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VIRIALIZATION IN *N*-BODY MODELS OF THE EXPANDING UNIVERSE. II. ALL PAIRS

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ABSTRACT

A previous investigation of the virialization of isolated pairs of galaxies, by testing the end product of the *N*-body simulations of Efstathiou and Eastwood, is extended to include all pairs, regardless of isolation. It is found that the nonisolated pairs do not satisfy the classical virial theorem as well as isolated pairs, but masses can be estimated to an accuracy of factors of 1.5 for the $\Omega_0 = 0.1$ model and 2 for the $\Omega_0 = 1.0$ model, over a range of separations $25 < H_0 r < 100$ km s^{-1}. Peebles's "cosmic virial theorem" is found to give mass estimates to somewhat better accuracy on the same scales. The calculation of the relative acceleration between pairs of mass points in the model from the three-point correlation function, using its empirical relation to the two-point correlation function, is shown to be valid to an accuracy of a factor ~ 1.5. Therefore, if the same results hold for the real universe, estimates of Ω_0 based on such virial analyses should be accurate to better than a factor of 2. Comparison with observations then leads to the conclusion that $\Omega_0 < 1$.

A direct comparison of the relative velocity dispersion of pairs of mass points in the model with the observed dispersion of galaxies also favors an open universe. Furthermore, as long as the mass is distributed as the galaxies, it is unlikely that this conclusion will be altered by more elaborate and realistic models.

Subject headings: cosmology — galaxies: clustering — numerical methods

I. INTRODUCTION

In regions of space that can be considered to be in hydrostatic equilibrium, mass estimates are obtained from the equation of hydrostatic equilibrium, or some integral of it. This method can be applied in the standard way to isolated bodies, such as globular clusters, but its application in cosmology is somewhat problematic. Material continues to rain in on clusters after their centers have collapsed (Gunn and Gott 1972), and the new arrivals in fact constitute the bulk of the matter in clusters (Rivolo and Yahil 1983).

Over the past decade or so there has been considerable effort, pioneered by Peebles (1973), to study the dynamics of clustering in the universe through the positional and velocity correlations of random (nonisolated) pairs of galaxies. This line of investigation complements the direct study of well-defined entities, such as isolated binary galaxies and groups and clusters. The aim is to avoid the difficult question of the extent of clusters, and the degree of their separation from the rest of the universe, by treating clustering as a statistical process.

There has been an extensive observational effort to determine the positional two-point, three-point, and four-point correlation functions (Groth and Peebles 1977; Fry and Peebles 1978; Davis and Peebles 1983; Bean *et al.* 1983; and references therein to earlier work). Unfortunately, it is very difficult to extract from the positional information alone unique dynamical consequences, as *N*-body models of the expanding universe have repeatedly shown (e.g., Gott, Turner, and Aarseth 1979; Efstathiou and Eastwood 1981; Frenk, White, and Davis 1983). Attention has therefore focused on velocity data for dynamical discrimination. Global energy theorems or estimates (Irvine 1961, 1965; Layzer 1963; Dmitriev and Zel'dovich 1964; Fall 1975; Saslaw and Aarseth 1982) are as yet of little practical interest for statistical studies, because the total peculiar kinetic energy is dominated by large-scale streaming motions, which are difficult to measure observationally.

Peebles (1976*a*, *b*) has argued that for small separations dynamical equilibrium should be expected. Therefore, the relative force between pairs of galaxies, i.e., the sum of the direct two-body force and the tidal force due to all other galaxies, must be balanced by a pressure gradient induced by the relative velocity dispersion. This hypothesis of local dynamical equilibrium has been dubbed the "cosmic virial theorem" (CVT). Furthermore, except for a normalization constant, which is proportional to the cosmological density parameter Ω_0, the mean tidal force between pairs of galaxies can be calculated from the three-point correlation function. It is therefore possible to determine this normalization constant, and hence Ω_0, by measuring the relative velocity dispersion of pairs of galaxies. However, the application of the CVT to the determination of Ω_0 has been somewhat problematic (see § III below).

In a companion paper (Evrard and Yahil 1985, hereafter Paper I) we examined mass estimates for binary, i.e., isolated, galaxies in the end product of the *N*-body simulations of Efstathiou and Eastwood (1981, hereafter EE). A variation of the classical virial theorem was used, and although the theorem was there applied to isolated pairs, it does not require the pairs to be isolated. Hence, if its assumptions are satisfied for all pairs, most of which are nonisolated, then the virial relation should hold for these pairs as well. It could then be the basis for estimates of galactic masses and Ω_0.

In this paper we extend the *N*-body investigation of Paper I to cover all pairs, whether isolated or not, with the aim of determining how well both the classical virial theorem and the CVT are satisfied. The same qualifications mentioned in Paper I apply here as well. Since the models start from cold Poisson distributions at the relatively recent redshifts of 19.2 ($\Omega_0 = 0.1$) and 9.9 ($\Omega_0 = 1.0$), there is not enough time to develop rich clusters and surrounding superclusters as we know them in nature. Hence, the two-point correlation function (Fig. 3 below) both is steeper than the observed one and has a shorter

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correlation length [the radius at which $\xi(r) = 1$]. In addition, the softening of the two-body force at small separations causes a truncation of ξ, which is not seen in nature (Gott and Turner 1979).

Nevertheless, the models may be studied in their own right. Some indication may thus be obtained of the reliability of mass estimates obtained through the use of such virial analyses in observational programs. If, in addition, the N-body models do bear some similarity to the real universe, the velocity dispersion found in them may also be compared directly with the observed one, without recourse to virial analyses, giving a direct estimate of Ω_0. In a sense, the N-body simulation itself then becomes the virial analyzer.

