SYSTEM-LEVEL QUALITY PLANNING AND DIAGNOSIS FOR COMPLEX MULTISTAGE MANUFACTURING PROCESSES

by

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To my mother, grandmother, beloved wife Jing and son Jonathan

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CHAPTER 1

INTRODUCTION

1.1 Motivation

With the development of the advanced manufacturing technologies, new manufacturing paradigm, such as cellular manufacturing, flexible manufacturing and reconfigurable manufacturing are widely adopted by manufacturers to perform complex manufacturing operations. In order to ensure modularity, flexibility and reconfigurability of a manufacturing system, the manufacturing operations are often divided into interconnected groups and performed on a series of manufacturing workstations or work-stages. This strategy leads to the Multistage Manufacturing Processes (MMP's). MMP's are applied to deliver product with complex functionalities and/or high quality requirements. Examples of such processes include: (i) the automotive body assembly process that assembles multiple parts on multiple stations, (ii) the machining processes that manufacture engine heads through multiple operations performed in a series of stages, and (iii) the semiconductor manufacturing processes that fabricate integrated circuits (IC) through hundreds of steps. The common characteristics of MMP are as follows: (i) a process with a series of operations are performed at multiple manufacturing stages or stations, (ii) the designated features of a product are generated sequentially at more than one stages, (iii) the output features from an upstream stages are used as the input features for downstream stages, and therefore, (iv) there are complex interactions between the process and product quality, as well as between different stages.

Quality, productivity, and cost are the most critical performance measures of MMP's. Since all these measures are influenced by the variation of the key product characteristics (KPC's), it is crucial to improve the stability of manufacturing process, reduce the variation of KPC's and, thus, ensure the delivery of quality product. However, due to the increasing product functionality and product variety, MMP's are very complex and may introduce many sources that contribute to the large variation of product quality. The unprecedented process complexity makes the quality assurance of MMP's a very challenging engineering problem.

The rapid advancements in information and sensing technology have created a data-rich manufacturing environment where heterogeneous data, including qualitative and quantitative data, are generated and collected from different phases of the entire product realization process. For instance, computer aided design (CAD) data, together with the designated product functionalities and quality specifications will be generated in the product design phase. Process plan will be determined in the process design phase to describe inter-connected groups of manufacturing operations and their precedence relationship. In manufacturing phase, in order to monitor process stability and control product quality, in-line sensing data, in terms of the measurements of KPC's, will be collected at final and/or intermediate stages.

Heterogeneous data pose great challenges to traditional quality assurance methodologies, which are mainly based on statistical analysis of observational data. For instance, statistical process control (SPC) focuses on detecting changes from process measurements. With the advancements in multivariate statistical process control (MSPC), integration of SPC-APC (Automatic Process Control) and SPC for auto-correlated data, traditional quality control methodologies have been significantly improved and adopted in many applications. However, limited diagnostic information can be provided solely based on the out-of-control signals casted by the control charts. This is because that the traditional SPC methodologies focus only on the process measurement data, without systematically incorporating the product/process design information. The most recent decade witnessed the development of new industrial engineering tools for effective quality assurance of MMP's. These methodologies, characterized by the effective integration of engineering domain knowledge with in-process sensing data, fundamentally enhance the quality engineers' capability in process change detection, root cause diagnosis and manufacturing system maintenance. Most of these research efforts concentrate on the quality assurance during the manufacturing phase of the product realization. However, the product quality is determined not only by the stability of the manufacturing processes, but also by the design of product and processes. It is desirable to conduct a systematic research that addresses quality assurance in all phases of product realization.

Various types of readily available data create tremendous opportunities for the development of such a unified methodology. By combining engineering domain knowledge, control theory, optimization algorithm and multivariate statistical analysis, this methodology will fuse the product/process design data with in-process quality measurement data. From this data fusion, knowledge of variation reduction will be generated and/or discovered to deepen the understanding of processes, product quality, and their complex interactions. The knowledge will be used to derive generic variation propagation model, based on which quality assurance are improved in both design and manufacturing.

1.2 Dissertation Research Overview

From the perspective of quality assurance, an MMP can be described by a production stream and a data stream (Shi 2006), as whown Figure 1.1. The lower panel of Figure 1-1 illustrates the production stream of a typical MMP, with each stage in the series represented as a block. In a certain stage of an MMP, process imperfections, e.g., locator error, will introduce quality problems that manifest as random deviations of features' measurements from their nominal values. Compounded with the quality input from the preceding stages, these quality problems will be transmitted to the downstream stages and finally accumulated in the final product. This kind of product quality variation and its propagation in an MMP is discussed by Liu and Hu (1997), using the term stream of variation (SoV), as suggested by Koren (Hu 1997).

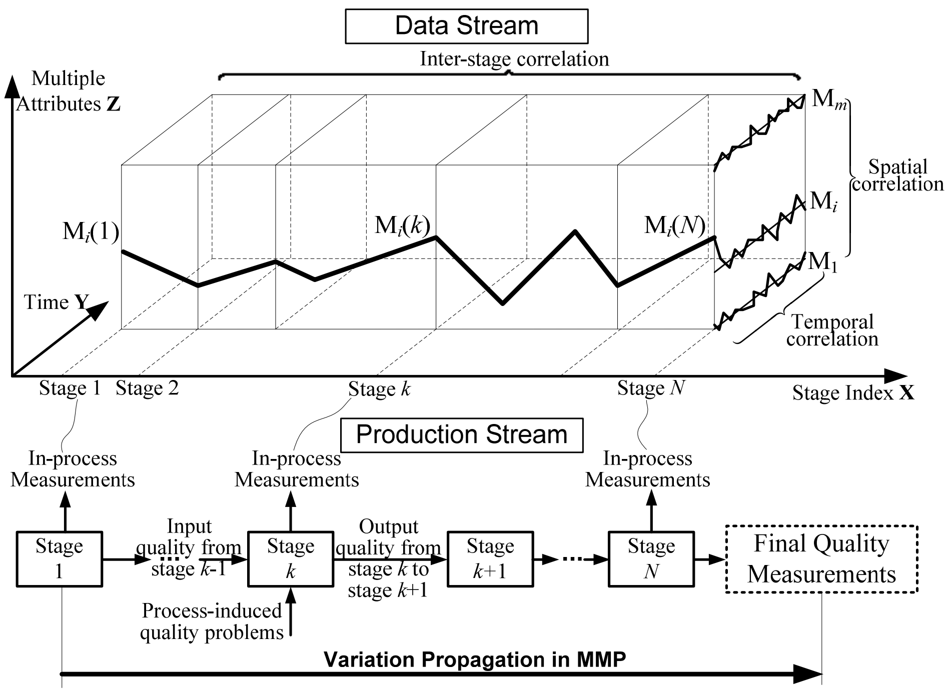

Figure 1-1. Production stream and data stream of MMP's.

Associated with production stream is the data stream, as shown in the upper panel of Figure 1-1. A data stream is formed by measurement data collected from stage to stage (the curve of $M_i(1)$, … $M_i(k)$, …, $M_i(N)$, along X-axis), from attributes to attributes (M_1, …, M_i, …, M_m along the Z-axis) and from time to time (the time series curve along Y-axis). For instance, in semiconductor industry, a wide range of measurements are made on each wafer fabricated in a lot (thousands of wafers in a lot), including particle data, in-line electrical measurements and final probe test data. As a result, approximately 1.5 Gb of data are collected per week. In automotive industry, a body-in-white (BIW) consists of 150-200 sheet-metal components that assembled together along over 60 assembly stations, involving 2,000 locating elements and more than 4,000 spot welds (Apley and Shi 2001). The optical coordinate measurement machines (OCMM) implemented in the plant provide capability to measure 100-150 points on each major assembly with a 100% inspection rate. The massive amount of data provides tremendous opportunity for more effective monitoring and diagnosis of the process. However, due to the complexity of the production stream and data stream in an MMP, quality control for reducing the product quality variation (variation reduction) is of great challenge. The challenges are mainly caused by the three types of correlation imbedded in the quality measurement data collected from MMP's: inter-stage correlation, spatial correlation and temporal correlation, as denoted in Figure 1-1.

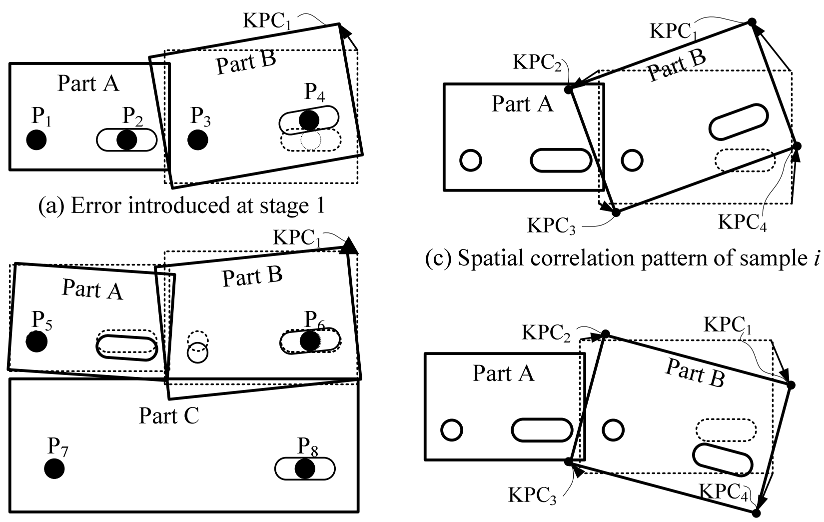

Figure 1-2. Correlations imbedded in the massive measurement data

The *inter-stage correlation* is the result of the interaction among the operations performed at different stages. As shown in Figure 1-2 (a) and (b) (Jin and Shi 1999), two-dimensional (2-D) parts, Part A and Part B, are fixed by a set of fixture locating pins, P_1 to P_4, and are assembled at stage 1. The error of P_4 makes Part B deviate from its nominal position, as depicted by dashed rectangular. Even if there is no error at stage 2, this deviation will be propagated to stage 2, where subassembly, A&B, is assembled with Part C. If the key product characteristics (KPC) on the upper-right corner of Part B, KPC_1, are measured at both stage 1 and stage 2, they are correlated to each other. The *Spatial correlation* refers to the correlation between the measurements of different KPC. Figure 1-2 (c) shows four KPC points, KPC_1 to KPC_4, to be measured after stage 1, as illustrated by the "●" in the layout. Corresponding to the deviation of P_4, these four points will deviate together, as shown by the arrows. The arrows visualize the patterns of the spatial correlations, with the direction indicating the deviation direction and the relative length indicating the deviation magnitudes. The *temporal correlation* reflects the random changes of the process from sample to sample. Figure 1-2 (c) and (d) shows different spatial patterns on sample *i* and sample *j*, which are different discrete samples assembled at different time. Their correlation reflects the changes of the process over time.

1.2.1 Research Problems

The development of quality assurance methodology focuses on the investigation of the three types of correlations imbedded in data stream. Several fundamental problems and challenges in both design and manufacturing need to be addressed.

(i) **Variation propagation modeling for MMP's**: To conduct variation reduction, it is necessary to understand the propagation of variation in MMP's by mathematically describing the propagation at the system level, in addition to models at the stage level, which is challenging. It is not sufficient to generate such a system-level model by simply grouping together models of individual stage. A generic methodology is desired to model 3-D variation propagation in a broad class of MMP's. The developed model will enable the investigation of various fundamental issues in variation reduction, including variation analysis, process monitoring, and variation source identification.

(ii) **Quality assured process planning for MMP's**. Quality assurance for advanced manufacturing is a continuous effort and should be systematically engaged in all the phases of product realization since the final product quality will be affected by the decisions made in different phases, including design and manufacturing. This is especially challenging for MMP's because of the complex variation propagation. It is crucial to make decisions in both design and manufacturing for a series of interrelated manufacturing stages, each of which is affected and/or affecting the others. Due to this stage-wise interrelation, experience-based approaches are widely adopted in initial product/process design phase. Variation propagation modeling provides capability to address this interrelation and to evaluate and benchmark alternative design candidates. With this capability, design of an MMP can be formulized as an optimal sequential decision making problem to identify the optimal plan that ensure the final product quality.

(iii) **Process variation source identification**. The quality assured product/process design and the generic variation propagation modeling will essentially improve the capability of process diagnosis in terms of variation sources identification. In order to improve its effectiveness and efficiency, variation source identification should take into account the situation that the process has significant deviation from designated target. Under this condition, diagnosis results based on pre-assumed *quantitative* variation propagation models may not truly reflect the real process faults and thus possibly lead to misleading conclusion. Therefore, the robustness issue of the variation source identification at production launch phase needs to be investigated through integration of appropriate representation of product/process design data and advanced multivariate statistical analysis methods.

1.2.2 Research Objectives

The objective of this dissertation research is to develop a unified methodology for quality assurance in MMP based on the fusion of various types of data from product design, process planning and in-process measurements. By fusing multidisciplinary techniques, knowledge of process variations and their propagation will be utilized in modeling, designing and diagnosing MMP's, as illustrated in Figure 1-3. More specifically, the following research will be conducted:

(i) *An innovative, generic 3-D state-space model*: A 3-D variation propagation model quantitatively describes complex Stream of Variation (SoV) in a broad class of MMP's. The potential contribution of this research is that it can quantitatively describe the complex interactions among quality characteristics and potential process variation sources and the variation propagation. The developed model enables the investigation of various fundamental issues in variation reduction, including variation analysis, process monitoring, and variation source identification. Especially, the modeling concept and techniques can be applied in the early phases of the product realization to effectively evaluate the product and process design alternatives.

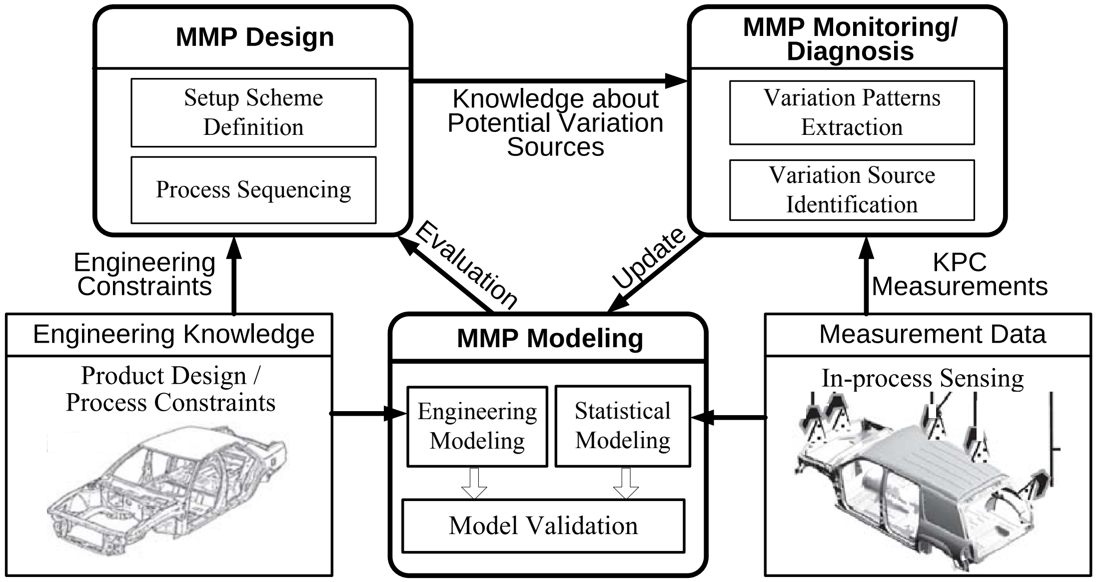

Figure 1-3. Paradigm of quality assurance in MMP

(ii) *An SoV-model based quality-assured setup planning methodology*: Such a methodology determines the optimal workpiece setups and operation sequences to reduce the cost related to process precision and ensure the delivery of quality product. The potential contribution of this research is that it addresses the setup planning by formulizing it as a sequential optimal decision making problem. Also, it creates the potential for concurrent development of system-level process planning, fixture design, process control strategy, maintenance planning and other critical aspects of the advanced manufacturing.

(iii) *A robust variation sources identification methodology*: The variation source identification methodology helps extract spatial variation patterns from multivariate measurement data and matches them with predefined *qualitatively* engineering patterns. The potential contribution of this research is that it provides a capability to diagnose process without complex quantitative engineering modeling. The diagnostic results are robust to frequent process changes and can be used to deepen quality engineers' understanding of the nature of variation patterns and variation propagation in MMP's, especially at production launch phase.

1.3 Related Work

Corresponding to the research objectives defined in the previous section, research work will be divided into three domains: variation propagation modeling for MMP's, process design for MMP's, and process diagnosis for MMP's. In this section, a comprehensive review of the existing methodologies in all the three domains will be presented.

1.3.1 Variation Propagation Modeling

The complex inter-stage correlation imbedded in the data stream makes it extremely difficult to monitor and diagnose MMP's, since the observed final product variation is an accumulation of variations from all stages. Therefore, in order to conduct variation reduction, it is important to understand how variation is introduced and transmitted across the stages. An N-stage manufacturing process and its variation propagation are illustrated in Figure 1-4, where $\mathbf{x}_k$ is a $p\times 1$ vector representing the quality of the p KPC at stage k, and k is the stage index, $k = 1, 2, \ldots, N$. $\mathbf{y}_k$ is a $m\times 1$ measurement vector of KPC quality at stage k. The system input vector, $\mathbf{u}_k$, contains the process-induced quality problems, including the potential process faults in fixture, machine tool and/or cutting tools. From a modeling perspective, the elements of $\mathbf{u}_k$ are the key control characteristics (KCC) or potential process faults at stage k. Vectors $\mathbf{w}_k$ and $\mathbf{v}_k$ are the un-modeled process error and measurement error at stage k, respectively.

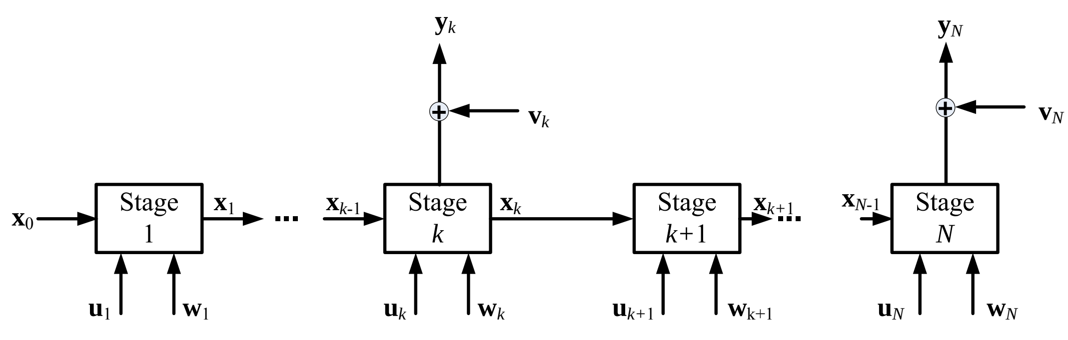

Figure 1-4. Stream of variation in an MMP

The purpose of the process modeling is to figure out the relationship between $\mathbf{y}_k$ and $\mathbf{x}_k$, $\mathbf{x}_k$ and $\mathbf{u}_k$, and especially, $\mathbf{x}_k$ and $\mathbf{x}_{k-1}$. Existing variation propagation modeling approaches can be divided into two categories: statistical modeling techniques and engineering modeling techniques.

Statistical modeling methods construct the variation propagation model from the historical quality measurement data collected at all the stages in an MMP. The model coefficients are estimated to describe the relationship between the data from two adjacent stages. Univariate KPC measurements collected from adjacent stages are used to fit a first order AR(1) model with the quality variable(s) from the downstream one of the two adjacent stages as the response and that from the upstream one as the predictor (Lawless *et al.* 1999). Fong and Lawless (1998) further extend this approach to cases where multiple KPC are considered. Since the proposed non-stationary AR(1) model is fitted from the true KPC measurement data, its approximate validity is of great concern. First, the existence of covariance of measurement error causes bias on the estimation of coefficient matrix $\mathbf{A}_k$ (Fong and Lawless 1998). Second, the model fitting is based on the assumption that the same KPC are measured at every stage and the inter-stage correlation exists only between the two adjacent stages. This was addressed by adopting the regression adjustment concepts introduced by (Mendel 1969), which regresses $\mathbf{x}_{i,k}$ on all the upstream $\mathbf{x}_{i,\kappa}$ ($\kappa = 1,2,\ldots, k$-1). Normality and constant variance are also important assumptions for fitting the AR(1) model, Agrawal *et al.*(1999) recommended various diagnostic model-checking techniques to verify the satisfaction of the assumption. For other industrial applications where these assumptions do not hold, generalized linear model (GLM) was utilized to deal with the count response (Skinner *et al.* 2003), gamma-distributed response (Jearkpaporn *et al.* 2003; Jearkpaporn *et al.* 2005), respectively. Finally, the variation propagation is formulated as a function of model coefficients that estimated from sample measurement data. Therefore, it is desirable to investigate the effects of sampling uncertainty. Agrawal *et al.* (1999) proposed two methods, parametric bootstrapping and normal approximation, to construct the confidence intervals of variance proportion.

Engineering modeling methods are based on engineering domain knowledge of product and process design. The essence of engineering modeling is to mathematically represent the knowledge in terms of relationships between potential process faults and quality of KPC's. Due to the complex inter-stage correlation, although the modeling and compensation for single-stage manufacturing process have been intensively studied (Ramesh *et al.* 2000), the efforts for MMP are relatively new and mainly focused on the model of geometrical and dimensional variation propagation. Jin and Shi (1999) applied the state space concept in system and automatic control theories to discrete-manufacturing process modeling, and developed a state space modeling technique to depict the variation propagation in 2-D automotive body assembly process. Mantripragada and Whitney (1998) proposed a "datum flow chain (DFC)" concept to identify and define the kinematics constraints and mates in the assembly process. This concept was further explicitly defined in a discrete state transition model to describe the variation propagation in assembly process, from a perspective of assembly structure design (Mantripragada and Whitney 1999). The state space model concept was also adopted by Huang *et al.*(2003) in modeling multistage machining processes. The implicit nonlinear nature of their method was addressed by (Djurdjanovic and Ni 2001) and a linear model was obtained by using Taylor series expansion. Zhou *et al.* (2003) further improved the modeling technique by providing explicit expression for derivation of all the system matrices in the model. Differential motion vector, a concept widely used in robotics, was adopted as the state vectors to represent the dimensional deviations of KPC.

Table 1-1 Comparison of variation propagation modeling techniques

	Statistical Modeling	**Engineering Modeling**
Model constructed from	Historical measurement data collected at all stages	Engineering domain knowledge on product/process design
Modeling assumption	Stable process, large sample size	Sufficient and accurate knowledge, small faults magnitudes
Interactions modeled	Inter-stage quality interaction	Inter-stage quality interaction, quality/fault interaction
Model performance	Coefficients estimation deteriorates as the number of stage, *N*, getting large	Only affected by in-accurate knowledge

The differences between the statistical modeling technique and engineering modeling technique are summarized in Table 1-1. These variation propagation modeling techniques provide the basis for the process monitoring and process fault diagnosis of MMP. In this dissertation research, a more generic state space model will be developed to describe the 3-D variations and their propagations in multistage assembly processes. Based on differential motion vector representation used in robotic manipulation, the new modeling technique provides more features to model a broad class of fixturing schemes and explicitly consider impacts of initial variations.

1.3.2 Design of MMP's

Successful design of MMP's will significantly reduce the potential quality problems at the early phase of product realization and thus improve the productivity of the manufacturing systems. Traditional process design approaches are frequently based on the evaluation of candidate design options by considering product dimensional variation. Among the evaluation strategies, process capability analysis is widely adopted to evaluate process stability with a set of indices, such as C_p or C_{pk} for single or multiple KPC's. Those indices can be directly computed from the KPC measurements collected from final product, but the relationship between the quality output and the process variation sources is not explicitly defined. Furthermore, the indices can only be evaluated when the process is under full production, not in the process design phase. Another category of process design methodologies are based on the Taguchi methods (Taguchi 1986), which emphasize on the direct experimentation and/or robust design with the aid of computer design models (Otto and Antonsson 1993; Parkinson *et al.* 1993). These approaches improve the process design's effectiveness and efficiency. However, some limitations are obvious. First, the statistical model achieved from physical experiments or computer simulation will only reflect limited number of situations that potentially present in a real process. There is no physical model that explicitly links the design alternatives with output. Second, the design of MMP should consider many factors simultaneously, including potential variation sources, potential fixturing schemes, and operation sequences. Without the integration of product and process data, efficiency and cost effectiveness of the robust design will be questionable.

Recently, incorporating variation propagation model with process design evaluation was investigated. Based on the analytical expression of key product/process design characteristics for MMP's, various indices were used to evaluate different design candidates and critical stages/operations that contribute most to the final product variations can be identified through variation analysis. This will be accomplished before the launch of a real production and thus, improve the process design strategy. However, the evaluation-oriented methodologies are still inefficient for the design of MMP's, since number of combination of design factors will be very large. The designer will only get the best result from options that have been tried. There is no systematic approach that analytically relates the design factors with the evaluation indices and formulizes the design as an optimization problem.

In this dissertation research, setup planning for MMP's will be investigated. Based on the generic 3-D state space model, this research focuses on optimally determining fixturing schemes and operation sequences to achieve designated product quality and at the same time release the tolerance requirements on process factors, such as fixture locator pin positions.

1.3.3 Process Variation Source Identification

A fundamental strategy to reduce variation in manufacturing phase is to first identify the sources of variation, i.e., process diagnosis, and then to take remedial action (Lawless *et al.* 1999). The diagnosis depends on the understanding of the process obtained either from historical data or from the engineering domain knowledge. The introduction of the engineering model enables effective diagnosis. According to dependence on engineering model, the existing approaches can be divided into three categories: direct estimation, engineering-model-based pattern matching and signature estimation.

Direct estimation approach requires an accurate engineering model developed off-line to describe the inter-stage correlation and spatial correlation imbedded in the data stream of an MMP. The model is then combined with in-line measurement data to estimate and infer the parameters of the distribution of variation sources. The state space engineering model is utilized to conduct diagnosis of MMP's. Based the model, Huang and Shi (2004) proposed a concept of "virtual operation measurements". This research provided a straightforward methodology to identify faulty stage(s) in an MMP. However, the variation sources can not be pin-pointed. This limitation was addressed by Zhou *et al.* (2004), who developed a statistical estimation methodology to identify the root-causes of dimensional variation in an MMP. The linear state space model is also transformed to a general linear mixed model (Rao and Kleffe 1988). Maximum likelihood estimation (MLE) method was selected to estimate the fixed effects (mean-shift) and random effects (increase of variance), because it provides estimation results with desirable statistical properties. In addition to the point estimation, hypothesis testing was conducted to provide the confidence level of the root-cause identification. Ding *et al.* (2005) investigated the interrelationship of different available estimation techniques and compared their properties, including pre-requisite conditions, bias and dispersion characteristics. Diagnosability is another critical concern about the direct estimation approach. This is, again, because of the complicated inter-stage correlations and spatial correlation. Ding *et al.* (2002) established an analogy between the concepts of diagnosability and the observability developed in classical control theory. Zhou *et al.* (2003) generalized Ding's work and proposed a concept of minimum diagnosable class(MDC).

Different from the direct estimation approach, the engineering-model-based pattern matching approaches do not directly use the engineering model. Instead, the model coefficients are treated as a library to store the expected spatial patterns of all the potential process variation sources. It is assumed that if a process variation source present in an MMP, a corresponding symptom will be reflected in the measurement of the final product or downstream intermediate products. The spatial pattern of this process variation source can be extracted from the analysis of the covariance matrix of the measurement data. The process fault identification can then be conducted by mapping the extracted patterns with expected patterns. Ceglarek and Shi studied the fixture fault diagnosis for autobody assembly (Ceglarek and Shi 1996) and sheet metal assembly (Ceglarek and Shi 1999), respectively. The geometric/dimensional relationship between fixture failure mode and KPC measurements are modeled quantitatively and the spatial patterns for potential fixture failure modes are determined. It was proved that under single fault situation, the spatial pattern defined in the model is coincident with the eigenvector associated with the largest eigenvalue of the covariance matrix. Therefore, principal component analysis (PCA) and the pattern recognition approach were used extract and match the spatial patterns with the expected ones for diagnosis purpose. This approach is further extended by Rong *et al.* (2000) to the diagnosis for compliant beam structure assemblies. Ding *et al.*(2002) adopted the same strategy to conduct the single fault diagnosis for MMP. Sample uncertainty and measurement noise are the two concerns about this type of approaches. Rong *et al.* (2000) investigated the sample uncertainty of the eigen-space analysis, assuming the simple structure of the measurement noise covariance. Ding *et al.* (2002) studied the impact of measurement noises with a general covariance structure on the pattern matching of single fault under the large sample assumption. Li *et al* (2004) further investigated the robustness issue by considering both the sample uncertainty and general structured measurement noise. The proposed robust method has higher identification probability when the two assumptions are not satisfied.

The signature-estimation-based process diagnosis approaches do not depend on pre-constructed engineering model. Since it is reasonable to assume that different process faults cause different spatial correlation patterns, process diagnosis can be accomplished by estimating, visualizing and interpreting the expected spatial patterns, or signature, imbedded in the variance-covariance of measurement data. Apley and Shi (2001) proposed a factor analysis method for multiple process faults diagnosis. Although the same linear model as defined in engineering model was assumed to describe the linear relationship between quality variables and process faults, the model coefficients are NOT known. The diagnosis is implemented by estimating the model coefficients with Factor Analysis. To avoid the unrealistic orthogonality assumption required by factor analysis, a ragged lower-triangular form of coefficient matrix was assumed. Physical interpretations of the extracted signature vectors were pursued after the estimation step. The assumption on the form of coefficient matrix was further released by Apley and Lee (2003) through adopting Independent component analysis (ICA) to estimate the spatial variation patterns from measurement data. Instead of focusing on the estimation of individual variation pattern, Jin and Zhou (2006) proposed a systematic technique to estimate the signatures of multiple process faults. The space spanned by the eigenvectors associated with large Eigenvalues of measurement covariance matrix was compared with the space spanned by the fault signatures stored in a process fault library, which was established by historical diagnostic results. Rigorous testing procedure was also developed to identify the variation sources.

