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SET IN STONE:
An Analemma in Northern Italy.

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On a recent trip through the less traveled parts of northern Italy (less traveled compared to the crowds encountered in Rome, Florence, and Venice) our traveling party found expected, and surprising, evidence of the great contributions to our modern world made by Italians in the first and second millennia. The evidence is set in stone.

In the fifth and sixth centuries A.D. Ravenna, on the Adriatic Sea, was the seat of authority of the emergent church at the time the division was opening between the eastern and western churches. Evidence of both Byzantine and western Christian traditions are preserved in the beautiful mosaics of the churches, mausoleums, and other religious places in Ravenna. The images, made of stone and colorful glass, have remained bright and clear over time. The interior of Basilica di Santo Vitale (6th century) is an example. Many mosaics in the basilica cover walls, ceilings, and floors as do others in various religious structures throughout the city. Ravenna has been continuously occupied; it was an intellectual center during the Renaissance. Dante's tomb next to the old churches is evidence of the importance of the city in the thirteenth and fourteenth centuries (Dante 1265-1321).

Urbino is a well-preserved Renaissance town in the mountains to the south of Ravenna. It was prominent in Italian history for two short centuries after which fortunes declined and it was annexed to the Papal States in 1631 and left to languish in obscurity. Its rise to fame was due to Duke Federico di Montefeltro (1444-1482) who was most influential in advancing its political, artistic, and intellectual achievements. The value of this history to us is that the magnificent Ducal Palace (the linked photo shows its courtyard) and other renaissance buildings in the city were not demolished or scavenged in the process of building new structures suited to subsequent eras. In the decline, the Palace fell into a state of abandonment and many of its art treasures were scattered to Rome, Florence and other seats of power. Restoration began in the early part of the 20th century and the Palace is now the National Gallery of the Marche. Also, fortunately, the Palace appears to have escaped the ravages of World War II despite the fact that heavy fighting occurred in the vicinity. Urbino was by-passed by the battles of that conflict.

Restoration of the Ducal Palace, along with the new acquisition of Renaissance art, has created a magnificent museum of the Renaissance. Here, within limestone and marble walls more delicate evidence of high civilization can be seen in the paintings, sculptures, and inlaid wooden cabinets and doors. The Duke's study, paneled with inlaid wood, celebrates the discovery of perspective views used by artists to depict, with mathematical precision, three dimensional images on two dimensional surfaces. The panels in the Duke's study have several versions of the illusion of three dimensions executed on a flat surface.

Padua is another mid-sized Italian city just to the north of Ravenna with architecture and art dating back before the renaissance. It is also the site of some of Galileo's experiments with telescopes and of the first scientific studies of the human body carried out by careful dissection of cadavers. These medical demonstrations took place on the stage of a steep-sided amphitheater located at the University of Padua, one of the oldest universities in Europe, founded in 1222 (Leslie Nystuen, M.D. attempts to enter the amphitheater). The careful, dispassionate demonstrations by professors in front of medical students were meant to impart knowledge to medical students as it was being created through use of the new empirical tradition of science. The walls of the loggia enclosing the old courtyard of the Bo Palace, one of the core buildings of the university, are lined with stone seals and crests of graduates of the medical school.

Bergamo is a city located at the foot of the Alps northeast of Milan. The city is divided into the old, Alti Calli (high city) located high up steep-sloped hills with the new town spread out across more level surfaces at lower elevations. The central piazza might serve as a setting for a Shakespearean play (photo; sketch by author). A medieval cathedral and a renaissance church stand close together at one end of the main piazza and are separated from it by an open-sided arcade covered by a high arched and vaulted ceiling.

An unusual feature located under this covered space is an analemma (and meridian) made of marble inlaid in the stone floor. An analemma is a graduated plot of the declination of the sun observed at solar noon throughout the year. A beam of sunlight passes through a small hole in a shield mounted high on the south facing wall. Through the course of a year the beam traces out the equation of time in an elongated, asymmetric figure 8 . The months and days of the year are marked on the analemma and each day is illuminated

in turn by the sunbeam as it traces out a calendar year. The centerline of the structure marks the line of the meridian (north/south direction). At solar noon on the day of the Winter Solstice (December 20 or 21) the sun is at its lowest declination and casts its pencil of sunlight onto the crossing of the meridian by the analemma trace at the extreme northern extent of the larger of the loops of the figure 8 shape. On the summer solstice (June 20 or 21) the sun is at its highest
point in the sky and the pencil of sunlight illuminates the crossing of the analemma trace with the meridian on the southern extreme of the smaller of the loops of the figure 8. The light beam crosses the meridian twice more during the year; in the center of the figure 8, once on the Spring Equinox and again on the Fall Equinox but with the pencil of light approaching from opposite directions.

On our visit, we immediately recognized the inlaid figure on the floor to be an analemma but we were puzzled because it was located under a roof and surrounded by large buildings. How could direct sunlight fall on it? Close observation revealed that to the south just enough open sky existed to permit the sun to shine through the shield and onto the floor at noon on any day of the year (with a view of the shield in mind, now check back to the broader general picture to see it in context). Analemmas are specific to the latitude at which they are located. Inscribed in the floor along with the analemma is a record of the latitude, longitude and elevation of the figure. The inscription reads, <‘Latitude 45° 12’ 11” Nord, - Longitude 9° 39’ 46” Est> and on another line, <Altitudine M 360.85 sul livello Dell Adriatico> .