In § II we briefly discuss the classical virial theorem and whether or not its assumptions are likely to be satisfied for clustering galaxies in an expanding universe. Section III outlines Peebles's cosmic virial theorem, and its somewhat problematic history. The results of our investigations are reported in § IV, and the conclusions are summarized in § V.

II. CLASSICAL VIRIAL THEOREM REVISITED

In Paper I we presented a new version of the classical virial theorem, in which the effect of the expanding cosmological substratum was subtracted out. For any pair of mass points the following identity holds:

$$\frac{d}{dt}(\boldsymbol{r}\cdot\boldsymbol{u}) = u^2 + \boldsymbol{r}\cdot\boldsymbol{g}\,, \tag{1}$$

where $\boldsymbol{r}$, $\boldsymbol{u}$, and $\boldsymbol{g}$ are the separation, relative peculiar velocity, and relative peculiar acceleration of the pair, respectively. If for a sample of pairs the ensemble average of the left-hand side of equation (1) can be shown to be zero, then one obtains the virial theorem

$$\langle u^2\rangle = -\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle\,. \tag{2}$$

It was shown in Paper I that in the EE N-body simulations the sample of all bound isolated pairs satisfied the assumptions of the virial theorem, and hence its corollary, equation (2). Here we seek to test the virial theorem for all pairs, regardless of isolation. The concept of binding of the pair to each other loses meaning here, since the relative acceleration is dominated by collective forces, and not by the two-body force (see Paper I, Fig. 1). Similarly, we cannot easily test for homogeneity in the distribution of orbital phases, as we did for the bound isolated pairs, because of the complex orbits resulting from the stochastic nature of the accelerations of pairs. The validity of the virial theorem can therefore be tested only by whether or not equation (2) is satisfied.

In spite of the formal difficulties in identifying a proper ensemble for which equation (2) should be satisfied, it is possible to estimate under which conditions the left-hand or right-hand side is definitely larger. At the centers of clusters, for example, the velocity dispersion is high, but $\boldsymbol{r}\cdot\boldsymbol{g}$ approaches zero in the limit $r \to 0$. For large separations, on the other hand, at least one of the members of the pair might be expected to be in its initial infall toward the nearest density perturbation, in which case $\boldsymbol{u} \sim \boldsymbol{g}t_0$, so that

$$\frac{u^2}{\boldsymbol{r}\cdot\boldsymbol{g}} \sim \frac{gt_0^2}{r} \sim \frac{4\pi G}{3}\,\delta\rho\, t_0^2\,, \tag{3}$$

and this quantity can clearly be less than unity if $\delta\rho$ is small enough. The agreement between u^2 and $-\boldsymbol{r}\cdot\boldsymbol{g}$, found in Paper I to hold for isolated pairs over a significant range in separation, is therefore not expected to hold for the non-isolated pairs. Whether some cumulative averages of the two quantities are going to be approximately equal will be seen in the results of § IV.

III. PEEBLES'S "COSMIC VIRIAL THEOREM"

As mentioned in § I, Peebles (1976*a*, *b*) suggested that the equation of hydrostatic equilibrium be applied to an ensemble average of galaxies, which can be described by the correlation functions. The first step is to write the equation of hydrostatic equilibrium, which is expected to hold at sufficiently small separations, at which the galaxies may be presumed to be "virialized":

$$\frac{d}{dr}(\xi\langle u_r^2\rangle) + \frac{2\xi}{r}(\langle u_r^2\rangle - \langle u_t^2\rangle) = \xi\langle g_r\rangle\,. \tag{4}$$

Here ξ is the two-point correlation function, $\langle u_r^2\rangle$ is the mean square radial velocity, $\langle u_t^2\rangle$ is the mean square relative tangential velocity (*one*-dimensional), and $\langle g_r\rangle$ is the mean relative radial acceleration. (Note that in our convention $\boldsymbol{g}$ is the *relative* acceleration, and is therefore twice as large as Peebles's $\boldsymbol{g}$, which is defined as the acceleration of only one of the two galaxies in the pair.)

Equation (4) in its differential form is a statement of an "average hydrostatic equilibrium," and can be tested directly in the N-body simulation, since all quantities in the equation can be evaluated. It will prove convenient, because of the wide range of $\xi(r)$, to multiply equation (4) by $-r/\xi(r)$, and study the relation

$$-\frac{r}{\xi}\frac{d}{dr}(\xi\langle u_r^2\rangle) - 2(\langle u_r^2\rangle - \langle u_t^2\rangle) = -\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle\,. \tag{5}$$

Except for the contribution of the direct two-body force, the mean of the remaining (tidal) relative acceleration of a pair of galaxies is given by an integral over the three-point correlation function:

$$-\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle = \frac{2Gm}{r} + \frac{2G\rho}{\xi(r)}\int d^3z\,\frac{\boldsymbol{r}\cdot\boldsymbol{z}}{z^3}\,\zeta(r, z\,|\,\boldsymbol{z}-\boldsymbol{r}\,|)\,. \tag{6}$$

Beyond a minimum separation, the second, tidal, term in equation (6) dominates (see Fig. 1 below).