These quantitative, model-based techniques fundamentally improve the capability of variation sources identification. In this dissertation research, an engineering-driven factor analysis will be developed to identify multiple variation sources present in an MMP. Different from existing approaches, the proposed one uses qualitative engineering indicator vectors to direct the rotation of factor loadings and estimate the true spatial patterns of variation sources. The estimated spatial patterns can be used to update the engineering domain knowledge.

1.4 Organization of the Dissertation

This dissertation presents the initial research efforts in developing a unified methodology for quality assurance of MMP's. The proposed methodologies are mainly applied in both design and manufacturing phases of product realization. Chapter 2, Chapter 3 and Chapter 4 are organized as individual research papers, addressing inter-connected research problems.

Chapter 2 presents a 3-D generic state space model of variation propagation in multistage assembly processes. Potential applications of the model are discussed. Chapter 3 developed a quality assured setup planning methodology for MMP's, based on the quality evaluation capability provided by the generic modeling technique introduced in Chapter 2. The optimal setup planning decisions made in the design phase provide not only information of fixturing schemes and operation sequences, but also knowledge about the spatial patterns of potential variation sources. This type of engineering knowledge will be used in Chapter 4 to direct the rotation of the factor loadings that extracted from the factor analysis of multivariate KPC measurement data. The rotation results best estimate the true spatial patterns of variation sources that actually present in an MMP and will be used as diagnosis tool in manufacturing phase of product realization. Finally, Chapter 6 summarizes the dissertation and its original contributions. Potential future study is also discussed.

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CHAPTER 2

STATE SPACE MODELING FOR 3-DIMENSIONAL VARIATION PROPAGATION IN MULTISTAGE ASSEMBLY PROCESSES[1]

Abstract

Dimensional variation propagation modeling is a critical enabling technique for product quality variation reduction in a multistage assembly process. However, the complicated interrelationships among different stages make the modeling extremely difficult. This paper aims to improve the existing modeling techniques by developing a more generic approach to covering broader assembly processes. In the paper, a systematic approach is proposed to describe the dimensional variations induced by various sources in the process and model their transmission along a sequence of stages in a state space format. A concept of differential motion vector is adopted to represent deviations with respect to four types of coordinate systems and formulate the deviation propagation as a series of homogeneous transformation among these different coordinate systems. A generic mechanism is proposed in this chapter to first time represent the effect of part feature variations induced from the initial part fabrication processes.

[1] Liu, J., Jin, J., and Shi, J., 2007, submitted to IEEE Transactions on Automation Science and Engineering, under revision.

2.1 Introduction

Multistage assembly processes (MAP's) are widely adopted in manufacturing industries to deliver products with complex designated functionalities and/or high quality requirements. In an MAP, parts or subassemblies from disparate sources are assembled together through a series of operations executed at multiple stages to form a final assembled product. The quality of the final products are reflected by a set of measurements of Key Product Characteristics (KPC's) which are defined as dimensions relating datums or features on different parts of an assembly. Quality KPC's can be achieved when all parts are located in the correct design position and orientation with respect to (w.r.t.) the six spatial degrees of freedom (dof). However, due to imperfections in assembly stages, e.g., fixture locators' deviations from the nominal positions and/or part features' deviations from the nominal geometry, assembly errors may be introduced and cause random deviations of KPC measurements from their designated nominal positions. As some features generated in upstream stages of an MAP are used as datum features in downstream stages, these deviations will propagate and accumulate, leading to large variations on the KPC's of final assembled products, or even severe interferences among parts and/or subassemblies.

Reducing the dimensional variation is, in practice, an essential issue for improving product quality as well as the productivity of an MAP. It is greatly beneficial to develop an automatic variation reduction strategy by monitoring process consistency and identifying the variation sources based on available online KPC measurements. The availability and accessibility of such data has been significantly improved with the advancements of sensing technology and information techniques. The implementation of in-line automatic measurement devices has created a data-rich environment which is capable of performing 100% inspection in an MAP with tremendous data streams (Shi 2006). However, complex interactions between KPC measurements and potential process variation sources and the inter-correlation among different assembly stages make it extremely challenging to effectively utilize those data streams for variation reduction purposes. Therefore, the development of a mathematical model, which can fully represent the product-process interactions and variation propagation, is considered as the first critical step of variation reduction.

The recent decade has witnessed the development of variation propagation modeling methodologies, which can be divided into two categories: statistical modeling and engineering modeling. Statistical modeling methods construct variation propagation models from historical quality measurement data collected at all stages in an MAP. Lawless *et al.* (1999) and Agrawal *et al.* (1999) investigated the variation transmission issues in multistage manufacturing processes by constructing an AR(1) model in a state space format. The estimated model coefficients are used to describe the inter-correlation among the KPC's from two adjacent stages. Based on KPC measurements, these statistical models aided quality engineers in understanding inter-stage correlations under certain faulty conditions in an MAP. However, the process-quality interactions are not considered in these models, which limited their diagnostic capability in linking KPC variations with process variation sources.

Engineering variation propagation modeling is conducted based on engineering domain knowledge of product/process design, rather than fitting the measurement data collected from real manufacturing processes. Different from statistical modeling techniques, the essence of engineering modeling is to mathematically represent quality-process interactions and inter-stage correlation in a series of equations based on the engineering design knowledge, e,g, computer aided design (CAD) and computer aided manufacturing (CAM).

For assembly process, Mantripragada and Whitney (1998) proposed a concept of datum flow chain (DFC) to capture the underlying datum logic, at an abstract level, by classifying assembly processes into two types: (i) Type-I, which assembles parts together according completely to their pre-fabricated mating features, e.g., furniture assembly; and (ii) Type-II, which achieves final assembly by welding, riveting, etc., parts located by fixtures according to a pre-defined DFC, e.g., automotive and aircraft body assembly. Method for modeling the variation propagation in these two types of assembly process was introduced by Mantripragada and Whitney (1999) from a perspective of assembly structure design. However, the explicit model derivation procedure was not discussed. Jin and Shi (1999) proposed state space modeling techniques for 2-Dimensional (2D) sheet metal assembly, without taking into account the 3D Type-I assembly process. State space model was adopt by Huang *et al.* (2003) to model the variation propagation in multistage machining processes, with an approximate linearization strategy. Zhou *et al.* further improved the modeling technique by providing explicit expressions for deriving all the system matrices in the model. Differential motion vectors (DMV), a concept widely used in robotics, was adopted as the state vectors to represent the location and orientation deviations of KPC's (Zhou *et al.* 2003). In machining process modeling, fixture locators are reasonably assumed to be deployed on a single datum surface. However, this assumption does not hold for a general assembly process. Recently, Huang *et al.* (2007) developed a generic 3D variation model for a group of locating schemes. However, the deviations of features on parts induced from part fabrication processes are not explicitly considered. Therefore, a more systematic coordinate system definition mechanism is needed to reduce the modeling complexity and increase its generality.

In this chapter, an explicit modeling technique is developed to mathematically describe *3D variation propagation* in a general MAP. The proposed state space model adopts DMV as the components of the state vectors to describe parts' dimensional deviations. System matrices in state transition and observation equations are explicitly derived by mathematically representing geometric relationships among parts, fixtures and reorientation operations. Section 2.2 introduces the mathematical representations of various types of deviations in an MAP. The details of the state space modeling procedure are presented in Section 2.3. Concluding remarks and potential applications of the proposed modeling techniques are discussed in Section 2.4.

2.2 Geometrical Representation for Deviation Modeling

An assembly process is used to attach interchangeable parts together in a sequential manner to create a finished product with complicated structure which cannot be obtained by other fabrication processes. Depending on the complexity of the product, an assembly process consists of multiple stages so the operation related movements are reduced to the minimum. Every stage of an MAP assembles two or more parts and/or subassemblies, which are located by either fixtures or the constraint surfaces or features, e.g., hole or plane, of other parts. If any deviation occurs on a fixture locator, or a constraint surface of a part, a dimensional deviation will be generated on some features on the subassembly. These faulty features will transmit such deviations to downstream stages through DFC Assembly quality is characterized by KPC measurements taken at the intermediate or final stage. This series of KPC measurements data carries all the information of process deviations and their transmission along multiple stages, as illustrated by Figure 1-1. To study the nature of this complicated deviation transmission, it is necessary to mathematically represent all the deviation information.

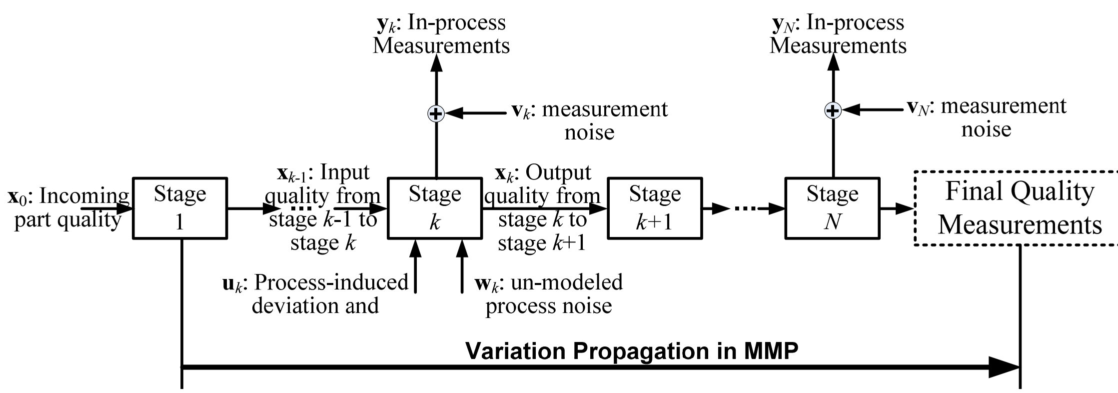


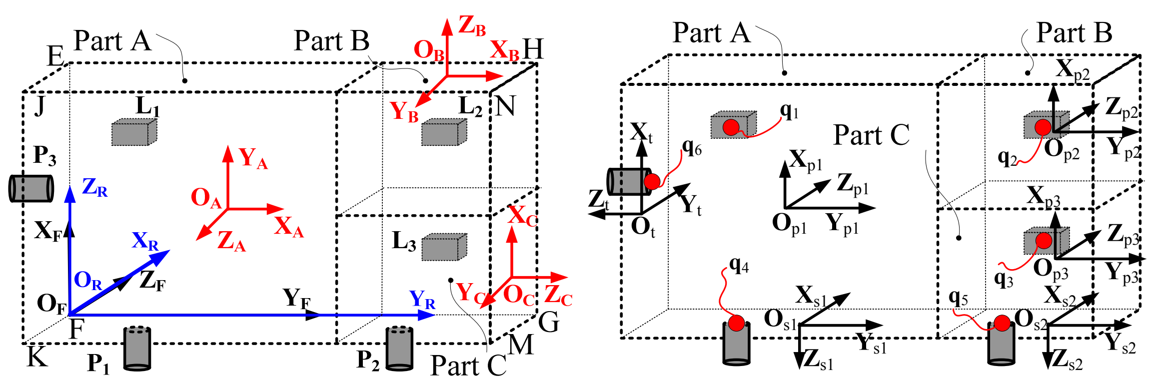
Figure 2-1. Illustration of deviation transmission in an MAP.

2.2.1 Coordinate Systems Definition

As aforementioned, there are three types of elements involved in the deviation transmission: fixtures, parts located by fixtures, and features on parts. Describing their geometric deviations and interrelationship is essential for variation propagation modeling. These deviations can be defined w.r.t. four types of different coordinate systems (CS).

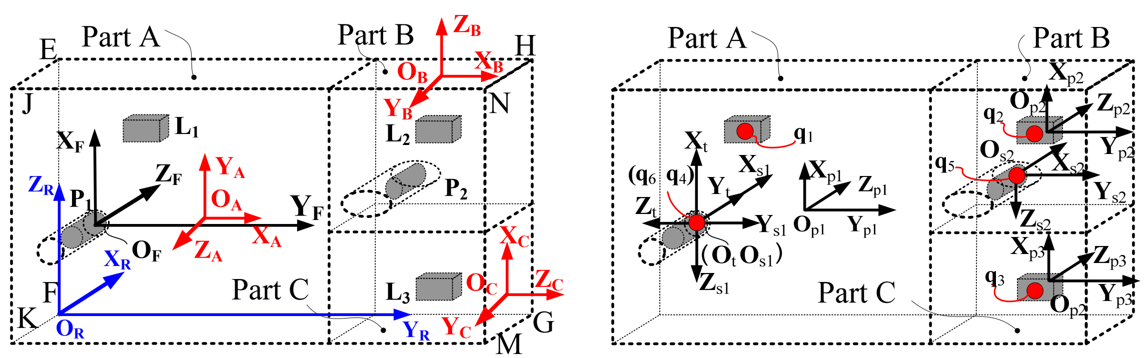
(i) RCS denotes the reference coordinate system which globally defines the reference for all the elements in an MAP. In this chapter, an RCS, as defined as $O_RX_RY_RZ_R$ in Figure 2-2, is assumed to be error free and unchanged for the entire process.

(ii) FCS denotes the fixture coordinate system attached to individual fixtures. Every fixture used in MAP is represented as an FCS and its deviation is represented as the deviation of true FCS from nominal ^{0}FCS, where the left superscript 0 represents the *nominal* CS. In an assembly process, the deviation of an FCS is caused by two factors: datum surfaces deviation and fixture locator deviation. Two most commonly used 3-2-1 fixturing schemes are considered in this chapter. A general 3-2-1 layout is shown in Figure 2-2 (a), which holds a subassembly composed of three parts. The six degrees of freedom of an subassembly can be fully confined by a general 3-2-1 fixture by supporting its ① primary datum surface (PD), e.g., EFGH, with three locators L_1, L_2 and L_3; ② seco ndary datum surface (SD), e.g., FKMG,



a.1 Definition of RCS, FCS and PCS   a.2 Definition of LCS

(a) General 3-2-1 fixturing scheme



b.1 Definition of RCS, FCS and PCS   b.2 Definition of LCS

(b) Pin-hole fixturing scheme

Figure 2-2. Illustration of 3-2-1 fixturing scheme and CS definition.

with two locators P_1 and P_2; and ③ tertiary datum surface (TD), e.g., EFKJ, with locator P_3. The FCS, $O_FX_FY_FZ_F$ is defined as shown in Figure 2-2 (a). The origin O_F is located within the primary datum surface and at the intersection of secondary and tertiary datum surfaces. Three axes are determined according to right-hand-rule (RHR), with Z_F, X_F and Y_F perpendicular to PD, SD and TD, respectively. A common alternative of the general 3-2-1 fixture setup is shown in Figure 2-2 (b). The three locators attaching SD and TD are replaces by a four-way pin, P_1, inserted in a hole, and a two-way pin, P_2, inserted in a slot. For this alternative, the origin O_F is located at the intersection of the PD and the center line of the short hole. Y_F axis is within PD and parallel to the long axis of the slot. Z_F is coincident with the norm of PD and X_F is determined according to RHR.

(iii) PCS denotes the part coordinate system attached to an individual part. Every part to be assembled in an MAP is represented as a unique PCS and its deviation is represented as the deviation of the true PCS from nominal ^{0}PCS. A PCS will be attached to the PD of a part when it is assembled. As shown in Figure 2-2 (a.1) and Figure 2-2 (b.1), $O_AX_AY_AZ_A$ is part A's PCS, which is located at the center of the PD of part A. Z_A is coincident with the norm of PD, and X_A and Y_A are determined following RHR.

(iv) LCS denotes the local coordinate system attached to individual features. According to their functionalities in an MAP, all features can be classified into two categories, which are: ① Datum features or components of datum surfaces. For instance, in Figure 2-2 (a.2) and (b.2), surface features p1, p2 and p3 (marked as $O_{p1}X_{p1}Y_{p1}Z_{p1}$, $O_{p2}X_{p2}Y_{p2}Z_{p2}$, $O_{p3}X_{p3}Y_{p3}Z_{p3}$, respectively, in both Figure 2-2 (a.2) and Figure 2-2 (b.2)) compose the PD of the subassembly, surface features s1 and s2 ($O_{s1}X_{s1}Y_{s1}Z_{s1}$ and $O_{s2}X_{s2}Y_{s2}Z_{s2}$ in Figure 2-2.) compose the SD, and surface feature t ($O_tX_tY_tZ_t$ in Figure 2-2) serves as the TD. These datum features will transmit deviations introduced at the upstream stages to the current stage. ② Components of KPC's. For instance, features s1 and s2 in Figure 2-2 (b.2) also serve as the components of a KPC defined as the distance between the center of the hole, s1, and that of the slot, s2. To construct the model, an LCS will be assigned to every feature that serves above two functional roles.

2.2.2 Vectorial Deviation Representation

Vectorial representation proposed by Yau (Yau 1997) is adopted in this chapter to represent the three elements in an MAP. It can directly link KPC measurements with design requirements based on the same dimensional representation principles of coordinate measurement machine (CMM) and computer-aided-design / computer-aided-manufacturing (CAD/CAM) systems. The position and orientation of an element can be defined by a vector that consists of a location vector and an orientation angular vector w.r.t. a certain CS. For instance, part A represented by PCS, $O_AX_AY_AZ_A$ can be defined w.r.t. FCS as $\mathbf{r}_A^F = \left[\left(\mathbf{t}_A^F\right)^{\mathrm{T}} \ \left(\boldsymbol{\omega}_A^F\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{t}_A^F = [x_A^F \ y_A^F \ z_A^F]^{\mathrm{T}}$ and $\boldsymbol{\omega}_A^F = [\phi_A^F \ \theta_A^F \ \psi_A^F]^{\mathrm{T}}$. This indicates that the projections of O_A on X_F, Y_F and Z_F are x_A^F, y_A^F and z_A^F, respectively; and the orientation of axes X_A, Y_A and Z_A can be obtained by sequentially rotating FCS around Z_F, Y_F and the new Z_F with Euler angles of ϕ_A^F, θ_A^F and ψ_A^F, respectively. For the part A shown in Figure 2-2 (a.1), $\phi_A^F = 0$, $\theta_A^F = \pi$, and $\psi_A^F = \pi/2$. With this mechanism, the interactions among features, parts and fixtures in an MAP can be described with the geometrical relationships among their CS's, by directly using homogeneous transformation matrix (HTM). Assuming the vectorial representation of CS1 w.r.t. CS2 is $\mathbf{r}_1^2 = \left[\left(\mathbf{t}_1^2\right)^{\mathrm{T}} \ \left(\boldsymbol{\omega}_1^2\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, an HTM, $\mathbf{H}_1^2$, is defined as:

$$\mathbf{H}_1^2 = \begin{bmatrix} \mathbf{R}_1^2 & \mathbf{t}_1^2 \\ \mathbf{0} & 1 \end{bmatrix}, \tag{2.1}$$

where

$$\mathbf{R}_1^2 = \begin{bmatrix} c\phi_1^2 c\theta_1^2 c\psi_1^2 - s\phi_1^2 s\psi_1^2 & -c\phi_1^2 c\theta_1^2 s\psi_1^2 - s\phi_1^2 c\psi_1^2 & c\phi_1^2 s\theta_1^2 \\ s\phi_1^2 c\theta_1^2 c\psi_1^2 + c\phi_1^2 s\psi_1^2 & -s\phi_1^2 c\theta_1^2 s\psi_1^2 + c\phi_1^2 c\psi_1^2 & s\phi_1^2 s\theta_1^2 \\ -s\theta_1^2 c\psi_1^2 & s\theta_1^2 s\psi_1^2 & c\theta_1^2 \end{bmatrix}, \tag{2.2}$$

is a rotation matrix corresponding to the orientation vector, $\boldsymbol{\omega}_1^2$; "c" and "s" denote "cos" function and "sin" function, respectively.

Based on the CS definition, the deviation of the three elements in an MAP can be represented by DMV's (Paul 1981) defined in their own CS's. For instance, the deviation of part A is represented as $\mathbf{x}_\mathrm{A} = [\mathbf{d}_\mathrm{A}^\mathrm{T}\ \boldsymbol{\delta}_\mathrm{A}^\mathrm{T}]^\mathrm{T}$, where $\mathbf{d}_\mathrm{A}$ contains three small translational deviations ($d_{\mathrm{X_A}}$, $d_{\mathrm{Y_A}}$ and $d_{\mathrm{Z_A}}$, along $\mathrm{X_A}$, $\mathrm{Y_A}$ and $\mathrm{Z_A}$, respectively) whereas $\boldsymbol{\delta}_\mathrm{A}$ contains three small rotational deviations ($\delta_{\mathrm{X_A}}$, $\delta_{\mathrm{Y_A}}$ and $\delta_{\mathrm{Z_A}}$, around $\mathrm{X_A}$, $\mathrm{Y_A}$ and $\mathrm{Z_A}$, respectively). These deviations will be further transformed w.r.t. different CS's to describe the deviation transmission and accumulation, according to the geometrical interrelationships among the three different elements in an MAP. The inter-CS transformation follows Corollary 1(Zhou *et al.* 2003).

Corollary 1 : consider three different coordinate systems, CS_1, CS_2 and CS_3. Given the deviations of CS_3 w.r.t. CS_2, $\mathbf{x}_3^2$, the deviations of CS_2 w.r.t. CS_1, $\mathbf{x}_2^1$, and the corresponding HTM, ${}^0\mathbf{H}_2^1$ and ${}^0\mathbf{H}_3^2$, the deviation of CS_3 w.r.t. CS_1 can be derived as

$$\mathbf{x}_3^1 = \mathbf{M}_3^2 \cdot \mathbf{x}_2^1 + \mathbf{x}_3^2, \tag{2.3}$$

where,

$$\mathbf{M}_3^2 = \begin{bmatrix} \left(\mathbf{R}_3^2\right)^\mathrm{T} & -\left(\mathbf{R}_3^2\right)^\mathrm{T} \cdot \left(\hat{\mathbf{t}}_3^2\right) \\ \mathbf{0} & \left(\mathbf{R}_3^2\right)^\mathrm{T} \end{bmatrix},$$

and $\mathbf{R}_3^2$ is the rotation matrix associated with the *nominal* orientation vector, $\boldsymbol{\omega}_3^2$, i.e., the nominal orientation of CS_3 w.r.t. CS_2. And skew symmetric matrix, $\hat{\mathbf{t}}_3^2$, is determined by the *nominal* location vector, $\mathbf{t}_3^2$ ($\mathbf{t}_3^2$=[$x_3^2\ \ y_3^2\ \ z_3^2$]$^\mathrm{T}$), as

$$\hat{\mathbf{t}}_3^2 = \begin{bmatrix} 0 & -z_3^2 & y_3^2 \\ z_3^2 & 0 & -x_3^2 \\ -y_3^2 & x_3^2 & 0 \end{bmatrix}. \tag{2.4}$$

This corollary can be used to derive the deviation *translation* among different CS's. For instance, given the deviation of a feature (CS_3 in *corollary* 1) w.r.t. a PCS (CS_2), and the deviation of the PCS (CS_2) w.r.t. RCS (CS_1), the deviation of the feature (CS_3) w.r.t. RCS (CS_1) can be derived from the nominal location and orientation of the PCS in RCS, as defined in *corollary* 1.

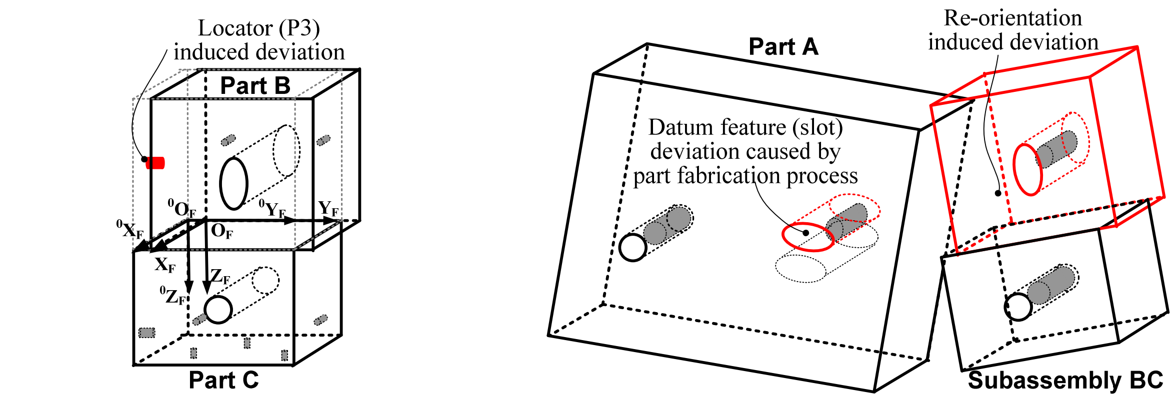
2.3 State Space Modeling for Multistage Assembly Processes

Variation propagation modeling is a procedure of describing the random deviation translation among different CS's. Given the product/process design information, model coefficients are determined by the part design, process sequences, fixturing schemes and geometric relationships among fixture locators, parts and features on the parts. The entire model can be constructed by modeling the deviation components, deviations propagation and their observations.

2.3.1 Modeling Deviation Components

At the stage *k* of an MAP, overall dimensional deviations consist of components that are contributed by three types of sources:

(i) *Assembly process induced deviations*: including the error of fixture locators and operation deviations caused by other assembling devices. Operation deviations refer to the deviations contributed by the operations *after* the part/subassembly being fixed by the fixtures. Thus, this type of deviations is represented as the deviation of PCS w.r.t. the true FCS. As discussed by Camelio *at al*. (2003), operation deviation is a composition of many possible deviation sources, which are quite dependent on particular operation situation. Detailed modeling of certain types of operations have been thoroughly studied for single-stage level in (Liu and Hu 1997; Shiu *et al.* 1997). To simplify the problem and focus on the variation propagation modeling, this paper assumes that the operation deviations are given modeling inputs. Fixture induced deviations, e.g., locator P3's error in Figure 2-3 (a), will be represented as the deviation of true FCS ($O_FX_FY_FZ_F$) w.r.t. nominal ^{0}FCS ($^0O_F{}^0X_F{}^0Y_F{}^0Z_F$).



(a) Process induced deviation at stage 1 (b) Datum induced deviation at stage 2

Figure 2-3. Illustration of deviation sources in an MAP

Assuming that there are FX_k fixtures used at the stage k to locate FX_k parts/subassemblies, each fixture, as illustrated in Figure 2-3 (a), is made up of six fixture locators. The coordinates of L_1, L_2 and L_3 in ${}^0F_{k,j}$ are $(L_{hx}^{{}^0F_{k,j}}, L_{hy}^{{}^0F_{k,j}}, L_{hz}^{{}^0F_{k,j}})$, h=1,2,3, and that of P_1, P_2 and P_3 are $(P_{wx}^{{}^0F_{k,j}}, P_{wy}^{{}^0F_{k,j}}, P_{wz}^{{}^0F_{k,j}})$, where w=1,2,3. For the fixture scheme in Figure 2-3 (b), five locators are used and can be described in a similar format. In this chapter, the fixture induced deviation is represented as the deviation of the FCS of the j^{th} fixture at the stage k, $F_{k,j}$, from its nominal ${}^0F_{k,j}$, i.e., $\mathbf{x}_{F_{k,j}}^{{}^0F_{k,j}}$. This deviation is caused by small deviations on the fixture locators denoted as

$$\mathbf{u}_{k,j}^{{}^0F_{k,j}} = \begin{bmatrix} \triangle L_{1z}^{{}^0F_{k,j}} & \triangle L_{2z}^{{}^0F_{k,j}} & \triangle L_{3z}^{{}^0F_{k,j}} & \triangle P_{1x}^{{}^0F_{k,j}} & \triangle P_{2x}^{{}^0F_{k,j}} & \triangle P_{3y}^{{}^0F_{k,j}} \end{bmatrix}^{\mathrm{T}}. \tag{2.5}$$

Analytical research has been conducted by Cai *et al.* (1997) to study the infinitesimal error of rigid body fixturing scheme. Based on their results and the fixture error analysis strategy in (Zhou *et al.* 2003), fixture induced deviation is modeled as a linear transformation of the fixture locator deviation,

$$\mathbf{x}_{F_{k,j}}^{{}^0F_{k,j}} = \mathbf{T}_{k,j}^{{}^0F} \cdot \mathbf{u}_{k,j}^{{}^0F_{k,j}}, \tag{2.6}$$

where the coefficient matrix $\mathbf{T}_{k,j}^{{}^0F}$ for general and pin-hole fixturing schemes are defined in Appendix I.

