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DIFFRACTION AND SCATTERING BY REGULAR BODIES

by

R. E. Kleinman

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ABSTRACT

This report summarizes the results of the research carried out at the Radiation Laboratory of The University of Michigan under contract AF 19(604)-6655 with the Air Force Cambridge Research Laboratories. The report covers activities during the period February 1960 to January 1965 and includes work on the preparation of reports presenting the electromagnetic scattering properties of regular bodies as well as original research in basic diffraction theory and nonlinear modeling.

INTRODUCTION

This is the final report on Contract AF 19(604)-6655, summarizing the results of the research carried out under this contract during the period February 1960 through January 1965. At its inception, the goal of the contract was the preparation of a series of reports presenting the electromagnetic scattering properties of regular bodies. These reports are intended to be exhaustive summaries of existing knowledge with research to be conducted in those areas where gaps in this knowledge are revealed. In March, 1962, emphasis on original research in basic diffraction theory was increased, though a primary task in the program continued to be the preparation of reports on scattering by regular shapes. This work was a continuation of that performed under contracts AF 19(604)-4993 and AF 19(604)-8030 and is now continuing under contract AF 19(628)-4328, all with Air Force Cambridge Research Laboratories.

The results of work on this contract are contained in the reports and journal articles listed in the appendix and will be briefly summarized here.

Three comprehensive reports on scattering by simple shapes, covering the sphere, cone, and prolate spheroid, were completed and a fourth, on cylinders, was brought to an advanced stage of preparation. This work provided the impetus for a number of original investigations concerning spheres, spheroids, cones and cylinders which resulted in additional technical reports and journal articles as reported below. The areas in which basic research was conducted, in addition to that related to work on scattering by simple shapes, included diffraction by a strip, general low frequency scattering, and nonlinear modeling. For reporting purposes the work performed under the contract will be described under these headings. It should be borne in mind however, that this separation is largely a reporting convenience since an effort was made to coordinate and relate the various phases of work performed under the contract. Thus, for example, the basic investigations of scattering by a spheroid (3648-4-T) derived motivation from and contributed results to the handbook on spheroids (3648-6-T).

I

DIFFRACTION AND SCATTERING BY REGULAR BODIES

The first comprehensive report on scattering by regular bodies (3648-1-T) was concerned with scattering of a plane wave by a sphere. A detailed description of the Mie solution was presented together with extensive cross section curves calculated from the Mie series. A guide to existing computations was also provided. The field scattered by a sphere was analyzed by alternate means in both low and high frequency regimes. For low frequencies the field components were expressed as series in ascending positive integral powers of ka where k is the wave number and a is the radius of the sphere. Two derivations of this series were given, one based on expanding various terms in the Mie solution and the other a quasi-static approach essentially independent of specific knowledge of the Mie series. The high frequency analysis was based on the Watson transform which forms the basis for the asymptotic solution of a large class of diffraction problems in terms of creeping waves. In addition, the classic and still useful physical optics approximation was discussed.

Preparation of the sphere report stimulated research in a number of areas. In presenting the Mie series solution, use was made of Hansen's vector wave functions. The divergence-free Hansen functions are constructed with vector operations on products of vectors with solutions of the scalar Helmholtz equation. The question of the degree of arbitrariness permitted in the choice of the vector used in this construction was investigated by Senior (1960a). Since the Hansen vector wave functions form the basis for most attempts at the solution of vector scattering problems, the choice of this vector vitally effects the ease with which a solution may be obtained in any particular case. It was proven that the possible choices of this vector are severely limited, including only radial or constant vectors. In the first case (radial vector) the Hansen vector wave functions take a form particularly convenient for solving the problems of scattering by a sphere and in fact are precisely Debye

potentials. In the second case (constant vector), the Hansen vector wave functions may be identified with electric and magnetic Hertz vectors.

