 90th meridian well enough, it marks neither the point midway between the equator and the pole nor the center of the northern half of the western hemisphere. The claims of the marker that it is at "the exact center of the northern half of the Western Hemisphere ..." and "... is exactly halfway between the North Pole and the Equator, ..." are simply not true.

### The Center of Population

When calculating the center of population of the United States, the Bureau of the Census explicitly states that it has assumed a spherical Earth (U.S. Bureau of the Census, 1973). But the Earth is ellipsoidal in shape, not spherical. The formulæ used by the Census Bureau for the center of population calculation are not particularly suitable for the computation of the center of populations on a sphere, let alone an ellipsoid. As has been previously pointed out in considerable detail (Barmore, 1991), the Census Bureau formulæ do not take the curvature of Earth's surface into account in an appropriate way. But, however the center of population is calculated for populations on the surface of a sphere, the questions remains: What will be the center of population for populations on the surface of an ellipsoid? As indicated in the previous section, there are two methods of computing centers on spherical surfaces and the procedures can be extended to the problem of calculating the center of population of the United States on the surface of an ellipsoid.

I have calculated the center of population of the United States for 1980 using Clarke's (1866) ellipsoid and the ellipsoidal geometry given in Bomford (1977) two ways: 1) in the Earth in three dimensions and 2) on the surface in two dimensions as outlined in a previous paper (Barmore, 1991). The same example data set was used in all cases. The results of these computations as well as previously derived results for the spherical case are shown in Table 2 and Figure 3.

When these results are considered it is clear that the difference between the center obtained with the Bureau of the Census formulæ and the center obtained using the preferred method (or the other reasonable alternative) is substantial. However, the error in ignoring the ellipsoidal shape of the earth is smaller — less than a minute of arc difference in the location of the center of population.

The Bureau of the Census gives the center of population to the nearest second of arc of latitude and longitude. If one wishes to pursue the location of the center of population of the United States to an accuracy of one second of an arc then the ellipsoidal shape of the

Table 2.

The Center of Population for 1980 for the United States calculated by various methods for the same example data set previously used (Barmore, 1991).

Center of Population				
Method of computation	label	latitude	longitude	depth
Bureau of the Census formulæ	<i>BC</i>	38.1376	90.5737	—
In three dimensions for a sphere	<i>s</i>	39.1823	90.3477	165 km
In three dimensions for an ellipsoid	<i>e</i>	39.1887	165 km	
On the surface of a sphere	<i>COP</i>	39.1980	90.4978	0
On the surface of an ellipsoid	<i>COP - E</i>	39.2045	90.4969	0

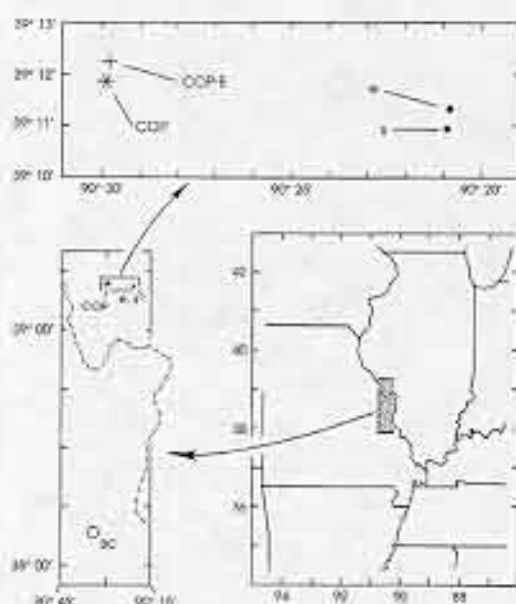


Figure 3. The "Center of Population" of the United States for 1980 calculated by various methods. The place shown as an open circle and labeled *BC*, is the center determined by the U.S. Bureau of the Census (1983). As discussed previously (Barmore, 1991) this place should not be called the center of population. The places shown as solid circles and labeled *s* and *e*, mark the centers calculated in three dimensions assuming the population is on the surface of a sphere or on the surface of Clarke's (1866) ellipsoid, respectively. The calculated centers lie ca. 165 km below the places marked. The places shown as an asterisk or a plus and labeled *COP* or *COP - E* are the centers calculated using the preferred method (Barmore, 1991) and assumes the population is on the surface of a sphere or on the surface of Clarke's (1866) ellipsoid, respectively. The preferred method restricts the computation and results to the surface (sphere or ellipsoid) containing the population.



*Summer, 1993*

Earth (and a host of other considerations) should be taken into account.

