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UM AC 92-6

December 1992

Structure in a Loitering Universe

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We study the formation of structure for a universe that undergoes a recent loitering phase. We compare the nonlinear mass distribution to that in a standard, matter dominated cosmology. The statistical aspects of the clustered matter are found to be robust to changes in the expansion law, an exception being that the peculiar velocities are lower by a factor of ~ 3 in the loitering model. Further, in the loitering scenario, nonlinear growth of perturbation occurs more recently ($z \sim 3-5$) than in the matter dominated case. Differences in the high redshift appearances of the two models will result but observable consequences depend critically on the chosen form, onset and duration of the loitering phase.

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I. INTRODUCTION

The standard Freedman–Robertson–Walker (FRW) universe with a hot big bang is simple and quite successful in predicting the Hubble expansion, the abundances of light elements and other phenomena. The current standard theories for structure formation, in particular inflation and cold dark matter (CDM), seem to give a reasonable explanation as to the formation of large scale structure in the Universe.

The standard cosmology invokes gravitational instability as the mechanism for structure formation. Forming structure gravitationally is very attractive, in numerical simulation structure that is remarkably similar to the one observed seems to form. Further, the recent COBE [1,2] observations of anisotropy of the microwave background radiation (CMBR) suggests that the primordial Universe was indeed slightly inhomogeneous and anisotropic. These tiny inhomogeneities may have been seeds that grew gravitationally to become the structure we see today.

As successful as it is, the standard model may be in apparent conflict with some cosmological observations. Recent findings suggest more inhomogeneities [3] on large scales $\gtrsim$10Mpc and smaller peculiar velocities [4] on small scale $\sim$ 1Mpc than the unbiased CDM scenario predicts. It might be possible to reconcile some of the observations with the model, *e.g.,* a biased CDM model gives a better agreement with small scale streaming velocities, but it makes the large scale power discrepancy worse. Another example is a flat cosmology with a nonvanishing cosmological constant which helps ameliorate the large scale velocities [5], however it conflicts with lensing statistics [6]. Thus it is not clear that CDM can be reconciled with observations of large scale structure and the cosmic microwave background radiation temperature variations.

Although gravitational instability has been tested for structure formation given standard cosmological background, *i.e.,* FRW with CDM [4], only a few studies of other cosmologies have been done [7,8]. In this paper, we present results for structure formation in a universe obeying a non–standard expansion law [9]. The motivation is to test the 'stability' of gravitational instability to changes, not in the initial conditions, but in the expansion law of the background metric. A universe which undergoes a period of 'loitering' is an attractive alternative to standard cosmologies [9]. The loitering universe scenario is an expanding Friedmann cosmology that undergoes a fairly recent ($z \sim 3-5$) phase of slow expansion. It is during this semi–static phase that large–scale structure is formed. The model is useful in providing explanations both for the small amplitude of CMBR fluctuations and for the so–called 'age problem' of globular clusters. Although a period of loitering can be achieved in a cosmological constant dominated universes [7], Durrer & Kovner [8] have noted that there are serious problems with these models.

Some researchers [10] claim that the ages of the oldest globular clusters are 1.6×10^{10} years, whereas the estimation of the Hubble constant today varies between $\mathrm{H}_o^{-1} = 1-2\times 10^{10}$ years. Since, in a matter dominated universe, the age of the universe $t_o \leq \mathrm{H}_o^{-1}$, and in particular, $t_o = 2/3\mathrm{H}_o^{-1}$ for flat ($\Omega = 1$) cosmologies, [younger for an open ($\Omega_o < 1$) universe]. The ages of globular clusters pose a real problem, especially if H_o is large (small H_o^{-1}) as some recent observations suggest [11]. The loitering scenario allows for a universe where $t_o > \mathrm{H}_o^{-1}$ as can be seen from the expansion law in figure 1 below.

Until the recent apparent detection of anisotropy in the CMBR [1,2], it was essential to investigate theories capable of producing structure from very low amplitude initial perturbations.Although the COBE result seems to confirm the standard model, it remains the only detection of the microwave anisotropy. Until confirmation of this detection and further detections on different angular scales, alternatives to the standard model should be considered viable. During the loitering phase, the density contrast of initial perturbations grows semi–exponentially with expansion factor, much faster than the power law dependence in standard FRW cosmologies. The rapid growth in amplitude allows non–linear structures to form at present without producing large Sacks–Wolfe anisotropies [9]. However, detailed properties of clustering in the non–linear regime have not yet been investigated for this model.

