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CLUSTERING OF DARK MATTER HALOS ON THE LIGHT-CONE: SCALE-, TIME- AND MASS-DEPENDENCE OF THE HALO BIASING IN THE HUBBLE VOLUME SIMULATIONS

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ABSTRACT

We develop a phenomenological model to predict the clustering of dark matter halos on the light-cone by combining several existing theoretical models. Assuming that the velocity field of halos on large scales is approximated by linear theory, we propose an empirical prescription of a scale-, mass-, and time-dependence of halo biasing. We test our model against the Hubble Volume N-body simulation and examine its validity and limitations. We find a good agreement in two-point correlation functions of dark matter halos between the phenomenological model predictions and measurements from the simulation for $R > 5h^{-1}$Mpc both in the real and redshift spaces. Although calibrated on the mass scale of groups and clusters and for redshifts up to $z \sim 2$, the model is quite general and can be applied to a wider range of astrophysical objects, such as galaxies and quasars, if the relation between dark halos and visible objects is specified.

Subject headings: cosmology: theory – dark matter – large-scale structure of universe – galaxies: halos – methods: numerical

1. INTRODUCTION

Clustering properties of luminous objects such as galaxies, clusters of galaxies and QSOs are useful tools not only in studying the nature of those objects but also in probing the cosmology. Current popular models predict that the cosmic structures evolved by gravitational instability from primordial fluctuations of mass density field generated through an inflationary epoch. The strongest support for this picture comes from recent detections of multiple peaks in the angular power spectrum of the cosmic microwave background radiation by the Boomerang (Netterfield et al. 2001) and MAXIMA-I (Lee et al. 2001) experiments. On the other hand, our knowledge about cosmic structures after the last scattering epoch, especially at high redshifts $z > 1$, is relatively poor, mostly because of observational costs associated with mapping the structure of many distant faint objects. Thanks to recent developments in instrument technology, this situation is improving dramatically. Large flows of data from the on-going wide-field galaxy and QSO redshift surveys, e.g., the Two-degree field (2dF) and the Sloan Digital Sky Surveys (SDSS), promise a new era of *precision cosmology.*

How accurately can we understand the nature of clustering of objects that will be precisely measured by these on-going surveys ? To construct a theoretical model of clustering of visible objects (galaxy, cluster of galaxies and QSOs) is not simple because it requires a detailed understanding of the biasing relation between those objects and the distribution of underlying dark mass. Popular models of the biasing based on the peak (Kaiser 1984; Bardeen et al. 1986) or the Press-Schechter theory (Mo & White 1996) are successful in capturing some essential features of biasing (Jing & Suto 1998). None of the existing models of bias, however, seems to be sophisticated enough for the coming precision cosmology era. Development of a more detailed theoretical model of bias is needed.

One way to understanding the clustering of objects is to describe it in terms of *dark matter halos*. The standard picture of structure formation predicts that the luminous objects form in a gravitational potential of dark matter halos. Therefore, a detailed description of halo clustering is the most basic step toward understanding the clustering of those objects. Eventually the halo model can be combined with the relation between the halos and luminous objects which has been separately investigated numerically and/or (semi-)analytically, e.g., Kauffmann & Haehnelt (2000).

The purpose of this paper is to improve theoretical predictions for clustering of luminous objects in large observational catalogs by developing a theoretical model of clustering of *dark matter halos* expected along the past light-cone of an observer. A special attention is payed to the scale-, time-, and mass-dependence of halo biasing. To do this, we combine several existing theoretical models including nonlinear gravitational evolution, the peculiar velocities of halos, and halos biasing. We also include the light-cone effect which is crucial when one analyzes data distributing over a broad redshift range. We then test the resulting predictions directly against a light-cone output from large N-body simulations. This work presents a natural generalization of our previous paper (Hamana, Colombi, & Suto 2001; hereafter HCS) discussing the clus-

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tering of dark matter on the light cone. Nevertheless this line of research benefits greatly from the modeling huge spatial volumes in simulations, a situation which has become possible only recently.