A compass rose is also present to which is affixed the presumed date of the work, 1857. That date is 137 years after telescope-equipped theodolites came into wide use for accurate measurement of angles (1720) (Wilford, p. 97). In that era, more accurate and stable surveying equipment was introduced and used for land surveys and earth measurements. In 1666, Isaac Newton had predicted that the earth might be better modeled as an ellipsoid or oblate spheroid (a solid figure generated by rotating an ellipse around its short, or minor, axis) than as a sphere (Wilford p.99). Using the improved surveying instruments and techniques, the French investigated this hypothesis by undertaking to measure an arc of the earth near the North Pole along the Meridian of Kitts (Lapland) (1736-37) and similarly along the Meridian of Quito (Ecuador) (1735-43) (Wilford, p. 101). The arc nearer the pole proved flatter than the arc at the equator, that is, a degree at high latitudes is longer than a degree near the equator. The curvature of the earth is greater at the equator and flatter near the poles. The technology increasing the precision in the measurement of angles and distances had been put to great scientific purpose, establishing the shape of the earth by empirical means in support of theory.

That more precise earth model was needed for the Bergamo analemma to be constructed with sufficient precision of shape and position for the sunbeam to stay on the track laid in stone. The Italians had undertaken cadastral and topographic surveys of northern Italy by the time the Bergamo analemma was constructed. Knowledge from such surveys would have been used to fix the location and altitude of the analemma. The two decimal figures for altitude implied high order geodetic control.

I wonder though whether theory alone was sufficient to predict the placement of the analemma relative to the shield mounted on the wall. It could be closely predicted but I speculate that the theory might have been backed up by empirical observations made throughout the seasons. Theory would direct how the construction should proceed; practice, on sunny days throughout the year, would suggest the location of the beam of light that could then be checked and recorded empirically. This presumes that they could determine the moment that high noon occurred. Over the months the precise path of the spot of light could be traced out.

Our overnight (May 15) visit to Bergamo was too short for us to learn of the history of the construction. A large library located on the piazza no doubt has a record of the project and perhaps on another day we will return and look into it. We were able to conduct some empirical observations of our own. Jeffrey Nystuen, one of our party, had a portable GPS (global positioning system) receiver. The instrument was a hand held Magellan GPS receiver that he named Enrico, after Henry the Navigator (1394-1460). Despite being under cover with little open sky visible, Enrico could fix the location and altitude of the analemma. The receiver recorded the same latitude to the exact second as that written in the stone. There was one-second difference in longitude. Enrico reported elevation as five meters higher than that recorded in stone. Fixing elevation with a GPS is less exact than fixing position.

The length of one second of longitude at 45 degrees north can be approximated by multiplying the cosine of latitude by the length of a second of longitude at the equator of the authalic sphere (a sphere with the same surface area of the ellipsoidal model of the earth) (Robinson, et al.). This works out to be about 22 meters . Using the GPS Precise Positioning Service the positioning error of the Magellan GPS receiver should be less than 2 meters. [The circumference of the authalic sphere is 40,030.2 kilometers; thus, dividing the circumference by 360 yields one degree as equal to 111.195 kilometers and, subdividing further, one second as 30.89 meters. Cosine 45 degrees times 30.89 meters equals approximately 22 meters (Robinson, et al.).]

In May of 2000, the Selective Availability feature of the GPS was turned off by the U.S. government. Prior to this date, for military purposes, deliberate degrading of the stability of the on-board atomic clocks in the GPS satellites degraded the GPS signals. The Standard Positioning Service that had been previously available (worldwide) provided at best 100 meter accuracy. As it was, this degradation could by overcome by using differential GPS procedures in which a base receiver station with a well-known position could be used in conjunction with a roving receiver. This procedure was being routinely applied by a variety of civilian users.

Today relatively inexpensive GPS receivers are being employed in myriad tasks not at all envisioned by the designers of the system. For example Enrico, the hand-held receiver, provides the basic latitude, longitude, and altitude with error terms attached but it can also indicate direction and speed of movement -- even walking speed. One can fix a position in Enrico's memory, such as the location of the parked car, and then wander off through a maze of medieval Italian streets and Enrico can show the path to take to return to the car. It has in its memory the locations of most towns and cities with population greater than twenty thousand, at least, in North America and Europe where Enrico has been put to use.

Precise agreement in latitude and one second discrepancy in longitude seems very good for the performance of a hand-held GPS receiver and for the older methods that were employed to locate and design the analemma. Yet given the assumed accuracies, a 22 meter discrepancy may be too large. The likely explanation is in the differences in the theories applied. The Italians in 1858 modeled the earth as an ellipsoid using the best available datum (values for earth radii and eccentricity) perhaps the Bessel, 1841 model that is widely used in Europe. This datum differs from WGS84 (World Geodetic System, 1984) which now is used in conjunction with modeling the satellite orbits. Those orbits respond to the earth's center of gravity, not its geometric center. The same basic assumption must be made to achieve identical results. What the good correspondence we observed indicates is that both models are very good.

The Italians have made many contributions to Western Civilization and to our modern global society. Fortunately records of some of these achievements have been set in stone. I wonder, in this new informational age that we live in today and that is so rapidly becoming digital and electronic, are we leaving imprinted forms so durable that after one hundred fifty years, a thousand years, or fifteen hundred years they will be fresh and bright, full of grace and beauty and intellectual achievement? Will they charm passing travelers?

