# Los Angeles, 1994–A Spatial Scientific View

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An algorithm discussed by Maria Hasse (Hasse, 1961; Harary, Norman, and Cartwright, 1965) offers a method for finding the shortest distance between any two nodes in a network of $n$ nodes when given only distances between adjacent nodes. The algorithm is one that focuses on structure alone, and it is therefore spatial. The procedure is similar in form to that used to multiply matrices, given two $n \times n$ matrices $A$ and $B$. To find the entries in their Hasse-sum, matrix $C$, take the minimum of the row-by-column sums; thus, the entry

$$c(21) = \min\{a(21) + b(12), a(22) + b(22), a(23) + b(32), ..., a(2n) + b(n2)\}.$$

The results below show the outcome of applying this tool from theoretical spatial science to the real-world: to one change in the Los Angeles freeway pattern following the recent devastating earthquake (January 17, 1994).

## Los Angeles, 1994.

When a recent earthquake caused a disastrous collapse of a span of the Santa Monica freeway, according to television reports the world's busiest freeway (carrying an estimated 300,000 vehicles per weekday), municipal authorities managed to keep the city moving. They employed a well-balanced combination of alternate routing using intelligent vehicle highway systems (IVHS) in which traffic lights along surface routes paralleling freeways were coordinated in response to user demand, together with media messages urging people to stay off roadways and the effective dispersal of information concerning alternate routes. Outside forecasters of doom predicted massive gridlock that did not occur in regions where alternate routing was available.

In the analysis below, we test Hasse's algorithm against a changed adjacency configuration and interpret the results. Indeed, what would a forecaster using the Hasse algorithm have predicted in this situation?

The map in Figure 1 shows a portion of the Los Angeles freeway system, and nearby major surface arterials, linking Los Angeles International Airport (LAX) to the Central City (CC). We tightened our focus to consider what sort of impact the partial closing of the Santa Monica freeway might have on travel times to and from the airport and the downtown region. The routing in Figure 1 is along freeways, only, that form a square envelope around the direct diagonal route (that does not exist in the real world) linking LAX to the CC. Any rupture along this circuit will completely destroy one of the two possible routes, sending all the traffic along one path only. Thus, when the Santa Monica freeway was ruptured (Figure 2)–cross-hatched area on Figure 1 – all the traffic would have been forced due east and then north, if only freeway linkages were employed.

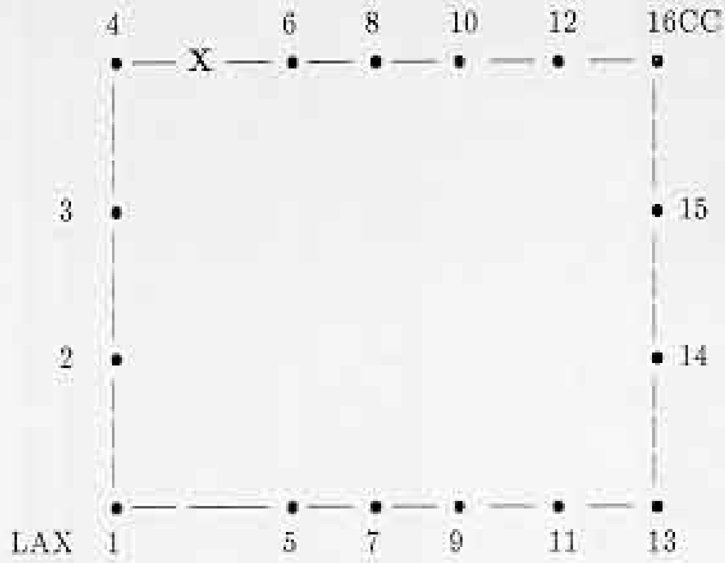


**Figure 1.** LAX denotes Los Angeles International Airport. CC denotes the Central City. Routes are along major expressways. The X indicates the rupture in the Santa Monica freeway caused by the January 17, 1994 earthquake. Consider that all lines, whether dashed or solid, represent continuous graphical linkage between adjacent nodes. The only break in the freeway is at the X.

To overcome this apparently disastrous traffic situation, it is natural to introduce alternate routes along roads that are already present. Indeed, in earlier mathematical references there is consideration of this sort of rerouting problem after some edges of a network have been deleted (Menger, 1927; Ford and Fulkerson, 1962). One set of major surface routes is added to the map in Figure 1 to offer a number of different routes (Figure 3). The matrix $A$ (Figure 4) displays time-distances in tabular form across the network shown in Figure 3. The entry of 12 in the first row, second column indicates that it takes 12 quarter-minutes to travel from the node labelled 1 to the node labelled 2. A zero in this matrix indicates that there are zero quarter-minutes required as travel time–thus, zeroes appear in this matrix only along the main diagonal. Nodes are treated as points within which no travel is possible. An asterisk indicates that there is no direct linkage between corresponding entries– an asterisk in the (1,3) position indicates that there is no single edge of the graph linking node 1 to node 3. All numerical entries are expressed in quarter-minutes; the Pascal program (Figure 5), was written to display integral results. (Use of a spreadsheet is possible but is far more time-consuming.) Travel times were calculated from distances in the 1993 Rand McNally Road Atlas, assuming (from field experience) an average speed of 40 mph.

Higher powers of the matrix $A$ count numbers of paths of longer length. The matrix $A^2$ counts paths of 2 edges as well as those of one edge. Thus, in $A^2$ one would expect to find an entry indicating the total time to travel from node 1 to node 3, as well as entries representing travel times across single graphical edges from node 1 to node 2 and from node

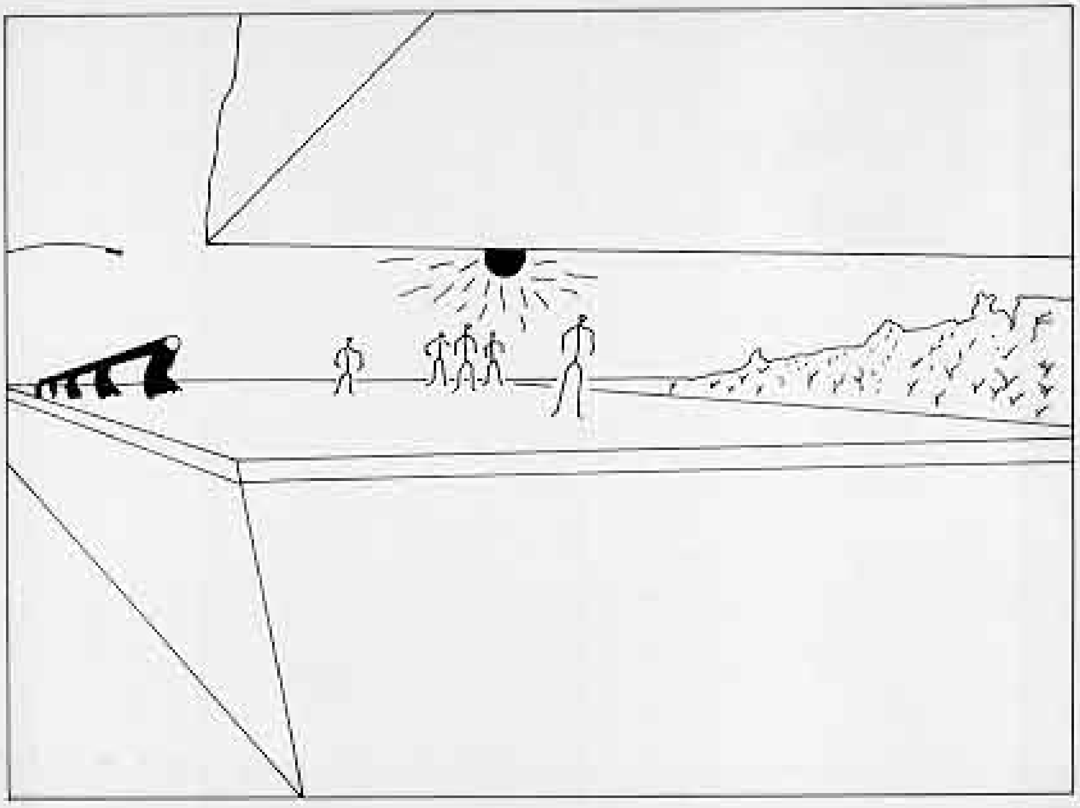

**Figure 2.** Drawing based on a photo, showing damage to the Los Angeles freeways, from the *New York Times*, Tuesday, January 18, 1994.