2. LIGHT-CONE OUTPUT AND SNAPSHOT DATA FROM THE HUBBLE VOLUME SIMULATION

In the following analysis, we use both "light-cone output" and snapshot data produced from the Hubble Volume ΛCDM simulation (Evrard et al. 2001). The initial CDM power spectrum is computed by CMBFAST (Seljak & Zaldarriaga 1996) assuming that $\Omega_{\rm b} = 0.04$ and $\Omega_{\rm CDM} = 0.26$, and is normalized so that $\sigma_8 = 0.9$. The background cosmology is spatially-flat with matter density $\Omega_{\rm m} = \Omega_{\rm CDM} + \Omega_{\rm b} = 0.3$, cosmological constant $\Omega_\Lambda = 0.7$ and the Hubble constant $H_0 = 70$ km·s^{-1}·Mpc^{-1}. The simulation employs $N = 10^9$ dark matter particles in a box of length $3000h^{-1}$Mpc on a side. The mass per particle is $2.25 \times 10^{12} h^{-1} M_\odot$. The light-cone output is generated in the following manner; we locate a fiducial observer at a corner of the simulation box at $z = 0$. The position and velocity of each particle are recorded whenever it crosses the past light-cone of this observer, and these coordinates are accumulated in a single data file. We use the "deep wedge" output[5] which subtends a 81.45 square-degree field directed along a diagonal of the simulation box up to $z = 4.4$. These data automatically include the evolution of clustering with look-back time (distance from the observer), which is essential in comparing models and observations of objects distributed over a broad range of redshift.

We identify dark matter halos on the light-cone using the standard friends-of-friends algorithm with a linking parameter of $b = 0.164$ (in units of the mean particle separation). Jenkins et al. (2001) show that such an algorithm produces a set of clusters whose mass function is well fit by a single functional form. We set the minimum mass of the halos as $2.2 \times 10^{13} h^{-1} M_\odot$, which consists of 10 simulation particles.

We find that the Press-Schechter model underpredicts the cumulative mass function of our halos with $M > 2.2 \times 10^{13} h^{-1} M_\odot$ at $z > 1$, while Sheth & Tormen (1999, hereafter ST99) overpredicts beyond $z \sim 1.5$. This tendency is consistent with the previous finding of Jenkins et al. (2001) that ST99 overestimates the number of halos when $\ln(\sigma^{-1})$ becomes large.

In §3.2 we also use the halo catalogue identified in the $z = 0$ snapshot data of the Hubble volume simulation to study the mass- and scale-dependence of halo biasing in detail. The halos are identified in the same manner as described above except that the minimum halo mass is $6.8 \times 10^{13} h^{-1} M_\odot$ (30 simulation particles). The total number of the identified halos in the $(3000h^{-1}\mathrm{Mpc})^3$-cube is 1,560,995. We note that the mass function and clustering of this this halo catalogue at $z = 0$ were already studied by (Jenkins et al. 2001), and by (Colberg et al. 2000), respectively. Our analysis below aims at a detailed modeling of the halo biasing properties at $z = 0$ in order to calibrate the empirical halo biasing model on the light-cone.

3. STATISTICS OF HALOS ON THE LIGHT-CONE

3.1. *Theoretical predictions of two-point correlation functions on the light-cone*

As emphasized by Suto et al. (1999), for instance, observations of high-redshift objects are carried out only through the past light-cone defined at $z = 0$, and the corresponding theoretical modeling should properly take account of relevant physical effects. Those include (i) nonlinear gravitational evolution, (ii) linear and nonlinear redshift space distortion, (iii) selection function of the target objects, and (iv) scale-, mass- and time-dependent biasing of those objects. In the present section we describe a model for the two-point statistics for dark matter halos with *all* the above effects properly considered. In what follows, we briefly describe the outline of our modeling (see HCS for details) focusing on those issues specific to dark matter halos.

Gravitational evolution of mass fluctuations can be accurately modeled by adopting a fitting formula of Peacock & Dodds (1996) for the nonlinear power spectrum in real space, $P^{\rm R}_{\rm PD}(k, z)$. Then the nonlinear power spectrum *in redshift space* is given as (Kaiser 1987; Peacock & Dodds 1996):

$$P^{\rm S}(k, \mu, z) = P^{\rm R}_{\rm PD}(k, z)[1 + \beta_{\rm halo}\mu^2]^2 D_{\rm vel}[k\mu\sigma_{\rm halo}], \quad (1)$$

where μ is the direction cosine in k-space, $\sigma_{\rm halo}$ is the one-dimensional *pair-wise* velocity dispersion of halos, and $\beta_{\rm halo} \equiv f(z)/b_{\rm halo}$. In the above expression, $f(z)$ is the logarithmic derivative of the linear growth rate $D(z)$ with respect to the scale factor, and $b_{\rm halo}$ is the halo bias factor. While both $\sigma_{\rm halo}$ and $b_{\rm halo}$ depend on the halo mass M, separation R, and z in reality, we neglect their scale-dependence in computing the redshift distortion, and adopt the halo number-weighted averages:

