THE COMPUTATION OF HIGH FREQUENCY SCATTERING

This is the Final Report on Contract F19628-73-C-0126, the main purpose of which was the development of computationally effective and efficient techniques for predicting high frequency scattering. A method is described for combining physical optics with first order diffraction theory and the effectiveness of the resulting program is demonstrated using data for a cone, cone-cylinder and cone-cylinder-flare. Other analytical and experimental investigations were also performed aimed at increasing the accuracy of the underlying theoretical model and some of the possible improvements and extensions of the computer program are indicated.
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I
INTRODUCTION

This is the Final Report on Contract F19628-73-C-0126 covering the period 1 January to 30 June 1973. The main purpose of the contract was to develop numerical techniques which are effective for the calculation of electromagnetic scattering by objects large compared to wavelength. It was required that the techniques should be capable of providing both the amplitude and phase of the far zone field scattered by a class of rotationally symmetric "edged" bodies over a wide range of aspect angles, and should be illustrated by explicit application to a right circular cone and a cone-cylinder combination.

These objectives have been substantially achieved and the procedures that were developed are described in the second interim report (Knott and Senior, 1973b) issued under the contract. But before proceeding to a presentation of this, and other, work we have performed, a few words of background are desirable to indicate why we chose to follow the approach that we did.

For purposes of routine scattering calculations, the physical optics method continues to be the most widely used. This is primarily because of its simplicity and ease of computation: the scattered field is approximated by a surface integral that is well suited to digital evaluation, and in many instances the approximation is sufficient to reproduce the dominant features of the actual scattering patterns. Nevertheless, the method is deficient in several respects, most strikingly as regards the polarization predictions and in those aspect ranges where the scattering is not dominated by specular contributions. Unfortunately, the absence of speculars is typical of the patterns of many aerospace vehicles at aspects of most interest.

Many of these deficiencies can be overcome using the geometrical theory of diffraction (GTD). This is a ray method which differs from geometrical optics in its inclusion of diffracted rays, but in spite of its demonstrated effectiveness particularly for "edged" bodies, it is not as widely used in routine scattering predictions as its capability would suggest. Two reasons for this are its computational inconvenience and the necessity for correcting the predicted scattering in the vicinity of the caustics, difficulties which are common to all ray techniques.
An approach which circumvents these difficulties while still retaining the proven effectiveness of GTD was developed under the predecessor contract (F19628-68-C-0071) and is based on the concept of equivalent filamentary currents existing at any line element of a surface singularity. The corresponding diffracted field is then given by a line integral along the surface singularity, and by demanding that this integral, when evaluated by the stationary phase method, yield the precise first order GTD expression for the wide angle far zone field for all angles of incidence and diffraction, the currents can be determined. The analysis is presented in Knott and Senior (1973d) and applied to a right circular cone. In the special case of bistatic scattering from a disk in the plane of incidence, it is shown that the integral yields a caustically corrected expression analogous to that originally derived by Ufimtsev (1957).

The concept of equivalent currents provides a simple means for combining caustically-corrected GTD estimates of diffraction from line elements of surface singularities with the physical optics predications of scattering from extended portions of the surface, and the whole can be done within the context of a straightforward computer program for the evaluation of the explicit integrals involved. Such a program has been developed for rotationally symmetric bodies having wedge-like singularities. Because the physical optics approximation itself provides contributions from singularities (which are the end points of the ranges of integration), the equivalent currents must be modified to avoid the inclusion of these erroneous returns. The modification is rather trivial and is described in Chapter II where the entire analytical and computational approach is summarized. More details can be found in Knott and Senior (1973b). The program capability is illustrated using the examples of a 15-degree half-angle right circular cone and a cone-cylinder combination, and the principal plane backscattering patterns for a 90-degree range of aspects are compared with experimental data.

Although the computer program is an effective tool as it stands at the moment, there are certain developments and extensions which are desirable if we
are to realize the full potential of our approach. In some cases the predicted
data are less accurate than we desire because of a shortcoming in the GTD method
itself, or in the analytical foundations of the approach. Further theoretical work
is then required and must be carried out first. An example is the transitional
effect which occurs when a ray passes close to an extended portion of the surface,
and some of the studies which we have made of this effect are documented in
Chapter III. In other cases, the inaccuracies are attributable to a limitation of the
present program and require no further theoretical work for their removal. Shadow-
ing is an example. At the moment the program includes only "self" shadowing and
the necessity of a more complete consideration is evidenced by the comparison of
computed and measured data for a cone-cylinder-flare presented in Chapter III.
Other possible and desirable extensions of the program are also discussed.
II