In a generic type I assembly process, it is not unusual to use some features on certain parts as fixture locators (called feature locators in this chapter) to locate other parts. In these cases, deviations of those feature locators w.r.t. to their own PCS should be transformed to that w.r.t. ^{0}FCS. A collection of six feature locators' DMV's w.r.t. their own PCS's is denoted as ${}^{L}\mathbf{x}_{k,j} = \left[\left(\mathbf{x}_{T_{L_1,k,j}}^{P_{k,i(L_1,k,j)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_2,k,j}}^{P_{k,i(L_2,k,j)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_3,k,j}}^{P_{k,i(L_3,k,j)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_1,k,j}}^{P_{k,i(P_1,k,j)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_2,k,j}}^{P_{k,i(P_2,k,j)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_3,k,j}}^{P_{k,i(P_3,k,j)}}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$, where subscript index for each DMV, "●,*k*,*j*", indicates the feature functions as the locator "●" of the fixture *j* at the stage *k*, i.e., $\bullet \in \{\mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3, \mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3\}$. Superscript "*i*(●,*k*,*j*)" is a function indicating the index of the part that contains feature "●,*k*,*j*", and $\mathbf{x}_{T_{\bullet,k,j}}^{P_{k,i(\bullet,k,j)}}$ represents the deviation of the feature "●,*k*,*j*" in the subscripts w.r.t. the PCS's of "*i*(●,*k*,*j*)" in the superscripts. Also, let ${}^{L}\mathbf{x}_{k,j}^{R} = \left[\left(\mathbf{x}_{i(L_1,k,j)}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(L_2,k,j)}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(L_3,k,j)}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(P_1,k,j)}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(P_2,k,j)}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(P_3,k,j)}}^{R}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$ be a collection of six parts' DMV's w.r.t. RCS, according to the *corollary* 1, a collection of six feature locators' DMV w.r.t. RCS can be derived as

$${}^{FL}\mathbf{x}_{k,j}^{R} = \mathbf{B}_{k,j}(1)\,{}^{L}\mathbf{x}_{k,j}^{R} + {}^{L}\mathbf{x}_{k,j}$$

where ${}^{FL}\mathbf{x}_{k,j}^{R} = \left[\left(\mathbf{x}_{T_{L_1,k,j}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_2,k,j}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_3,k,j}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_1,k,j}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_2,k,j}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_3,k,j}}^{R}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$, $\mathbf{B}_{k,j}(1) = diag\left\{\mathbf{M}_{T_{\bullet,k,j}}^{P_{k,i(\bullet,k,j)}}\right\}$, is a diagonal block matrix with diagonal blocks as $\mathbf{M}_{T_{\bullet,k,j}}^{P_{k,i(\bullet,k,j)}}$. Since it is rational to assumed that ${}^{0}F_{k,j}$ has no deviation w.r.t. RCS, i.e., DMV $\mathbf{x}_{{}^{0}F_{k,j}}^{R} = \mathbf{x}_{R}^{{}^{0}F_{k,j}} = \mathbf{0}$. According to *corollary* 1, $\mathbf{x}_{T_{\bullet,k,j}}^{{}^{0}F_{k,j}} = \mathbf{M}_{T_{\bullet,k,j}}^{R}\mathbf{x}_{R}^{{}^{0}F_{k,j}} + \mathbf{x}_{T_{\bullet,k,j}}^{R} = \mathbf{x}_{T_{\bullet,k,j}}^{R}$. Thus, a collection of six feature locators' DMV's w.r.t. ${}^{0}F_{k,j}$ can be derived as ${}^{FL}\mathbf{x}_{k,j}^{{}^{0}F_{k,j}} = \left[\left(\mathbf{x}_{T_{L_1,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_2,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{L_3,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_1,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_2,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{P_3,k,j}}^{{}^{0}F_{k,j}}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$. ${}^{FL}\mathbf{x}_{k,j}^{{}^{0}F_{k,j}}$ enables the derivation of $\mathbf{u}_{k,j}^{0F}$ from DMV's of individual features. Referring to its definition, only the deviation along a particular axis of a fixture locator is considered as a component in $\mathbf{u}_{k,j}^{0F}$. Therefore,

$$\mathbf{u}_{k,j}^{0F} = \mathbf{S}_{0,k,j} \cdot {}^{FL}\mathbf{x}_{k,j}^{0F_{k,j}}, \tag{2.7}$$

where $\mathbf{S}_{0,k,j}$ is a diagonal selector matrix, i.e., $\mathbf{S}_{0,k,j} = diag\{\mathbf{s}_{\bullet}(*)\}$, $\mathbf{s}_{\bullet}(*)$ is an 1×6 vector with the $*^{\text{th}}$ element equal to 1 and others equal to 0, and $\bullet \in \{\mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3, \mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3\}$.

(ii) *Datum features induced deviations*: including the datum features deviations caused by preceding assembly processes (or called re-orientation induced deviation (Jin and Shi 1999)), as the subassembly BC's deviation shown in Figure 2-3 (b); and that caused by the part fabrication processes, as the deviation of slot feature on part A in Figure 2-3 (b).

For a stage k with FX_k parts or subassemblies to be assembled, a fixturing schemes with three datum surfaces is defined for each general 3-2-1 fixture j, j=1,2,…, FX_k, as illustrated in Figure 2-2 (a), The PD associated with the fixture j at the stage k, $\mathrm{PD}_{k,j}$, is composed of three features, $p_{1,k,j}$, $p_{2,k,j}$, and $p_{3,k,j}$, which are in touch with locators L_1, L_2 and L_3 at datum points q_1, q_2 and q_3, respectively. With the same notation format, the $\mathrm{SD}_{k,j}$ consists of two features, $s_{1,k,j}$ and $s_{2,k,j}$, touching locators P_1 and P_2 at datum points q_4 and q_5, respectively. The $\mathrm{TD}_{k,j}$ that touches locator P_3 at datum point q_6 is denoted as $t_{k,j}$. Representing the deviations of the six datum features w.r.t. RCS with six DMV's as $\mathbf{x}_{T_{p_1,k,j}}^{R}$, $\mathbf{x}_{T_{p_2,k,j}}^{R}$, $\mathbf{x}_{T_{p_3,k,j}}^{R}$, $\mathbf{x}_{T_{s_1,k,j}}^{R}$, $\mathbf{x}_{T_{s_2,k,j}}^{R}$ and $\mathbf{x}_{T_{t,k,j}}^{R}$, datum induced deviations for the fixturing scheme j at the stage k is modeled as the deviations of true FCS ($F_{k,j}$) w.r.t. RCS (R):

$$\begin{aligned} \mathbf{x}_{F_{k,j}}^{R} &= \mathbf{T}_{1,k,j}\mathbf{x}_{T_{p_1,k,j}}^{R} + \mathbf{T}_{2,k,j}\mathbf{x}_{T_{p_2,k,j}}^{R} + \mathbf{T}_{3,k,j}\mathbf{x}_{T_{p_3,k,j}}^{R} \\ &\quad + \mathbf{T}_{4,k,j}\mathbf{x}_{T_{s_1,k,j}}^{R} + \mathbf{T}_{5,k,j}\mathbf{x}_{T_{s_2,k,j}}^{R} + \mathbf{T}_{6,k,j}\mathbf{x}_{T_{t,k,j}}^{R}, \end{aligned} \tag{2.8}$$

where $\mathbf{T}_{1,k,j}$ through $\mathbf{T}_{6,k,j}$ are determined by the nominal location of the six datum points, q_1 through q_6, and that of fixture locators, as shown in Figure 2-2 (a.2). The derivation procedure and values of $\mathbf{T}_{1,k,j}$ through $\mathbf{T}_{6,k,j}$ can be found in Appendix II.

(iii) *Noises*: including the system noise that cannot be modeled with the linear state space representation and the noises introduced by measurement devises.

2.3.2 Modeling Deviation Propagation

Based on the geometric deviation representation of the three elements in an MAP, and the modeling of deviation components that induced by datum features and fixture locators, variation (deviation) propagation can be described in a state space model. As denoted in Figure 1-1, the deviation of an assembly after the stage k is represented by a state vector, $\mathbf{x}_k$, which is a stack of the MDV's of all parts assembled. With the deviation of the part r (r=1,2,…, S) at the stage k being represented as a 6×1 vector, $\mathbf{x}_{k,r}^{R}$ (the deviation of PCS$_r$ from its nominal, w.r.t. RCS, at stage k), the state vector of a final assembly with S parts will be $\mathbf{x}_k = \left[\left(\mathbf{x}_{k,1}^{R}\right)^{\mathrm{T}} \quad \left(\mathbf{x}_{k,1}^{R}\right)^{\mathrm{T}} \quad \cdots \quad \left(\mathbf{x}_{k,S}^{R}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$. Before the first stage, there is no process operated and therefore no deviations, i.e., $\mathbf{x}_0 = \mathbf{0}_{6S\times 1}$. As parts or subassemblies pass through an MAP, the elements corresponding to the assembled parts may be changed to reflect their quality deviations, whereas the elements corresponding to unassembled parts remain zeros. At each stage k of an MAP, dimensional deviations may come from three types of sources.

Denoting deviations of all parts after the stage k-1 as state vector $\mathbf{x}_{k\text{-}1}$, all deviations induced by part fabrication processes and assembly process performed at the stage k as $\mathbf{u}_k$, and deviations of in-process KPC's measured at the stage k as $\mathbf{y}_k$, the variation (deviation) propagation in an MAP can be formulated in a linear discrete state space model:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k, \\ \mathbf{y}_k &= \mathbf{C}_k \mathbf{x}_k + \mathbf{D}_k \mathbf{u}_k + \mathbf{v}_k, \end{aligned} \tag{2.9}$$

where $\mathbf{A}_k\mathbf{x}_{k-1}$ represents the deviations transmitted from upstream stages by re-orientation movements; $\mathbf{B}_k\mathbf{u}_k$ represents the quality deviations introduced from the stage k; $\mathbf{C}_k\mathbf{x}_k$ reflects that the KPC deviations are calculated from the linear combination of quality deviation; the KPC deviations contributed from deviations of features on incoming parts are captured by $\mathbf{D}_k\mathbf{u}_k$; $\mathbf{w}_k$ and $\mathbf{v}_k$ are the un-modeled system noise and measurement noise.

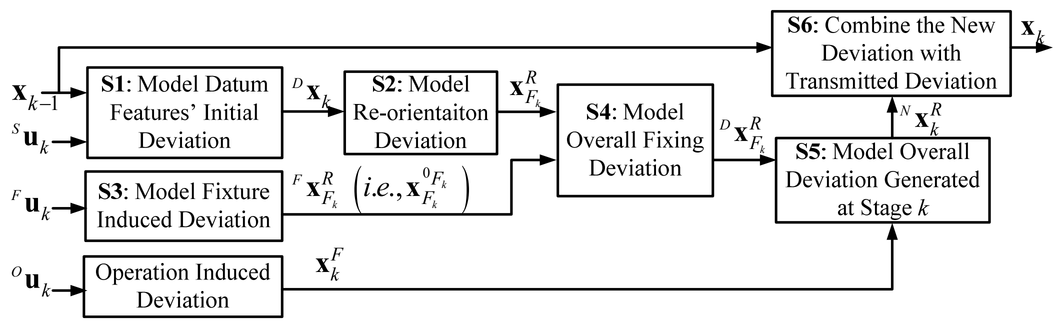


Figure 2-4. Procedure of derivation of deviation propagation model

The modeling of deviation components that induced by datum features and fixture locators on a single-stage fixturing scheme provides the basis for deviation propagation modeling on a multi-stage level, which is a procedure of deriving system matrices in the state transition equations in Eq.(2.9). Figure 2-4 shows the major steps to determine the system matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ in the state transition equation of the stage k. These steps are the mathematical representation of the physical operations at each stage of an MAP.

S1: *Modeling initial deviations of datum features.* For every part/subassembly fixed by fixture j at the stage k, there are six datum features. Their deviations due to re-orientation movements, i.e., generated and transmitted from preceding stage k-1, can be derived from the parts' deviations generated at preceding stages, i.e.,

$$\begin{aligned} {}^{P}\mathbf{x}_{k,j}^{R} &= \mathbf{S}_{k,j}(1)\cdot\mathbf{x}_{k-1} \\ &= \left[\left(\mathbf{x}_{P_{k,i(p_1,k,j)}}^{R}\right)^{\mathrm{T}}\ \left(\mathbf{x}_{P_{k,i(p_2,k,j)}}^{R}\right)^{\mathrm{T}}\ \left(\mathbf{x}_{P_{k,i(p_3,k,j)}}^{R}\right)^{\mathrm{T}}\ \left(\mathbf{x}_{P_{k,i(s_1,k,j)}}^{R}\right)^{\mathrm{T}}\ \left(\mathbf{x}_{P_{k,i(s_2,k,j)}}^{R}\right)^{\mathrm{T}}\ \left(\mathbf{x}_{P_{k,i(t,k,j)}}^{R}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \end{aligned}$$

where ${}^{P}\mathbf{x}_{k,j}^{R}$ is an intermediate vector containing the deviations of all parts (w.r.t. RCS) that provide datum features for fixture j at the stage k, where the subscript, $i(\bullet,k,j)$, is a function indicating the index of the part that contains datum feature ●, $\bullet \in \{p_1, p_2, p_3, s_1, s_2, t\}$, for fixture j at stage k. the selector matrix, $S_{k,j}(1)$, is determined by the datum scheme defined in the process design,

$$\mathbf{S}_{k,j}(1) = \left[\boldsymbol{\theta}^{\mathrm{T}}_{i(p1,k,j)} \ \boldsymbol{\theta}^{\mathrm{T}}_{i(p2,k,j)} \ \boldsymbol{\theta}^{\mathrm{T}}_{i(p3,k,j)} \ \boldsymbol{\theta}^{\mathrm{T}}_{i(s1,k,j)} \ \boldsymbol{\theta}^{\mathrm{T}}_{i(s2,k,j)} \ \boldsymbol{\theta}^{\mathrm{T}}_{i(t,k,j)} \right]^{\mathrm{T}}. \tag{2.10}$$

$\boldsymbol{\theta}_{i(\bullet,k,j)}$ is a 6×6S matrix of S 6×6 square sub-matrices, where the $i(\bullet,k,j)$ one is an identity matrix and others are sub-matrices of zeros.

Datum features deviations may also be contributed by the part fabrication processes. They are represented as ${}^{S}\mathbf{x}_{k,j} = \left[\left(\mathbf{x}^{P_{k,i(p_1,k,j)}}_{T_{p_1,k,j}} \right)^{\mathrm{T}} \ \left(\mathbf{x}^{P_{k,i(p_2,k,j)}}_{T_{p_2,k,j}} \right)^{\mathrm{T}} \ \left(\mathbf{x}^{P_{k,i(p_3,k,j)}}_{T_{p_3,k,j}} \right)^{\mathrm{T}} \ \left(\mathbf{x}^{P_{k,i(s_1,k,j)}}_{T_{s_1,k,j}} \right)^{\mathrm{T}} \ \left(\mathbf{x}^{P_{k,i(s_2,k,j)}}_{T_{s_2,k,j}} \right)^{\mathrm{T}} \ \left(\mathbf{x}^{P_{k,i(t,k,j)}}_{T_{t,k,j}} \right)^{\mathrm{T}} \right]^{\mathrm{T}}$, where $\mathbf{x}^{i(\bullet,k,j)}_{\bullet}$ is the DMV of datum feature, ●, w.r.t the PCS of part $i(\bullet,k,j)$, and $\bullet \in \{\mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3, \mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3\}$. These initial deviations can be translated to that w.r.t. the RCS by applying *corollary* 1,

$${}^{D}\mathbf{x}^{R}_{k,j} = \mathbf{A}_{k,j}(1) \cdot {}^{P}\mathbf{x}^{R}_{k,j} + {}^{S}\mathbf{x}_{k,j}, \tag{2.11}$$

where $\mathbf{A}_{k,j}(1) = diag\{\mathbf{M}^{P_{k,i(\bullet,k,j)}}_{T_{\bullet,k,j}}\}$ is a diagonal block matrix with diagonal blocks as $\mathbf{M}^{P_{k,i(\bullet,k,j)}}_{T_{\bullet,k,j}}$ given in Eq.(2.3).

Eq. (2.11) models the initial deviations of datum features for fixturing scheme j at the stage k. It can be applied to all the FX_k fixturing schemes used at the stage k. A stack of all ${}^{P}\mathbf{x}^{R}_{k,j}$, i.e., ${}^{P}\mathbf{x}^{R}_{k} = \left[\left({}^{P}\mathbf{x}^{R}_{k,1} \right)^{\mathrm{T}} \ \left({}^{P}\mathbf{x}^{R}_{k,2} \right)^{\mathrm{T}} \ldots \left({}^{P}\mathbf{x}^{R}_{k,FX_k} \right)^{\mathrm{T}} \right]^{\mathrm{T}}$, is obtained by

$${}^{P}\mathbf{x}^{R}_{k} = \mathbf{A}_{k}(0)\mathbf{x}_{k-1}, \tag{2.12}$$

where $\mathbf{A}_k(0) = \left[\left(\mathbf{S}_{k,1}(1)\right)^{\mathrm{T}} \quad \left(\mathbf{S}_{k,2}(1)\right)^{\mathrm{T}} \quad \ldots \quad \left(\mathbf{S}_{k,FX_k}(1)\right)^{\mathrm{T}} \right]^{\mathrm{T}}$. Similarly, denoting ${}^D\mathbf{x}_k = \left[\left({}^D\mathbf{x}_{k,1}^R\right)^{\mathrm{T}} \left({}^D\mathbf{x}_{k,2}^R\right)^{\mathrm{T}} \ldots \left({}^D\mathbf{x}_{k,FX_k}^R\right)^{\mathrm{T}} \right]^{\mathrm{T}}$, and ${}^S\mathbf{u}_k = \left[\left({}^S\mathbf{x}_{k,1}\right)^{\mathrm{T}} \left({}^S\mathbf{x}_{k,1}\right)^{\mathrm{T}} \ldots \left({}^S\mathbf{x}_{k,FX_k}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$, by repetitively applying *corollary* 1, we can have

$$ {}^D\mathbf{x}_k = \mathbf{A}_k(1) \cdot {}^P\mathbf{x}_k^R + {}^S\mathbf{u}_k , \tag{2.13} $$

where $\mathbf{A}_k(1) = diag\left\{\mathbf{A}_{k,j}(1)\right\}$. In Eq.(2.13), ${}^D\mathbf{x}_k$, ${}^P\mathbf{x}_k^R$ and ${}^S\mathbf{u}_k$ are all in the space of $\Re^{6 \cdot 6 \cdot FX_k \times 1}$, and $\mathbf{A}_k(1) \in \Re^{6 \cdot 6 \cdot FX_k \times 6 \cdot 6 \cdot FX_k}$. Eq. (2.13) implies that the initial datum features' deviations are contributed by two sources: the deviations generated at the preceding stages and that generated when the incoming parts are fabricated.

S2: *Modeling re-orientation deviation.* For the j^{th} fixturing scheme at the stage *k*, datum induced re-orientation deviations are derived by plugging ${}^D\mathbf{x}_{k,j}^R$ into Eq.(2.8),

$$ {}^D\mathbf{x}_{F_{k,j}}^R = \mathbf{A}_{k,j}(2) \cdot {}^D\mathbf{x}_{k,j}^R , \tag{2.14} $$

where $\mathbf{A}_{k,j}(2) = [\mathbf{T}_{1,k,j} \ \mathbf{T}_{2,k,j} \ \mathbf{T}_{3,k,j} \ \mathbf{T}_{4,k,j} \ \mathbf{T}_{5,k,j} \ \mathbf{T}_{6,k,j}]$ for general 3-2-1 fixturing scheme, or $\mathbf{A}_{k,j}(2) = [\mathbf{T}_{1,k,j} \ \mathbf{T}_{2,k,j} \ \mathbf{T}_{3,k,j} \ \mathbf{T}_{4,k,j}^* \ \mathbf{T}_{5,k,j}^* \ \mathbf{T}_{6,k,j}^*]$ for pin-hole fixturing scheme. The datum induced re-orientation deviations for all FX_k fixturing schemes can be obtained by stacking up all ${}^D\mathbf{x}_{F_{k,j}}^R$. Denoting deviations (caused by datum deviations) of all $F_{k,j}$, j=1,2,…,FX_k, w.r.t. RCS as ${}^D\mathbf{x}_{F_k}^R = \left[\left({}^D\mathbf{x}_{F_{k,1}}^R\right)^{\mathrm{T}} \left({}^D\mathbf{x}_{F_{k,2}}^R\right)^{\mathrm{T}} \ldots \left({}^D\mathbf{x}_{F_{k,FX_k}}^R\right)^{\mathrm{T}} \right]^{\mathrm{T}}$, we have

$$ {}^D\mathbf{x}_{F_k}^R = \mathbf{A}_k(2) \cdot {}^D\mathbf{x}_k . \tag{2.15} $$

where F_k represents all FX_k fixtures at the stage *k*, $\mathbf{A}_k(2) = diag\left\{\mathbf{A}_{k,j}(2)\right\}$, and $\mathbf{A}_k(2) \in \Re^{6 \cdot FX_k \times 6 \cdot 6 \cdot FX_k}$.

S3: *Modeling fixture induced deviation.* For the j^{th} fixture used at the stage k, fixture induced deviation can be computed by plugging the fixture locators' deviation defined by Eq. (2.5) or Eq. (2.7) into Eq. (2.6), i.e., $\mathbf{x}_{F_{k,j}}^{{}^0F_{k,j}} = \mathbf{T}_{k,j}^{F} \cdot \mathbf{u}_{k,j}^{{}^0F}$. In this chapter, nominal FCS's are assumed to be deviation-free, i.e., DMV $\mathbf{x}_{{}^0F_{k,j}}^{R} = \mathbf{0}$. Thus, by applying *corollary* 1, the fixture induced deviations can be further represented as deviation of FCS w.r.t., RCS, i.e., ${}^{F}\mathbf{x}_{F_{k,j}}^{R} = \mathbf{M}_{F_{k,j}}^{{}^0F_{k,j}} \mathbf{x}_{{}^0F_{k,j}}^{R} + \mathbf{x}_{F_{k,j}}^{{}^0F_{k,j}} = \mathbf{x}_{F_{k,j}}^{{}^0F_{k,j}}$, and

$$ {}^{F}\mathbf{x}_{F_{k,j}}^{R} = \mathbf{A}_{k,j}(3) \cdot \mathbf{u}_{k,j}^{{}^0F}, \tag{2.16} $$

where $\mathbf{A}_{k,j}(3) = \mathbf{T}_{k,j}^{F}$. Let ${}^{F}\mathbf{x}_{F_k}^{R} = \left[\left({}^{F}\mathbf{x}_{F_{k,1}}^{R} \right)^{\mathrm{T}} \ \left({}^{F}\mathbf{x}_{F_{k,2}}^{R} \right)^{\mathrm{T}} \ \ldots \ \left({}^{F}\mathbf{x}_{F_{k,FX_k}}^{R} \right)^{\mathrm{T}} \right]^{\mathrm{T}}$ denote the deviations of all $F_{k,j}$, j=1,2,…,FX_k, w.r.t. R that caused by fixture locators' deviations represented by ${}^{F}\mathbf{u}_{k} = \left[\left(\mathbf{u}_{k,1}^{{}^0F} \right)^{\mathrm{T}} \ \left(\mathbf{u}_{k,2}^{{}^0F} \right)^{\mathrm{T}} \ldots \left(\mathbf{u}_{k,FX_k}^{{}^0F} \right)^{\mathrm{T}} \right]^{\mathrm{T}}$, we have

$$ {}^{F}\mathbf{x}_{F_k}^{R} = \mathbf{A}_{k}(3) \cdot {}^{F}\mathbf{u}_{k}, \tag{2.17} $$

where $\mathbf{A}_k(3) = diag\left\{ \mathbf{A}_{k,j}(3) \right\}$, ${}^{F}\mathbf{u}_k \in \Re^{6FX_k \times 1}$, and $\mathbf{A}_k(3) \in \Re^{6FX_k \times 6FX_k}$.

S4: *Modeling overall fixturing deviations*. Since both datum induced deviations and fixture induced deviations affect assembly quality through fixturing scheme, their deviations derived in steps S2 and S3 can be added together to form the overall fixturing deviations, i.e.,

$$ \mathbf{x}_{F_{k,j}}^{R} = {}^{D}\mathbf{x}_{F_{k,j}}^{R} + {}^{F}\mathbf{x}_{F_{k,j}}^{R}, \tag{2.18} $$

and by stacking up deviations of all $F_{k,j}$, j=1,2,…,FX_k, we have

$$ \mathbf{x}_{F_k}^{R} = {}^{D}\mathbf{x}_{F_k}^{R} + {}^{F}\mathbf{x}_{F_k}^{R}, \tag{2.19} $$

where $\mathbf{x}_{F_k}^{R}, {}^{D}\mathbf{x}_{F_k}^{R}, {}^{F}\mathbf{x}_{F_k}^{R} \in \Re^{6FX_k \times 1}$.

S5: *Calculate overall deviations at the stage k*. The overall deviations at the stage k are the combination of overall fixturing deviations and operations induced deviations. As aforementioned, operations deviations are assumed given by the deviations of parts w.r.t. true FCS. Denoting $l_{k,j}$ as the number of parts on a subassembly located by fixture j at the stage k ($l_{k,j}$=1 for a single part), operation deviations are represented as ${}^{O}\mathbf{u}_{k,j}=\left[\mathbf{0}^{\mathrm{T}} \ldots \left(\mathbf{x}_{P_{k,r(1,k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \left(\mathbf{x}_{P_{k,r(c,k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \left(\mathbf{x}_{P_{k,r(l_{k,j},k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \mathbf{0}^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{0}$ is a 6×1 vector of zeros, subscript $r(c,k,j)$ is a function indicating the indices of parts on the subassembly c, c=1,2,…, $l_{k,j}$, and $r(c,k,j) \in \{1,2,...,S\}$. According to *corollary* 1, overall parts deviations caused by a single fixture j are represented as deviations of parts w.r.t. RCS, i.e.,

$$ {}^{N}\mathbf{x}_{k,j}^{R}=\mathbf{A}_{k,j}(4)\cdot\mathbf{x}_{F_{k,j}}^{R}+{}^{O}\mathbf{u}_{k,j}, \tag{2.20}$$

where $\mathbf{A}_{k,j}(4)=\left[\mathbf{\Theta}^{\mathrm{T}} \ldots \left(\mathbf{M}_{P_{k,r(1,k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \left(\mathbf{M}_{P_{k,r(c,k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \left(\mathbf{M}_{P_{k,r(l_{k,j},k,j)}}^{F_{k,j}}\right)^{\mathrm{T}} \ldots \mathbf{\Theta}^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\mathbf{\Theta}$ is a 6×6 matrix of zeros. Assuming that deviations of $F_{k,j}$ from ${}^{0}F_{k,j}$ are very small compared to the dimensional of fixture layout, according to Zhou *et al.* (2003), $\mathbf{M}_{P_{k,r(c,k,j)}}^{F_{k,j}}$ can be replaced by $\mathbf{M}_{P_{k,r(c,k,j)}}^{{}^{0}F_{k,j}}$.

Eq. (2.20)models overall deviations from a single fixture j at the stage k. For all FX_k fixtures, ${}^{N}\mathbf{x}_{k}^{R}$, is the summation of ${}^{N}\mathbf{x}_{k,j}^{R}$, i.e.,

$$ {}^{N}\mathbf{x}_{k}^{R}=\mathbf{A}_{k}(4)\cdot\mathbf{x}_{F_{k}}^{R}+{}^{O}\mathbf{u}_{k}, \tag{2.21}$$

where ${}^{N}\mathbf{x}_{k}^{R}=\sum_{j=1}^{FX_k}\left({}^{N}\mathbf{x}_{k,j}^{R}\right)$, $\mathbf{A}_{k}(4)=\left[\mathbf{A}_{k,1}(4)\ \mathbf{A}_{k,2}(4)\ \ldots\ \mathbf{A}_{k,FX_k}(4)\right]$, ${}^{O}\mathbf{u}_{k}=\sum_{j=1}^{FX_k}{}^{O}\mathbf{u}_{k,j}$, ${}^{N}\mathbf{x}_{k}^{R}$ and ${}^{O}\mathbf{u}_{k} \in \Re^{6S\times 1}$ and $\mathbf{A}_{k}(4)\in\Re^{6S\times 6FX_k}$.

S6: *Calculate overall deviations after the stage k*. After assembly operations at the stage k, the deviations of parts w.r.t. RCS are the combinations of the deviations transmitted from preceding stages and that generated at the current stage k, i.e.,

$$\mathbf{x}_k = \mathbf{x}_{k-1} + {}^N\mathbf{x}_k^R . \quad (2.22)$$

The intermediate results listed in Eq. (2.11)-(2.21) are the building-blocks of the system matrices in model (2.9). Plugging them into Eq. (2.22), the state transition equation in model (2.9) can be represented as

$$\begin{aligned} \mathbf{x}_k &= \left[\mathbf{I}_{6S\times 6S} + \mathbf{A}_k(4)\cdot\mathbf{A}_k(2)\cdot\mathbf{A}_k(1)\cdot\mathbf{A}_k(0)\right]\mathbf{x}_{k-1} \\ &\quad + \left[\mathbf{A}_k(4)\cdot\mathbf{A}_k(2) \quad \mathbf{A}_k(4)\cdot\mathbf{A}_k(3) \quad \mathbf{I}_{6S\times 6S}\right]\cdot\begin{bmatrix} {}^S\mathbf{u}_k \\ {}^F\mathbf{u}_k \\ {}^O\mathbf{u}_k \end{bmatrix} + \mathbf{w}_k , \end{aligned} \quad (2.23)$$

The detailed derivations are presented in Appendix III.