$$\sigma^2_{\rm halo}(>M, z) \equiv \frac{\displaystyle\int_M^\infty 2D^2(z)\sigma^2(M, z=0)\, n_{\rm J}(M, z) dM}{\displaystyle\int_M^\infty n_{\rm J}(M, z) dM} \quad (2)$$

$$b_{\rm halo}(>M, z) \equiv \frac{\displaystyle\int_M^\infty b_{\rm ST}(M, z)\, n_{\rm J}(M, z) dM}{\displaystyle\int_M^\infty n_{\rm J}(M, z) dM}, \quad (3)$$

where we adopt the halo mass function $n_{\rm J}(M, z)$ fitted by Jenkins et al. (2001) and the mass-dependent halo bias factor $b_{\rm ST}(M, z)$ proposed by ST99. The value of $\sigma(M, z = 0)$, the halo center-of-mass velocity dispersion at $z = 0$, is modeled following Yoshida, Sheth & Diaferio (2001):

$$\sigma(M, z = 0) = \frac{430/\sqrt{3}}{1 + (M/2.487 \times 10^{16} h^{-1} M_\odot)^{0.284}} \ \mathrm{km\ s^{-1}}. \quad (4)$$

In deriving equation (2), we follow Sheth & Diaferio (2001) and assume that the evolution of halo velocities is well approximated by linear theory, and in order to relate the halo velocity dispersion to the pairwise velocity dispersion, we have assumed no velocity correlation between halos. We use the Lorentzian damping function $D_{\rm vel}$ in k-space for mass (Magira, Jing, & Suto 2000), and the Gaussian for halos (Ueda, Itoh, & Suto 1993; Yoshida, Sheth & Diaferio 2001).

[5] For details of the output formats of the Hubble volume simulation, see the Virgo archive web site http://www.mpa-garching.mpg.de/Virgo/hubble.html

3.2. *Modeling scale and mass dependence of the halo biasing*

The most important ingredient in describing the clustering of halos is their biasing properties. The mass-dependent halo bias model was developed by Mo & White (1996) based on the extended Press-Schechter theory (Lacey & Cole 1993). This biasing model is improved empirically by Jing (1998) and ST99 so as to more accurately reproduce the mass-dependence in N-body simulation results. We further attempt to incorporate the scale-dependence on the basis of the results of Taruya & Suto (2000, hereafter TS00) in which the scale-dependence arises as a natural consequence of the formation epoch distribution of halos. Yoshikawa et al. (2001) exhibit that the scale-dependence of the TS00 model agrees with their numerical simulations although the amplitude is larger because the volume exclusion effect is not properly taken into account in the model. Therefore we construct an empirical halo bias model of the two-point statistics which reproduces the scale-dependence of the TS00 bias with the amplitude fixed by the mass-dependent ST99 bias on linear scales. We find that the required behavior is well described by the following simple fitting formula:

$$b_{\rm halo}(M,R,z) = b_{\rm ST}(M,z)\left[1.0 + b_{\rm ST}(M,z)\sigma_R(R,z)\right]^{0.15} \quad (5)$$

for $R > 2R_{\rm vir}(M,z)$, and otherwise 0, where $R_{\rm vir}(M,z)$ is the virial radius of the halo of mass M at z and $\sigma_R(R,z)$ is the mass variance smoothed over the top-hat radius R. While the above cutoff below $2R_{\rm vir}(M,z)$ is simply intended to incorporate the halo exclusion effect very roughly, we find it a reasonable approximation as will be shown below.