THE COMPUTER PROGRAM

The approach taken to carry out the contract objectives was to combine the equivalent edge currents developed by Knott and Senior (1973d) with physical optics currents on surfaces. The equivalent currents represent a significant improvement in the ease of applying the geometrical theory of diffraction, but because of the combined formulation, it is necessary to modify them. The modification was prompted by the knowledge that the equivalent currents implicitly include physical optics contributions so that, at least in any generalized procedure, the physical optics returns would be tallied twice. Thus, in order to reserve the equivalent currents for modeling the effects of edges alone, and physical optics for surfaces alone, we sought to remove the additional components.

The modification can be worked out by extending Ufimtsev's results (1957) and we showed how to do so in the second interim report (Knott and Senior, 1973b). It consists of defining a pair of physical optics "diffraction coefficients" which are then subtracted from the usual diffraction coefficients appearing in the equivalent current expressions. The modified equivalent currents are thus represented as the difference between a pair of terms, and this fact is enlightening as well as useful: both terms are singular along certain caustic directions, but the singularities cancel each other. The result is that the net scattered fields are finite everywhere, in contrast to the caustics predicted by the use of GTD alone.

The formulation is an improvement over physical optics and/or GTD because, in addition to its improved accuracy, it accommodates bistatic scattering, arbitrary profiles, arbitrary incidence and mixed polarizations. In the present instance, however, backscattering is of far more interest than bistatic and it is sufficient to compute only the principal plane patterns (electric vector parallel and perpendicular to the plane of incidence). Furthermore, an arbitrary profile can be well approximated by a collection of straight line and circular arc segments. This means that with but a small sacrifice in generality, many cases of practical interest may be studied by computing only the principal plane patterns of metallic
bodies of roll symmetry whose profiles are describable in terms of straight or circular line segments. Such bodies include cones, cylinders, disks, frusta, ogives, spheres, and combinations thereof.

The numerical procedure we devised sums the modified equivalent currents along all edges and the physical optics currents over all surfaces. Since the body is taken to be one of revolution, all edges are circumferential and the summation is carried out over a body-fixed azimuthal coordinate, $\phi$. This coordinate is also used, in addition to an axial coordinate $z$, to sum the physical optics surface currents. The summation along the $z$ coordinate is carried out numerically over curved surfaces, but is accomplished analytically over straight line segments of the profile. Surface elements and edge elements which are turned away from the direction of incidence are not included in the summations and thus the effects of self shadowing are automatically accounted for.

The computer program we developed creates the body profile from input data supplied by the user. The profile is represented as a finite collection of abutting line segments and if there are $j$ segments, $j+1$ endpoints are required to specify the profile. (Since an edge is formed by the junction of a pair of adjacent segments, there will be $j-1$ edges whose contributions must be computed.) The program assembles the profile on the basis of the endpoint coordinates and, if a segment is specified by the user to be a curved one, it breaks the segment into $M$ cells (also specified by the user) and generates the coordinates of the midpoint of each cell. The final endpoint of the profile of the body must lie on the axis.

The flow of computations is roughly as follows: after reading the input data the program creates the body profile, assigns names (quite literally) to all segments, sets the angle of incidence to the initial value specified and sets the body angle $\phi$ to 2 degrees. The physical optics contributions of all illuminated curved surface elements centered on this body angle are tallied at once. The program then indexes through all straight line segments and adds their contributions to running sums established for each class of scatterer, including the returns from edges which are based on the modified equivalent currents. After all contributions have
been accounted for, with shaded segments and elements being ignored, the angle \( \phi \) is advanced 4 degrees and the process is repeated until the entire circle has been swept out. The returns are converted to decibels and printed out, and the running sums are cleared to zero in preparation for the next aspect angle.

The number of aspect angles used in the computations is controlled by the input specification of the initial and final angles, along with the increment desired for indexing through the range. If fewer than 53 angles are required, the total output of the program will occupy no more than two printed pages for a single obstacle. The results are packed on the left side of the sheet so that the cumbersome computer sheets may be trimmed down to conventional size for filing and storage. The first page of the output summarizes the salient obstacle parameters, including the names and locations of segments, the axial locations of the edges, the frequency of the incident wave and other such information. Since many individuals often misplace the input cards used to run their programs, the input data are echoed at the top of the first page for reference.

