

VIRIALIZATION IN N -BODY MODELS OF THE EXPANDING UNIVERSE. I. ISOLATED PAIRS

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ABSTRACT

The degree of virialization of isolated pairs of galaxies is investigated in the N -body simulations of Efstathiou and Eastwood for open ($\Omega_0 = 0.1$) and critical ($\Omega_0 = 1.0$) universes, utilizing the three-dimensional information available for both position and velocity. Roughly half of the particles in the models form isolated pairs whose dynamics is dominated by their own two-body force. Three-quarters or more of these pairs are bound, and this ensemble of bound isolated pairs is found to yield excellent mass estimates upon application of the virial theorem. Contamination from unbound pairs introduces error factors smaller than 2 in mass estimates, and these errors can be corrected by simple methods. Orbits of bound pairs are highly eccentric, but this does not lead to serious selection effects in orbital phases, since these are uniformly distributed. The relative velocity of these pairs of mass points shows a Keplerian falloff with separation, contrary to observational evidence for real galaxies. All the above results are independent of the value of Ω_0 , but may be sensitive to initial conditions and the point-mass nature of the particles.

Subject headings: cosmology — galaxies: clustering — numerical methods

I. INTRODUCTION

The suggestion that galaxies possessed massive unseen halos which extended well beyond their visible range (Ostriker, Peebles, and Yahil 1974; Einasto, Kaasik, and Saar 1974) has led to intense interest in the determination of galactic masses. Optical measurements have now firmly established that the rotation curves of galaxies are flat out to the Holmberg radius (Rubin, Ford, and Thonnard 1980). Radio measurements have extended rotation curves beyond the Holmberg radius, but there have been some questions about warping and observational difficulties (see the review by Bosma 1983). In any event, no definite Keplerian falloff has been seen, and the evidence is consistent with a flat rotation curve throughout the observed range of galactocentric radii. Hence, there is indeed more mass in the outer parts of galaxies than is indicated by the light distribution.

In order to extend the mapping of the mass distribution to larger radii, it is necessary to study binary galaxies, or groups and clusters. Binary galaxies have always played an important role in the determination of galactic masses. Recent work (Turner 1976*a, b*; Peterson 1979*a, b*; Karachentsev 1981*a-d*; Rivolo and Yahil 1981; White *et al.* 1983; Tift 1985) has concentrated on obtaining unbiased samples, and extending the measurements to larger separations, up to 700 kpc (throughout this paper we assume $H_0 = 50 \text{ km s}^{-1}$). The relative velocity of the binaries shows no Keplerian falloff with separation, suggesting that galaxies extend out to those distances.

In addition to difficulties encountered when working in redshift space, actual determinations of binary masses are hindered by our ignorance of their dynamics (cf. Turner 1976*b*; Peterson 1979*b*; White *et al.* 1983; Sharp 1984). It is not known to what extent the two-body force dominates the relative acceleration of galaxy pairs designated as binaries according to some algorithm. Even if it is known that the two-body force dominates, the distribution of eccentricities in the selected sample, as well as the uniformity of orbital phases, must in

practice also be known. Indeed, Burbidge (1975) has argued that most binaries might not even be bound, and their relative velocities might simply be a measure of the velocity dispersion of field galaxies. The hope that bound binaries might be identified by larger relative velocities than field galaxies (Yahil 1977) has not been realized; field galaxies show a velocity dispersion at least as large as that of isolated binaries (Rivolo and Yahil 1981; Davis and Peebles 1983; Bean *et al.* 1983).

To further our understanding of the underlying dynamics of binary galaxies, we have examined the N -body simulations of Efstathiou and Eastwood (1981, hereafter EE). These simulations contain 20,000 equal point masses, describing the evolution of structure in universes with $\Omega_0 = 0.1$ and $\Omega_0 = 1.0$, starting from a cold Poisson distribution at redshifts ~ 20 and ~ 10 , respectively. Using the three-dimensional information in the simulation, available by the courtesy of Dr. Efstathiou, we seek to establish what fraction of isolated pairs are indeed bound by their mutual attraction, and how accurately mass estimates can be made through application of the virial theorem. The hope is that the answers to these questions, determined within the context of the models, will carry over to more realistic models, and to the real universe. A companion paper (Evrard and Yahil 1985, hereafter Paper II) examines virial analyses of samples of all pairs, which are dominated by the nonisolated pairs.

No attempt is made here to consider projection effects. First, projection effects can be calculated only in the context of a specific survey, after the luminosity function and apparent magnitude completeness function have been determined. Second, the simulations only partially take into account the finite extent of galaxies by softening the two-body force at short separations (see § II*a* below). As a result, binaries with separations exceeding ~ 200 kpc show a Keplerian falloff of relative velocity with separation, contrary to observational evidence. Third, the simulation is limited to equal point masses, and we do not know what effect unequal point masses would have. We have therefore shied away from a detailed comparison of the model with observations, and consider it instead in its own right.

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The structure of the paper is as follows. The models are described in § II, including the question of scaling the model to the real universe and the method of extracting pair samples. Section III is devoted to a reformulation of the classical virial theorem in an expanding universe, by explicitly subtracting out the underlying Hubble expansion. The results of our investigation are presented in § IV, followed by a discussion in § V.

II. MODELS

a) Description

We utilize the final configurations of the 20,000-body galaxy clustering simulations of EE for both an open universe, ($\Omega_0 = 0.1$) and a critically dense universe ($\Omega_0 = 1.0$). The initial conditions, except density, are identical for both models. At the start of the integration, 20,000 equal mass points are distributed with a Poissonian power spectrum ($|\delta_k|^2 \propto k^n$, with $n = 0$) and zero peculiar velocities. Edges of the cube are handled by periodic boundary conditions. The particles are assumed to interact only through Newtonian gravity of point masses, i.e., the galaxies are assumed to have formed prior to the epoch at which the integration is begun. The method of integration is the particle-particle/particle-mesh (P³M) scheme (Hockney and Eastwood 1981), which divides the force on a particle into a long-range smooth component and a small-scale fluctuating component. This method possesses extensive dynamic range, being able to track both the flow of large-scale matter currents, and the two-body orbit of an isolated binary, for a large number ($\sim 10^4$) of particles (see Efstathiou *et al.* 1985 for a comparison of N -body integration methods). It is thus capable of producing accurate statistics of local dynamical properties.

In order to mimic the finite extent of galaxies, and to reduce the computational effort of tracking close binaries, the gravitational force is softened at separations smaller than a softening radius d_m , which is held fixed in comoving coordinates at $d_m = 0.0024$ in units of the cube length. Using the scaling relations presented in § IIb below, this corresponds to 440 kpc at the present epoch. The softened force deviates significantly from an inverse square law at separations $r \lesssim d_m/2 \approx 200$ kpc. Note that this force is conservative, implying that collisions between overlapping galaxies are elastic. There are currently no reliable methods for modeling inelastic, tidal behavior between pairs of galaxies in N -body systems with such large numbers of particles.

