NON-SPECULAR RADAR CROSS SECTION STUDY

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FOREWORD

This report documents work performed by the Radiation Laboratory of the University of Michigan, Ann Arbor, Michigan under USAF Contract F33615-72-C-1439 "Non-Specular Radar Cross Section Study." The work was sponsored by the Electromagnetic Division, Air Force Avionics Laboratory and the technical monitor was Dr. Charles H. Krueger, AFAL/WRP.

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This technical report has been reviewed and is approved for publication.
This Technical Report has been reviewed and is approved for publication.

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ABSTRACT

The significance of the research summarized herein is the development of techniques for reducing the radar cross sections of non-specular types of scatterers. For the specific purpose of optimizing camouflage treatments a computer program is used which digitally solves the surface field integral equations for an impedance boundary condition. The program, furnished by the Air Force Avionics Laboratory and later modified by the Radiation Laboratory to handle variable impedances, computes the currents induced on the surfaces of two-dimensional obstacles by an incident plane wave and sums the currents to obtain the far bistatic field. Results for two different bodies are presented showing that control of the surface impedance can produce substantial radar cross section reductions, but that the impedance must not be changed too rapidly. Specific results are included showing the effects of different loading locations and impedance variations on the surface and far bistatic fields for two selected shapes.
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I INTRODUCTION

The objective of the research reported herein is to investigate non-specular scattering and to develop techniques for suppressing it, with the expectation that absorbing materials will be the most likely means of doing so. To carry out the investigation, three simple scattering obstacles are used that embody the features of non-specular scattering to be treated. The primary tool used to accomplish the work is a digital computer program based on the integral equation solution of a two-dimensional obstacle over which a surface impedance boundary condition is imposed.

The suppression of scattering is a camouflage problem, but over the years the development of techniques for the reduction of specular contributions has received most attention. This is in part because specular echoes tend to dominate the net return wherever they are present, and though they tend to be restricted in the aspect angles for which they occur, they are amenable to suppression using radar absorbents. In contrast, non-specular scattering is more pervasive and even if all specular echoes are suppressed, we are still faced with this other form of scattering which can generate returns of an unacceptable magnitude and do so over a wide range of aspects. Indeed, modern aerospace vehicles are seldom seen at specular aspects in a tactical environment, yet their radar cross sections can attain levels that increase the probability of detection. It is therefore important to address the problem of reducing non-specular returns.

In Chapter II we discuss the types of non-specular scattering to be treated and describe a computer program used to explore effective surface loadings. The original program was somewhat mismatched to the task required of it — it assumed a constant impedance over the entire surface, for example, although this rarely occurs in nature — and we had to modify the program in order to accomplish our task. The final form of the program is listed in the Appendix.
along with a short description of its operation.

Because the computer program employs an impedance boundary condition, whereas we must ultimately deal with real material properties such as permittivity, permeability and conductivity, it is important to relate the surface impedance to the electrical characteristics of realistic substances. The translation from one to the other is not simple for non-trivial geometries and we explore in Chapter III the conditions under which an identification might be made. A laboratory experiment has been planned in which the ratio of measured E and H fields will be compared against the predictions of the formulae given in Chapter III.

Although the computer program was designed to handle either parallel or perpendicular polarizations (E- and H-polarizations, respectively), we noted anomalous results for E-polarization when the obstacle has sharp edges. The problem has not been fully resolved and for this reason we concentrate in this report on H-polarization only, using an ogival cylinder and a wedge-cylinder as test obstacles. These are, of course, two-dimensional analogs of the ogive (spindle) and the cone-sphere. Chapter IV focusses on the metallic versions of these bodies using high frequency techniques such as the geometrical theory of diffraction (GTD), mainly to establish quantitative checks on the results of the computer program. The results of the surface impedance variations studies are summarized in Chapter V and data are presented showing possible trade-offs in performance. We emphasize that this represents "optimization" in terms of surface coverage only, since we do not yet have in hand the characteristics of real materials (i.e., density, structural integrity, etc.) necessary to optimize on any other basis.
II NON-SPECULAR SCATTERING

2.1 The Nature of Non-Specular Scattering

The double-barrelled objective of this research is "to investigate non-specular scattering and to develop techniques for suppressing it", but we have already accomplished most of the first task under previous contracts. Non-specular scattering is literally any that is not specular and, as such, is invariably produced either by geometrical discontinuities or by discontinuities in the illumination of the surface. Edges and shadow boundaries are examples of each kind.

Non-specular scattering can be immediately grouped into two classes, one embracing waves that travel over or along surfaces and the other due to direct diffraction from discontinuities such as edges. Creeping waves and traveling waves share a common inclination to hug a smooth surface and are traditionally distinguished from each other according to the surface illumination: traveling waves occur in lit regions and creeping waves in geometrically shaded regions. By running across a lit surface, a traveling wave tends to feed on the incident wave, building up strength as it goes. A creeping wave, being consigned to the shadows, tends to expire, the total decay being greater for larger radii of surface curvature.

The simplest imaginable obstacle has at least two scattering "mechanism", one of which is a non-specular contribution, and only slightly more complex shapes have even more. The sphere exhibits the classic specular return plus creeping wave contributions; the ogive has non-specular scatterers localized at both tips; the cone-sphere displays no fewer than four contributions (some of which are admittedly small): tip, join, creeping wave and, depending on the angle of incidence, a possible specular echo. Nonetheless, by carefully choosing the geometrical characteristics of test obstacles, we can study ways of suppressing the non-specular scattering.
As will be shown in a moment, the tool with which we studied non-specular scattering suppression was a computer program designed for two-dimensional bodies (cylinders), and we therefore chose two-dimensional analogs of the above obstacles. The ogival cylinder has a pair of edges of which only one -- the front for E-polarization and the rear for H-polarization -- is a dominant non-specular scatterer. When incidence is not too far out of the plane of symmetry for H-polarization, the ogival cylinder supports a traveling wave which, when reflected from the far edge, is a substantial source of echo. For E-polarization the front edge diffracts the incident wave and this is the most difficult of non-specular scattering to suppress.

The circular cylinder is the simplest two-dimensional body supporting creeping waves, but since it has a specular contribution that tends to mask the creeping wave effect in the far field, it is an unsuitable model with which to study creeping waves. The specular component can be removed by mating a segment of the circular cylinder with a wedge and we then have the two-dimensional analog of the cone-sphere. This body has, in addition to the creeping wave, a join contribution and a diffractive component from the apex of the wedge. The edge reduces to a vertex if we pass from the two-dimensional to the three-dimensional case, but since the edge is a large contributor, whereas the vertex is not, we must bear in mind that a new source of scattering has been introduced into the problem.

These two shapes -- the two-dimensional counterparts of the ogive and the cone-sphere -- thus contain all three kinds of non-specular scattering mechanisms. The infinite strip was also studied (see Chapter IV) but only because there are more strip than ogival cylinder data in the literature against which to test the computer program. Another shape which has been considered is the prism (the analog of a right circular cone), but we have not progressed far enough to have generated any data for this body.
Having determined the kinds of non-specular scattering we must be concerned with, and the shapes of the obstacles they may be studied with, we must now address the second part of the contract objective: developing techniques to suppress such scattering. Previous experience has shown that the application of absorbent materials is the most likely approach and it has been demonstrated that any material of sufficient loss does a good job, at least in reducing creeping and traveling waves. But what has not been established is the optimum configuration or material properties, given the constraints of the optimization. These constraints could take the form of a minimum surface coverage that would produce, say, a 13-dB cross section reduction or, on the other hand, finding the reduction achievable if, say, only 10 percent of the surface were available for treatment. Other constraints might be specifiable in terms of maximum material thickness, maximum weight per square foot, and so on.

2.2 The Digital Approach

There are three conceivable ways to approach the optimization task before us: theoretical, experimental or digital. Naturally, each of these has its own virtues and shortcomings, and we have in fact embarked on a fourth approach using a meld of all three, although with a heavy reliance on the third method. The digital solution to electromagnetic scattering is the most attractive at present because of the speed with which accurate results may be obtained, although we must note that it is restricted to bodies of modest size. The purely theoretical approach also produces accurate results, but only for a restricted class of simple shapes such as spheres and circular cylinders, and even these must bear uniform coatings. Experimental measurements by their very nature directly address the practicality required of the physical world, but for our particular task they would be far too slow. Thus we have relied on the digital method for the bulk of the research, but have not abandoned theory and experiment.
There is, in fact, a strong connection between the results of a computer run and a laboratory experiment performed on the same body. One represents a numerical solution of the wave equation while the other is an analog solution, and if both are carried out carefully enough, the results will be indistinguishable. The digital solution can be had much quicker, however, with the added attraction that nonexistent materials can be simulated. But curiously enough, a single digital "experiment" yields little more information than a single laboratory experiment. In both cases the extraction of useful information usually requires varying one or more parameters, implying that several experiments must be run.

In this contract we have relied mainly on a computer program furnished by AFAL and later modified. The program is designed to compute the surface fields and far-zone scattering from cylinders of arbitrary cross section and assumes that an impedance boundary condition connects the tangential electric and magnetic fields on the cylinder surface. The program is based on the integral equations for the total surface field components and the cylinder profile is approximated by a finite collection of straight line segments. The accuracy of the program is determined by the size of each element (in wavelengths) and the number of sampling points available limits the size of cylinder that can be accommodated.

Early in the contract we were furnished two programs, RAM1A and TWOD, each of which treated two-dimensional scattering bodies. Program RAM1A utilized a constant impedance boundary condition for either H- or E-polarized incident plane waves while TWOD allowed the user to specify the permittivity of a layer of material covering a conducting cylinder for E-polarization. Neither program was quite suitable for our specific needs since the impedance had to be variable along the surface if we were to use RAM1A, or both the permittivity and permeability had to be specifiable for either polarization if we were to use TWOD.
We would have preferred to expand a program like TWOD, in which material properties could be specified, but it appeared at the time to be too great a task to make such a change. On the other hand, RAM1A could be quickly fixed merely by creating an impedance array so that each surface element could be assigned any desired impedance value.

RAM1A had two options for the specification of body profile: i) the program generated a right circular cylinder of specified size having a constant surface impedance, or ii) it read in the surface coordinates, point-by-point, of the profile specified by the user. Since we contemplated running the program for several shapes and a wide variety of loading schemes, it seemed advisable to devise more efficient ways of specifying both the profile and the impedance than by reading in the data point-by-point. We therefore expanded the program so that the impedance over a given portion of the surface could be generated internally with the specification of two or three constants of a mathematical expression. Similarly, relatively complicated profiles could be specified by inputting the end points of circular arcs and straight lines along with the angles they subtend. For the purpose of obtaining quick-look plots of results, a printer plot routine was also added to the program. The program was now sufficiently different from the original that we renamed it RAM1B.

We subsequently ran the modified program in an extensive study of the H-polarized returns from an ogival cylinder and a wedge-cylinder, producing over 115 sets of data whose results are summarized in Chapter V. We discovered that the maximum number of sampling points is limited by internal core storage to about 120 which, at the modest rate of 16 samples per wavelength, permits us to treat obstacles with perimeters up to about 7.5 wavelengths.

Our success with H-polarization did not carry over to E-polarization, however. Eventually we discovered a faulty statement in subroutine FIELD, but when this was corrected we still obtained anomalous results for structures
with sharp edges. For blunt shapes such as circular and elliptic cylinders, on the other hand, RAM1B performs admirably, but it appears that the opposite sides of a thin structure such as exist in the vicinity of a wedge can approach no closer than 0.05\(\lambda\) or 0.10\(\lambda\) of each other if the program is to produce accurate results. This in turn suggests that the fields in the vicinity of a sharp edge are not being accurately computed and that (perhaps) the edge should be blunted.

Although this can certainly be done, it represents a modification of the geometry that could possibly destroy the characteristics we are seeking to study. A preliminary test of edge rounding gave us quite acceptable surface current distributions except at the rearmost point of the body. As the radius of curvature becomes smaller, the front edge currents increase while the rear edge currents decrease. But when the rounding is carried to the limit, i.e., a true edge, the current distribution suddenly reverts to an unacceptable form. At the time of this writing, a redistribution of sampling points is being studied to determine whether a denser packing of the points in the vicinity of an edge can overcome the difficulty.
III SURFACE IMPEDANCE APPROACH

The computer program referred to above postulates a cylinder of arbitrary cross section at the surface of which an impedance boundary condition is imposed. By application of the moment method to the corresponding integral equations (see, for example, Andreasen, 1965), the surface and far bistatic fields produced by an incident plane electromagnetic wave are determined.

As a result of the extension carried out at the commencement of this study, the surface impedance can be a function of position on the profile \( \rho = \rho(\theta) \), where \( \rho, \theta, z \) are cylindrical polar coordinates with \( z \) axis parallel to the generators of the cylinder. Since our task is to use this approach to gather information about the scattering by finite, three-dimensional objects, our concern is limited to the case in which the direction of the incident wave is perpendicular to the \( z \) axis. The integral equations, which are coupled for oblique angles of incidence, now become

\[
H_z^1(s') = \frac{1}{2} K_s(s') + \frac{1}{4} \int_{C} \eta^{(1)}(s) K_s(s) H^{(1)}_0(kr) d(ks) + \frac{1}{4} \int_{C} K_s(s)(\hat{\mathbf{r}} \cdot \hat{n}) H^{(1)}_1(kr) d(ks)
\]

\[
Y \mathbf{E}_z^1(s') = \frac{1}{2} \eta^{(2)}(s') K_z(s') + \frac{1}{4} \int_{C} K_z(s) H^{(1)}_0(kr) d(ks) + \frac{1}{4} \int_{C} (s) K_z(s)(\hat{\mathbf{r}} \cdot \hat{n}) H^{(1)}_1(kr) d(ks)
\]

(Andreasen, 1965), where \( \mathbf{r} = \mathbf{r} - \mathbf{r}' \), \( s \) is the surface distance along the profile, \( \hat{n} \) is a unit outward normal to the cylinder, \( \eta^{(1),(2)} \) are scalar surface impedances such that at the surface
\[ E_s = \eta^{(1)} \frac{1}{Z_o} H_z, \quad E_z = -\eta^{(2)} \frac{1}{Z_o} H_s, \quad (3.3) \]

and \( K_s = H_z, \quad K_z = -H_s \) are the transverse and axial surface current densities. Thus, for incidence in a plane perpendicular to the z axis, the equations are decoupled, and a simpler and more efficient computer program could have been constructed if attention had been confined to this case at the outset. Moreover, it is now sufficient to take the incident field polarized with either its magnetic or electric vector in the z direction (H- or E-polarization, respectively).

An impedance boundary condition can be written in the form

\[ E - (\hat{n} \cdot E) \hat{n} = \eta_s Z_o \hat{n} \times H \quad (3.4) \]

where \( \hat{n} \) is a unit outward normal and \( E, H \) are the total fields at the surface. In this equation, \( \eta_s \) is the surface impedance relative to the impedance \( Z_o \) of free space and can be either a scalar or, as in eqs. (3.3), a dyadic. Such a boundary condition is often used as a means of simulating an imperfect surface and a variety of scattering problems have been analyzed on this basis. Two examples are diffraction by a half plane (Senior, 1952, 1959a) and by a wedge (Senior, 1959b; Maliuzhinets, 1958a,b, 1960) whose constant impedance may, in the case of Maliuzhinets' treatment, differ on the two faces. For these geometries the solutions can also be obtained if the surface impedance varies in either of two particular ways. The results of these analyses have been used to find high frequency approximations to the back scattering cross sections of absorbing strips (Bowman, 1967) and absorbing finite cones at axial incidence (Bowman, 1970).