The second page lists the computed scattering cross sections in decibels above a square wavelength. In the interest of presenting as much scattering information as is concisely possible, the phase of the various contributors is not listed on output, but such data could easily be displayed, since the scattering data are stored internally as complex numbers. Only principal plane cross sections are computed but the output list includes the individual returns of all of the classes of scatterer making up the complete obstacle. Four columns of data are devoted to the physical optics returns from disks, cylinders, frusta and curved portions ("ogives"), two columns list the E- and H-polarization echoes from edges and two columns are reserved for the total cross section. Thus the output list is helpful in analyzing the importance of the various contributions to the net backscattering pattern.

The radar cross section predictions of the computer program were compared with measurements of several obstacles as a means of testing. Comparisons were presented in our second interim report for two right circular cones and a sphere-capped frustum, of which we repeat in Figures 1 through 4 the data for the
FIG. 1: COMPARISON OF PHYSICAL OPTICS (●) WITH THE MEASURED PATTERN (—) OF A 15-DEGREE HALF-ANGLE CONE AT 9.6 GHz FOR H-POLARIZATION.
FIG. 2: COMPARISON OF PHYSICAL OPTICS (•) WITH THE MEASURED PATTERN (——) OF A 15-DEGREE HALF-ANGLE CONE AT 9.6 GHz FOR E-POLARIZATION.
FIG. 3: COMPARISON OF COMPUTED VALUES (●), INCLUDING PHYSICAL OPTICS AND EDGE CONTRIBUTIONS, WITH THE MEASURED PATTERN (—) OF A 15-DEGREE HALF-ANGLE CONE AT 9.6 GHz FOR H-POLARIZATION.
FIG. 4: COMPARISON OF COMPUTED VALUES (○), INCLUDING PHYSICAL OPTICS AND EDGE CONTRIBUTIONS, WITH THE MEASURED PATTERN (-----) OF A 15-DEGREE HALF-ANGLE CONE AT 9.6 GHz FOR E-POLARIZATION.
15-degree half angle cone previously given. Since the program does not include
the second order diffraction known to occur across the base of the cone, an absorber
pad was cemented to the base during the measurements in order to decrease the
additional contribution. The electrical size of the cone is \( \text{ka} = 10.06 \).

The shortcoming of the physical optics approximation alone is demonstrated
in Figures 1 and 2, for H- and E-polarization respectively. The solid datum points
represent the physical optics values obtained from the program and the solid traces
are the measured patterns. Since physical optics is independent of polarization, the
computed data in both figures are the same. It is evident that physical optics is a
good approximation only within 15 degrees or so of the specular flash and becomes
progressively poorer as the aspect angle swings closer to the body axis. The mea-
sured data are much larger than the physical optics predictions for H-polarization
(Fig. 1) and much smaller for E-polarization (Fig. 2) in the aspect angle range 20
to 60 degrees. In both cases physical optics underestimates the strength of the
axial lobe by 12 dB. Thus physical optics fails by a wide margin to provide even
the gross characteristics of the true scattering pattern out to 60 degrees in aspect
angle.

When edge returns are included in addition to the "standard" physical optics
terms by means of the modified equivalent edge currents, the predicted results are
markedly improved, as may be seen in Figs. 3 and 4 for H- and E-polarization,
respectively. The edge contributions raise the axial lobe to within 2 dB of the
measured value and in the intermediate aspect angle range they produce the true
pattern characteristic that physical optics alone cannot. However, there is now a
differential error in the axial region for H-polarization (Fig. 3) with the computed
main and first side lobes being higher and lower, respectively, than the measured
values. This is typical of GTD estimates for narrow cones whose electrical dimen-
sions are not large and can be attributed to the effect that the generators of the cone
have on the field reaching, and being diffracted from, the base. This "transitional"
effect is examined in the next Chapter.

Comparisons of measured and computed patterns for a cone-cylinder are
shown in Figures 5 and 6. The half angle of the cone was 15.1 degrees and the
FIG. 5: COMPARISON OF COMPUTED VALUES (●), INCLUDING PHYSICAL OPTICS AND EDGE CONTRIBUTIONS, WITH THE MEASURED PATTERN (—) OF A CONE-CYLINDER AT 9.0 GHz FOR H-POLARIZATION.
FIG. 6: COMPARISON OF COMPUTED VALUES (●), INCLUDING PHYSICAL OPTICS AND EDGE CONTRIBUTIONS, WITH THE MEASURED PATTERN (—) OF A CONE-CYLINDER AT 9.0 GHz FOR E-POLARIZATION.
length of the cylinder was 3,066 wavelengths. The cone and cylinder had common base diameters $ka = 5.76$, considerably smaller than that of the 15-degree cone discussed above. The comparison is quite good for $H$-polarization (Fig. 5), even to the extent that the shoulders in the lobes at 50 and 75 degree aspects are predicted. (Near broadside incidence, the shoulder is produced by interference between the sidelobes of the specular flashes from the cone and cylinder.) The agreement for $E$-polarization (Fig. 6) is not quite as good, possibly because of measurement errors. Note, for example, that the measured nose-on values differ by 1.5 dB for the two polarizations.