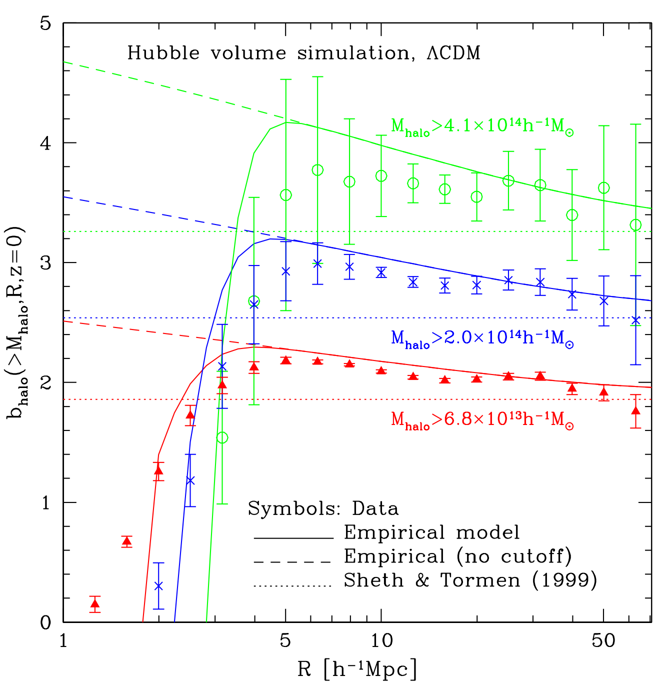

FIG. 1.— Halo biasing parameter defined by the two-point correlation functions on the constant-time hyper-surface in real space ($z = 0$); solid and dotted lines indicate our empirical model [eq. (5)], and the scale-independent model by ST99, respectively. The dashed lines correspond to our model without correction for the halo exclusion effect.

We test this empirical bias model against the halo catalogue generated from the snapshot data at $z = 0$. To do this, we compute the two-point correlation functions of halos of mass $M_{\rm halo} > M_{\rm min}$ in real space, then we divide them directly by the corresponding mass correlation function. We adopt the estimator $\xi = (DD - 2DR+RR)/RR$ (Landy & Szalay 1993) with the standard bootstrap method with 200 random re-samplings. Figure 1 compares the resulting bias factor of halos, $b_{\rm halo}(>M,R,z=0)$; open circles, crosses, and filled triangles for $M_{\rm min} = 4.1\times10^{14}h^{-1}M_\odot$, $2.0\times10^{14}h^{-1}M_\odot$, and $6.8\times10^{13}h^{-1}M_\odot$, respectively. The dotted horizontal lines indicate the $b_{\rm ST}(M,z)$, and our model predictions (eq.[5]) are plotted in solid lines. Given the simple formula that we adopted, the agreement with the numerical simulations at $z = 0$ is satisfactory.

Then our empirical halo bias model can be applied to the two-point correlation function of halos at z in redshift space as

$$\xi_{\rm halo}(M,R,z) = b_{\rm halo}^2(M,R,z)\int_0^\infty P^{\rm S}(k,z)\frac{\sin kR}{kR}\frac{k^2dk}{2\pi^2}. \quad (6)$$

Finally, the correlation function of halos on the light-cone is computed by averaging over the appropriate halo number density and the comoving volume element between the survey range $z_{\rm min} < z < z_{\rm max}$ (Matarrese et al. 1997; Moscardini et al. 1998; Yamamoto & Suto 1999; Suto et al. 2000):

$$\xi_{\rm halo}^{\rm LC}(>M,R) = \frac{\displaystyle\int_M^\infty dM\int_{z_{\rm min}}^{z_{\rm max}} dz\frac{dV_{\rm c}}{dz}\; n_{\rm J}^2(M,z)\xi_{\rm halo}(M,R,z)}{\displaystyle\int_M^\infty dM\int_{z_{\rm min}}^{z_{\rm max}} dz\frac{dV_{\rm c}}{dz}\; n_{\rm J}^2(M,z)} \quad (7)$$

where $dV_{\rm c}/dz$ is the comoving volume element per unit solid angle (HCS). Although our modeling is not completely self-consistent in the sense that the scale-dependence of the halo biasing factor is neglected in describing the redshift distortion (§3.2), the above prescription is supposed to provide a good approximation since the scale-dependence in the biasing is of secondary importance in the redshift distortion effect of halos.

3.3. *Clustering on the light-cone*

The two-point correlation functions on the light-cone are plotted in Figure 2 for halos with $M > 5.0\times10^{13}h^{-1}M_\odot$, $M > 2.2\times10^{13}h^{-1}M_\odot$ and dark matter from top to bottom. The range of redshift is $0 < z < 1$ (*Left panel*) and $0.5 < z < 2$ (*Right panel*). Predictions in redshift and real spaces are plotted in dashed and solid lines, while simulation data in redshift and real spaces are shown in filled triangles and open circles, respectively.