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Related articles from the internet:

Sundials on the internet

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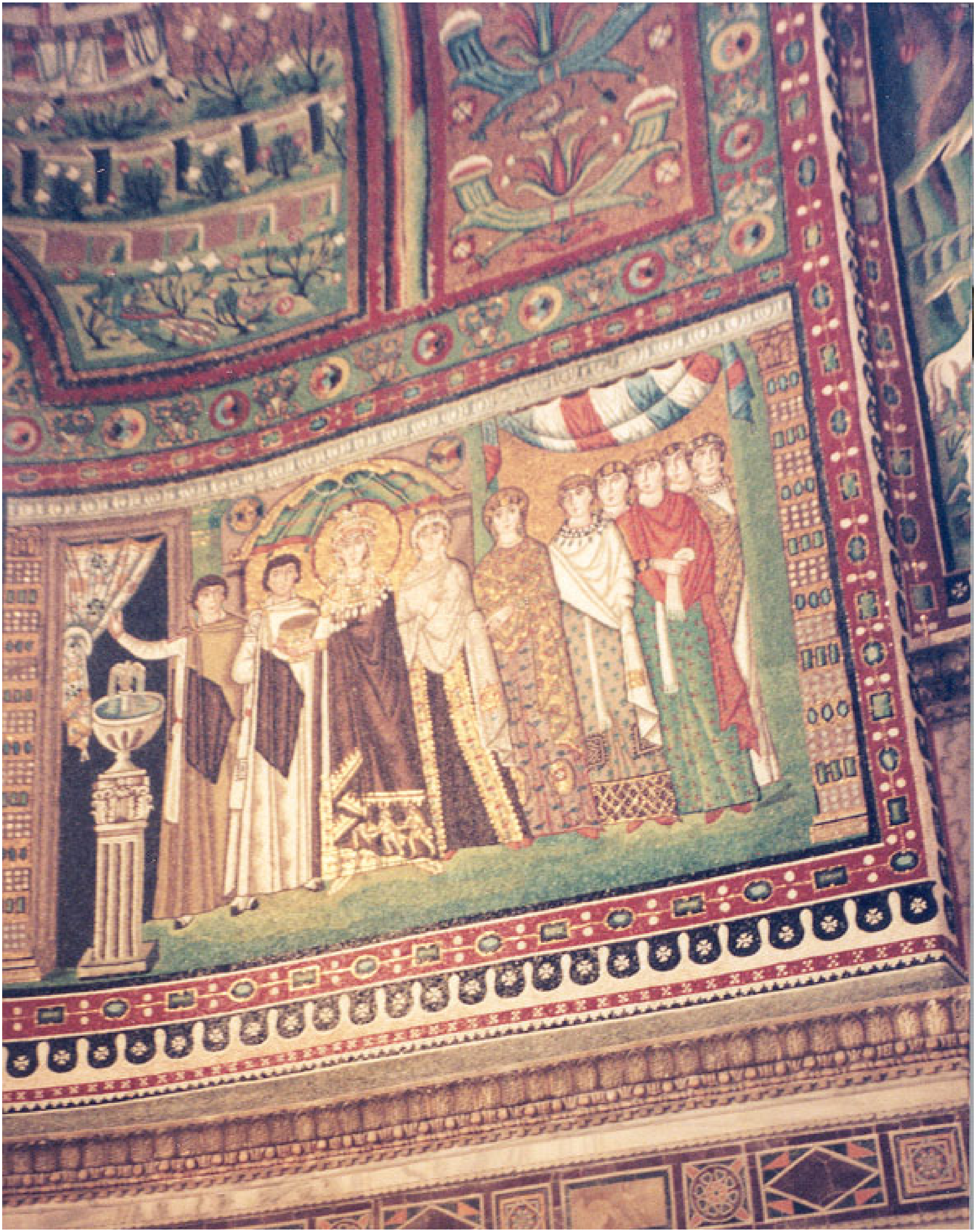


