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ON AVERAGE OUTGOING QUALITY

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ABSTRACT

All manufacturing and assembly operations have an associated inspection operation to ensure that the product conforms to specifications. For a product which has several quality characteristics each of which must be inspected for conformity, it is sometimes necessary to inspect the part more than once. This is especially true for cases where the inspection procedures cannot be automated. A model is proposed in this paper to determine the minimum number of repeat inspections that would be required to achieve a desired level of outgoing quality. Two types of inspection error and multiple inspections are considered in model development. The application of the model is demonstrated through several examples.

NOMENCLATURE

- $(AOQ)_i$ = average outgoing quality after stage i
 e_{1i} = probability that a conforming item is classified as non-conforming at stage i
 e_{2i} = probability that a non-conforming item is classified as conforming at stage i
 $(e_1)_{ij}$ = probability of a type I error occurring at inspection stage i ($1 \leq i \leq n$) when inspecting characteristic \bar{j} ($1 \leq j \leq J$)
 $(e_2)_{ij}$ = probability of a type II error occurring at inspection stage i ($1 \leq i \leq n$) when inspecting characteristic \bar{j} ($1 \leq j \leq J$)
 J = number of characteristics to be inspected per item
 n = number of stages of inspection necessary
 λ_j = mean number of non-conformities for characteristic j [Poisson parameter for characteristic j ($1 \leq j \leq J$)]

INTRODUCTION

The productivity of manufacturing and other production or assembly operations can be gaged along several dimensions. A somewhat simplistic indicator of an increase in productivity is an increase in the number of units of product produced or a decrease in unit cost of product. More realistically, however, measures of productivity must account for product quality which, when considered together with unit cost, will determine whether or not a product is competitive.

It is well recognized that quality must be built into a product. It should be assured at every stage of the production cycle, from initial design to final delivery. Quality cannot be inspected into a product. However, inspection becomes necessary to ensure that a product conforms to specifications. It is therefore performed at several stages of the production cycle.

Inspection can be by variables or by attributes and is often based on a sampling plan which can be designed in many ways. Sampling plans have been developed that make appropriate allowances for errors of judgement made by human inspectors (1-5,7). Automated or machine inspection, however, is considered to be relatively free of error. In each case, the accept-reject decision of the inspection device significantly determines the overall quality of the outgoing product, and the effectiveness of a sampling plan can be indicated by a measure of the Average Outgoing

Quality (AOQ) of the product inspected.

Another method of improving AOQ is to subject a product lot to repeat inspections, preferably by independent inspectors. Thus each screening would eliminate a certain percentage of the non-conforming items. This procedure usually results in an improvement in the overall outgoing product quality by reducing the effects of human errors of judgement (8).

There are two types of errors of judgement associated with an inspection task: rejecting a conforming item (type I error) or accepting a non-conforming item (type II error). Although rejecting a conforming item may not make the outgoing quality poorer, it has the effect of increasing costs due to unnecessary rework. Accepting a non-conforming item reduces AOQ and is manifest, eventually, as an increase in unit cost of product.

It has been found that the probability of rejecting a non-conforming item is significantly increased if additional inspectors are included in the inspection sequence (8). This occurs without an associated increase in the number of conforming items being rejected incorrectly. This is particularly true if an item has several quality characteristics that need to be correctly classified as acceptable for the total item to be considered acceptable. In this paper we discuss the case of a production process where, for reasons of precision, safety or performance it is necessary to ensure that each of the

several characteristics of each item conforms to specifications.

According to this inspection scheme, all the characteristics in an item, whether these be critical, major, or minor are examined by an inspector. When the first non-conforming characteristic is detected the item is rejected. The item is accepted as conforming only if all characteristics are classified as conforming. Accepted items are sent to the next stage for inspection by a second inspector. The procedure is continued until a specified AOQ is achieved.

In this paper, expressions are derived to determine the AOQ at the end of each stage of inspection. A computer program is developed to determine the minimum number of stages of inspection required to meet a desired AOQ level.

MATHEMATICAL DEVELOPMENT

Assumptions

Each characteristic is produced by a separate and independent process. For example, the inner and outer diameter, and length of a bushing are three characteristics that result from three different processes, i.e. turning, drilling and parting. As such, it may be assumed that the number of non-conformities for each characteristic in the item is Poisson distributed, and that, the distribution for each characteristic is independent of any other distribution. It follows that the total number of non-

conformities per product is also Poisson distributed.