The models are evolved until the cosmological scale factor has increased by factors of 19.2 for $\Omega_0 = 0.1$ and 9.9 for $\Omega_0 = 1.0$. Unfortunately, for the Poissonian initial distribution these expansion factors are insufficient to allow the formation of superclusters such as are observed in the real universe at the present epoch, containing rich clusters having at least ~ 100 galaxies at density contrasts $\delta\rho/\rho_0 \sim 10^4$. Instead, as noted by Davis and Peebles (1983), smaller groups and clusters form, which are only loosely coupled to one another. Accordingly, the two-point correlation function has a steeper slope than observed, with γ in the range 2–3. We feel that detailed large-scale agreement is not a prerequisite for undertaking the present study of principally small-scale behavior.

b) Scaling

The EE N -body calculation uses the dimensionless normalization $G = m = L = 1$, where G is the gravitational constant, m is the mass of each particle, and L is the comoving length of the integration cube, equated to the proper length at the start

of the integration. In converting these to dimensional scales appropriate to galaxies, we use G and the Hubble constant at the present epoch H_0 (taken to be $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) to scale two of the three dimensions. For a given Ω_0 this establishes the mean density of the universe, ρ_0 . The third dimension is scaled by assigning a mean intergalactic separation, or, alternatively, the mass of a galaxy.

Observationally, the mean intergalactic separation can be defined only for galaxies brighter than some limiting absolute magnitude M_c , since the luminosity function has been observed to converge only at very faint (dwarf) luminosities (Sandage, Binggeli, and Tammann 1985). For a typical Schechter luminosity function,

$$\phi(M) = cx^\beta e^{-x}, \quad (1)$$

where $x(M) = 10^{-0.4(M-M_0)}$, and the parameters, $M_0 = -20.7$, $c = 2.45 \times 10^{-3} \text{ Mpc}^{-3}$, and $\beta = 0$, are taken from the analysis of the *Revised Shapley-Ames Catalog* (RSA) (Tammann, Yahil, and Sandage 1979; Yahil, Sandage, and Tammann 1980), we can evaluate the mass per galaxy to be

$$M_{\text{gal}} = 2.6 \times 10^{13} \Omega_0 / \text{Ei}(x_c) M_\odot, \quad (2)$$

where $x_c = x(M_c)$, and $\text{Ei}(x_c)$ is the exponential integral of this variable. In order to avoid the difficulty of assigning equal masses to dwarfs and regular galaxies, we have set the cutoff magnitude at $M_c = -19$. This gives a mass per galaxy of

$$M_{\text{gal}} = 2.2 \times 10^{13} \Omega_0 M_\odot, \quad (3)$$

and establishes the length of the computational cube² to be

$$L = 185 \text{ Mpc}. \quad (4)$$

Because of the exponential integral in equation (2), any reasonably dimmer cutoff would yield a mass and length differing by only a small factor.

c) Pair Samples

The *main* sample we use in this study, and in the study of all pairs (Paper II), consists of pairs having comoving separations $x < 0.001$ (proper distance $r < 1.85$ Mpc). In order to reduce the total pair count to a more manageable size, we eliminated all pairs with either member within a distance of 0.1 from any of the simulation boundaries. This excludes roughly half the total volume, leaving us with a total pair count of 18,231 for $\Omega_0 = 0.1$ and 35,022 for $\Omega_0 = 1.0$.

In order to determine the relative acceleration and degree of isolation of a pair having separation x , we need to identify surrounding neighbors, including those whose separation from the pair is larger than x . We have therefore constructed an *extended* sample consisting of all pairs having comoving separations $x < 0.05$ ($r < 9.3$ Mpc), for which at least one member lies within the inner $(0.8)^3$ region. Using this set, we generate a list of neighbors for each pair of the main sample.

The *relative* acceleration of a pair of particles consists of its direct two-body force, augmented by the collective sum of the *tidal* forces exerted by their neighbors. If the contribution of the neighbors to the tidal force is added up in order of increasing distance from the pair, one finds quick convergence once this distance is somewhat larger than the separation of the pair. We calculate the tidal force using the list of pairs from the

² Unless otherwise noted, all proper distances quoted are at the present epoch.

extended sample, thus guaranteeing that we include all particles whose distance from the nearest member of the pair is at least 5 times greater than the separation of the pair (and in most cases the cutoff distance is much larger than that). This provides adequate convergence.

The isolation parameter y is defined for each pair i, j as

$$y = \min(x_{ik}, x_{jk})/x_{ij}, \quad (5)$$

where k runs over all the neighbors of the pair. Pairs are termed isolated if $y > 1$, i.e., if the members the pair are each other's nearest neighbors, and nonisolated if $y < 1$. Observational selections of binary galaxies on the sky have typically employed stricter isolation criteria (e.g., Turner 1976a).

III. VIRIAL THEOREM IN THE EXPANDING UNIVERSE

The application of the virial theorem in cosmology is rife with uncertainty, caused mainly by the fact that most dynamical systems are not completely decoupled from the expanding substratum of the universe. The centers of clusters may be "virialized," i.e., in hydrostatic equilibrium, but the classical virial theorem is an integral relation between kinetic and gravitational energies over an entire system. When attempting to apply the classical virial theorem over the "entire" cluster, however, one runs into the practical difficulty of identifying its edge, and the dynamical difficulty of continual inflow into the cluster. Hence the coupling to the rest of the universe.

For isolated pairs it may seem that the classical approach of the virial theorem would be adequate. However, when we take a snapshot of the universe at a given epoch, we do not know what fraction of the isolated pairs are still in their initial orbit, and hence affected by the Hubble flow. We have therefore sought to redefine the virial theorem in a manner that subtracts out the effects of the underlying Hubble expansion.

Consider the time derivative of the scalar product of the separation of a pair \mathbf{r}_{ij} and their relative peculiar velocity \mathbf{u}_{ij} ,

$$d(\mathbf{r}_{ij} \cdot \mathbf{u}_{ij})/dt = \dot{\mathbf{r}}_{ij} \cdot \mathbf{u}_{ij} + \mathbf{r}_{ij} \cdot \dot{\mathbf{u}}_{ij}. \quad (6)$$

The equation of motion for the peculiar velocity can be written in the form

$$\dot{\mathbf{u}}_{ij} = -H\mathbf{u}_{ij} + \mathbf{g}_{ij}, \quad (7)$$

where \mathbf{g}_{ij} is the excess relative acceleration of the pair, i.e., the full relative acceleration minus the contribution of the smooth cosmological substratum. So, using equation (7) in equation (6), along with the defining relation between peculiar and proper velocity, yields

$$d(\mathbf{r}_{ij} \cdot \mathbf{u}_{ij})/dt = u_{ij}^2 + \mathbf{r}_{ij} \cdot \mathbf{g}_{ij}. \quad (8)$$