2.3.3 Modeling Deviation Measurements

The derivation of the deviation observation equation can be conducted in a similar way. Since a KPC is a dimensional deviation relating a datum or a features on different parts. Consider a stage where the number of KPC's being measured is M_k, the m^{th} KPC, m=1,2,…, M_k, involves $f_{m,k}$ features on $f_{m,k}$ different parts. An intermediate vector, ${}^S\mathbf{x}_{k,m}^R$, which represents deviations of all $f_{m,k}$ parts that are involved with the m^{th} KPC at the stage k, can be derived as

$${}^S\mathbf{x}_{k,m}^R = \mathbf{S}_{k,m}(2)\cdot\mathbf{x}_k = \left[\left(\mathbf{x}_{P_{k,i(1,k,m)}}^R\right)^{\mathrm{T}} \left(\mathbf{x}_{P_{k,i(2,k,m)}}^R\right)^{\mathrm{T}} \cdots \left(\mathbf{x}_{P_{k,i(f_{m,k},k,m)}}^R\right)^{\mathrm{T}}\right]^{\mathrm{T}},$$

where the subscript, $i(\bullet,k,m)$, is a function indicating the part that contains feature ●, ●=1,2,…, $f_{m,k}$, and selector matrix, $\mathrm{S}_{k,m}(2)$, is determined by the measurement strategy with a form similar to $\mathrm{S}_{k,j}(1)$ defined in Eq. (2.10). Corresponding to ${}^S\mathbf{x}_{k,m}^R$, the feature deviations caused by part fabrication processes are

$${}^P\mathbf{x}_{k,m} = \left[\left(\mathbf{x}_{T_{1,k,m}}^{P_{k,i(1,k,m)}}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{2,k,m}}^{P_{k,i(2,k,m)}}\right)^{\mathrm{T}} \cdots \left(\mathbf{x}_{T_{f_{m,k},k,m}}^{P_{k,i(f_{m,k},k,m)}}\right)^{\mathrm{T}}\right]^{\mathrm{T}},$$

where $\mathbf{x}_{\bullet}^{i(\bullet,k,j)}$ is the deviation of the feature, ●, w.r.t the PCS of part *i*(●,*k*,*m*). These features' deviations w.r.t. RCS can be achieved by applying *corollary* 1,

$$^{M}\mathbf{x}_{k,m}^{R} = \mathbf{C}_{k,m}(1)\cdot {}^{P}\mathbf{x}_{k,m} + {}^{S}\mathbf{x}_{k,m} = \left[\left(\mathbf{x}_{T_{1,k,m}}^{R}\right)^{\mathrm{T}} \left(\mathbf{x}_{T_{2,k,m}}^{R}\right)^{\mathrm{T}} \dots \left(\mathbf{x}_{T_{f_{m,k},k,m}}^{R}\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{2.24}$$

where, $\mathbf{C}_{k,m}(1) = diag\{\mathbf{M}_{T_{\bullet,k,m}}^{P_{k,i(\bullet,k,m)}}\}$. The Linear combination of the $f_{m,k}$ elements in $^{M}\mathbf{x}_{k,m}^{R}$ features is implemented by a row vector $\mathbf{c}_{k,m}$ to represent the deviation of the m^{th} KPC, i.e.,

$$y_{m,k} = \mathbf{c}_{k,m}\, {}^{M}\mathbf{x}_{k,m}^{R}, \tag{2.25}$$

Eq. (2.25)models the deviations of the m^{th} KPC at the stage *k*, which can be applied to all M_k KPC's measured at the stage *k*. Let $^{PM}\mathbf{x}_{k}^{R}$ be a stack of all $^{P}\mathbf{x}_{k,m}$, *m*=1,2,…, M_k, i.e., $^{PM}\mathbf{x}_{k}^{R} = \left[\left({}^{P}\mathbf{x}_{k,1}\right)^{\mathrm{T}} \left({}^{P}\mathbf{x}_{k,2}\right)^{\mathrm{T}} \dots \left({}^{P}\mathbf{x}_{k,M_k}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, we have

$$^{PM}\mathbf{x}_{k}^{R} = \mathbf{C}_{k}(0)\mathbf{x}_{k}, \tag{2.26}$$

where $\mathbf{C}_{k}(0) = \left[\left(\mathbf{S}_{k,1}(1)\right)^{\mathrm{T}} \left(\mathbf{S}_{k,2}(1)\right)^{\mathrm{T}} \dots \left(\mathbf{S}_{k,M_k}(1)\right)^{\mathrm{T}}\right]^{\mathrm{T}}$. Also, denoting $^{M}\mathbf{x}_{k} = \left[\left({}^{M}\mathbf{x}_{k,1}^{R}\right)^{\mathrm{T}} \left({}^{M}\mathbf{x}_{k,2}^{R}\right)^{\mathrm{T}} \dots \left({}^{M}\mathbf{x}_{k,M_k}^{R}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, and $^{M}\mathbf{u}_{k} = \left[\left({}^{S}\mathbf{x}_{k,1}\right)^{\mathrm{T}} \left({}^{S}\mathbf{x}_{k,1}\right)^{\mathrm{T}} \dots \left({}^{S}\mathbf{x}_{k,M_k}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$, by repetitively applying *corollary* 1, we can have

$$^{M}\mathbf{x}_{k} = \mathbf{C}_{k}(1)\cdot {}^{PM}\mathbf{x}_{k}^{R} + {}^{M}\mathbf{u}_{k}, \tag{2.27}$$

where $\mathbf{C}_{k}(1) = diag\left\{\mathbf{C}_{k,m}(1)\right\}$. Finally, by staking up all M_k KPC's, $\mathbf{y}_k$ will be

$$\mathbf{y}_{k} = \mathbf{C}_{k}(2)\, {}^{M}\mathbf{x}_{k}. \tag{2.28}$$

where $\mathbf{C}_k(2)=\left[\left(\mathbf{c}_{k,1}\right)^{\mathrm{T}}\ \left(\mathbf{c}_{k,2}\right)^{\mathrm{T}}\cdots\left(\mathbf{c}_{k,M_k}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$. By plugging the intermediate results in Eq. (2.24)-(2.28), the deviations of KPC measurements are

$$\mathbf{y}_k=\mathbf{C}_k(2)\cdot\mathbf{C}_k(1)\cdot\mathbf{C}_k(0)\cdot\mathbf{x}_k+\mathbf{C}_k(2)\cdot{}^{M}\mathbf{u}_k\,. \tag{2.29}$$

It should be noted that the term $\mathbf{C}_k(2)\cdot{}^{SM}\mathbf{u}_k$ reflects the KPC deviations contributed by the incoming parts fabricated before the assembly processes. Denoting $\mathbf{u}_k=\left[\left({}^{S}\mathbf{u}_k\right)^{\mathrm{T}}\ \left({}^{F}\mathbf{u}_k\right)^{\mathrm{T}}\ \left({}^{O}\mathbf{u}_k\right)^{\mathrm{T}}\ \left({}^{SM}\mathbf{u}_k\right)^{\mathrm{T}}\right]^{\mathrm{T}}$ as the combination of all the deviations induced by part fabrication processes and assembly process, the system matrices in model (2.9) are:

$$\mathbf{A}_k=\mathbf{I}_{6S\times 6S}+\mathbf{A}_k(4)\cdot\mathbf{A}_k(2)\cdot\mathbf{A}_k(1)\cdot\mathbf{A}_k(0)\,, \tag{2.30}$$

$$\mathbf{B}_k=\left[\mathbf{A}_k(4)\cdot\mathbf{A}_k(2)\quad \mathbf{A}_k(4)\cdot\mathbf{A}_k(3)\quad \mathbf{I}_{6S\times 6S}\quad \mathbf{0}_{6S\times 6M_k}\right], \tag{2.31}$$

$$\mathbf{C}_k=\mathbf{C}_k(2)\cdot\mathbf{C}_k(1)\cdot\mathbf{C}_k(0)\text{, and} \tag{2.32}$$

$$\mathbf{D}_k=\left[\mathbf{0}_{M_k\times(36FX_k+6FX_k+6S)}\quad \mathbf{C}_k(2)\right], \tag{2.33}$$

where $\mathbf{I}_{a\times a}$ is an $a\times a$ identity matrix and $\mathbf{0}_{c\times d}$ is an $c\times d$ matrix with all zero elements.

2.4 Conclusion

Complicated correlations between different stages of an MAP make it difficult for engineers to understand the product quality variation propagation along stages and thus significantly impede the alternative product/process design evaluation and process variation sources identification. In this chapter, an analytical variation propagation modeling technique is proposed based on DMV representation. The model is presented in a state space format, for which a systematic procedure has been developed. Compared with existing MAP modeling techniques, the proposed one is more generic by covering both Type-I and Type-II assembly and by including the impact of part fabrication errors on assembly quality.

The proposed state space model for MAP has great potentials for various applications to achieve quality assurance in MAP: (i) Since the interactions between process variables (i.e., fixture locators deviations) and quality variables (i.e., KPC deviations) are mathematically represented, variation sources identification can be conducted by collecting KPC measurement data and solving the equations. (ii) This model can also be used as a tool for quality assured product or process design. By providing an analytical prediction of quality variations, alternative design options will be evaluated to choose the one that can cost-effectively deliver good quality. This strategy moves the quality assurance to the early stage of production realization and will reinforce total quality management. (iii) The structure of state space model makes it possible to adopt some of the classical control theories to solve engineering problems related to quality engineering, e.g., diagnosability study and sensor placement. (iv) Since the deviations of features that caused by part fabrication process are explicitly modeled, the variation sources can be traced back to upstream productions so that product designer can either use the model to assign tolerances to different part suppliers, or conduct tolerance synthesis among different parts to improve quality and reduce manufacturing costs.

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CHAPTER 3

QUALITY ASSURED SETUP PLANNING BASED ON THE STREAM-OF-VARIATION MODEL FOR MULTISTAGE MACHINING PROCESSES[2]

Abstract

Setup planning is a set of activities used to arrange manufacturing features into an appropriate sequence for processing. It has significant impacts on the product quality, which is often measured in terms of dimensional variation in the Key Product Characteristics (KPC). Current approaches to setup planning are experience-based and tend to be conservative by selecting unnecessarily precise machines and fixtures to ensure final product quality. This is especially true in multistage machining processes (MMP's) because it has been difficult to predict the variation propagation and its impact on the KPC quality of final product. In this chapter, a new methodology is proposed to realize cost-effective, quality assured setup planning for MMP's. Setup planning is formulated as an optimization problem based on quantitative evaluation of variation propagations. The optimal setup plan minimizes the cost related to process precision (CRPP) and satisfies the quality specifications. The proposed approach can significantly improve the effectiveness as well as the efficiency of the setup planning for MMP's.

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[2] Liu, J., Shi, J., and Hu, S.J., 2007, accepted by IIE Transactions, Quality and Reliability Engineering.

3.1 Introduction

Process planning is the systematic determination of the steps by which a product is manufactured. It is a key element that bridges activities in design and manufacturing. In the past decades, process planning and its automation enablers have received extensive study and made significant progress (Maropoulos 1995). Many reported approaches of process planning include conceptual process planning, setup planning and detailed process planning, as shown in Figure 3-1. Conceptual process planning includes engineering feature recognition, process selection, and machine/tooling selection. Detailed process planning includes fixture design, quality-assurance-strategy selection, and cost analysis.

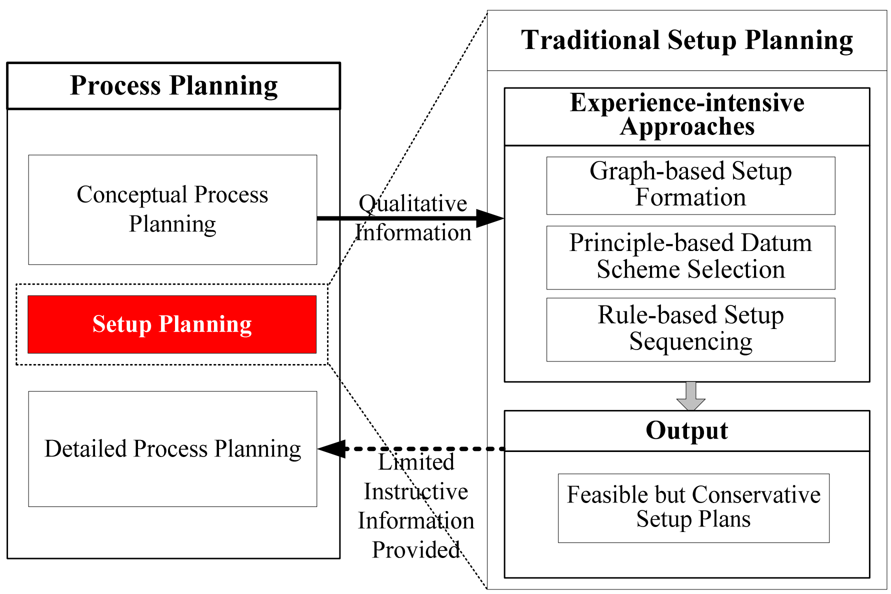

Figure 3-1. The existing commonly used setup planning approaches

Setup planning constitutes a critical component that connects conceptual process planning and detailed process planning. Conceptual process planning provides qualitative information to setup planning, including designated features, selected processes and datum scheme constraints. The purpose of setup planning is to arrange manufacturing features into an appropriate sequence of setups in order to ensure product quality and productivity (Huang and Liu 2003). A setup plan is comprised of setup formation, datum scheme selection and setup sequencing (Huang 1998). It defines a series of datum/fixturing schemes for a multi-stage machining process (MMP), as shown in Figure 3-1. However, the setup plan obtained from those traditional methods provides limited instructive information to subsequent planning activities in detailed process planning.

Product quality is one of the main concerns of setup planning. A well defined setup plan should be able to satisfy quality specifications under normal manufacturing conditions. Product quality is affected by the outcome of setup planning since the series of datum and fixtures defined by a specific setup plan may introduce errors which will propagate along the machining stages and accumulate in the final product. Different setup plans specify different datum/fixturing schemes, lead to different variation propagation scenario, and result in different product quality. Thus, one of the major tasks in setup planning is to identify the optimal setup from multiple alternatives to ensure product quality.

Some research has been conducted in quality assured setup planning, addressing issues in setup formation, datum scheme selection and setup sequencing. Zhang *et al.* (1996) proposed principles for achieving tolerance control proactively via appropriately grouping and sequencing features according to their tolerance relationships. Mantripragada and Whitney (1998) presented the "datum flow chain" concept to relate datum logic explicitly with product KPC tolerances and assembly sequences. Quantitative approaches were also developed to evaluate variation stack-up associated to different process design. Rong and Bai (1996) presented a method to verify machining accuracy corresponding to fixture design. Song *et al.* (2005) developed a Monte Carlo simulation-based method to analyze the quality impact of production planning. Xu and Huang (2006) modeled the simulated quality distributions in multiple attributes utility (MAU) function. Besides the simulation-based approaches, analytical methods were also studied to investigate the interactions between product quality and process variability. For a given setup plan, Stream of Variation (SoV) methodologies (Shi 2006) and state space modeling techniques were developed to model the dimensional variation propagation along different setups (Ding *et al.* 2002; Hu 1997; Huang *et al.* 2007; Jin and Shi 1999; Zhou *et al.* 2003).

Cost-effectiveness is another critical concern in setup planning. It can be evaluated in terms of the cost related to process precision (CRPP), such as the cost to achieve necessary fixture precision to satisfy product quality requirements. The precision refers to the inherent variability in an MMP and CRPP is the cost for achieving necessary precision level to ensure the satisfaction of product quality requirements. CRPP is assumed to be inversely proportional to the necessary process precision. Corresponding to different setup plans, different process precision is required and thus different costs are incurred. Therefore, setup planning should be a discrete constrained optimization procedure. Ong *et al* (2007) considered various cost factors in the optimization index, including the cost of machines and fixtures. However, these cost factors are not directly linked with process precision.

It is desirable that the optimal setup plan is the one that satisfies product quality specification with relatively imprecise fixtures and machines to minimize the CRPP. However, the setup plans developed solely based on principles and experiences could be very conservative. Although they are generally feasible with respect to the quality consideration, cost-effectiveness may not be optimal. For instance, in order to ensure the final product quality, engineers tend to conservatively select unnecessarily precise fixtures and thus cause unnecessary CRPP. This is especially true for the upstream stages of an MMP because of the lack of the capabilities for variation propagation evaluation. Furthermore, in order to automate the process planning, the outcomes of setup planning should be effectively integrated with other activities of detailed process planning, e.g. fixture design. Fixture layout design for a particular setup is critical input information for setup planning, whereas the setup planning results determine the MMP whose fixture system should be optimized on process level. However, although the fixture layout design has been successfully investigated on both single stage level (Cai *et al.* 1997) and process level (Kim and Ding 2004), effective setup/fixture planning study is still primitive. This is because that the qualitative-principle based setup planning provides limited potential for specifying quantitative precision requirements of fixture design. In addition, conservative process precision requirements will make the designed fixture unnecessarily expensive. This functional limitation of conventional setup planning significantly hinders the implementation of the process planning automation.

Existing setup planning approaches are summarized in Table 3-1. As can be seen, most reported research has focused on the evaluation of setup plan alternatives. Some work exists for conducting optimal setup planning, based on qualitative or simulation-based evaluation of product quality. Although the simulation provides an effective strategy to compare alternative setup plans regarding their output product quality, it consumes a substantial amount of time and computational resources.

Table 3-1. Approaches for setup planning

	Evaluation-Oriented Setup Planning	**Optimization-Oriented Setup Planning**
Qualitative Quality Evaluation	Zhang *et al*. (1996); Mantripragada and Whitney (1998).	Ong *et al.*, (2007)
Simulation-based Quality Evaluation	Song *et al.*, (2005)	Xu and Huang, (2006)
Analytical Quality Evaluation	Hu, S.J. (1997); Ding *et al.*, (2002); and Zhou *et al.* (2003), etc.	*To be studied in this chapter*

This chapter adopts an integrated setup/fixture planning strategy in process planning. It focuses on the systematic development of a cost-effective, quality assured setup planning, which is a fundamental enabler of the integrated setup/fixture planning. Because of the complexity of the integrated problem and the overwhelming computational requirements, an iterative approach is appropriate. As illustrated in Figure 3-2, the stage/setup level optimal fixture layouts for all candidate datum schemes are first determined and fixed. In each stage, different datum scheme options may be assigned with different fixture layouts. These stage/setup level fixture layouts are the inputs to the setup planning, together with the information on feature representation, design specification, constraints on datum scheme and setup sequence. As shown in Figure 3-2, the development of the proposed setup planning consists of three steps: (i) Candidate setup formations and datum schemes are formulated based on input information. Their potential variation stack-up can be analytically predicted by the SoV model. (ii) Based on those candidate setups defined in step (i), the setup planning is formulated as a sequential decision making on an optimal series of setups that cost-effectively satisfies product quality specifications. A cost criterion is defined to evaluate the optimality of

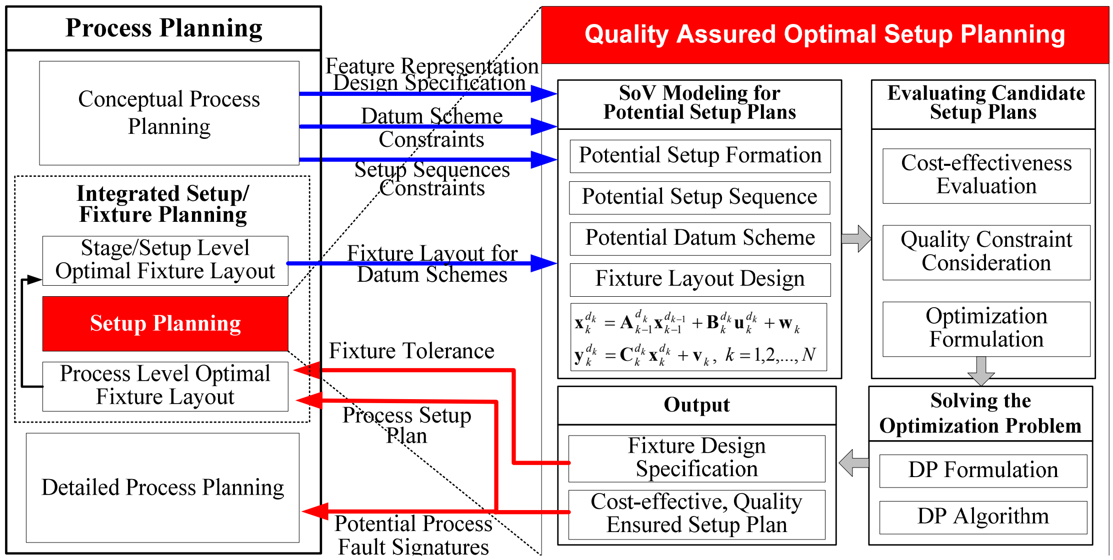

Figure 3-2. Overview of the proposed "SoV-based, quality assured setup planning"

candidate setup plans under the constraints of product quality specifications. (iii) Dynamic Programming (DP) is used to solve the optimal sequential decision making problem and generate the optimal setup plan, which provides setup information for subsequent activities in process planning. Based on analytical quality evaluation strategy, the proposed optimal setup planning methodology will be effective and efficient. When the optimal setup plan is determined, Kim and Ding (2004)'s approach can be applied to achieve process-level optimal fixture layout, which will be used to update the stage/setup level fixture layouts for repeating the iterative optimization procedure.

The remainder of this chapter is organized as follows. The SoV-based optimal setup planning methodology is introduced in Section 3.2. Section 3.3 presents a case study by applying the proposed approach to generate setup plan for MMP's. Conclusions and future works will be discussed in Section 3.4.

3.2 Quality Assured, Cost Effective Setup Planning

The design specifications of a machined product are often satisfied by machining operations performed on a series of stages. In each stage, a set of features will be generated with a specific setup. Due to the variation of the machining operations, the dimensional precision of the final product is affected by three major variation sources:

(i) *Machine and cutting tool*, which refers to the random deviation of the cutting tools from their nominal paths.

(ii) *Fixture*, which refers to the random deviation of the fixture locators from their nominal positions.

(iii) *Datum,* which refers to the random deviation of the datum features, generated in previous stages, from their nominal positions and/or dimensions.

Both (i) and (ii) are treated as random process deviation. The third source exists because some features generated in the upstream stages are used as the datum features in the downstream stages according to the setup plan. Thus, the dimensional variation, which is introduced by fixtures and/or machine and cutting tools in the upstream stages, will be propagated through datum features and accumulated in the features generated in the downstream stages through datum features. Different setup plans, i.e., different datum schemes and different setup sequences, lead to different variation propagation scenarios, and thus, result in different final product quality. In order to compare candidate setup plans, an effective method is needed to evaluate the impacts of potential datum schemes and setup sequences on the quality of final product.

3.2.1 Variation Propagation Model for Setup Planning

One effective tool to model the variation propagation in MMP's is the state space modeling technique (Shi 2006). Zhou *et al*. (2003) presented a detailed derivation and validation of the model with given process/product design, including the information of setup formation, datum scheme selection and setup sequence. However, some additions are necessary due to the following unique characteristics in setup planning:

(i) *Multiple datum scheme options:* In setup planning, every stage has a set of candidate datum schemes. Different datum schemes support different operations that generate different features, which further constrain the pool of candidate datum schemes for downstream stages. Also, datum scheme selection is directly related to the fixture design and thus significantly affects the CRPP. Thus, there is a need for explicit representation of the selected datum scheme for every stage.

(ii) *Setup precedence requirements:* According to the design specifications, some features must be fabricated in a stage with comparatively precise datum features, which may be machined in an upstream stage. This kind of precedence relationships is not often straightforward to determine, especially when the tolerance interdependences among features are complicated. Therefore, the capability to explicitly represent the sequence of setups and the chain of datum schemes is needed to evaluate different setup precedence options.

(iii) *Tracing the setup chain:* Since the CRPP is inversely proportional to the precision of fixtures, process planners tend to select less precise fixtures to reduce the cost. However, this will increase the dimensional variation of the generated features and increase the datum variation if some of them are used as datum in the downstream stages. As a result, the datum features with large variation force the downstream fixtures to be very precise to satisfy quality specifications. In other words, due to the complex variation propagation, relaxing the upstream process precision may result in the need for tighter tolerances in the downstream processes and thus increase the total CRPP. Therefore, to achieve an overall cost-effectiveness, the variation propagation of the setup chain must be traced and explicitly modeled.

Figure 3-3 illustrates variation propagation scenario of the setup plan of an MMP. The nomenclature is explained as follows:

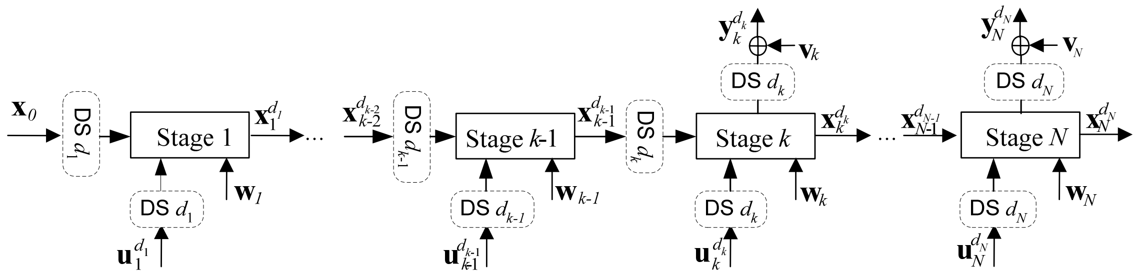

Figure 3-3. Variation propagation in a setup plan

- The datum scheme (DS) of stage k (k = 1, 2, …, N) is denoted as DS d_k (d_k = 1, 2, …, D_k, where D_k is the total number of feasible datum scheme options for stage k). A datum scheme refers to the coordinate system specified by a group of datum surfaces, within which the machining process can be performed. Datum scheme is very important to the variation propagation modeling since all those three aforementioned variation sources affect the quality of newly generated features through datum, as shown in Figure 3-3.

- Corresponding to a selected datum scheme d_k in stage k, the quality of all features are denoted by a state vector $\mathbf{x}_k^{d_k}$, with each element represents the dimensional deviation from its nominal value.

- The random deviation of process variables associated with a selected datum scheme d_k in stage k is denoted as $\mathbf{u}_k^{d_k}$. Corresponding to the major variation sources, $\mathbf{u}_k^{d_k}$ models the random process deviations of both machine/cutting tools and fixture locators, as defined in (Zhou *et al.* 2003). Represented as deviations of the tool path from its nominal path, $\mathbf{u}_k^{d_k}$ models many types of sources, including geometric and kinematics errors, thermal errors, cutting force induced errors and tool-wear induced errors (Zhou *et al.* 2003). The elements in $\mathbf{u}_k^{d_k}$ are called process variables and are treated as independent system input following multivariate normal distribution.

- The un-modeled system noises due to the model linearization are represented by $\mathbf{w}_k$. Compared to the deviations modeled in $\mathbf{u}_k^{d_k}$ and $\mathbf{x}_k^{d_k}$, the elements in w_k are higher order small values. $\mathbf{w}_k$ are assumed to be independent of any component of $\mathbf{u}_k^{d_k}$ (k = 1, 2, …, N; d_k = 1, 2, …, D_k). And, the elements of $\mathbf{w}_k$ are assumed to be independent of each other and have zero mean.

- Since the features are measured in the coordinate system defined by the selected datum scheme d_k, the measurements of quality are denoted as $\mathbf{y}_k^{d_k}$. In this chapter, the measurements are assumed to be multivariate normal.

- The measurement noise is denoted by a random vector $\mathbf{v}_k$, which is independent of $\mathbf{x}_k^{d_k}$, $\mathbf{u}_k^{d_k}$ and $\mathbf{w}_k$ (k = 1, 2, …, N; d_k = 1, 2, …, D_k). The components of $\mathbf{v}_k$ are assumed to be independent of each other and have zero mean. And the magnitudes of $\mathbf{v}_k$ components are determined by the accuracy/precision of the measurement device, which are usually on the level of 1μm.

Adopting the assumptions of rigid part and small error, a linear state space model can be constructed to associate the product quality with a sequence of setups according to the setup plan, as shown in Eq. (3.1),

$$\begin{aligned} \mathbf{x}_k^{d_k} &= \mathbf{A}_{k-1}^{d_k}\mathbf{x}_{k-1}^{d_{k-1}} + \mathbf{B}_k^{d_k}\mathbf{u}_k^{d_k} + \mathbf{w}_k \\ \mathbf{y}_k^{d_k} &= \mathbf{C}_k^{d_k}\mathbf{x}_k^{d_k} + \mathbf{v}_k, \quad k = 1,2,...,N, \end{aligned} \tag{3.1}$$

where $\mathbf{A}_{k-1}^{d_k}\mathbf{x}_{k-1}^{d_{k-1}}$ represents the datum induced random deviation corresponding to the selected datum scheme d_k in stage k, and $\mathbf{x}_{k-1}^{d_{k-1}}$ is the quality, in terms of dimensional deviation, transmitted from upstream stages. $\mathbf{B}_k^{d_k}\mathbf{u}_k^{d_k}$ describes the impact of deviation from the process variables, corresponding to the selected datum scheme d_k, in the quality of features generated in stage k. $\mathbf{C}_k^{d_k}$ is the observation matrix mapping features' quality to the measurements. A validation of this SoV modeling in (Zhou *et al.* 2003) demonstrates that the SoV model can adequately represents the process errors and their propagations in the multistage machining process. Ren *et al.* (2006) further demonstrated that the model linearization is valid when number of stage is moderate.

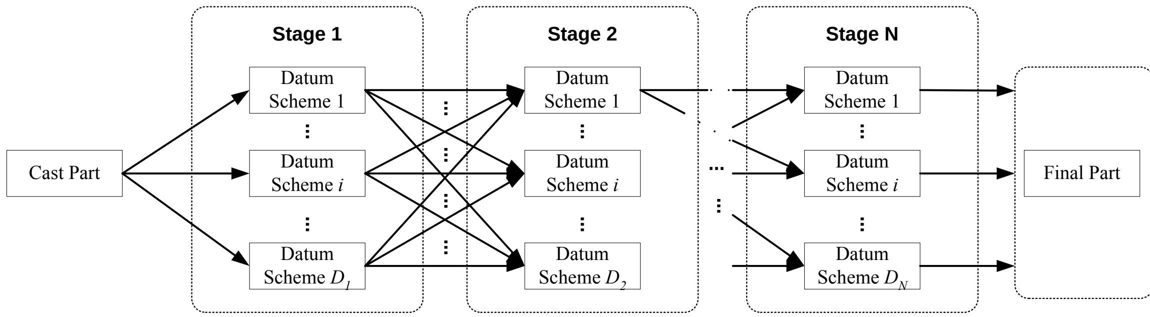

Figure 3-4. Datum scheme alternatives for sequential decision making

As aforementioned, setup planning is a series of decisions based on alternative datum schemes for multiple stages, as illustrated in Figure 3-4. For the optimal datum scheme selected for stage *k*, Eq. (3.1) can be reformulated as:

$$\begin{aligned} \mathbf{x}_k^{d_k^*} &= \mathbf{A}_{k-1}^{d_k^*}\mathbf{x}_{k-1}^{d_{k-1}^*} + \mathbf{B}_k^{d_k^*}\mathbf{u}_k^{d_k^*} + \mathbf{w}_k, \\ \mathbf{y}_k^{d_k^*} &= \mathbf{C}_k^{d_k^*}\mathbf{x}_k^{d_k^*} + \mathbf{v}_k, \end{aligned} \qquad (3.2)$$

where $d_k^* \in \{d_k \mid d_k = 1,2,...,D_k\}$, for $k = 1, 2, \ldots, N$, represents the index of the selected optimal datum scheme in stage *k*. Please note that d_k^* is one link of the optimal datum scheme chain $\left(d_1^* \quad d_2^* \quad \ldots \quad d_N^*\right)$ that is determined through considering all the stages in the entire processes. Thus, d_k^* may not necessarily be optimal for a single stage *k*.