Only in certain very special cases can the impedance be related to the geometry and material of the surface and, hence, interpreted in terms of these quantities alone. The simplest case of all is that of a plane wave in free space.
incident on a half space composed of a material whose permittivity and permeability are $\epsilon$ and $\mu$ respectively. If the electric vector is in the plane of incidence (H-polarization)

$$\eta_s = \left\{ 1 - \left( \frac{\mu}{\mu} \eta \sin \theta \right)^2 \right\}^{1/2} \eta ,$$  \hspace{1cm} (3.5)

but if the electric vector is perpendicular to this plane (E-polarization)

$$\eta_s = \left\{ 1 - \left( \frac{\mu}{\mu} \eta \sin \theta \right)^2 \right\}^{-1/2} \eta$$  \hspace{1cm} (3.6)

where $\theta$ is the angle which the incident field direction makes with the normal to the surface and

$$\eta = \sqrt{\frac{\mu \epsilon}{\mu \epsilon}}$$  \hspace{1cm} (3.7)

is the intrinsic or bulk impedance of the material. The surface impedance is independent of the polarization and, hence, a scalar if and only if

$$\left| \frac{\mu}{\mu} \eta \sin \theta \right|^2 \ll 1 .$$  \hspace{1cm} (3.8)

This also serves to suppress the dependence on the incident field direction and the surface impedance then reduces to the bulk impedance.

If the material is confined to a planar slab of thickness $d$ backed by a perfect conductor, the analogous expressions for $\eta_s$ are

$$\eta_s = \left\{ 1 - \left( \frac{\mu}{\mu} \eta \sin \theta \right)^2 \right\}^{1/2} \eta \tanh \left[ -ikd \left( 1 - \left( \frac{\mu}{\mu} \eta \sin \theta \right)^2 \right)^{1/2} \right]$$  \hspace{1cm} (3.9)

($\pm$ for H- and E-polarizations, respectively), where $k$ is the propagation constant
in the material. Under the condition (3.8), the dependence on both the polarization and direction of the incident field disappears, and the surface impedance becomes

$$\eta_s = \eta \tanh(-ikd) \quad .$$  

(3.10)

This reduces to the bulk impedance if

$$\text{Im. } (kd) \gg 1 \quad ,$$  

(3.11)

i.e. for sufficiently large attenuation in the layer. A multi-layered slab can be treated in a similar manner.

The more general applicability of an impedance boundary condition has been examined by a number of authors, but each analysis is based on a specific model and is predicated on certain simplifying assumptions about the field. The most common assumptions are that the surface is exposed to an incident plane wave and that the surface field depends only on the local geometry. Rytov (1940) and Leontovich (1948) have then shown that at the curved surface of a homogeneous isotropic medium, eq. (3.4) is applicable with \( \eta_s = \eta \) if, at each point,

$$|k R| \gg 1$$

where \( R \) is any radius of curvature. The need to supplement this restriction under certain circumstances has been pointed out by Senior (1960) who has also shown that the boundary condition is unaffected if the material properties vary slowly as a function of position. In this case the surface impedance varies in a like manner.

In the above examples we have been at pains to arrive at a surface impedance which is independent of the field characteristics and, where possible, of the geometry as well. In general, however, the surface impedance does depend on these quantities, and any realistic simulation of an actual physical problem
should take this into account. It is not hard to see how this dependence comes about. After all, the surface impedance is a function of the local surface field and this usually depends on the incident field characteristics, the global and local geometry and material, etc. By virtue of this dependence on the local field, the impedance cannot be specified at the outset of an analysis or computation whose objective is, among other things, to calculate the local surface field; and if the dependence is ignored, the impedance will no longer simulate the type of surface assumed, and may not even be compatible with any physically realisable material.

These facts can be illustrated by the problem of a plane wave incident on a metallic sphere of radius a clad with a uniform coating. Each of the spherical modes, electric and magnetic, into which the surface field can be decomposed has associated with it a specific surface impedance which is constant over the sphere. These modal impedances can be determined from an exact solution of the problem and differ markedly from one another. The net surface field is a sum over these modes whose individual weights are specified by the incident field, and though it in turn implies a (dyadic) surface impedance at each point, this impedance is a function of position and no longer a property of the local geometry alone. If the sphere is electrically large, it would seem reasonable that over the illuminated hemisphere the surface impedance could be estimated using the slab formulae (3.5) and (3.6), but it is by no means obvious that this will also apply over the shadowed half where the surface field is made up of creeping waves.

Similarly, if a cone is covered with a uniform coating, the surface impedance obtained from measured data is a function of distance from the tip. This is true even for a plane wave which is incident axially and therefore illuminates each element of the surface identically. The variation is particularly rapid within a wavelength or two from the tip and since no method has yet been found for predicting this impedance from the properties of the coating, we are
equally at sea in looking for a coating that provides a given surface impedance.

Although an impedance boundary condition is a very useful concept, its value lies more in its simulation of a generic class of imperfect surfaces than in the modeling of any single and specific departure from perfect conductivity. Its general relevance to all entails particular relevance to none, and to compute the effect of a given surface impedance provides very little understanding of how any actual coating will perform. This is certainly so in the situations of prime interest in this study where the surface field departs drastically from the simple form characteristic of a planar surface.

Having said all this, it must now be admitted that the consideration of bodies with specific surface impedances does represent a possible (if not the only) non-experimental approach to the design of absorbers for non-specular performance, and the computer program mentioned earlier is the only program which has the potential for the task. From its use so far, we have already obtained significant quantitative information about the effect of different impedance variations on the scattering produced by traveling, creeping and edge waves, and some of these results are discussed in Chapter V. But even the formulation of the problem in terms of an impedance boundary condition has some interesting consequences which can give considerable insight.

An alternative but entirely equivalent version of the boundary condition (3.4) is (Senior, 1962)

\[ \hat{H} - (\hat{n} \cdot \hat{H}) \hat{n} = -\frac{1}{\eta_s} Y_o \hat{n} \wedge E \]  

(3.12)

which can be obtained from (3.4) by the transformation \( \hat{H} \rightarrow Y_o \hat{E}, \ \hat{E} \rightarrow -Z_o \hat{H}, \ \eta_s \rightarrow \frac{1}{\eta_s} \). This transformation is a result of the duality of Maxwell's equations and is reflected in eqs. (3.1) and (3.2). As inspection shows, (3.2) follows from (3.1) on making the transformation
\[
\frac{H^1_z}{E^i_z} \rightarrow Y \frac{E^i_z}{Y^i_z}, \quad K \rightarrow \eta^{(2)} \eta^{(1)} K, \quad \eta^{(1)} \rightarrow \frac{1}{\eta^{(2)}},
\]
(3.13)

and hence, if eq. (3.1) is solved for a particular surface impedance \(\eta^{(1)}(s)\) to yield the bistatic scattered field \(H^S_z\), this also represents the bistatic scattered field \(Y \frac{E^S_z}{Y^i_z}\) for the analogous E-polarized incident field and for the surface impedance \(\eta^{(2)}(s) = \frac{1}{\eta^{(1)}(s)}\). Insofar as we are concerned only with surface impedances which are nowhere zero or infinite, we can therefore confine our attention to (say) eq. (3.1) and treat the case of E-polarization by solving eq. (3.1) for the reciprocal surface impedance.

A more important consequence of the duality is the information it provides about the effects of different impedances. This can be illustrated by considering some form of scattering contributor which is much greater for E-polarization than for H, e.g. the edge of a wedge for near-symmetrical illumination. We can attempt to reduce the H-polarized contribution by loading the surface, and small values of \(\eta^{(1)}\) may, indeed, reduce the scattering, but if \(\eta^{(1)}\) is increased further, the scattering must ultimately increase to approach that for E-polarization.

To use this information most effectively, it is necessary to know the types and magnitudes of the scattering contributions which any simple metallic body generates. This topic is taken up in the next Chapter.
IV SCATTERING BY METALLIC BODIES

The obstacles of prime interest in this contract are the ogival cylinder and the combination wedge-cylinder, which are two-dimensional analogs of the ogive and the cone-sphere. These cylindrical shapes support the same kinds of scattering mechanisms as do their three-dimensional cousins, and the ogival cylinder itself can be a good representation of an airfoil. Unfortunately, the cylindrical shapes have edges which do not exist in the three-dimensional case and the presence of these edges must always be borne in mind. Both the two and three-dimensional versions of the ogive support traveling waves near end-on regions of incidence, but the ogival cylinder has, in addition, edge diffraction components. The wedge-cylinder supports all three types of non-specular scattering, including creeping waves, but it requires a selection of polarization and incidence angles to bring them all into play simultaneously.

In this chapter we use the geometrical theory of edge diffraction to estimate the high frequency scattering of the two metallic cylindrical bodies. Such estimates have proved valuable as checks at various stages of our work with the program, and GTD is the most easily applied method for obtaining them.

In applying the theory, we assume that an E- or H-polarized plane wave impinges on the body at right angles to the cylindrical axis. The complex amplitude \( P(\phi', \phi) \) of the far zone bistatic field is to be determined, where \( \pi - \phi' \) and \( \pi - \phi \) are the angles made by the incident and scattered field direction with the longitudinal plane of symmetry of the body profile. The far zone field is then

\[
P(\phi', \phi) \sqrt{\frac{2}{\pi kR}} e^{i(kR - \pi/4)}
\]  

(4.1)
and the scattering cross section $\sigma$ per unit length is

$$\sigma(\phi_o, \phi) = \frac{4}{k} \left| P(\phi_o, \phi) \right|^2,$$

(4.2)

i.e.

$$\frac{\sigma}{\lambda} = \frac{2}{\pi} \left| P \right|^2.$$

(4.3)

4.1 Ogival Cylinder

We consider an ogival cylinder whose profile in the $\rho, \phi$ plane of the cylindrical coordinate system $\rho, \phi, z$ is an arc of a circle and the reflection of that arc in its chord (see Fig. 4-1). If the radius of the circle is $a$ and the

angle which each arc subtends at its center of curvature is $2\Omega$,

the arc length along upper or lower surface is $s = 2a\Omega$

the body length is $l = 2a \sin \Omega$

Fig. 4-1: Ogival geometry
and the maximum width is \( w = 2a(1 - \cos \Omega) \), so that the length-to-width ratio is \( \frac{L}{w} = \cot \Omega \).

To produce a slender shape favoring traveling wave contributions, we chose the apex half angle \( \Omega \) to be 12.5 degrees. Due to sampling limitations in the computer program that restrict the maximum size of the body, we selected a body length near 3\( \lambda \) so as to be within the program's capabilities and yet be large enough for high frequency approximations to be useful. For \( L = 3\lambda \) and \( \Omega = 121/2^\circ \),

\[
s = 3.024\lambda, \quad w = 0.3286\lambda, \quad \frac{L}{w} = 9.130.
\]

The cylinder is illuminated by an E- or H-polarized plane wave incident at an angle \( \pi - \phi_o \) to edge on and the scattering is observed at an angle \( \pi - \phi \).

If \( \phi_o \), \( \phi < \pi - \Omega \) so that the rear edge is directly illuminated and fully visible, and if \( \phi >> \pi - 2\Omega - \phi_o \) so that the direction of observation is well away from the direction of specular scattering off the upper surface, the scattering at high frequencies is mainly due to edge diffraction. Since this is a local phenomenon and the two arcs form wedges of total included angle 2\( \Omega \) at their points of intersection, we can estimate the scattering using the known wedge diffraction coefficients

\[
P^{E,H}_{\text{wedge}}(\alpha, \theta) = \frac{i}{2\nu} \sin \frac{\pi}{\nu} \left( \left( \cos \frac{\pi}{\nu} - \cos \frac{\alpha - \theta}{\nu} \right)^{-1} \mp \left( \cos \frac{\pi}{\nu} + \cos \frac{2\pi - \alpha - \theta}{\nu} \right)^{-1} \right)
\]

(Bowman et al., 1969; pp. 258, 263) valid in directions away from any optical boundaries, where \( \nu = 2(1 - \Omega/\pi) \) and the geometry is as shown in Fig. 4-2.

For the wedge modeling the front edge of the body, \( \alpha = \phi_o \), \( \theta = \phi \), and if the phase origin is taken at this point, the contribution of the front edge is

\[
P^{E,H}_{\text{front}} = \frac{i}{2\nu} \sin \frac{\pi}{\nu} \left( \left( \cos \frac{\pi}{\nu} - \cos \frac{\phi_o - \phi}{\nu} \right)^{-1} \mp \left( \cos \frac{\pi}{\nu} + \cos \frac{2\pi - \phi_o - \phi}{\nu} \right)^{-1} \right)
\]
In the particular case of backscattering with $\theta_o = 155^\circ$ and $\Omega = 12^{1/2}_{\circ}$ (implying $\nu = 67/36$),

\[
\begin{align*}
\mathbf{P}_\text{front}^E &= -i0.583, \quad \mathbf{P}_\text{front}^H = i0.105. \tag{4.6}
\end{align*}
\]

Since the ogival cylinders of interest to us are relatively thin, it is natural to ask if they can be approximated by a thin strip. This would enable us to use the simpler version of eq. (4.5) appropriate to a half plane for which $\nu = 2$, viz.

\[
\begin{align*}
\mathbf{P}_\text{front}^{E,H} &= -\frac{i}{4} \left\{ \sec \frac{\theta - \phi}{2} + \sec \frac{\theta + \phi}{2} \right\}. \tag{4.7}
\end{align*}
\]

For backscattering at the same angle of incidence, $\phi = 155^\circ$, as before, eq. (4.7) gives

\[
\begin{align*}
\mathbf{P}_\text{front}^E &= -i0.526, \quad \mathbf{P}_\text{front}^H = i0.026. \tag{4.8}
\end{align*}
\]

and comparison with (4.6) shows that the half plane yields a good approximation.
to the E-polarized scattering from the edge, but an inadequate approximation for H-polarization.

For the trailing edge the required identification in eq. (4.4) is $\alpha = \pi - \phi$, $\theta = \pi - \phi$, and on inserting the phase factor necessary to transfer the phase origin to the front edge of the cylinder, we have

$$P^{E,H}_{\text{rear}} = \frac{i}{2\nu} \sin \frac{\pi}{\nu} \left[ \left( \cos \frac{\pi}{\nu} - \cos \frac{\phi - \phi}{\nu} \right)^{-1} + \left( \cos \frac{\pi}{\nu} + \cos \frac{\phi + \phi}{\nu} \right)^{-1} \right] \cdot \exp \left\{ ik\ell \left( \cos \frac{\phi}{\nu} + \cos \phi \right) \right\}. \quad (4.9)$$

Again, for $\phi = 155^0$ and $\Omega = 12^0$, $\frac{1}{2}$,

$$\left| P^{E}_{\text{rear}} \right| = 0.006 \quad , \quad \left| P^{H}_{\text{rear}} \right| = 0.484$$

whereas for a half plane

$$\left| P^{E}_{\text{rear}} \right| = 0.026 \quad , \quad \left| P^{H}_{\text{rear}} \right| = 0.526 \quad .$$

The inadequacy of the half plane approximation is now more pronounced for E-polarization than for H.