2 to node 3; indeed, as would be hoped the travel time of 30 quarter-minutes from nodes 1 to 3 is the sum of the travel times from 1 to 2 (12 quarter-minutes) and from 2 to 3 (18 quarter-minutes).

The Hasse operator (erroneously referred to as the Hedetniemi operator in some earlier work, corrected by F. Harary who also notes that this procedure may also be present in literature earlier than Hasse's 1961 article) always selects the shortest path if more than one is available. Other algorithms execute similar calculations; however, Floyd's algorithm provides easy display of lengths only (and not the components that compose them), while Dijkstra's algorithm is not designed for easy display of results but does permit the determination of the actual position of the shortest path. Both of these algorithms require the same number of steps independent of the actual data; Hasse's does not – it stops shorter than would Floyd's or Dijkstra's in many situations. Further detail has been published elsewhere (Harary, Norman, and Cartwright, 1960; Arlinghaus, Arlinghaus, and Nystuen, 1990).

The matrices $A$ through $A^8$ show travel times across paths of varying length for the

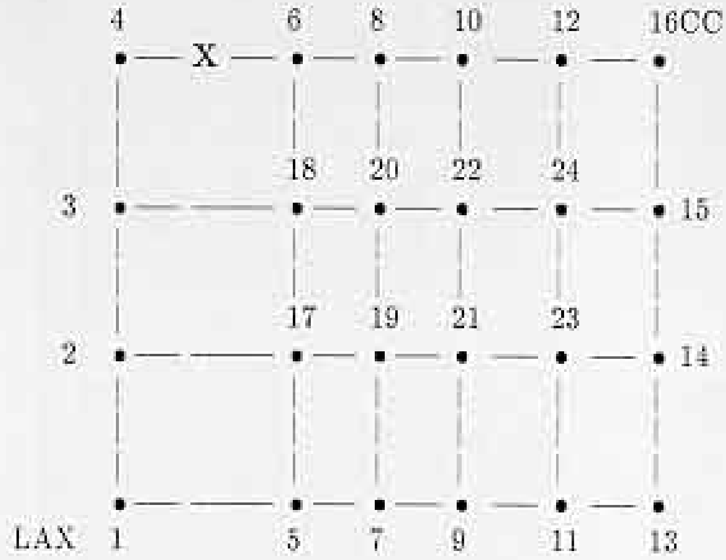


**Figure 3.** Same basic pattern as Figure 1, with surface routes added, and intersections of surface routes added as nodes in the graph. LAX denotes Los Angeles International Airport. CC denotes the Central City. Routes are along major expressways. The X indicates the rupture in the Santa Monica freeway caused by the January 17, 1994 earthquake. Consider that all lines, whether dashed or solid, represent continuous graphical linkage between adjacent nodes. The only break in the freeway is at the X.

freeway system prior to the earthquake (Figure 4a). The algorithm stops when $A^{n+1} = A^n$; in this case, therefore, the last matrix with new entries is $A^8$–the matrix $A^9$ is calculated to know when to stop the iteration. A different initial matrix is required to capture the linkage pattern between LAX and CC following the 1994 earthquake (Figure 4b)–the Santa Monica freeway was shattered between nodes 4 and 6 on the graph in Figure 3. The matrix $B$ in Figure 6 indicates a new adjacency pattern; in $A$, the 4th row, 6th column contained a value of 30 to represent the direct linkage between nodes 4 and 6. The corresponding entry in matrix $B$ is an asterisk – that is, there is no path, of a single graphical edge, available between nodes 4 and 6. When Hasse's algorithm is run using $B$ (Figure 4b), instead of $A$ (Figure 4a), as the initial matrix, the iteration requires the same number of stages; however, some of the entries are larger in $B$ than in $A$, reflecting the need for longer paths to provide alternate routes around the earthquake-altered freeway. In the eighth iteration, the $B$-iterate contains entries in the (4,6), (4,8), (4,10), (4,12), and (4,16) positions that are about 30 quarter-minutes larger than are the entries in the corresponding eighth A-iterate. This increase in the structural model comes purely from spatial pattern–it does not address the natural increase in congestion that one would also expect.

The surface route pattern that was introduced permitted all turns at each of the surface route intersections; this sort of strategy appears desirable, but because turns (especially U.S. left-hand turns onto two way streets) generally force additional slowing of the traffic

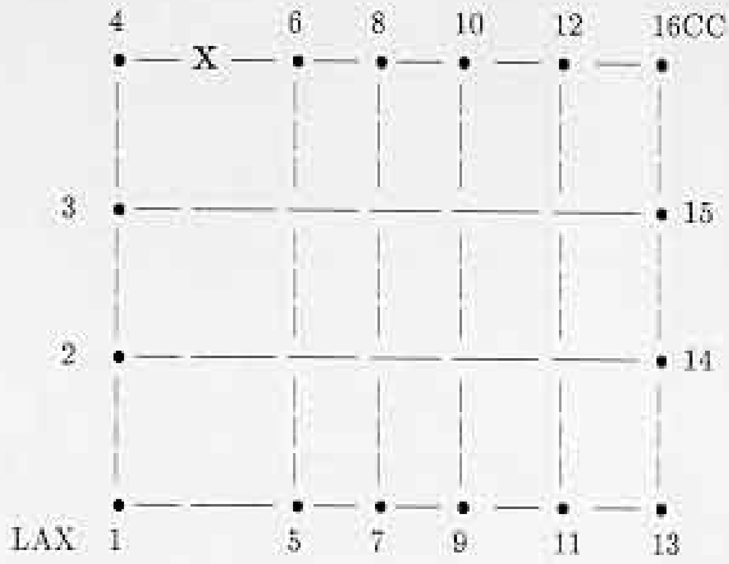


**Figure 6.** Same basic pattern as Figure 1, with surface routes added: unlike Figure 2, in this case intersections of surface routes are NOT added as nodes in the graph. The surface routes have limited access. LAX denotes Los Angeles International Airport. CC denotes the Central City. Routes are along major expressways. The X indicates the rupture in the Santa Monica freeway caused by the January 17, 1994 earthquake. Consider that all lines, whether dashed or solid, represent continuous graphical linkage between adjacent nodes. The only break in the freeway is at the X.

one might consider further alteration of the structural model.

Figure 6 shows a modified form of the map in Figure 3; in it, the nodes 17 through 24 have been omitted. This graphical omission corresponds to the real-world notion of preventing intersecting traffic flows within the interchanges. One way to reduce congestion is to prohibit all turns. Another is to use traffic lights in a manner that responds to the traffic itself, rather that to estimates of traffic. The structural model in Figure 6 represents this sort of approach; the north-south route from node 5 to node 6 does not intersect any of the east-west surface vehicular flows.

Figure 7a shows the initial matrix $C$ representing this particular structural model that permits restricted pre-earthquake travel across surface routes. Figure 7b shows the initial matrix $D$ representing the model with the rupture in the Santa Monica freeway. When Hasse's algorithm is run, there are clearly once again a number of locations that stand out: the (4,6) entry, for example, goes from 30 quarter-minutes to 114 quarter-minutes in this case. Indeed, there is not even any path available of length less than 5 for this entry: there is an asterisk in this position in $D^4$ –the only asterisk for this entry in the $C$ iteration sequence, with the bridge in, is in the first matrix. The last entries to come into the $D$ sequence iteration are (4,8) and (4,9)–this situation tallies with the relationships shown on the map in Figure 3. Traffic engineers might choose this latter model during times of the day when volumes are not high at the nodes showing large increases, or some other strategy that responds to traffic history.

The path structure from node 1 (LAX) to node 16 (CC) is not altered; the Santa Monica freeway was not the shortest route from LAX to CC although its closure no doubt adds to the congestion along shorter routes. Most of the entries in the fourth row of $D^7$ to the right of, and including, the sixth column show increases in time – some only slight and some substantial. Only the fourth row and the fourth column show altered time patterns, pre- and post-earthquake – Hasse's algorithm shows that the underlying spatial structure of the road network is sufficient to provide alternate routing to and from LAX to CC and between many of the intervening locations. This finding matches what has apparently happened in the actual post-earthquake environment.

