MICROCELL HEX-NETS?

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The ongoing revolution in electronic communications offers exciting opportunities to realize geographic ideas in perhaps unimagined electronic realms. It is well-known, throughout governmental, business, and academic communities, that the cartographer can make a map from hundreds of electronic layers in a Geographic Information System (GIS), in which the data behind the map work interactively with the map, so that upgraded data produces an upgraded map. GIS is certainly one exciting result of the interaction between traditional science and electronics.

Cordless telephones offer other prospects: networks of mobile terminals can be linked together in networks across city streets as well as within office skyscrapers. Chia (1992) notes that the Research on Advanced Communications for Europe (RACE) initiative of 1988, to study techniques to implement a third generation Universal Mobile Telecommunications System by the year 2000, is a significant step toward unifying communications and fixed networks.

The concept of a mobile telecommunication is straightforward (Chia 1992). Simply stated, a set of microcell base stations, each of which can transmit and receive electronic information, is spread across a geographic space as a network of stations, each with its own tributary area, a microcell, with which it communicates. Typically, one might think of the microcell base station as the center of a circular tributary area, with circular areas packed to cover a larger circular area. At the center of the larger circular area, a macrocell base station serves as an "umbrella" to relay information to the microcell base stations under it, and from one network of microcells to the next (Chia 1992). Within this sort of "mixed cell architecture," a vehicle carrying a terminal onboard passes through the microcells and receives information on a continual basis from the base station associated with the microcell it is currently traversing. This sort of hand-off of information in order to traverse a network is not new; indeed, the Rohrpost — an underground network of pneumatic tubes used for message transmission in Berlin in the early 1900s — was composed of energy regions in which pneumatic carriers were handed off from one region to the next in order to transmit messages across a fairly large geographic area (Arlinghaus 1986). More commonly, a relay foot race involves the handing off of a baton from one runner to a second, once the first runner has expended much energy to traverse some specified geographic space. There are a host of other illustrations of this sort.

There are apparently numerous engineering concerns associated with the optimal positioning of the base stations: antenna radiation patterns, natural terrain features, interference from tall buildings, and interference and signal attenuation of all sorts, including difficulties when the mobile unit turns a corner (Chia, 1992). The geometry of directional paths through Manhattan space (Krause 1975), based on number of vehicle turns can then also become of concern (Arlinghaus and Nystuen 1989).

It is the geographic issues of street patterns and building position that are fundamental to the engineering concerns in implementing these networks in which moving vehicles interchange information with a fixed network of base stations (Chia 1992). Even a brief glance at an atlas shows the range of variation in street pattern — from the predominantly rectilinear grid of Manhattan, to the polar-coordinate style of rotary and radial evident in Washington D.C. Thus, many studies involving microcell networks are done, initially, in an abstract environment (Chia, 1992) — as a benchmark against which to evaluate others in less than optimal environments. It is within this spirit that a microcell system, composed of layers of microcells of varying size, is viewed.

Lattices

Viewed broadly, microcell base stations are a set of lattice points. The way in which the lattice is constructed can affect all other considerations of the functioning of the consequent microcell network. There are an infinite number of "general" environments that one might use in which to construct benchmark networks. When the size of the microcells is sufficiently "large," the microcell tributary areas might be viewed as curved surfaces which when pieced together form a set of plates composing a broad continental (for example) surface. When the size of the microcells (or macrocells) is "local" rather than "large," curvature may not be an issue and the cells might be treated as plane regions. (What is "local," and what is not, is a significant problem for pragmatic implementation; at the abstract level it is of importance to note it, but not necessarily to deal with it directly.) And, if the line-of-sight geometry is one that excludes parallelism, or that permits more than one parallel, then it may be suitable to view microcell network architecture/geography from the non-Euclidean vantage point of elliptic or hyperbolic geometry (Arlinghaus 1990).