In the course of the high frequency analysis a fundamental error in the creeping wave formalism as applied to three-dimensional bodies was discovered. This was pointed out by Kazarinoff and Senior (1962). They analyzed the field distribution on the surface of a sphere in the shadow region using both the exact Mie series and creeping wave theory. Whereas the major components were in good agreement, the minor components were completely different, particularly in the neighborhood of the caustic and the discrepancy could not be accounted for by introducing higher order terms into the expressions for the launch and decay factors of the creeping wave. The error occurred because of an incorrect inference as to the relative magnitude of the minor creeping wave deep in the shadow based on an estimate of this magnitude near the shadow.

The details of the high frequency analysis leading to expressions for the far field scattered from a sphere were presented by Senior and Goodrich (1964). Erroneous implications of the unwarranted assumption as to the behavior of the minor creeping wave were corrected. Analysis of the scattered field resulting in a decomposition into optics and creeping wave terms was presented and the resulting formulas were given in a form suitable for computation. Numerical results for the amplitude of the first few creeping waves as well as the shadow boundary contribution to the scattering amplitude were included.

The creeping wave formalism is used to describe high frequency scattering, where all radii of curvature of the scattering object are much greater than the wavelength of the incident radiation. To see how this description is modified when the smallest radius of curvature is much smaller than the wavelength while the largest radius of curvature remains much greater than the wavelength, Goodrich and Kazarinoff (1963a) derived the leading term in the asymptotic expansion of the field on the surface of a long thin prolate spheroid. The scalar scattering problem was solved

for both Dirichlet and Neumann boundary conditions when the incident field consisted of a point source located on the axis of symmetry. It was found that the field thus obtained could be decomposed into terms representing various modes of waves. These waves had properties which closely identified them with traveling waves and at the same time had properties which identified them as descendants of creeping waves. Each wave mode had a representation as an infinite sum of waves, the m th term of which was interpretable as a sum of two waves that had traveled m times about the spheroid in opposite directions.

A similar analysis was carried out for a thin oblate spheroid illuminated by a point source on the axis of symmetry by Goodrich, Kazarinoff and Weston (1963). The field on the surface of the spheroid, for both Dirichlet and Neumann boundary conditions, was determined using asymptotic theory of differential equations with turning points. This surface field was interpreted in terms of traveling waves which were reflected at the rim of the spheroid and the reflection coefficients were determined.

The second report in the series summarizing the scattering properties of simple shapes dealt with the cone (3648-2-T). The exact solution of the problem of scalar scattering from a semi-infinite cone for both Dirichlet and Neumann boundary conditions and either plane wave or point source excitation was presented in detail. In addition existing expressions for scattered field and cross section for both scalar and vector scattering were systematically listed. Various approximations were given, i. e. thin cone formulae, thick cone formulae, far field expressions, etc., together with the restrictions of the parameters under which the approximations are valid. Approximate results were presented for scattering from finite cones with various base terminations. Modified Rayleigh formulas were presented for use at low frequencies and available high frequency approximations, i. e. physical optics, modified geometric optics, modified wedge theory, and impulse approximation, were also recorded. Efforts aimed at predicting finite cone scatter-

ing behavior in the resonance region were discussed and the results reported. In addition all available experimental data on scattering from cone-like structures were presented.

The prolate spheroid was the subject of the third report in the series on scattering by regular bodies. The prolate spheroidal wave functions were discussed and a catalog of existing numerical tables listing parameter ranges and indices covered was included. The exact solution of scalar scattering problems in terms of these spheroidal functions was described in detail. The corresponding form of the solution of the vector problem was presented together with a demonstration of the "inseparability" of this case, that is, the fact the the coefficients in the spheroidal function expansions of the vector wave functions are determined only to the extent that they are solutions of an infinite system of simultaneous linear algebraic equations. Descriptions of various approximations were presented including frequency-restricted methods (modifications of Rayleigh scattering, variational techniques at low frequencies; physical optics, modified geometric optics, and various asymptotic theories at high frequencies), eccentricity-restricted approximations (perturbations of spheres and thin rods) and approximations for weak scatterers where the material properties of the spheroid were close to those of the surrounding medium. The principal end results of the various analyses together with available information on accuracy and range of validity were listed. In addition, there was included a compilation of quantitative data including the majority of the curves and points, both theoretical and experimental, obtained and published by the principal investigators of the problem to date.