**Policy Implications**

In order to turn the elegant theoretical tool of Hasse into one a traffic engineer might actually employ, there are a number of policy implications to consider – policy changes can put real-world teeth into theory.

1. No turns except onto expressways means maximum flow; however, this strategy is awkward for those living in the area. Indeed, even if it is assumed that people can turn off onto minor streets but cannot turn at major intersections, these local turns cause a lower average speed.
2. Permit right hand turns only –not too disruptive of flow so speed is maintained. The algorithm still holds, even with an asymmetric matrix.
3. Permit all turns – there are a number of engineering strategies that might have corresponding structural components in a graphical model. Left hand turns slow the system. Insert different average speeds or times on the edges of the structural model.
4. Use one-way streets–this strategy equalizes left and right turns; it, too, produces asymmetric adjacency matrices.

**References**

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## Figures containing tables

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**Figure 4a.i This is the initial matrix,** *A*. Figure 4a contains a set of nine tables (i to ix) illustrating the use of Hasse's algorithm on part of the LA freeway/surface route system, shown in Figure 3, prior to the earthquake of January 17, 1994. Travel times are in one-quarter minutes. An asterisk indicates that the travel time between locations is too large to enter the matrix. A double-zero indicates an entry of 0.

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| * | 42 | 24 | 00 | * | 30 | * | 42 | * | * | * | * | * | * | * | * | * | 42 | * | * | * | * | * | * |
| 18 | 18 | * | * | 00 | * | 12 | * | 18 | * | * | * | * | * | * | * | 12 | 24 | 21 | * | * | * | * | * |
| * | * | 39 | 30 | * | 00 | * | 12 | * | 21 | * | * | * | * | * | * | 33 | 21 | * | 30 | * | * | * | * |
| 30 | * | * | * | 12 | * | 00 | * | 6 | * | 12 | * | * | * | * | * | 24 | * | 15 | 27 | 21 | * | * | * |
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**Figure 4a.ii. This is the power 2 matrix, $A^2$.**

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| 54 | 42 | 24 | 00 | * | 30 | * | 42 | * | 51 | * | * | * | * | * | * | 48 | 42 | * | 51 | * | * | * | * |
| 18 | 18 | 36 | * | 00 | 45 | 12 | * | 18 | * | 24 | * | * | * | * | * | 12 | 24 | 21 | 33 | 30 | * | * | * |
| * | 39 | 39 | 30 | 45 | 00 | * | 12 | * | 21 | * | 27 | * | * | * | * | 33 | 21 | 42 | 30 | * | 39 | * | * |
| 30 | 30 | * | * | 12 | * | 00 | 48 | 6 | * | 12 | * | 18 | * | * | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | * |
| * | * | 48 | 42 | * | 12 | 48 | 00 | * | 9 | * | 15 | * | * | * | 21 | 42 | 30 | 33 | 21 | 42 | 30 | * | 36 |
| 36 | * | * | * | 18 | * | 6 | * | 00 | 48 | 6 | * | 12 | 24 | * | * | 30 | * | 21 | 33 | 15 | 27 | 21 | 33 |
| * | * | * | 51 | * | 21 | * | 9 | 48 | 00 | * | 6 | * | * | 30 | 12 | * | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| * | * | * | * | 24 | * | 12 | * | 6 | * | 00 | 48 | 6 | 18 | 30 | * | * | * | 27 | * | 21 | 33 | 15 | 27 |
| * | * | * | * | * | 27 | * | 15 | * | 6 | 48 | 00 | * | 36 | 24 | 6 | * | * | * | 36 | 39 | 27 | 33 | 21 |
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| * | * | * | * | * | * | * | 21 | * | 12 | * | 6 | 45 | 33 | 21 | 00 | * | * | * | * | * | 33 | 36 | 24 |
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| * | * | 36 | * | * | 39 | 33 | 30 | 27 | 21 | * | 27 | * | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
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**Figure 4a.iii This is the power 3 matrix, $A^3$.**

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| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | * | 57 | * | * | * | * | * | * | 24 | 18 | 33 | 27 | 42 | 36 | * | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | * | 42 | * | 51 | * | 57 | * | * | * | * | 48 | 42 | 57 | 51 | * | 60 | * | * |
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| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | * | 21 | * | 27 | * | * | * | 33 | 33 | 21 | 42 | 30 | 51 | 39 | * | 48 |
| 30 | 30 | 48 | * | 12 | 57 | 00 | 48 | 6 | 54 | 12 | * | 18 | 30 | * | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| * | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | * | 15 | * | * | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | * | * | 18 | * | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | * | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| * | * | 57 | 51 | * | 21 | 54 | 9 | 48 | 00 | 54 | 6 | * | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | * | * | * | 24 | * | 12 | * | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | * | 27 | 39 | 21 | 33 | 15 | 27 |
| * | * | * | 57 | * | 27 | * | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | * | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| * | * | * | * | 30 | * | 18 | * | 12 | * | 6 | 48 | 00 | 12 | 24 | 45 | * | * | 30 | * | 21 | 33 | 15 | 27 |
| * | * | * | * | * | * | 30 | * | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | * | 18 | 30 | 9 | 21 | 3 | 15 |
| * | * | * | * | * | * | * | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | * | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | * | * | * | 33 | * | 21 | * | 12 | 51 | 6 | 45 | 33 | 21 | 00 | * | * | * | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | * | * | 27 | * | * | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | * | 45 | * | * | 33 | * | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | * | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | * | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | * | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| * | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| * | 30 | * | * | 36 | * | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | * | 15 | 27 | 6 | 18 | 00 | 12 |
| * | * | 48 | * | * | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | * | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.iv This is the power 4 matrix,** $A^4$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | * | 42 | * | 48 | * | * | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | * |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | * | * | 33 | * | * | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | * | 63 | * | * | 51 | * | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | 72 | 42 | * | 51 | * | 57 | * | * | * | 63 | 48 | 42 | 57 | 51 | 66 | 60 | * | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | * | 30 | 39 | * | * | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | 63 | 21 | * | 27 | * | * | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | * | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | * | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| * | 57 | 57 | 51 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | * | * | 24 | * | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| * | * | 63 | 57 | * | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | * | * | * | 30 | * | 18 | * | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | * | 30 | * | 21 | 33 | 15 | 27 |
| * | 33 | * | * | 39 | * | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| * | * | 51 | * | * | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | * | 63 | * | 33 | * | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | * | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | * | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | * | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | * | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| * | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.v This is the power 5 matrix, $A^5$.**

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | * | 48 | 45 | * | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | * | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | * | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | 72 | 42 | 78 | 51 | * | 57 | * | * | 75 | 63 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | * | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | * | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 51 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | * | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| * | 63 | 63 | 57 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | * | * | 30 | * | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | * | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| * | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | 69 | 63 | * | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.vi This is the power 6 matrix,** $A^6$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | 72 | 42 | 78 | 51 | 84 | 57 | * | 75 | 75 | 63 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 51 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 57 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | * | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | 66 | 69 | 63 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.vii This is the power 7 matrix,** $A^7$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | 78 | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | 72 | 42 | 78 | 51 | 84 | 57 | 87 | 75 | 75 | 63 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 51 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 57 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | 87 | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| 78 | 66 | 69 | 63 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.viii This is the power 8 matrix,** $A^8$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | 78 | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 30 | 72 | 42 | 78 | 51 | 84 | 57 | 87 | 75 | 75 | 63 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 30 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 42 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 51 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 57 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | 87 | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| 78 | 66 | 69 | 63 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4a.ix This is the power 9 matrix,** $A^9$. It is identical to the matrix in Figure 4a.viii and so the algorithm terminates.