Our model and simulation data also show quite good agreement for dark halos at scales larger than $5h^{-1}$Mpc. Below that scale, they start to deviate slightly in a complicated fashion depending on the mass of halo and the redshift range. This discrepancy may be ascribed to both the numerical limitations of the current simulations and our rather simplified model for the halo biasing (eq.[5]). Nevertheless the clustering of *clusters* on scales below $5h^{-1}$Mpc is difficult to determine observationally anyway, and our model predictions differ from the simulation data only by

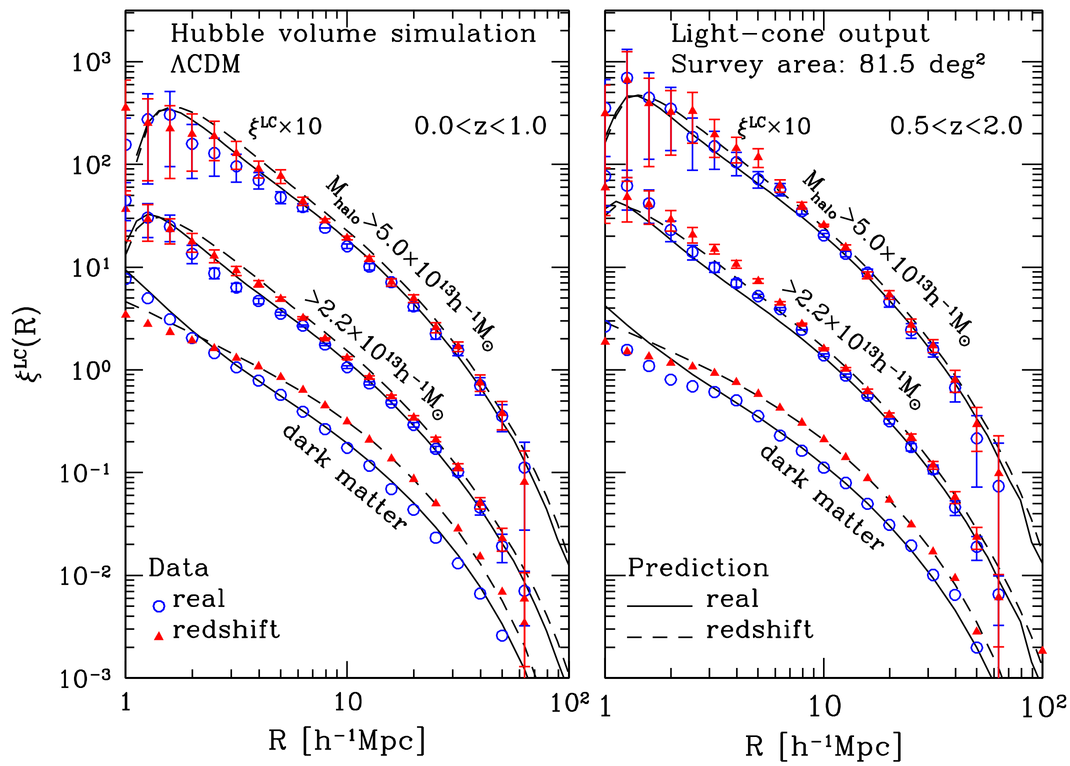

FIG. 2.— Two-point correlation functions of halos on the light-cone; simulation results (symbols; open circles and filled triangles for real and redshift spaces, respectively) and our predictions (solid and dotted lines for real and redshift spaces, respectively). The error bars denote the standard deviation computed from 200 random re-samplings of the bootstrap method. Upper set is for halos with $M_{\rm halo} \geq 5.0\times10^{13}h^{-1}M_{\odot}$, middle set is for $M_{\rm halo} \geq 2.2\times10^{13}h^{-1}M_{\odot}$, and lower set is for the dark matter. Notice that the amplitude of the upper sets is increased by an order of magnitude for clarity.

~ 20 percent at most. Therefore we conclude that in practice our empirical model provides a successful description of halo clustering on the light-cone.

4. CONCLUSIONS AND DISCUSSION

We develop a phenomenological model to predict the clustering of dark matter halos on the light-cone by combining several existing theoretical models. Combining the TS00 bias model with the ST99 mass function model, we are, for the first time, able to construct a halo biasing model that reproduces well the mass- and radial-dependence measured in the Hubble Volume simulation output data. Once calibrated with the $z = 0$ snapshot data, we find that our model agrees well with the two-point correlation functions of the simulated halos up to $z = 2$ in both real and redshift spaces. Although we show that this phenomenological model of halo clustering provides accurate predictions for the two-point correlation function of halos over a limited range in mass and redshift, we anticipate that it can be applied to a wider range of scales. This opens up application to modeling observations of various astrophysical objects, such as galaxies, clusters of galaxies and quasars, under model-specific assumptions for the relation between dark halos and luminous objects.

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