Empirically, the three-point correlation function has been found to be related to the two-point correlation function (Peebles and Groth 1975; Groth and Peebles 1977):

$$\zeta(r, z, |\,\boldsymbol{z}-\boldsymbol{r}\,|) = Q[\xi(r)\xi(z) + \xi(r)\xi(|\,\boldsymbol{z}-\boldsymbol{r}\,|) + \xi(z)\xi(|\,\boldsymbol{z}-\boldsymbol{r}\,|)]\,, \tag{7}$$

with $Q \sim 1$. If the above relation holds with $\xi(r) \propto r^{-\gamma}$, then the tidal acceleration is given by

$$-\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle = \tfrac{3}{2}K_\gamma Q r_0^\gamma \Omega_0 H_0^2 r^{2-\gamma}\,, \tag{8}$$

where K_γ is a constant depending only on γ. Further assumptions about the form of the velocity anisotropy term allow equation (4) to be integrated, yielding a relation for the velocity dispersion, from which Ω_0 can be determined. For example, assuming isotropic orbits gives

$$\langle u_r^2(r)\rangle = \frac{3}{4(\gamma-1)}K_\gamma Q r_0^\gamma \Omega_0 H_0^2 r^{2-\gamma}\,. \tag{9}$$

The application of the CVT to estimate Ω_0 has been somewhat problematic. First, the estimated Ω_0, scaling as Δv^2, is sensitive to the actual observational value of Δv, over which there is some debate. Davis and Peebles (1983) argue for $\Delta v = 340 \pm 40(H_0 r/100)^{0.13}$ km s^{-1}, while Rivolo and Yahil (1981) found $\Delta v = 100 \pm 15$ km s^{-1}, and Bean *et al.* (1983) quote 250 ± 50 km s^{-1}, both independent of separation. Rivolo and Yahil (1981), using their value of Δv, obtained $\Omega_0 = 0.01$, which they rejected outright as being too low compared with determinations from Virgocentric inflow (Yahil 1981; Aaronson *et al.* 1982). They suspected that either equation (7) or the CVT, or both, might not hold, although their low value of Δv clearly weighed heavily upon these concerns.

Davis and Peebles (1983) noted that, with equation (7) as it stands, the relative force between galaxies is dominated by each one's close satellites (see also Fig. 1 of Peebles 1976*a*). Believing this to be an unphysical peculiarity, they replaced the last term in equation (7) with the middle one when calculating the relative acceleration via equation (6). This reduces the estimate of the collective force by a factor of ~3, resulting in $\Omega_0 = 0.2$. Although this value is in reasonable agreement with the value found from the Virgocentric inflow, and with other estimates contained in their paper, the somewhat arbitrary nature of the applied modification, for which there is no justification of which we are aware, again calls into question the basic tenets of the CVT.

On the other hand, Bean *et al.* (1983), with only a moderate change to the basic theory, found $\Omega_0 = 0.14$. They inserted a

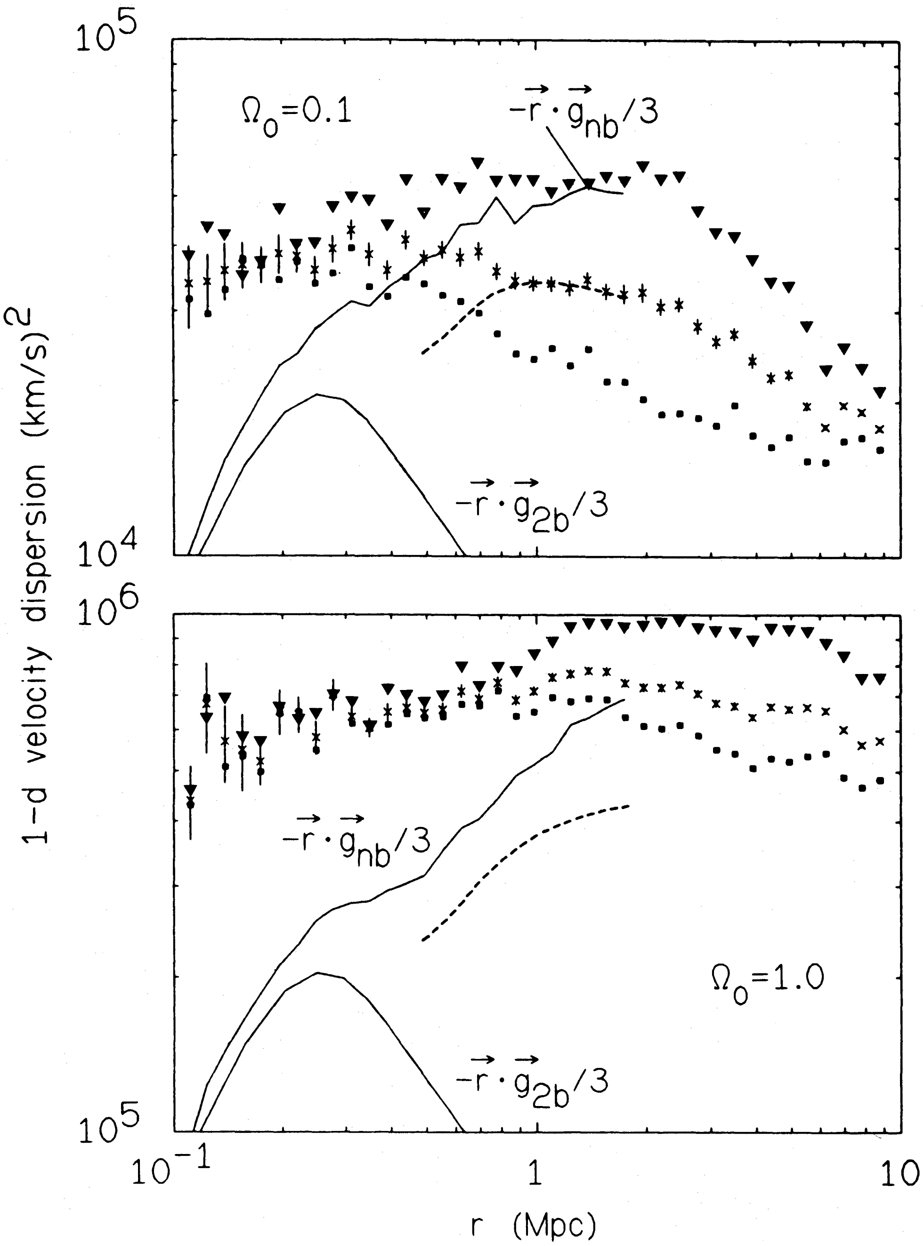

FIG. 1.—Mean square relative peculiar velocity components (*crosses*), radial components (*inverted triangles*), and tangential components (*squares*) are plotted vs. separation. All velocities are normalized to *one* dimension. The upper solid line shows the measured values of the virial variable $-\langle \boldsymbol{r} \cdot \boldsymbol{g} \rangle/3$. The dashed line gives the correlation function estimate of that quantity using equations (6) and (7) with $Q = 1$. The lower solid line shows the virial variable with only the direct two-body contribution to $\boldsymbol{g}$.

softening term into the Newtonian gravity term in equation (6), transforming z^{-3} to $(z + \epsilon)^{-3}$ to represent the finite extent of galaxies.