In this paper we employ N–body simulations in both loitering and matter dominated

(MD) cosmologies to test statistical properties of the large–scale matter distribution in the non–linear regime. We find that the loitering scenario produces structure remarkably similar to that formed in the MD scenario, with some interesting differences. For the loitering scenario the Sacks–Wolfe $\Delta T/T$ is smaller, small scale streaming velocities agree better with observations, and the universe is old enough to account for the age of the oldest globular clusters.

The paper is organized as follows: In section II we review the loitering universe model. In section III, we present details of the initial conditions and results of the simulations. We conclude in section IV.

II. THE LOITERING MODEL

For the purpose of testing the large scale structure predictions of the loitering model, we developed a simple model that has all the interesting features of the loitering scenario [9] without any of the more complicated dynamics involved in a more sophisticated theory. Consider a closed universe where, in addition to the conventional matter (*i.e.,* matter with both p and $\rho \geq 0$), there exists a source term having an equation of state $p_s = \gamma \rho_s$ where $-1 \leq \gamma < -\frac{1}{3}$. The energy density of such matter scales as $\rho_s \propto \frac{1}{a^n}$ where $0 \leq n < 2$, $[n = 3(1+\gamma)]$.

If we assume that the additional source term does not interact with ordinary matter, the only thing that will affect the formation of structure is the different expansion law, which is why we can use the simplistic theory as a good approximation for structure formation studies. This effect is the one we study here. The Einstein equations for a Friedman cosmology with an additional source term are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\rho_{\mathrm{T}} - \frac{k}{a^2}\right) = \frac{8\pi}{3}\left(\frac{\rho_o}{a^3} + \frac{\rho_s}{a^n} - \frac{k}{a^2}\right) \tag{1}$$

and

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}(\rho_{\mathrm{T}} + 3p) \ = \frac{4\pi G}{3}\left(\frac{\rho_o}{a^3} + \frac{(n-2)\rho_s}{a^n}\right) \ , \tag{2}$$

where ρ_o and ρ_s are constants and $k = 1$ (closed universe). There is an implicit assumption that at late times the conventional matter content of the universe is in the form of cold matter, *i.e.,* $\rho_{\rm cm} \propto a^{-3}$. When all the terms in the right hand side of Eq. (1) are of comparable magnitudes, a period of coasting may result during which $\dot{a} \simeq$ constant $\ll 1$ and $\ddot{a} \simeq 0$. After the 'loitering' phase the universe goes into a late, power law 'inflationary' phase.

The expansion law of the model we consider here and the standard FRW cosmology expansion law are shown in Figure 1. For the run described in the paper we chose the parameters to be $n = 1.2$ and $\rho_s/\rho_o = 0.32$. This choice of parameters implies that the age of the universe is $t_o \simeq 8/3{\rm H}_o^{-1}$ whereas in a matter dominated universe $t_o = 2/3{\rm H}_o^{-1}$, (see Figure 2) this solves the 'age problem'. Clearly the form of the expansion law depends on the choice of these parameters. Ideally, the values of these parameters should be determined from the nature of the matter described by the source term. The parameters we chose lead to a rather short and recent period of loitering. The onset, duration and form of the expansion law can be manipulated by choosing different values of n and ρ_s.

We assume that the matter which undergoes gravitational clustering has negligible pressure so that the gravitational instability is not hindered by sound waves or free–streaming. We parameterize the amount of the clustering matter (CM) by

$$\Omega_{\rm cm} = \frac{\rho_{\rm cm}}{\rho_{\rm crit}} \; ; \qquad \rho_{\rm crit} \equiv \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \; , \tag{3}$$

where $\rho_{\rm cm}$ is the density of CM. We use as our working hypothesis [12] that $\Omega_{\rm cm} \geq 0.1$. Since, during the late inflationary stage, $\Omega_{\rm cm}$ starts to drop, we define today by the requirement that the ratio between the amount of matter that clusters to the critical density is not be smaller than that. Our constraint on $\Omega_{\rm cm}$ means that today cannot be too late in this model.