Evident in these reports on spheres, cones and spheroids is the inadequacy of existing theory to predict scattering behavior in the resonance region. A detailed discussion of attempts to obtain results for back scattering cross section in this region for flat-backed cones and thin spheroids was presented by Crispin, Siegel, and Sleator (1963). Various attempts to extend low and high frequency approximations

using both analytic and heuristic means were reported and the results compared with existing experimental data. It was found that the available techniques were considerably more effective in predicting the cross sections of thin cones than thin spheroids and a physical mechanism giving rise to this discrepancy was proposed.

A similar study of the back scattering cross section of a cone-sphere (smooth join) was given by Senior (1965) who presented some new experimental data for both back scattered and surface fields at nose-on incidence and deduced therefrom a modification to accepted theory sufficient to explain the results. It was found that while use of physical optics on the cone and creeping waves on the sphere did not suffice to predict the observed far field behavior, the discrepancy was traceable to an increase in the far field contribution of the creeping wave. This enhancement of the creeping wave was larger than could be accounted for by the presence of a singularity, albeit at weak one, at the cone-sphere join and indicated an additional enhancement due to a traveling wave on the cone portion which flows over the join. Quantitative measures of the enhancement were derived which bring theoretical values of cross section into better agreement with existing experimental data.

Preparation of the prolate spheroid report revealed some gaps in existing tables of spheroidal functions which prevent the calculation of the scattering characteristics of spheroids for cases of practical interest. To correct this deficiency, a computational program was carried out which provided eigenvalues, normalization constants, radial and angular spheroidal functions and their derivatives. The results of this calculation are to be presented in two reports, 7133-1-T and 7133-2-T, to be issued under contract AF 19(628)-4328.

The fourth report in the series on scattering from regular bodies deals with circular cylinders: infinite, semi-infinite and finite. This work was brought to an advanced stage under the subject contract but will be completed and the report issued under the continuation.

Related to this cylinder study was the work of Plonus (1960) who calculated the far field and back scattering cross section of a perfectly conducting cylinder surrounded by a dielectric sleeve. When the thickness of the surrounding shell was small, a simplified expression for the back scattered field was obtained in the form of a term associated with the conducting cylinder plus a perturbation due to the shell. Another simplification of the exact result was obtained for the case when the propagation constant characterizing the material of the layer was much larger than the free space propagation constant. Numerical results were presented for the back scattered cross section as a function of layer thickness for a number of conducting cylinders of radius large with respect to wavelength and with layers of resonant thickness.

II DIFFRACTION BY A STRIP

Work on this problem received its impetus from the results of Kleinman and Timman (3648-3-T) who presented a closed form expression for the solution of the problem of diffraction of a line source by a soft strip (Dirichlet boundary condition). They derived a general class of solution of the two-dimensional Helmholtz equation which satisfied particularly simple boundary conditions. These were used to construct a (double) integral equation for the Green's function of the first kind (vanishes on the boundary) for a line segment. In the particular case when the line segment was infinite, a new integral representation of the line source and its image was obtained. When the line segment was semi-infinite, this integral equation was used to cast the well known Sommerfeld solution for the half plane scattering problem into a more complicated but nevertheless useful form. The solution of the integral equation for a line segment was found by comparison with the analogous form into which the known solution of the half plane problem had been cast. The solution was sufficiently complicated in form, a triple integral expression with a complicated though well-defined integrand, so that any calculation represented a considerable task.

An extension of this result to the case of plane wave incidence was presented by Kleinman (1963). The integral representation of the solution for line source excitation was of such complexity that a direct determination of the asymptotic behavior for a source located far from both strip and field point was not trivial. This was circumvented by using the two-dimensional form of the Helmholtz integral representation of a field in terms of its values on a boundary to obtain the plane wave result. Although this result was slightly simpler in form than the line source expressions, further simplification was still required for it to be comparable with the elegant and useful form of the half plane solution.