The sum of (4.5) and (4.9) constitutes the first order approximation to the far field amplitude in which all interaction between the front and rear edges is ignored. Thus

$$P^{E,H}(\phi, \phi) = \frac{i}{2\nu} \sin \frac{\pi}{\nu} \left\{ (C_1 + C_2) + (C_1 - C_3) \exp \left[ ik\ell \left( \cos \phi + \cos \phi \right) \right] \right\} \quad (4.10)$$

where

$$C_1 = \left( \cos \frac{\pi}{\nu} - \cos \frac{\phi - \phi}{\nu} \right)^{-1}$$

$$C_2 = \left( \cos \frac{\pi}{\nu} + \cos \frac{2\pi - \phi - \phi}{\nu} \right)^{-1} \quad (4.11)$$

$$C_3 = \left( \cos \frac{\pi}{\nu} + \cos \frac{\phi + \phi}{\nu} \right)^{-1} .$$
Fig. 4-3: Edge contributions for an ogival cylinder with $\phi_o = 155^\circ$, $\Omega = 12^{1/2}_0$ and $t = 3\lambda$. 
Fig. 4-4: E- and H-polarized bistatic scattering patterns for an ogival cylinder with $\phi_o = 155^\circ$, $\Omega = 12^{1/2}\circ$ and $t = 3\lambda$ as given by GTD.
Typical values of $\sigma/\lambda$ due to the front and rear edge contributions alone are plotted as functions of $\phi$ in Fig. 4-3. Their relative magnitudes show that the scattering is determined primarily by the front edge for E-polarization and by the rear edge for H-polarization. The total scattering for each polarization is shown in Fig. 4-4 and note that while the rear edge has very little effect on the E-polarized pattern, the front edge return is strong enough, when combined with the rear edge return, to produce the interference pattern of the lowermost trace in Fig. 4-4.

4.2 Wedge Cylinder

The second shape of interest is a wedge cylinder consisting of a wedge of half angle $\Omega$ smoothly joined to a portion of a circular cylinder of radius $a$. The profile is then as shown in Fig. 4-5 and some of the pertinent dimensions are indicated. The length-to-width ratio is $\frac{1}{2} (1 + \cosec \Omega)$ and the arc length along the upper or lower surface is $a (\cos \Omega + \frac{\pi}{2} + \Omega)$.

A plane E- or H-polarized wave is incident in the x direction and we seek an expression for the high frequency scattering in a direction making an angle $\phi$ with the positive x axis. The requirement that both wedge-cylinder joins be
directly visible restricts the validity of our results to the angular range \( \phi > \pi - \Omega \) and, in practice, interest centers on the special case of backscattering for which \( \phi = \pi \).

If the origin is taken at the edge of the wedge, a high frequency approximation to the complex far field amplitude is

\[
P^{E,H}(\pi, \phi) = P_{\text{front}}^{E,H} + \left( P^{E,H}_{\text{join}(1)} e^{-ika \cos \Omega \sin \phi} + P^{E,H}_{\text{join}(2)} e^{ika \cos \Omega \sin \phi} \right) \\
\cdot e^{ika \cos \Omega \cot \Omega (1 + \cos \phi)} + P_{\text{cw}}^{E,H} e^{ika \cosec \Omega (1 + \cos \phi)} \tag{4.12}
\]

where the four terms on the right hand side are, respectively, the contributions of the front edge, the upper and lower wedge–cylinder joins and the creeping waves. The first of these is given by eq. (4.4) with \( \alpha = \pi \) and \( \theta = \phi \) and is

\[
P_{\text{front}}^{E,H} = \frac{i}{2 \nu} \sin \frac{\pi}{\nu} \left\{ \left( \cos \frac{\pi}{\nu} - \cos \frac{\pi - \phi}{\nu} \right)^{-1} + \left( \cos \frac{\pi}{\nu} + \cos \frac{\pi - \phi}{\nu} \right)^{-1} \right\} \tag{4.13}
\]

where \( \nu = 2(1 - \Omega/\pi) \), as before. In the particular case of backscattering \( (\phi = \pi) \) with \( \Omega = 12^{1/2} \) (implying \( \nu = 67/36 \)),

\[
P_{\text{front}}^{E} = -i0.541 \quad , \quad P_{\text{front}}^{H} = i0.063
\]

Each join of the wedge and the cylinder forms a line discontinuity in curvature and the diffraction coefficient for such a surface singularity has been derived by Senior (1972). The expression is

\[
P^{E,H}_{\text{join}}(\alpha, \theta) = - \frac{a_2 - a_1}{2k} \frac{1 + \cos(\alpha + \theta)}{(\cos \alpha + \cos \theta)^3} \tag{4.14}
\]

where \( a_1 \) and \( a_2 \) are the curvatures of the abutting surfaces and the geometry is as shown in Fig. 4–6. For the upper join (1), the necessary identification is
Fig. 4-6: Geometry for a curvature discontinuity

\[ a_2 = 0 \quad a_1 = \frac{1}{a} \quad \alpha = \Omega \quad \theta = \pi + \Omega - \phi \]

giving

\[ P_{\text{join}}^{E,H}(1) = \frac{1}{2ka} \frac{1 - \cos(\phi - 2\Omega)}{\cos \Omega - \cos(\phi - \Omega)} \right) \right)^3 \]  

(4.15)

and in the particular case of backscattering

\[ P_{\text{join}}^{E,H}(1) = -\frac{1}{8ka} \sec \Omega \tan^2 \Omega \quad P_{\text{join}}^{H}(1) = \frac{1}{8ka} \sec \Omega (1 + \sec^2 \Omega) \]   

(4.16)

For the lower join(2), the appropriate expression is obtained by replacing \( \phi \) by \( 2\pi - \phi \) in eq. (4.15) and is

\[ P_{\text{join}}^{E,H}(2) = \frac{1}{2ka} \frac{1 - \cos(\phi + 2\Omega)}{\cos \Omega - \cos(\phi + \Omega)^3} \]

(4.17)
which reduces to (4.16) when \( \phi = \pi \).

If \( \phi > \pi - \Omega \) the creeping wave contribution will be the same (to the first order) as it would be for a complete circular cylinder of radius \( a \). Hence

\[
P_{\text{cw}}^{E,H} = \frac{1}{2} C \, m \, e^{5i\pi/6} \frac{\cos \nu (\pi - \phi)}{\sin \nu \pi}
\]  

(Bowman et al, 1969; pp. 102, 111), where we have omitted higher order terms corresponding to waves which skirt the rear of the cylinder more than once. For E-polarization

\[
C = \left( \text{Ai}'(-\alpha) \right)^{-2} \left( 1 + e^{i\pi/3} \frac{\alpha}{30} m^{-2} + O(m^{-4}) \right)
\]

\[
\nu = ka + e^{i\pi/3} \, \alpha \, m^{-1} e^{-i\pi/3} \frac{2}{60} m^{-1} + O(m^{-3})
\]  

(4.19)

and \( m = \left( \frac{ka}{2} \right)^{1/3} \). Since

\[
\alpha = 2.338107..., \quad \text{Ai}'(-\alpha) = 0.7012108..., \n\]

approximations which are adequate for most practical purposes are

\[
C = 2.033774 \left( 1 + e^{-0.0779369} m^{-2} \right)
\]

(4.20)

\[
\nu = ka + e^{i\pi/3} \, 2.338107 \, m - e^{-i\pi/3} \, 0.0911124 \, m^{-1}.
\]

For H-polarization,

\[
C = \frac{1}{\beta} \left( \text{Ai}(-\beta) \right)^{2} \left( 1 + e^{i\pi/3} \frac{\beta}{30} \left( 1 - \frac{3}{\beta^2} \right) m^{-2} + O(m^{-4}) \right)
\]

\[
\nu = ka + e^{i\pi/3} \, \beta \, m - e^{-i\pi/3} \frac{\beta}{60} \left( 1 + \frac{6}{\beta^3} \right) m^{-1} + O(m^{-3})
\]

(4.21)
and since

\[ \beta = 1.018792\ldots, \quad \text{Ai}(-\beta) = 0.5356566\ldots \]

we can approximate \( C \) and \( \nu \) as

\[ C = 3.420915 \left( 1 - e^{i\pi/3} \right) 0.06238566 \text{ m}^{-2} \]

\[ \nu = ka + e^{i\pi/3} 1.018792 \text{ m} - e^{-i\pi/3} 0.1154545 \text{ m}^{-1} . \]

This completes the specification of the various terms in eq. (4.12).

Although the contributions of the front edge, the two joins and the creeping wave are individually functions of the aspect angle \( \phi \), most of the aspect variation of \( P^{E,H}(\pi, \phi) \) at high frequencies is attributable to constructive and destructive interference, i.e. to the explicit exponential factors in eq. (4.12). Over the small angular range \( \pi - \Omega < \phi \leq \pi \) for which (4.12) is applicable, \( P_{\text{front}}^{E,H} \) etc. vary relatively little, and it is therefore sufficient to use the simple case of backscattering to compute the magnitudes of the individual contributions to the scattering.

When \( \phi = \pi \), we have

\[ P^{E,H}(\pi, \pi) = P_{\text{front}}^{E,H} + 2 P_{\text{join}}^{E,H} e^{2ika \cos\Omega \cot\Omega} + P_{\text{cw}}^{E,H} e^{2ika \cosec\Omega} \]

and the magnitude of each term's contribution to \( \frac{\sigma}{\lambda} \) is plotted as a function of \( ka \) in Fig. 4-7. For E-polarization the creeping wave contribution is entirely negligible (it is of order -55 dB), and since even the join contribution is 25 dB or more below that of the edge, the scattering is dominated by the frequency-independent edge return. To achieve a 13 dB reduction in the backscattering cross section, it is now only necessary to find a method for adequately suppressing the front edge return. For H-polarization, on the other hand, all three sources
Fig. 4-7: Axial backscattering contributions for a wedge cylinder having $\Omega = 12^{1/2}^\circ$. 
of scattering are comparable in magnitude, leading to a rather complicated variation of the total scattering as a function of frequency. To suppress any one source would not significantly reduce the net return, and for this polarization it is necessary to devise means for suppressing all of the three sources. It is true that as $ka$ increases, the join and creeping wave contributions decrease whereas the edge return remains constant, but since each of the first two is more than $13\,\text{dB}$ below the edge return only if $ka > 37, 24$ respectively, $ka$ must be large indeed to justify ignoring them completely.

4.3 Numerical Comparisons

The geometrical theory of diffraction is a valuable tool in studies of high frequency scattering but its usefulness is primarily restricted to metallic bodies. Nevertheless, its conceptual simplicity is very helpful in diagnosing sources of scattering and under a wide variety of circumstances, its accuracy has been found adequate for most practical purposes.

The present contract is vitally concerned with the effect of non-uniform surface impedances on the scattering properties of selected bodies. Since GTD has neither the accuracy nor versatility for accomplishing this task, it has been necessary to rely on the digital techniques embodied in the RAM1B program, but even so GTD can still play an important role. Specifically, it affords a check on the gross accuracy of the computer program in at least the particular case of a metallic body and it can also help quantify our deductions from the computed data for non-metallic bodies.

To illustrate this role, consider an ogival cylinder illuminated by an E-polarized plane wave. For $\phi = 155^0$, $\omega = 12^{1/2}$, and $l = 3\lambda$, the bistatic scattering predicted by RAM1B is compared with the GTD prediction in Fig. 4-8. It will be recalled that the GTD analysis was carried out under the assumption that the rear edge is fully visible, and the discrepancy between the curves for $\phi \geq 165^0$
Fig. 4-8: Comparison of the GTD (-----) and RAM1B (-----) predictions of the bistatic scattering from an ogival cylinder for $\phi_0 = 155^\circ$, $\Omega = 12^{1/2}_2^\circ$ and $\ell = 3\lambda$. 
is obviously due to the violation of this assumption. For smaller values of $\phi$
the agreement is rather good and gives us confidence in the accuracy of GTD
as well as in the performance of the computer program. What discrepancy there
is is substantially removed if the rear edge contribution in eq. (4.7) is multi-
plied by a factor $A$ where $A \simeq 1.08 e^{-10.44}$. Such a modification could be
attributed to either the incident ray or the diffracted ray, or both. It is equiva-
 lent to a small amplitude enhancement and phase delay and both of these are con-
sistent with the 'transitional effect' (Senior and Uslenghi, 1973) occurring when
a ray passes close to a surface.

The agreement shown in Fig. 4–8 confirms that the scattering from the
ogival cylinder is due to both the front and rear edges. The rear edge is the
dominant contributor (see Fig. 4–3) for this polarization, and were we to suppress
this contributor entirely, the reduction in $\frac{\sigma}{\lambda}$ over the range $120^\circ \leq \phi \leq 180^\circ$ could
not average more than about 13 dB. The obvious way to reduce (say) the rear
dge return is to introduce a non-zero surface impedance over a portion of the
rear half of the ogive. If this is fairness to the metallic surface such that no new
source of scattering is created, the expected scattering pattern should be con-
sistent with eq. (4.7) with an unknown factor $A$ multiplying the second term. By
fitting the resulting version of (4.7) to the scattering pattern obtained from the
computer program, the coefficient $A$ can be assigned a value appropriate to the
chosen surface impedance variation. The magnitude of $A$ is then a quantitative
measure of the effectiveness of the surface impedance, and this procedure has
been implemented for a variety of impedance variations.

When the ogival cylinder is illuminated by an E-polarized plane wave,
the dominant scatterer is the front edge, and we may expect to produce a 13 dB
reduction in the total scattering by attacking the front edge alone. Our initial
attempts were entirely unsuccessful and the reason became evident when the

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computer prediction for a metallic body was compared with the GTD estimate. There was no agreement whatsoever, and our confidence in the GTD approach now led us to a detailed examination of the computer program. This revealed an error in the incident field specification for E- (but not H-) polarization. The error was duly corrected and the program re-run. The agreement with GTD was only slightly improved, and since the scattering pattern was characterized by a deep null, it was apparent that a strong rear edge contribution was being obtained. Several procedures have been used to try to overcome this difficulty and although we shall not know the full results in time for inclusion in this report, the importance of having some method for checking the computer results has been demonstrated.
V SUPPRESSION OF TRAVELING AND CREEPING WAVES

5.1 Surface Impedance Loading for H-Polarization

We used two gauges in this investigation to assess the effect of impedance loading schemes: the surface field behavior and bistatic far field patterns. Although one might think that the backscattering, instead of the bistatic, patterns would have been more desirable for this purpose, this is not necessarily the case. Firstly, the bistatic pattern is very much like an expanded version of the backscattering pattern and consequently there are fewer peaks and nulls to deal with. Secondly, it is more costly to obtain backscattering patterns because a new incident field value must be computed at each surface point for each new incident direction. Finally, since many subroutines in RAM1B perform more than one function, most of the subroutines would have had to have been modified in order to obtain the desired result.

A third measure of the efficiency of surface loading was the determination of a complex weighting factor A, described in Chapter IV, but A is obtained by means of a far field pattern-matching procedure. This factor is therefore an auxiliary measure rather than one of the primary gauges of impedance performance. It was used only for the ogival cylinder as an estimate of the traveling wave suppression, and was not applied to the wedge-cylinder studies.

In the following two sections we discuss the suppression of traveling waves on the ogival cylinder and creeping waves on the wedge-cylinder. Both types of waves are favored by H-polarization and both are, for most practical purposes, insignificant for E-polarization. Thus it is sufficient to consider only the former.
5.2 The Ogival Cylinder

In commencing work with the ogival cylinder, one must first realize that the backscattering pattern in regions centered on the longitudinal plane of symmetry is marked by strong interference, and is therefore permeated with deep nulls. These are the characteristics of traveling wave contributions and although it was the bistatic, and not the backscattering, pattern that would be used to judge performance, we sought to choose an incidence angle lying near the peak of the first lobe off the plane of symmetry. It is necessary to choose this location carefully, since the amplitudes of other positions (i.e., a few dB down the steep sides of a lobe) are sensitive to slight shifts in the angular distribution of a pattern as well as to changes in amplitude.