This equation is not as trivial as might seem at first glance, because $\mathbf{u}_{ij} \neq \dot{\mathbf{r}}_{ij}$, and $\mathbf{g}_{ij} \neq \dot{\mathbf{u}}_{ij}$, but the extra terms cancel. Equation (8) is the natural generalization of the similar relation for proper coordinates. Its advantage over the latter is that the effects of the Hubble expansions have been explicitly subtracted out, and it can therefore be applied equally well in a domain of separations where the Hubble velocity is comparable to the peculiar velocity. This application follows the same reasoning as is used for the more familiar versions of the virial theorem in classical mechanics, arguing that if the time average of the left-hand side of equation (8) is zero, then the time average of the right-hand side must also be zero, and hence

$$\langle u_{ij}^2 \rangle = -\langle \mathbf{r}_{ij} \cdot \mathbf{g}_{ij} \rangle. \quad (9)$$

In practice the time average needs to be replaced with an ensemble average over many pairs. In order for this substitution to be valid, we need to be assured that (1) the orbits are finite, and there is no net expansion or contraction, and (2) our sample is a fair sample of orbital phases. Note that equation (8) is a general identity, and hence under assumptions 1 and 2 its corollary, equation (9), applies equally well to isolated or non-isolated pairs. The case of nonisolated pairs is treated in Paper II.

IV. RESULTS

a) Does the Two-Body Force Dominate in Isolated Binaries?

In Figure 1 we present the mean ratio $\langle q_{ij} \rangle$ between the collective tidal force on a pair in the main sample and its two-body force,

$$q_{ij} = \left| \sum_k (\mathbf{g}_{ik} - \mathbf{g}_{jk}) \right| / |\mathbf{g}_{ij}|, \quad (10)$$

where k runs over all neighbors of the pair out to the limiting extended separation $x = 0.05$. Note that the collective force of distant neighbors represents only a small fractional perturbation on the two-body force for pairs with $y > 3$. Even those less isolated, with $1 < y < 3$, have $\langle q_{ij} \rangle < 0.5$ in both models. In contrast, the force between nonisolated pairs ($y < 1$) is dominated by the collective effect of neighbors, with $\langle q_{ij} \rangle$ as large as 30.

Encouraged by the finding that the two-body force dominates the relative acceleration of isolated pairs, we have allowed ourselves to use all the isolated pairs in the extended sample for the statistical tests in this paper. We have thus enlarged our isolated sample to separations $x < 0.05$ ($r < 9.3$ Mpc), although the number of isolated pairs with wide separations is not very large.

b) Are Isolated Pairs Bound?

For isolated pairs whose dynamics is dominated by their own self-interaction, we can make the distinction between bound and unbound pairs by measuring the energy of the two-body system in its center-of-mass frame:

$$E = v^2/4 + V(r), \quad (11)$$

Figure 2 shows a histogram of the sample broken into bound (shaded) and unbound (unshaded) portions as a function of separation. Unbound pairs constitute only 15% of the pairs for the $\Omega_0 = 0.1$ model, and 26% for the $\Omega_0 = 1.0$ model. The fraction of unbound pairs is sensibly larger at larger separations.

We can estimate the separation beyond which we would expect isolated pairs to be unbound by solving equation (11) for the limiting case $E = 0$. Starting at the big bang, $t = 0$, with separation $r = 0$, one finds that, at the present epoch t_0 , the separation of pairs with zero total energy is

$$r_{ZE} = 3(Gmt_0^2)^{1/3} = \begin{cases} 4.4 \text{ Mpc} & \Omega_0 = 0.1 \\ 7.7 \text{ Mpc} & \Omega_0 = 1.0, \end{cases} \quad (12)$$

where we have ignored the softening of the two-body force. These estimates are in good agreement with the maximum radial extent of the bound pairs in Figure 2.

Clearly, we do not expect the ensemble of unbound pairs to satisfy the virial theorem, since the finite orbit condition is violated. We turn now to verifying whether the ensemble of

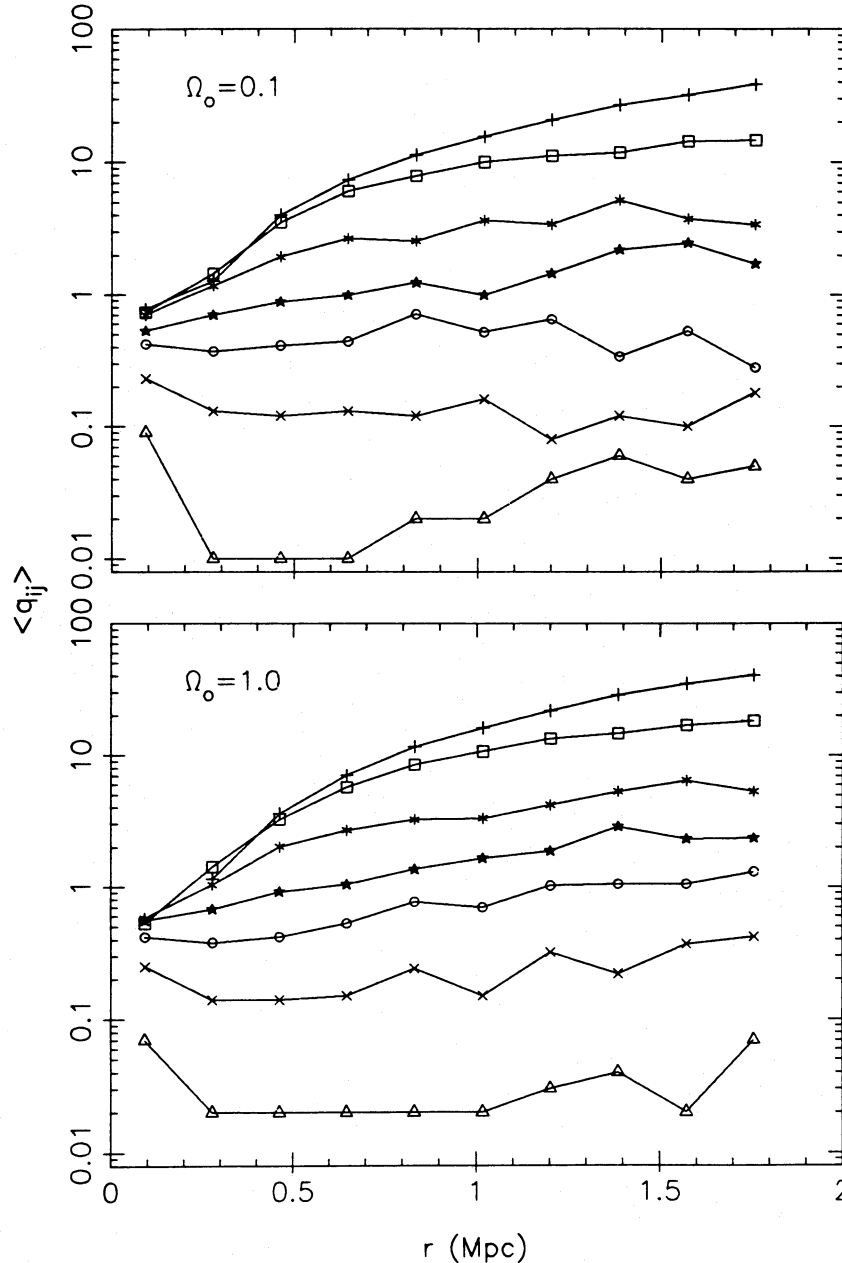


FIG. 1.—Ratio of the collective to the two body force (eq. [10]) for all pairs in the main sample. Symbols correspond to different ranges of isolation parameter y : $0.0 < y < 0.25$ (plus signs); $0.25 < y < 0.5$ (squares); $0.5 < y < 0.75$ (asterisks); $0.75 < y < 1.0$ (stars); $1.0 < y < 1.5$ (circles); $1.5 < y < 3.0$ (crosses); $y > 3.0$ (triangles).