**Figures containing tables**

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | * | * | 18 | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * |
| 12 | 00 | 18 | * | * | * | * | * | * | * | * | * | * | * | * | * | 6 | * | * | * | * | * | * | * |
| * | 18 | 00 | 24 | * | * | * | * | * | * | * | * | * | * | * | * | * | 18 | * | * | * | * | * | * |
| * | * | 24 | 00 | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * |
| 18 | * | * | * | 00 | * | 12 | * | * | * | * | * | * | * | * | * | 12 | * | * | * | * | * | * | * |
| * | * | * | * | * | 00 | * | 12 | * | * | * | * | * | * | * | * | * | 21 | * | * | * | * | * | * |
| * | * | * | * | 12 | * | 00 | * | 6 | * | * | * | * | * | * | * | * | * | 15 | * | * | * | * | * |
| * | * | * | * | * | 12 | * | 00 | * | 9 | * | * | * | * | * | * | * | * | * | 21 | * | * | * | * |
| * | * | * | * | * | * | 6 | * | 00 | * | 6 | * | * | * | * | * | * | * | * | * | 15 | * | * | * |
| * | * | * | * | * | * | * | 9 | * | 00 | * | 6 | * | * | * | * | * | * | * | * | * | 21 | * | * |
| * | * | * | * | * | * | * | * | 6 | * | 00 | * | 6 | * | * | * | * | * | * | * | * | * | 15 | * |
| * | * | * | * | * | * | * | * | * | 6 | * | 00 | * | * | * | 6 | * | * | * | * | * | * | * | 21 |
| * | * | * | * | * | * | * | * | * | * | 6 | * | 00 | 12 | * | * | * | * | * | * | * | * | * | * |
| * | * | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 12 | * | * | * | * | * | * | * | 3 | * |
| * | * | * | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 21 | * | * | * | * | * | * | * | 3 |
| * | * | * | * | * | * | * | * | * | * | * | 6 | * | * | 21 | 00 | * | * | * | * | * | * | * | * |
| * | 6 | * | * | 12 | * | * | * | * | * | * | * | * | * | * | * | 00 | 12 | 9 | * | * | * | * | * |
| * | * | 18 | * | * | 21 | * | * | * | * | * | * | * | * | * | * | 12 | 00 | * | 9 | * | * | * | * |
| * | * | * | * | * | * | 15 | * | * | * | * | * | * | * | * | * | 9 | * | 00 | 12 | 9 | * | * | * |
| * | * | * | * | * | * | * | 21 | * | * | * | * | * | * | * | * | * | 9 | 12 | 00 | * | 9 | * | * |
| * | * | * | * | * | * | * | * | 15 | * | * | * | * | * | * | * | * | * | 9 | * | 00 | 12 | 6 | * |
| * | * | * | * | * | * | * | * | * | 21 | * | * | * | * | * | * | * | * | * | 9 | 12 | 00 | * | 12 |
| * | * | * | * | * | * | * | * | * | * | 15 | * | * | 3 | * | * | * | * | * | * | 6 | * | 00 | 12 |
| * | * | * | * | * | * | * | * | * | * | * | 21 | * | * | 3 | * | * | * | * | * | * | 12 | 12 | 00 |

**Figure 4b.i This is the initial matrix,** $B$. Figure 4b contains a set of nine tables (i to ix) illustrating the use of Hasse's algorithm on the LA freeway system following the earthquake of January 17, 1994. Travel times are in one-quarter minutes. An asterisk indicates that the travel time between locations is too large to enter the matrix. A double-zero indicates an entry of 0.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | * | 18 | * | 30 | * | * | * | * | * | * | * | * | * | 18 | * | * | * | * | * | * | * |
| 12 | 00 | 18 | 42 | 18 | * | * | * | * | * | * | * | * | * | * | * | 6 | 18 | 15 | * | * | * | * | * |
| 30 | 18 | 00 | 24 | * | 39 | * | * | * | * | * | * | * | * | * | * | 24 | 18 | * | 27 | * | * | * | * |
| * | 42 | 24 | 00 | * | * | * | * | * | * | * | * | * | * | * | * | * | 42 | * | * | * | * | * | * |
| 18 | 18 | * | * | 00 | * | 12 | * | 18 | * | * | * | * | * | * | * | 12 | 24 | 21 | * | * | * | * | * |
| * | * | 39 | * | * | 00 | * | 12 | * | 21 | * | * | * | * | * | * | 33 | 21 | * | 30 | * | * | * | * |
| 30 | * | * | * | 12 | * | 00 | * | 6 | * | 12 | * | * | * | * | * | 24 | * | 15 | 27 | 21 | * | * | * |
| * | * | * | * | * | 12 | * | 00 | * | 9 | * | 15 | * | * | * | * | * | 30 | 33 | 21 | * | 30 | * | * |
| * | * | * | * | 18 | * | 6 | * | 00 | * | 6 | * | 12 | * | * | * | * | * | 21 | * | 15 | 27 | 21 | * |
| * | * | * | * | * | 21 | * | 9 | * | 00 | * | 6 | * | * | * | 12 | * | * | * | 30 | 33 | 21 | * | 27 |
| * | * | * | * | * | * | 12 | * | 6 | * | 00 | * | 6 | 18 | * | * | * | * | * | * | 21 | * | 15 | 27 |
| * | * | * | * | * | * | * | 15 | * | 6 | * | 00 | * | * | 24 | 6 | * | * | * | * | * | 27 | 33 | 21 |
| * | * | * | * | * | * | * | * | 12 | * | 6 | * | 00 | 12 | 24 | * | * | * | * | * | * | * | 15 | * |
| * | * | * | * | * | * | * | * | * | * | 18 | * | 12 | 00 | 12 | 33 | * | * | * | * | 9 | * | 3 | 15 |
| * | * | * | * | * | * | * | * | * | * | * | 24 | 24 | 12 | 00 | 21 | * | * | * | * | * | 15 | 15 | 3 |
| * | * | * | * | * | * | * | * | * | 12 | * | 6 | * | 33 | 21 | 00 | * | * | * | * | * | * | * | 24 |
| 18 | 6 | 24 | * | 12 | 33 | 24 | * | * | * | * | * | * | * | * | * | 00 | 12 | 9 | 21 | 18 | * | * | * |
| * | 18 | 18 | 42 | 24 | 21 | * | 30 | * | * | * | * | * | * | * | * | 12 | 00 | 21 | 9 | * | 18 | * | * |
| * | 15 | * | * | 21 | * | 15 | 33 | 21 | * | * | * | * | * | * | * | 9 | 21 | 00 | 12 | 9 | 21 | 15 | * |
| * | * | 27 | * | * | 30 | 27 | 21 | * | 30 | * | * | * | * | * | * | 21 | 9 | 12 | 00 | 21 | 9 | * | 21 |
| * | * | * | * | * | * | 21 | * | 15 | 33 | 21 | * | * | 9 | * | * | 18 | * | 9 | 21 | 00 | 12 | 6 | 18 |
| * | * | * | * | * | * | * | 30 | 27 | 21 | * | 27 | * | * | 15 | * | * | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| * | * | * | * | * | * | * | * | 21 | * | 15 | 33 | 15 | 3 | 15 | * | * | * | 15 | * | 6 | 18 | 00 | 12 |
| * | * | * | * | * | * | * | * | * | 27 | 27 | 21 | * | 15 | 3 | 24 | * | * | * | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.ii. This is the power 2 matrix,** $B^2$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | * | 30 | * | 36 | * | * | * | * | * | * | * | 18 | 30 | 27 | * | * | * | * | * |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | * | * | * | * | * | * | * | * | * | 6 | 18 | 15 | 27 | 24 | * | * | * |
| 30 | 18 | 00 | 24 | 36 | 39 | * | 48 | * | * | * | * | * | * | * | * | 24 | 18 | 33 | 27 | * | 36 | * | * |
| 54 | 42 | 24 | 00 | * | 63 | * | * | * | * | * | * | * | * | * | * | 48 | 42 | * | 51 | * | * | * | * |
| 18 | 18 | 36 | * | 00 | 45 | 12 | * | 18 | * | 24 | * | * | * | * | * | 12 | 24 | 21 | 33 | 30 | * | * | * |
| * | 39 | 39 | 63 | 45 | 00 | * | 12 | * | 21 | * | 27 | * | * | * | * | 33 | 21 | 42 | 30 | * | 39 | * | * |
| 30 | 30 | * | * | 12 | * | 00 | 48 | 6 | * | 12 | * | 18 | * | * | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | * |
| * | * | 48 | * | * | 12 | 48 | 00 | * | 9 | * | 15 | * | * | * | 21 | 42 | 30 | 33 | 21 | 42 | 30 | * | 36 |
| 36 | * | * | * | 18 | * | 6 | * | 00 | 48 | 6 | * | 12 | 24 | * | * | 30 | * | 21 | 33 | 15 | 27 | 21 | 33 |
| * | * | * | * | * | 21 | * | 9 | 48 | 00 | * | 6 | * | * | 30 | 12 | * | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| * | * | * | * | 24 | * | 12 | * | 6 | * | 00 | 48 | 6 | 18 | 30 | * | * | * | 27 | * | 21 | 33 | 15 | 27 |
| * | * | * | * | * | 27 | * | 15 | * | 6 | 48 | 00 | * | 36 | 24 | 6 | * | * | * | 36 | 39 | 27 | 33 | 21 |
| * | * | * | * | * | * | 18 | * | 12 | * | 6 | * | 00 | 12 | 24 | 45 | * | * | * | * | 21 | * | 15 | 27 |
| * | * | * | * | * | * | * | * | 24 | * | 18 | 36 | 12 | 00 | 12 | 33 | * | * | 18 | * | 9 | 21 | 3 | 15 |
| * | * | * | * | * | * | * | * | * | 30 | 30 | 24 | 24 | 12 | 00 | 21 | * | * | * | 24 | 21 | 15 | 15 | 3 |
| * | * | * | * | * | * | * | 21 | * | 12 | * | 6 | 45 | 33 | 21 | 00 | * | * | * | * | * | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | * | * | * | * | * | * | * | 00 | 12 | 9 | 21 | 18 | 30 | 24 | * |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | * | 39 | * | * | * | * | * | * | 12 | 00 | 21 | 9 | 30 | 18 | * | 30 |
| 27 | 15 | 33 | * | 21 | 42 | 15 | 33 | 21 | 42 | 27 | * | * | 18 | * | * | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| * | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | * | 36 | * | * | 24 | * | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| * | 24 | * | * | 30 | * | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | * | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| * | * | 36 | * | * | 39 | 33 | 30 | 27 | 21 | * | 27 | * | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| * | * | * | * | * | * | 27 | * | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | * | 15 | 27 | 6 | 18 | 00 | 12 |
| * | * | * | * | * | * | * | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | * | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.iii This is the power 3 matrix, $B^3$.**

