Interruption!

Sandra Lach Arlinghaus
The University of Michigan

Interruption, from the Latin–*rumpere* (to break) plus *inter* (between, among), means literally "to break into (between)." The concept of "interruption" can be employed to guide research direction between apparently disparate objects of study; "interruption" is a meta-concept like "symmetry," "duality," and a host of others. We are all familiar with flat maps of the Earth that are interrupted. Indeed, all flat maps of the Earth are interrupted; the one-point compactification of the sphere guarantees that this is so from a topological standpoint. From a more pragmatic standpoint, we know that it is not possible to remove the peel from an orange and place it flatly in the plane – the peel will rip.

Classical interruption in mapping

It is this pragmatic view of mapping the Earth into the plane that conjures up most visual images of an "interrupted" map projection – one in which some cuts have been made (typically in the oceans) in order to preserve some degree of a desirable property, such as conformality or equality of area. Philbrick's (1963) Sinu-Mollweide has the northern hemisphere continuous with slits in the oceans in the southern hemisphere; Goode's Homolosine Equal Area projection (Goode, various years) has interruptions in oceans in both hemispheres. Either of these projections would be viewed, clearly, as an "interrupted" projection.

However, would all who see these as interrupted also view a cylindrical projection (Miller, for example) as "interrupted"? Of course it is, for once the sphere is projected onto the surface of the cylinder, the cylinder must then be "developed" or unrolled into a section of the plane. The development of a surface in the plane is a cut – a form of breaking into the cylinder – an interruption. The difference is that the interruption in a Miller cylindrical projection often determines the boundary of the map in the plane – our eye seeks closure and when the cut coincides with the map boundaries we use for closure, the visual effect is less jarring; the interruption is masked by the boundary.

Abstract variants on interruption and mapping

Going farther abstractly, one might consider rather than a map on a cylinder, a map on a Möbius strip; Tobler (1961) described a scheme in which a pin, poked through a map on a Möbius strip, emerges at its antipodal point. When this procedure is continued a finite number of times, the boundaries of a region and its antipodal region are traced out simultaneously on this one-sided map. This novel approach suggests ways to trace out partial, discrete, boundaries. Spilhaus (1979) suggests that to construct a continuous map of the antipodes one "show which land is opposite other land ... by taking a pair of maps of two hemispheres and putting them back to back with the North Pole covering the South Pole." Neither construction touches on deeper non-Euclidean aspects of this style of construction (Arlinghaus, 1987).

From the viewpoint of interruption, however, what is interesting is the mere idea of considering a map on a Möbius strip. The cylinder and the Möbius strip are both developable surfaces in the plane and they are but two members of a broader class. Because developable surfaces, when interrupted and placed in the plane, are those whose boundaries can easily mask the cuts of interruption, they are a class of particular interest. This broader class of surface may be viewed as composed of two structurally parallel sequences of transformations

– one easily visualized and the other visualized easily only by analogy with the first (Figure 1). (This sort of characterization is common in a variety of books that deal with elementary topology, as for example in Courant and Robbins, 1941.)

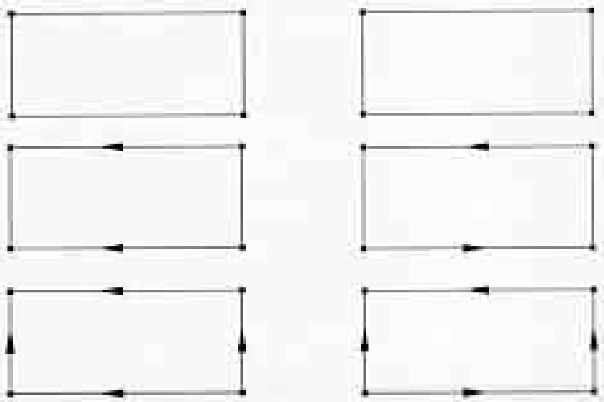

Figure 1. Two sequences: on the left, a rectangle is rolled up into a cylinder, and then the cylinder is joined, end-to-end, to form a torus. On the right, a rectangle, given a half-twist, is rolled up into a Möbius strip, and then joined (with another half twist), end-to-end, to form a Klein bottle.

Visual sequence:

1. A plane rectangle may be rolled into a cylinder by gluing together the upper left to the upper right corners and the lower left to the lower right corners. The result is a cylinder with diameter that of the length of the top of the rectangle.
2. A cylinder may be rolled into a torus by gluing one circular end of the cylinder to the other – the seam along which gluing takes place is the circle that matches the ends of the straight line seam along the length of the cylinder.

Abstract sequence:

1. A plane rectangle may be rolled into a Möbius strip by gluing together the upper left to the lower right corners of the rectangle and the lower left to the upper right corners of the rectangle. The result is a Möbius strip; the gluing action imparts a half-twist to the rectangular strip.
2. A Möbius strip may be rolled into a Klein bottle by gluing one "circular" end of the Möbius strip to the other, as with the torus.

What can be glued can be unglued (in this context); thus, cylinder, torus, Möbius strip, and Klein bottle are developable surfaces in the plane. One can view each of them as a surface on which to map; difficulty in such an approach is encountered only when the need to visualize physical objects is relied upon. Conceptually, from a structural viewpoint, the Möbius strip is no more difficult to consider than is the cylinder; the Klein bottle no more difficult than is the torus.