The angular position of the first traveling wave lobe of a thin wire is given approximately by the formula (Senior and Knott, 1968)

\[ \theta \approx 49.4 \sqrt{\frac{\lambda}{l}} \]  \hspace{1cm} (5.1)

where \( l \) is the length of the wire, \( \lambda \) is the wavelength of the impinging plane wave and \( \theta \) is measured in degrees from end-on incidence. For most of the ogival cylinder studies we chose a cylinder width (corresponding to the axial length of a true ogive) of 3.0\( \lambda \), which is large enough that high frequency prescriptions, such as GTD, apply, yet small enough to ensure adequate sampling by the computer program. Assuming that eq. (5.1) holds for the cylinder as well as the wire, we find that \( \theta \approx 28.5 \) degrees. This value was used for our first set of computations with program RAM1B. We found, however, that the finite thickness of the cylinder, coupled with the curvature of the surface, required that the angle be reduced to about 25 degrees. All the results shown below are for this angle of incidence, except where noted otherwise.

The computed surface currents induced on the metallic ogival cylinder are shown in Fig. 5-1. Since the magnetic vector is parallel to the cylinder axis,
Fig. 5–1: Computed surface currents on metallic ogival cylinder.
the physical optics currents are 2.0 and zero on the lit and shadowed side, respectively, and are independent of position along the surface. Note from the uppermost set of datum points that it requires at least one wavelength of surface before the actual currents build up to the physical optics value. On the shadowed side the actual currents are much smaller than on the lit side, but they are substantially different from the (zero) value predicted by physical optics.

On both sides there is a standing wave pattern betraying the existence of a surface wave reflected from the back toward the front edge (the periodicity is very nearly 0.5λ). The amplitude of the interference pattern increases toward the rear of the cylinder, showing that the wave traveling away from the rear edge is decaying and in this respect the reflected surface wave is similar to a creeping wave. Note that the mean surface current intensity on the shadowed side decays rearward from the front edge, showing that the ogival cylinder supports creeping waves on the shadowed side as well as traveling waves on the lit side.

When the rear half of the cylinder is given a constant surface impedance of 1.0 + j0 (normalized to that of free space), the surface currents over that half are greatly reduced, as illustrated in Fig. 5–2. However, the abrupt change in impedance produces a substantial reflection, as evidenced by the persistence of a standing wave pattern on the lit side. On the shadowed side, however, we find a marked reduction of reflected waves and, at least over the forward half, the surface currents trace out the mean of those in Fig. 5–1. Nonetheless, even over the loaded portion of the surface, we can perceive a small oscillation signifying that some small reflected waves are still present. The strong perturbations in the surface field over the forward half of the lit side of the cylinder demonstrate that abrupt changes in impedance are to be avoided.

Tapering the impedance from zero at the forward edge of the loaded
Fig. 5-2: Computed surface current for ogival cylinder with rear half given a surface impedance $Z_s = 1.0$. 
region to a value of about $1.07 + i 0$ at the rear edge produces a far more
dramatic change, as shown in Fig. 5-3. The impedance was increased with
the square of distance along the surface, and this has the effect of sharply
reducing the rear edge reflections without introducing any reflections at the
loading edge of the loading zone itself. The rear edge contribution was not
eradicated completely, however, because a small ripple can still be seen on the
surface current behavior. The ripple is more pronounced on the shadowed
than on the lit side because the mean level is appreciably smaller there. This
figure shows that impedance tapering is a highly desirable technique and in
a moment we will explore several tapering rates to see which is best for a
given size of loading zone.

Bistatic scattering patterns produced by the surface field distributions
in Figs. 5-1 through 5-3 are presented in Fig. 5-4 for comparison. The upper-
most and lowermost patterns are for the bare, conducting obstacle and for the
$\frac{s}{2}$ loading over its rear half, respectively. Note that these patterns have the
same form, except that the null near 38 degrees is much deeper for the loaded
object. This is the result of reducing the amplitude of the dominant scatterer
(the rear edge) to the extent that it approaches the small residual return from
the front edge. The center pattern, on the other hand, has its peaks and nulls
displaced: its form is consistent with the notion that, while the rear edge scat-
ttering has been greatly reduced, a third scatterer has been created. The angular
distance between the first peak and the first null is approximately twice that of
the other two patterns, suggesting that two scatterers producing the pattern are
separated by half the distance between the original pair of scattering centers.
Thus, the far field data, as well as the surface field behavior, lead to the same
conclusion: a sharp impedance jump is a source of scattering.

There is only one angle on the plot where backscattering occurs (namely,
Fig. 5-3: Computed surface currents for ogival cylinder with impedance load over the rear half varying as $s^2$. 

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Fig. 5-4: Computed bistatic scattering for two types of loading over rear half of ogival cylinder.
at $\theta = 25$ degrees), but the patterns over a small angular region centered on
this angle are indicative of what may be expected of a true backscattering
pattern. The cross section reduction available can be estimated by measuring
how much the first peak has been suppressed; this amounts to $9.0$ dB for the
$s^2$ loading and a scant $2.8$ dB for the constant load. These levels do not meet
the desired requirement of $13$ dB reduction and we studied other loading rates,
including linear and $s^3$. The results are given in Fig. 5-5.

The quantity plotted in Fig. 5-5 is the level of the peak in the bistatic
scattering pattern lying nearest the backscattering direction. The rear half
of the cylinder was given a general surface impedance variation described by

$$Z_s = Z_{\text{max}} s^m,$$  \hspace{1cm} (5.2)

where $Z_{\text{max}}$ is a real impedance value desired of the surface impedance at
the rear of the body, $s$ is a distance measured along the surface commencing
at the midpoint on upper or lower profiles and increasing to the rear, and $m$
is an integer. The constant load ($m = 0$) is the poorest of the four studied
because the jump in impedance at the leading edge of the load introduces a
source of scattering. The linear loading rate ($m = 1$) is the best of the four,
at least for $Z_{\text{max}} \ll 1$, but performs poorer than the square ($m = 2$) or the
cubic ($m = 3$) rate. As the steepness of the ramp function increases, so does
the discontinuity (in first derivative) of the impedance at its start, and ultimately
the scattering from this discontinuity becomes the dominant source.

The best rate appears to be the square law variation and for $Z_{\text{max}} = 3.0$, it produces a cross section matching that obtainable if the rear edge contribution
were totally wiped out. This level is shown as the horizontal line near the
bottom of the figure and was obtained via the high frequency GTD analysis given
previously. On the basis of this figure we would conclude that square law loading
is optimum for $H$-polarization incident on an ogival cylinder, provided an impedance
Fig. 5-5: Comparison of scattering cross sections for several loading rates applied to the rear half of an ogival cylinder.
as high as 3 could be synthesized and provided the entire rear half of the body could be treated. The maximum reduction available is of the order of 15 dB.

However, it might legitimately be asked, "What if such a high impedance is unavailable? Or how much do we sacrifice by using less than 50 percent of the surface of the body?" The answer to the first can be read from Fig. 5-5; if impedances greater than \( Z_{\text{max}} \sim 1.0 \) cannot be realized, then a linear loading is appropriate and, if the rear half of the body remains accessible, then a 9 dB reduction can be achieved. Although it is not obvious how much more than this could be achieved, it is our hope that the residual effects of a specular absorber could be used to advantage, for then the impedance would not have to be commenced at a level of zero, but a finite level appropriate to the material already present.

The answer to the second question can be obtained from inspection of Fig. 5-6. A portion \( f \) of both the upper and lower surfaces, extending to the rear edge of the body, was given a surface impedance varying as the square of distance and the figure confirms what one would predict in advance: performance falls off if less surface is available. Although we present only \( s^2 \) loadings, the results for other variations follow the same general trend. An example of what the chart tells us is as follows: assume that a 10 dB reduction is desired. Such a reduction is achievable over a 1.5\( \lambda \) segment of surface for \( Z_{\text{max}} = 1.25 \), or over 1.0\( \lambda \) for \( Z_{\text{max}} = 1.6 \), or over 0.75\( \lambda \) for \( Z_{\text{max}} = 2.3 \). The reduction apparently cannot be achieved using only a 0.25\( \lambda \) length of surface, unless the impedance is carried to very high values not shown on the chart.

The maximum available scattering cross section reduction is about 15 dB if the rear edge contribution is completely obliterated. Even more can be obtained if the front edge can be attacked in addition so we attempted to load the entire surface of the cylinder to see how much the cross section could be
Fig. 5-6: Comparison of scattering cross sections for varying surface coverage of the rear part of an ogival cylinder; the surface impedance varied as $s^2$ in all four cases, and both the upper and lower surfaces were treated.
further reduced. Four constant values of impedance were selected and the surface currents produced by two of them are compared against those of the metallic body in Fig. 5-7; only the currents on the illuminated side of the cylinder are shown. Note that the currents become progressively less intense as the surface impedance is increased.

When the far field scattering from the fully loaded cylinders is computed, however, the presence of relatively small currents does not guarantee that the scattering has been reduced, as is evidenced by the data in Fig. 5-8. Increasing the surface impedance from zero to 0.5 produces an overall reduction in excess of 12 dB, but further increases in impedance serve only to increase the scattering. Moreover, the pattern becomes sensibly constant and the lobing structure disappears; the pattern takes on the characteristics of a single scatterer.

This is consistent with the duality noted in Chapter III between the integral equations for the two polarizations. Although we are dealing here exclusively with H-polarization, the increased surface loading enhances the front edge contribution with the result that the far field scattering has E-polarization characteristics. The far fields become stronger, even though the surface currents are weaker, because the currents are multiplied by the impedance before they are summed. Thus the magnitudes of the surface currents alone are not reliable indicators of the strength for field scattering. We conclude that, for H-polarization, the leading edge is best given a coating of small, but finite, impedance.

Other tests have also been performed, but the results are not included in this report. One of them involved the specification of a non-zero reactive component of surface impedance and the results show that such a component tends to degrade any of the benefits bestowed by the real part. Small imaginary components can be tolerated, but if they become too large, then the benefits of
Fig. 5-7: Computed surface currents over the lit side of a completely loaded ogival cylinder.
Fig. 5-8: Comparison of computed scattered fields for a completely covered ogival cylinder.
cross section reduction are lost. Depending on the sign of the reactive component, the lobe structure due to the traveling wave contributions is shifted in aspect angle. We conclude that it is best, if possible, to use complex surface impedances whose imaginary parts are small compared to their real parts.

5.3 The Wedge–Cylinder

We chose the wedge–cylinder as a test obstacle with which to study creeping waves, with the wedge being smoothly mated to a segment of a circular cylinder in order to remove the specular scattering normally produced by a purely circular cylinder. The wedge angle was chosen to match that of the ogival cylinder and we found that when the radius, \( a \), of the cylinder joined to the wedge is taken such that \( ka = 3 \), the surface sampling rate also matched that of the ogival cylinder. Although the maximum length of the wedge–cylinder is some 10 percent less than that of the ogival cylinder, the perimeters of the two bodies are very nearly the same.

The electrical size \( ka = 3 \) is important for another reason, and can be seen from Fig. 4–7. The far field scattering along the plane of symmetry is near a peak value there, so that the reduction of any of the three components producing the net scattering is likely to reduce the net return. Although this is not precisely the case -- the presence of a third scatterer (the front edge) tends to confuse the interpretation of the results -- the H-polarized front edge return is smaller than either the join or creeping wave contribution. The edge contribution is not necessarily negligible.

Our studies of the wedge cylinder were carried out primarily for incidence lying in the longitudinal plane of symmetry and impinging on the front edge. We chose incidence in the plane of symmetry because the radar cross section attains its highest value at this point in the end-on region. When the aspect angle
swings far enough to shadow one of the joins between wedge and cylinder, this contribution is lost, as is the direct contribution from the creeping wave. Thus, if the scattering can be suppressed for incidence in the plane of symmetry, the effect will carry over to other angles.

The behavior of the surface currents on the bare, metallic shape is plotted in Fig. 5-9 along with those computed for two selected impedances covering the entire profile. The ripples on the surface of the wedge show that a wave propagates forward toward the edge due to reflections from the join and to the creeping wave that has traversed the shadowed half of the cylinder. There are two creeping waves traveling in opposite directions in the shadow, of course, and they produce a standing wave pattern in the shadowed region that rapidly decays. The currents peak at the very rear of the cylinder, which is characteristic of axial incidence, and independent of $ka$.

Even on the metallic body the currents do not attain the physical optics value; this is due to a wave launched rearward by the edge that partially cancels the incident wave. The edge-launched wave decays slowly because the wedge face is flat and it requires about $4\lambda$ or more of surface before the currents approach an amplitude of 2.0. This is in contrast to the ogival cylinder currents (Fig. 5-1) which require only a wavelength or so. The difference between the two cases is the curvature of the ogival cylinder surface; this attenuates the edge-launched wave fairly rapidly, hence the wave quickly becomes too weak to buck the effect of the incident wave. On the wedge-cylinder these waves travel along the surface in nearly the same direction as that of the incident wave and, in the absence of a reflection from the rear of the body, they produce a standing wave pattern with a period that far exceeds the length of the body.

The mean surface field intensity at the join, as read from Fig. 5-9, is about 1.5 times the strength of the incident field, whereas physical optics predicts a value of precisely 2.0. This suggests that the join and creeping wave scattering
Fig. 5-9: Computed surface currents for a wedge-cylinder at axial incidence.
for the metallic obstacle in Chapter IV were overestimated by a factor of $2/1.5 = 4/3$. The radar cross sections shown for these two scatterers are therefore too large by a factor of $16/9$, and a more accurate estimate can be obtained from Fig. 5-9 by lowering their values 2.5 dB at $ka = 3.0$. The precise amount of correction needed is frequency sensitive, of course, because the length of the wedge face increases with frequency, thus permitting the surface fields to more closely approach, and indeed exceed, their physical optics values.

Loading the entire surface of the wedge-cylinder reduces the currents as in the case of the ogival cylinder. Creeping waves still exist, as evidenced by the gentle undulations in the shadow region, but they decay rapidly enough that no effect on the fields on the wedge can be detected. The far zone scattered fields produced by several loadings are shown in Fig. 5-10 and note that the same behavior emerges as was illustrated in Fig. 5-8 for the ogival shape. Coating the entire body with a surface impedance of 0.5 reduces the end-on cross section by a healthy 13 dB, although the reduction is less than this for larger bistatic angles. Further loading serves only to increase the cross section until finally, for a surface impedance of 3.0, the scattering is greater than that of the metallic object. The increase is wholly due to covering the front edge and we reach the same conclusion as with the ogival cylinder: for H-polarization the leading edge is best covered with but a low impedance surface.

The creeping wave skirting the shadowed part of the cylinder should yield to a surface impedance treatment in much the same way as did the rearward traveling wave on the lit surface of the ogival cylinder. The loading must be symmetrical over the rear of the wedge-cylinder because a pair of waves traverse the shadows in opposite directions, and this suggests an impedance variation similar to that of Fig. 5-11. Note that any impedance varying as $s^m$ produces a cusp at the rear of the body (for $m = 0$). Our experience with impedance discontinuities on the ogival cylinder suggested that such a cusp should
Fig. 5-10: Computed far field scattering of a wedge-cylinder with surface impedance loading over entire profile.
Fig. 5-11: Impedance varying with $s^m$ produces a cusp at the rear of the wedge-cylinder.

be avoided, whereupon we devised a generalized Gaussian loading having the form shown in Fig. 5-12. The mathematical form we used was

$$Z_s = Z_a \exp \left( -\beta (s - s_o)^2 \right), \quad (5.3)$$

where $s_o$ is the surface location desired of the peak in the curve.