bound, isolated pairs is a good candidate for virial analysis, by examining the two requirements posed at the end of § III above.

c) *Are Isolated Pairs a Fair Ensemble for the Virial Theorem?*

Binaries in initially cold N -body models such as these tend to have very eccentric orbits (Gott *et al.* 1979). This is shown dramatically in Figure 3, in which we plot scatter diagrams of the eccentricity against separation, along with a histogram of eccentricities. The eccentricity is determined from the relative separation and velocity of the pair, assuming it to be governed by the Newtonian two-body force alone. This is a very good approximation in the case of isolated pairs, except that the

eccentricity loses meaning for $r < d_m$, where the softened force deviates from the Newtonian one. For that reason we have excluded pairs with such small separations from Figure 3.

With such strongly eccentric orbits, the question naturally arises whether or not there might be a selection effect, such that a significant number of pairs are included in the “isolated” category only in the part of their orbit in which they are close to perigalacticon, and are deemed “nonisolated” when closer to apogalacticon. An additional question is whether or not a significant fraction of the pairs are in their initial infall toward each other, such as is apparently the case with M31 and the Galaxy.

In Figure 4 we plot the radial relative velocity of the bound

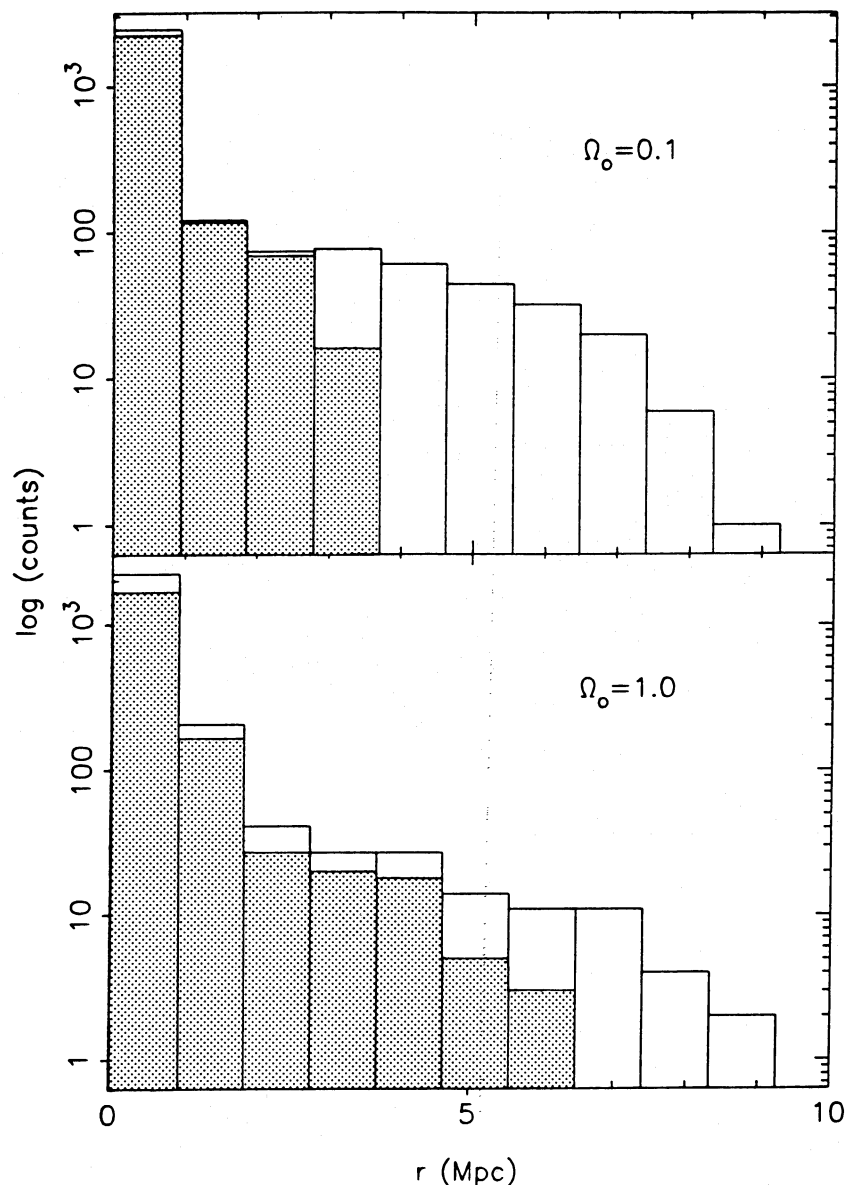


FIG. 2.—Histogram of isolated pair counts against separation. Bound pairs are shaded. Most unbound pairs are found at wider separations.

isolated pairs against their separation (cf. the similar diagram presented by Rivolo and Yahil 1983 for simulations of rich clusters). It is immediately seen that the pairs with separations larger than about 1.0–1.1 Mpc for $\Omega_0 = 0.1$ and 1.7–1.8 Mpc for $\Omega_0 = 1.0$ are in their initial infall, but at shorter separations we already have some galaxies that have had at least one rebound.

For separations that are smaller than the observed maximum extent after rebound, taken to be 1.0 Mpc ($\Omega_0 = 0.1$) and 1.7 Mpc ($\Omega_0 = 1.0$), there are both outgoing and ingoing pairs, and orbital phases are therefore expected to be more homogeneously distributed. This is indeed seen to be the case in Figure 5, in which we plot a scatter diagram of the orbital phase against separation, along with a histogram for phases of pairs with separations less than the above limits. Again, we need to exclude separations smaller than d_m , where we cannot employ Newtonian two-body physics. This exclusion appears

to be the cause of the diminution of phases close to zero (and π) in the $\Omega_0 = 0.1$ model, but the effect is not large.

We conclude that these isolated bound pairs are good candidates for the virial theorem, provided that pairs whose separation exceeds that of the maximum extent of rebounding galaxies are excluded. Precisely the same conclusion was reached by Rivolo and Yahil (1983) in the case of rich clusters. We therefore turn next to test the validity of the virial theorem for this sample.

d) Is the Virial Theorem Satisfied for Isolated Pairs?