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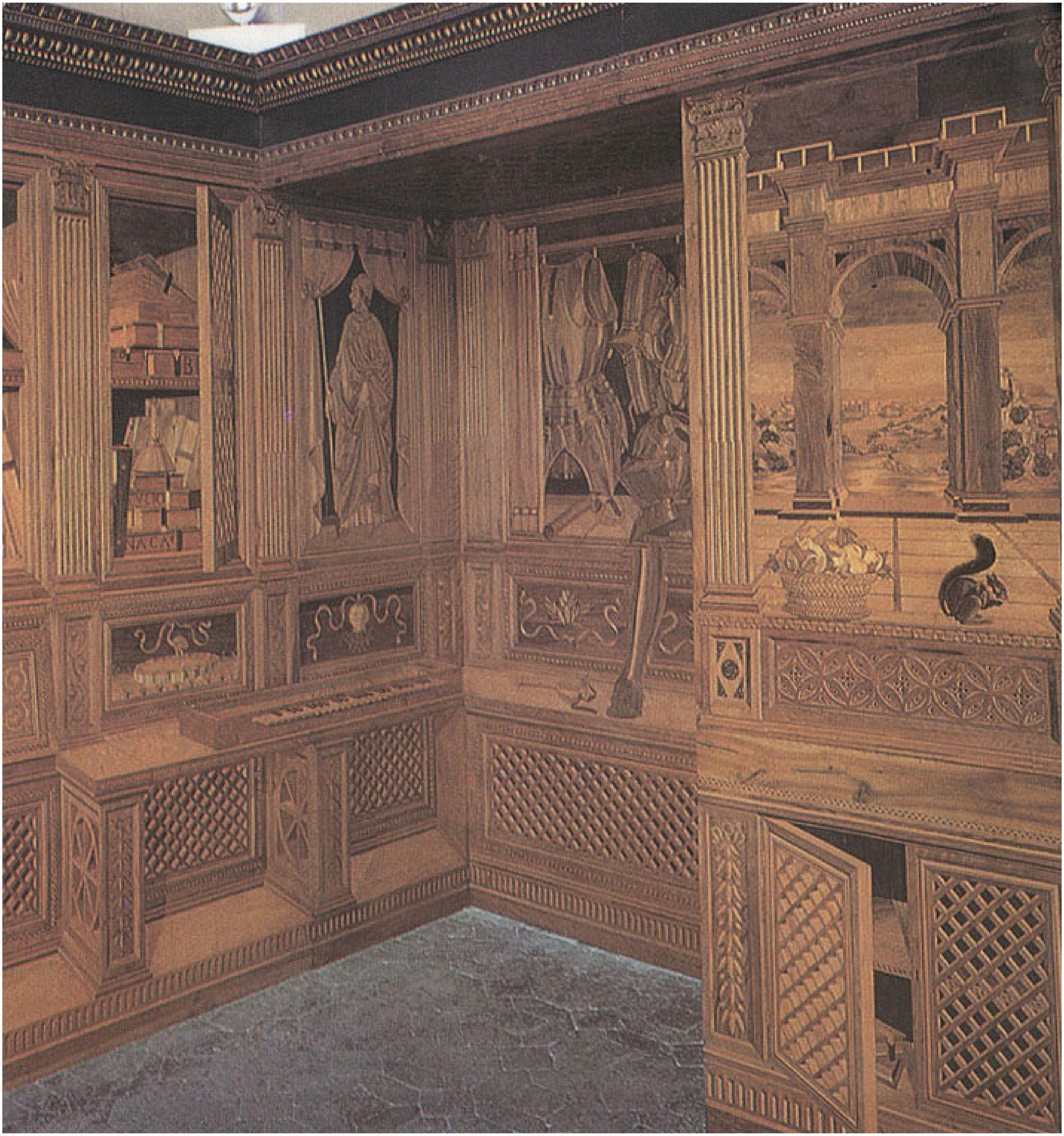
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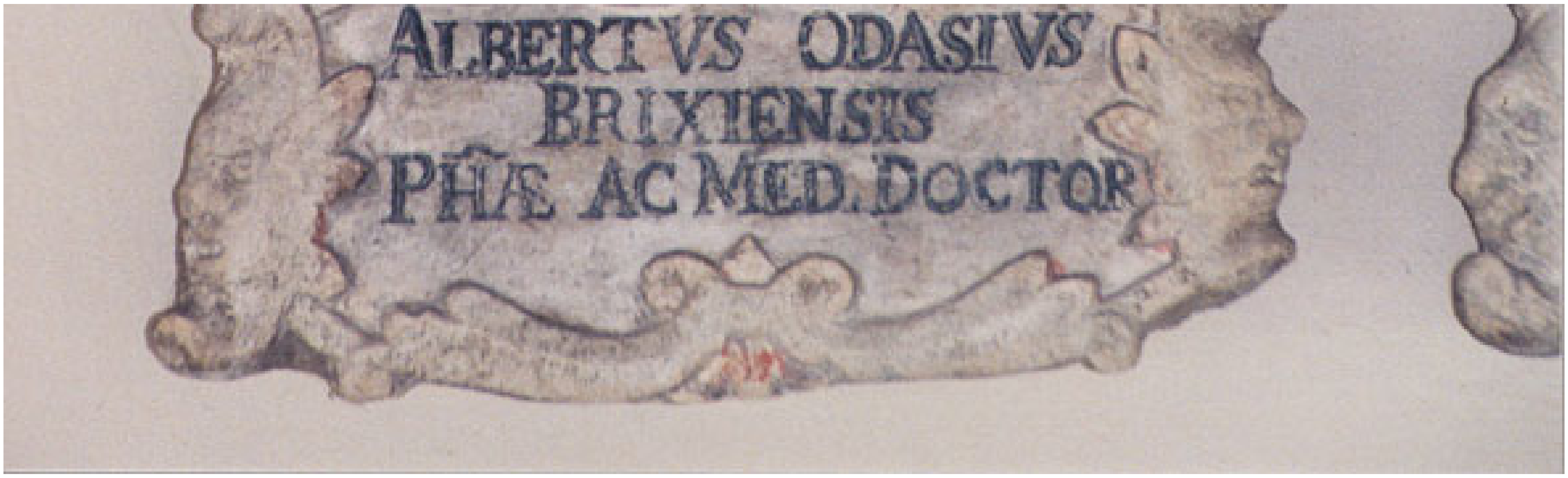
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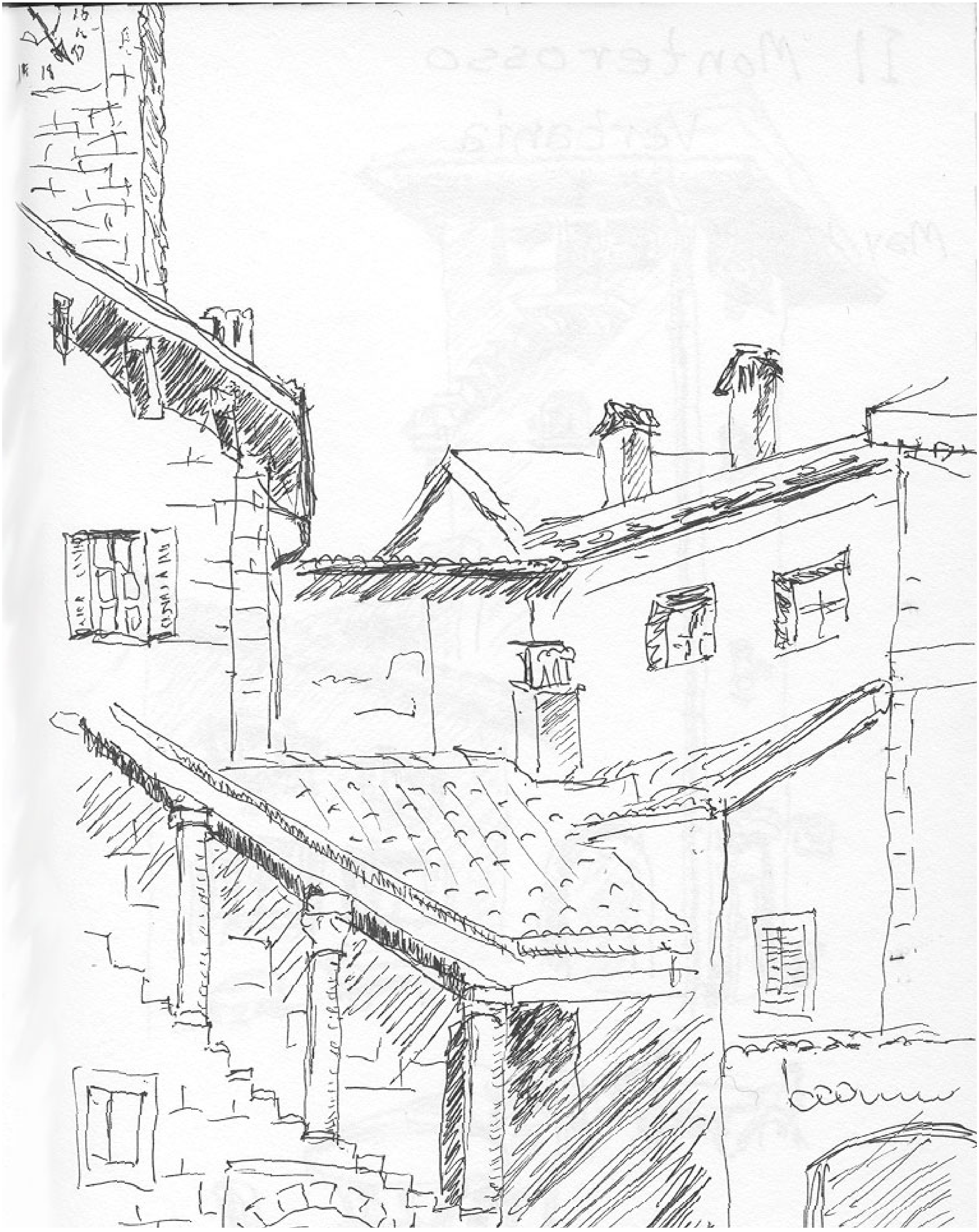