### **Summary**

The Earth is not spherical. The Earth is ellipsoidal in shape. When computations are done without due regard for the ellipsoidal shape of the Earth they may be in error by amounts on the order of 1/10 degree. This paper points out: 1) that errors of ca. 1/10 degree result in qibla (and other azimuths calculated on a sphere, 2) that errors of ca. 1/10 degree result in the location of the geographic center of very large areas calculated on a sphere, but 3) that the error in the location of United States population center when properly calculated on a sphere is less than one minute of an arc.

## Appendix A

Because the Earth is not a sphere (nor, for that matter, exactly an ellipsoid of revolution) a certain amount of care will be needed in using the terms *azimuth* and *distance*. This paper uses several terms (described below) which correspond closely to those defined and used by Bomford (1977). Also, several other concepts deserve additional comment.

**ASTRONOMICAL AZIMUTH:** For places on the physical surface of the Earth, the astronomical azimuth of one place from another corresponds to what would be measured with an accurate instrument located on the surface of the Earth.

**GEODETIC AZIMUTH:** For places on the surface of an ellipsoid representing the Earth, the geodetic azimuth of one place from another is what would be measured with an accurate instrument located on the surface of the ellipsoid, the instrument being "leveled" relative to the ellipsoid's normal at the instrument's location rather than the "gravitational field."

**SIMPLE AZIMUTH:** For places on the surface of a sphere, the simple azimuth of one place from another corresponds to what would be measured with an accurate instrument located on the surface of the sphere, the instrument being "leveled" relative to the sphere's normal at the instrument's location rather than the "gravitational field."

**DISTANCE:** For places on the surface of an ellipsoid, distances between places are often measured along the "normal sections" rather than along geodesics. For places on the surface of a sphere, distances between places are almost always measured along geodesics, called great circles.

On the sphere simple azimuths and great circle distances are easily calculated with spherical trigonometry. On the ellipsoid geodetic azimuths and normal section distances are determined by more complex calculations. In this paper Cunningham's formula was used for Geodetic Azimuth (Bomford, 1977, Eq. 2.23) and Rudee's "9-figure" formula was used for distances along the normal section (Bomford, 1977, p. 136).

**LOCATION:** Places are located on a sphere, an ellipsoid or an accurate map according to their geographic (that is, geodetic) coordinates.

**ACCURACY:** Roughly speaking, calculations done on a sphere will represent distances and direction on the real surface of the Earth with an accuracy of one degree or more. Calculations done on a suitable ellipsoid will represent distances and direction on the real surface of the Earth with an accuracy of one minute of arc or more. For an accuracy of one second of arc or more, details such as the choice of the ellipsoid parameters, the Earth's gravitational field and heights of the various places must be taken into account. For the purposes of this paper (accuracy of one minute of arc) geodetic azimuths and distances in the normal sections represent the real case well enough. It is a rare case indeed that the difference between the geodetic and the astronomical quantities would be so large as one minute of arc (Bomford, 1977, p. 115, 528). In the main text of the paper results are often stated to the nearest second of arc (or 0.0001 degree). It should be kept in mind that these results are the geodetic results. This level of accuracy is justified for comparisons of similar results but it is not the absolute accuracy of quantities on the physical surface of the Earth.

**ELLIPSOID:** All the calculations involving the ellipsoid and discussed in the main part of the text used Clarke's (1866) Ellipsoid,  $a=6378.2064$  km. and  $e=0.08227185$ . The geometry of the ellipsoid and various series expansions for some of the relationships were those given by Bomford (1977, Appx. A, C).

NOTATION: All azimuths are measured from the North toward the East and are always positive (i.e., SW = +225 degrees, never -135). Angles are given in degrees and decimal degrees (sometimes without the unit name or symbol) or in degrees and minutes of arc (and sometimes seconds of arc) and always with the symbols: dd°mm'ss").

COMPUTATIONS: All computations were done on an Apple IIGS computer using the spreadsheet in AppleWorks 3.0 (Claris Corp.).

Appendix B  
Half-way Point Calculation.

(added to this reprinting at the request of the Editor.)

If the distance from the equator to the pole measured along a meridian on the surface of the ellipsoid is  $s$ , then:

$$s = \int_{\text{equator}}^{\text{pole}} ds.$$

Rewriting this in terms of the radius of curvature,  $\rho$ , and the geographic (geodetic) latitude,  $\phi$ , the latitude of the half-way point,  $\Phi$ , is then given by:

$$\int_0^{\Phi} \rho \cdot d\phi = \frac{1}{2} \int_0^{\pi/2} \rho \cdot d\phi = \frac{1}{2}s.$$