It is assumed that the inspection of each of the J characteristics in a product takes place independently and an item is rejected when a non-conforming characteristic is detected. In practice, however, some of the characteristics may be correlated due to correspondences in equipment or within the process. On the other hand, an inspector's perception of the behavior of the process may affect his/her judgement and make him/her perform as if the characteristics are not independent. In such instances, if an inspector finds the already inspected characteristics to be good, he/she may not inspect the remaining ones as thoroughly as he/she should. This simplifying assumption is necessary since correlations among human inspectors are very difficult to determine, although quite possible for machine inspection.

It is further assumed that the probabilities of making a type I or a type II error when inspecting the j th characteristic at stage i , i.e., $(e_1)_{ij}$ and $(e_2)_{ij}$ are known and independent of $(e_1)_{ik}$ and $(e_2)_{ik}$ for $k \neq j$. The probabilities of committing a type I or a type II error are a function of several factors that affect an inspector's performance. In general, these factors include the perceived incoming product quality, the payoff matrix associated with an inspector's performance, the type of inspection plan and instruments used, individual factors like age, experience, and morale, stimulus-response compatibility, and environmental factors like temperature, humidity, lighting

and noise.

A final assumption for the mathematical development is that the performance of inspectors at any stage is independent of other stages, i.e., $(e_1)_{ij}$ for $j=1,2,\dots,J$ is independent of $(e_1)_{k,j}$ for $k \neq i$, but may be a function of the factors mentioned above. In other words, an inspector does not know if the item at hand has already been inspected by other inspectors or whether it is going through more inspection stages.

Model Development

As mentioned earlier, the number of non-conformities for each characteristic is assumed to be Poisson distributed with mean λ_j ($j=1,2,\dots,J$). Assuming independence of these distributions, we have

$$\begin{aligned} P(\text{an item being conforming}) &= \prod_{j=1}^J \exp(-\lambda_j) \\ &= \exp\left(-\sum_{j=1}^J \lambda_j\right) \\ &= P_c \end{aligned}$$

$$P(\text{an item being non-conforming}) = 1 - \exp\left(-\sum_{j=1}^J \lambda_j\right) = p$$

where p is the overall incoming fraction of non-conforming items.

We can now compute the probabilities of type I and type II errors at each inspection stage. The incoming fraction of non-conforming items, p , is assumed to be a

known constant. In a later work we will address the case when p is a random variable.

Inspection Stage i . A type I error is incurred whenever a conforming characteristic is classified as non-conforming. A conforming item is accepted only if all J characteristics are classified as conforming. Thus, the probability of rejecting a conforming item at stage i is :

$$e_{1i} = 1 - \prod_{j=1}^J (1 - e_{1ij}) = 1 - P_i \quad (1)$$

A type II error is incurred whenever a non-conforming item is classified as conforming. This happens when one or more characteristics are non-conforming. Thus all non-conforming characteristics are incorrectly classified as conforming. Therefore, type II error probability at stage i , e_{2i} , is the probability of accepting k non-conforming characteristics incorrectly and accepting the remaining $(J - k)$ conforming characteristics correctly. This can be expressed as

$$e_{2i} = \sum_{k=1}^J P(E_k) \quad (2)$$

where $P(E_k) = P(\text{at stage } i \text{ there are } k \text{ non-conforming characteristics present which are all misclassified and the remaining } J - k \text{ conforming characteristics are correctly classified})$

For example, for $k=1$ only one of the J characteristics, say J_1 , is non-conforming. This characteristic is misclassified while the others are correctly classified. Hence

$$P(E_1) = \sum_{J_1=1}^J P(\text{characteristic } J_1 \text{ non-conforming}) \times \\ P(\text{char. } J_1 \text{ misclassified}) \times \\ P(\text{all other char. are conforming}) \times \\ P(\text{all other correctly classified})$$

For $j \neq J_1$:

$$P(E_1) = \sum_{J_1=1}^J (1 - \exp(-\lambda_{J_1})) \cdot e^{2iJ_1} \cdot \exp\left(-\sum_{j=1}^J \lambda_j\right) \cdot \prod_{j=1}^J (1 - e^{-1ij}) \\ = \sum_{J_1=1}^J \frac{(1 - \exp(-\lambda_{J_1})) \cdot e^{2iJ_1}}{(1 - e^{-1iJ_1}) \cdot \exp(-\lambda_{J_1})} \cdot P_c \cdot P_i$$

For $k=2$ any pair of the J characteristics can be non-conforming. These two are misclassified and the rest are correctly classified. All possible combinations of two out of J characteristics must be considered. Hence

$$P(E_2) = \sum_{J_1=1}^{J-1} \sum_{J_2=J_1+1}^J P(\text{Char. } J_1 \text{ \& } J_2 \text{ non-conforming}) \times \\ P(\text{char. } J_1 \text{ \& } J_2 \text{ misclassified}) \times \\ P(\text{all other char. are conforming}) \times \\ P(\text{all other correctly classified})$$