Fig. 5-12: Surface impedance whose distance dependence is a Gaussian shape.

The impedance of eq. (5.3) is zero only for infinite values of $s$ (for $Z_a \neq 0$), and mathematically we can never taper to a metallic surface. However,
the exponent $\beta(s - s_o)^2$ can be chosen, via a specification of the constant $\beta$, such that $Z_s$ can be brought down to, say, 1 percent of the peak value within a specifiable distance. Three selections of $\beta$ were employed to load the rear of the cylinder and the behavior of $Z_s/Z_a$ for the three choices are illustrated in Fig. 5-13. The shadow boundaries, if they were to be plotted, would lie at

![Graph showing impedance variations]

**Fig. 5-13:** Three impedance variations were used to suppress creeping waves.

$|s - s_o| = 0.75\lambda$, consequently the loaded surface lies entirely within the geometric shadow.

It can be seen that these are not perfectly Gaussian distributions and the reason is that surface impedances can be specified in the program only at the endpoints of the straight line segments that approximate the body profile. The program subsequently computes an impedance at each segment midpoint by taking the arithmetic mean of the two values at the endpoints, hence the actual distribution is somewhat different than that intended.
The surface fields produced by these distributions when placed at the rear of the wedge-cylinder are very much like those of the uncoated object (the uppermost trace in Fig. 5–9), except that the amplitudes of the ripples on the wedge face are more gentle and the peak at the antipode is lower. The effects of the loading on the far field scattering is not particularly impressive, as might be judged by Figure 5–14 which compares the scattering for the best case used \( \beta = 18.4 \) and \( Z_a = 2.0 \) in eq. (5.3)) with that of the untreated obstacle. The entire bistatic pattern is reduced, although not uniformly, with the end-on (backscattering) cross section being about 6.5 dB lower than that of the bare body. The patterns corresponding to all the remaining choices of \( \beta \) and \( Z_a \) \( (Z_a < 2.0) \) lie between the two plotted in Fig. 5–14.

A summary of the end-on cross sections for these loadings is shown in Fig. 5–15 as functions of \( \beta \) and \( Z_a \). There is very little difference between two uppermost curves, and, as noted a moment ago, the lower one represents only a 6.5 dB reduction for the maximum impedance used. A glance at Fig. 5–13 shows that the transition loading zone, within which the impedance rises from minimum to maximum values, is less than 0.5\( \lambda \) wide. Since the geometric shadow is 1.5\( \lambda \) wide, this transition region could be expanded to about 0.75\( \lambda \) on either side of the antipode and still lie within the shadow. We do not yet know how effective a more extensive surface coverage will be, because such tests have not yet been run. But even if we could totally obliterate the creeping wave, we would be left with the join return which, on the basis of Fig. 4–7, can be assigned a scattering amplitude of about \(-18 \) dB\( \lambda \) for \( ka = 3.0 \). Obviously we must suppress the join contribution as well as that of the creeping wave.

We attempted to load the region of the join with the same Gaussian distributions that were used at the rear of the wedge-cylinder (see Fig. 5–13) but these turned out to be utterly disappointing. The maximum suppression obtained
Fig. 5-14: Far fields produced by the best Gaussian distribution used thus far for creeping waves.
Fig. 5-15: End-on cross sections of the wedge-cylinder as functions of $Z_a$ in eq. (5.3).
was a scant 5 dB and, whereas these impedances produced a finite, if small, 
reduction when placed at the rear of the body, some of them increased the 
scattering when placed at the join! We attribute the enhancement to an ex-
cessively steep rate of change in impedance, much as was found for a sharp 
linear taper in the surface impedance of an ogive.

It might be argued that the enhancement came about because one of the 
dominant contributors was out of phase with the other, but this is not so because, 
as mentioned in connection with Fig. 4-7, the two are nearly in phase to pro-
duce the peak return near ka = 3.0. Moreover, we did produce a reduction when 
the other contributor, the creeping wave, was suppressed. That there was an 
enhancement can also be seen from the surface current plots in Fig. 5-16. The 
current distribution for the metallic wedge-cylinder is shown as the solid trace 
for reference, and that produced by the surface impedance variation portrayed 
in the upper right corner is shown as the dashed trace. Note that the loading 
generates a stronger reflected current wave than exists on the bare body and this 
in turn will produce a stronger scattered field. There is a marked reduction in 
the amplitude of the creeping wave but in the far field the reduction will be 
totally swamped by the enhancement of the join return.

The end-on cross sections of the join-loaded wedge-cylinder are plotted 
in Fig. 5-17. It is only when the loading is spread over a wavelength or so of 
surface (β = 293) on either side of the join that we obtain a cross section reduc-
tion, as evidenced by the lowermost curve. The maximum reduction available 
with this particular load was only 5 dB and, although we have no data to show it, 
we believe that more reduction can be obtained by spreading the impedance over 
even more surface. Both of the other two Gaussian loadings enhanced the cross 
section, one of them by as much as 9 dB.

The Gaussian distribution is the only one we studied for the wedge-cylinder

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Fig. 5-16: The impedance variation shown to scale at upper right creates a reflection.
Fig. 5-17: Computed end-on cross sections of the join-loaded wedge-cylinders as a function of $Z_a$ in eq. (5.3).
and we used it separately for the join and creeping wave contributions. Its effect is markedly different for the two, always producing a reduction, albeit a small reduction for narrow loads, in the creeping wave and, if poorly chosen, an enhancement of the join return. The distinction between the two cases is the illumination and it appears that a rapid rise in surface impedance can be tolerated if it occurs in the shadow but not if it is placed on a lit portion of the body. At the time of this writing other variations of impedance are being studied to see if any improvement can be obtained.

The results cited above confirm our previous notions that it requires at least $0.5\lambda$ of surface transition region to be available for the application of loading. As the transition region is shortened, the loading ceases to be distributed and takes on the characteristics of a lumped or discrete load. In the limit as the transition region vanishes, the load is applied at a single location and the result amounts to nothing more than a cancellation scheme. We have already seen in other contracts that such methods suffer severe bandwidth limitations (for passive loads) and are of little practical value, especially if a large reactive component is required of the load. Thus we ought to expect to consume some finite area in the surface impedance loading process and it is our hope that a physical realization of that impedance could be packed into a space not more than $0.5\lambda$ wide.
VI SUMMARY, CONCLUSIONS AND FUTURE PLANS

The repeated use of program RAM1B has already delineated some of the surface impedance values needed to control non-specular scattering, but bridging the gap between the abstraction of impedance and the reality of the physical world is another matter. As pointed out in Chapter III, we have made a start in this direction by examining the slab model (of material), which may be of use providing certain restrictions are imposed on the electrical characteristics of physical materials. The model can be improved if we can include the effects of surface curvature and some progress has been made in this direction also. Using the exact solution of a limited class of boundary value problems (such as coated sphere or cylinder) we hope to isolate, or at least quantify, the role played by curvature.

Detailed studies of the influence of surface impedance on traveling and creeping waves have been carried out, although we have accomplished more on the former than the latter. Both sources of non-specular scattering are favored by H-polarization and, aside from the program difficulties noted earlier, we have not studied the effects of surface impedance for E-polarization. Salient results are that traveling and creeping waves are best suppressed by smoothly increasing the surface impedance with increasing distance, taking the impedance to as high a value as is possible. Equally important, however, is to insure that the net change in impedance level not take place in too short a distance, lest a new source of scattering be created.

Tentative plans for the remainder of the contract period cover several tasks. For one, we shall examine other variations in the H-polarized surface impedance imposed on a wedge-cylinder than the Gaussian distributions used thus far; attention will be focussed on the join contribution because the scattered fields seem to be quite sensitive to the impedances placed there. The surface impedance computer program will be evaluated when applied to sharp loading
edges for E-polarization, and the improvements, if any, obtained by dense sampling at, or rounding of, the edge will be determined. If the outcome of these tests is favorable, we shall use the program to determine if the surface impedance concept can be used to control leading edge diffraction.

In parallel, another program designed for bodies with resistive sheets attached will be implemented. As with RAM1A, this program will require some modification but it is expected that our experience with RAM1A will simplify the task. A laboratory experiment using a finite section of ogival cylinder will be carried out. In this experiment a layer of material will be applied to the surfaces and the tangential electric and magnetic fields measured. The results will be used to study the relation between surface impedance and material properties, and will also serve as a useful check on the efficacy of the surface impedance computer program.
APPENDIX

--Description of Program RAM1B--

We received a source deck for program RAM1A in early May, 1972. Due to software incompatibilities at the University of Michigan Computing Center, RAM1A would not run at first but the problems were soon resolved. We then made three major changes in order to produce a program more suitable to the investigation at hand:

1) The surface impedance was arrayed so that it could vary from point to point over the obstacle profile (RAM1A accommodated only constant impedance).

2) Subroutine GEOM was expanded to facilitate easy input of relatively complicated profiles and surface impedance variations.

3) A subroutine (GPM) was added to provide "quick look" printer plots of surface current and impedance.

A fourth change was made some months later after an error was discovered in subroutine FIELD. The modified program was dubbed RAM1B and a listing of it is reproduced on pages 68 to 95.

The general structure of RAM1B is shown in Figure A-1. In addition to the main program there are 10 subroutines, all of which are called exclusively by only one other subroutine. Large blocks of storage are shared by them (by means of COMMON statements) to reduce core requirements. As indicated in the figure, the main program calls but three subroutines, and two of the three in turn call three others.

The main program does little more than read control information and most of the input/output operations take place in subroutines GEOM, PASS2, and GPM. The obstacle profile is set up in GEOM and the elements of several arrays
are created, such as the coordinates of sampling points, the values of surface impedance, and the components of surface normal and tangential unit vectors. These data must be established before subroutine ALLLK is called.

ALLLK uses the geometrical information provided by GEOM to create two matrices, ABO1 and ABO2, as required by the integral equation formulation. They depend, of course, on the polarization of the incident wave. The matrix elements contain Hankel functions and ALLLK calls on subroutines HANK or HANKY, depending on the size of the argument, to compute these functions. Subroutine HANKY (argument less than or equal to 0.90) computes the Bessel functions $J_n(x)$ and $Y_n(x)$, from which the Hankel function is constructed; subroutine HANK (argument greater than 0.90) calls upon subroutine BESSJN to accomplish this task. If the surface elements have been subdivided into 3 or more subelements (for greater accuracy in computing the matrices ABO1 and ABO2), then subroutine SIMP is called to sum the contributions of the sub-elements. After ALLLK has finished creating the two matrices it returns control to the main program.

MAIN then calls subroutine PASS2 to compute the target scattering. PASS2 accomplishes this in several stages. First, it calls subroutine FIELD, which computes the incident field components, depending on polarization, at each of the surface elements. FIELD also adds the two matrices ABO1 and ABO2, then passes control back to PASS2. PASS2 next calls subroutine ZVO8, which has two functions to perform. ZVO8 inverts the matrix handed to PASS2 by FIELD and, using the incident field quantities generated by FIELD, it generates the surface current distribution.

After ZVO8 passes control back to PASS2, the surface currents are summed for each of several bistatic angles (specified on input to MAIN). Depending on the options chosen on input, any of several quantities may be written on the output record. Examples are printer plots of surface currents and impedances.
(for which purpose subroutine GPM is called), the matrix elements, and the
bistatic scattering cross section. Other data, such as lists of the coordinates
of surface elements and the impedance assigned to each element, are printed
out by subroutine GEOM.

The main program contains a large number of initial comment cards
explaining the input format of the data cards. Since these comments are self-
exploratory, no further description of them will be made. The reader will note
that several FORTRAN statements have been rendered inoperable by converting
them to comment statements. We include them in the listing because they were
part of the source deck furnished by AFAL.

A final note should be made regarding the method by which GEOM
creates the target profile and the coordinates of the straight line segments
that approximate the profile. We assume that the profile can be described by
an assembly of circular arcs and that the body has a plane of mirror symmetry.
Thus we need specify the shape of only half the body and let the computer find the
image. The program expects to read the coordinates of the endpoints of arcs,
the angles subtended by those arcs at the center of the generating circles, and
the number of elements the arcs are to contain. This is sufficient information
to allow the computation of the radius of each arc and consequently the coordi-
nates of the midpoint of each element. It is assumed that the arcs lie in the
upper half plane $y \geq 0$ and that they are concave down, so that the resulting
profile is convex.
Fig. A-1: General Structure of RAM1B
RAMIE RCS PROGRAM
THIS IS A MODIFIED VERSION OF RAMIA SUPPLIED BY "PAF" (AFAL). THE PROGRAM HANDLES ARBITRARY SURFACES SPECIFIED BY SEGMENTS OF ARCS, VARIABLE SURFACE IMPEDANCE, AND VARIABLE SAMPLING DENSITY. TIME CONVENTION IS EXP(-IWT), (Z=J-B IS INDUCTIVE).
ALL IMPEDANCES ARE NORMALIZED TO 20

***INPUT DATA DESCRIPTION***

(1) CYLINDER KA=1. HORIZ POL Z.S=0.,1.
TITLE CARD USE UP TO 72 COLUMNS

(2) M N
M 1-5 TOTAL NUMBER OF CELLS
N 6-10 NUMBER OF SU. CELLS PER CELL, 1-5

(3) IOPT IPP KODE W. IT ALPHA RADIUS CENTX CENY ZC
IOPT 1-5 -1= COMPUTES COMPLEX GEOMETRY, VARYING SURFACE IMPEDANCE, AND VARYING SAMPLING DENSITY
0= SURFACE POINTS AND IMPEDANCES ARE READ IN
1= SURFACE COMPUTED FOR A CYLINDER, CONSTANT ZS
IPP 6-10 0= PARALLEL POLARIZATION
1= PERPENDICULAR POLARIZATION
2= PAR POL, PERFECT CONDUCTOR
3= PERP POL, PERFECT CONDUCTOR
KODE 11-15 0= GEOMETRY COMPUTED, MATRIX INVERTED
1= STORED GEOMETRY USED, NO INVERSION
W. IT 16-20 1= NUMBER OF BISTATIC DIRECTIONS
ALPHA 21-30 INCIDENT DIRECTION IN DEGREES
RADIUS 31-40 RADIUS OF CYL IN WAVELENGTHS, USE WHEN IOPT=1
CENTX 41-50 CENTER OF CYL, USE WHEN IOPT=1
CENY 51-60 CENTER OF CYL, USE WHEN IOPT=1
ZC 61-70, 71-80 CONST SURFACE IMPEDANCE, USE WHEN IOPT=1

### NO MORE DATA NEEDED WHEN IOPT=1

(4) L IPRINT W
L 1-5 NUMBER OF SEGMENTS IN THE UPPER GEOMETRY
THE LOWER PART IS AN IMAGE OF THE UPPER
IPRINT 6-10 -1= PRINTS OUT LOTS OF STUFF
0= LESS STUFF, NO PLOT
1= LESS STUFF, PLUS A PLOT OF SURFACE CURRENT AND SURFACE IMPEDANCE
W 11-15, 16-20 STARTING POINT OF THE FIRST SEGMENT, IN CM
MAIN