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | * | 36 | * | 42 | * | * | * | * | * | 18 | 30 | 27 | 39 | 36 | * | * | * |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | * | * | * | * | * | * | * | 6 | 18 | 15 | 27 | 24 | 36 | 30 | * |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | * | 57 | * | * | * | * | * | * | 24 | 18 | 33 | 27 | 42 | 36 | * | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | * | 72 | * | * | * | * | * | * | * | * | 48 | 42 | 57 | 51 | * | 60 | * | * |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | * | 24 | * | 30 | * | * | * | 12 | 24 | 21 | 33 | 30 | 42 | 36 | * |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | * | 21 | * | 27 | * | * | * | 33 | 33 | 21 | 42 | 30 | 51 | 39 | * | 48 |
| 30 | 30 | 48 | * | 12 | 57 | 00 | 48 | 6 | 54 | 12 | * | 18 | 30 | * | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| * | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | * | 15 | * | * | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | * | * | 18 | * | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | * | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| * | * | 57 | * | * | 21 | 54 | 9 | 48 | 00 | 54 | 6 | * | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | * | * | * | 24 | * | 12 | * | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | * | 27 | 39 | 21 | 33 | 15 | 27 |
| * | * | * | * | * | 27 | * | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | * | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| * | * | * | * | 30 | * | 18 | * | 12 | * | 6 | 48 | 00 | 12 | 24 | 45 | * | * | 30 | * | 21 | 33 | 15 | 27 |
| * | * | * | * | * | * | 30 | * | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | * | 18 | 30 | 9 | 21 | 3 | 15 |
| * | * | * | * | * | * | * | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | * | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | * | * | * | 33 | * | 21 | * | 12 | 51 | 6 | 45 | 33 | 21 | 00 | * | * | * | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | * | * | 27 | * | * | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | * | 45 | * | * | 33 | * | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | * | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | * | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | * | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| * | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| * | 30 | * | * | 36 | * | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | * | 15 | 27 | 6 | 18 | 00 | 12 |
| * | * | 48 | * | * | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | * | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.iv This is the power 4 matrix,** $B^4$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | * | 42 | * | 48 | * | * | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | * |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | * | * | 33 | * | * | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | * | 63 | * | * | 51 | * | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | 72 | 72 | * | 81 | * | * | * | * | * | * | 48 | 42 | 57 | 51 | 66 | 60 | * | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | * | 30 | 39 | * | * | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | 63 | 21 | * | 27 | * | * | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | * | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | * | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | * | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| * | 57 | 57 | 81 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | * | * | 24 | * | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| * | * | 63 | * | * | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | * | * | * | 30 | * | 18 | * | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | * | 30 | * | 21 | 33 | 15 | 27 |
| * | 33 | * | * | 39 | * | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| * | * | 51 | * | * | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | * | * | * | 33 | * | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | * | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | * | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | * | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | * | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| * | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.v This is the power 5 matrix,** $B^5$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | * | 48 | 45 | * | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | * | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | * | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | 72 | 72 | 78 | 81 | * | 87 | * | * | 75 | * | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | * | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | * | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 81 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | * | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| * | 63 | 63 | 87 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | * | * | 30 | * | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | * | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| * | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | * | 69 | * | * | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.vi This is the power 6 matrix, $B^6$.**

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | * | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | 72 | 72 | 78 | 81 | 84 | 87 | * | 75 | 75 | 93 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 81 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 87 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | * | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| * | 66 | 69 | 93 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.vii This is the power 7 matrix, $B^7$.**