In view of this muddled observational situation, we seek in this paper to analyze the underlying principles of the CVT. Specifically, we test in § IV whether the average hydrostatic equilibrium assumed in equation (4) holds, and how accurately $\langle \boldsymbol{r} \cdot \boldsymbol{g} \rangle$ is determined by equation (6), using equation (7) to represent the three-point correlation function.

IV. RESULTS

The mean square relative peculiar velocity (*one-dimensional*), as well as the directly measured virial variable $-\langle \boldsymbol{r} \cdot \boldsymbol{g} \rangle/3$, is plotted against separation in Figure 1. The mean square radial and mean square tangential (also *one-dimensional*) components are also given. The scaling of the dimensionless N-body model to the real universe is the same as in Paper I, namely, $H_0 = 50$ km s^{-1} Mpc^{-1}; the mass per galaxy is $2.2 \times 10^{13}\ \Omega_0\ M_{\odot}$, and the length of the computational cube at the present epoch is 185 Mpc.

a) Relative Velocity Dispersion

The distribution of mean square relative velocity with separation is seen in Figure 1 to be fairly flat and very anisotropic in both models. The anisotropy extends well interior to the region of infall, which is apparent upon comparison with Figure 2, where the mean radial component of pairs' proper velocity is plotted. This feature arises from the "cold" (i.e., zero peculiar velocity) initial state of the models and is analogous to the high orbital eccentricities found for isolated binaries in Paper I.

The rms relative velocity for all separations smaller than

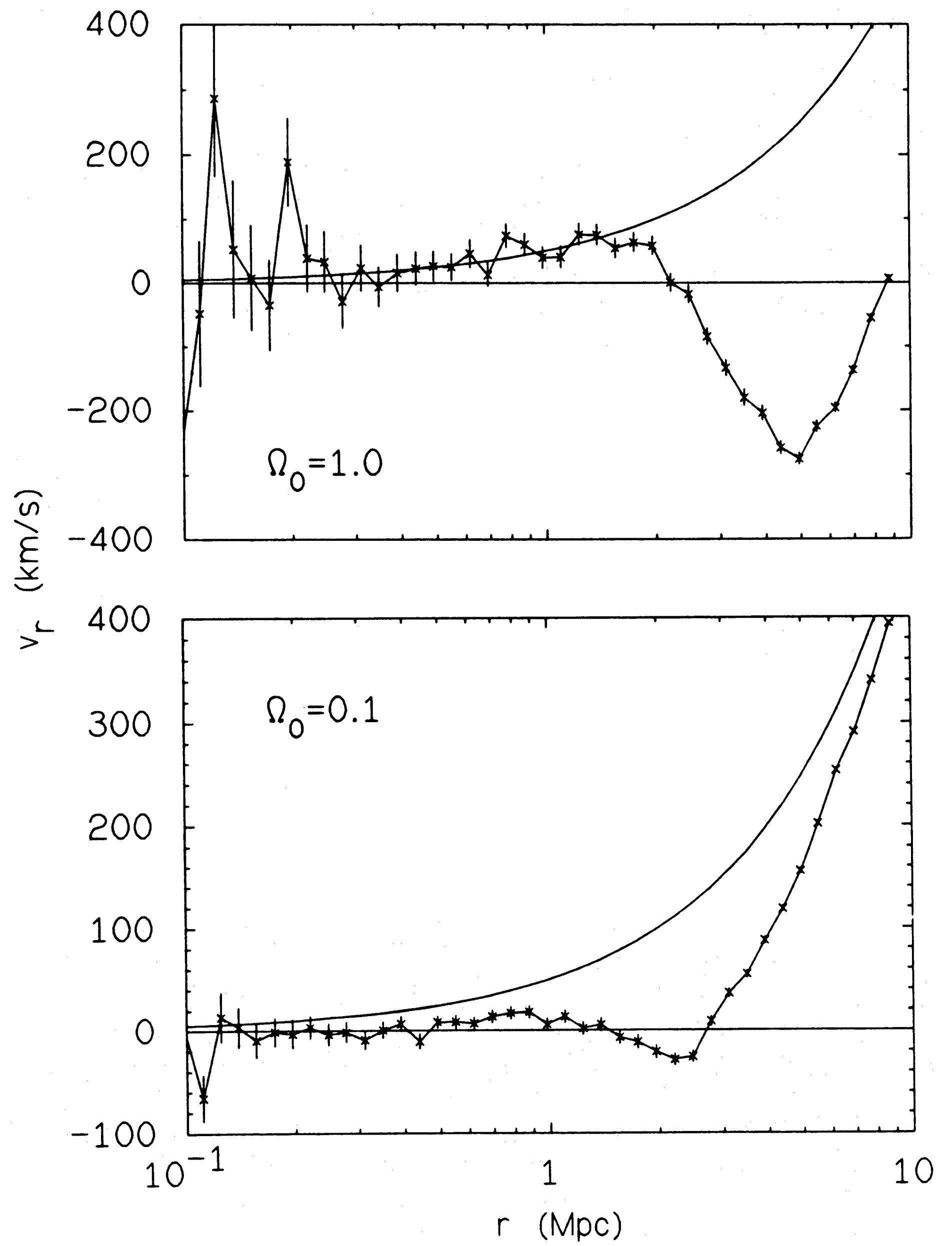

FIG. 2.—Mean value of the radial component of the pairs' *proper* relative velocity is shown plotted vs. separation. The solid line is the Hubble relation, $v_r = H_0 r$. Regions of infall are evident in both models, although they are much weaker in the low-density simulation, where the growth of clustering has nearly stopped.

