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WEB FRACTALS  
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Documents written in HTML (Hyper-Text Markup Language) for WebSites not only offer the capability to place Home Pages on the World Wide Web, but also extend the dimension in which typesetting takes place. Broadly viewed, from a typesetting standpoint, HTML appears to be equivalent to a subset of commands of Plain TeX. One notable exception arises, however, in the commands in HTML that enable the user to specify links to other documents, graphical or textual. A Home Page, or any other page or set of pages (paper or electronic), might be viewed as a one dimensional string of letters broken into words. Graphics offer an opportunity to extend the text into other dimensions. So too do the links from one web page to another. They offer an extraordinary capability to reach out within the text itself: an opportunity to “fill” a text-shed beyond the linear text-stream. And of course, one can mix and match links, graphics, text, and whatever is available on the web. Without loss of generality, interest might be confined to text because it is with text that the extension into extra dimensions appears straightforward.

Thus, it becomes of interest to ask how much of a text-shed is filled by the activity of creating links and how one might measure such filling of text-space. Concepts from fractal geometry and from chaos theory will be employed to consider these issues.

#### GLOBAL WEB VIEW

The global perspective employed in much of modern mathematics focuses not on the individual objects being studied but on the sets of transformations that enable one to move from one object, or type of object, to another. Thus, one learns group theory by studying the various morphisms that link groups, rather than by focusing on the tables that describe individual groups. Linkages expose structure.

Thus, one view of the Web would be to shrink to nodes all home pages and focus only on the links that exist between sites. The set of nodes is finite, but it is unbounded--one can imagine going beyond any upper bound on the number of sites, simply by adding one more. The set of links, too, is finite and unbounded. The morass of links defies good graphical description. Search engines try to make sense of the Web through various organizational schemes. One view of the pattern is as a cataloguer’s nightmare; another is as a chance to introduce order into seeming abstract chaos.

Yet another might be to engage in graphical analysis based on some sort of ordering scheme that permits the assessment of extent of space filled. The concept of attractors and repellers seems an important one to search engines. One such ordering scheme might be such as suggested below. Of course there are many others, too.

$y=x$  means link...both to and from me to you

$y>x$  means link...me to you

$y<x$  means link...me from you

The algebraic, geometric, and topological structure of the web are important to analyze: the space-filling ideas represented in fractal geometry suggest one style of approach. Any deep, careful mathematical analysis should, however, offer a systematic means to evaluate the current status of web content, and more important, provide a continuing framework in which to guide and understand web development. These comments offer mere hints at directions substantive work might take; hopefully, they offer encouragement to consider the web itself as a source of various styles of research opportunity.

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