It is interesting that in the nose-on region the computed radar cross sections also differ for the two polarizations, although the error is less than 0.5 dB. This is characteristic of computations made for scatterers containing cylindrical sections. In the nose-on region the trailing edge of the cylinder is illuminated at very nearly grazing incidence, and the shadow boundary changes quite rapidly, whereas the leading edge of the cylinder (where it joins the cone) remains fully illuminated. The net result is a slight broadening of the pattern which, depending on the strengths of other scatterers on the body, may or may not be noticeable. To rectify this defect, the illumination of the trailing end of the cylinder should be modified as described in Chapter III.
III
FURTHER STUDIES

In addition to the development and construction of the particular computer program described, a number of other investigations were carried out under the contract aimed at improving our ability to calculate high frequency scattering behavior. Some of these were brought to fruition, whilst others were only partially completed and will not be discussed here; some were of a theoretical nature aimed, for example, at improving the capability of the GTD method, while others were specifically concerned with increasing the scope and accuracy of the computer program, but even in the former case it was our hope that the theoretical advances would ultimately be reflected in a "better" version of the program.

One of the more successful theoretical investigations was prompted by the amplitude discrepancies between the computed and measured data for a narrow angle cone in the near-axial region. These discrepancies were noted by Senior and Uslenghi (1973) when comparing the predictions of second order GTD with measured data. For a 15-degree half angle cone with $ka$ near 10, the difference between the axial cross sections is about 2 dB. Covering the back of the experimental model to reduce the cross base interaction removed most of the frequency variation, but since the measured data now exceeded the first order GTD prediction by about the same amount, it appeared that at least a portion of the error lay in the GTD assumption of the field strength reaching the diffracting singularity at the rim of the base.

To explore these and other effects associated with right circular cones, a series of detailed measurements was performed for cones of half angles 15 and 40 degrees in the $ka$ range 8 to 20. The resulting patterns were published as the first interim report (Knott and Senior, 1973a) under this contract. The discrepancy between theory and experiment is illustrated in Fig. 7 where the measured axial cross sections for the cones are compared with the second order GTD predictions. For the narrow cone in the range $ka = 8$ to 12 the measured data exceed the theoretical values, but for the larger cone with $12 < ka < 20$ the reverse is true. Rather
FIG. 7: COMPARISON OF MEASURED (*) AXIAL BACKSCATTERING CROSS SECTION OF 15- AND 40-DEGREE HALF-ANGLE CONES WITH THE SECOND-ORDER GTD PREDICTIONS (——) WITHOUT MODIFICATION.
interestingly, for $\gamma = 15$ degrees the discrepancies are even larger for $ka$ near 20, but are smaller for $ka$ near 12. This is the opposite of what would be expected of a truly asymptotic theory.

The fact that the field on the surface of a cone can differ significantly from its physical (or geometrical) optics value is apparent from the exact calculations by Senior and Wilcox (1967) for a semi-infinite cone at axial incidence. It is there noted that at distances of more than a fraction of a wavelength from the tip, the field behavior is similar to that on a wedge of the same included angle, and this is also evident from asymptotic analyses (see, for example, Bowman, et al., 1969, p. 647 et seq.) of the fields close to the surfaces of hard and soft cones. For a metallic wedge illuminated by an E- or H-polarized plane wave, exact expressions for the total field are, of course, available (Bowman et al., 1969, pp 256-262), and if attention is confined to the "transition region" close to the reflected wave boundary and, hence, close to the surface of a narrow angle cone, the effective incident field strength exceeds the true value by a factor

$$f = \frac{1}{2} e^{\pm i k p} \hat{p} \left\{ 1 + \frac{2e^{-i \pi/4}}{\sqrt{\pi}} F \left( \sqrt{2kp \sin \frac{\psi}{2}} \right) \right\}$$