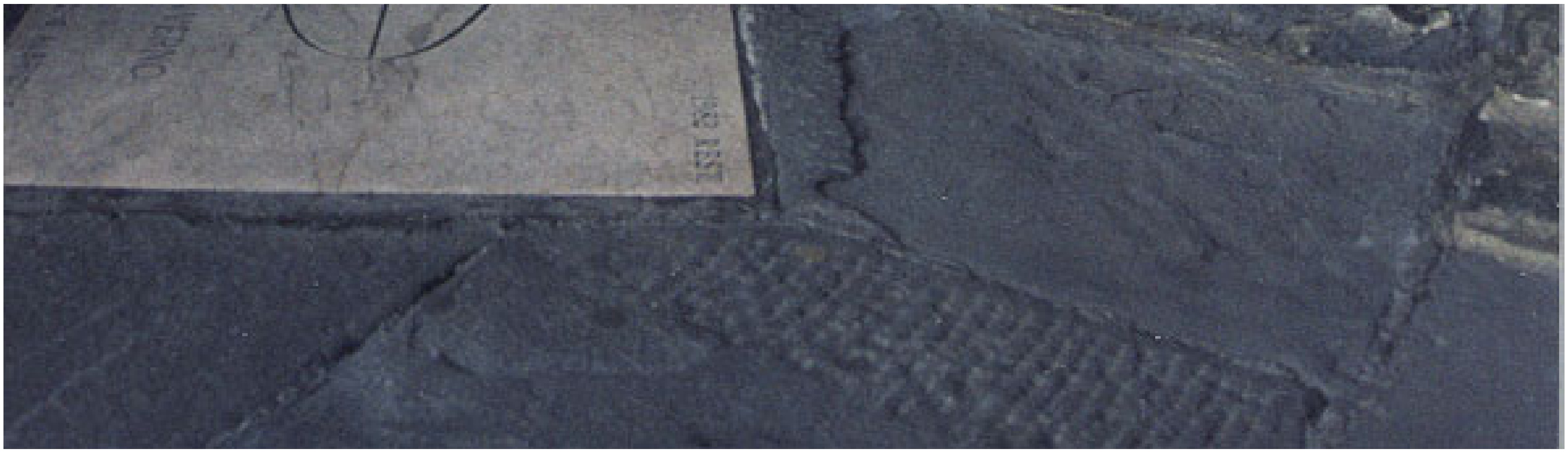


ALTITVDINE M. 360.85 SUL LIVELLO DELL'ADRIATICO
LATITVDINE 45°42'11" NORD - LONGITVDINE 9°39'46" EST


LATITVDINE 45°42'11" NORD - LONGITVDINE 9°39'46" EST
ALTITVDINE M. 360.85 SUL LIVELLO DELL'ADRIATICO


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RINNOVATA NEL 1857


ALTITVDINE M. 360.85 SUL LIVELLO DELL'ADRIATICO
LATITVDINE 45° 42' 11" NORD - LONGITVDINE 9° 39' 46" EST

Quotation from PC Magazine, online.