For $j \neq J_1 \text{ \& } J_2$

$$P(E_2) = \sum_{J_1=1}^{J-1} \sum_{J_2=J_1+1}^J ((1-\exp(-\lambda_{J_1}))(1-\exp(-\lambda_{J_2}))) \times \\ (e^{2iJ_1} \cdot e^{2iJ_2}) \exp(-\sum_{j=1}^J \lambda_j) \cdot \prod_{j=1}^J (1-e_{1ij})$$

$$P(E_2) =$$

$$\sum_{J_1=1}^{J-1} \sum_{J_2=J_1+1}^J \frac{(1-\exp(-\lambda_{J_1}))(1-\exp(-\lambda_{J_2})) \cdot e^{2iJ_1} \cdot e^{2iJ_2}}{\exp(-(\lambda_{J_1} + \lambda_{J_2})) \cdot (1-e_{1iJ_1}) \cdot (1-e_{1iJ_2})} \cdot P_c \cdot P_i$$

In general we have

$$P(E_i) = \sum_{J_1=1}^{J-k+1} \sum_{J_2=J_1+1}^{J-k+2} \dots \dots \dots \quad (3)$$

$$\dots \sum_{J_k=J_{k-1}+1}^J \frac{\prod_{j=1}^k (1-\exp(-\lambda_{J_j}))}{\exp(-\sum_{j=1}^k \lambda_{J_j})} \cdot \frac{\prod_{j=1}^k e^{2iJ_j}}{\prod_{j=1}^k (1-e_{1iJ_j})} \cdot P_c \cdot P_i$$

We now proceed to determine the expected number of correctly and incorrectly accepted items at the end of each stage to determine AOQ.

Stage 1. If the total number of incoming items is N , then

$$E(\# \text{ conforming items}) = \exp(-\sum_{j=1}^J \lambda_j) \cdot N \quad (4)$$

$$E(\# \text{ non-conforming items}) = (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot N \quad (5)$$

$$E(\# \text{ items incorrectly rejected}) = e_{11} \cdot \exp(-\sum_{j=1}^J \lambda_j) \cdot N \quad (6)$$

$$E(\# \text{ items incorrectly accepted}) = e_{21} \cdot (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot N \quad (7)$$

$$E(\# \text{ conforming items correctly accepted}) = (1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) \cdot N \quad (8)$$

Therefore, the expected number of items both correctly and incorrectly accepted at stage 1 is

$$(1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) \cdot N + e_{21} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot N \quad (9)$$

AOQ is basically a ratio of the expected number of non-conforming items that have completed inspection but have not been detected to the expected number of items that are accepted. We have

$$(AOQ)_1 = \frac{e_{21} \cdot (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot N}{[(1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) + e_{21} \cdot (1 - \exp(-\sum_{j=1}^J \lambda_j))] \cdot N}$$

$$= e_{21} \cdot (1 - \exp(-\sum_{j=1}^J \lambda_j)) \times$$

$$\left[(1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) + e_{21} \cdot (1 - \exp(-\sum_{j=1}^J \lambda_j)) \right]^{-1} \quad (10)$$

Stage 2. The input to stage 2 is all the items that are accepted at stage 1, as given by equation (9). The first term represents the conforming items and the second represents the incorrectly accepted non-conforming items.

Expected number of correctly accepted conforming items

$$= (1 - e_{12})(1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) \cdot N \quad (11)$$

Expected number of incorrectly accepted items that are non-conforming

$$= e_{21} \cdot e_{22} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot N \quad (12)$$

Thus, expected number of both correctly and incorrectly accepted items at stage 2 is (from (11) and (12))

$$[(1 - e_{12})(1 - e_{11}) \exp(-\sum_{j=1}^J \lambda_j) + e_{21} \cdot e_{22} (1 - \exp(-\sum_{j=1}^J \lambda_j))] \cdot N \quad (13)$$

Therefore

$$(AOQ)_2 = e_{21} \cdot e_{22} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \times$$

$$\left[(1 - e_{12})(1 - e_{11}) \cdot \exp(-\sum_{j=1}^J \lambda_j) + e_{21} \cdot e_{22} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \right]^{-1} \quad (14)$$

By analogy, the expression for $(AOQ)_3$ after stage 3 is:

$$(AOQ)_3 = e_{21} \cdot e_{22} \cdot e_{23} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \times$$

$$\left[(1 - e_{13})(1 - e_{12})(1 - e_{11}) \exp(-\sum_{j=1}^J \lambda_j) + e_{21} \cdot e_{22} \cdot e_{23} (1 - \exp(-\sum_{j=1}^J \lambda_j)) \right]^{-1}$$

and $(AOQ)_n$ after stage n is

$$(AOQ)_n = (1 - \exp(-\sum_{j=1}^J \lambda_j)) \prod_{i=1}^n e_{2i}$$

$$\times \left[\exp(-\sum_{j=1}^J \lambda_j) \cdot \prod_{i=1}^n (1 - e_{1i}) + (1 - \exp(-\sum_{j=1}^J \lambda_j)) \cdot \prod_{i=1}^n e_{2i} \right]^{-1} \quad (15)$$

Equation (15) gives the expression for the average outgoing quality where e_{1i} and e_{2i} are given by equations (1) and (2).