regardless of polarization, provided the direct edge diffraction is ignored. The quantities in eq. (1) are shown in Fig. 8 and $F(x)$ is the finite Fresnel integral

$$F(x) = \int_{0}^{x} e^{i \mu^2} d\mu .$$

By reciprocity, the field diffracted by a surface singularity on the cone will be affected in the same manner, and the incorporation of the factors $f$ into the GTD prescription leads to a marked improvement in the agreement between measurement and theory for a right circular cone (Knott and Senior, 1973c). This is true near the axis as well as on-axis and, were these factors to be included in the computer program, it is expected that many of the discrepancies noted in Chapter II for both the cone-cylinder and the cone would be removed.
The same factor (1) is also effective in removing the discontinuity predicted by GTD when a flash point becomes shadowed. Since its magnitude decreases rapidly with increasing negative $\psi$ from the value $1/2$ for grazing incidence ($\psi = 0$), the factor is consistent with the continuous but rapid change in the illumination that is expected on physical grounds; nevertheless, for $\psi < 0$ there is no theoretical justification for the application of eq. (1) to a cone.

In the computer program discussed in Chapter II, only that portion of a line singularity which is directly illuminated is excited and over the remaining portion of the singularity the excitation is assumed zero. Although the resulting diffracted field is continuous as a function of angle by virtue of the integration inherent in the equivalent current approach, the discontinuity in the currents does generate an erroneous contribution which is evident, for example, in the backscattering from a right circular cone at aspects just outside the backward cone, i.e. $\phi > \gamma$. The inclusion of the factor $f$ even for $\psi < 0$ does improve matters somewhat, but a close examination of measured and computed patterns still shows some discrepancies in the lobing structure when $\phi$ exceeds $\gamma$ by a few degrees.

To pinpoint the actual contributors to the scattering in this aspect range, the measured data for the 15-degree half-angle cone (Knott and Senior, 1973a) have been analyzed for $15^o < \phi < 25^o$, both as a function of $\phi$ for fixed $ka$, and as a function of $ka$
for fixed $\phi$. Although the study has not yet been completed, it has indicated which ray paths are important at these aspects and shown the necessity for including tip diffraction in any valid treatment of the scattering.

Turning now to the computer program, one of the desirable extensions is in the treatment of shadowing. Since geometrical and physical optics both prescribe that when a surface element is shielded from the incident field it does not contribute to the scattering, the program must decide which elements are shadowed. There are two kinds of shadowing: self shadowing, which depends only on the orientation of a given element with respect to the incident field, and mutual shadowing, which involves the relative orientation of pairs of elements.

Self shadowing is easy to visualize if we assume that the scatterer is formed by a closed, singly connected perfectly conducting shell. Every element of the shell then has inner and outer surfaces. The inner surface is always shielded from the incident field (whether the scatterer is convex or not) and the outer surface is shielded if $\hat{n} \cdot \hat{i} > 0$, where $\hat{n}$ is the outward normal of the element and $\hat{i}$ is the direction of propagation of the incident field. This provides a very simple basis for deciding whether a surface element of the obstacle shadows itself.

Mutual shadowing can occur if a pair of surface elements is aligned along an incident ray such that one blocks the incident ray from reaching the other. In this case the shadowing decision is also a simple one to make, requiring only that $(\vec{r}_2 - \vec{r}_1) \cdot \hat{i} > 1 - \delta$, where $\vec{r}_2$ and $\vec{r}_1$ are the position vectors of the two elements and $\delta$ is a small number associated with the physical size of an element. However, since all possible combinations of element pairs must be examined, the decision-making process may take a great deal of machine time. Although the total number of tests to be made can be reduced somewhat on the basis of other facts—for example, self-shadowed elements do not participate in mutual shadowing—schemes should be found to speed up the process. Mutual shadowing does not occur for convex obstacles, and since our present program does not consider it, the program naturally works best for bodies of this type.
Nevertheless, it is important that mutual shadowing be included if the program is to have the generality we desire, and the effects produced by its omission can be illustrated using the example of the flared shape shown in Fig. 9. Measured data for H- and E-polarizations at a frequency of 11.495 GHz, corresponding to \( ka = 12.208 \), where \( a \) is the maximum body diameter, are compared with the values predicted by the present program in Figs. 10 and 11, respectively. The considerable discrepancies, particularly for E-polarization, in the aspect range \( 10^\circ < \phi < 30^\circ \) are mainly due to the erroneous retention of contributions from the remote portion of the flare, and would be substantially reduced if mutual shadowing were taken into account.