The Global Positioning System is now ten times more accurate than before.

By Alfred Poor — May 18, 2000

"The Global Positioning System (GPS)... has become an important part of our business and recreational activities. With the flick of a switch in May, the U.S. government instantly made it about ten times more accurate.

Originally designed to provide navigation information for the military services, the GPS depends on a constellation of 24 satellites and five ground stations. In order to prevent the system from being so accurate that enemies could use it against us, the Department of Defense had been using selective availability (SA) to degrade the signal. As a result, civilian applications were limited to about 100-meter accuracy, while the military's own equipment was able to eliminate the SA errors.

Turning off SA has improved GPS accuracy for all users to from 10 to 20 meters. With SA enabled, reported positions could be as much as 300 feet distant from the actual ones--enough to cause a mapping program to place your car on a street parallel to the one you were actually on. With SA turned off, the resulting 30-foot accuracy is enough to determine whether your car is on one side of a divided highway or the other. This level of accuracy will provide many benefits, including faster response times by emergency services.

...Although the United States intends to make the GPS system available for free for worldwide use, the military retains the ability to enable SA distortion 'on a regional basis' if required for national security."

Sundials on the Internet

For a full overview **click here**

sundial trail of Milan and Bergamo

These notes are based on a very short visit to Northern Italy in October 1999.

Milan

The Duomo (cathedral) of Milan has a magnificent tiled floor, interrupted by a meridian line which runs nearly the full width of the building, and goes on for 3m. up the left hand wall. You will find it just inside the main entrance doors, in front of the small publications booth. The line is a brass strip, with white marble on either side. The signs of the zodiac are marked at appropriate intervals along the meridian line. There is a small hole in the wall high up on your right hand side as you enter the cathedral; a spot of light shines through the hole and casts a small circle of light on the floor around the time of solar noon. There is a printed table on the left hand wall showing the time of solar noon (together with sunrise and sunset)

According to the guide book, there is another sundial outside the cathedral as well, but I was not able to locate it.

While other sundials in Milan appear to be very few, there is a magnificent collection of sundials and other scientific instruments concerned with the sun at the Museo Poldi-Pezzoli which is quite close to the Duomo. There are some 230 exhibits in all, well displayed in three cases on the first floor. The museum is a delight - not a bit like the formal public art galleries (such as the Pinotecha Brera nearby) but with some really nice paintings displayed in fairly small rooms with furniture, china cabinets, etc, and some displays of other classes of objects such as sundials, clocks, and weapons.

The picture shows an ivory sun-dial in the shape of a ship known as a navicella. Posters of this sundial are sold in the museum.

The sundial display has many attractive portable dials, including two sundials combined with a small cannon fired by the sun's rays focused through a magnifying glass at noon, a curious horizontal sundial with a gnomon adjustable for latitude (though, since the lines on the dial plate are fixed, it would not be very accurate at any location other than the latitude it was designed for), and some shepherd's dials and astrolabes. Altogether unmissable!

Bergamo

Bergamo is a very interesting town located some 40 km. east of Milan, and accessible either by the blue motorway buses which leave from the Piazza Castello, or by train from the Central Station. The bus station and train station in Bergamo are very close together. Catch a no. 1 or 1A bus to the foot of the funicular railway, and take the funicular to ascend into the old town. From the terminus, walk directly across the square and up some steps into another small square. There is a group of 4 large sundials on the wall high up to your right.

Carry on in the same direction until you reach the Duomo (cathedral). In the colonnade opposite the main door of the cathedral is a very handsome meridian line. The central line is marked with the dates and months throughout the year. There are two subsidiary lines fanning out which indicate 15 minutes before noon, and 15 minutes after noon. Between these two outer lines, an analemma is engraved in the stone. High up in one of the arches of the collonnade is a disk with a central hole. Towards solar noon, a spot of light shines through the hole on the pavement, and gradually moves across the meridian line. When it is on the meridian line, it is exactly solar noon. When it crosses the hour-glass shape of the analemma, it is exactly local noon, which is about 11.35 by your watch (or 12.35 when daylight saving is in operation). This difference is because Bergamo is some 9 degrees west of the standard meridian for Central European Time, which is at the longitude of Prague.

Sundials on the Internet

For a full overview **click here**

Sundials on the Internet - the Equation of Time

"Sun time" and "clock time"

Sundials tell "sun time". Clocks and watches tell "clock time". Neither kind of time is intrinsically "better" than the other - they are both useful and interesting for their separate purposes.

"Sun time" is anchored around the idea that when the sun reaches its highest point (when it crosses the meridian), it is **noon** and, next day, when the sun again crosses the meridian, it will be noon again. The time which has elapsed between successive noons is sometimes more and sometimes less than 24 hours of clock time. In the middle months of the year, the length of the day is quite close to 24 hours, but around 15 September the days are only some 23 hours, 59 minutes and 40 seconds long while around Christmas, the days are 24 hours and 20 seconds long.

"Clock time" is anchored around the idea that each day is exactly 24 hours long. This is not actually true, but it is obviously much more convenient to have a "mean sun" which takes exactly 24 hours for each day, since it means that mechanical clocks and watches, and, more recently, electronic ones can be made to measure these exactly equal time intervals.