As the expansion of the universe slows down, the growth of the density contrast: $\delta = \delta\rho_{\rm cm}/\rho_{\rm cm}$ speeds up. This can be readily seen from the Bonner equation describing the growth of the density contrast in an expanding universe:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho_{\rm cm}\delta \; , \tag{4}$$

a change in variables $\delta = \tilde{\delta}/a$ transforms this equation to

$$\ddot{\tilde{\delta}} - (4\pi G\rho_{\rm cm} + \frac{\ddot{a}}{a})\tilde{\delta} = 0 \ . \tag{5}$$

If both $\frac{\dot{a}}{a}$ and $\frac{\ddot{a}}{a}$ are small compared to $4\pi G\rho_{\rm cm}$, Eq. (5) can be solved in the WKB approximation, giving

$$\delta = \frac{1}{a}\exp\left(\int\sqrt{4\pi G\rho_{\rm cm}}\,dt\right) = \frac{1}{a}\exp\left(\int\sqrt{\frac{3}{2}\Omega_{\rm cm}}\,d\ln a\right) \ . \tag{6}$$

Thus, as long as $\Omega_{\rm cm} \gg 1$ ($\dot{a}/a \ll \rho_{\rm cm}$), the perturbations grow rapidly. This represents an enormous speed up over the growth rate of perturbations in a matter dominated FRW universe, in which $\delta \propto a$.

III. SIMULATION RESULTS

We use N–body simulations to investigate details of the non–linear mass distribution. Initial conditions were constructed by random sampling a scale–free power spectrum $P(k)d^3k = Ak^n$ with spectral index $n = -1$ and amplitude A discussed below. The spectral index chosen $n = -1$ is approximately the slope of the CDM spectrum on the scale of rich clusters of galaxies $\sim 10^{15}\mathrm{M}_\odot$. Sampling is done at 64^3 discrete modes appropriate to the Fourier domain of a periodic, 64^3 spatial grid. A displacement field is generated on this grid from the spectral information via use of the Zel'dovich approximation [13]. The initial particle positions and momenta are fed as initial conditions into a P3M N–body code [14]. The code calculates gravitational forces using Fourier transforms on large scales and pairwise summations on small scales. Pairwise forces are softened on small scales using a Plummer potential with softening parameter $\varepsilon = 0.00125L$ where L is the length of the periodic cube. Energy was conserved to better than 1% in the Layzer–Irvine equation [13] for both of the runs discussed below.

Two simulations were performed using initial conditions differing only in their *rms* displacement field amplitudes. During the loitering phase, perturbations will grow at a greatly enhanced rate relative to the MD case. Since we did not know *a priori* the endpoint of the loitering run, we arbitrarily scaled down the initial amplitude of the displacement field by a factor 16 for the loitering run relative to the MD run. Time was used as the expansion variable with 1500 time steps used to evolve the systems to their endpoints. The endpoint for the MD run was chosen when the variance of mass fluctuations within a sphere of radius $L/8$ was unity. The endpoint of the loitering model was chosen as the epoch in which the mass correlation function matched that of the MD run. This process involved some trial and error using 32^3 particle runs.

Figure 3 shows projections of a slice of dimension $L/2 \times L/2 \times L/5$ for the two runs at the final time and at an epoch corresponding to a redshift $z = 4$. Note the suppression of clustering in the loitering model at $z = 4$. As can be seen from Figure 1, this epoch is close to the beginning of the loitering phase when dramatic perturbation growth sets in. By the final epoch, structure in the two models is nearly identical. Figure 4 displays the mass correlation function and pairwise velocity statistic in dimensionless units. Remarkably, the correlation functions of the loitering and MD runs are almost identical. The small difference at separations $r/L < 0.002$ are not resolved by these experiments. Comparison between the 32^3 and 64^3 loitering runs indicates that convergence is achieved beyond about two softening lengths.