The utility of considering various mapping surfaces–GIS

A current maxim of those concerned with the protection of various elements of the environment is "to think globally, act locally." While this may have fine implications for landfill management, it is a dangerous cartographic practice. Globally we should think of a sphere or some other approximation of the Earth's surface that is topologically equivalent (homeomorphic) to the sphere. Locally we tend to think of our immediate part of the Earth as flat; recently, Barmore (1992; 1994) has shown the difficulty in determining geographic centers of various sorts when concerns for curvature are not involved in policy decisions. In earlier times, this sort of lack of tying knowledge of the earth as a sphere to a local plane environment was evident: from Eratosthenes' measurement of the Earth to the great voyages undertaken at the end of the Middle Ages and beginning of the Renaissance in Western Europe.

Most mapping is done from the global/spherical viewpoint to the local/planar viewpoint; it need not be, and when the mapping is from developable surface to plane, or from sphere to object homeomorphic to the sphere, then maps that hide interruption can be constructed. One place where this issue has, for the most part, not been addressed at all, is in the electronic environment of the Geographic Information Systems (GISs). In a recent paper, Tobler (1993) speaks to this issue at some length and notes, in particular, that of the hundreds of GISs available, "The one exception, explicitly designed to consider the spheroidal earth, is the 'Hipparchus' system developed by Hrvoje Lukatela of Calgary, Alberta (Lukatela 1987)." GISs such as this apparently offer a way to make maps directly from spherical data, eliminating the middle step of imitating the traditional drafting processes of the human arm and the planar decisions associated with those. This sort of idea seems quite natural–why should we use the computer to imitate the classical drafting process; why not use it to take advantage of the underlying mathematical characteristics of the real problems of dealing with surfaces?

Another route to this sort of end might be to construct data structures in the environment of the mathematics of the Klein bottle, torus, Möbius strip, or cylinder, and then to develop (as in "unroll") the mathematics to make plane maps. Either way – from sphere to sphere homeomorph, or from developable surface to plane, one might look forward to more elegantly constructed electronic programs for executing mapping – with the usual hoped-for consequence that elegance in theory leads to leaps in practice.

Future directions

What is important to consider for maps is important to consider for other representations of the earth's surface. Cartographic considerations can guide disparate research projects of spatial character.

Structural models (Harary, Norman, and Cartwright, 1965), one form of abstract graphs (Harary 1969), can offer yet another way to map the Earth. These abstract graphs serve as "maps" whenever any discrete set of real-world locations and flows can be captured in channels linking locations: the locations serve as the nodes for the graph and the channels serve as edges linking nodes. Thus, a set of cities and the railroad tracks joining them may be represented visually as a structural model – the cities are nodes and the tracks are edges of the model. Indeed, a set of individuals, at least some of whom share a common belief, may also be represented as a structural model; the individuals are nodes and the belief, if shared, is represented along edges linking appropriate individuals. There are numerous examples one might construct. What is important is that these models, as are maps, are also subject to interruption. Because it is abstractly preferable to avoid or to mask interruption, it is important to know how it arises.

References

Arlinghaus, Sandra L. 1987. Terrae Antipodum. In *Essays on Mathematical Geography – II*, Monograph #5, Institute of Mathematical Geography, Ann Arbor, MI, 33-40.

Barmore, Frank. 1992. Where are we? Comments on the concept of the "center of population." *Solstice: An Electronic Journal of Geography and Mathematics*. Monograph #16, Institute of Mathematical Geography, Ann Arbor, MI, 22-38.

Barmore, Frank. 1994. Center here; center there; center, center everywhere. *Solstice: An Electronic Journal of Geography and Mathematics*. Monograph #18, Institute of Mathematical Geography, Ann Arbor, MI, 12-25.

Courant, R. and Robbins, H. 1941. *What Is Mathematics?* Oxford University Press, London.

Goode's School Atlas. Various editions. Rand McNally. First copyrighted, 1922; Golden Anniversary Edition, Espenshade, E. B. , Jr. and Morrison, J. L. *Rand McNally World Atlas*. Rand Mc Nally, Chicago.

Harary, F. 1969. *Graph Theory*. Addison-Wesley, Reading, Mass.

Harary, F., Norman R., and Cartwright, D. 1965. *Structural Models: An Introduction to the Theory of Directed Graphs*. Wiley, New York.

Lukatela, H. 1987. Hipparchus Geopositioning Model: An Overview, Proceedings, Auto-Carto 8:87-96, ASPRS & ACSM, Baltimore.

Philbrick, Allen K. 1963. *This Human World*. Wiley, New York. Reprinted, Institute of Mathematical Geography.

Spilhaus, Athelstan. 1979. To see the oceans, slice up the land. *Smithsonian Magazine*, Nov. 1979, 116-122.

Tobler, Waldo R. 1961. World map on a Möbius strip. *Surveying and Mapping*, XXI, p. 486.

Tobler, Waldo R. 1993. Global spatial analysis. In Tobler, *Three Presentations on Geographical Analysis and Modeling*, National Center for Geographic Information and Analysis, Technical Report 93-1.

Webster's Seventh New Collegiate Dictionary. 1965. G. & C. Merriam Co., Springfield Mass.