Within a plane region, there are two basic ways of creating an evenly-spread lattice: one with the lattice points lying in a grid pattern, and the other with the lattice points lying in a triangular / hexagonal grid pattern (Coxeter 1961). The differences between the two should be clear to anyone who has played the game of checkers on both a square board and on a "Chinese" board. What is not evident, though, is the sorts of patterns that emerge when one stacks layers of square or hexagonal cells of different sizes in varying orientations. When a square lattice is chosen, one style of space-filling by tributary regions emerges; when an hexagonal lattice is chosen, another appears.

Microcell hex-nets

Walter Christaller (1933, 1966) grappled with the problem of overlays of hexagonal nets; he did so in the German urban environment. One might question some of the interpretations of the patterns, but his analysis of the actual patterns of overlays is correct. There are numerous discussions of this problem, often under the heading of "central place theory" — or, how cities might share interstitial space (Christaller 1933, 1966; Dacey 1965). When the focus is on the extent to which space is filled by portions of the hexagonal outlines, as it might be when signal attenuation and interference of radio waves are an issue, then the fractal approach which permits the easy measurement of the extent to which an infinitely iterated overlay of nets will fill space is useful.

One way to look at the complicated issue of visualizing overlays of hexagonal nets is simply to think of a central hexagon surrounded by six hexagons of the same size — each of these is centered on a microcell base station. The central hexagon is also centered on a macrocell base station which serves the entire set of seven hexagons and has as its own larger tributary macrocell, a hexagon formed by joining pairs of vertices (separated by two intervening vertices) of the perimeter of this snowflake region. When these microcells and macrocells are iterated across the plane, a stack of two layers of hexagonal cells emerges, with the orientation of one relative to the other at an angle that insures that each of the macrocells contains the geometric equivalent of 7 microcells. If one zooms in or out, to generate other layers of larger or smaller hexagons, the stack may be increased; as long as the angle of orientation is fixed by the first two, the value of "7" will be a constant of the hierarchy —no matter which two adjacent layers of the hierarchy are considered, a large cell will contain the equivalent of 7 smaller cells. In the literature, this is often referred to as the "$K = 7$" hierarchy.

When one chooses different orientations of the nets, different K values emerge; indeed, there are an infinite number of possibilities. When it is also required that vertices of smaller hexagons coincide with those of larger hexagons, there are still an infinite number of hierarchies with the K values generated by the Diophantine equation $x^2 + xy + y^2 = K$ (Dacey 1965) where x and y are the coordinates of the lattice points arranged in a triangular lattice (and so relative to a coordinate system with the y-axis inclined at 60° to the x-axis).

A structurally identical process may be employed to make similar calculations for layers of squares centered on a square lattice. Relationships which show the number of small square microcells within a larger square macrocell are also constant between adjacent layers of a hierarchy formed from a single orientation criterion ("J" value).

Fractal geometry may be used to generate any of these hierarchies: hexagonal or square. All that is needed is to know the number of sides in a fractal generator and the self-similarity pattern desired (K- or J-value). From these, one can determine completely and uniquely the entire hierarchy–both cell size within a layer and orientation of layer (Arlinghaus, 1985; Arlinghaus and Arlinghaus 1989). The fractal dimension measures the extent to which parts of the boundary of the hexagons or squares remain under infinite iteration. When the results of the calculations are displayed in a table, it appears that hexagonal nets consistently fill less space (Arlinghaus, 1993).

This Table suggests that individuals actually implementing microcell systems might wish to first consider shape, size, and orientation of layers of a mixed cell architecture prior to superimposing any of the geographic concerns of street networks, or engineering concerns caused by interference and signal attenuation. A mixed cell architecture of low fractal dimension might be one that reduces interference, to some extent, just by the relative positions of microcells to macrocells.

Table: Comparison of fractal dimensions

Lattice coordinates of microcell base station adjacent to microcell base station at (0, 0)	Fractal Dimension	
	Squares	Hexagons
(1,1)	2.0	1.262
(1,2)	1.365	1.129
(0,2)	2.0	1.585
(0,3)	1.465	1.262
(0,4)	1.5	1.161
(0,5)	1.365	1.209
(0,6)	1.387	1.161
(0,7)	1.318	1.129
(0,8)	1.333	1.153
(0,9)	1.290	1.131
(0,10)	1.301	1.114

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