A big difference, however, exists in the pairwise velocity statistic. The amplitude of velocities in the loitering run is suppressed by a factor ~ 3 relative to the MD run. It is well known that $\Omega = 1$ MD models have a 'velocity problem' on small scales [4]. The problem can be expressed very simply along the following lines. If galaxies trace mass so that a critical density of dark matter is in halos around bright galaxies then the expected velocity on a spatial scale R can be inferred from application of the virial theorem

$$v \simeq \sqrt{\frac{3\Omega_{\rm cm}}{8\pi n_* R^3}} \mathrm{H}_o R \qquad (7)$$

where $n_* \simeq 0.002h^3$ Mpc^{-3} is the observed number density of bright galaxies, $h = \mathrm{H}_o/(100\mathrm{km/s/Mpc})$. This leads to the result that one expects velocity amplitudes of

$\sim$ 1000km/s on scales of 1Mpc. This rough calculation is supported by numerical experiments [14,15] which invoke biased galaxy formation as a partial solution. Even biased models do not bring the velocities down to the observed level of $\sim$ 300km/s. The loitering model has a low density of clustered matter and therefore behaves like a low Ω universe from the point of view of velocities. Linear perturbation analysis [16] confirms that the amplitude of the velocity field depends almost entirely on the value of $\Omega_{\rm cm}$ and only weakly on the value of the source term which acts as an effective cosmological constant at every time slice. Although attractive on small scales, this feature of the model may be a liability on 10Mpc scales where velocity fields of $\gtrsim$500km/s are claimed to exist [17].

In Figure 5, we plot the group multiplicity function for the two models at two different redshifts. The groups have been defined using a traditional friends–of–friends algorithm which joins common members of pairs with separations $r < \eta N^{-1/3}$. We use $\eta = 0.15$ to pick out groups at a density contrast of several hundred. Although there is a large difference between the cumulative numbers of collapsed objects at $z > 3$, the numbers are similar at $z = 0$. However, the loitering model has a slightly flatter low mass slope and has somewhat more high mass systems at the present.

IV. SUMMARY AND DISCUSSION

We have investigated the formation of structure for a universe that undergoes a recent loitering phase and compared it to a conventional, matter dominated universe. The universe under investigation is much older than the matter dominated one, $t_o \simeq 8/3{\rm H}_o^{-1}$. Further, we find that despite the different rates of perturbation growth in the linear regime, the non–linear structure in the two models is remakably similar. In particular, the shape of correlation function of the clustered matter and the mass functions of collapsed objects are nearly identical at the present epoch. However, because the loitering universe contains a factor ten less clustered matter than the flat, MD model, the peculiar velocities are reduced by a factor of $\sqrt{10}$.

Observed peculiar velocities on small scales (1Mpc) favour models where the density of clustered matter is low. However, observed velocities on large scales ($\gtrsim$10Mpc) favour high density models. The loitering model fails to generate high enough velocities on large scales but provides good agreement with observations on small scales. The flat MD model has the inverse problem of generating high amplitude velocities on all scales. At present, there are no models capable of reproducing the observed velocities on all scales.

At high redshift, differences in the mass distributions between the loitering and MD models become apparent. For example, Figure 5 shows an order of magnitude difference in the expected abundance of massive, collapsed objects. The abundance of quasars at high redshift may, in principle, provide a distinguishing signature between the models. However, detailed predictions from the loitering universe will depend critically on the choice of parameters.

In a broader context, this study addresses the question of the 'stability' of gravitational instability as a mechanism for structure formation under changes in the assumed expansion law of the universe. We have found the resultant non–linear structure to be remarkably robust. To distinguish between models invoking different expansion laws, observations of structure at both low and high redshifts is required.

Acknowledgement: H.A.F. was supported in part by National Science Foundation grant NSF–92–96020. A.E.E. was supported in part by NASA Theory grant NAGW–2367.

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FIGURES

FIG. 1. The expansion laws for the two simulations.

FIG. 2. The age of the universe for the two simulations in units of $2/3\mathrm{H}^{-1}$.

FIG. 3. Projections of a slice of dimension $L/2 \times L/2 \times L/5$ for the loitering run (left) and the MD run (right) at the final time (bottom) and at an epoch corresponding to a redshift $z = 4$. Note the suppression of clustering in the loitering model at $z = 4$ (top).

FIG. 4. Displays the mass correlation function (top) and pairwise velocity statistic (bottom) in dimensionless units. The correlation functions of the loitering and MD runs are almost identical. The small difference at separations $r/L < 0.002$ are not resolved by these experiments as comparison between the 32^3 and 64^3 loitering runs indicates. The pairwise velocity statistics show that the amplitude of velocities in the loitering run is suppressed relative to the MD run.

FIG. 5. The group multiplicity function for the two models at two different redshifts. We pick out groups at a density contrast of several hundred. Although there is a large difference between the cumulative numbers of collapsed objects at $z > 3$, the numbers are similar at $z = 0$.