Equation (15) can be written as

$$(AOQ)_n = \left[\frac{\exp(-\sum_{j=1}^J \lambda_j) \prod_{i=1}^n (1 - e_{1i})}{1 - \exp(-\sum_{j=1}^J \lambda_j) \prod_{i=1}^n e_{2i}} + 1 \right]^{-1} \quad (16)$$

For any value of i , if $e_{2i} = 0$ then $(AOQ)_n = 0$, i.e. if the probability of a non-conforming item being incorrectly classified as conforming is zero at any inspection stage,

then no non-conforming items pass any stage and, naturally, $(AOQ)_n = 0$. Also, for any i , if $e_{1i} = 1$, then $(AOQ)_n = 1$ i.e. all conforming items are incorrectly classified as non-conforming and all items at the end of inspection stage n are non-conforming.

From equation (16), for $\prod_{i=1}^n (1 - e_{1i}) > \prod_{i=1}^n e_{2i}$

then, with an increase in n , $(AOQ)_n$ decreases i.e. the outgoing quality improves. Thus, for a given set of values of e_{1i} and e_{2i} the variation of $(AOQ)_n$ shows an exponential decay with an increase in n .

Again, from equation (16), the case $\prod_{i=1}^n e_{2i} > \prod_{i=1}^n (1 - e_{1i})$ represents a non-practical situation since the probability of accepting a non-conforming item cannot be greater than the probability of rejecting a non-conforming item for any or all inspection stages.

RESULTS

Examples are presented to illustrate the use of the model. An interactive FORTRAN program was developed and implemented on the AMDAHL 470 computer. The inputs to this program are (i) a desired AOQ level for the product, (ii) Poisson parameters for each quality characteristic to be inspected, and (iii) the known type I and type II error probabilities for each characteristic. The outputs of the program indicate the number of stages of inspection required to meet the desired AOQ, and the value of the achieved AOQ. A sample of the program output is shown in the

appendix.

Example 1

Determine the minimum number of inspection stages required for AOQ of (1) .01% (2) .0001% and (3) .00001% for an item which has 5 characteristics with the following values for the Poisson parameters and error probabilities

Char. #	1	2	3	4	5
λ	.30	.35	.25	.30	.25
(e_1, e_2)	(.05, .10)	(.05, .10)	(.05, .10)	(.10, .05)	(.10, .05)

The minimum number of inspections required, as determined from the computer program, is (1) 4 (2) 5 (3) 6.

When the quality of the incoming products is improved (λ for the five characteristics is .05, .15, .15, .10, .20) the new values for the minimum number of inspections required are (1) 3 (2) 4 (3) 5. Figures 1(a) and (b) (Appendix) show the decrease of AOQ with an increase in the number of inspections required.

Example 2

Determine the minimum number of inspection stages required for a desired AOQ of (1) .01% (2) .001% and (3) .0001% for an item with 3 characteristics with the following values for the Poisson parameters and error probabilities

Char. #	1	2	3
λ	.30	.25	.20
(e_1, e_2)	(.05, .10)	(.01, .05)	(.10, .10)

The minimum number of inspections required, as determined by the computer program, is (1) 3 (2) 4 and (3) 5.

If the quality of the incoming product does not change, but error probabilities increase to (.10,.15), (.05,.10) and (.15,.15) then, the number of inspections required increases to (1) 4 (2) 5 and (3) 6. Figures 2(a) and (b) (Appendix) show the decrease of AOQ with an increase in the number of inspections required.

DISCUSSION

An analytical model has been developed to determine the minimum number of inspections required in a situation where, because of safety or performance requirements, items need to be 100% inspected. There is, of course, no guarantee that even with several 100% inspections done by skillful and diligent inspectors, all non-conforming items would be eliminated. Human inspectors are subject to fatigue and variability. As such, the belief in perfection through 100% inspection is mistaken for both psychological and statistical reasons.