Another desirable extension of the program would be the inclusion of surface singularities which are other than wedge-like, e.g., discontinuities in curvature. Since both the exact and physical optics expressions for the diffraction coefficients associated with this form of singularity are available (Senior, 1972) and since the "modified" coefficients appear explicitly in the integral expressions for the diffracted fields, it would be quite easy to generalize the program in this manner. A more realistic treatment of certain types of capped bodies would then be possible.

In any simple application of the physical optics method, one of the main sources of error in the predicted scattering is the contribution from the abrupt discontinuity in the postulated surface field at the shadow boundary. This is evident in the physical optics estimate for the backscattering cross section of a sphere as a function of frequency, and if \( ka \) is not so large that the shadow boundary contribution can be neglected in comparison with the specular return, the former beats with the latter to produce an interference which is quite distinct from that which exists in practice. At least in an average sense, the accuracy would be improved were the erroneous shadow boundary contribution neglected and this could be achieved by postulating a "cancelling" line source at the boundary. Such a concept of cancelling sources at the shadow boundaries could be incorporated in
FIG. 9: GEOMETRY OF THE FLARED SHAPE. ALL LINEAR DIMENSIONS ARE GIVEN IN INCHES.
FIG. 10: COMPARISON OF THE COMPUTED (○) AND MEASURED BACKSCATTERING OF THE FLARED SHAPE AT 11.495 GHz FOR H-POLARIZATION.
FIG. 11: COMPARISON OF THE COMPUTED (○) AND MEASURED BACKSCATTERING OF THE FLARED SHAPE AT 11.495 GHz FOR E-POLARIZATION.
in the present program and should lead to improved predictions of the scattering, particularly if the frequency were not high. The improvement should be evident, for example, in the scattering of a conical body at aspects which are just outside the backward cone.

A further development of this idea is now the inclusion of the correct shadow boundary contribution whenever this is known. A case in point is that of a cone-sphere (or other spherically-capped body) where the true shadow boundary effect is attributable to creeping waves on the sphere, and can be simulated using a ring source of appropriate strength at the shadow boundary. The procedure then would be to insert an equivalent source at the boundary that not only cancels the physical optics effect but also reproduces the theoretical creeping wave return. This, together with the inclusion of scattering from discontinuities in curvature, would certainly be necessary for predicting the scattering from cone-spheres, and amounts to the consideration of effects which are no longer entirely local in origin. In the same vein we could, of course, consider second order effects in general, but each development of this type would diminish the general purpose nature of the program, while improving the predictions for the specific class of body for which the extension is appropriate.
IV

CONCLUSIONS

The objective that we set ourselves at the outset of the contract was the development of a relatively general program for predicting high frequency scattering. Consistent with the accuracy desired, it was important that the program be as efficient as possible, and this was one of the reasons for restricting attention to rotationally symmetric bodies and assuming the body profile to be composed of straight line and/or circular arc segments alone. The results we have presented show that our objective has, in large measure, been achieved, and the program described in the second interim report (Knott and Senior, 1973b) should be of value to all who are concerned with the routine prediction of the scattering from a class of aerospace vehicles.

Nevertheless, there are certain extensions of the existing program that are necessary if the data are to have the accuracy desired over a wide range of circumstances, and which are compatible with its general purpose nature. The three extensions which we regard as most essential and, requiring no further theoretical work as such, would be relatively straightforward to implement, are:

(i) The incorporation of transition functions to account for the changing illumination of edges as shadowing is approached. This is particularly important at aspects close to axial and has already been included in restricted versions of the program.

(ii) The consideration of shadowing other than self-shadowing: this is necessary for an adequate treatment of bodies portions of which are concave and/or flared.

(iii) The admission of surface singularities which are more general than wedge-like, e.g., discontinuities in the curvature of the body profile instead of in its slope.

Although there are many other developments and extensions which would be possible, some of which were mentioned in Chapter III, the three which we have listed would not destroy the general purpose nature of the program and would not
markedly affect its efficiency. They could be implemented with a rather small expenditure of time (and money) and we would then be in possession of a program whose utility was comparable to that of the standard physical optics program, but whose accuracy was greatly superior. As such, it would be the natural tool for routine scattering calculations in the years to come.
REFERENCES


Knott, E. F. and T. B. A. Senior (1973a), "CW measurements of right circular cones", University of Michigan Radiation Laboratory Report No. 011758-1-T.