C (5) MM ANG W ZFORM ZDIST ZA ZS ZEX
C MM  1-5 NUMBER OF CELLS IN THE SEGMENT
C ANG  6-10 ANGLE IN DEGREES SUBTENDED BY THE ARC
C W  11-15,16-20 END POINT OF THE SEGMENT IN CM
C ZFORM 21-25 -1= SPECIAL FORM--USER MUST SUPPLY
C FUNCTION ZFUNCA(ZA,ZS,ZEX,S)
C 0 = ZS=ZA+Z..*S**ZEX
C 1 = ZS=ZA+Z..*EXP(-ZEX*S)
C ZDIST 26-30 -1= S=X-DISTANCE FROM THE LEFT SIDE OF THE BODY
C 0 = S STARTS FROM THE LEFT SIDE OF THE BODY
C 1 = S STARTS FROM EACH SEGMENT
C ZA  31-35,36-40 SURFACE IMPEDANCE TERM, COMPLEX
C ZS  41-45,46-50 SURFACE IMPEDANCE TERM, COMPLEX
C ZEX  51-55 SURFACE IMPEDANCE TERM, REAL
C ### REPEAT CARD (5) FOR EACH ADDITIONAL SEGMENT
C
C (6) MORE WAVEL ZFAC
C MORE  1-5 1= REPEAT CARD (6) FOR NEW WAVELENGTH
C OR ZFAC
C WAVEL  6-10 END OF COMPUTATION FOR THIS GEOMETRY
C ZFAC 11-15,16-20 COMPLEX FACTOR THAT MULTIPLIES IMPEDANCE
C SPECIFIED BY ZA,ZS,ZEX
C
C ***** THE FOLLOWING CARDS ARE USED WHEN IOPT=0--READ IN POINTS
C
C (4) X Y ZS
C X  1-10 X-COORDINATE OF THE CELL ENDPOINT, IN WAVELENGTHS
C Y  11-20 Y-COORDINATE OF THE CELL ENDPOINT, IN WAVELENGTHS
C ZS 21-30,31-40 SURFACE IMPEDANCE AT THE POINT, COMPLEX
C ### REPEAT CARD (4) FOR EACH POINT, + THE FIRST POINT
C
C ***** EXAMPLE INPUT DATA FOR COMPUTATION OF J AND S FOR A SQUARE CYLINDER, SIDE=2, USING IOPT=-1 OPTION
C MARKERS(****) ARE POINTERS FOR COLUMNS 5,10,15,20, ETC
C
C (1) SQUARE KA=1. PERP POL ZS=(0.,0.1)
C (2) 16
C (3) 1
C (4) -1 -1 0 37180.
C (5) 2.0001-.999 1. 0 0 0 .1
C (6) 4.0001-999 1. 0 0 0 .1
C (7) 2.00011. 0. 0 0 0 .1
C (8) 06.283 1.
C *** * * * * * * * * * * * * * * * *
MAIN PROGRAM

COMMON /B06/DSQ(50),X(50),Y(50),TH(50),S(50),LL(50),PH(50)
       1, DELX(50,5),DELY(50,5),DELS(50,5)
       2, DELTS(50),XN(50),YN(50),XL(50),YL(50),SITH(50),COTH(50)
       COMMON /B04/AC(50),A(50),Q(50),THQ(50)
       COMMON /B03/ID(50),PI(50),PHI(50),AA(50,51),AMAG(50),PHASL(50)
       COMMON /B02/ACL,AK,ALPHA,EX,EY,EZ,COSL,SINL
       COMMON /B01/M,V,M2,KODE,IAIT,HPAR,HPER,N,NI
       COMMON /B00/IDC,MORE,IPRINT,IOPT,PI,WAVEL
       COMPLEX A:01,A:02,ZS,WS1,WS1,WTB,WS3,WS3,GTLLP,DI23,K1
       COMPLEX WC010,WCl01,WCl02,PI'NC,PHI,AA,A,ZC
       1 READ (5,5) (ID(I),I=1,36)
       5 FORMAT (36A2)
       IAT = 1
       BETA = 0.
       READ(5,9) M,N
       9 FORMAT (2I5)
       READ(5,10) IOPT,IPP,KODE,IAIT,ALPHA,RAIUS,CEIX,CELT,Y,ZC
       10 FORMAT (4I5,6F10.5)
       WRITE (6,6) (ID(I),I=1,36)
       6 FORMAT (1HI,3SA2)
       WRITE(6,15) IOPT,M,PIP,RADIUS,CEIX,CELT,IPP,ALPHA,BETA,ZC,KODE
       15 FORMAT (1H0,6I0PT,13,2X3HM,14,2X3HN,14,2X8HRADIUS,F7.5,
            1 2X7HCEIX=F7.3,2X7HCELT=F7.3,2X5HIPP=F7.3,2X8HALPHA=F7.3/27H BETA =F7.3,2X3X,4HZC =F2.10,3X,6HKODE=F13)
       MORE = 0
       HPAR = +1.0
       HPER = -1.0
       COSL = COSD(BETA)
       SINL = SIND(BETA)
       AK = 6.2831853
       AY = N
       N1 = +1
       IF (IOPT.NE.-1) IPRINT = -1
       IF (N.LT.3) N1 = N
       M2 = M
      REWIND 3
       IF (KODE.EQ.1) IAT = 2
       IF (KODE.EQ.1) GO TO 93
       91 CALL GEOM (M, N, IAT, RADIUS, CEIX, CEYT)
       IF (IAIT.LE.1) WRITE (3) X,Y,XN,YN,COTH,SITH,DSQ,TH,THQ
       CALL ALL7K(M,IPP,4)
       93 CONTINUE
       CALL PASS2 (IPP)
       IF (MORE.EQ.1) GO TO 91
       GO TO 1
       END

70
SUBROUTINE GEOM (M, N, WHIT, RADIUS, CELTX, CEHTY)
COMMON /BOA/DSQ(50), X(50), Y(50), TH(50), S(50), LL(50), PH(50)
1, DELX(50, 5), DELY(50, 5), DELS(50, 5)
2, DELTS(50), XN(50), YN(50), XL(50), YL(50), STH(50), COSH(50)
COMMON /BOE/ZC, MORE, IPRINT, IOPT, PI, WAVEL
COMPLEX ZS, ZSF, ZC, ZS, ZFAC, ZA, ZFUN
COMPLEX W, WA, WB, WC, WP, WW, ROI
AK=6.2831853
PI=AK/2.
RAD=PI/180.
AVN=N
VI=M+1
IF (N.LT.3) VI=-N
MI=M+1
IF (IOPT) 10, 40, 20
20 CONTINUE
WRITE (6, 25)
25 FORMAT(10H0, 4X2HXX, 11X2HYY, 11X4HREZS, 11X4HIMZS, 11X3HTHI,
13X14H---COMPUTED---)
THM=M
DHM=AK/THM
DO 35 I=1, MI
ZS(I)=ZC
XJ=I-1
TH(I)=3.14159-XJ*DHM
IF (TH(I).LE.0.) TH(I) = 0.0
IF (TH(I).GE.0.) TH(I) = TH(I)*180.0/3.14159
STH = SIN(TH(I))
CTH = COS(TH(I))
XX(I)=RADIUS*CTH
YY(I)=RADIUS*STH
WRITE (6, 30) XX(I), YY(I), ZS(I), TH(I)
35 CONTINUE
30 FORMAT (1H ,5E13.5)
GO TO 61
40 READ (5, 60) (XX(I), YY(I), ZS(I), I=1, MI)
45 WRITE (6, 50)
50 FORMAT(140, 4X2HXX, 11X2HYY, 11X4HREZS, 11X4HIMZS,
11X13H---READ IN---)
55 FORMAT (1H ,4E13.5)
60 FORMAT (4F10.5)
GO TO 61
10 IF (MORE.EQ.1) GO TO 62
READ(5, 135) L, IPRINT, WE
135 FORMAT(215, 2F5.3)
MSTART=1
MEND=0

71
ARC=0.
IF (IDPT, M, -1) GO TO 19
WRITE(6,51)
51 FORMAT(1H-,9X4HSEGM,1X5HZFORM,1X5HZDIST,4X8H---WA---,11X3H---W---,
1,6X8H SL ,1X7H AVG ,7X8H---ZA---,11X8H---Z---,
26X7H ZEX ,/
19 CONTINUE
DO 100 I=1,L
READ(5,145) MM, AVG, W, IZFORM, IZDIST, ZA, ZW, ZEX
145 FORMAT(15,3F5.2,2I5,5F5.2)
DANG=ANG/MM/RAD
RGT=COMPLEX(COS(DANG),-SIN(DANG))
WA=W3
WS=W
UA=WA
VA=WA*(0.,-1.)
UB=W3
VU=W3
ANGT=ANG/2.*RAD
DU=UA-UB
DV=VA-VU
R=SQRT(DU**2+DV**2)/2./SIN(ANGT)
DARC=R*DANG
D=(UB-UA)
D=D/ABS(D)
CT=1./TAN(A NGT)
UC=(UA+UB-D*(VA-VU)*CT)/2.
VC=(VA+VB-D*(UB-UA)*CT)/2.
WC=COMPLEX(UC,VC)
WP=WA-UC
MEVD=MEVD+MM
AM=MM
ARCL=0.
IF (1.EQ.L) MEND=MEND+1
DO 75 J=MSTART,MEND
WW=WP+WC
XD=WW
XF(J)=XD
YF(J)=WW*(0.,-1.)
SF(J)=ARC
XARC=XD-XF(J)
ARC=ARC+DARC
ARCL=ARCL+DARC
IF (IZDIST) 2,3,4
2 SD=XARC
GO TO 5
3 SD=ARC
GO TO 5
4 SD=ARCL
5 CONTINUE
IF (IZFORM) 6, 7, 6
6 ZSF(J) = ZFUNC(ZA, ZB, ZEX, SD)
   GO TO 9
7 ZSF(J) = ZA + ZB*SD**ZEX
   GO TO 9
8 ZSF(J) = ZA + ZB*EXP(-ZEX*SD)
9 CONTINUE
   WP = WP*ROI
   LL(J) = I
75 CONTINUE
   MSTART = MEND + 1
   IF (I. EQ. L) ARCL = ARCL-DARC
   IF (IOPT .NE. -1) GO TO 21
   WRITE (6, 52) I, IZFORM, IZDIST, WA, WC, ARCL, ANG, ZA, ZB, ZEX
21 CONTINUE
100 CONTINUE
   K = MEND
   SEND = SF(K)
   MEND = 2*K - 1
   DO 150 J = MSTART, MEND
      K = K - 1
      XF(J) = XF(K)
      YF(J) = -YF(K)
      SF(J) = 2.*SEND - SF(K)
      ZSF(J) = ZSF(K)
      LL(J) = LL(K - 1)
150 CONTINUE
62 READ(5, 175) MORE, WAVEL, ZFAC
175 FORMAT (I5, 3F5.2)
   IF (IOPT .NE. -1) GO TO 53
   WRITE (6, 23) ZFAC, WAVEL
23 FORMAT (1H0, 10X5HZFAC=, F8.3, 1H, , F8.3, 7X11HWAVELENGTH=, F7.3)
53 CONTINUE
   IF (IPRINT) 101, 102, 102
101 CONTINUE
   WRITE (6, 155)
155 FORMAT (1H0, 4X1HJ, 4X2HHX, 11X2HYY, 11X1HS, 11X4HREZS, 11X4HIMZS, 11X23H*** READ IN SURFACE ***)
102 CONTINUE
   DO 250 I = 1, MEND
      XX(I) = XF(I)/WAVEL
      YY(I) = YF(I)/WAVEL
      S(I) = SF(I)/WAVEL
      ZSI(I) = ZSF(I)*ZFAC
      IF (IPRINT) 111, 112, 112
111 CONTINUE
   WRITE (6, 160) I, XX(I), YY(I), S(I), ZSI(I)
160 FORMAT (1H , I3, 3E13.5)
112 CONTINUE

250 CONTINUE
61 CONTINUE
121 CONTINUE
WRITE (6,65)
65 FORMAT(1H0,4X1HX,12X1HY,12X1H5,12X1HA,12X2HRQ,11X3HTHQ,10X3HD5Q,
112X4HREZS,10X4HIMZS//)
122 CONTINUE
ZB=ZS(I)
SB=SI
DO 70 I=1,M
ZA=ZB
Z3=ZS(I+1)
ZS(I)=(ZA+Z3)/2.
SA=SI
SB=SI(I+1)
S(I)=(SA+SB)/2.
XI(I)=(XX(I)+XX(I+1))/2.0
YI(I)=(YY(I)+YY(I+1))/2.0
D=XI(I)-CENTX
A=YI(I)-CENTY
RQ(I)= SQRT(A*A+3*B)
THQ(I)= QATAVD(A,B)
DSQ(I)= SQRT((XX(I)-XX(I+1))**2+(YY(I)-YY(I+1))**2)
131 CONTINUE
WRITE (6,31) XI(I),YI(I),A,RQ(I),THQ(I),DSQ(I),ZS(I)
31 FORMAT (1H9,9E13.5)
132 CONTINUE
70 CONTINUE
DO 80 I = 1,M
A = YY(I+1)-YY(I)
B = XX(I+1)-XX(I)
THN(I)= 90.0+QATAVD(A,B)
XL(I)=-A/DSQ(I)
YL(I)=-B/DSQ(I)
80 CONTINUE
DO 300 I=1,M
DELTX=(XX(I+1)-XX(I))/AN
DELY=(YY(I+1)-YY(I))/AN
DELX(I,1)=XX(I)
IF (N.EQ.1) DELX(I,1)=DELTX/2.+XX(I)
DELY(I,1)=YY(I)
IF (N.EQ.1) DELY(I,1)=DELTY/2.+YY(I)
DELS(I,1)=DSQ(I)/A4
IF (N.LT.3) GO TO 300
DO 301 J=2,N
DELEX(I,J)=DELEX(I,J-1)+DELTX
301 CONTINUE
DELY(I,J) = DELY(I,J-1) + DELTY
DELS(I,J) = DELS(I,J-1)

301 CONTINUE
300 CONTINUE
141 IF (IPRINT) 141, 142
142 CONTINUE

WRITE (6, 5150)
5150 FORMAT (14HDELX, DELY, DELS//)

DO 5151 I = 1, M
DO 5151 J = 1, N
WRITE (6, 5152) I, J, DELX(I,J), DELY(I,J), DELS(I,J), XN(I), YN(I), XL(I)
5152 FORMAT (1H, 214, 7E13.5)