1 Mpc is 190 km s^{-1} for the $\Omega_0 = 0.1$ model, and 825 km s^{-1} for the $\Omega_0 = 1.0$ model. The observed velocity dispersion measurements for separations under 1 Mpc range between 100 and 310 km s^{-1} (see § III). Taking any value in this range, comparison with the N-body models clearly favors the case of an open universe. This conclusion is unlikely to be altered by more realistic modeling of extended galaxies and stronger positional correlation, as long as all mass is assumed to be distributed as the galaxies. A velocity dispersion of 825 km s^{-1} (the $\Omega_0 = 1.0$ case) cannot be dissipated by galaxies whose internal dispersion or rotation velocities are ~ 150 km s^{-1} (one-dimensional), and stronger clustering will only increase the velocity dispersion.

b) *Mean Relative Acceleration*

In Figure 1 and in what follows, we use the actual mean relative acceleration, $\langle \boldsymbol{r} \cdot \boldsymbol{g} \rangle$, calculated by explicitly adding to the direct force between each pair the tidal contributions of its neighbors. In observational studies, on the other hand, this quantity has been estimated from the two and three-point correlation functions, using equations (6) and (7). It is clearly worth checking how good this estimate is in the models. Note, however, that the two-point correlation function, shown in Figure 3, is not well described by a single power law. Instead, since force softening below $r = d_m = 440$ kpc (see Paper I) causes severe flattening of the correlation function below these

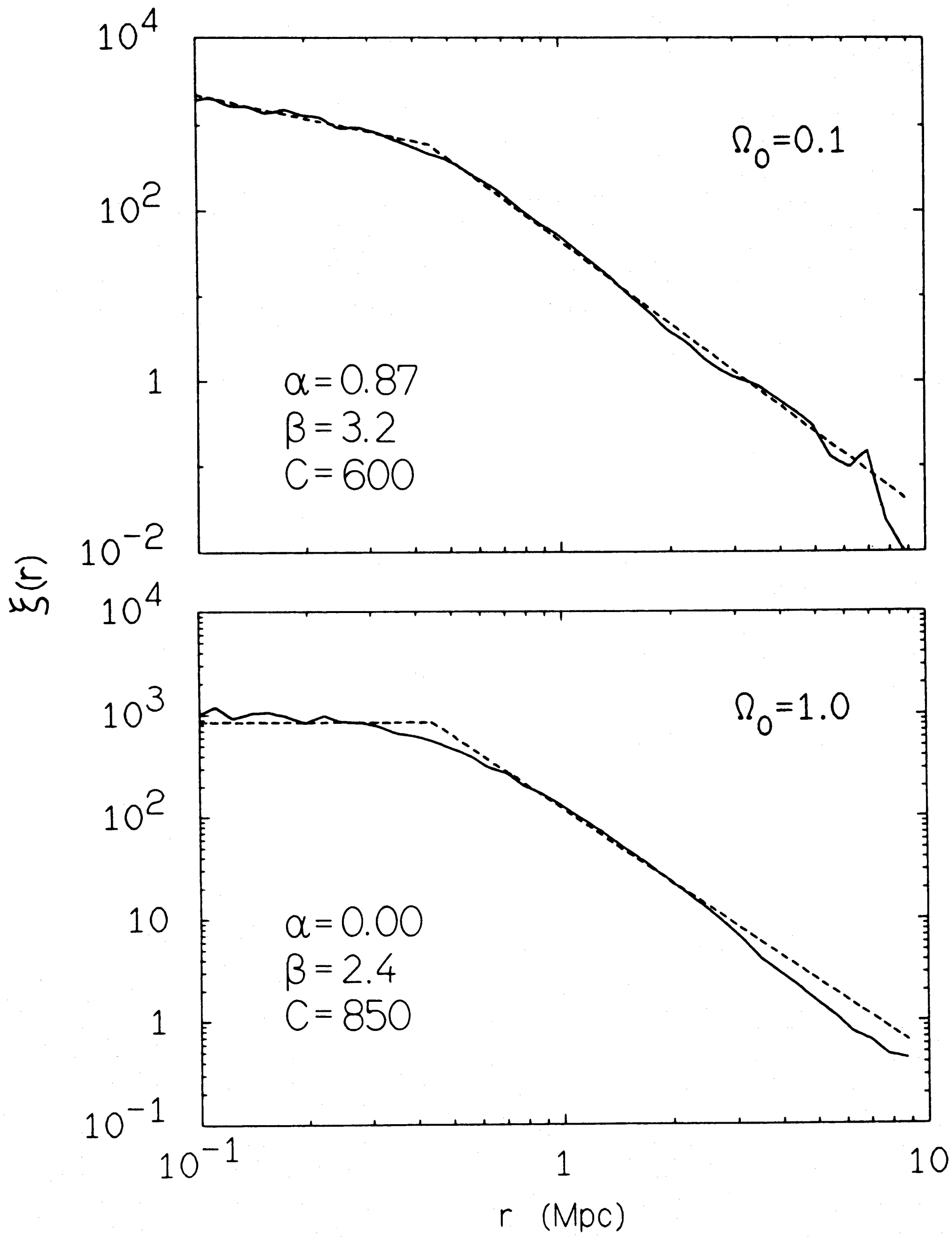

FIG. 3.—Two-point correlation function $\xi(r)$ as a function of separation, along with double power-law fit (eq. [10]). Force softening at small separations severely flattens the correlation function scales on $r < d_m = 440$ kpc.