In Figure 6 we plot the mean square relative peculiar velocity of the bound isolated pairs (normalized to one dimension) as a function of separation. Both the radial and the tangential square components are presented, with the high eccentricity of the orbit manifested by the higher radial components. Also shown are the directly measured values of $-r_{ij} \cdot g_{ij}/3$. In addi-

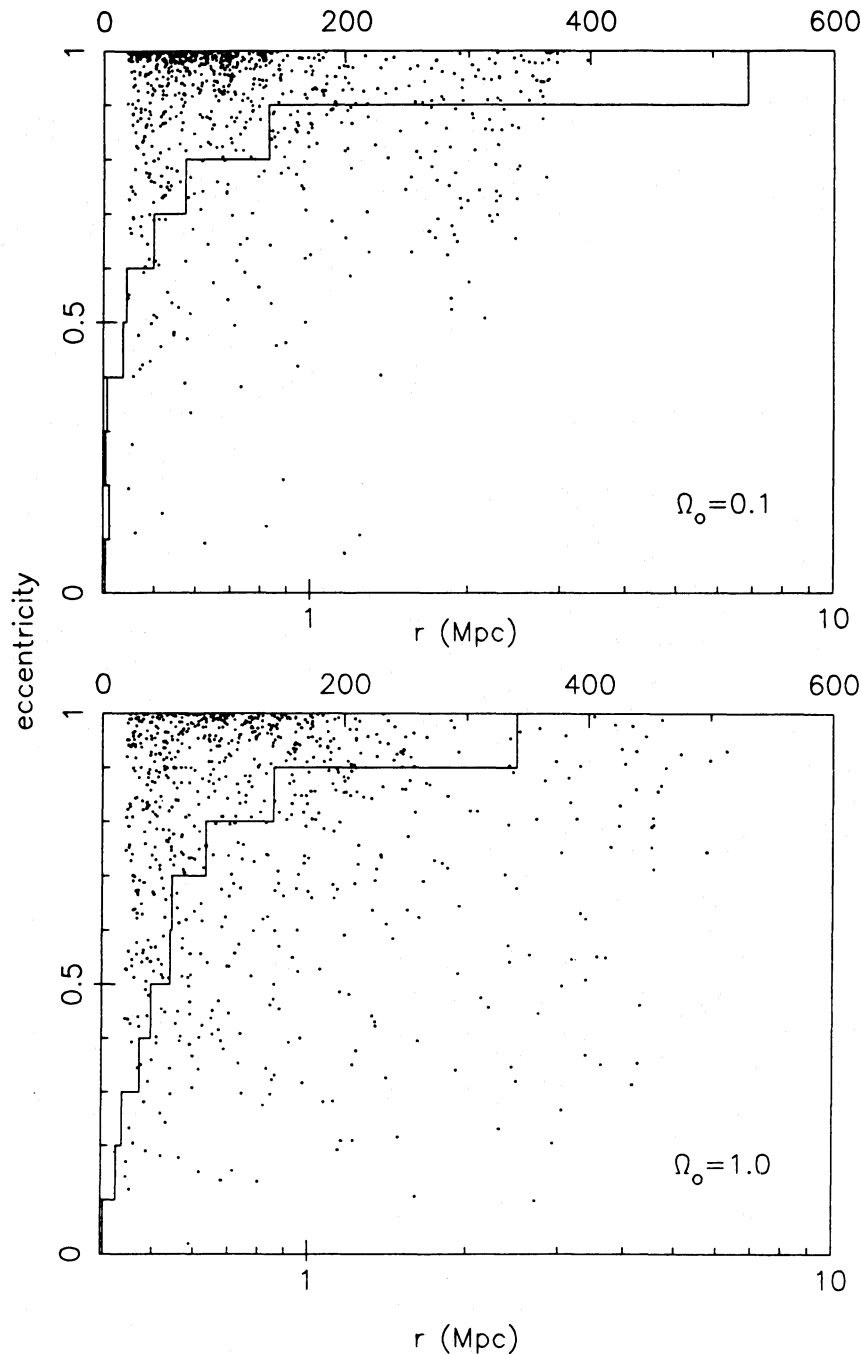


FIG. 3.—Scatter plot of orbital eccentricities vs. pair separation along with a histogram of eccentricities for bound isolated pairs. (Note histogram scale on top of graph.) Pairs with separations $r < d_m$ are excluded because of force softening on these scales.

tion, the plots include the pure two-body value of this latter quantity, showing explicitly how small is the effect of the collective force of neighbors.

Figure 6 shows a flat velocity distribution for separations smaller than $\sim \frac{1}{2}d_m \approx 200$ kpc, but a clear Keplerian falloff at larger separations. The one-dimensional rms relative velocity at small separations is 150 km s^{-1} for the $\Omega_0 = 0.1$ model and 500 km s^{-1} for the $\Omega_0 = 1.0$ model. This may be compared with the observational determinations of $\sim 100 \text{ km s}^{-1}$ (see references cited in § I), but such direct comparison should be viewed with caution—the Keplerian falloff in the models

beyond ≈ 200 kpc is not seen in the data for *projected* separations $\lesssim 700$ kpc.

In Figure 7 we plot the mass estimator, the *cumulative* ratio $\langle u_{ij}^2 \rangle / \langle -\mathbf{r}_{ij} \cdot \mathbf{g}_{ij} \rangle$, as a function of separation. The ratio is seen to be higher than unity for small separations, where most pairs are near perigalacticon, and, in fact, within the softened part of the two-body force, but converges to unity around $r_{ij} = d_m$, and remains at that value. The inclusion of pairs which are on their initial infall makes little difference to the cumulative ratio, because of the small number of bound systems at these larger separations. The final dimensionless mass estimates deter-