If costs associated with the overall inspection process are taken into consideration, as they well should be, the significance of 100% inspection for some situations can be better appreciated. For example, the major inspection costs are (i) labor or machine operations costs, (ii) cost of shipping an unacceptable item to a customer, (iii) cost of rejecting an acceptable item, and (iv) cost of sorting a lot

after it has been rejected. If, for example, the cost under item (ii) is very high, one must invariably repeat a 100% inspection.

Inputs to the model can be quantified, making the model useful. It might initially appear that specifying the type I and type II errors for each characteristic would be difficult, if not impossible. But much work has already been reported in the literature on the evaluation of inspector error as a function of incoming fraction defective. For example, Biegel (4) proposes a linear relationship between incoming fraction defective and the two types of error. Work is presently in progress to determine how the two types of error can be established for each characteristic. Methods to determine this are based on simulated laboratory inspection situations. These inputs can be estimated with an acceptable degree of accuracy.

The two types of error can be reduced, to some extent, by motivating the inspector, improved lighting and other environmental conditions, improved test equipment, and by providing timely and accurate feedback information. Wherever possible, engineers look into the possibility of automating the inspection procedures to reduce the two types of error. However, the present state of technology has not reached a point at which the probability of committing the two types of error can be reduced to zero.

The assumptions of independence of inspection for all characteristics and inspectors, although convenient, are not

practical. Nevertheless, an attempt has been made in this paper to understand some of the basic processes involved. In earlier work, an even more restrictive assumption of identical type I and type II error probabilities, and identical values of the λ parameter for each of the J characteristics had been made (6). This has been relaxed in the present work. Further improvements can be made and more usable models developed by relaxing the assumption of independence in inspection of the different characteristics. Future work is planned to address these issues.

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APPENDIX

Sample Output of the Computer Program

ENTER NUMBER OF CHARACTERISTICS PER ITEM (MAXIMUM OF 5)