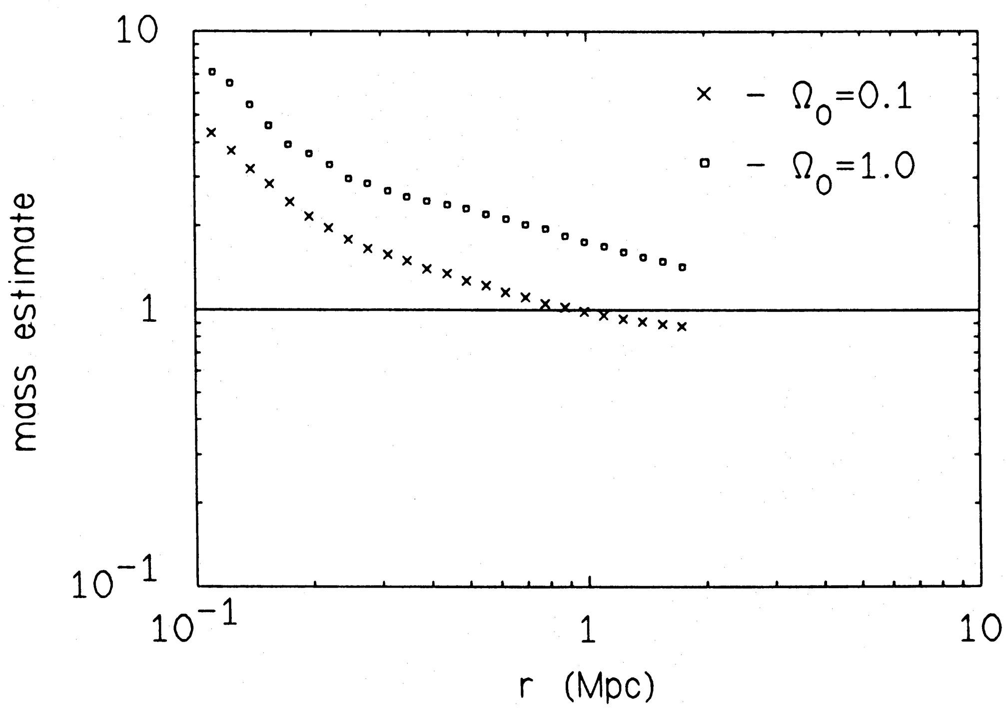

FIG. 4.—Mass estimates obtained by the cumulative ratio $-\langle u^2\rangle/\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle$. Note the steady decline in this ratio, as opposed to the rapid convergence found for bound isolated pairs in Fig. 6 of Paper I. Yet the accuracy is better than a factor of 2 for separations 500 kpc < 1.85 Mpc.

scales, we adopt a double power law:

$$\xi(r) = \begin{cases} C(d_m/r)^\alpha, & r < d_m \\ C(d_m/r)^\beta, & r > d_m \end{cases}, \qquad (10)$$

with the values of the parameters α, β, and C given in Figure 3.

The calculation of the integral in equation (6) therefore no longer yields the simple power law, equation (8). In Figure 1 the dashed line shows values of $-\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle$ calculated from equation (6), using equations (7) and (10) with $Q = 1$. The results show moderate agreement in shape, but the magnitude is low by a factor of ~ 1.5. We consider this reasonable agreement, since it is within the error margin of the virial analysis itself, which is a factor of ~ 2. Also, Q has not been reliably determined for the 20,000-body models; similar 1000-body models have $Q = 1.2$ (EE). Thus, our Q-value may be low by a small factor.

Note that we have explicitly included the two-body force in the calculation. This facilitates comparison with the directly measured relative acceleration, in which the two-body virial variable $\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle/3$ is also plotted in Figure 1, so that its contribution to the total may be assessed.

c) *Classical Virial Theorem*

Figure 1 shows that the agreement between the mean square relative velocity $\langle u^2\rangle/3$ and the virial variable $-\langle \boldsymbol{r}\cdot\boldsymbol{g}\rangle/3$ is not nearly as good as in the case of isolated pairs (Paper I). As expected from the discussion in § II above, at small separations the mean square velocity exceeds the virial variable, and the ratio of the two decreases with increasing separation, without reaching any asymptotic value. Despite this lack of convergence, we find in Figure 4 that, for separations between 500 kpc and 2 Mpc, the cumulative virial ratio is equal to unity within factors of 1.5 for the $\Omega_0 = 0.1$ model and 2 for the $\Omega_0 = 1.0$ model.

It is tempting to identify the scale at which the cumulative virial ratio equals unity as the size of a "typical" cluster. Inspection of Figure 4 shows these clustering scales to be 1 Mpc for $\Omega_0 = 0.1$ and 4 Mpc for $\Omega_0 = 1.0$, using direct extrapolation of the data in the latter case. We can compare these scales with two other scales of interest. The standard clustering length in these models, i.e., the distance at which $\xi(r) = 1$, is 3 Mpc for $\Omega_0 = 0.1$ and 6 Mpc for $\Omega_0 = 1.0$. Another way to define a cluster size is from the phase-space diagram of the radial component of the relative velocities of the pairs versus their separations (cf. Rivolo and Yahil 1983). Figure 2 for $\Omega_0 = 0.1$ that there is modest infall between 1.5 and 2.0 Mpc, indicating that the largest clusters are of this size. In the $\Omega_0 = 1.0$ model, tremendous streaming motions exist between 2.0 and 9.0 Mpc, with the maximum infall velocity occurring at about 5 Mpc.

d) *Peebles's "Cosmic Virial Theorem"*

Consider first the differential form of the cosmic virial theorem, equation (5). Both sides of the equation are plotted in Figure 5. The results show a systematic underestimate of the pressure term (left-hand side) relative to the gravitational term (right-hand side) over all scales measured. This is not the result of a general contraction, as can readily be seen from Figure 2, and must therefore be due to time-dependent terms that are ignored by the CVT. The disagreement, however, is comparable to the one noted for the classical virial theorem. Outside of the softening range of the two-body force, $r > 440$ kpc, the two sides of equation (5) do not differ by more than a factor of ~ 1.5 for both models. Interior to the softening region, the velocity data become more noisy, and therefore the unsmoothed derivatives on the left-hand side show more scatter. Generally, agreement is not as good in this region.