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | 78 | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | 72 | 72 | 78 | 81 | 84 | 87 | 87 | 75 | 75 | 93 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 81 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 87 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | 87 | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| 78 | 66 | 69 | 93 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.viii** This is the power 8 matrix, $B^8$.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 51 | 30 | 60 | 36 | 69 | 42 | 75 | 48 | 45 | 57 | 78 | 18 | 30 | 27 | 39 | 36 | 48 | 42 | 54 |
| 12 | 00 | 18 | 42 | 18 | 39 | 30 | 48 | 36 | 57 | 42 | 63 | 45 | 33 | 45 | 66 | 6 | 18 | 15 | 27 | 24 | 36 | 30 | 42 |
| 30 | 18 | 00 | 24 | 36 | 39 | 48 | 48 | 54 | 57 | 60 | 63 | 63 | 51 | 51 | 69 | 24 | 18 | 33 | 27 | 42 | 36 | 48 | 48 |
| 54 | 42 | 24 | 00 | 60 | 63 | 72 | 72 | 78 | 81 | 84 | 87 | 87 | 75 | 75 | 93 | 48 | 42 | 57 | 51 | 66 | 60 | 72 | 72 |
| 18 | 18 | 36 | 60 | 00 | 45 | 12 | 54 | 18 | 63 | 24 | 69 | 30 | 39 | 51 | 72 | 12 | 24 | 21 | 33 | 30 | 42 | 36 | 48 |
| 51 | 39 | 39 | 63 | 45 | 00 | 57 | 12 | 63 | 21 | 69 | 27 | 72 | 60 | 51 | 33 | 33 | 21 | 42 | 30 | 51 | 39 | 57 | 48 |
| 30 | 30 | 48 | 72 | 12 | 57 | 00 | 48 | 6 | 54 | 12 | 60 | 18 | 30 | 42 | 63 | 24 | 36 | 15 | 27 | 21 | 33 | 27 | 39 |
| 60 | 48 | 48 | 72 | 54 | 12 | 48 | 00 | 54 | 9 | 60 | 15 | 63 | 51 | 39 | 21 | 42 | 30 | 33 | 21 | 42 | 30 | 48 | 36 |
| 36 | 36 | 54 | 78 | 18 | 63 | 6 | 54 | 00 | 48 | 6 | 54 | 12 | 24 | 36 | 57 | 30 | 42 | 21 | 33 | 15 | 27 | 21 | 33 |
| 69 | 57 | 57 | 81 | 63 | 21 | 54 | 9 | 48 | 00 | 54 | 6 | 54 | 42 | 30 | 12 | 51 | 39 | 42 | 30 | 33 | 21 | 39 | 27 |
| 42 | 42 | 60 | 84 | 24 | 69 | 12 | 60 | 6 | 54 | 00 | 48 | 6 | 18 | 30 | 51 | 36 | 48 | 27 | 39 | 21 | 33 | 15 | 27 |
| 75 | 63 | 63 | 87 | 69 | 27 | 60 | 15 | 54 | 6 | 48 | 00 | 48 | 36 | 24 | 6 | 57 | 45 | 48 | 36 | 39 | 27 | 33 | 21 |
| 48 | 45 | 63 | 87 | 30 | 72 | 18 | 63 | 12 | 54 | 6 | 48 | 00 | 12 | 24 | 45 | 39 | 51 | 30 | 42 | 21 | 33 | 15 | 27 |
| 45 | 33 | 51 | 75 | 39 | 60 | 30 | 51 | 24 | 42 | 18 | 36 | 12 | 00 | 12 | 33 | 27 | 39 | 18 | 30 | 9 | 21 | 3 | 15 |
| 57 | 45 | 51 | 75 | 51 | 51 | 42 | 39 | 36 | 30 | 30 | 24 | 24 | 12 | 00 | 21 | 39 | 33 | 30 | 24 | 21 | 15 | 15 | 3 |
| 78 | 66 | 69 | 93 | 72 | 33 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 | 60 | 51 | 51 | 42 | 42 | 33 | 36 | 24 |
| 18 | 6 | 24 | 48 | 12 | 33 | 24 | 42 | 30 | 51 | 36 | 57 | 39 | 27 | 39 | 60 | 00 | 12 | 9 | 21 | 18 | 30 | 24 | 36 |
| 30 | 18 | 18 | 42 | 24 | 21 | 36 | 30 | 42 | 39 | 48 | 45 | 51 | 39 | 33 | 51 | 12 | 00 | 21 | 9 | 30 | 18 | 36 | 30 |
| 27 | 15 | 33 | 57 | 21 | 42 | 15 | 33 | 21 | 42 | 27 | 48 | 30 | 18 | 30 | 51 | 9 | 21 | 00 | 12 | 9 | 21 | 15 | 27 |
| 39 | 27 | 27 | 51 | 33 | 30 | 27 | 21 | 33 | 30 | 39 | 36 | 42 | 30 | 24 | 42 | 21 | 9 | 12 | 00 | 21 | 9 | 27 | 21 |
| 36 | 24 | 42 | 66 | 30 | 51 | 21 | 42 | 15 | 33 | 21 | 39 | 21 | 9 | 21 | 42 | 18 | 30 | 9 | 21 | 00 | 12 | 6 | 18 |
| 48 | 36 | 36 | 60 | 42 | 39 | 33 | 30 | 27 | 21 | 33 | 27 | 33 | 21 | 15 | 33 | 30 | 18 | 21 | 9 | 12 | 00 | 18 | 12 |
| 42 | 30 | 48 | 72 | 36 | 57 | 27 | 48 | 21 | 39 | 15 | 33 | 15 | 3 | 15 | 36 | 24 | 36 | 15 | 27 | 6 | 18 | 00 | 12 |
| 54 | 42 | 48 | 72 | 48 | 48 | 39 | 36 | 33 | 27 | 27 | 21 | 27 | 15 | 3 | 24 | 36 | 30 | 27 | 21 | 18 | 12 | 12 | 00 |

**Figure 4b.ix This is the power 9 matrix,** $B^9$. It is identical to the matrix in Figure 4b.viii and so the algorithm terminates.

## Figure for Hasse algorithm in Pascal

```
program hasse(input,output);
const max=999999;
                    n=24;
type hed=array[1..n,1..n] of integer;
var a:array[1..n] of hed;
        done:boolean;
        i,j,k,num:integer;
procedure print(matrix:hed);
begin
        for i:=i to n do
            begin
            for j:=1 to n do
                if matrix[i,j]=max then write (' *')
                    else write(matrix[i,j]:4);
            writeln
            end
end;
procedure hedsum(power, init:hed;var next:hed;var flag:boolean);
var row,col,min,middle,temp:integer;
begin
        flag:=true;
        for row:=1 to n do
        for col:=1 to n do
        begin
            min:=power[row,col];
            for middle:=1 to n do
            begin
                temp:=power[row,middle]+init[middle,col];
                if temp¡min the min:=temp
            end;
            next[row,col]:=min;
            if next[row,col]¡¿power[row,col] then flag:=false;
        end
end;
{main program}
begin
        for i:=1 to n do for j:=1 to n do a[1][i,j]:=max;
```

```
        for i:=1 to n do a[1][i,i]:=0;
        repeat
              readln(i,j,num);
              a[1][i,j]:=num;
              a[1][j,i]:=num;
        until eof;
        page; writeln('this is the initial matrix');writeln;
        print(a[1]);
        k:=0;
        repeat
              k:=k+1;
              hedsum(a[k],a[1],a[k+1],done);
              page; writeln('this is power',k+1:5); writeln;
              print(a[k+1]);
        until (done) or (k=n-1);
        writeln;
        writeln('the number of steps was', k:5)
end.
```

**Figure 5.** Computer program, written in Pascal, of W. C. Arlinghaus; originally presented on a poster by Arlinghaus, Arlinghaus, and Nystuen, "Elements of Geometric Routing Theory–II" Association of American Geographers, National Meetings, Toronto, Ontario, April 1990.

## Figures containing tables

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | * | * | 18 | * | * | * | * | * | * | * | * | * | * | * |
| 12 | 00 | 18 | * | * | * | * | * | * | * | * | * | * | 33 | * | * |
| * | 18 | 00 | 24 | * | * | * | * | * | * | * | * | * | * | 45 | * |
| * | * | 24 | 00 | * | 30 | * | * | * | * | * | * | * | * | * | * |
| 18 | * | * | * | 00 | 42 | 12 | * | * | * | * | * | * | * | * | * |
| * | * | * | 30 | 42 | 00 | * | 12 | * | * | * | * | * | * | * | * |
| * | * | * | * | 12 | * | 00 | 45 | 6 | * | * | * | * | * | * | * |
| * | * | * | * | * | 12 | 45 | 00 | * | 9 | * | * | * | * | * | * |
| * | * | * | * | * | * | 6 | * | 00 | 45 | 6 | * | * | * | * | * |
| * | * | * | * | * | * | * | 9 | 45 | 00 | * | 6 | * | * | * | * |
| * | * | * | * | * | * | * | * | 6 | * | 00 | 45 | 6 | * | * | * |
| * | * | * | * | * | * | * | * | * | 6 | 45 | 00 | * | * | * | 6 |
| * | * | * | * | * | * | * | * | * | * | 6 | * | 00 | 12 | * | * |
| * | 33 | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 12 | * |
| * | * | 45 | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 21 |
| * | * | * | * | * | * | * | * | * | * | * | 6 | * | * | 21 | 00 |