The state space model in Eq. (3.2) can be transformed into a linear input-output model as:

$$\mathbf{y}_k^{d_k} = \sum_{i=1}^{k} \mathbf{C}_k^{d_k}\mathbf{\Phi}_{k,i}^{(\bullet)}\mathbf{B}_i^{d_i}\mathbf{u}_i^{d_i} + \mathbf{C}_k^{d_k}\mathbf{\Phi}_{k,0}^{(\bullet)}\mathbf{x}_0 + \sum_{i=1}^{k} \mathbf{C}_k^{d_k}\mathbf{\Phi}_{k,i}^{(\bullet)}\mathbf{w}_i + \mathbf{v}_k, \qquad (3.3)$$

where $\mathbf{\Phi}_{k,i}^{(\bullet)}$ is the state transition matrix tracing the datum schemes transformation from stage *i* to *k*-1; and $\mathbf{\Phi}_{k,i}^{(\bullet)} = \mathbf{A}_{k-1}^{d_{k-1}}\mathbf{A}_{k-2}^{d_{k-2}}\cdots\mathbf{A}_i^{d_i}$ for $i < k$, and $\mathbf{\Phi}_{k,k}^{(\bullet)} = \mathbf{I}$. Initial state vector $\mathbf{x}_0$ represents the original quality of the part that enters the first stage of the process. Without loss of generality, $\mathbf{x}_0$ is set to 0. Then Eq. (3.3) changes to

$$\mathbf{y}_k^{d_k} = \sum_{i=1}^{k} \mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i} \mathbf{u}_i^{d_i} + \sum_{i=1}^{k} \mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{w}_i + \mathbf{v}_k, \tag{3.4}$$

For a selected datum scheme, d_k, and the decisions on datum schemes for upstream stages $\{d_1 \quad d_2 \quad \ldots \quad d_{k-1}\}$, the coefficient matrices, $\mathbf{A}_k^{d_k}$, $\mathbf{B}_k^{d_k}$, $\mathbf{C}_k^{d_k}$ and $\mathbf{\Phi}_{k,i}^{(\bullet)}$ ($i = 1,2,\ldots,k$), can be derived following the same procedure as presented in (Zhou *et al.* 2003). This variation propagation modeling technique provides the setup planer a tool to predict the product quality of candidate datum schemes and alternative setup sequences of an MMP. Compared to the method, proposed by Xu and Huang (2006), that can only assess the quality after the whole setup plan is defined, state space modeling provides the capability to assess product quality for each intermediate setup. This modeling technique can be effectively incorporated into the decision making process for the optimal setup plan determination.

3.2.2 Setup Plan Evaluation Strategy

Different setup plan will result in different product quality in terms of KPC variation and incur different CRPP. From the optimization point of view, setup planning can be formulated as a discrete constrained optimization problem.

3.2.2.1 Optimization of the Setup Planning

In this chapter, the objective of setup planning is to minimize the CRPP while satisfying the KPC quality constraints. The mathematical representation is defined as:

$$\begin{aligned} &\min_{\mathbf{T_u}} \quad \{C_{\mathbf{Tu}}(\mathbf{T_u})\} \\ &\text{s.t.} \quad \frac{USL_i - LSL_i}{\sigma_{y_i}} \geq \tau_i \;, \quad i = 1, 2, \ldots, M. \end{aligned} \tag{3.5}$$

where $\mathbf{T_u} = [T_{u_1} \quad T_{u_2} \quad \ldots \quad T_{u_P}]^{\mathrm{T}}$ is a P×1 vector with each element T_{u_j} represents the tolerance of a corresponding process variable u_i defined in u, and $\mathbf{u} = [\mathbf{u}_1^T \quad \mathbf{u}_2^T \quad \ldots \quad \mathbf{u}_N^T]^T$, with $\mathbf{u}_k$ ($k = 1,2,\ldots,N$) as a $p_k \times 1$ vector representing the process variables (i.e., fixture locator deviations) in stage k. Please note that

$P=\sum_{k=1}^{N} p_k$. M is the total number of KPC and P is the total number of process variables. USL_i and LSL_i are the predefined Upper Specification Limit and Lower Specification Limit of KPC y_i, respectively. σ_{y_i} is the standard deviation of KPC y_i and τ_i is a constant, $i=1,2,...,M$. $C_{\mathbf{Tu}}(\mathbf{T_u})$ is the CRPP function of process tolerance. Various cost functions have been proposed for different tolerance synthesis. Considering the structural simplicity, a reciprocal function is adopted in this chapter:

$$C_{\mathbf{Tu}}=\sum_{j=1}^{P}\frac{w_j}{T_{u_j}}, \tag{3.6}$$

where w_j 's, $j=1,2,...,P$, are weighting coefficients. These weighting coefficients should be determined according to practical situation. For instance, coefficients assigned to the fixtures used in the same stage can be equal to each other; fixtures or machine tools manufactured by the same supplier or used in the same stage may be assigned with the same value. More discussions on the selection of those weighting coefficients are provided in the case studies in Section 3.3.

For a complicated MMP, there always exist multiple quality characteristics. It is desirable to define a multivariate process capability index for process quality control. However, at the setup planning stage, there is no *a priori* information of the correlations between quality characteristics. A scalar multivariate process capability index may be misleading if it is defined without appropriate consideration of correlations between quality characteristics. Thus, in industrial applications, for the sake of convenience, most of the tolerance regions are specified as a collection of individual specifications for each variable, as defined in Eq. (3.5). The intersection of these specifications would form a rectangular solid zone (Jackson 1991). Chen (1994) proposed a multivariate process capability index over a rectangular solid tolerance zone $V=\{\mathbf{y}\in R^M$: $\max(|y_i-\mu_i|/r_i, i=1,2,\ldots,M)\leq 1\}$. Based on this definition, a necessary condition for a process to be capable over a rectangular solid zone is that each individual univariate process is capable with respect to the corresponding specification limits. In addition, according to the discussion of Chen (1994), correlations between quality characteristics make the process more capable over a rectangular tolerance zone. Therefore, in this chapter, individual process capability constraints are adopted to conservatively ensure that the setup plan is capable to satisfy to the specifications on all quality characteristics.

Ding *et al*. (2005) studied the relationship between tolerance and variation of process variables through examining the clearance of the pin-hole locating pair. In this chapter, the process capability ratio, $\eta_j = T_{u_j} / \sigma_{u_j}$, are assumed to be constants. Therefore, the tolerance of a process variable can be replaced by its standard deviation. Recall that the elements in $\mathbf{u}_k^{d_k}$ are defined as the deviations of fixture locators with zero mean, thus their variances $\sigma_{u_j}^2 = E(u_j^2)$, $j = 1,2,...,P$. Let $\mathbf{\Xi_u} = [\sigma_{u_1} \quad \sigma_{u_2} \quad \cdots \quad \sigma_{u_P}]^T$, the tolerance of process variables can be defined by $\mathbf{T_u} = \begin{bmatrix} T_{u_1} & T_{u_2} & \ldots & T_{u_P} \end{bmatrix}^T = diag\{\eta_1, \quad \eta_2 \,, \quad \ldots, \quad \eta_P\} \cdot \mathbf{\Xi_u}$. Then the objective function, $C_{\mathbf{Tu}}(\mathbf{T_u})$, in Eq. (3.5) can be transformed to:

$$C_{\mathbf{u}}(\mathbf{u}) = \sum_{j=1}^{P} \frac{w_j}{\eta_j \cdot \sigma_{u_j}} \quad . \tag{3.7}$$

3.2.2.2 Dynamic Programming formulation

Previous sections present the techniques that enable: (i) the description of the impacts of datum scheme selection and setup sequencing on the variation of product quality, (ii) the modeling of the variation propagation, and (iii) the quantitative evaluation of the candidate setup plans. Based on these enablers, setup planning can be formulated as a sequential decision making on the selections of datum schemes in all stages to satisfy quality specifications with overall cost-effectiveness. In this sequential decision making problem, the datum scheme selected for stage *k* is affected by that selected for the upstream stages and will affect that selected for the downstream stages. This characteristic is identical to that of dynamic programming (DP) problem. Therefore, DP methodology is adopted to solve the optimization problem.

Figure 3-5 illustrates a sequential decision process for a chain of datum scheme selection. There are N+1 columns in the diagram, representing the N stages of the machining processes, and an initial DP state $(\bullet, \mathbf{x}_0)$. Each column k (k = 1, 2,…, N) consists of D_k nodes, corresponding to D_k feasible datum schemes. A node $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$, d_k =1, 2,…, D_k, in Figure 3-5 is a DP state that represents the datum scheme selection in stage k, where $\mathbf{Q}_k$ defines the in-process quality specifications for the features generated from stage 1 to stage k. Since the quality specifications for the incoming part is not related to the quality consideration of the machining process, it is set to "$\bullet$" in the initial DP state, i.e., not specified. According to Eq. (3.5), $\mathbf{Q}_k$ is an $M \times M$ matrix with the diagonal elements $q_{k,i,i} = \left[\left(USL_{k,i} - LSL_{k,i} \right) / \tau_i \right]^2$, $i = 1,2,...,M$; $k = 1,2,...,N$. $USL_{k,i}$ and $LSL_{k,i}$ are the given in-process specification limits for KPC i in stage k. The off-diagonal elements of $\mathbf{Q}_k$ can also be specified regarding to the covariance matrix structure of $\mathbf{y}_k^{d_k}$ for a given d_k. The connections linking nodes in column k-1 to those in column k reflect state transitions.

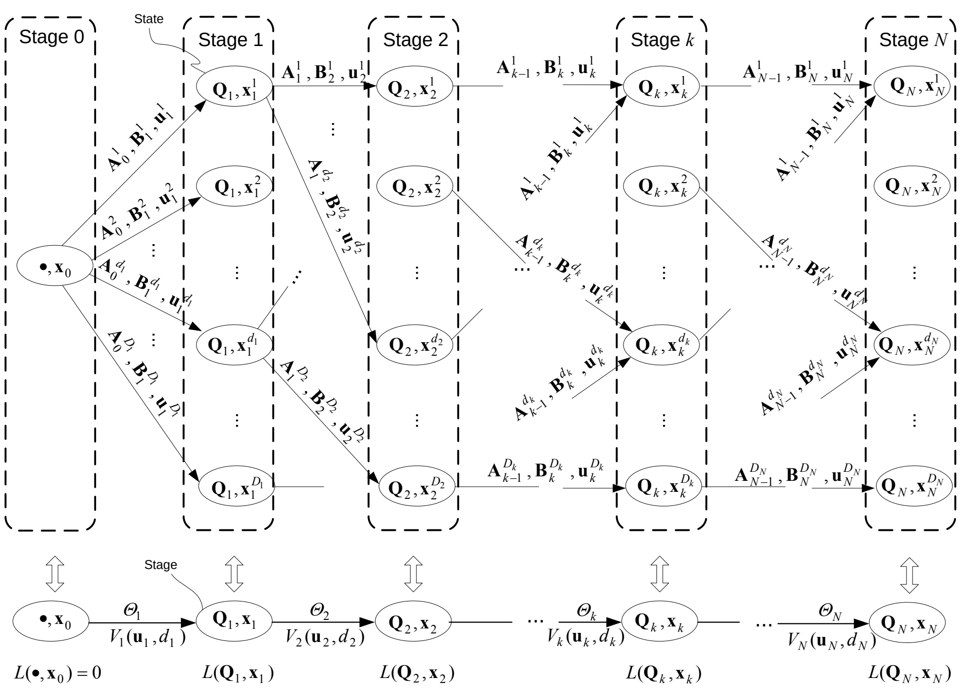

Figure 3-5. DP network of setup planning decision sequence

Given the datum scheme and setup sequence selected for upstream stages, different nodes from two neighbor stages are connected or disconnected, according to the pre-defined datum scheme constraints. Although there are D_k potential DP states for each stage, the process planner observes only the one that is finally selected. Therefore, the concept of "DP-stage" $(\mathbf{Q}_k, \mathbf{x}_k)$ is defined to "contain" all the possible states, $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$, d_k =1, 2,…, D_k, in a column k (Denardo 2003). As shown in the bottom portion of Figure 3-5, the $\mathbf{u}_k$ "contains" all the possible $\mathbf{u}_k^{d_k}$'s, d_k =1, 2,…, D_k. Associated with each DP-stage is a set of decisions Θ_k on datum scheme selection.

Selecting datum scheme, d_k, incurs cost $V_k(\mathbf{u}_k, d_k)$ and implements transition from DP-stage $(\mathbf{Q}_{k-1}, \mathbf{x}_{k-1})$ to DP-stage $(\mathbf{Q}_k, \mathbf{x}_k)$. Let $\mathbf{q}_k(\mathbf{u}_k, d_k)$ be the constraints on the KPC variations generated in stage k if datum scheme d_k is selected. In other words, $\mathbf{q}_k(\mathbf{u}_k, d_k)$ is the maximum KPC variations that can be allowed after the fabrication performed in stages 1 through k. Also let $t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})$ be the state transition function linking $\mathbf{x}_{k-1}^{d_{k-1}}$ and $\mathbf{x}_k^{d_k}$, then Eq. (3.1) can be of the form $\mathbf{x}_k^{d_k} = t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1}) = \mathbf{A}_{k-1}^{d_k}\mathbf{x}_{k-1}^{d_{k-1}} + \mathbf{B}_k^{d_k}\mathbf{u}_k^{d_k} + \mathbf{w}_k$. The decision-making on d_k's, (k=1,2,…, N) repeats itself for all stages, following $t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})$. The cost of decision d_k in stage k is defined as

$$V_k(\mathbf{u}_k, d_k) = C_{\mathbf{u}_k^{d_k}}(\mathbf{u}_k^{d_k}) = \sum_{j=1}^{p_k} \frac{w_j}{\eta_j \cdot \sigma_{u_{k,j}^{d_k}}} \quad , \tag{3.8}$$

where p_k ($k = 1,2,...N$) is the dimension of $\mathbf{u}_k^{d_k}$ and $P = \sum_{k=1}^{N} p_k$. $\sigma_{u_{k,j}^{d_k}}$ is the standard deviation of the j^{th} element of $\mathbf{u}_k^{d_k}$, j = 1,2,..., p_k. This cost can be interpreted as the cost consumed to provide enough process precision for stage k, corresponding to the selected datum scheme d_k. Let $L(\mathbf{Q}_k, \mathbf{x}_k)$ be the minimum CRPP that is consumed from stage 1 to stage k by selecting datum schemes d_1, d_2,…, d_k, and generating quality variation at most $\mathbf{Q}_k$, the DP function can be defined as:

$$L(\mathbf{Q}_k, \mathbf{x}_k) = \begin{cases} \min\limits_{\substack{d_k \in \Theta_k, d_{k-1} \in \Theta_{k-1} \\ q_k(\mathbf{u}_k, d_k) \leq \mathbf{Q}_k}} \{L(\mathbf{Q}_k - \mathbf{q}_k(\mathbf{u}_k, d_k), t((\mathbf{Q}_k, \mathbf{x}_k), d_k, d_{k-1})) \\ \qquad + V_k(\mathbf{u}_k, d_k)\}, \qquad \mathit{for}\ k = 1, \ldots, N, \\ 0, \qquad \mathit{for}\ \ k = 0, \end{cases} \tag{3.9}$$

where $\mathbf{Q}_k$ is pre-defined, $\mathbf{q}_k(\mathbf{u}_k, d_k)$ and $V_k(\mathbf{u}_k, d_k)$ can be derived based on the state space model (1). According to Eq. (3.4), the covariance matrix of $\mathbf{y}_k^{d_k}$ is:

$$\begin{aligned} \mathbf{\Sigma}_{\mathbf{y}_k^{d_k}} &= \sum_{i=1}^{k-1} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}\right) \mathbf{\Sigma}_{\mathbf{u}_i^{d_i}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}\right)^{\mathrm{T}} + \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}\right) \mathbf{\Sigma}_{\mathbf{u}_k^{d_k}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}\right)^{\mathrm{T}} \\ &\quad + \sum_{i=1}^{k} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)}\right) \mathbf{\Sigma}_{\mathbf{w}_i} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)}\right)^T + \mathbf{\Sigma}_{\mathbf{v}_k}, \end{aligned} \tag{3.10}$$

where $\mathbf{\Sigma}_{\bullet}$ is the covariance matrix for variable "$\bullet$". Eq. (3.10) shows that the KPC covariance can be treated as the accumulated covariance of all process variables used from stage 1 to stage *k*, plus the covariance of the un-modeled process variations and the variance of measurement noise. In order to ensure that the product quality generated from stage 1 to stage *k* satisfies the specifications, $\mathbf{\Sigma}_{\mathbf{y}_k^{d_k}}$ should satisfy the specification

$$\sigma^2_{\mathbf{y}_{k,i}^{d_k}} \leq s \cdot q_{k,i,i}, \ i = 1, 2, \ldots, M, \tag{3.11}$$

where $\sigma^2_{\mathbf{y}_{k,i}^{d_k}}$ is the *i*th diagonal element of matrix $\mathbf{\Sigma}_{\mathbf{y}_k^{d_k}}$, $q_{k,i,i}$ is the *i*th diagonal element of matrix $\mathbf{Q}_k$ and the scalar *s* is a safety-factor ($0 \leq s \leq 1$). Since $\mathbf{w}_k$ and $\mathbf{v}_k$ contains second or higher order of small values whose magnitudes are much smaller than that of $\mathbf{x}_k^{d_k}$ and $\mathbf{u}_k^{d_k}$, their contributions to the $\mathbf{\Sigma}_{\mathbf{y}_k^{d_k}}$ can be ignored. Thus, by eliminating the third and the fourth terms of the right-hand-side of Eq. (3.10), $\mathbf{\Sigma}_{\mathbf{y}_k^{d_k}}$ can be approximated by

$$\begin{aligned} \tilde{\mathbf{\Sigma}}_{\mathbf{y}_k^{d_k}} &= \sum_{i=1}^{k-1} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}\right) \mathbf{\Sigma}_{\mathbf{u}_i^{d_i}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i}\right)^{\mathrm{T}} \\ &\quad + \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}\right) \mathbf{\Sigma}_{\mathbf{u}_k^{d_k}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k}\right)^{\mathrm{T}}. \end{aligned} \tag{3.12}$$

The first term on the right-hand-side of Eq. (3.12) stands for the quality covariance (measured based on datum scheme d_k) accumulated from stage 1 to stage k, whereas the second term stands for the quality covariance generated in stage k by selecting datum scheme d_k. Let

$$\tilde{\mathbf{\Sigma}}_{\mathbf{y}_{k-1}^{\bullet}} = \sum_{i=1}^{k-1} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i} \right) \mathbf{\Sigma}_{\mathbf{u}_i^{d_i}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,i}^{(\bullet)} \mathbf{B}_i^{d_i} \right)^{\mathrm{T}}, \tag{3.13}$$

be the quality covariance accumulated from stage 1 to stage k-1, the amount of newly generated quality covariance can be derived as:

$$\mathbf{\Sigma}_{d_k} = \tilde{\mathbf{\Sigma}}_{\mathbf{y}_k^{d_k}} - \tilde{\mathbf{\Sigma}}_{\mathbf{y}_{k-1}^{\bullet}} = \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} \right) \mathbf{\Sigma}_{\mathbf{u}_k^{d_k}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} \right)^{\mathrm{T}} . \tag{3.14}$$

Since the process cost modeled in Eq. (3.7) is inversely proportional to the process variations. In order to minimize the process cost, process variations, the diagonal elements in $\mathbf{\Sigma}_{\mathbf{u}_k^{d_k}}$, k=1,2,…N, should be relaxed as much as possible. This will lead to the increase of the KPC variations defined by the diagonal elements in $\tilde{\mathbf{\Sigma}}_{\mathbf{y}_k^{d_k}}$. Considering the quality constraints specified by Q$_k$, $\tilde{\mathbf{\Sigma}}_{\mathbf{y}_k^{d_k}}$ should satisfy

$$\tilde{\mathbf{\Sigma}}_{\mathbf{y}_k^{d_k}} = s\mathbf{Q}_k, \tag{3.15}$$

where s is the same as that defined in Eq. (3.11). Given the $\mathbf{Q}_k$'s, $k = 1,2,...,N$, the constraints $\mathbf{q}_k(\mathbf{u}_k, d_k)$ has the form

$$\begin{aligned} \mathbf{q}_k(\mathbf{u}_k, d_k) &= \left(s\mathbf{Q}_k - \tilde{\mathbf{\Sigma}}_{\mathbf{y}_{k-1}^{\bullet}} \right) \\ &= \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} \right) \mathbf{\Sigma}_{\mathbf{u}_k^{d_k}} \left(\mathbf{C}_k^{d_k} \mathbf{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} \right)^{\mathrm{T}} . \end{aligned} \tag{3.16}$$

From Eq. (3.16), the covariance matrix of $\mathbf{u}_k^{d_k}$ can be derived as

$$\mathbf{\Sigma}_{\mathbf{u}_k^{d_k}} = \left(\mathbf{\Gamma}_k^{(\bullet)} \right)^{-} \left(s\mathbf{Q}_k - \tilde{\mathbf{\Sigma}}_{\mathbf{y}_{k-1}^{\bullet}} \right) \left[\left(\mathbf{\Gamma}_k^{(\bullet)} \right)^{\mathrm{T}} \right]^{-}, \tag{3.17}$$

where $\boldsymbol{\Gamma}_k^{(\bullet)} = \mathbf{C}_k^{d_k} \boldsymbol{\Phi}_{k,k}^{(\bullet)} \mathbf{B}_k^{d_k} = \mathbf{C}_k^{d_k} \mathbf{B}_k^{d_k}$, and $(\boldsymbol{\Delta})^{-}$ denotes the Moore-Penrose inverse of the rectangular matrix $\boldsymbol{\Delta}$. $\widetilde{\boldsymbol{\Sigma}}_{\mathbf{y}_{k-1}^{\bullet}}$ contains the variation propagation information and is determined by the datum scheme selection and sequencing decisions made for upstream stages. When $\boldsymbol{\Gamma}_k^{(\bullet)}$ is column-wise full rank, Eq. (3.17) can give a real solution of $\boldsymbol{\Sigma}_{\mathbf{u}_k^{d_k}}$. Assuming that the process variables are mutually independent, the tolerance specification for $\mathbf{u}_k^{d_k}$ can be obtained by a $p_k \times 1$ vector

$$\mathbf{T}_{\mathbf{u}_k^{d_k}} = [\eta_k \sigma_1^{d_k} \quad \eta_k \sigma_2^{d_k} \quad \ldots \quad \eta_k \sigma_{p_k}^{d_k}]^{\mathrm{T}} , \tag{3.18}$$

where $\left(\sigma_j^{d_k}\right)^2$ is the j^{th} diagonal elements of $\boldsymbol{\Sigma}_{\mathbf{u}_k^{d_k}}$, j=1,2,…,p_k, k=1,2,…,N and d_k= 1,2,…D_k. According to the definition of $\mathbf{u}_k^{d_k}$, $\mathbf{T}_{\mathbf{u}_k^{d_k}}$ contains the tolerance of machining/cutting tools and fixture locators. In order to increase the exchangeability of fixture locators, improve maintainability of the fixture system, and reduce the "Long-Run Overall Production Cost," different locators on the same fixture are assigned with the same tolerance, as discussed by Chen *et al.* (2006). Therefore, fixture locators' tolerances can also be specified as $\eta_k \sigma_*^{d_k}$, where $\sigma_*^{d_k} = \min_{j \in J_f} \{\sigma_j^{d_k}\}$ and J_f is a set containing all the index of fixture locators in $\mathbf{u}_k^{d_k}$. With Equations from Eq. (3.8) to Eq. (3.18), setup planning can be formulated as solving a series of DP functional equations

3.2.2.3 Optimization algorithm

Reaching algorithm (Denardo 2003) is used to solve the dynamic programming problem defined in Eq. (3.9). According to Figure 3-5, the value of each DP-state node $(\mathbf{Q}_k, \mathbf{x}_k^{d_k})$ is denoted as s_{k,d_k}, which represents the minimum process precision cost incurred so far from stage 1 to stage k by selecting datum scheme d_k in stage k. Let $v_{k,d_k}^{d_{k-1}}$ denote the corresponding cost incurred in stage k corresponding to datum selections of the upstream stage k-1 and that of the stage k, and $v_{k,d_k}^{d_{k-1}} = V_k(\mathbf{u}_k, d_k)$. The pseudo code of the reaching algorithm is defined as:

(i) Set $s_{0,\bullet} = 0$ and $s_{k,d_k} = +\infty$ for $k = 1, 2, \ldots, N$; $d_k = 1, 2, \ldots, D_k$,

(ii) DO for k = 1, 2,…, N

(iii) DO for dk = 1, 2, …, Dk

$$s_{k,d_k} \leftarrow \min\left\{ s_{k,d_k}, \inf_{d_{k-1}\in\Theta_{k-1}} \{ s_{k-1,d_{k-1}} + v_{k,d_k}^{d_{k-1}} \} \right\}.$$

In this algorithm, $v_{k,d_k}^{d_{k-1}}$ will be set to ∞ for an infeasible datum scheme selection. This value indicates that, given the variation accumulated in upstream stages, the selected datum scheme at current stage cannot meet the quality specification. The final results include (i) the minimized total CRPP, $L(\mathbf{Q}_N, \mathbf{x}_N)$; (ii) a sequence of decisions $\left(d_1^* \quad d_2^* \quad \ldots \quad d_N^*\right)$ on datum schemes for a sequence of stages, which is the optimal setup plan; and (iii) the tolerance specifications, $\mathbf{T_u}$, of the fixtures used in all stages.

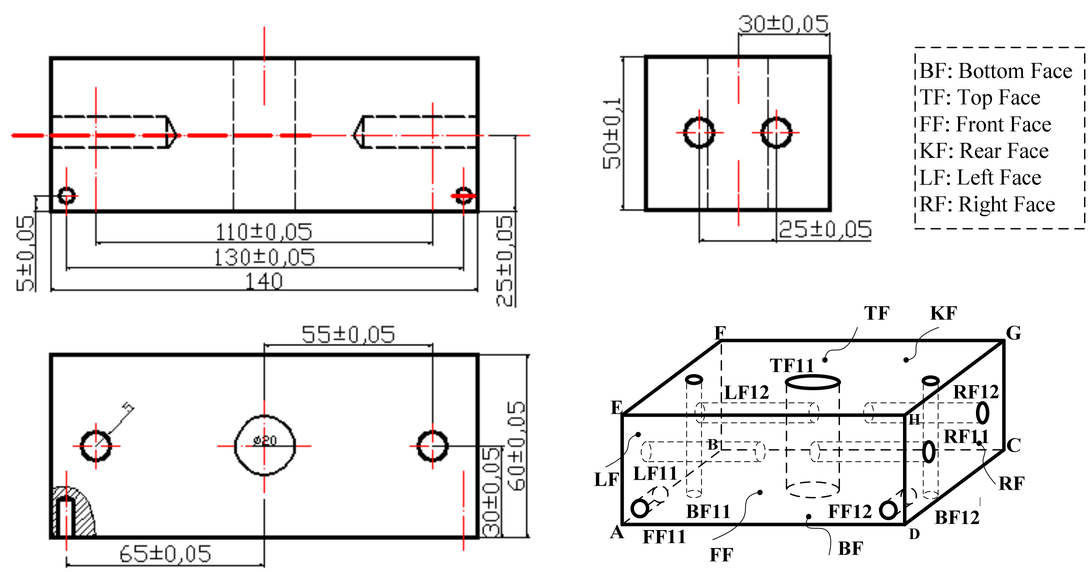

Figure 3-6. Part drawing and KPC specifications

3.3 Case Study

A case study is conducted to demonstrate the SoV-model based, quality assured optimal setup planning for an MMP. The product KPC and their associated design specifications are defined in Figure 3-6. Based on the analysis of features locations and tooling approaching directions, a 3-stage machining process is proposed. The candidate

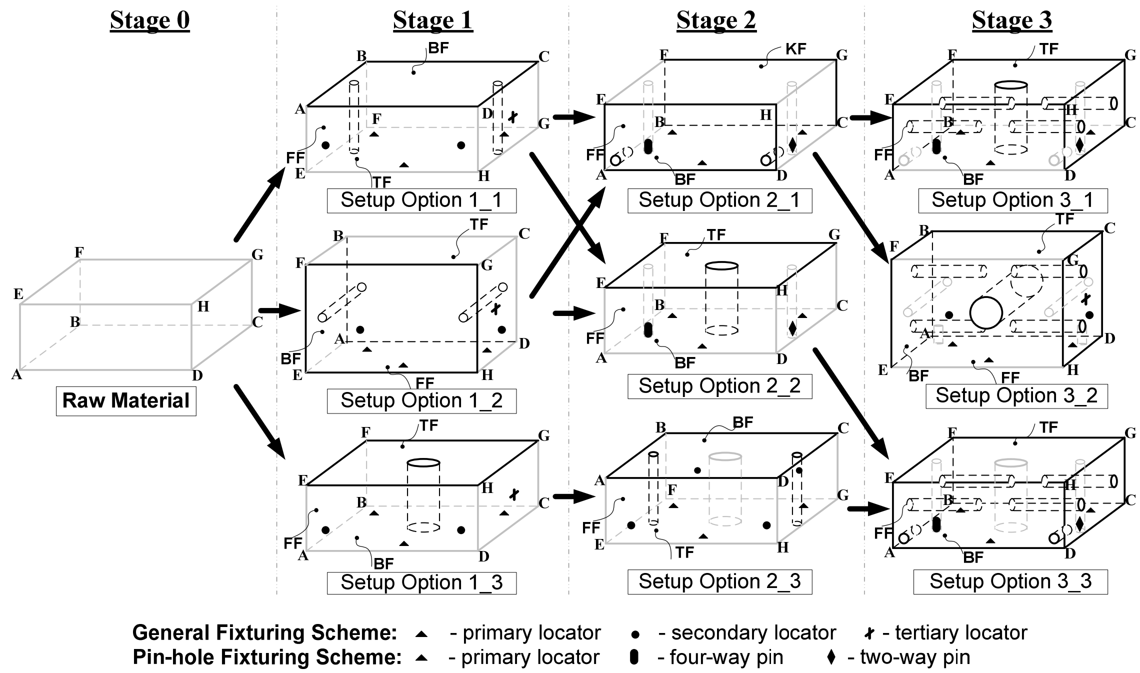

Figure 3-7. Setup options for a 3-stagae machining process

datum schemes for each stage are proposed and shown in Figure 3-7. Correspondingly, stage/setup level fixture layouts are assumed as given. These include general 3-2-1 fixturing schemes (e.g., Setup Option 1_1) and pin-hole fixturing schemes (e.g., Setup Option 2_1), as discussed by Zhou *et al.*(2003).