Since the differential form of the CVT, equation (5), seems to hold rather well, we have also tried to test an integral form, by

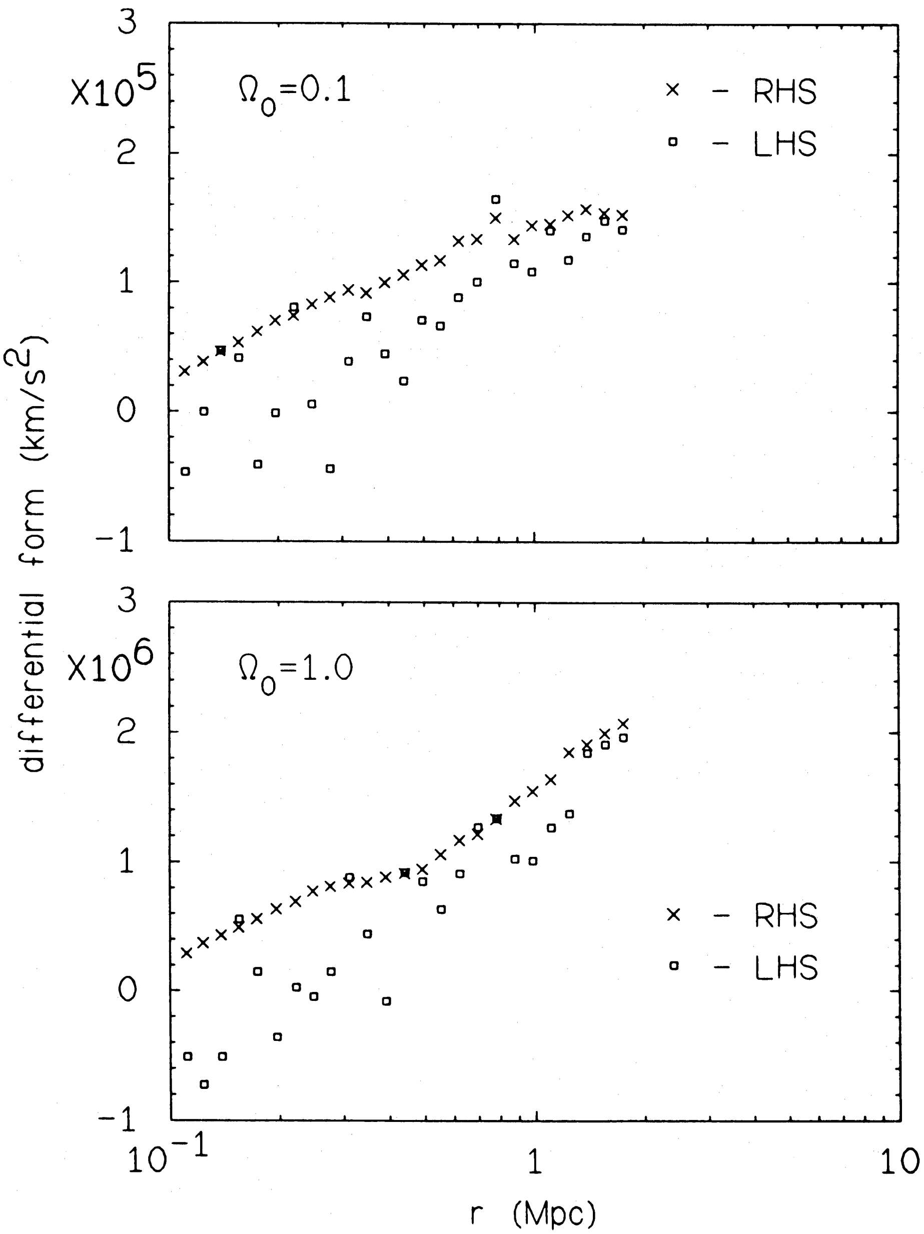

FIG. 5.—Measured values of the left-hand and right-hand sides of the differential form of Peebles's cosmic virial theorem (eq. [5]).

integrating equation (4) between r and the maximum separation for which we have measured g, 1.85 Mpc. The integral relation is more useful for mass estimates, because in practical cases the samples are not likely to be as rich as the ones in the N-body simulations, and the statistical noise would make the use of the differential form of equation (5) more difficult. The results are presented in Figure 6, in which we have divided the integral of equation (4) by ξ, for the same reasons for which we preferred equation (5) to equation (4) (see § III). The systematic bias in the differential form causes similar discrepancies in the estimated velocities. Outside of the softening range, the two sides agree to within a factor of 1.5; below this range, the discrepancy is larger.

V. CONCLUSIONS

We conclude that, for nonisolated pairs in the EE N-body simulations, both the classical virial theorem, and the "cosmic virial theorem" (CVT) proposed by Peebles (1976*a*, *b*), provide reasonable mass estimates, which are accurate to better than a factor of 2 on scales $25 < H_0 r < 100$ km s^{-1}. If virial analyses are similarly accurate for the real universe, results of their application to galaxies (Rivolo and Yahil 1981; Davis and Peebles 1983; Bean *et al.* 1983) should reinforce the conclusion that the universe is open.