Bomford (1977, eq. A.53) gives the radius of curvature in terms of the semi-major axis  $a$ , the eccentricity  $e$ , and the geographic latitude. Then:

$$\int_0^{\Phi} \frac{a(1-e^2)d\phi}{(1-e^2\sin^2\phi)^{3/2}} = \frac{1}{2} \int_0^{\pi/2} \frac{a(1-e^2)d\phi}{(1-e^2\sin^2\phi)^{3/2}}$$

Cancelling common terms, using the binomial expansion ( $e$  is small), and evaluating the resulting series of definite integrals on the right hand side (RHS) one finds:

$$\text{RHS} = \frac{\pi}{4} \left[ 1 + \frac{3}{2} \cdot e^2 \cdot \frac{1}{2} + \frac{3 \cdot 5}{2 \cdot 4} \cdot e^4 \cdot \frac{1 \cdot 3}{2 \cdot 4} + \dots \right].$$

The left hand side (LHS) integrals can be reduced (with a certain amount of algebraic and trigonometric manipulation) to:

$$\text{LHS} = \Phi + \frac{3}{2} \cdot e^2 \cdot \left( \frac{\Phi}{2} - \frac{\sin 2\Phi}{4} \right) + \frac{3 \cdot 5}{2 \cdot 4} \cdot e^4 \cdot \left( \frac{3}{8} \Phi - \frac{\sin 2\Phi}{4} + \frac{\sin 4\Phi}{32} + \dots \right).$$

Ignoring the smaller terms — terms containing  $e^4$ ,  $e^6$  etc. (using the eccentricity for Clarke's 1866 ellipsoid) yields:

$$\Phi = 0.787923557 = 45.1447^\circ = 45^\circ 08' 41''.$$

Including terms containing  $e^4$  and  $e^6$  yields:

$$\Phi = 0.787945019 = 45.145924^\circ = 45^\circ 08' 45''.$$



## References

- Barmore, Frank E. 1991. "Where Are We? Comments on the Concept of the 'Center of Population'." *The Wisconsin Geographer*, Vol. 7, 40-50. (Reprinted (with the example data set used and with several corrections) by the Institute of Mathematical Geography, Ann Arbor, MI, in their journal, *Solstice: An Electronic Journal of Geography and Mathematics*, Vol. III, No. 2, 22-38, Winter, 1992.)
- Barmore, Frank E. 1985. "Turkish Mosque Orientation and The Secular Variation of the Magnetic Declination." *The Journal of Near Eastern Studies*, Vol. 44, No. 2, 81-98.
- Berryman, Peter. 1989. "PONIATOWSKI." *The New Berryman Berryman Songbook*, Madison, WI, Lou and Peter Berryman.
- Bomford, G. 1977. *Geodesy*. Oxford UK, Clarendon Press. A reprinting (with corrections) of the 1971 3rd Ed.
- Head, J. W., et al. 1981. "Topography of Venus and Earth: A Test for the Presence of Plate Tectonics." *American Scientist*, Vol. 69, 614-623.
- King, D. A., 1972. "Kibla." *The Encyclopædia of Islam*, 2nd ed., Vol. 5, 83-88.
- King, D. A., 1978. "Three Sundials from Islamic Andalusia." *Journal for the History of Arabic Sciences*, Vol. 2, 358-392.
- King, D. A., 1982a. "Astronomical Alignments in Mediæval Islamic Religious Architecture." *Annals of the New York Academy of Sciences*, Vol. 385, 303-312.
- King, D. A., 1982 b. "The World about the Kaaba." *The Sciences*, Vol. 22, 16-20.
- Mendelev, D. I. 1907. *K Poznaniyu Rossii*, 5th ed. St. Petersburg, A. S. Suvorina, p. 139.
- Mutch, T. A., et al. 1976. *The Geology of Mars*. Princeton NJ, Princeton University Press, pp. 61-63, 209, and 213.
- U. S. Bureau of the Census. 1983. *1980 Census of Population, Vol. 1, Chapter A, Part 1 (PC80-1-A1)*. Washington DC; U.S. Dep't. of Commerce, Bureau of the Census. Appendix A, p. A-5 and Table 8, p. 1-43.
- U. S. Bureau of the census. 1973. *1970 Census of Population and Housing: Procedural History (PHC(R)-1)*. Washington DC; U.S. Dep't. of Commerce, Bureau of the Census. Appendix B (Computation of the 1970 U.S. center of population), pp. 3-50.
- The Times Atlas of the World*, 8th Comp. Ed. 1990. New York, Times Books / Random House.
- Mecca al-Mukarrama, 1:15000 1970? (Riyadh, Kingdom of Saudi Arabia, Ministry of Petroleum and Resources, Aerial Survey Dep't.)

\* Frank E. Barmore, Department of Physics, Cowley Hall, University of Wisconsin, La Crosse, La Crosse, WI 54601