In pursuing this goal, further analysis revealed an apparent discrepancy between these integral representations of the strip solution and the classic Mathieu

function series representation at low frequencies. This low frequency analysis gave rise to some striking new results, described in the next section, which proved so fruitful that work on the strip problem was temporarily set aside, though it is to be resumed under contract AF 19(628)-4328.

A different extension of strip diffraction was presented by Goodrich and Kazarinoff (1963b) who considered the problem of scalar diffraction of cylindrical waves from a thin elliptic cylinder on which either Dirichlet or Neumann boundary conditions are imposed. They derive the asymptotic behavior of the surface field distribution for any source position. The surface field representation is decomposed into two parts, one representing the field on a strip and the second representing a perturbation taking into account the cylindrical nature of the boundary. The methods employed, analogous to those successfully applied to prolate and oblate spheroids discussed previously, are applications of the asymptotic theory of second order differential equations with turning points.

III SCATTERING AT LOW FREQUENCIES

In the sphere report (3648-1-T), a derivation of the Rayleigh series was presented which involved an expansion of the terms of the Mie series in powers of the wave number, k . The corresponding analysis for spheroids was presented by Senior (1960b). For the scalar problem of diffraction of a plane wave incident along the axis of symmetry of a spheroid, prolate or oblate, the low frequency expansion of the scattered far field was obtained, for terms up to and including k^6 , by expanding the exact series of spheroidal wave functions. Results were obtained for both Dirichlet and Neumann boundary conditions. The simplifications resulting when the spheroid degenerates into a sphere or a disk were demonstrated.

Further analysis of low frequency scalar scattering by spheroids was pursued by Senior (3648-4-T) wherein the radius of convergence of the Rayleigh series expansion of the far field was determined. As before, the incident excitation was a plane wave propagating along the axis of symmetry and both Dirichlet and Neumann boundary conditions were treated. The radius of convergence is determined by locating the poles of the coefficients of the far field expansion; the values of the expansion parameter corresponding to these poles are ordered according to the magnitude of their moduli and the smallest one corresponds to the radius of convergence of the series. If the exact expressions for the individual coefficients are not known or if their complication is such that the location of singularities is not practicable a new method of estimating the radius of convergence is presented. This involves a comparison of numerical coefficients in expansions of the individual coefficients. Estimates of the radius of convergence of the Rayleigh series are found for spheroids of all eccentricities and it is shown that of all spheroidal bodies the sphere has the least such radius.

The problem of determining terms in the Rayleigh series when the scattering surface is not "separable" (i. e. not a level surface of a coordinate system in which the Helmholtz equation succumbs to the method of separation of variables) but is the

intersection of such separable surfaces is considered by Darling and Senior (1965). Using a method developed by Darling for finding solutions of potential problems for such surfaces, they derive higher order terms in the Rayleigh series in the particular case when the scattering surface is a spherically capped cone, that is, the intersection of a cone and a sphere.

A new approach to the solution of problems of scattering by three dimensional finite bodies was found by Kleinman (3648-7-T). A method was presented whereby the solution of the static potential problem for a Dirichlet boundary condition on a particular surface was transformed by successive operations into the solution of the scalar Helmholtz equation which also satisfies a Dirichlet condition on the same boundary. A new integral equation for the scattered field is derived whose kernel is the potential Green's function for the surface instead of the free space Green's function for the Helmholtz equation. Despite the fact that the integral operator operates over all space, rather than just the scattering surface, and is really an integro-differential operator, it is still possible to solve the equation iteratively in a standard Neumann expansion. This series has a nonzero radius of convergence and is in a sense a partial summation of the low frequency expansion. The explicit relation between this Neumann series and the Rayleigh expansion is known. A proof of the convergence of the Neumann series is provided.

A revised version of these results is contained in Kleinman (1965a). The only substantial change concerned an improvement in the proof of convergence of the series of iterates. This required obtaining a bound on the norm of the integral operator involved which was accomplished rigorously.

Extension of this method to scalar scattering from surfaces on which Neumann and mixed boundary conditions are prescribed is receiving continuing attention. Preliminary results for the Neumann case were reported by Kleinman (1964). Application of the technique to specific calculations of terms in the low frequency expansion for particular objects as well as possible extension to two-dimensional and vector problems will be considered under contract AF 19(628)-4328.