The CVT is really a statement of *local* hydrostatic equilibrium, and as such may be somewhat better to use. Its accu-

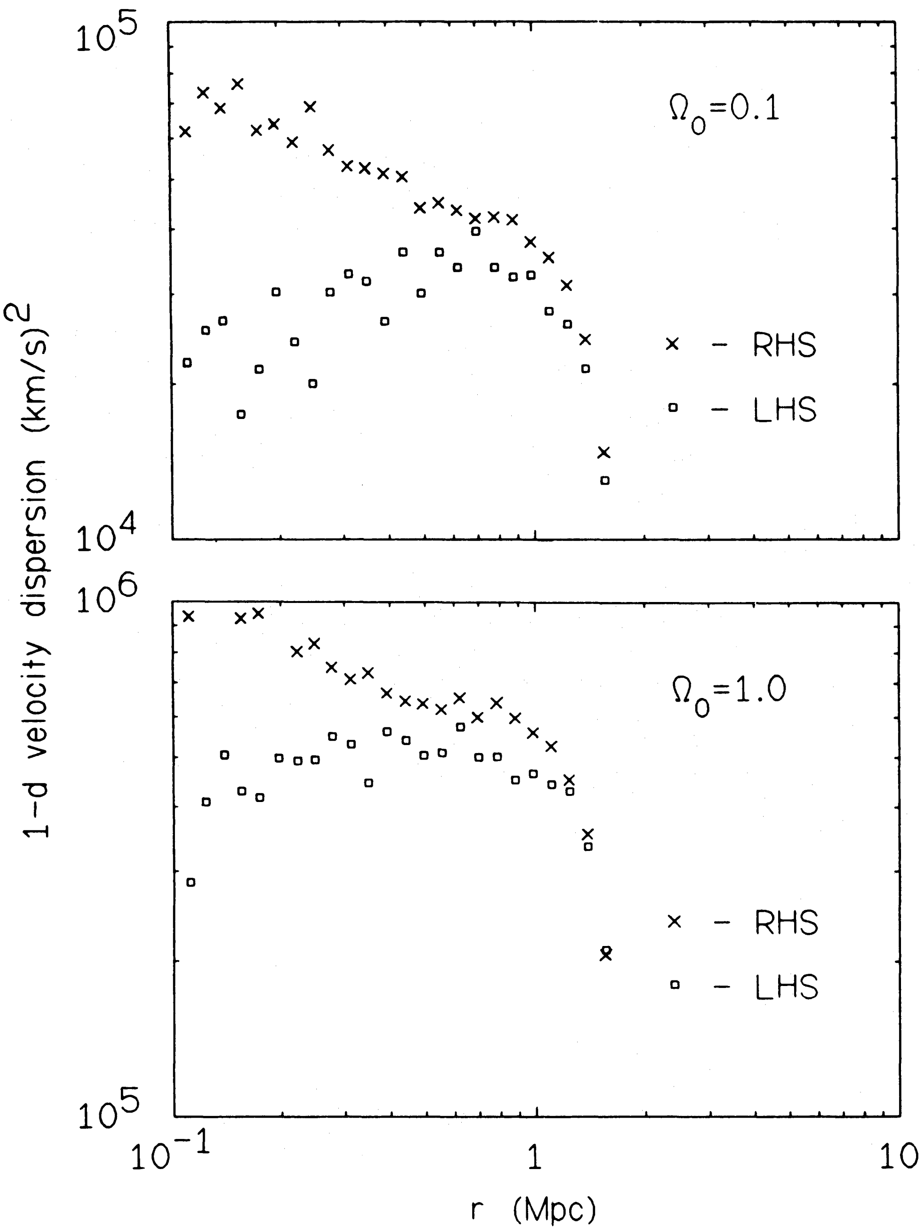

FIG. 6.—Measured values of the integral form of Peebles's cosmic virial theorem. Plotted is the integral of eq. (4) (multiplied by ξ). from the sample limit, $r = 1.85$ Mpc, to smaller r.

racy is seen not to depend on separation, and is somewhat better than that of the classical virial theorem. The latter, being a global measure, is sensitive to large-scale currents at large separations (infall into clusters), and is also not satisfied at very small separations inside clusters, where the tidal potential between neighboring galaxies is small compared with the global depth of the gravitational potential.

The calculation of the mean relative acceleration of pairs of galaxies from the three-point correlation function, equation (6), using its empirical relation to the two-point correlation function, equation (7), was shown to be valid to within a factor of ~1.5. However, the usually deduced power-law relation between the relative velocity dispersion and separation, equation (9), does not apply to the EE N-body models, in which the two-point correlation function is sufficiently different from a simple power law.

The one-dimensional velocity dispersions found in the models for all pairs with separations smaller than 1 Mpc are 190 km s^{-1} ($\Omega_0 = 0.1$) and 825 km s^{-1} ($\Omega_0 = 1.0$). If the models bear any resemblance to the real universe, then these values may be directly compared with the observed ones. While there is a disagreement about the observed velocity dispersion, an open universe is inferred from any value in the quoted range of $\Delta v = 100$–310 km s^{-1}. More realistic N-body calculations might include better modeling of extended galaxies, and initial conditions resulting in stronger correlation functions, more closely resembling the real universe. However, extended galaxies, with internal velocities ~150 km s^{-1} (one-dimensional), are unlikely to dissipate the high velocity dispersion obtained in the $\Omega_0 = 1.0$ model, and stronger clustering will only increase it. As long as mass is distributed as the galaxies, such refinements are therefore unlikely to create critical or closed models consistent with the observed velocity dispersion.

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