Table 3-2. Setup options for the 3-stage machining process

Index	**Stage**					
	1		2		3	
1	DS: SF:	TF-FF-RF BF, BF11, BF12	DS: SF:	BF-BF11-BF12 FF, FF11, FF12, KF	DS: SF:	BF-BF11-BF12 TF, TF11, LF, LF11, LF12, RF, RF11, RF12
2	DS: SF:	FF-TF-RF BF, BF11, BF12	DS: SF:	BF-BF11-BF12 TF, TF11	DS: SF:	FF-FF11-FF12 FF, FF11, FF12, KF, LF, LF11, LF12, RF, RF11, F12
3	DS: SF:	BF-FF-LF TF, TF11	DS: SF:	TF-FF-RF BF, BF11, BF12	DS: SF:	BF-BF11-BF12 TF, TF11, LF, LF11, LF12, RF, RF11, RF12, KF

Table 3-2 summarizes the alternative datum schemes (DS) and setup formations (SF) for each stage. Corresponding to these datum scheme candidates d_k's (k=1, 2, 3), the coefficient matrices in state space models, $\mathbf{A}_k^{d_k}$, $\mathbf{B}_k^{d_k}$ and $\mathbf{C}_k^{d_k}$, are generated. According to the constraints on datum scheme and datum sequence, the DP network is established, as shown in Figure 3-8.

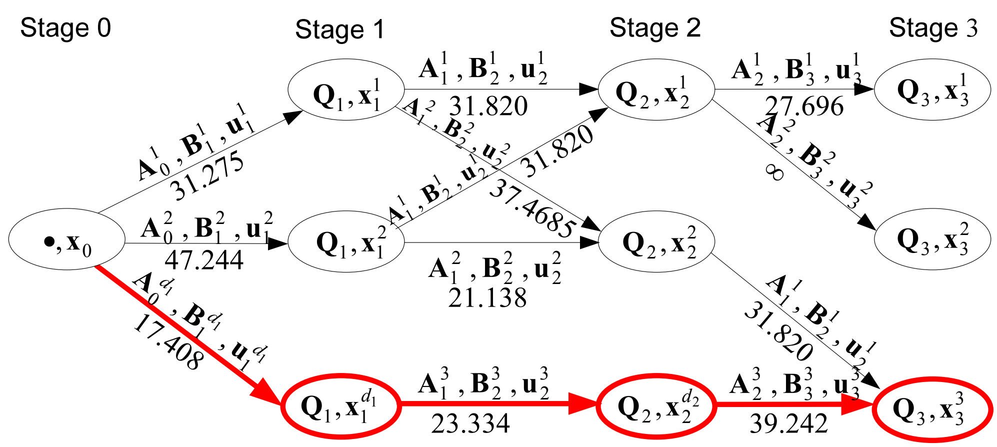

Figure 3-8. DP network for the 3-stagae machining process

The intermediate results are summarized in Table 3. The optimal setup plan is identified and highlighted, in Figure 3-8, as the bold path. The optimal setup plan is: (i) in the 1st stage, the part is fixed with datum features BF, FF and LF, and features TF, TF11 are generated; (ii) in the 2nd stage, the part is fixed with datum features TF, FF and TF, and features BF, BF11 and BF12 are generated; (iii) The remaining features will be generated in the 3rd stage with the part being fixed on datum features BF, BF11 and BF12. This optimal setup plan can be denoted as a DFC: {{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}}, with the total CRPP of 79.983.

Table 3-3. Intermediate results of reaching algorithm

s_{k,d_k}		Stage 1	Stage 2	Stage 3
Setup option d_k	1	31.275	60.095	87.791
	2	47.244	68.744	98.085
	3	17.408	40.742	**79.983***

One of the by-products of the SoV-based setup planning methodology is the tolerance specifications for the fixture design. In this case study, based on the $\boldsymbol{\sigma}_{\mathbf{u}_i^*}$'s, $\mathbf{T}_{\mathbf{u}_i^*}$ (i=1, 2, 3) are given as $\mathbf{T}_{\mathbf{u}_1^*}$=[0.086 0.086 0.086 0.086 0.086 0.086]^T, $\mathbf{T}_{\mathbf{u}_2^*}$ = [0.037 0.037 0.037 0.037 0.037 0.037]^T, and $\mathbf{T}_{\mathbf{u}_3^*}$ = [0.019 0.019 0.019 0.019 0.019 0.019]^T. The fixture design that meets these specifications will be cost-effective and sufficiently precise to ensure the product quality. The results show that the fixtures for upstream stages, i.e., stage 1 and stage 2, are not required to be as precise as that for the downstream operations, i.e. the optimal setup plan is not conservative.

Sensitivity analysis was also conducted to examine the impact of the assignments of weighting coefficients' values on the optimization results. It is assumed that: (i) the weighting coefficients assigned to the locators belonging to the same fixture are the same; (ii) fixtures used at stage 1 will be assigned different weighting coefficients from that assigned to fixtures used at stage 2 and stage 3, and (iii) the weighting coefficients assigned to fixtures used in stage 2 and stage 3 are the same. For instance, if a sum of weighting coefficients, 0.1 (0.1/6 for each locator) is assigned to stage 1 fixture, that for fixtures in stage 2 and stage 3 will be 0.45 (0.45/6 for each locator).

Table 3-4. Impact of sum weighing coefficients on optimization results

Case Number	**Sum Weighting Coefficients for Stage 1**	**Optimal Setup Plan**
1	0.1	{{FF-TF-RF},{ BF-BF11-BF12},{BF-BF11-BF12}}
2	0.2	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
3	0.3	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
4	0.4	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
5	0.5	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
6	0.6	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
7	0.7	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
8	0.8	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}
9	0.9	{{BF-FF-LF},{TF-FF-TF},{BF-BF11-BF12}}

Table 3-4 shows the optimization results associated to different combinations of the coefficients assignments. The optimal setup plans are consistent, except for case 1, where fixture in stage 1 is significantly under-weighted with a weighting coefficient 0.1. This indicates that the optimization result for this case study is not sensitive to the value of weighting coefficient. This is because the datum scheme option 3 for stage 1 significantly out-performs the other two options in terms of CRPP. The differences among those three options dominate the whole optimization of the three stages, as shown in Figure 3-8.

3.4 Conclusion

This chapter proposed a methodology for optimal setup planning for MMP's. Based on the SoV concept, state space modeling technique is expanded to be applicable to datum selection and setup sequencing decisions. The SoV model provides the basis for quantitative, analytical evaluation of the quality impacts of candidate setup plans. This evaluation capability enables the formulation of the setup planning as an optimization problem that minimizes the CRPP with the final product quality as constraints. DP is employed to solve this sequential optimal decision making problem.

In the proposed method, setup planning is formulated as a Dynamic Programming problem, which provides a nice representation of the sequential decision making procedure. However, one disadvantage of DP is that it needs intensive computational resources. When the number of stages and the number of alternative datum schemes are getting large, the cost for obtaining an optimal solution will be unaffordable. Potential solutions include: (i) Using different formulation, such as reinforcement learning, neurodynamic programming or approximate dynamic programming; (ii) Incorporating engineering domain knowledge to decouple an MMP into smaller segments of sub-processes and/or add more constraints to reduce the number of alternative datum schemes. These topics will be investigated in our future work.

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CHAPTER 4

ENGINEERING-DRIVEN FACTOR ANALYSIS FOR VARIATION SOURCES IDENTIFICATION IN MULTISTAGE MANUFACTURING PROCESSES[3]

Abstract

Variation source identification is an important task of quality assurance in multistage manufacturing processes (MMP). However, existing approaches, including the quantitative engineering-model-based methods and the data-driven methods, provide limited capabilities in variation sources identification. This chapter proposes a new methodology that does not depend on accurate quantitative engineering models. Instead, engineering domain knowledge about the interactions between potential variation sources and product quality variables are represented as qualitative indicator vectors. These indicator vectors guide the rotation of the factor loading vectors that are derived from factor analysis of the multivariate measurement data. Based on this engineering-driven factor analysis, a procedure is presented to identify multiple variation sources that present in an MMP. The effectiveness of the proposed methodology is demonstrated in a case study of a three-stage assembly process.

[3] Liu, J., Shi, J., and Hu, S.J., 2007, accepted by ASME Transactions, Journal of Manufacturing Science and Engineering,

Nomenclature

y: a $p\times 1$ vector containing random deviations of p KPC's

$\mathbf{u}_s$: an $s\times 1$ vector containing random deviations of s process variation sources that present in the process

$\boldsymbol{\Gamma}_s$: a $p\times s$ matrix with each column a spatial pattern vector of a variation source

p: the number of KPC's

s: the number of variation sources that present in the process

L: a $p\times s$ initial loading matrix with each column vector, $\mathbf{l}_j$, an initial factor loading vector

$\mathbf{l}_j$: an $p\times 1$ initial factor loading vector, and $\mathbf{l}_j$ is orthogonal to $\mathbf{l}_t$ when $j\neq t$, $j,t=1,2,\ldots, s$

$\mathbf{L}^*$: a $p\times s$ rotated loading matrix with each column vector, $\mathbf{l}_j^*$, an rotated factor loading vector

$\mathbf{l}_j^*$: a $p\times 1$ rotated factor loading vector, $j=1,2,\ldots, s$

T: a $p\times M$ indicator matrix with each column vector, $\boldsymbol{\tau}_m$, an indicator vector derived from engineering knowledge representation

$\boldsymbol{\tau}_m$: a $p\times 1$ indicator vector, $m=1,2,\ldots, M$

M: the number of potential variation sources

$F_{k:i_k}$: the i_k^{th} feature in stage k, $i_k=1, 2, \ldots, n_k$

n_k : the number of features in stage k

$FX_{k:l_k}$:the l_k^{th} fixtures used in stage k, $l_k=1, 2,\ldots, t_k$

t_k: the number of fixtures used in stage k

4.1 Introduction

Variation source identification is an important task of quality assurance in multistage manufacturing processes (MMP's) (Ferrell and Elmaghraby 1990). The variation sources are special causes of variation in key product characteristics (KPC's). Since KPC's variation affects the final product quality as well as the manufacturing system productivity (Heikes and Montgomery 1981), it is essential to conduct variation reduction by detecting process variation changes and identifying the underlying variation sources. This is especially challenging for MMP's, where multiple stages are involved in generating designated KPCs or functionality of a product. As illustrated in Figure 4-1, in stage *k* of an MMP, special causes (Montgomery 2004) (e.g., excessive variation of the locations of locating pins) will increase the variation of some KPC's measurements to a level that exceeds their tolerances. Compounded with the input quality transmitted from preceding stages, these quality problems will be further propagated to the downstream stages and accumulated to the final product. In order to trace the variation propagation and monitor an MMP, it is ideal to take measurements in every stage. However, to reduce the inspection cost, it is customary to measure KPC's only after the final stage, e.g., stage *N*, as shown in Figure 4-1. Therefore, the complex variation propagation and the lack of in-process measurements make it extremely difficult to identify the variation sources in an MMP (Shi 2006).

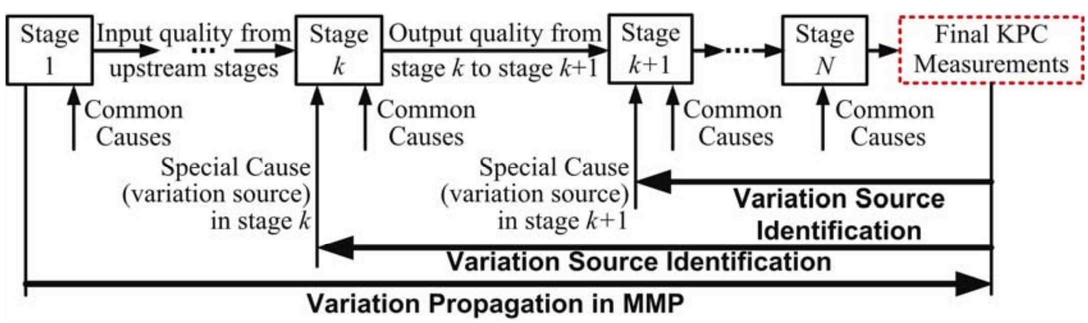


Figure 4-1. Complex variation propagation scenario in an MMP

Variation source identification can be accomplished by investigating multiple KPC's and their spatial patterns, since the spatial patterns reflect the characteristics of the variation sources. As shown in Figure 4-2 (a), part A and part B are fixed by fixture locating pins, P_1 to P_4, and are assembled in stage 1. P_1 and P_3 are 4-way pins that restrain the degree of freedoms of the parts along X and Z axes, whereas P_2 and P_4 are 2-way pins that restrain the parts from rotating around the axis perpendicular to the XZ plane. The subassembly is then assembled with Part C in stage 2. Five (5) features, two locating holes, F_1 and F_3, and three corner points, F_2, F_4 and F_5, are considered and their position along X and Z axes are the KPC's. The variations of the locating pins are the variation sources since they cause excessive variation in the KPC's. For instance, in stage 1, the variation of P_4 along z direction, defined as a variation source P_4_z, leads to orient-

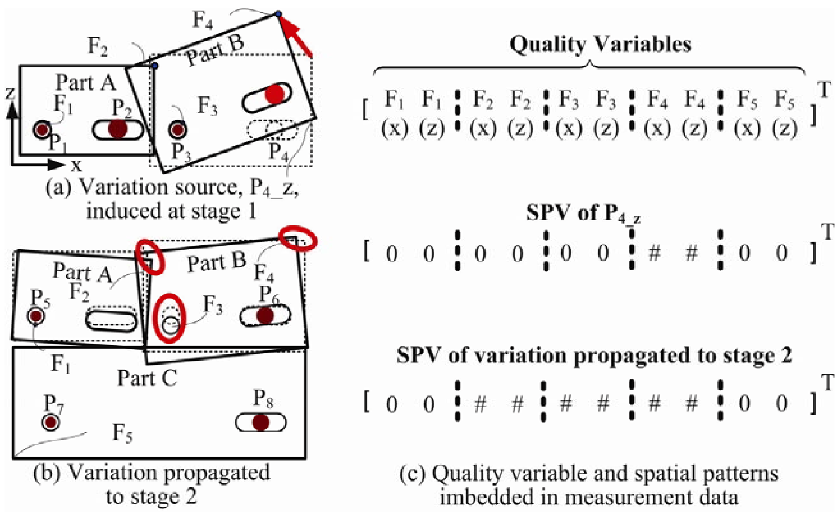


Figure 4-2. KPC's, variation source and their SPV's

ation variation of Part B and thus causes the position variation of F_4. KPC's are grouped together to form spatial pattern vectors (SPV's), which describe the effects of a variation sources. As shown in Figure 4-2 (c), P_4_z's SPV is a 10×1 vector corresponding to 10 KPC's. In stage 1, only F_4 are affected by P_4_z, thus F_4(x) and F_4(z) in the SPV will be non-zero values, denoted as "#". Because of the variation propagation caused by parts' reorientation, P_4_z's SPV in stage 2 is different, indicating that F_2, F_3 and F_4 are all affected by P_4_z introduced in preceding stage 1. Figure 4-2 shows that each individual potential variation source in a stage of an MMP will have its particular SPV. This one-to-one relationship makes it possible to identify the variation sources by investigating the SPV's, which are either derived from engineering domain knowledge about product/process design, or estimated from multivariate measurements of KPC's.

Table 4-1. Summary of reported variation source identification approaches

	Engineering-Driven	Data-Driven
Features	• SPV's are derived from accurate *a priori* engineering knowledge of product/process design. • SPV's are defined with exact values. • Variation sources are identified by engineering-model-based direct estimation.	• SPV's are estimated from multivariate quality measurement data. • Exact values of true SPV's are unknown. • Variation sources are identified by pattern interpretation according to engineering knowledge.
Limitations	• Comprehensive and accurate engineering knowledge is mandatory. • Diagnostic results are not robust to unknown tooling position due to adjustments or worn out.	• Engineering knowledge is not directly involved in the estimation of SPV's. • Achieved spatial patterns may not be the best estimates of true SPV's.

Progresses have been made in recent years in developing methodologies to implement this basic idea in MMP's (Shi 2006). Existing approaches can be divided into two categories: engineering-driven approaches and data-driven approaches, as summarized in Table 4-1.

Engineering-driven approaches intend to directly link the engineering knowledge of the process variation sources with the KPC measurements through mathematical modeling. Jin and Shi (1999) developed the state space models to represent the geometrical relationships between KPC and process key control characteristics (KCC's) according to product/process design information. Mantripragada and Whitney proposed a "datum flow chain (DFC)" concept for an assembly (1998) and explicitly defined it in a discrete state transition model to describe the variation propagation in assembly process (Mantripragada and Whitney 1999). State space modeling techniques are further investigated and applied in assembly processes (Camelio *et al.* 2003; Ding *et al.* 2002) and machining processes (Djurdjanovic and Ni 2001; Huang *et al.* 2000; Zhou *et al.* 2003). These modeling approaches lead to a generic linear model:

$$\mathbf{y} = \mathbf{\Gamma u} + \mathbf{v}, \tag{4.1}$$

where $\mathbf{y}$ ($\mathbf{y} \in \Re^{p\times 1}$) is a vector of dimensional deviations of p KPC's. Vector $\mathbf{u}$ ($\mathbf{u} \in \Re^{M\times 1}$) consists of the deviations of M potential process variation sources. $\mathbf{\Gamma}$ ($\mathbf{\Gamma} \in \Re^{p\times M}$) in engineering-driven approaches is a constant coefficient matrix derived from the product/process design and it describes the linear interactions between process (potential variation sources) and product (KPC's). The negligible un-modeled factors and measurement noise are represented by vector $\mathbf{v}$ ($\mathbf{v} \in \Re^{p\times 1}$). Based on the linear model (4.1), diagnostic approaches have been developed (Ding *et al.* 2002; Zhou *et al.* 2004). These engineering-model-based techniques fundamentally improve the capability of variation sources identification. However, significant effort is needed to derive and validate the exact values of the coefficients of the model, i.e., the values of the elements in matrix Γ. The dependence on comprehensive and accurate *a priori* engineering knowledge also impacts the robustness of the diagnosis approach. When unknown tooling adjustments are performed, the engineering knowledge will be either incomprehensive or inaccurate. Thus the model will no longer reflect the true linear interactions between process and quality and may lead to unreliable or misleading diagnostic results.

Data-driven approaches avoid the dependence on accurate *a priori* engineering knowledge and complex model derivation. Wolbercht *et al.* (2000) developed a system to implement real time monitoring an diagnosis of MMP's using Bayesian networks. The method depends on in-process measurement data collected at several points throughout the process. When the KPC measurements are only available at the final stage, diagnosis is conducted by directly estimating the SPV's imbedded in the multivariate measurements of KPC's and matching them to the expected SPV's of potential variation sources. This is equivalent to estimating a matrix $\mathbf{\Gamma}_s$ whose s column vectors are the SPV's of the s variation sources that actually present in an MMP. Ceglarek and Shi (1996) employed a principal component analysis (PCA) technique to extract a single spatial pattern from KPC covariance matrix and compare them with predefined spatial patterns of potential variation sources. Liu *et al.* (1997) proposed a factor analysis (FA) based method to diagnose an MMP through investigating multiple orthogonal SPV's. The diagnosis capability of those approaches is confined by the assumptions of orthogonality. Apley and Shi (2001) developed an FA method to extract and interpret the SPV's of the variation sources. An assumption a ragged lower-triangular Γ matrix is mandatory. Independent component analysis techniques were also used by Apley and Lee (2003) to separate variation sources "blindly," with constraints on auto-correlation and distribution conditions. Jin and Zhou (2006) developed a method to identify variation sources through analyzing fault space, which is spanned by the eigenvectors of the covariance matrix of measurement data. However, when there is more than one variation sources present in a process, the estimated SPV's will be different from the true SPV's and thus insufficient to guide corrective actions. This is because that the existing data-driven methods do not directly use the engineering knowledge when analyzing the multivariate statistics of KPC measurements.

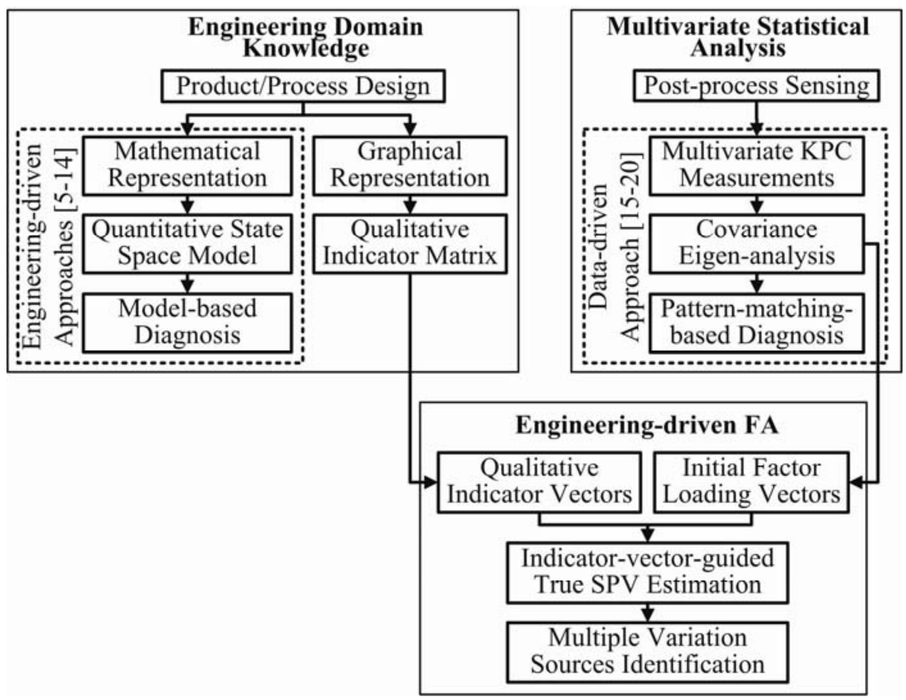


Figure 4-3. Overview of the proposed approach

In order to improve the interpretation of data-driven approaches and avoid dependence on accurate models, it is important to directly use the engineering knowledge to direct statistical data analysis. Liu and Hu (1997) and Camelio and Hu (2003) developed designated component analysis (DCA) approach based on mutually orthogonal designated variation patterns that are derived directly from knowledge of product/process design. This approach facilitates the diagnosis of multiple fixture faults that present in a single stage of an assembly process. However, it has limited potential to be applied in MMP's. This is because that variation propagation along stages will make the designated variation patterns more complicated and thus un-orthogonal to each other. And the orthonomalization of those patterns will change their physical interpretations and result in misleading diagnostic conclusion. In this chapter, an engineering-driven FA method is proposed for estimating multiple non-orthogonal true SPV's and identifying their underlying variation sources in MMP's. This method involves four steps: (i) converting engineering domain knowledge into qualitative indicator vectors, (ii) deriving the initial principal factor loading vectors, (iii) rotating the initial factor loading vectors to estimate the true SPV's, and (iv) identifying of multivariate variation sources, as shown in Figure 4-3. Different from existing data-driven methods, the engineering knowledge of the impacts of potential variation sources is systematically represented in a set of qualitative indicator vectors to directly guide the factor rotation. The rotated factor loading vectors are the best estimation of true SPV's and therefore, can be used to best interpret the true nature of the variation sources and to direct corrective actions. Compared to the engineering-driven methods, it is robust to process changes since no exact model coefficient values are needed. This method effectively integrates the engineering knowledge with statistical analysis of multivariate KPC measurements. It combines the advantages of the engineering-driven approaches and data-driven approaches, and overcome their limitations.

The remainder of this chapter is organized as follows. Section 4.2 provides an overview of the proposed methodology and presents the necessity of the engagement of engineering domain knowledge in guiding SPV estimation. Section 4.3 introduces the qualitative representation of engineering domain knowledge and engineering-driven factor analysis for variation sources identification. Case study results are presented in Section 4.4 to demonstrate the capability of the proposed method. Conclusions and future works are discussed in Section 4.5.

4.2 Overview of the Methodology

This chapter focuses on directly incorporating the engineering knowledge in guiding the estimation of SPV's. The variation sources identification starts from statistically analyzing multivariate KPC measurements, collected at the final stage of an MMP. The objective is to detect large variations, estimate their SPV's, and interpret or map them with the expected spatial patterns. This can be achieved by analyzing the covariance matrix of KPC measurements with the FA method (Lawley and Maxwell 1971) that adopts the linear model defined in Eq. (4.1). Following assumptions about Eq. (4.1) are made:

(i) $\mathbf{y}$ follows p-dimensional multivariate normal distribution, i.e., $\mathbf{y} \sim N_p(\boldsymbol{\mu}_\mathbf{y}, \boldsymbol{\Sigma}_\mathbf{y})$. Since only the quality problem manifested as increased KPC variations are considered in this chapter, it is assumed that $\boldsymbol{\mu}_\mathbf{y} = \mathbf{0}$.

(ii) $\mathbf{u}_s$ is an $s \times 1$ ($s < p$) unknown vector containing the deviations of s variation sources that present in an MMP. It follows an s-dimensional multivariate normal distribution, i.e., $\mathbf{u}_s \sim N_s(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{u}_s})$. Since the variation sources, e.g., fixture locators, are often fabricated, installed and maintained separately for different stations, it is reasonably to assume that $\boldsymbol{\Sigma}_{\mathbf{u}_s}$ is a diagonal matrix.

(iii) Different from that defined in Eq. (4.1), FA assumes that $\boldsymbol{\Gamma}_s$ ($\boldsymbol{\Gamma}_s = [\boldsymbol{\gamma}_1\ \boldsymbol{\gamma}_2\ \ldots\ \boldsymbol{\gamma}_s]$) is an *unknown* constant $p \times s$ matrix that reflects the linear impacts of $\mathbf{u}_s$ on $\mathbf{y}$. Each column vector $\boldsymbol{\gamma}_i$ is called a loading vector corresponding to the i^{th} element in $\mathbf{u}$, and in this chapter, the column vector $\boldsymbol{\gamma}_i$ is an SPV of the i^{th} variation source. The objective of the proposed method is to estimate the $\boldsymbol{\gamma}_i$'s, $i=1,2,\ldots,s$, in $\boldsymbol{\Gamma}_s$ to investigate the natures of the variation sources and identify them.

(iv) $\mathbf{v}$ follows a p-dimensional multivariate normal distribution, i.e., $\mathbf{v} \sim N_p(\mathbf{0}, \boldsymbol{\Sigma}_\mathbf{v})$. In this chapter, it is assumed that all KPC's are measured by the same measuring device. Thus, measurement noises are independent of each other, i.e., $\boldsymbol{\Sigma}_\mathbf{v} = \sigma^2\mathbf{I}$, where σ^2 is a scalar and $\mathbf{I}$ is an identity matrix with an appropriate dimension. It is also reasonable to assume that $\mathbf{v}$ is independent of $\mathbf{u}_s$, i.e., the measurement noises are independent of the variation sources.

With the above assumptions, the covariance matrix of $\mathbf{y}$ can be computed as

$$\boldsymbol{\Sigma}_{\mathbf{y}} = \boldsymbol{\Gamma}_s \boldsymbol{\Sigma}_{\mathbf{u}_s} \boldsymbol{\Gamma}_s^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathbf{v}}\,, \tag{4.2}$$

The first step in estimating $\boldsymbol{\Gamma}_s$ is to determine the number of variation sources present in an MMP, i.e., s. Based on the eigenvalue-eigenvector pairs derived from eigen-decomposition of $\boldsymbol{\Sigma}_{\mathbf{y}}$, s can be determined by Akaike Information Criterion (AIC) and Minimum Description Length (MDL) criterion, as defined in Eq. (4.3) and Eq.(4.4),

$$\mathrm{AIC}(q) = n(p-q)\log(a_q / g_q) + q(2p-q), \text{ and} \tag{4.3}$$

$$\mathrm{MDL}(q) = n(p-q)\log(a_q / g_q) + q(2p-q)\log(n)/2, \tag{4.4}$$

where n is the sample size, and q is the number of the largest eigenvalues considered. a_q and g_q are the arithmetic mean and the geometric mean of the rest (p-q) smallest eigenvalues of $\boldsymbol{\Sigma}_{\mathbf{y}}$, respectively. In order to determine s, AIC(q) and MDL(q) are iteratively evaluated for q=1, …, p-1, and s is equal to the q* that minimizes AIC(q*) or MDL(q*). As recommended by Apley and Shi (Apley and Shi 2001), when small magnitudes of variations are expected, adopting AIC will result in a high probability of correct number estimation. Otherwise, MDL criteria should be adopted to achieve consistent estimation of q*.