Obviously, these small differences in the lengths of "sun days" and "mean days" build up to produce larger differences between "sun time" and "clock time". These differences reach a peak of just over 14 minutes in mid-February (when "sun time" is slow relative to "clock time") and just over 16 minutes at the beginning of November (when "sun time" is fast relative to "clock time"). There are also two minor peaks in mid-May (when "sun time" is nearly 4 minutes fast) and in late July (when sun time is just over 6 minutes slow) (These minor peaks have the fortunate effect, in the Northern hemisphere, that the differences are relatively minor during most of the months when there is a reasonable amount of sunshine).

The differences do not cumulate across the years, because "clock time" has been arranged so that, over the course of a four year cycle including a leap year, the two kinds of time very nearly come back to the same time they started. (The "very nearly" is because "clock time" still has to be adjusted by not having a leap year at the turn of each century, except when the year is exactly divisible by 400, so 1900 was not a leap year, but 2000 will be). Even with this correction, we had an extra second added to "clock time" recently.

The reasons for these differences are discussed below, followed by some information on what the differences are at given times of year.

Why the days are of different lengths

These differences arise from two quite separate causes. The first is that the plane of the Equator is not the same as the plane of the Earth's orbit around the sun, but is offset from it by the **angle of obliquity.**

The second is that the orbit of the Earth around the sun is an ellipse and not a circle, and the apparent motion of the sun is thus not exactly equal throughout the year. The sun appears to be moving fastest when the Earth is closest to the sun.

These two effects are explained in more detail in a leaflet of the Royal Greenwich Observatory and in Art Carlson's excellent article on the subject at the end of this page.

The sum of the two effects is the Equation of Time, which is the red curve with its characteristic twin peaks shown below. (Many thanks to Patrick Powers for providing this graph from his own sundial page).

Graph of the Equation of Time

Some people like such information presented in tables rather than in graphs, so two tables are presented for your information below. These are both handy summary tables, which will give you a different view of the Equation of Time, and may help you to remember some key features, for example, that between the end of March and mid-September the sun is never more than 6 minutes away from "clock time", and for the whole of February it is 13 or 14 minutes slow! If you want to know the Equation of Time for every day of the year, there is a table in Appendix A of the book by Waugh.

Table showing the dates when "Sun Time" is (nearly) exactly a given number of minutes fast or slow on "Clock Time"

Minutes Fast		
16	Nov 11	Oct 27
15	Nov 17	Oct 20
14	Nov 22	Oct 15
13	Nov 25	Oct 11
12	Nov 28	Oct 7
11	Dec 1	Oct 4
10	Dec 4	Oct 1
9	Dec 6	Sep 28

8	Dec 9			Sep 25
7	Dec 11			Sep 22
6	Dec 13			Sep 19
5	Dec 15			Sep 16
4	Dec 17			Sep 13
3	Dec 19	May 4	May 27	Sep 11
2	Dec 21	Apr 25	Jun 4	Sep 8
1	Dec 23	Apr 21	Jun 9	Sep 5
0	Dec 25	Apr 15	Jun 14	Sep 2
1	Dec 28	Apr 12	Jun 19	Aug 29
2	Dec 30	Apr 8	Jun 23	Aug 26
3	Jan 1	Apr 5	Jun 29	Aug 22
4	Jan 3	Apr 1	Jul 4	Aug 18
5	Jan 5	Mar 29	Jul 9	Aug 12
6	Jan 7	Mar 26	Jul 18	Aug 4
7	Jan 9	Mar 22		
8	Jan 12	Mar 19		
9	Jan 15	Mar 16		
10	Jan 18	Mar 12		
11	Jan 21	Mar 8		
12	Jan 24	Mar 4		
13	Jan 29	Feb 27		
14	Feb 5	Feb 19		

Table showing the Equation of Time on the 5th, 15th and 25th of each month, together with the average daily change in seconds (given in minutes and second, + = "Sun time" is fast on "clock time"

Eq.of time on the:	5th	15th	25th	Av. change (secs)
January	-5m03	-9m10	-12m12	20
February	-14m01	-14m16	-13m18	5
March	-11m45	-9m13	-6m16	16
April	-2m57	+0m14	+1m56	18

May	+3m18	+3m44	+3m16	4
June	+1m46	-0m10	-2m20	16
July	-4m19	-5m46	-6m24	20
August	-5m59	-4m33	-2m14	11
September	+1m05	+4m32	+8m04	20
October	+11m20	+14m01	+15m47	13
November	+16m22	+15m28	+13m11	10
December	+9m38	+5m09	+0m13	27

The equation of time

Written by Art Carlson . October, 1995 .

The rotation of the Earth makes a good clock because it is, for all practical purposes, constant. Of course, scientists are not practical and care about the fact that the length of the day increases by one second every 40 000yrs. For the rest of us, it's just a matter of finding a convenient way to determine which way the Earth is pointing. Stars would be good, but they are too dim (and too many) at night and go away during the day. A useful aid is the Sun, which is out and about when we are and hard to overlook. Unfortunately, the apparent position of the sun is determined not just by the rotation of the Earth about its axis, but also by the revolution of the Earth around the Sun. I would like to explain exactly how this complication works, and what you can do about it.