**Figure 7a.i This is the initial matrix,** $C$. Figure 7a contains a set of seven tables (i to vii) illustrating the use of Hasse's algorithm on the LA freeway system and the limited access surface route network (Figure 6) prior to the earthquake of January 17, 1994. Travel times are in one-quarter minutes. An asterisk indicates that the travel time between locations is too large to enter the matrix. A double-zero indicates an entry of 0.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | * | 18 | 60 | 30 | * | * | * | * | * | * | 45 | * | * |
| 12 | 00 | 18 | 42 | 30 | * | * | * | * | * | * | * | 45 | 33 | 45 | * |
| 30 | 18 | 00 | 24 | * | 54 | * | * | * | * | * | * | * | 51 | 45 | 66 |
| * | 42 | 24 | 00 | 72 | 30 | * | 36 | * | * | * | * | * | * | 69 | * |
| 18 | 30 | * | 72 | 00 | 42 | 12 | 48 | 18 | * | * | * | * | * | * | * |
| 60 | * | 54 | 30 | 42 | 00 | 51 | 6 | * | 15 | * | * | * | * | * | * |
| 30 | * | * | * | 12 | 51 | 00 | 45 | 6 | 51 | 12 | * | * | * | * | * |
| * | * | * | 36 | 48 | 6 | 45 | 00 | 51 | 9 | * | 15 | * | * | * | * |
| * | * | * | * | 18 | * | 6 | 51 | 00 | 45 | 6 | 51 | 12 | * | * | * |
| * | * | * | * | * | 15 | 51 | 9 | 45 | 00 | 51 | 6 | * | * | * | 12 |
| * | * | * | * | * | * | 12 | * | 6 | 51 | 00 | 45 | 6 | 18 | * | 51 |
| * | * | * | * | * | * | * | 15 | 51 | 6 | 45 | 00 | 51 | * | 24 | 6 |
| * | 45 | * | * | * | * | * | * | 12 | * | 6 | 51 | 00 | 12 | 24 | * |
| 45 | 33 | 51 | * | * | * | * | * | * | * | 18 | * | 12 | 00 | 12 | 33 |
| * | 45 | 45 | 69 | * | * | * | * | * | * | * | 27 | 24 | 12 | 00 | 21 |
| * | * | 66 | * | * | * | * | * | * | 12 | 51 | 6 | * | 33 | 21 | 00 |

**Figure 7a.ii. This is the power 2 matrix, $C^2$.**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | * | * | * | 57 | 45 | 57 | * |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | * | * | * | 51 | * | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 54 | * | 60 | * | * | * | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 30 | 81 | 36 | * | 45 | * | * | * | 75 | 69 | 90 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | * | * | 63 | * | * |
| 60 | 72 | 54 | 30 | 42 | 00 | 51 | 6 | 57 | 15 | * | 21 | * | * | 99 | * |
| 30 | 42 | * | 81 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | * | * | * |
| 66 | * | 60 | 36 | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | * | * | * | 21 |
| 36 | * | * | * | 18 | 57 | 15 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | * | 57 |
| * | * | * | 45 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | * | 33 | 12 |
| * | 51 | * | * | 24 | * | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| * | * | 72 | * | * | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 57 | 45 | 63 | * | * | * | 18 | * | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 63 | * | * | * | 24 | * | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | * | 99 | * | * | * | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| * | 66 | 66 | 90 | * | * | * | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7a.iii This is the power 3 matrix,** $C^3$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | * | 57 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | * | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 54 | 60 | 60 | * | 69 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 30 | 81 | 36 | 87 | 45 | * | 51 | 87 | 75 | 69 | 90 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 63 | 75 | * |
| 60 | 72 | 54 | 30 | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | * | 105 | 99 | 27 |
| 30 | 42 | 60 | 81 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | * | 63 |
| 66 | 78 | 60 | 36 | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | * | 42 | 21 |
| 36 | 48 | * | 87 | 18 | 57 | 15 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | * | 69 | 45 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | * | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| * | 72 | 72 | 51 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 57 | 45 | 63 | 87 | 30 | * | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 63 | 105 | 30 | * | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 75 | 99 | * | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 90 | * | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7a.iv This is the power 4 matrix, $C^4$.**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | 81 | 48 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | 78 | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 54 | 60 | 60 | 66 | 69 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 30 | 81 | 36 | 87 | 45 | 93 | 51 | 87 | 75 | 69 | 57 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 63 | 75 | 69 |
| 60 | 72 | 54 | 30 | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | 69 | 105 | 48 | 27 |
| 30 | 42 | 60 | 81 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | 42 | 63 |
| 66 | 78 | 60 | 36 | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | 54 | 42 | 21 |
| 36 | 48 | 66 | 87 | 18 | 57 | 15 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | 78 | 69 | 45 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | 93 | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| 81 | 72 | 72 | 51 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 48 | 45 | 63 | 87 | 30 | 69 | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 63 | 105 | 30 | 54 | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 75 | 48 | 42 | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 57 | 69 | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7a.v This is the power 5 matrix,** $C^5$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | 81 | 48 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | 78 | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 54 | 60 | 60 | 66 | 69 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 30 | 81 | 36 | 87 | 45 | 93 | 51 | 87 | 75 | 69 | 57 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 42 | 54 | 69 |
| 60 | 72 | 54 | 30 | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | 69 | 60 | 48 | 27 |
| 30 | 42 | 60 | 81 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | 42 | 63 |
| 66 | 78 | 60 | 36 | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | 54 | 42 | 21 |
| 36 | 48 | 66 | 87 | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | 78 | 69 | 45 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | 93 | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| 81 | 72 | 72 | 51 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 48 | 45 | 63 | 87 | 30 | 69 | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 42 | 60 | 30 | 54 | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 54 | 48 | 42 | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 57 | 69 | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7a.vi This is the power 6 matrix, $C^6$.**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | 81 | 48 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | 78 | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 54 | 60 | 60 | 66 | 69 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 30 | 81 | 36 | 87 | 45 | 93 | 51 | 87 | 75 | 69 | 57 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 42 | 54 | 69 |
| 60 | 72 | 54 | 30 | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | 69 | 60 | 48 | 27 |
| 30 | 42 | 60 | 81 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | 42 | 63 |
| 66 | 78 | 60 | 36 | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | 54 | 42 | 21 |
| 36 | 48 | 66 | 87 | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | 78 | 69 | 45 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | 93 | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| 81 | 72 | 72 | 51 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 48 | 45 | 63 | 87 | 30 | 69 | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 42 | 60 | 30 | 54 | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 54 | 48 | 42 | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 57 | 69 | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7a.vii This is the power 7 matrix,** $C^7$. The algorithm terminates in six steps; this matrix is identical to $A^6$.

## Figures containing tables

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | * | * | 18 | * | * | * | * | * | * | * | * | * | * | * |
| 12 | 00 | 18 | * | * | * | * | * | * | * | * | * | * | 33 | * | * |
| * | 18 | 00 | 24 | * | * | * | * | * | * | * | * | * | * | 45 | * |
| * | * | 24 | 00 | * | * | * | * | * | * | * | * | * | * | * | * |
| 18 | * | * | * | 00 | 42 | 12 | * | * | * | * | * | * | * | * | * |
| * | * | * | * | 42 | 00 | * | 6 | * | * | * | * | * | * | * | * |
| * | * | * | * | 12 | * | 00 | 45 | 6 | * | * | * | * | * | * | * |
| * | * | * | * | * | 6 | 45 | 00 | * | 9 | * | * | * | * | * | * |
| * | * | * | * | * | * | 6 | * | 00 | 45 | 6 | * | * | * | * | * |
| * | * | * | * | * | * | * | 9 | 45 | 00 | * | 6 | * | * | * | * |
| * | * | * | * | * | * | * | * | 6 | * | 00 | 45 | 6 | * | * | * |
| * | * | * | * | * | * | * | * | * | 6 | 45 | 00 | * | * | * | 6 |
| * | * | * | * | * | * | * | * | * | * | 6 | * | 00 | 12 | * | * |
| * | 33 | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 12 | * |
| * | * | 45 | * | * | * | * | * | * | * | * | * | * | 12 | 00 | 21 |
| * | * | * | * | * | * | * | * | * | * | * | 6 | * | * | 21 | 00 |