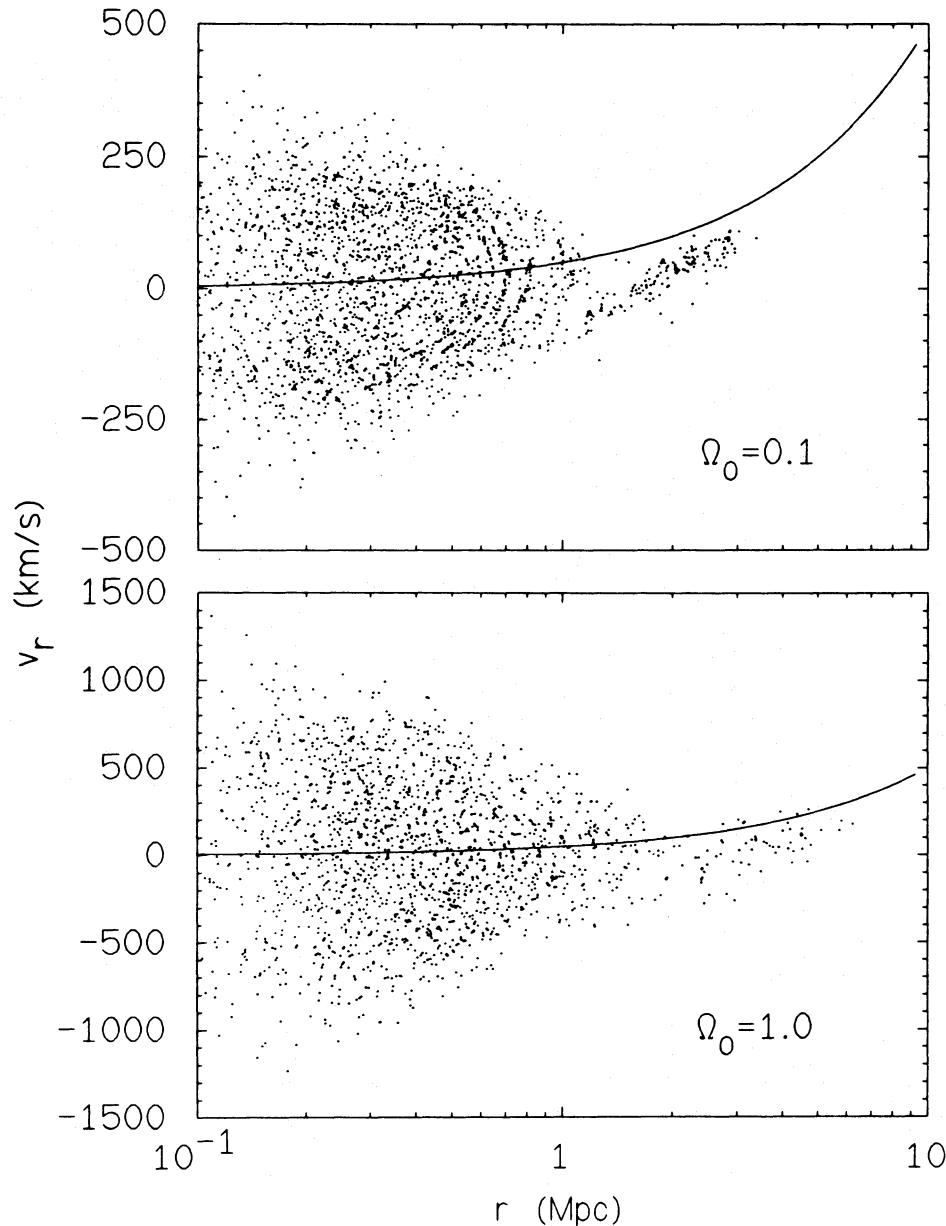


FIG. 4.—Scatter plot of radial velocity vs. separation for bound isolated pairs. The “tail” of pairs is just separating out of the Hubble flow (solid line: $v_r = H_0 r$).

mined from the asymptotic ratios are 0.93 for the $\Omega_0 = 0.1$ model and 1.02 for the $\Omega_0 = 1.0$ model, with statistical uncertainties of a few percent in both numbers. This should be viewed as excellent agreement.

In real applications, of course, it is all but impossible to identify bound isolated pairs. The observed sample will be contaminated by unbound pairs, which will tend to increase the ratio $\langle u_{ij}^2 \rangle / \langle -r_{ij} \cdot g_{ij} \rangle$. It is therefore interesting to measure the error which the unbound pairs introduce into the virial estimates.

Repeating the above analysis using the entire sample of isolated pairs, including the unbound ones, results in $\langle u_{ij}^2 \rangle / \langle -r_{ij} \cdot g_{ij} \rangle$ reaching an asymptotic value ~ 1.2 for the $\Omega_0 = 0.1$

model and ~ 2.2 for the $\Omega_0 = 1.0$ model. To see how well this error can be minimized, we have tested an iterative process for determining masses. One starts by using the mass estimate derived from the whole sample of isolated pairs to determine orbital energies for each pair. One then repeats the mass estimate using only those pairs which are bound by this zeroth-order mass. The process is repeated until convergence. We find rapid convergence (within ~ 5 iterations) from the initial values of 1.2 and 2.2 above to final estimates of 0.92 ($\Omega_0 = 0.1$) and 1.2 ($\Omega_0 = 1.0$). A simpler way to eliminate contamination from unbound pairs is to employ a stricter isolation criterion. By excluding pairs with isolations $y < 2$, where most of the unbound pairs can be found, we obtain mass estimates of 0.95