5

ENTER E1, E2, AND LAMBDA FOR CHARACTERISTIC # 1

SEPARATED BY COMMAS, E.G. 0.05,0.10,0.15

.05,.10,.05

ENTER E1, E2, AND LAMBDA FOR CHARACTERISTIC # 2

.05,.10,.15

ENTER E1, E2, AND LAMBDA FOR CHARACTERISTIC # 3

.05,.10,.15

ENTER E1, E2, AND LAMBDA FOR CHARACTERISTIC # 4

.10,.05,.10

ENTER E1, E2, AND LAMBDA FOR CHARACTERISTIC # 5

.10,.05,.20

ENTER THE DESIRED AOQ, E.G. 0.001

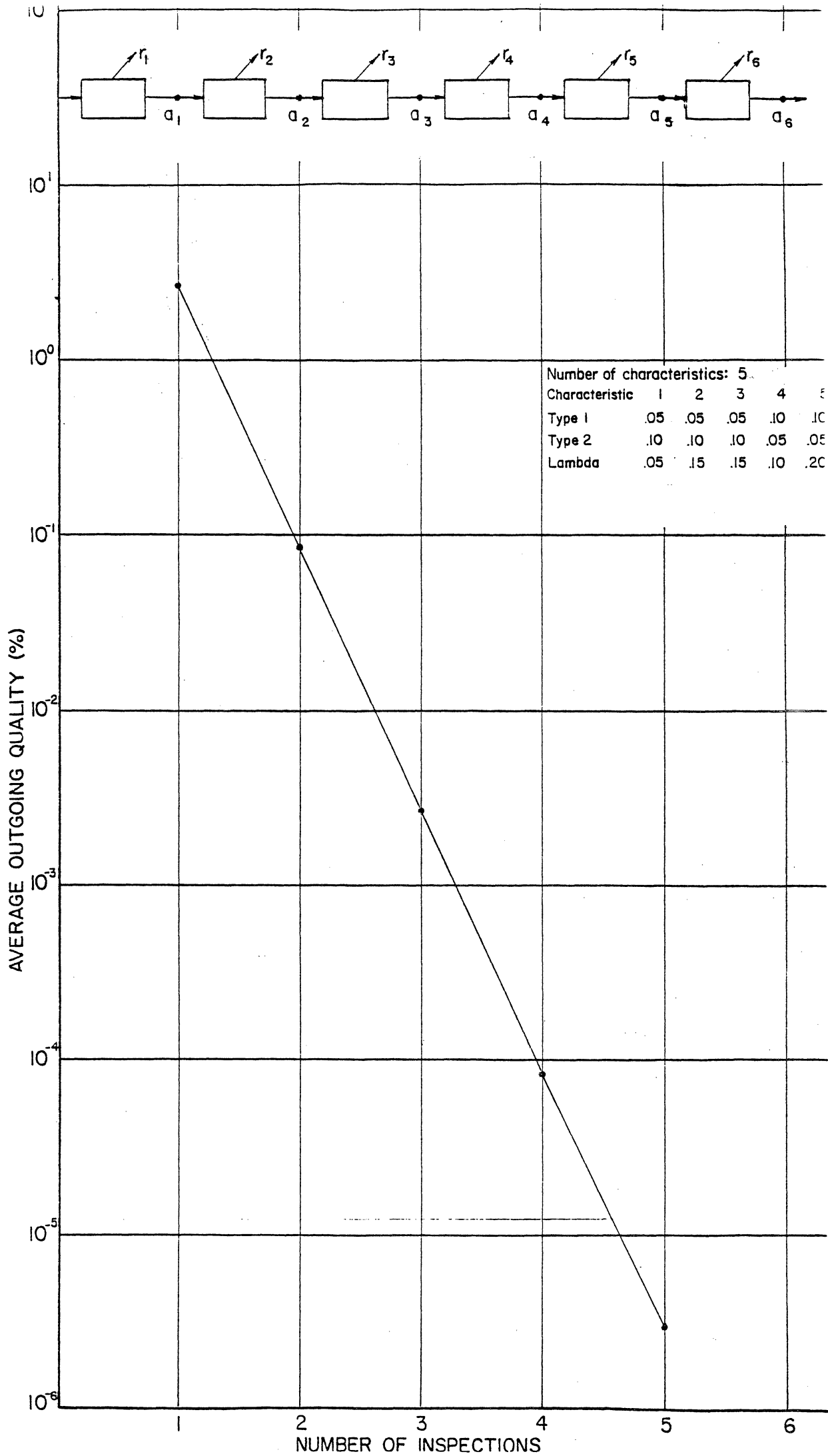
0.0001

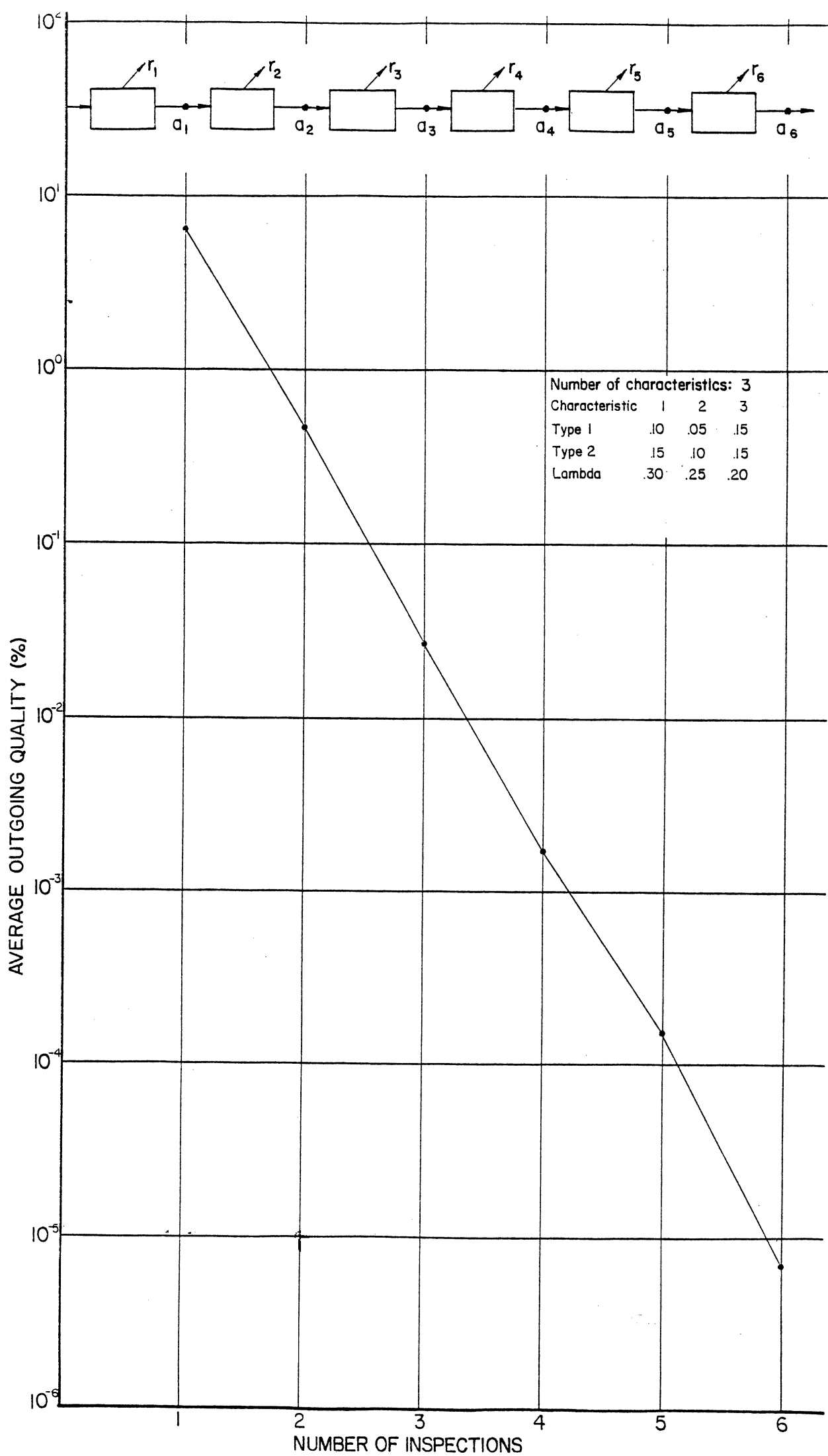
CHARACTERISTIC	TYPE1	TYPE2	LAMBDA
1	0.05000	0.10000	0.05000
2	0.05000	0.10000	0.15000
3	0.05000	0.10000	0.15000
4	0.10000	0.05000	0.10000
5	0.10000	0.05000	0.20000