**Figure 7b.i This is the initial matrix,** $D$. Figure 7b contains a set of seven tables (i to vii) illustrating the use of Hasse's algorithm on the LA freeway system and the limited access surface route network (Figure 6) following the earthquake of January 17, 1994. Travel times are in one-quarter minutes. An asterisk indicates that the travel time between locations is too large to enter the matrix. A double-zero indicates an entry of 0.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | * | 18 | 60 | 30 | * | * | * | * | * | * | 45 | * | * |
| 12 | 00 | 18 | 42 | 30 | * | * | * | * | * | * | * | 45 | 33 | 45 | * |
| 30 | 18 | 00 | 24 | * | * | * | * | * | * | * | * | * | 51 | 45 | 66 |
| * | 42 | 24 | 00 | * | * | * | * | * | * | * | * | * | * | 69 | * |
| 18 | 30 | * | * | 00 | 42 | 12 | 48 | 18 | * | * | * | * | * | * | * |
| 60 | * | * | * | 42 | 00 | 51 | 6 | * | 15 | * | * | * | * | * | * |
| 30 | * | * | * | 12 | 51 | 00 | 45 | 6 | 51 | 12 | * | * | * | * | * |
| * | * | * | * | 48 | 6 | 45 | 00 | 51 | 9 | * | 15 | * | * | * | * |
| * | * | * | * | 18 | * | 6 | 51 | 00 | 45 | 6 | 51 | 12 | * | * | * |
| * | * | * | * | * | 15 | 51 | 9 | 45 | 00 | 51 | 6 | * | * | * | 12 |
| * | * | * | * | * | * | 12 | * | 6 | 51 | 00 | 45 | 6 | 18 | * | 51 |
| * | * | * | * | * | * | * | 15 | 51 | 6 | 45 | 00 | 51 | * | 27 | 6 |
| * | 45 | * | * | * | * | * | * | 12 | * | 6 | 51 | 00 | 12 | 24 | * |
| 45 | 33 | 51 | * | * | * | * | * | * | * | 18 | * | 12 | 00 | 12 | 33 |
| * | 45 | 45 | 69 | * | * | * | * | * | * | * | 27 | 24 | 12 | 00 | 21 |
| * | * | 66 | * | * | * | * | * | * | 12 | 51 | 6 | * | 33 | 21 | 00 |

**Figure 7b.ii. This is the power 2 matrix,** $D^2$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | * | * | * | 57 | 45 | 57 | * |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | * | * | * | 51 | * | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | * | * | * | * | * | * | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | * | * | * | * | * | * | * | * | * | 75 | 69 | 90 |
| 18 | 30 | 48 | * | 00 | 42 | 12 | 48 | 18 | 57 | 24 | * | * | 63 | * | * |
| 60 | 72 | * | * | 42 | 00 | 51 | 6 | 57 | 15 | * | 21 | * | * | * | * |
| 30 | 42 | * | * | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | * | * | * |
| 66 | * | * | * | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | * | * | * | 21 |
| 36 | * | * | * | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | * | 57 |
| * | * | * | * | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | * | 33 | 12 |
| * | 51 | * | * | 24 | * | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| * | * | 72 | * | * | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 57 | 45 | 63 | * | * | * | 18 | * | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 63 | * | * | * | 24 | * | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | * | * | * | * | * | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| * | 66 | 66 | 90 | * | * | * | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7b.iii This is the power 3 matrix,** $D^3$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | * | 57 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | * | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 90 | 60 | * | * | 78 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | * | * | * | * | * | * | 96 | 87 | 75 | 69 | 90 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 63 | 75 | * |
| 60 | 72 | 90 | * | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | * | 105 | * | 27 |
| 30 | 42 | 60 | * | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | * | 63 |
| 66 | 78 | * | * | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | * | 42 | 21 |
| 36 | 48 | * | * | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | * | 78 | * | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | * | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| * | 72 | 72 | 96 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 57 | 45 | 63 | 87 | 30 | * | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 63 | 105 | 30 | * | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 75 | * | * | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 90 | * | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7b.iv This is the power 4 matrix,** $D^4$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | 81 | 48 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | 78 | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 90 | 60 | 87 | 66 | 78 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 114 | 84 | * | * | 102 | 93 | 96 | 87 | 75 | 69 | 90 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 42 | 75 | 69 |
| 60 | 72 | 90 | 114 | 42 | 00 | 51 | 6 | 57 | 15 | 63 | 21 | 69 | 105 | 48 | 27 |
| 30 | 42 | 60 | 84 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | 42 | 63 |
| 66 | 78 | 87 | * | 48 | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | 54 | 42 | 21 |
| 36 | 48 | 66 | * | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | 78 | 78 | 102 | 57 | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | 93 | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| 81 | 72 | 72 | 96 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 48 | 45 | 63 | 87 | 30 | 69 | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 42 | 105 | 30 | 54 | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 75 | 48 | 42 | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 90 | 69 | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7b.v This is the power 5 matrix,** $D^5$.

```
00 12 30 54 18   60 30 66 36 75   42 81 48 45 57 78
12 00 18 42 30   72 42 78 48 78   51 72 45 33 45 66
30 18 00 24 48   90 60 87 66 78   69 72 63 51 45 66
54 42 24 00 72   11484 11190 102 93 96 87 75 69 90
18 30 48 72 00   42 12 48 18 57   24 63 30 42 54 69
60 72 90 11442   00 51 6  57 15   63 21 69 60 48 27
30 42 60 84 12   51 00 45 6  51   12 57 18 30 42 63
66 78 87 11148   6  45 00 51 9    57 15 63 54 42 21
36 48 66 90 18   57 6  51 00 45   6  51 12 24 36 57
75 78 78 10257   15 51 9  45 00   51 6  57 45 33 12
42 51 69 93 24   63 12 57 6  51   00 45 6  18 30 51
81 72 72 96 63   21 57 15 51 6    45 00 51 39 27 6
48 45 63 87 30   69 18 63 12 57   6  51 00 12 24 45
45 33 51 75 42   60 30 54 24 45   18 39 12 00 12 33
57 45 45 69 54   48 42 42 36 33   30 27 24 12 00 21
78 66 66 90 69   27 63 21 57 12   51 6  45 33 21 00
```

**Figure 7b.vi This is the power 6 matrix,** $D^6$.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 12 | 30 | 54 | 18 | 60 | 30 | 66 | 36 | 75 | 42 | 81 | 48 | 45 | 57 | 78 |
| 12 | 00 | 18 | 42 | 30 | 72 | 42 | 78 | 48 | 78 | 51 | 72 | 45 | 33 | 45 | 66 |
| 30 | 18 | 00 | 24 | 48 | 90 | 60 | 87 | 66 | 78 | 69 | 72 | 63 | 51 | 45 | 66 |
| 54 | 42 | 24 | 00 | 72 | 11484 | | 11190 | | 102 | 93 | 96 | 87 | 75 | 69 | 90 |
| 18 | 30 | 48 | 72 | 00 | 42 | 12 | 48 | 18 | 57 | 24 | 63 | 30 | 42 | 54 | 69 |
| 60 | 72 | 90 | 11442 | | 00 | 51 | 6 | 57 | 15 | 63 | 21 | 69 | 60 | 48 | 27 |
| 30 | 42 | 60 | 84 | 12 | 51 | 00 | 45 | 6 | 51 | 12 | 57 | 18 | 30 | 42 | 63 |
| 66 | 78 | 87 | 11148 | | 6 | 45 | 00 | 51 | 9 | 57 | 15 | 63 | 54 | 42 | 21 |
| 36 | 48 | 66 | 90 | 18 | 57 | 6 | 51 | 00 | 45 | 6 | 51 | 12 | 24 | 36 | 57 |
| 75 | 78 | 78 | 10257 | | 15 | 51 | 9 | 45 | 00 | 51 | 6 | 57 | 45 | 33 | 12 |
| 42 | 51 | 69 | 93 | 24 | 63 | 12 | 57 | 6 | 51 | 00 | 45 | 6 | 18 | 30 | 51 |
| 81 | 72 | 72 | 96 | 63 | 21 | 57 | 15 | 51 | 6 | 45 | 00 | 51 | 39 | 27 | 6 |
| 48 | 45 | 63 | 87 | 30 | 69 | 18 | 63 | 12 | 57 | 6 | 51 | 00 | 12 | 24 | 45 |
| 45 | 33 | 51 | 75 | 42 | 60 | 30 | 54 | 24 | 45 | 18 | 39 | 12 | 00 | 12 | 33 |
| 57 | 45 | 45 | 69 | 54 | 48 | 42 | 42 | 36 | 33 | 30 | 27 | 24 | 12 | 00 | 21 |
| 78 | 66 | 66 | 90 | 69 | 27 | 63 | 21 | 57 | 12 | 51 | 6 | 45 | 33 | 21 | 00 |

**Figure 7b.vii This is the power 7 matrix,** $D^7$. The iteration terminates after 6 steps; this matrix is identical to $D^6$.