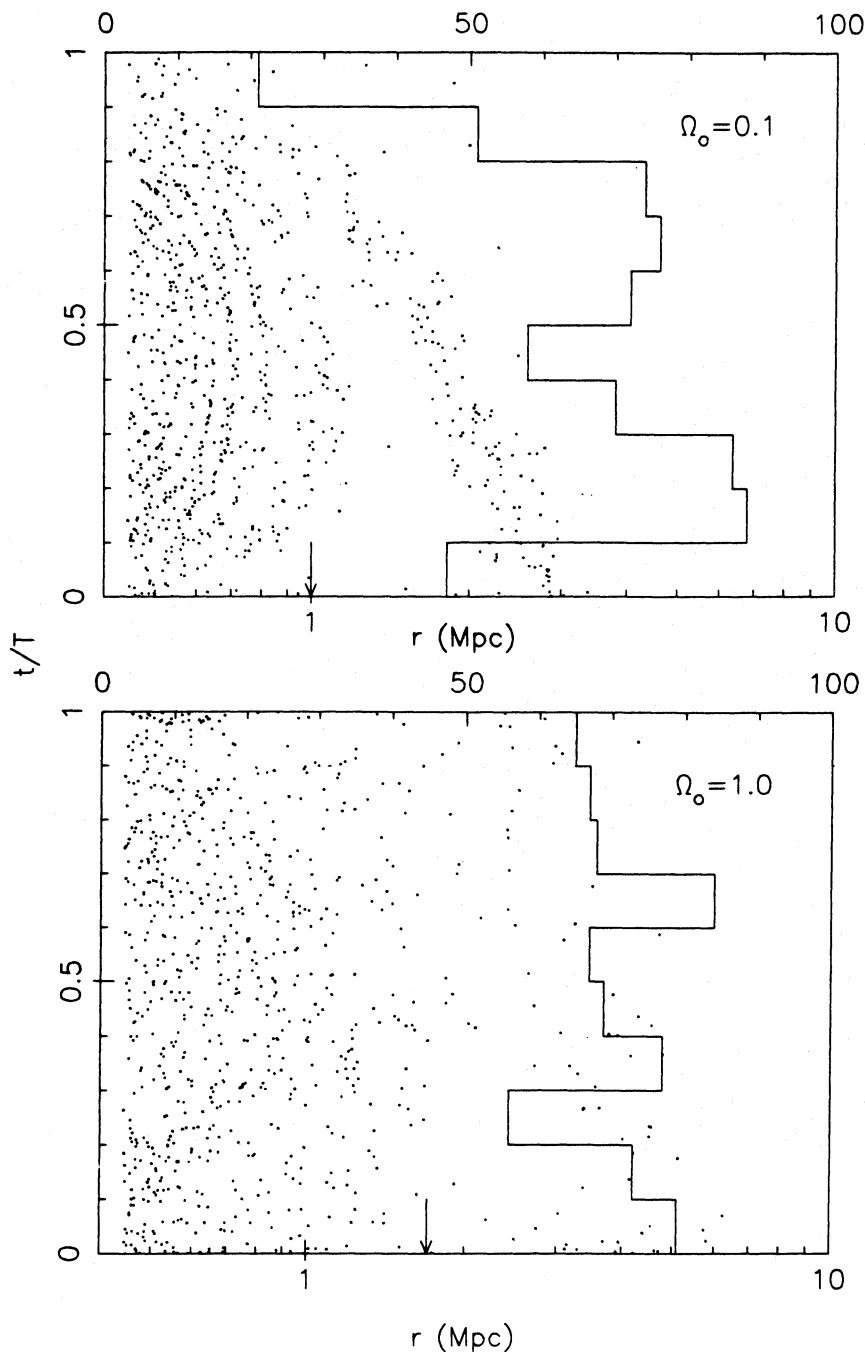


FIG. 5.—Scatter plot of orbital phases vs. separation along with a histogram of phases for bound isolated pairs. Times $t = 0, T$ correspond to perigalacticon. Pairs with separations $r < d_m$ are excluded as in Fig. 3. Also, only pairs with $r < 1.0$ Mpc ($\Omega_0 = 0.1$) and $r < 1.7$ Mpc ($\Omega_0 = 1.0$) are used in the histogram to eliminate any bias from pairs just beginning their initial infall.

in the $\Omega_0 = 0.1$ model and 1.5 in the $\Omega_0 = 1.0$ model. Further exclusion of unbound pairs by iteration gives final mass estimates of 0.86 and 1.04, respectively.

V. DISCUSSION

We have shown that nearly half of the point masses in the EE N -body simulations double up to form isolated pairs (each other's nearest neighbors), whose dynamics is dominated by their own two-body force. More than three-quarters of these

pairs are bound in very eccentric orbits. The high eccentricity means that most pairs should be found near apgalacticon, but this does *not* imply serious selection effects in orbital phases, since we have found these to be uniformly distributed.

As expected from the boundedness and uniformity of pairs' orbital phases, the ensemble of bound isolated pairs satisfies the virial theorem nearly exactly. Mass estimates are accurate to within 7% for the $\Omega_0 = 0.1$ model and 2% for $\Omega_0 = 1.0$, statistical uncertainty being of this order. Contamination from

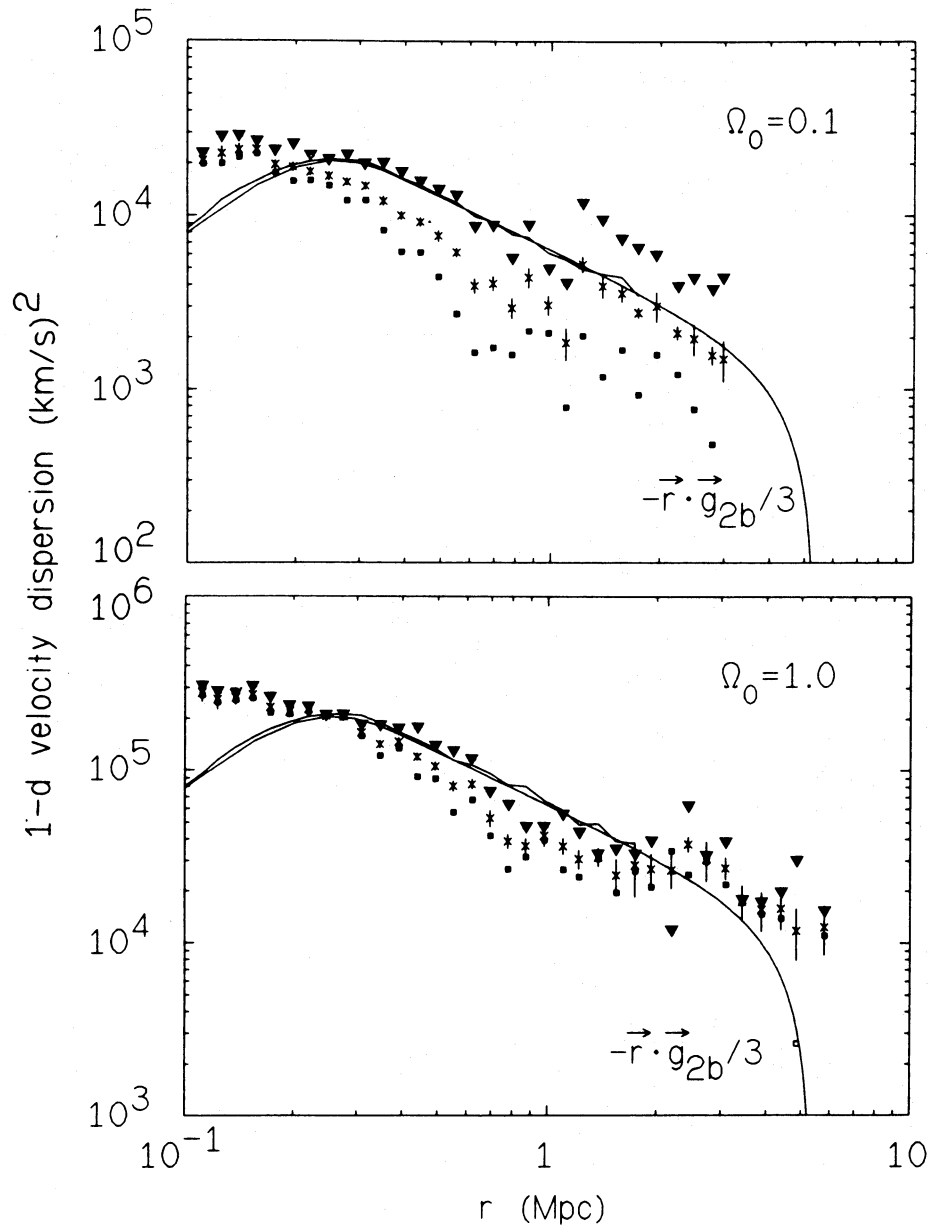


FIG. 6.—Mean square relative peculiar velocity components (*crosses*), radial components (*inverted triangles*), and tangential components (*squares*) plotted vs. separation for bound isolated pairs. The smooth curve is the normalized virial variable $-\vec{r} \cdot \vec{g}_{2b}/3$, with \vec{g} the softened two-body acceleration, while the more jagged curve is similar but with the full N -body acceleration \vec{g} measured from the simulation. Statistical error bars are plotted on the mean velocity components only.

unbound pairs introduced errors smaller than a factor of 2 in mass estimates, and these errors can be readily corrected by simple methods.

The “cold starts” used in the models favor the formation of long-lived binaries. N -body models with “warm starts” will produce a considerably smaller fraction of bound objects (Gott *et al.* 1979). Rivolo and Yahil (1981) found that 348 out of 864 galaxies, or roughly 40% of all galaxies in the RSA sample, were members of pairs with projected isolation parameters $s > 1.5$. (This corresponds roughly to $y > 1$ because they measured neighbor distances from the centroid of the pair.) This is interestingly close to the fraction $\sim 50\%$ found in the simulations, lending some support to the “cold start” scenario, but nothing more detailed can be said without including the effects of projection in analyzing the models.