142 CONTINUE
RETURN
END
SUBROUTINE PASS2(IPP)

COMMON /30B/DSQ(50),X(50),Y(50),THN(50),S(50),LL(50),PH(50)
1, DELX(50, 5), DELY(50, 5), DELS(50, 5)
2, DELS(50), XN(50), YN(50), XL(50), YL(50), SITH(50), COTH(50)
COMMON /30A/AA(50, 50), A0(50, 50)
COMMON /30A/AA(50, 50), TH(50), XX(50), YY(50), RR(50), THQ(50)
COMMON /30A/IN(60), PI(50), PHI(50), AA(50, 51), AMAX(50), PHASE(50)
COMMON /30A/AM, AK, ALPH, EX, EY, EZ, COS, SIN
COMMON /30S/IM, J, MM, KODE, IAT, HPAR, HPER, WIN
COMMON /30E/ZA, ZT, MOTE, IPRINT, IOPT, PI, WAVE
COMPLEX A301, A302, ZS, WS1, WT1, WT3, WS4, WS30, WS3, GILLP, UI, SI, BI
COMPLEX WC100, WC101, WC102, PINC, PHI, AA, AB, SUM, ZC
DIMENSION PTS(100, 5), A(6)
DATA A/1H-,1H-,1H-,1H-,1H-,1H-/IF (KODE.NE.0)READ (3) X, Y, XN, YN, COTH, SITH, DSQ, TH, THQ
CALL FIELD (IPP, HPAR, HPER)
IF (IOPT.EQ.0 .OR. IOPT.EQ.1) WAVE1=1.0
M2=M
M1=M+1
IF (KODE.NE.0) READ (3) AA
IF (IPRINT) 101, 102, 102
101 RENORM=0.5*WAVE1/PI
WRITE (6,155)
155 FORMAT (1X0,74A(I,J)/)
DO 5046 J=1,M2
5046 WRITE (6,5030) (AA(I,J),J=1,M2)
5030 FORMAT (1H0,8E13.5)
102 RENORM=0.5*WAVE1/PI
CALL ZV08 (AA, M2, PINC, PHI, IAT)
IF (IPRINT) 111, 112, 112
111 CONTINUE
IF (KODE.EQ.0) WRITE (3) AA
IF (IPP.EQ.0) WRITE (6,110)
IF (IPP.EQ.1) WRITE (6,125)
IF (IPP.EQ.2) WRITE (6,140)
IF (IPP.EQ.3) WRITE (6,155)
110 FORMAT (1H1,30X20HCURRENT DISTRIBUTION,3X14H---PARALLEL---//)
125 FORMAT (1H1,30X20HCURRENT DISTRIBUTION,3X19H---PERPENDICULAR?---//)
140 FORMAT (1H1,30X20HCURRENT DISTRIBUTION,3X28HPERFECT CONDUCTOR---P
1PARALLEL//)
155 FORMAT (1H1,30X20HCURRENT DISTRIBUTION,3X33HPERFECT CONDUCTOR---P
1PERPENDICULAR//)
171 FORMAT (1H0,24X14HINCIDENT FIELD)
112 CONTINUE
DO 172 I=1,M2
TI=PINC(I)
T2=PINC(I)*(0.,-1.)
PHI(I)=0.
IF (T2.NE.0.) PHI(I)=QATAND(T2, TI)

76
IF (IPRINT) 121,122,122
121 CONTINUE
WRITE (6,30) PIVC(I)
30 FORMAT (1H,20X,6E13.6)
122 CONTINUE
172 CONTINUE
IF (IPRINT) 131,132,132
131 CONTINUE
WRITE (6,175)
175 FORMAT (1H0,22X5HTHETA,10X9HMAGNITUDE,10X5HPHASE//)
132 CONTINUE
DO 180 I=1,92
  T1=REAL(PHI(I))
  T2=AIMAG(PHI(I))
  AMAG(I)=SQRT(T1*T1+T2*T2)
  PHASE(I)=0.
  IF (T2,VE.0.0) PHASE(I)=360*ATAND(T2,T1)
  IF (IPRINT) 141,142,142
141 CONTINUE
WRITE (6,185) THET(I),AMAG(I),PHASE(I)
185 FORMAT (1H0,20X,7.2,3X3E18.8)
142 CONTINUE
180 CONTINUE
IF (IoPT,VE.-1) GO TO 19
WRITE (6,10)
10 FORMAT(///,10X4HSEG,M,3XHJ,J,6XIX,I,8XHY,Y,8XHS,5XHJMOD,
      14XHJPHASE,3XHJS,REAL,3XHJS,IMAG,2XHJPHASE,/) 
      WRITE(6,11)LL(I),I,X(I),Y(I),S(I),AMAG(I),PHASE(I),ZS(I),PH(I),
      II=1,9)
11 FORMAT(1H0,113,15,3F9.3,4F9.3)
> THIS PORTION OF THE PROGRAM PLOTS CURVES
> IF (IPRINT.EQ.0) GO TO 19
> SUBROUTINE GPM IS REQUIRED
WRITE (6,12)
12 FORMAT(1H1)
DO 13 I=1,6
  DO 14 J=1,92
    IF(I.EQ.1) PTS(J,I)=0.5
    IF(I.EQ.2) PTS(J,I)=1.0
    IF(I.EQ.3) PTS(J,I)=2.0
    IF(I.EQ.4) PTS(J,I)=REAL(S(J))/2.+5
    IF(I.EQ.5) PTS(J,I)=AIMAG(S(J))/2.+5
    IF(I.EQ.6) PTS(J,I)=AMAG(S(J))
14 CONTINUE
13 CONTINUE
CALL GPM(S,PTS,5,10,M,6,M,51,A)
19 CONTINUE
WRITE (6,200)
200 FORMAT (1H1,10X33H:ISTATIC SCATTERING CROSS SECTION//
      1 27X5HTHETA,6X5HSGM,6X8HSGM D5)
TAU=180.0
DTAU= 90./FLOAT(WIT-1)
DO 85 I=1,WIT
TH(I)=TAU
SITH(I)=SIN(WIT(I))
COTH(I)=COSD(TH(I))
85 TAU=TAU-DTAU
WIT=WIT+1
TH(WIT)=ALPHA
SITH(WIT)=SIN(WIT(WIT))
COTH(WIT)=COSD(TH(WIT))
IF (IPP.EQ.0 .OR. IPP.EQ.2) GO TO 205
IF (IPP.EQ.1 .OR. IPP.EQ.3) GO TO 225
205 DO 215 L=1,WIT
IF (IPP.EQ.2) GO TO 301
GO TO 305
301 DO 302 I=1,MI
302 ZS(I)=(0.,0.)
305 CONTINUE
SUM=(0.,0.)
DO 210 J=1,WIT
RDOT=TH(L)*X(J)+SITH(L)*Y(J)
OS=RDOT
ARCSOS=-(COTH(L)*X(J)+SITH(L)*Y(J)) *360.*COS(PI/PHASE(J))
SUM=SUM+AMAG(J)*DSQ(J)*(OS-ZS(J)-1.)*CMPLX(COSD(ARGCOS),SIND(1/ARGCOS))
210 CONTINUE
SACST=9.8696*CABS(SUM)**2.
SACST=SACST*RENUM
BACDB=10.ALOG10(SACST)
215 WRITE (*,220) TH(L),SACST,BACDB
220 FORMAT (1H ,25XF7.2,F13.8,F13.8)
GO TO 240
225 DO 235 L=1,WIT
IF (IPP.EQ.3) GO TO 311
GO TO 315
311 DO 312 I=1,MI
312 ZS(I)=(0.,0.)
315 CONTINUE
SUM=(0.,0.)
DO 230 J=1,WIT
RDOT=TH(L)*X(J)+SITH(L)*Y(J)
OS=RDOT
ARCSOS=-(COTH(L)*X(J)+SITH(L)*Y(J)) *360.*COS(PI/PHASE(J))
SUM=SUM+AMAG(J)*DSQ(J)*(OS-ZS(J))*CMPLX(COSD(ARGCOS),
1*SIND(ARGCOS))
230 CONTINUE
SACST=9.8696*CABS(SUM)**2.
SACST=SACST*RENUM
BACDB=10.ALOG10(SACST)
235 WRITE (*,220) TH(L),SACST,BACDB
> 240 CONTINUE
>        REVERSE 3
>        RETURN
>        END
SUBROUTINE ALLLK(M, IPP, N)
  COMMON /30/ DSQ(50), X(50), Y(50), TH(50), S(50), LL(50), PH(50)
  1, DELX(50, 5), DELY(50, 5), DELS(50, 5)
  2, DELTS(50, X(50), Y(50), XL(50), YL(50), SITH(50), COTH(50)
  COMMON /30A/ A(50, 50), A..2(50, 50)
  COMMON /30B/ S(50), TH(50), XX(50), YY(50), RQ(50), THQ(50)
  DIMENSION XS(10), YS(10), XSS(10), YSS(10), XI(10), YT(10)
  1, XU(10), YU(10), GTLLP(50)
  COMMON /BOC/ AK, AK, ALPHA, EX, EY, EZ, COS., SIN.
  REAL JO, J1, JO, W1, J2, W2, J3, W3
  DIMENSION JO(10), JW(10), J1(10), W1(10), J2(10), W2(10), J3(10), W3(10)
  COMPLEX A01, A02, ZS, W1, WT1, WT8, W8, WS30, W53, GTLLP, ... I5, ... K1
  S1VA=CSS
  AK=0.2831853*STWA
  AK4=AK/4.
  IF(Z1.GE.3) N= N+1
  W53 = CMPLX(0., 25)*AK
  WAI=AK
  DO 100 I = 1, M
  GTLLP(I) = CMPLX(0., 0.)
  DO 100 J = 1, M
  IF (1.E0.J) GO TO 85
  TX = X(I) - X(J)
  TY = Y(I) - Y(J)
  APP = SQRT(TX*TX + TY*TY)
  CWR = ((X(I) - X(J))*X(I) + (Y(I) - Y(J))*Y(J))/APP
  CRP = ((X(J) - X(I))*X(I) + (Y(J) - Y(I))*Y(J))/APP
  CWS = SQRT(A01 - CWR*CWR)
  CRSP = SQRT(A02(1. - CRP*CRP))
  CLR = ((X(I) - X(J))*XL(I) + (Y(I) - Y(J))*YL(I))/APP
  CLRP = ((X(J) - X(I))*XL(J) + (Y(J) - Y(I))*YL(J))/APP
  DO 300 L = 1, N
  DY = (Y(I) - DELY(J, L))
  DX = (X(I) - DELX(J, L))
  P = SQRT(DX*DX + DY*DY)
  SMLG = (-DY*DX*XL(J)*YL(I) + DY*DX*YL(J)*XL(I) + DX*DY*XL(J)*YL(I)
                   - DX*DX*YL(J)*YL(I))/P**3
  RANS = WAI*P
  IF (RANS.LE.90) CALL HANK (RANS, I, X, Y, R)
  IF (RANS.GT.90) CALL HANKY (RANS, I, X, Y, R)
  W1 = CMPLX(X, Y, R)
  W51 = WT1*DSQ(J)
  J1(L) = X, Y
  J0(L) = Y
  IF (RANS.LE.90) CALL HANK (RANS, 0, X, Y, R, Y)
  IF (RANS.GT.90) CALL HANKY (RANS, 0, X, Y, R, Y)
  W8 = CMPLX(X, Y, R, Y)
  WS8 = WT8*DSQ(J)
  J08(L) = X, Y, R
  N0(L) = YR, R
IF (IPP.EQ.1.OR.IPP.EQ.2) GO TO 300
IF (RANS.LE.90) CALL HANK (RANS,2,X2,Y2)
IF (RANS.GT.90) CALL HAVY (RANS,2,X2,Y2)
WS30=CMPLX(X2,Y2)
E(1)=WT1*SMLG+CLRP*(+WT8*WAI-WT1/P)*CLR
BIGG=WAI*(X(I)*X(J)+Y(I)*Y(J))*CMPLX(XRR,YRR)+.K1
IF (N.LT.3) GLLLP(I)=GLLIP(I)+.K1*DSQ(J)/4.
J3(I)=REAL (E(1))
V3(I)=AIMAG (E(1))
WS30=BIGG*DSQ(J)
J2(I)=REAL (BIGG)
V2(I)=AIMAG (BIGG)
CONTINUE
GO TO 300
IF (N.LT.3) GO TO 15
DS=DELS(J,1)
CALL SIMP(J,1,0,0,DS,V,XS,0,1)
CALL SIMP(J,1,0,0,DS,V,YS,0,1)
WS1 = CMPLX(XS(1),YS(1))
CALL SIMP(J,0,0,0,DS,V,XSS,0,1)
CALL SIMP(J,0,0,0,DS,V,YSS,0,1)
WS8 = CMPLX(XSS(1),YSS(1))
IF (IPP.EQ.1.OR.IPP.EQ.2) GO TO 15
CALL SIMP(J,0,0,0,DS,V,XT,0,1)
CALL SIMP(J,0,0,0,DS,V,YT,0,1)
WS30 = CMPLX(2*XT(1),YT(1))
CALL SIMP(J,0,0,0,DS,V,XU,0,1)
CALL SIMP(V,0,0,0,DS,V,YU,0,1)
GLLIP(I)=GLLIP(I)+CMPLX(XU(1),YU(1))/4.
CONTINUE
GO TO 15
IF (IPP.EQ.3) GO TO 75
IF (IPP.EQ.2) GO TO 70
IF (IPP.EQ.1) GO TO 80
A0(I,J)=WS1*CVR*WS3
A02(I,J)=.25*WS30
GO TO 100
70 A0(I,J)=WS1*CVR*WS3
70 CONTINUE
A02(I,J)=WS8*AK4
GO TO 100
75 A0(I,J)=WS1*CVR*WS3
75 CONTINUE
A02(I,J)=.25*WS30
GO TO 100
80 A0(I,J)=WS1*CVR*WS3
A02(I,J)=WS8*AK4
GO TO 100
85 A0(I,J)=CMPLX(5,0)
85 CONTINUE
A02(I,J)=CMPLX(1.57079,ALOG (1.57079*DSQ(J)*COS(J)-.4223)*DSQ(J))
IF (IPP.EQ.3.0 .OR. IPP.EQ.0) ABO2(I,J)=-ABO2(I,J)
100 CONTINUE
IF (IPP.EQ.1.0 .OR. IPP.EQ.2) GO TO 201
DO 200 I=1,M
200 ABO2(I,I)=ABO2(I,I)+GTLLP(I)
201 CONTINUE
5030 FORMAT (1H ,8E13.5)
RETURN
END
SUBROUTINE GPM(X, Y, L, S, M, N, W, LW, A)
*C THIS SUBROUTINE HAS BEEN ESPECIALLY MODIFIED FOR USE WITH RAMI.
*C** CONTROL
*C
*C CALL GPM(X, Y, L, S, M, N, W, LW, A)
*C** WHERE
*X = ARRAY OF INDEPENDENT VALUES, DIMENSIONED X(M).
*Y = ARRAY OF SETS OF DEPENDENT VALUES, DIMENSIONED Y(M,N).
*L = NUMBER OF LINES TO BE SKIPPED BEFORE DISPLAY.
*S = NUMBER OF SPACES FROM LEFT SIDE OF PAGE TO
** BE SKIPPED BEFORE DISPLAY.
*M = NUMBER POINTS IN EACH SET.
*N = NUMBER OF SETS OF POINTS.
*W = WIDTH OF DISPLAY IN PRINT SPACES.
*LW = LENGTH OF DISPLAY IN PRINT LINES.
*A = ARRAY OF SINGLE CHARACTERS, DIMENSIONED A(N), TO
** REPRESENT THE TREND FOR EACH SET (EX. - DATA A/1HA,
*IHs,...Etc.)
*C**

DIMENSION X(M), A(N)
DIMENSION PLOT(51,100),Y(100,6)

INTEGER S, W, WI

DATA BLANK/1H/,VERT/1H/,HORIZ/1H/=/
*C** CHECK MAXIMUM WIDTH AND LENGTH REQUESTED AND
** EXIT IF NOT CORRECT
*C** IF (S+W .GT. 131) GO TO 900
*C** IF (L+LW .GT. 98) GO TO 800
*C** FIND MINIMUM AND MAXIMUM OF X AND Y
*C** XMAX=X(1)
*XMIN=X(1)
*C** DO 10 I=2,M
*C** IF (X(I) .GT. XMAX) XMAX=X(I)
*C** IF (X(I) .LT. XMIN) XMIN=X(I)
*C
*YMIN=0.
*YMAX=2.5
*C** COMPUTE SCALE FACTOR -- P FOR X, Q FOR Y
*C** P=FLOAT(W-1)/(XMAX-XMIN)