Apley and Shi (2001) showed that if s variation sources are present in the process, the enginvalues of $\boldsymbol{\Sigma}_{\mathbf{y}}$, λ_i (i=1, 2, …, p), will have a relationship such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_s > \sigma^2 = \lambda_{s+1} = \ldots = \lambda_p$. In addition, Jin and Zhou (2006) showed that the s eigenvectors associated with the largest s eigenvalues of $\boldsymbol{\Sigma}_{\mathbf{y}}$ span the same linear space of the s SPV's in $\boldsymbol{\Gamma}_s$. Therefore, it is applicable to estimate the true SPV's by performing an eigen-decomposition of $\boldsymbol{\Sigma}_{\mathbf{y}}$,

$$\mathbf{\Sigma}_{\mathbf{y}} = \mathbf{L}\mathbf{L}^{\mathrm{T}} + \mathbf{\Sigma}_{\mathbf{v}} = \sum_{i=1}^{s}\left(\lambda_i - \sigma^2\right)\mathbf{e}_i\mathbf{e}_i^{\mathrm{T}} + \sigma^2\sum_{i=1}^{p}\mathbf{e}_i\mathbf{e}_i^{\mathrm{T}} = \mathbf{E}_s\left[\mathbf{\Lambda}_s - \sigma^2\mathbf{I}\right]\mathbf{E}_s^{\mathrm{T}} + \sigma^2\mathbf{I}, \tag{4.5}$$

where $\mathbf{E}_s=[\mathbf{e}_1\ \ \mathbf{e}_2\ \ \ldots\ \ \mathbf{e}_s]$ and $\mathbf{\Lambda}_s=diag\{\lambda_1, \lambda_2, ..., \lambda_s\}$ are the matrix composed of eigenvectors $\mathbf{e}_i$ and λ_i, respectively. $\mathbf{L} = [\mathbf{l}_1\ \mathbf{l}_2\ \ldots\ \mathbf{l}_s] = \mathbf{E}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{1/2}$ is the loading matrix in FA, where $\mathbf{l}_i$'s, i=1,2,…, s, are initial loading vectors that are orthogonal to each other. Compared to Eq.(4.2), $\mathbf{l}_i$'s are the initial estimates of the scaled γ_i's since $span\{\mathbf{l}_1\ \mathbf{l}_2\ldots\mathbf{l}_s\} = span\{\gamma_1\ \gamma_2\ ...\ \gamma_s\}$. $\sigma^2\mathbf{I}$ is assumed to be the covariance matrix of measurement noise, i.e., $\mathbf{\Sigma}_{\mathbf{v}}$. However, in practice, the value of σ^2 is often not available, or the $\mathbf{\Sigma}_{\mathbf{v}}$ is not in such a simple structure. Therefore, an approximation of $\mathbf{L}$ can be achieved by

$$\mathbf{L} \approx \mathbf{E}_s\mathbf{\Lambda}_s^{1/2}. \tag{4.6}$$

This approximation will make the initial factor loading vectors deviate from their true values. According to the investigation conducted by Li, *et al.* [23], when σ^2 is much smaller than λ_i, i=1,2,…,s, the deviations will be negligible.

When there is a single variation source present in an MMP, i.e., s=1, the initial estimation, $\mathbf{l}_1$, is an effective estimation of the scaled true SPV of that variation source (Ceglarek and Shi 1996). However, when there are multiple variation sources, i.e., $s > 1$, the eigenvectors may not be coincident with SPV's of the s variation sources, unless those true SPV's are orthogonal to each other. Treating the eigenvector-based loading vectors as estimates of true SPV's may yield non-interpretable or even misleading results and consequently provide limited information for diagnosis and corrective actions. Thus, the orthogonal factor loading vectors should be rotated, obliquely, to estimate true SPV's of the variation sources. Various oblique factor rotation methods have been developed based on explanatory criteria to achieve simple structure and improved factor interpretability (Darton 1980). However, these criteria have limited justification for the diagnostic purpose, since the rotated loading vectors may not be effective in estimating the true SPV's of variation sources. Target rotation (Lawley and Maxwell 1971) provides directional information for factor rotation with hypothesized patterns. This technique is adopted in this chapter, with target patterns systematically defined by the engineering domain knowledge. Thus, the rotated factor loading vectors have the right structures that best estimate the true SPV's of the variation sources.

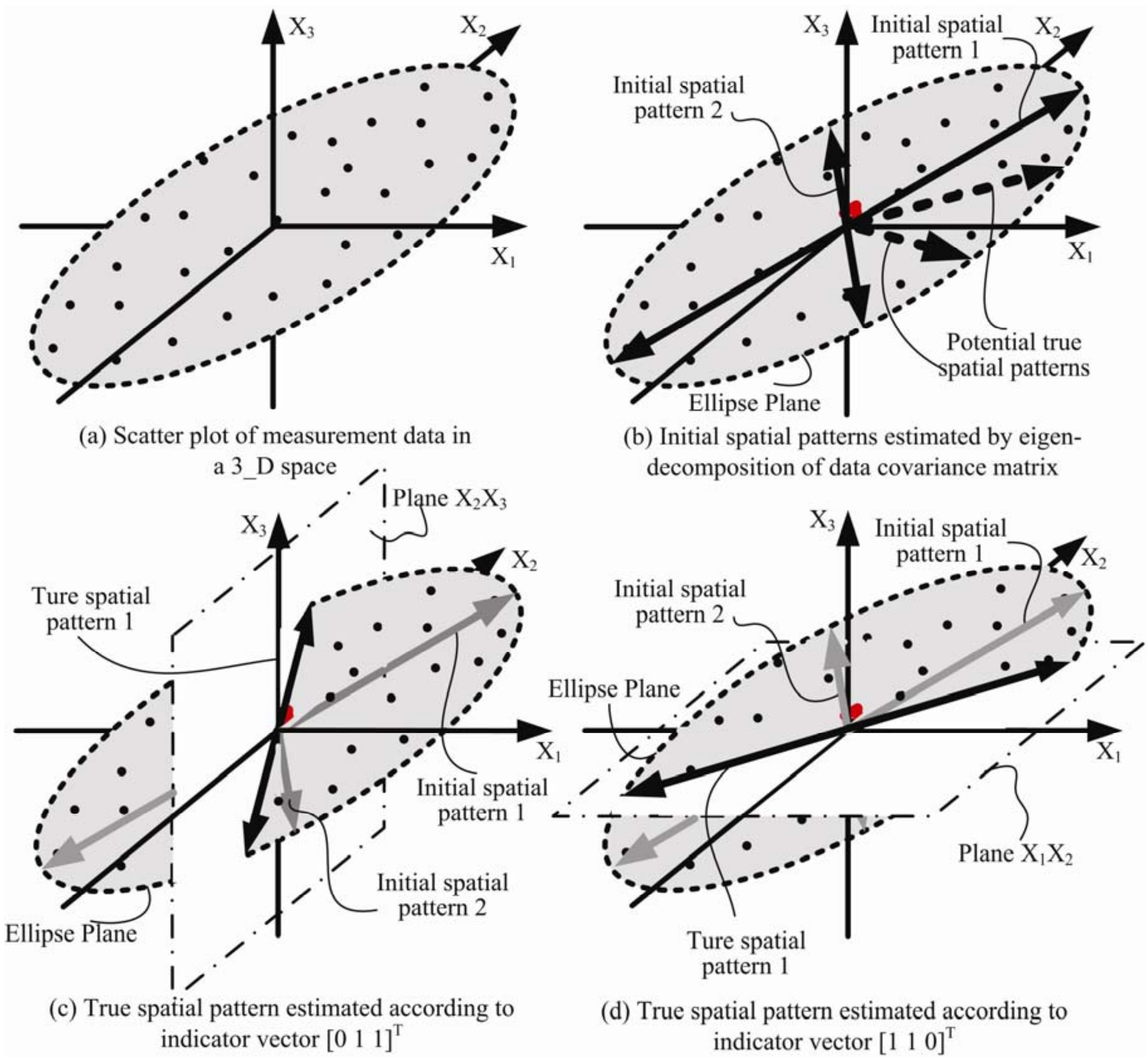


Figure 4-4. Conceptual illustration of indicator vector guided factor rotation

The underlying idea of the target guide rotation is illustrated in Figure 4-4. For a case where three KPC's, X_1, X_2, and X_3, are measured, the three-dimensional (3-D) measurement data can be shown in a 3-D scatter plot. Suppose that there are two (2) variation sources present in the process, the measurement data points will be scattered in a 2-D plane and within an ellipse, as shown in Figure 4-4 (a). Orthogonal factor loading vectors can be achieved from eigen-analysis of data covariance matrix. These orthogonal loading vectors and the true SPV's of the two variation sources will span the same 2-D plane. And the true SPV's could be any linear combinations (rotation) of the orthogonal loading vectors, as shown as the "potential true spatial patterns" in Figure 4-4 (b). Information that indicates the direction of the true SPV's is critically important for the rotation. For instance, if it is known that a certain variation source affects only X_2 and X_3, its true SPV must span the plane defined by axis X_2 and axis X_3, i.e., plane X_2X_3, and simultaneously span the ellipse plane, as shown in Figure 4-4 (c). This means that the true SPV is the intersection of the plane X_2X_3 and the ellipse plane. The direction information of each potential variation source can be represented with an indicator vector, e.g., $[0\ 1\ 1]^T$, where those three elements correspond to three KPC's, X_1, X_2, and X_3, and their values indicate whether the corresponding KPC is affected (1) or not affected (0) by the variation source. Therefore, in this case, if the direction information corresponding to the two potential variation sources are $[0\ 1\ 1]^T$ and $[1\ 1\ 0]^T$, respectively, the scaled true SPV's can be estimated, as shown in Figure 4-4 (c) and Figure 4-4 (d), respectively.

As illustrated in Figure 4-4, in order to estimate the true SPV's of variation sources from multivariate statistical analysis, it is crucial to (i) obtain engineering domain knowledge on interactions between product KPC's and potential variation sources and represent it in an appropriate form, and (ii) use the engineering knowledge in guiding the factor rotation to estimate the true SPV's. The variation sources are identified based on the interpretation of the estimated true SPV's.

4.3 Engineering-Driven Factor Analysis for Variation Sources Identification

This section introduces the proposed new methodology following the ideas illustrated in Section 4.2. Basically, the methodology includes three components: (i) qualitative engineering domain knowledge representation, (ii) indicator-vector-guided factor rotation for estimating the scaled true SPV, and (iii) the procedure of multiple variation sources identification.

4.3.1 Engineering Knowledge Representation

A successful engineering knowledge representation for variation sources identification needs a thorough description of an MMP, and a mechanism to interrelate potential variation sources with the KPC's. More importantly, it is necessary to provide directional indication of true SPV's.

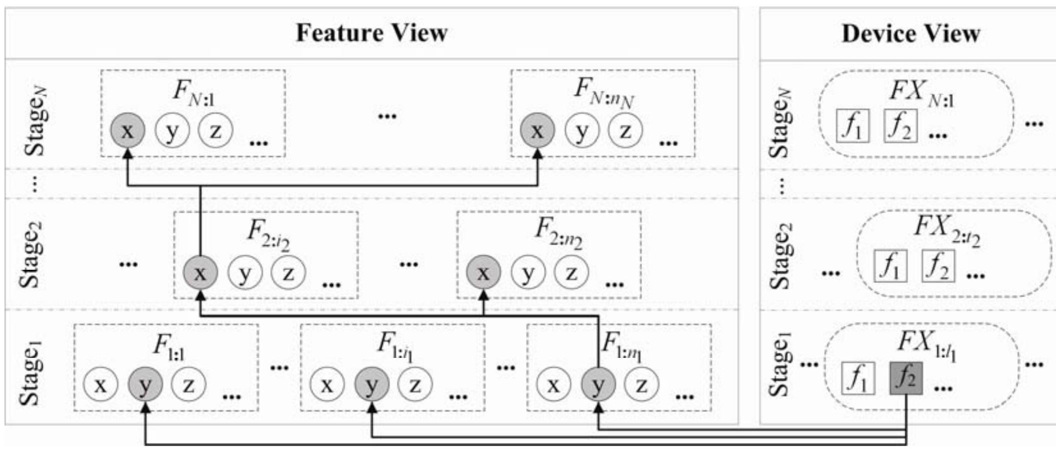


Figure 4-5. Graphical description of an MMP

Engineering knowledge about an MMP comes from the product and process design, which includes the process flow, the manufacturing datum schemes selected for each operation, the features generated in a series of operations, their precedence relationships, and the potential variation sources. The description of an MMP can be represented in a diagram, as shown in Figure 4-5.

This diagram contains a "Feature View" and a "Device View", and includes four categories of information about an MMP: (i) process flow and precedence relationships, (ii) potential variation sources, (iii) relationships between potential process variation sources and KPC's, and (iv) deviation propagation scenario.

The *process flow and precedence relationships* include process sequence information, operations information and datum flow information. As shown in Figure 4-5, process sequence is presented as N layers in the two views of the diagram, corresponding to the N stages of an MMP. Operations information indicates the features generated in each stage. Denoted as dashed boxes in Figure 4-5, $F_{k:i_k}$ represents the i_k^{th}

feature (i_k=1, 2, …, n_k) generated in stage k, and n_k is the number of features generated in stage k, k=1,2,…,N. For each feature, its KPC's are denoted as circles nodes, e.g., ⓩ, following vector feature representation (Zhou *et al.* 2003). Datum flow information is shown through the linkage between features. In feature view, features in a particular stage k are linked with some features generated in upstream stages. This means that those upstream features are used as the datum features in stage k.

Potential variation sources are in the machine tool, fixtures and datum features, as discussed by Jin and Shi (1999), and Zhou *et al.* (2003). In Figure 4-5, fixtures are denoted as dashed blocks, and are organized in different layers in the device view, where $FX_{k:l_k}$ represents the l_k^{th} fixtures (l_k=1, 2,…, t_k) used in stage k, and t_k is the total number of fixtures used in stage k. This chapter considers the most commonly used 3-2-1 fixturing scheme. Vectorial notation for fixture components used in (Jin and Shi 1999; Zhou *et al.* 2003) are adopted and its elements are represented as the square nodes within the blocks, e.g., $\boxed{f_1}$.

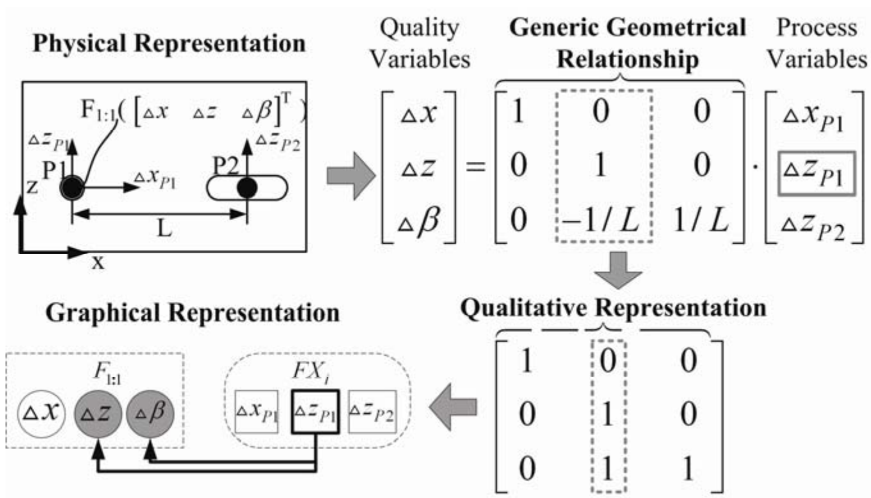


Figure 4-6. Qualitative representation of quality/process interaction

The *relationships between potential process variation sources and KPC's* describe the impacts of potential variation sources on KPC's. These impacts are denoted as the connections linking the process variables in the device view, e.g., f_2 of $FX_{1:l_1}$, and the KPC's in the feature view, e.g., y of $F_{1:1}$. With the fixtures and features represented in a vector form, the linear relationship between them can be described with a qualitative indicator matrix, as illustrated by Figure 4-6. The interactions between three KPC's, Δx, Δz, and $\Delta\beta$, of $F_{1:1}$ and three variation sources, Δx_{p1}, Δz_{p1}, and Δz_{p2}, of FX_i are considered. Their generic geometric relationship can be described in a matrix developed in (Jin and Shi 1999; Zhou *et al.* 2003), as shown in Figure 4-6. Qualitative representation matrix can be achieved by keeping zero elements in the generic geometric relationship matrix and replacing the non-zero elements with 1's. The remaining 0's in the pattern matrix are called *specified elements*, and the 1's are *unspecified elements*. In the qualitative indicator matrix, the i,j^{th} element indicates whether the j^{th} variation source has an impact on the i^{th} quality variable. When it is 1, there will be a connection between the two nodes in the graphical representation, and vise versa. As shown in Figure 4-6, according to the qualitative representation, Δz_{p1} of fixture FX_i affects Δz, and $\Delta\beta$ of $F_{1:1}$.

Deviation propagation is caused by the faulty datum features generated in upstream stages. It can be described by linking the datum features with the features generated based on them. As shown in Figure 4-5, feature $F_{1:n_1}$ is used as the datum in stage 2. Thus, if the f_2 of $FX_{1:k_1}$ causes an deviation of $F_{1:n_1}$ in the y direction, this deviation will be propagated to the x's of $F_{2:i_2}$ and $F_{2:n_2}$, and further propagated to the x's of $F_{N:1}$ and $F_{N:n_N}$, since $F_{2:i_2}$ is selected as the datum feature in stage N. Therefore, the graphical representation of deviation propagation is a path that connects the potential variation sources with all the affected KPC's. The rationale underlying the connections is the same as that of the state transition matrices proposed in (Jin and Shi 1999; Zhou *et al.* 2003).

4.3.2 Indicator Matrix Definition

The graphical description of an MMP in Figure 4-5 should be transformed to a qualitative indicator matrix to guide the SPV transformation. The feature view of an MMP contains p KPC's in $\mathbf{y}$ of Eq. (4.1), whereas the device view contains M potential variation sources in $\mathbf{u}$. Although matrix $\mathbf{\Gamma}$ is not known since the exact values of its elements are not derived, we can still use a $p \times M$ indicator matrix, $\mathbf{T} = [\tau_{im}]$, to link potential variation sources with KPC's, where

$$\tau_{im} = \begin{cases} 1, & \text{When there is a connection linking the } m^{\text{th}} \text{ potential} \\ & \text{variation source and the } i^{\text{th}} \text{ quality variable;} \\ 0, & \text{Otherwise;} \end{cases} \qquad (4.7)$$

for i=1,2,…,p, and m=1,2,…,M. The column vectors, $\boldsymbol{\tau}_m$, in matrix $\mathbf{T}$ are indicator vectors to indicate the direction information of true SPV's of variation sources, where $\boldsymbol{\tau}_m=[\tau_{1m}\ \tau_{2m}\ \dots\ \tau_{pm}]^{\mathrm{T}}$. The column vectors in $\mathbf{\Gamma}$ of Eq. (4.1) are the true SPV's of all potential variation sources, whereas the columns in pattern matrix $\mathbf{T}$ only partially reflect $\mathbf{\Gamma}$.

4.3.3 Indicator-Vector-Guided True SPV Estimation

Factor rotation is a transformation technique to improve the interpretability of factor loadings, i.e., the initial estimates of true SPV's. The objective is to find a rotation matrix $\mathbf{R}$ to transform the orthogonal initial factor loading vectors that are defined in Eq.(4.6), to

$$\mathbf{L}^* = \mathbf{L}\mathbf{R} \approx \mathbf{E}_s \mathbf{\Lambda}_s^{1/2} \mathbf{R}, \qquad (4.8)$$

which best estimates the scaled true SPV's of underlying variation sources. In this chapter, indicator matrix $\mathbf{T}$ works as a target that guides the factor rotation. Two assumptions are necessary to make the target factor rotation applicable (Lawley and Maxwell 1971):

(i) $\mathbf{u}_s$ is an $s \times 1$ ($s < p$) unknown vector containing the deviations of s variation sources that present in an MMP. It follows an s-dimensional multivariate normal distribution, i.e., $\mathbf{u}_s \sim N_s(\mathbf{0},\ \mathbf{\Sigma}_{\mathbf{u}_s})$. Since the variation sources, e.g., fixture locators, are often fabricated, installed and maintained separately for different stations, it is reasonably to assume that $\mathbf{\Sigma}_{\mathbf{u}_s}$ is a diagonal matrix.

(ii) Each indicator vector, $\boldsymbol{\tau}_m$, should contain at least s-1 specified elements, i.e., 0's;

(iii) The indicator vectors are different from each other. That is, if an operator, $\otimes$, is defined such that $1 \otimes 1 = 0$, $0 \otimes 0 = 0$, $1 \otimes 0 = 1$, and $0 \otimes 1 = 1$, hold for scalars, and $\boldsymbol{\tau}_k^{\mathrm{T}} \otimes \boldsymbol{\tau}_l = \sum_{t=1}^{p} (\tau_{tk} \otimes \tau_{tl})$, $k \neq l$, hold for vectors, the indicator vectors should satisfy

$$\boldsymbol{\tau}_k^{\mathrm{T}} \otimes \boldsymbol{\tau}_l \neq 0, \text{ for } k \neq l. \tag{4.9}$$

The initial $p \times s$ orthogonal factor loading matrix can be denoted as $\mathbf{L} = [l_{ij}]$, whereas the rotated factor loading matrix can be denoted as $\mathbf{L}^* = [l_{ij}^*]$, where i= 1,2, …, p, and j=1, 2, …, s. Let $\mathbf{r}_j$ denote the j^{th} column of the rotation matrix $\mathbf{R}$, then the j^{th} column of $\mathbf{L}^*$ can be achieved by

$$\mathbf{l}_j^* = \mathbf{L}\mathbf{r}_j,\ j = 1, 2, \ldots, s. \tag{4.10}$$

The objective of the indicator-vector-guided factor rotation is to achieve maximum agreement between the estimated SPV's and the spatial patterns specified by *a priori* knowledge. This means that, corresponding to the specified elements, , of a given $\boldsymbol{\tau}_{m_j}$ in $\mathbf{T}$, (i.e., $\tau_{i^* m_j}$=0, $i^* \in I_{m_j}$, $I_{m_j} \subset \{1,2,...,p\}$ and $m_j \subset \{1,2,...,M\}$), the elements, $l_{i^* j}^*$, of the rotated loading vector, $\mathbf{l}_j^*$, should also have values that are close to zero. This can be evaluated by the sum of squares of the elements corresponding to the specified elements in $\boldsymbol{\tau}_{m_j}$,

$$ag_j(m_j) = \sum_{i^* \in I_{m_j}} (l_{i^* j}^*)^2,\ j = 1, 2, \ldots, s, \text{ and } m_j \in \{1,2,...,M\}, \tag{4.11}$$

where $ag_j(m_j)$ is the agreement index of loading vector j with respect to indicator vector $\boldsymbol{\tau}_{m_j}$. In order to achieve best agreement, $ag_j(m_j)$, should be minimized. In other words, the factor rotation can be formulated as a problem to find a rotation vector $\mathbf{r}_j$ to minimize

Eq. (4.11), subject to the sum of squares of all elements being held constant. This is equivalent to maximizing the sum of squares of the unspecified elements (Lawley and Maxwell 1971). Given an indicator vector, $\boldsymbol{\tau}_{m_j}$, $\mathbf{L}_{m_j}^r$ denotes a restricted loading matrix that can be achieved by replacing all the elements in the $i^{*\text{th}}$ row of $\mathbf{L}$ with zeros (i = 1,2,…, p) whenever $\tau_{i^* m_j}$ =0 and keeping all the rows corresponding to unspecified elements in $\boldsymbol{\tau}_{m_j}$ unchanged. Consider vector $\mathbf{l}_j^r(m_j) = \mathbf{L}_{m_j}^r \mathbf{r}_j(m_j)$, $\mathbf{l}_j^r(m_j)$ and $\mathbf{r}_j(m_j)$ are the restricted rotated loading vector and rotation vector corresponding to $\boldsymbol{\tau}_{m_j}$, respectively. $\mathbf{l}_j^r(m_j)$ can be treated as the result of replacing all the specified elements in $\mathbf{l}_j^*$ with zeros. Thus, the sum of squares of the unspecified elements can be calculated accordingly and the factor rotation can be formulated as an optimization problem, i.e., for a given $\boldsymbol{\tau}_{m_j}$, $m_j \in \{1,2,...M\}$,

$$\begin{aligned} &\max_{\mathbf{r}_j(m_j)} \left(\mathbf{L}_{m_j}^r \mathbf{r}_j(m_j)\right)^{\mathrm{T}} \left(\mathbf{L}_{m_j}^r \mathbf{r}_j(m_j)\right) \\ &\text{s.t.} \quad \mathbf{l}_j^{*\mathrm{T}} \mathbf{l}_j^* = \mathbf{l}_j^{\mathrm{T}} \mathbf{l}_j. \end{aligned} \tag{4.12}$$

Lagrange multiplier method for solving (4.12) indicates that the optimal rotation vector $\mathbf{r}_j(m_j)$, is the eigenvector associated with the largest eigenvalue of matrix $\mathbf{H}_j(m_j)$, where

$$\mathbf{H}_j(m_j) = \left(\mathbf{L}^{\mathrm{T}}\mathbf{L}\right)^{-1}\left[\left(\mathbf{L}_{m_j}^r\right)^{\mathrm{T}} \mathbf{L}_{m_j}^r\right]. \tag{4.13}$$

Based on this relationship, for a given $\boldsymbol{\tau}_{m_j}$, the rotated loading vector that achieves maximum agreement can be defined as

$$\mathbf{l}_j^*(m_j) = \mathbf{L}\mathbf{r}_j(m_j), j = 1, 2, \ldots, s, \tag{4.14}$$

and the associated agreement coefficient, $ag_j(m_j)$, can be calculated.

4.3.4 Procedure of Multiple Variation Sources Identification

The indicator-vector-guided factor rotation defines the derivation procedure that determines the rotation vector to achieve a single rotated loading vector that maximizes the agreement with the given $\boldsymbol{\tau}_{m_j}$. Corresponding to the s variation sources that are present in an MMP, the s rotation vectors $\mathbf{r}_j(m_j)$ can be calculated successively for $j = 1, 2,\ldots, s$. The s indicator vectors, $\boldsymbol{\tau}_{m_j}$, are selected from $\mathbf{T}_j$, where $\mathbf{T}_j$ is the indicator matrix containing the indicator vectors for rotating $\mathbf{l}_j^*$. This means that m_j is also a decision variable. Rotation vectors $\mathbf{r}_j(m_j)$ ($m_j \in \{1,2,...M\}$) are determined and their agreement coefficients $ag_j(m_j)$ will be calculated for different $\boldsymbol{\tau}_{m_j}$. For each rotated loading vector, $\mathbf{l}_j^*$, j=1,2,…, s, the index m_j^* that minimizes agreement coefficient is determined by

$$m_j^* = \arg\min_{m_j}[ag_j(m_j)]. \qquad (4.15)$$

Let h be a predefined threshold value that reflects the desired agreement of the rotated loading vectors with respect to the indicator vectors. If $ag_j(m_j^*) < h$, it indicates that the $m_j^{*\,\text{th}}$ process variation source is identified. Otherwise, it indicates that an unknown variation source is present and its SPV is not pre-specified in $\mathbf{T}$. According to Eq. (4.1), $\mathbf{l}_j^*$ reflects the combined impact of the variation sources and the measurement noise. For the elements (of $\mathbf{l}_j^*$) corresponding to the specified elements (0's) of indicator vectors, their magnitudes should be in the same scale of measurement noise. Thus, the threshold value h can be determined as, $\bar{\sigma} < h < p\bar{\sigma}$, where $\bar{\sigma}$ is the average of the standard deviations of measurement noises, and p is the number of KPC's.

After identifying the j^{th} process variation source, its indicator vectors will be removed from $\mathbf{T}_j$. Thus, m_{j+1}^* can only be chosen from the reduced pattern matrix, $\mathbf{T}_{j+1}$, where $\mathbf{T}_{j+1}$ = $\{\boldsymbol{\tau}_k \mid k \neq m_1, m_2,..., m_j\}$. For each m_j^*, j = 1,2,…,s, the optimal rotation vectors, $\mathbf{r}_j(m_j^*)$, are determined and used to form rotation matrix $\hat{\mathbf{R}}$, where $\hat{\mathbf{R}} = [\mathbf{r}_1(m_1^*) \quad \mathbf{r}_2(m_2^*) \quad \ldots \quad \mathbf{r}_s(m_s^*)]$. In practice, it is convenient to rescale the columns of $\hat{\mathbf{R}}$ to make the rotation vectors have unit variances. This rescaling is accomplished by

$$\mathbf{R} = \hat{\mathbf{R}}\mathbf{D}, \tag{4.16}$$

where $\mathbf{D}$ is a diagonal matrix with positive diagonal elements, defined by $\mathbf{D}^2 = diag\left[\left(\hat{\mathbf{R}}^{\mathrm{T}}\hat{\mathbf{R}}\right)^{-1}\right]$, and $diag(\mathbf{X})$ denotes the diagonal part of matrix $\mathbf{X}$. Therefore, based on Eq. (8) through Eq. (4.16), rotated factor loading vectors of $\mathbf{L}^*$ give the best estimations of the true SPV's of the variation sources.

The procedure for process variation sources identification is illustrated in Figure 4-7. The four major steps are summarized as follows:

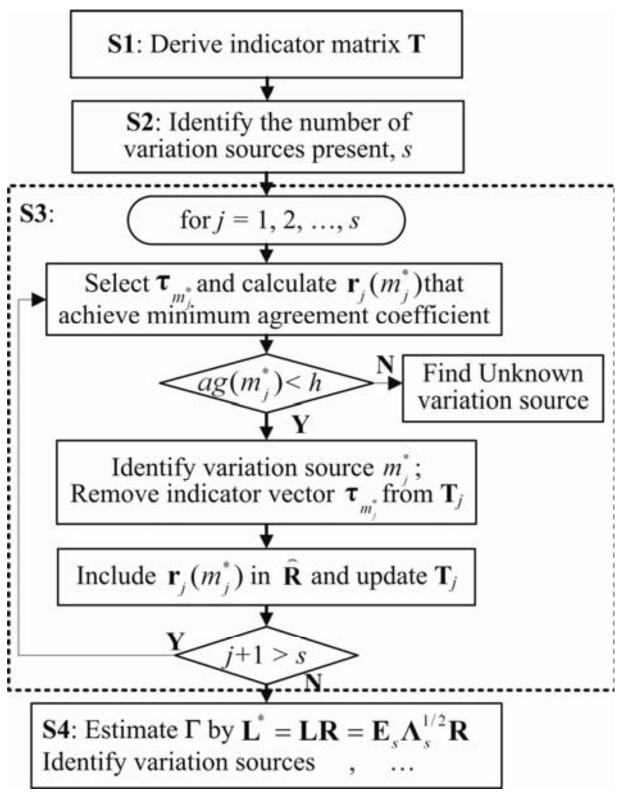


Figure 4-7. Procedure of multiple variation sources identification for an MMP

S1: An indicator matrix, **T**, is obtained from the engineering domain knowledge to represent the relationships between the potential variation sources and KPC's.

S2: The number of variation sources that present in an MMP is determined by using the eigen-decomposition of $\mathbf{\Sigma_y}$ and AIC or MDL criteria.