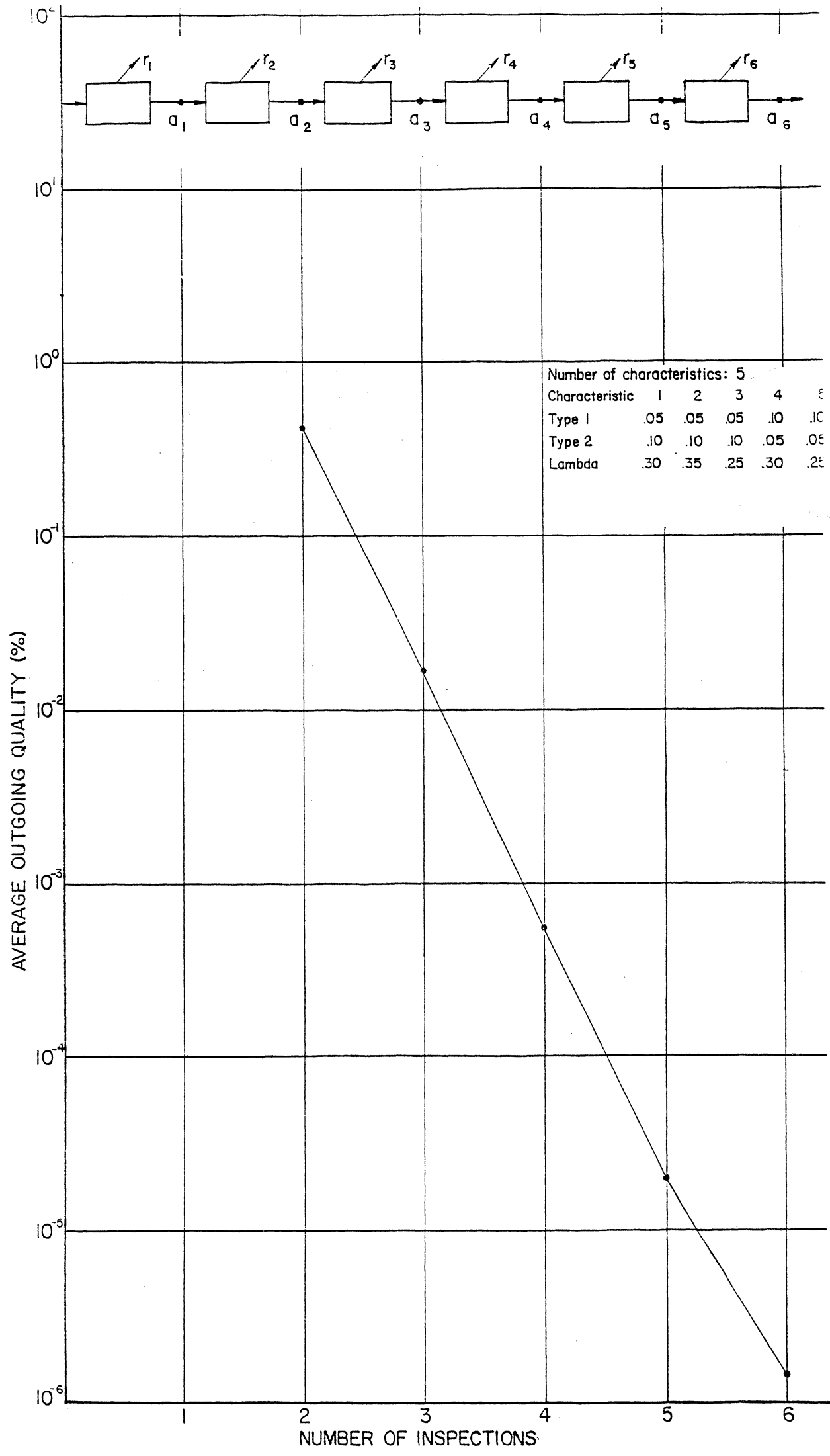
DESIRED AOQ IS:.0001

ACHIEVED AOQ=0.00002660

NUMBER OF INSPECTIONS: 3

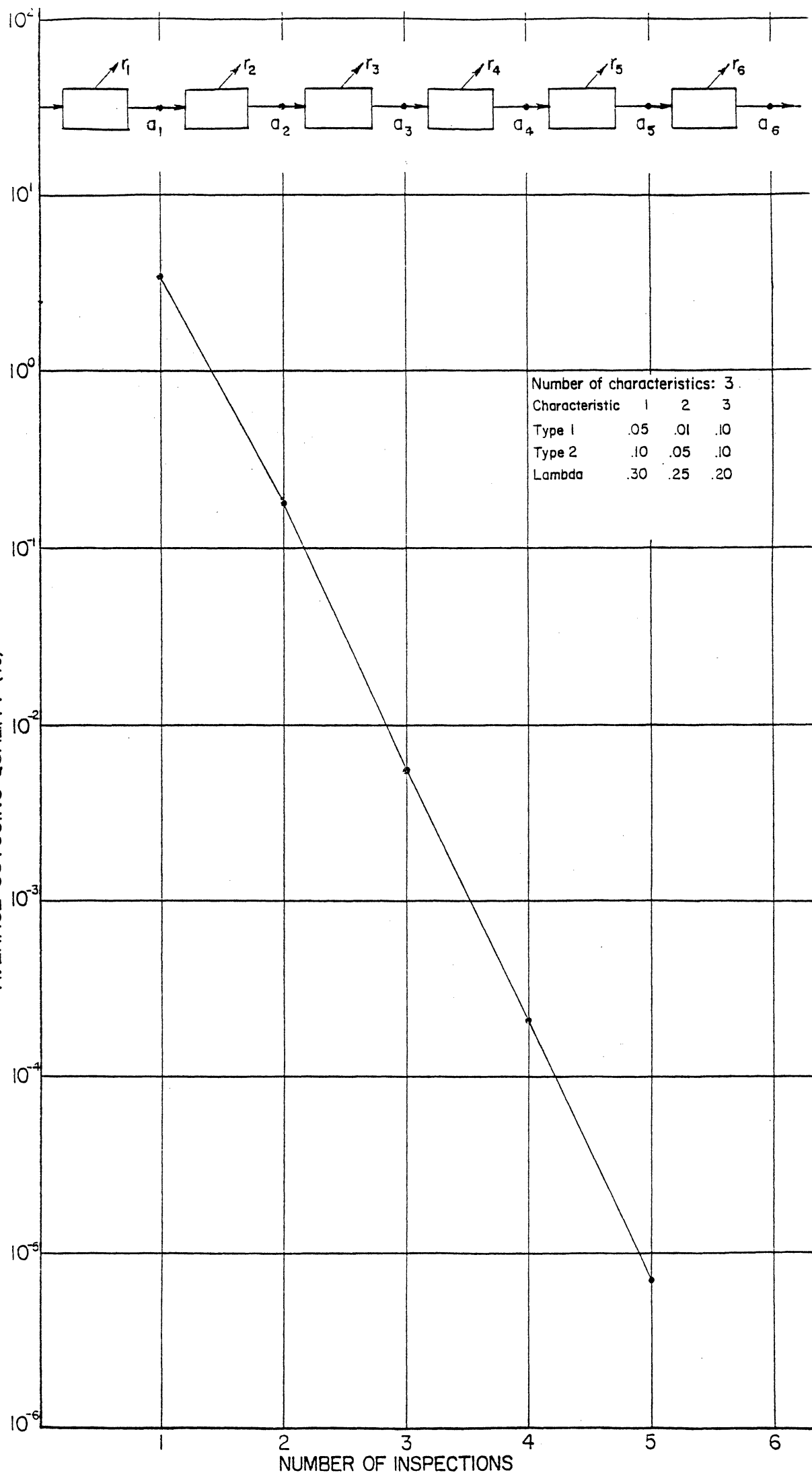






AVERAGE OUTGOING QUALITY (%)

NUMBER OF INSPECTIONS



AVERAGE OUTGOING QUALITY (%)

NUMBER OF INSPECTIONS

UNIVERSITY OF MICHIGAN



3 9015 04733 7731