The diameter of the Sun as seen from the Earth is 1/2 degree, so it moves by its own radius every minute.

```
 24hrs   60min   1
------ x ----- x -deg = 1min
360deg     1hr   4
```

That means it will be hard to read a sundial to better than the nearest minute, but then, we don't bother to set our clocks much more accurately than that either. Unfortunately, if we define the second to be constant (say, the fraction 1/31 556 925.974 7 of the year 1900, the "ephemeris second"), then we find that some days (from high noon to high noon) have more than 86,400 seconds, and some have less. The solar Christmas day, for example, is 86,430 seconds long. The discrepancy between "apparent time" and "mean time" can add up to +/- 15min. How does it come about?

The inclination of the ecliptic

First note that the Earth rotates on its axis not once in 24hrs but once in 23hrs 56min 4sec. It's just that in the course of a 365dy year, the Earth must turn an extra time to make up for its orbit about the sun.

```
1dy   24hrs   60min
--- x ----- x ----- = 3min 56sec
366    1dy     1hr
```

The trouble comes in because this 3min 56sec is only an average value. Think of an observer sitting at the north pole on a platform which rotates once every 23hrs 56min 4sec. She will see the stars as stationary and the sun as moving in a circle. The plane of this circle is called the "ecliptic" and is tilted by 23.45deg relative to the equatorial plane. The observer will see the sun move from the horizon, up to 23.45deg, then back down to the horizon. The sun will move at a constant speed (I'm lying, but wait till later) along its circle, but the shadow cast by the North Pole (the one with the red and white candy stripes) will *not* move at a constant rate. When the sun is near the horizon, it must climb at a 23.45deg angle, so that it has to move 1.09deg before the shadow moves 1deg.

```
    1deg
------------- = 1.0900deg
cos(23.45deg)
```

On the other hand, in the middle of summer, the sun is high in the sky taking a short cut, so it must move only 1deg along its circle to cause the shadow to move 1.09deg. This effect generalizes to more temperate climates, so that in spring and fall the 3min 56sec is reduced by the factor 1.09 to 3min 37sec, whereas in summer and winter it is correspondingly increased to 4min 17sec. Thus a sundial can gain or lose up to 20sec/dy due to the inclination of the ecliptic, depending on the time of year. If it is accurate on one day, six weeks later it will have accumulated the maximum error of 10min.

```
20sec            2    1min
----- x 45dys x -- x ----- = 10min
 1dy            pi   60sec
```

The seasonal correction is known as the "equation of time" and must obviously be taken into account if we want our sundial to be exact to the minute.

If the gnomon (the shadow casting object) is not an edge but a point (e.g., a hole in a plate), the shadow (or spot of light) will trace out a curve during the course of a day. If the shadow is cast on a plane surface, this curve will (usually) be a hyperbola, since the circle of the sun's motion together with the gnomon point define a cone, and a plane intersects a cone in a conic section (hyperbola, parabola, ellipse, or circle). At the spring and fall equinox, the cone degenerates to a plane and the hyperbola to a line. With a different hyperbola for each day, hour marks can be put on each hyperbola which include any necessary corrections. Unfortunately, each hyperbola corresponds to two different days, one in the first half and one in the second half of the year, and these two days will require different corrections. A convenient compromise is to draw the line for the "mean time" and add a curve showing the exact position of the shadow points at noon during the course of the year. This curve will take the form of a figure eight and is known as an "analemma". By comparing the analemma to the mean noon line, the amount of correction to be applied generally on that day can be determined. At the equinox, we found that the solar day is closer to the sidereal day than average, that is, it is shorter, so the sundial is running fast. That means in fall and spring the correct time will be earlier than the shadow indicates, by an amount given by the curve. In summer and winter the correct time will be later than indicated.

The eccentricity of the Earth's orbit

If you look at such a figure eight calculated correctly, you will see that the fall and winter loop is actually somewhat larger than the spring and summer loop. This is due to the lie I told above. The Earth does not actually orbit at a constant speed around the sun. On January 2, the Earth is 1.7% closer to the Sun than average and thus the angular velocity is 3.4% larger (conservation of angular momentum). This make the solar day longer than the sidereal day by about 8sec more than average,

```
3min 56sec
---------- x 0.034 = 8.0sec/dy
   1dy
```

and in the course of 3 months a sundial accumulates an error of 8min due to the eccentricity of the Earth's orbit.

```
8.0sec             2    1min
------ x 91dys x -- x ----- = 8min
  1dy             pi   60sec
```

Thus the correct time will be later than the shadow indicates at the spring equinox and earlier at the fall equinox. This shifts the dates at which the sundial is exactly right from the equinoxes into the summer, making the summer loop of the figure eight smaller.

The 20sec/dy error due to the inclination of the ecliptic and the 8sec/dy error due to the eccentricity work in the same direction around Christmas time and add up exactly (well, almost) to the 30sec/dy mentioned earlier. The accumulated errors of 10min and 8min due to these two effects don't add up quite so neatly, so the maximum accumulated error turns out to be somewhat less than 18min. If you calculate everything correctly, you find that during the course of a year a sundial will be up to 16min 23sec fast (on November 3) and up to 14min 20sec slow (on February 12).

Suppose in October you start a 15min coffee break at 10:45 by the wall clock. If you believe the sundial outside, without accounting for the equation of time. you will already be late for the 11:00 session as soon as you step out the door.

Other pages on the Internet which are concerned with the Equation of Time are from the Royal Greenwich Observatory
http://www.ast.cam.ac.uk/pubinfo/leaflets/equation/equation.html,
and from a number of individual pages, including
http://ourworld.compuserve.com/homepages/patrick_powers/sundials.htm , from
http://cpcug.org/user/jaubert/sundial.html (this link now changed or superseded)
http://www.ipp.mpg.de/~awc/sundial.html

We would appreciate an E-mail from you if you know of any others. Thank you