We also find it interesting that the fraction of points in isolated pairs is nearly the same in both models. The explanation for this lies in the initial particle distribution, which is the same in both models. Suppose that pairs whose initial separation was smaller than some critical fraction f of the mean interparticle distance tended to be dominated from the start by their two-body force, and remained binaries thereafter. (Again, it is the lack of initial thermal velocities which allows such a simplistic view.) The Poisson probability for *not* having a neighbor within a fraction f of the mean interparticle separation is $\exp(-f^3)$. Since $\sim 50\%$ of the particles seem to satisfy this condition, it follows that $f = [-\ln(0.5)]^{1/3} = 0.89$. Thus, in a statistical sense, particles that initially find a neighbor within about 90% of the mean interparticle spacing tend to stick to it. This fraction is the same in both models because the strength

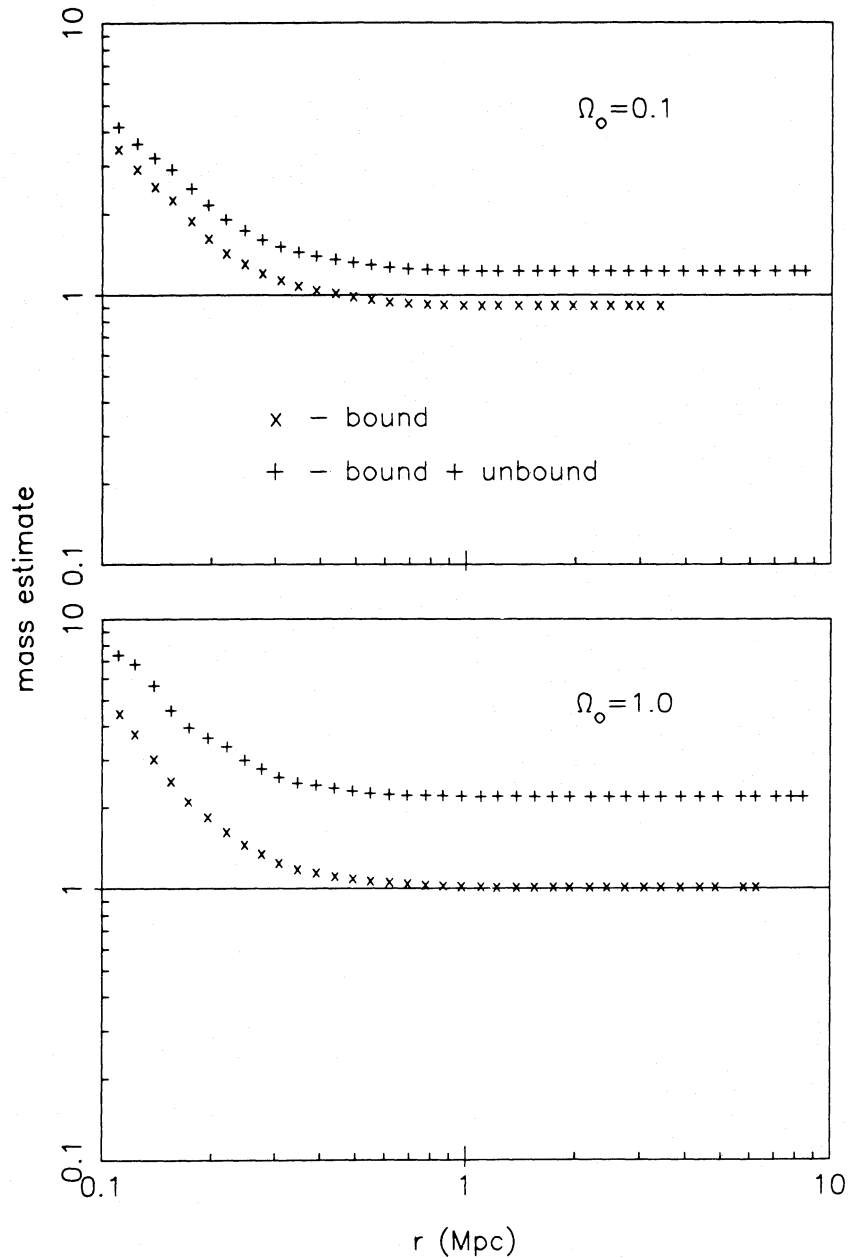


FIG. 7.—Mass estimates obtained from the cumulative values of the virial ratio $\langle u^2 \rangle / \langle -r \cdot g \rangle$ for bound only and for all (bound + unbound) isolated pairs. Bound isolated pairs yield extremely accurate masses. Contamination from unbound pairs affects mass estimates by a factor of roughly 2 in the critical model and by only 30% in the open case.

of the collective relative to the two-body forces is independent of the mass of the individual objects.

This point of view also explains the highly eccentric orbits of bound isolated pairs. The members of a bound pair of particles, which is initially dominated by its two-body force, will recede from the Hubble expansion, fall radially toward each other, and remain in an orbit with $e \approx 1$, unless strongly perturbed by neighbors. Moreover, if initial peculiar velocities were assigned to correspond to a purely growing gravitational instability, as they should if tidal forces between neighboring protogalaxies were negligible during the epoch of galaxy formation, then the tendency to form eccentric binaries would be

even stronger, since the peculiar motion of the members of a close pair would be in the direction of infall toward each other.

The distinct possibility that the members of isolated pairs of galaxies may fall radially toward each other, and perhaps collide, suggests that N -body models with more realistic galaxy-galaxy interactions, including tidal effects and mergers, are imperative in order better to model the properties of real binary galaxies, and to reproduce the observed statistical distributions of binary characteristics. This conclusion is strengthened by the discrepancy between the Keplerian falloff of relative velocity with separation in the models and the flat relationship in the observational data.

Simulations of galaxy collisions (see reviews by Tremaine 1981; Alladin and Narasimhan 1983; White 1983) show that the transfer of a pair's orbital energy into internal energy of the individual galaxies is quite significant, often resulting in capture or merger. Barnes (1984, 1985) has examined virial mass estimates for small groups with galaxies modeled as heavy "core" particles immersed in a background made up of many light "halo" particles, and found significant underestimation of the total group mass after only a few evolutionary time scales. Dynamical friction rapidly draws the core particles toward the center of the group, shrinking their mean radius, while their velocities remain roughly constant. Thus, the estimated mass $M \sim RV^2$ decreases as the system evolves. A similar effect is expected to be manifested in binary systems possessing a core-halo structure, although its magnitude will depend heavily on the assumed extent and other characteristics of the halo. (Note that dynamical friction can still occur between two galaxies even if their halos are not overlapping [Tremaine and Weinberg 1984].)

It is reassuring to find that, for the simple system studied here made up of equal mass particles interacting with a softened $1/r$ potential, virial mass estimates are extremely accurate. There remains the problem of understanding "virialization" for a system of extended objects interacting with excitable internal degrees of freedom.

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