83
Q = FLOAT(LN-1)/(YMAX-YMIN)

BLANK PLOT ARRAY

DO 30 I=1, W
DO 30 J=1, LV
30 PLOT(J, I) = BLANK

CONSTRUCT BORDER OF DISPLAY

DO 40 J=1, LV
I = 1
PLOT(J, I) = VERT
I = W
40 PLOT(J, I) = VERT

W1 = W-1
DO 50 I=2, W1
J = 1
PLOT(J, I) = HORIZ
J = LV
50 PLOT(J, I) = HORIZ

COMPUTE SUBSCRIPTS AND INSERT TREND CHARACTER IN
PLOT ARRAY

DO 60 I=1, M
DO 60 J=1, W
II = I+INT(0.5+P*(X(I)-XMIN))
J1 = LV-INT(0.5+Q*(Y(I, J)-YMIN))
60 PLOT(J1, II) = A(J)

SKIP L LINES BEFORE BEGINNING DISPLAY PRINTING

DO 70 K=1, L
70 PRINT 600
600 FORMAT (1H )

WRITE OUT PLOT ARRAY, SKIPPING S SPACES BEFORE PRINTING
EACH LINE OF DISPLAY

DO 80 J=1, LV
80 PRINT 601, (BLANK, K=1, S), (PLOT(J, I), I=1, W)
601 FORMAT (132AI)
PRINT 602, XMIN, XMAX, YMIN, YMAX
602 FORMAT (1H0, 5X, 6HXMIN =E16.8, 10X, 6HXMAX =E16.8, 10X,
6HYMIN =E16.8, 10X, 6HYMAX =E16.8)
RETURN

ERROR MESSAGES BEFORE TERMINATION
**C**

> 300 PRINT 603, L, LN
> 603 FORMAT (30HAL+LN IS GREATER THAN 58 L = I3, 5X, 4HLN = I5)
> CALL SYSTEM
> 900 PRINT 604, S, W
> 604 FORMAT (30HAS+W IS GREATER THAN 131 S = I3, 5X, 3HW = I3)
> CALL SYSTEM
> END
SUBROUTINE ZV08 (A, N, X, Y, IAT)
DIMENSION A(50,51), X(50), Y(50,50,50), M(50)
COMPLEX A, D, BIGA, HOLD, Y, X
INTEGER L, M
IF (IAT-1) 200,200,300
200 CONTINUE
D=CMPLX(1.0,0.0)
DO 60 K=1,N
L(K)=K
M(K)=K
BIGA=A(K,K)
DO 20 J=K,N
DO 20 I=K,N
10 IF (CABS(BIGA)-CABS(A(I,J))) 15,20,20
15 BIGA=A(I,J)
L(K)=I
M(K)=J
20 CONTINUE
J= L(K)
IF (J-K) 35,35,25
25 DO 30 I=1,N
HOLD=-ACK, I
ACK, I)=A(J,I)
30 A(J,I)=HOLD
35 I=M(K)
IF (I-K) 45,45,38
38 DO 40 J=1,N
HOLD=-ACK, K)
40 A(J,K)=A(J,I)
45 IF (CABS(BIGA)) 48,46,48
46 D=CMPLX(0.0,0.0)
WRITE (6,220) D
RETURN
48 DO 55 I=1,N
IF (I-K) 50,55,50
50 A(I,K)=A(I,K)/(-BIGA)
55 CONTINUE
C REDUCE MATRIX
 DO 65 I=1,N
 DO 65 J=1,N
 IF (I-K) 60,65,60
60 IF (J-K) 62,65,62
65 CONTINUE
C DIVIDE ROW BY PIVOT
 DO 75 J=1,N
 IF (J-K) 70,75,70
70 ACK, J)=ACK, J)/BIGA
75 CONTINUE
PRODUCT OF PIVOTS

D = D * HIGA
A(K, K) = (1.000, 0.000) / HIGA

80 CONTINUE

84 N = N

DMAG = C * ABS(D) * (2.0 ** N)
WRITE(6, 220) D, DMAG

K = N
100 K = K - 1

IF (K) 150, 150, 105

105 I = L(K)

IF (I - K) 120, 120, 108

108 DO 110 J = 1, N

HOLD = A(J, K)
A(J, K) = - A(J, I)

110 A(J, I) = HOLD

120 J = M(K)

IF (J - K) 100, 100, 125

125 DO 130 I = 1, N

HOLD = A(K, I)
A(K, I) = - A(J, I)

130 A(J, I) = HOLD

GO TO 100

150 CONTINUE

300 CONTINUE

DO 210 I = 1, N

Y(I) = CMPLX(0.0, 0.0)

DO 210 J = 1, N

210 Y(I) = A(I, J) * X(J) + Y(I)

RETURN

220 FORMAT(140, 2X, 'THE DETERMINANT IS ', E15.8, '4H + I ', E15.8, '20H. IT
1 MAGNITUDE IS ', E15.8)

END
SUBROUTINE FIELD (IPP, HPAR, HPER)
COMMON /B00/ DSN(50), X(50), Y(50), TH(50), S(50), LL(50), PH(50)
1, DELX(50, 5), DELY(50, 5), DELS(50, 5)
2, DELTS(50), XN(50), YN(50), XL(50), YL(50), SITH(50), COTH(50)
COMMON /B01/ AB01(50, 50), AB02(50, 50)
COMMON /B02/ X(50), Y(50), XX(50), YY(50), RR(50), TH(50)
COMMON /B03/ IDO(60), PI(50), PHI(50), AA(50, 51), AMAG(50), PHASE(50)
COMMON /B04/ AKP, AK, ALPHA, EX, EY, EZ, COSB, SINB
COMMON /B05/ M, N, KODE, IAT, DUM1, DUM2, DUM3
COMPLEX AB01, AB02, ZS, WS1, WT1, WT8, WS8, WS3, GILL, SIG0, SIG1
COMPLEX WC100, WC101, WC102, PINC, PHI, AA, AB, WS105, WS106
AK = 6.2831853
WS100 = COSD(ALPHA)
WS101 = SIND(ALPHA)
DO 30 I = 1, M
WS102 = AK * COSB * (WS100 * X(I) + WS101 * Y(I))
PINC(I) = CMPLX(COS(WS102), -SIN(WS102))
IF (IPP.EQ.1 .OR. IPP.EQ.3) GO TO 10
PINC(I) = -PINC(I) * COS0 * (WS100 * XN(I) + WS101 * YN(I))
10 DO 20 J = 1, M
AA(I, J) = -AB01(I, J)
IF (IPP.EQ.1) AA(I, J) = AA(I, J) + AB02(I, J) * ZS(J)
IF (IPP.EQ.0) AA(I, J) = AA(I, J) - AB02(I, J) * ZS(J)
20 CONTINUE
RETURN
END
SUBROUTINE HANK (X, Y, BESJ, BESY)
DIMENSION FACT(30), PHI(30)
C DIMENSION FACT(30), PHI(30)
IF (X.LE.0.0001) GO TO 600
605 CONTINUE
FACT(1) = 1.0
DO 10 I = 2, 30
FI = I - 1
10 FACT(I) = FACT(I - 1) * FI
PHI(I) = 0.0
SUM1 = 0.0
DO 21 I = 2, 25
XI = I - 1
PHI(I) = SUM1 + 1.0 / XI
21 SUM1 = PHI(I)
CALL BESJN (X, Y, BESJS)
HALFX = X / 2.0
IF (N) 20, 30, 40
20 WRITE (6, 500) N
500 FORMAT (10H BESYN N=I3)
RETURN
30 SUM1 = 0.0
TERM1 = 1.0
BESY = .63662*(ALOG(HALFX) + .5772157) * BESJS
DO 50 I = 1, 20
L = I + 1
GO TO (1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2), I
1 SIGN = -1.0
GO TO 3
2 SIGN = 1.0
3 EXN = 2*I
SAVE = (-.63662*SIGN*(HALFX**EXN))/FACTL)**2)*TERM1
IF (ABS(SAVE) - 1.E-9) 60, 60, 70
50 BESY = BESY + SUM1
RETURN
70 SUM1 = SUM1 + SAVE
RI = I + 1
50 TERM1 = TERM1 + 1.0 / RI
40 SUM2 = 0.0
SUM3 = 0.0
TOTAL = 0.0
BESY = 0.0
TERM1 = (ALOG(HALFX) + .5772157) * BESJS
DO 25 KP = 1, N
K = KP - 1
EXN = 2*K - N
FACT = N - K
TERM2 = TERM2 + (FACT* (FACT)*HALFX**EX2)/(2.0*FACT(KP))
25 SUM2 = SUM2 + TERM2
DO 31 KP = 1, 20
89
X=KP-1
GO TO (4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5,4,5),KP
4 HOLD=-1.0
GO TO 6
5 HOLD=1.0
6 NEXP=2*K+N
KSUB=KP+N
TERM3=(HOLD*(PHI(KP)+PHI(KSUB)))*HALFX**NEXP/(2.0*(FACT(KP))
1 FACT(KSUB))
SUM3=SUM3+TERM3
DESSY=.63662*(TERM1-SUM2+SUM3)
IF(DESSY-TOTAL)31,41,31
31 TOTAL=DESSY
41 RETURN
600 CONTINUE
WRITE (6,601) X
601 FORMAT (1HO,4HX = F10.5)
X=.0001
GO TO 605
END
SUBROUTINE HANKY(Z,W1,BESSJ,BESSY)
DIMENSION B(60),C(60)
IF (Z.EQ.0.0) Z=0.01
T=1.E-10
IHOPE=50
N=(12.65+SQRT(Z)/6.)*ALOG(Z+2.)+Z+45./(Z+7.5)
IF (N.GT.IHOPE) N=50
W1=W1+1
W2=W2+2
G=0.
S(W1)=T
XX=W2
S(W2)=T*X/(2.*XX)
DO 50 I=1,N
K=W2-1
XX=XX-1.
S(K-1)=(2.*XX-2.)**3/(Z-B(K+1))
G=G+(S(K-1))**2
G=(2.*G-G(1))**(0.5)
(S(W1))=0.
(S(W2))=0.
DO 40 I=1,N
S(I)=S(I)*G
AM=0.0
AX=0.0
SX=0.0
AY=0.0
LY=0.0
M=0
15 AM=AM+1.0
M=M+1
X=((-1.0)**(M-1))*(2.0*AM)/(AM*AM)**2*(2*M+1)
Y=((-1.0)**(M-1))*(1.0+2.0*AM)/(AM*(1.0+AM))**2*(2*M+2)
CX=BX
BX=AX
AY=AY+Y
GO TO 15
GO TO 15
41 Y1=S(1)*ALOG(Z)+AX
Y2=B(2)*(ALOG(Z)-1.0)-B(1)/Z+AY
Y1=(2./3.)*Y1-(2./3.)*Y2*(.69314718-.57721567)**(3(1))
Y2=(2./3.)*Y2-(2./3.)*Y1*(.69314718-.57721567)**(3(2))
C(1)=Y1
C(2)=Y2
DO 55 M=3,N
55 END
AX2 = M - 2

55 C(M) = ((2.0 * AM2) / Z) * C(M-1) - C(M-2)

C(N1) = 0.
C(N2) = 0.
NUM = I + NN.
BESSJ = B(NUM)
BESSY = C(NUM)
RETURN
END
SUBROUTINE BESSJN (X, Y, BESSJ)
DIMENSION FACT(30)
C
DIMENSION FACT(30)
FACT(1)=1.0
DO 10 I=2,30
FI=I-1
10 FACT(I)=FACT(I-1)*FI
HALFX=X/2.0
SUM=0.0
BESSJ=0.0
DO 20 K=1,10
GO TO (1,2,1,2,1,2,1,2,1,2),K
1 HOLD=1.0
GO TO 30
2 HOLD=-1.0
30 EXN=Y+2*(K-1)
KN=K+N
TERM1=1.0/(FACT(K)*FACT(KN))
IF (EXN.GT.80.0) GO TO 50
TERM2=HALFX**EXN
SAVE=TERM1*TERM2*HOLD
BESSJ=BESSJ+SAVE
COMP=A35(SAVE/BESSJ)
IF (COMP-0.0001) 40,40,20
20 SUM=BESSJ
40 RETURN
50 WRITE (6,51) EXN,K,N
51 FORMAT (1X0,6HEXN=F10.5,5X,4HK=I3,5X,4HN=I3)
STOP
END
SUBROUTINE SIMP (Y, X1, X2, DX, NP, A, CON, K)

MODIFIED SIMPSON'S RULE FOR INTEGRATION

(MORE COMMONLY KNOWN AS HEATH'S RULE)

DELTA X MUST BE CONSTANT BUT NUMBER OF X'S NEED NOT BE EVEN.

K=0 RUNNING INTEGRALS COMPUTED

K=1 COMPUTE TOTAL INTEGRAL ONLY

DIMENSION Y(10), A(10)

SUM=CON

A(1)=CON

IF (X2-X1) 1,2,2

1 DX=-1.0*ABS(DX)

2 IF (NP-3) 3,4,4

4 IF (K) 5,5,6

5 NDEL=1

M=2

GO TO 7

6 NDEL=0

M=1

7 LP= NP-2

HOLD=DX/12.0

DO 50 I=1,LP

SUM=SUM+HOLD*(5.0*Y(I)+8.0*Y(I+1)-Y(I+2))

A(M)=SUM

50 M=M+NDEL

A(M)=SUM+HOLD*(5.0*Y(NP)+8.0*Y(NP-1)-Y(NP-2))

RETURN

A(1)=0.0

RETURN

END
FUNCTION QATAN (A,B)
  IF(.T.) 2,1,2
  1 QATAN=0.
  RETURN
  2 CONTINUE
  QATAN=ATAN2(A,B)
  RETURN
END

FUNCTION QATAND(A,B)
  IF(.T.) 4,3,4
  3 QATAND=0.
  RETURN
  4 CONTINUE
  QATAND= ATAN2(A,B) * 57.2957795
  RETURN
END

FUNCTION SIND(A)
  SIND = S1W(.0174532925 * A)
  RETURN
END

FUNCTION COSD(A)
  COSD = COS(.0174532925 * A)
  RETURN
END

COMPLEX FUNCTION ZFUN(ZA,ZB,ZEX,ST)
*C THIS FUNCTION USES A GAUSSIAN SHAPE FOR THE IMPEDANCE
COMPLEX ZA,ZB
ST=(ST-REAL(ZB))/ZEX
ST=.5*ST*ST
IF (ST-ST.35.) GO TO 10
ZFUN=ZA*EXP(-ST)
RETURN
10 ZFUN=(0.,0.)
RETURN
END
REFERENCES


NON-SPECULAR RADAR CROSS SECTION STUDY

Technical Report, Scientific, Interim

Eugene F. Knott and Thomas B. A. Senior

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11. SUPPLEMENTARY NOTES

The significance of the research summarized herein is the development of techniques for reducing the radar cross sections of non-specular types of scatterers. For the specific purpose of optimizing camouflage treatments a computer program is used which digitally solves the surface field integral equations for an impedance boundary condition. The program, furnished by the Air Force Avionics Laboratory and later modified by the Radiation Laboratory to handle variable impedances, computes the currents induced on the surfaces of two-dimensional obstacles by an incident plane wave and sums the currents to obtain the far bistatic field. Results for two different bodies are presented showing that control of the surface impedance can produce substantial radar cross section reductions, but that the impedance must not be changed too rapidly. Specific results are included showing the effects of different loading locations and impedance variations on the surface and far bistatic fields for two selected shapes.
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