An expository description of presently available low frequency techniques was given by Kleinman (1965b). The "Rayleigh region" was defined as that frequency range wherein the Rayleigh or low frequency expansion converges. This is mathematically precise and consistent with, though may be considered an extension of, the usual meaning of "Rayleigh region" in radar scattering work. General methods for obtaining successive terms in low frequency expansions were outlined and those special shapes for which calculations have actually been carried out were enumerated.

In the course of the preparation of this description of low frequency techniques, a deficiency was found in the standard (Stevenson) low frequency treatment of electromagnetic scattering problems. It was discovered that the usual method explicitly determines only the first three terms of the low frequency expansion and the suggested modification for treating higher order terms is insufficient since it leads to discontinuous field quantities. This has gone apparently unnoticed since no calculations have ever been carried out with this method for terms of higher order than three. A means of surmounting this difficulty has been found and will be reported under the successor to the present contract.

IV
NONLINEAR MODELING

Work on nonlinear modeling was a continuation of efforts under previous contracts AF 19(604)-4993, -7428 and -8030. The ultimate goal was the derivation of analytical methods to transform a physical problem for which neither experimental nor theoretical techniques are appropriate into another problem which is more amenable to experiment or theory or both. In general such transformations of the mathematical description of one physical system into another will be nonlinear, giving rise to the descriptive phrase nonlinear modeling. An analysis of the mathematics underlying the application of nonlinear modeling techniques to practical physical problems was presented by Ruehr (3648-5-T). Nonlinear modeling of three different physical processes, described by three basic types of partial differential equations was considered. For the Helmholtz equation (elliptic) and the diffusion equation (parabolic) it was found that the allowable modeling functions must satisfy a particular nonlinear ordinary differential equation. The problem of similitude restrictions (conditions under which modeling functions exist) is reduced to the study of polynomials generated by Burmann series expansions of functions, $f(kx)$, in powers of $f(x)$, where f denotes a solution of the differential equation satisfied by the modeling function. It was also shown that it is not possible to derive a similar theory for the hyperbolic wave equation.

APPENDIX:
WORK PUBLISHED UNDER CONTRACT AF 19(604)-6655Scientific Reports

- 3648-1-T, "Studies in Radar Cross Sections XLVII - Diffraction and Scattering by Regular Bodies I: The Sphere". R. F. Goodrich, B. A. Harrison, R. E. Kleinman and T. B. A. Senior. December 1961.
- 3648-2-T, "Studies in Radar Cross Sections XLVIII - Diffraction and Scattering by Regular Bodies II: The Cone". R. E. Kleinman and T. B. A. Senior. January 1963.
- 3648-3-T, "Studies in Radar Cross Sections XLIV - Diffraction by a Strip". R. E. Kleinman and R. Timman. February 1961.
- 3648-4-T, "Studies in Radar Cross Sections XLVI - Convergence of Low Frequency Expansions in Scalar Scattering by Spheroids". T. B. A. Senior. August 1961.
- 3648-5-T, "Studies in Nonlinear Modeling V: Nonlinear Modeling Functions of a Special Type". O. G. Ruehr. August 1962.
- 3648-6-T, "Studies in Radar Cross Sections XLIX - Diffraction and Scattering by Regular Bodies III: The Prolate Spheroid". F. B. Sleator. February 1964.
- 3648-7-T, "Iterative Solution of the Helmholtz Equation". R. E. Kleinman. August 1964.

Journal Articles

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- Plonus, M.A. (1960) "Back Scattering from a Conducting Cylinder with a Surrounding Shell", Can. J. Phys. 38, 1665-1676.
- Senior, T.B.A. (1960a) "A Note on Hansen's Vector Wave Functions," Can. J. Phys. 38, 1702-1705.
- Senior, T.B.A. (1960b) "Scalar Diffraction by a Prolate Spheroid at Low Frequencies," Can. J. Phys. 38, 1632-1641.
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