S3: All the s initial estimates of SPV's are sequentially rotated based on indicator matrix, $\mathbf{T}_j$, $j = 1,2,\ldots,s$. The rotation results indicate that either the known variation sources are identified or unknown sources are found. Index m_j^* and rotation vectors $r_j(m_j^*)$, $j = 1,2, \ldots, s$, are recorded and in indicator matrix $\mathbf{T}_j$ is updated.

S4: Process variation sources $m_1^*, m_2^*, \ldots, m_s^*$ are identified and their true SPV's, vectors in $\mathbf{L}^*$, are estimated based on normalized rotation matrix **R**.

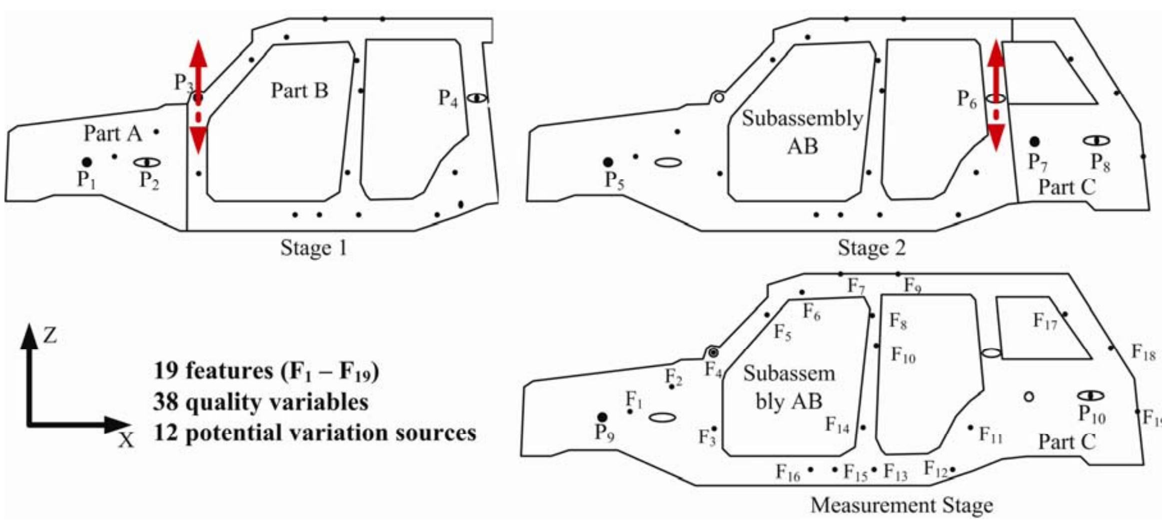


Figure 4-8. A three-stage assembly processes

4.4 Case Study

A case study of a three-stage assembly process, as shown in Figure 4-8, is conducted to demonstrate the effectiveness of the proposed methodology. In this process, three panel parts are assembled in the first two stages to form an automotive side aperture. As summarized in Table 4-2, in each stage, the parts and/or subassembly are fixed by fixtures with a 4-way pin (e.g., P_1) and a 2-way pin (e.g., P_2). In the third stage, nineteen

(19) KPC features, F_1 through F_{19}, are measured to monitor the dimensional quality of the final side-aperture ABC. These features are measured in terms of X-direction and Z-direction deviations from their nominal positions. Thus, there are totally 38 KPC's considered.

Table 4-2. Summary of the three-stage assembly process

Stage	Fixture	Operations
1	Fix part A by $FX_{1:1}$ (P_1 and P_2) Fix part B by $FX_{1:2}$ (P_3 and P_4)	Assemble part A and part B
2	Fix subassembly AB by $FX_{2:1}$ (P_5 and P_6) Fix part C by $FX_{2:2}$ (P_7 and P_8)	Assemble subassembly AB and part C
3	Fix side aperture ABC by $FX_{3:1}$ (P_9 and P_{10})	Measure 19 features (F_1~F_{19}) on side aperture ABC

As a preparation of variation source identification, graphical description of this 3-stage assembly process is developed, as shown in Figure 4-9. The nineteen KPC features in different stages are denoted in different layers in the Feature View. In the Device View, every fixture contains three elements, where f_1 and f_2, represent the potential X-direction and Z-direction variation sources of 4-way pins, respectively, and f_3 represents the potential Z-direction variation source of 2-way pins. The connections link the potential variation sources to some KPC's, indicating the product/process interactions. For instance, Figure 4-9 shows that if the potential variation source f_2 of $FX_{1:2}$ actually present in the process, all the features on part B ($F_{1:3}$ through $F_{1:16}$) will be affected in stage 1 and will deviate from their nominal positions. According to the process design, these random deviations will be propagated to stage 2 and reflected in the measurements. For simplicity, only the fixtures used in stage 1 and stage 2 are considered and thus there are 12 potential variation sources. Accordingly, a 38×12 qualitative indicator matrix $\mathbf{T}$ for the 12 potential variation sources is established. For instance, the indicator vector for f_2 of $FX_{1:2}$ is $\boldsymbol{\tau}_{FX_{1:2\_f_2}} = [\mathbf{0}_4^{\mathrm{T}} \quad \mathbf{1}_{28}^{\mathrm{T}} \quad \mathbf{0}_6^{\mathrm{T}}]$, where $\mathbf{0}_m$ is a m×1 column vector with all elements equal to 0, and $\mathbf{1}_n$ is a n×1 column vector with all element equal to 1.

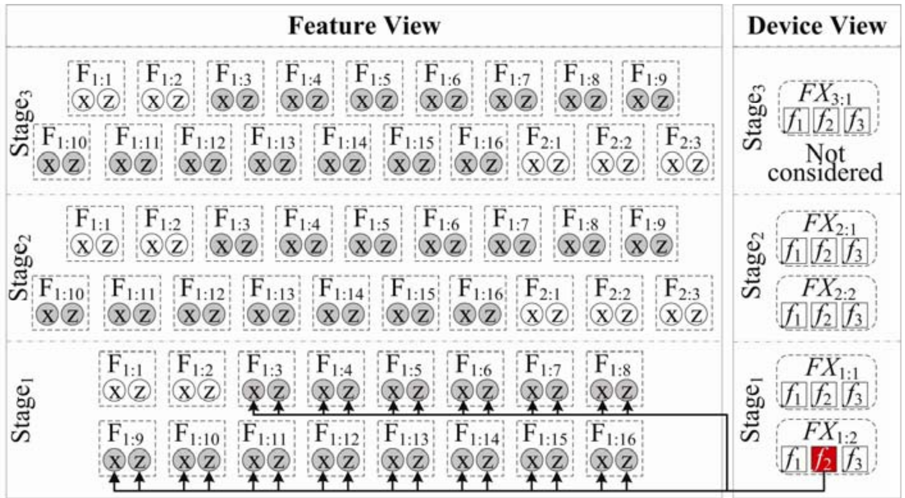


Figure 4-9. Graphical description of 3-stage assembly process

Monte Carlo simulation is performed to generate measurement data based on the linear model (1), where the matrix $\boldsymbol{\Gamma}$ is derived by state space model introduced in (Jin and Shi 1999). The $\boldsymbol{\Gamma}$ matrix contains 12 true SPV's, denoted as $\boldsymbol{\gamma}_{FX_{k:j\_f_t}}$, corresponding to 12 potential

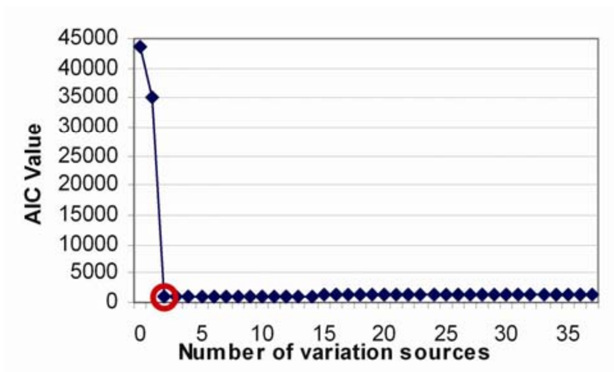
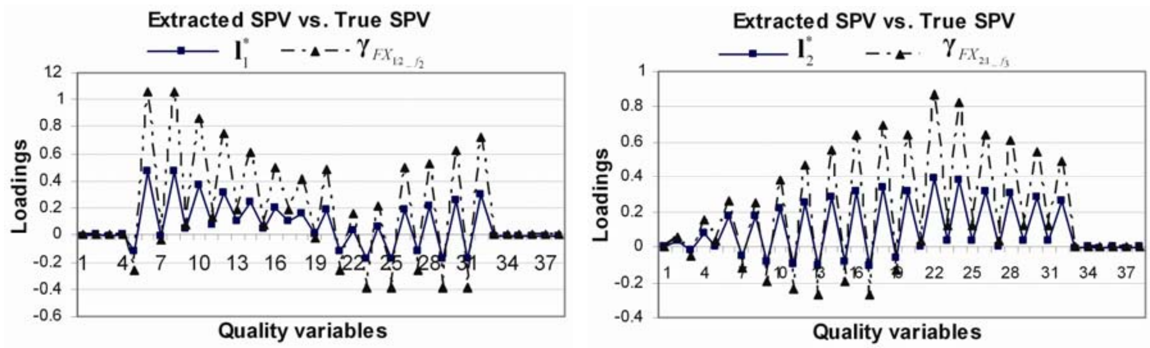


Figure 4-10. Determination of the number of variation sources according to AIC

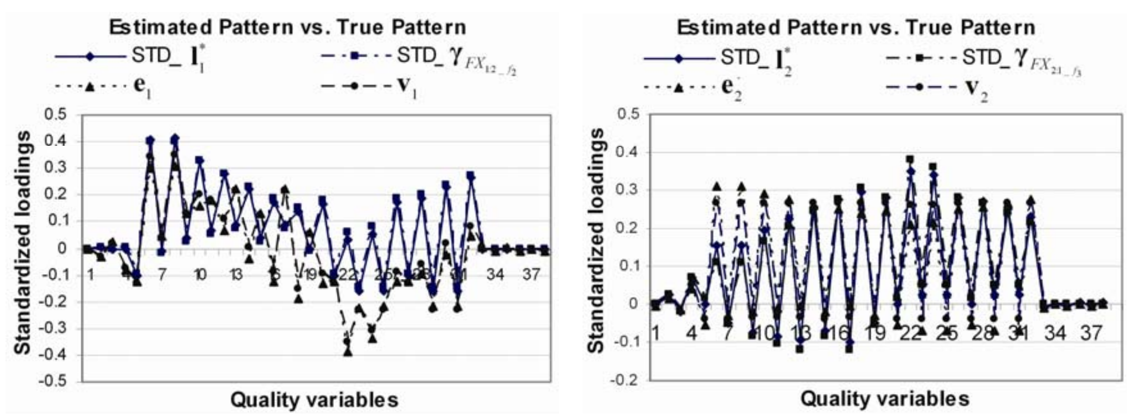
variation sources of f_t of fixture j used in stage k. The input vector $\mathbf{u}$ follows 12-variate normal distribution, i.e., $\mathbf{u}\sim N(\mathbf{0}_{12},\boldsymbol{\Sigma}_\mathbf{u})$. In this case, f_2 of $FX_{1:2}$ (i.e., P_3, Z-direction, denoted as $FX_{1:2\_f_2}$) and f_3 of $FX_{2:1}$ (i.e., P_6, Z-direction, denoted as $FX_{2:1\_f_3}$) are simulated as the variation sources, as shown in Figure 4-8. Thus, the 5th and the 9th diagonal elements of $\boldsymbol{\Sigma}_\mathbf{u}$ are 0.6 and 0.4, respectively, whereas the other diagonal elements of $\boldsymbol{\Sigma}_\mathbf{u}$ are set to be 0.01. The measurement noise vector $\mathbf{v}$ follows 38-variate normal distribution, i.e., $\mathbf{v} \sim N(\mathbf{0}_{38}, \boldsymbol{\Sigma}_\mathbf{v})$, where $\boldsymbol{\Sigma}_\mathbf{v}$ is a 38×38 diagonal matrix with its diagonal elements ranging from 0.001 to 0.01. There are 150 samples simulated to generate KPC measurements.



(a) (b)

Figure 4-11. Comparison of the rotated loading vectors with the true SPV

Following the procedure proposed in Section 3, the engineering-driven factor analysis was conducted. AIC value indicates that there are s=2 variation sources present, as shown in Figure 4-10. According to the agreement coefficients, the two aforementioned variation sources are identified. The two rotated factor loadings, $\mathbf{l}_1^*$, and $\mathbf{l}_2^*$, are compared with $\boldsymbol{\gamma}_{FX_{1:2\_f2}}$ and $\boldsymbol{\gamma}_{FX_{2:1\_f3}}$, respectively, in Figure 4-11.



(a) (b)

Figure 4-12. Comparison of standardized SPV's

Figure 4-12 shows the comparison of the standardized rotated loading vectors and the standardized true SPV's. This visual comparison shows that the rotated factor loading vectors agree with the true SPV's very well. Also shown in Figure 12 are the standardized SPV's estimated with methods in (Jin and Zhou 2006) ($\mathbf{v}_1$ and $\mathbf{v}_2$) and in (Liu and Hu 1997) ($\mathbf{e}_1$ and $\mathbf{e}_2$). There are significant discrepancies between $\mathbf{e}_1$, $\mathbf{v}_1$ and $\boldsymbol{\gamma}_{FX_{1:2}\_f_2}$, and between $\mathbf{e}_2$, $\mathbf{v}_2$ and $\boldsymbol{\gamma}_{FX_{2:1}\_f_3}$. This is because the angle between $\boldsymbol{\gamma}_{FX_{1:2}\_f_2}$ and $\boldsymbol{\gamma}_{FX_{2:1}\_f_3}$ is 51.58°, the orthogonal factor loading rotation cannot give an acceptable estimation of the true SPV's.

The estimated SPV's, $\mathbf{l}_1^*$, and $\mathbf{l}_2^*$, are visualized in Figure 4-13. The SPV of variation source 1, $\mathbf{l}_1^*$, shows that all the features on part B, F_3~F_{16}, deviate from their nominal positions along a circle centered at the slot S_1. This SPV indicates that P_3 used in stage 1 has an abnormally large variation along Z direction. The SPV of variation source 2, $\mathbf{l}_2^*$, shows that all the features on subassembly AB, F_1~F_{16} , deviate from their nominal positions along a circle centered at the hole H_1. This SPV indicates that P_6 used in stage 2 has an abnormally large variation along Z direction. In practice, although their true SPV's are unknown, the variation sources can still be identified by visualizing and interpreting the geometric implications of the estimated SPV's.

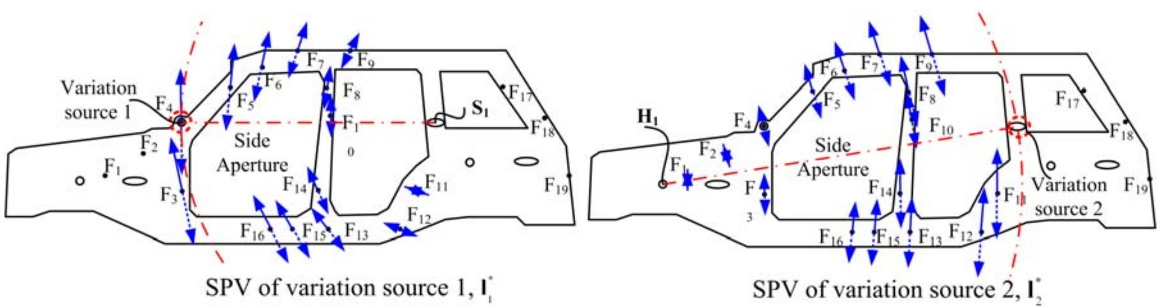


Figure 4-13. Visualization of estimated SPV's

A mathematical way to justify the agreement between loading vectors with true SPV is to calculate the angle between these two vectors, when the true SPV's in $\boldsymbol{\Gamma}$ are known. The smaller the angle, the better the two vectors match. Ding *et al*. (Ding *et al.* 2002) and Li *et al.* (Huang *et al.* 2007) used a method to define the boundary of the perturbation caused by measurement noise. This boundary is represented as an angle of the cone surrounding an SPV and can be calculated according to the eigenvalues associated with the SPV's, and eigenvalues of $\mathbf{\Sigma_v}$. The maximum and minimum eigenvalues of the matrix **A** are denoted as $\lambda_{max}(\mathbf{A})$ and $\lambda_{min}(\mathbf{A})$, respectively. According to the simulation conducted in the case study, $\lambda_{max}(\mathbf{\Sigma_v}) = 0.01$, $\lambda_{max}(\mathbf{\Sigma_v})/\lambda_{min}(\mathbf{\Sigma_v})$ is 10, and the ratio of eigenvalues associated with SPV's over $\lambda_{max}(\mathbf{\Sigma_v})$ is approximately equal to 20.350. Based on the equation (7) in (Huang *et al.* 2007), the boundary angle will be approximately equal to 11.28°. According to the standardized rotated loading vectors and true SPV's, the angle between $\mathbf{l}_1^*$ and $\boldsymbol{\gamma}_{FX_{1:2\_f2}}$ is 4.05°, and that between $\mathbf{l}_2^*$ and $\boldsymbol{\gamma}_{FX_{2:1\_f3}}$ is 6.21°, which means the rotated loading vectors fall in the boundary of true SPV's and thus, identify variation sources and give a best estimation of the true SPV's. It should be emphasized that these diagnosis results are obtained following the method proposed in this chapter, which does not require the accurate state space model. In other word, the proposed method is more robust to unknown process adjustments or tooling worn out and has more appealing capability in variation sources identification.

Although, for the sake of simplicity, a three-stage assembly process is used to demonstrate the effectiveness in identifying multiple variation sources in MMP's, the proposed methodology is applicable for more complex MMP's with more stages. For such a complex process, the graphical description of the MMP will contain more layers for more stages. In each layer, more feature nodes and fixture nodes may be considered. Their interactions will still be denoted as links connecting nodes in different views and different layers, as shown in Figure 4-5. More qualitative indicator vectors will be derived from the graphical knowledge presentation, corresponding to more potential variation sources that may present in the process. The dimension of the measurements of KPC's, **y**, may also increase. However, these changes will not affect the effectiveness of the proposed methodology. This is because that, the two necessary assumptions for indicator vectors are more likely to be satisfied, as the number of stages increases. For instance, if the process is properly designed, the random deviation of a locating pin used in stage 10 will less likely affect the features generated in stage 1. The efficiency of the proposed methodology will not be affected dramatically either. According to the procedure illustrated in Figure 4-7, if there are *s* variation sources that present in an MMP that contains M potential variation sources, $sM - \sum_{q=1}^{s}(q-1)$ iterations are needed to identify them. Whereas the method introduced in (Jin and Zhou 2006) considers all those s eigenvectors together to form a fault space and $\binom{s}{M}$ iterations to finish the variation sources identification. This will substantially increase the computational load.

4.5 Conclusion

The identification of process variation sources in an MMP demands the integration of engineering domain knowledge with appropriate multivariate statistical analysis. This chapter presents a methodology to implement this integration in estimating the true SPV's of the variation sources, without complex quantitative modeling of the interactions between process variation sources and KPC's. Instead, the engineering knowledge about those relationships is represented as a qualitative indicator matrix. The key element of this methodology is the indicator vectors defined based on product/process knowledge to guide the factor rotation, which significantly improves the diagnostic interpretability of factor loadings and thus ensures the applicability of FA in variation sources identification. A procedure based on this factor rotation technique is developed for diagnosing MMP's by identifying multiple process variation sources, whose SPV's are non-orthogonal. Although the effectiveness of the proposed methodology is demonstrated through a case study of dimensional variation sources identification in a manufacturing process, the method can also be used in other applications where the measurements of observable variables can be linearly linked with a set of latent variables, as defined in Eq.(4.1).

In order to implement the proposed methodology in a complex MMP, various resources are necessary. For instance, software is needed to aid engineers in converting product/process design information into graphical and qualitative representations. The estimation of SPV's also needs substantial amount of multivariate data. Thus, advanced measurement system, such as in-line Optical Coordinate Measuring Machines, are desirable to create a data-rich environment to reduce the uncertainty caused by the random samples. The developed methodology could be sensitive to data uncertainty or missing data. More research needs to be done to investigate the impacts of sample uncertainty on the effectiveness of the proposed methodology and improve its robustness.

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CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

The research presented in this dissertation is concluded by summarizing the main achievements and the original contributions. Potential future studies are also discussed in this section.

5.1 Conclusions

The massive, readily available data generated and collected from different phases of product realization create tremendous opportunity to improve the quality assurance of MMP's. In this dissertation, the framework of a unified methodology is proposed to effectively fuse the data by combining engineering domain knowledge, control theory, optimization algorithms and advanced multivariate statistical analysis and generate knowledge for the quality assurance purpose. The knowledge is then applied in different phases of product realization to systematically improve the quality as well as productivity of MMP's. Within this proposed framework, some initial research has been done in variation propagation modeling, process design and process variation source identification.

The original contributions of this dissertation can be summarized as the following key components.

(i) *An generic 3-D state space approach for analytical modeling of the stream of variation in MMP's*. Based on DMV representation, the model mathematically describes the process induced variations and their propagations along stages. Compared with existing modeling techniques, the proposed one extend current model to 3-D processes where it is crucial to investigate the dimensional variation in a more realistic 3-D space. Also, the model is able to cover a wider variety of processes, including both Type-I and Type-II assembly process. The impacts of part fabrication errors on product quality are explicitly incorporated in the model. Although the presented modeling technique is formulized for the multistage assembly processes, it has great potential to be extended to other categories of processes where dimensional variation is of great concern. The proposed model creates the scientific basis for developing novel research in product/process design, process monitoring and diagnosis. Especially, the process evaluation capability provided by this model enables the quality assurance in the early stage of production realization and significantly improve the current process design strategy.

(ii) *A quality assured setup planning methodology for MMP's*. Based on the quality evaluation capability provided by the SoV modeling, setup planning is formulized as an optimal sequential decision problem that minimizes the CRPP with the final product quality as constraints. This is the first optimization approach to setup planning based on analytical evaluation. Both the effectiveness and the efficiency have been significantly improved. Although only the setup planning of multistage machining process is investigated, the generic problem formulation and analytical evaluation techniques can also be applied in the process design of other types of MMP's. The outputs of the quality assured setup planning include an optimal setup plan and the tolerances of potential variation sources, the spatial patterns of which define their impacts on quality variables under normal production condition. These spatial patterns can further be utilized in process diagnosis in manufacturing phase of product realization. This research also enables the concurrent design of process operations and design of tooling, such as fixture layout design, etc.

(iii) *An innovative engineering-driven factor analysis method for variation source identification*. Different from existing engineering-driven approaches and data-driven approaches, the proposed one integrates the qualitatively-represented engineering domain knowledge with the multivariate statistical analysis, without the dependence on the complex, accurate quantitative SoV model. Instead, the engineering knowledge on product/process interactions is represented as a series of qualitative indicator vectors, which are used to direct the estimation of true spatial patterns from the eigen-analysis of multivariate measurement data. This approach significantly improves the robustness of the diagnostic results under the situation that the real process has significantly deviating from its nominal condition, and/or the accurate engineering domain knowledge is not available. The proposed generic approach can also be applied to the study of all the process whose input and output can be linearly related to each other.

Table 1-1 lists the research topics of quality assurance for MMP's in design and manufacturing phases, the existing methodologies and the new advancements that have been achieved based on this dissertation research. Also listed are the problems need to be addressed as future research, which will be discussed in the next section.

5.2 Discussion of Future Research

This dissertation presented initial research efforts in a general framework quality assurance in MMP's. In order to achieve the ultimate research objective of a unified methodology that can be applied to different phases of product realization, much more problems should be solved. Some issues worth investigating are:

(i) *Improving SoV model for covering more processes.* Current state space model are mainly based on the engineering knowledge about the Kinematic relationships between variation sources and KPC's. Dimensional variation can be modeled under certain assumptions, such as small variation magnitudes, acceptable linearization residuals, etc. With the development of new advanced manufacturing processes, more quality features will be considered and the interaction between process variation sources and quality features may not sufficiently described based solely on Kinematics. New modeling research should be addressed in three directions: a) including more quality features that is not currently covered; b) incorporating physical experiment results into model to

Table 5-1 Comparison of variation propagation modeling techniques

<table>
<tr><th colspan="2">Research Problems</th><th>Existing Methodologies</th><th>New Advancements</th></tr>
<tr><td rowspan="3">Design</td><td>Process Planning</td><td>Evaluation-oriented (Zhang, et al., 1996; Song, et al., 2005; etc.)
Simulation-based Optimization-oriented (Ong, et al., 2002; Xu and Huang, 2006; etc.)</td><td rowspan="2">Quality Assured Setup Planning for Multistage Machining Processes (Liu et al., 2007a)</td></tr>
<tr><td>Tolerance Synthesis</td><td>Process-oriented (Ding, et al., 2005)</td></tr>
<tr><td>Fixture Layout Design</td><td>Stage-Level (Cai, et al., 1997; Carlson, 2001; etc.)
Process-Level (Kim and Ding, 2004; etc.)</td><td>Future research:
Integrated setup and fixture planning.</td></tr>
<tr><td rowspan="4">Manufa-cturing</td><td>Variation Propagation Modeling</td><td rowspan="3">2D Assembly (Jin and Shi, 1999; etc.)
3D Machining (Zhou et al., 2003; etc.)
SPC (Skinner, 2006; etc.)
MSPC (Zantek, et al., 2006; etc.)
Engineering-driven (Zhou, et al., 2004; etc.)
Data-driven (Wolbrecht, et al., 2000; Li et al., 2008; etc.)</td><td rowspan="3">3D Variation Propagation Modeling for Multistage Assembly Process (Liu, et al., 2007b.)

Engineering-driven Factor Analysis for Variation Source Identification (Liu, et al., 2005.; Liu, et al., 2007c.)</td></tr>
<tr><td>Process Monitoring</td></tr>
<tr><td>Root Cause Diagnosis</td></tr>
<tr><td>Process Control</td><td>SPC-APC (Box and Kramer, 1992; Zhong and Shi, 2007; etc.)
Control and Compensation (Fenner et al., 2005; Izquierdo, et al., 2007; etc.)</td><td>Future Research:
Feed-Forward Control for Multistage Assembly Process with Consideration of SoV Model Uncertainty.</td></tr>
</table>

represent more relationships beyond kinematics; and thus need c) combining the results of data-based statistical modeling and engineering-knowledge-based physical modeling. The engineering-driven FA methodology presented in Chapter 4 creates potential of hybrid modeling effort, which is enabled by the integration of existing knowledge and the discovered knowledge.

(ii) *Considering data uncertainty in quality assurance decision making.* Since data plays an increasingly important role in the unified methodology of quality assurance in MMP's, the uncertainty induced by the heterogeneous data will significantly affect the decisions made based on it. For instance, as discussed in Chapter 4, sample uncertainty and measurement noise may alter the identification of variation sources and cause misleading diagnostic conclusion. Also, in aforementioned hybrid modeling, uncertainty induced by experimental data needs to be investigated to ensure the validity of the model. Other decisions, such as those made for fixture layout design and process control, need also consider the uncertainty induced by the model, or by the initial variation of parts. This is because the decision made only considering deterministic situation will not be robust to uncertainty.

(iii) *Developing unified, concurrent and responsive quality assurance methodology for MMP's.* This dissertation presented some initial research efforts. However, the framework based on the generic model and data infrastructure enables the future research works on the concurrent and responsive quality assurance. For instance, in addition to the integrated setup and fixture planning in design phase and the robust control strategy for process compensation in manufacturing phase, research are also needed in quality assured product design, responsive maintenance, and concurrent product/process improvements based on in-process inspection.

All these research will contribute to a potential new paradigm of quality assurance for MMP's. The characteristic of this new paradigm is that, knowledge of quality assurance will be generated from multidisciplinary investigation of various types of data and will be applied to different phases of product realization in a concurrent and responsive way.

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APPENDICES

Appendix I: Coefficient Matrix of Different Fixturing Schemes

For the general 3-2-1 fixture locating scheme, as shown in Figure 2-2 (a), coefficient matrix $\mathbf{T}_{k,j}^{{}^{0}F}$ is defined as

$$
\mathbf{T}_{k,j}^{{}^{0}F} = \left[\begin{array}{cccccc}
(L_{2y}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}})P_{1z}^{{}^{0}F_{k,j}} \big/ C_{11} & (L_{3y}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}})P_{1z}^{{}^{0}F_{k,j}} \big/ C_{11} & (L_{1y}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}})P_{1z}^{{}^{0}F_{k,j}} \big/ C_{11} & -P_{2y}^{{}^{0}F_{k,j}} \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & P_{1y}^{{}^{0}F_{k,j}} \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & 0 \\
(L_{3x}^{{}^{0}F_{k,j}} - L_{2x}^{{}^{0}F_{k,j}})P_{3z}^{{}^{0}F_{k,j}} \big/ C_{11} & (L_{1x}^{{}^{0}F_{k,j}} - L_{3x}^{{}^{0}F_{k,j}})P_{3z}^{{}^{0}F_{k,j}} \big/ C_{11} & (L_{2x}^{{}^{0}F_{k,j}} - L_{1x}^{{}^{0}F_{k,j}})P_{3z}^{{}^{0}F_{k,j}} \big/ C_{11} & P_{3x}^{{}^{0}F_{k,j}} \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & -P_{3x}^{{}^{0}F_{k,j}} \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & 1 \\
L_{3y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}} L_{3x}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{3x}^{{}^{0}F_{k,j}} L_{1y}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{2y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}} \big/ C_{11} & 0 & 0 & 0 \\
L_{2x}^{{}^{0}F_{k,j}} - L_{3x}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{3x}^{{}^{0}F_{k,j}} - L_{1x}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{1x}^{{}^{0}F_{k,j}} - L_{2x}^{{}^{0}F_{k,j}} \big/ C_{11} & 0 & 0 & 0 \\
L_{2y}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{3y}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}} \big/ C_{11} & L_{1y}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}} \big/ C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & -1 \big/ (P_{1y}^{{}^{0}F_{k,j}} - P_{2y}^{{}^{0}F_{k,j}}) & 0
\end{array} \right] \tag{A1}
$$

where

$$C_{11} = L_{3x}^{{}^{0}F_{k,j}} L_{1y}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}} + L_{3y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}} + L_{2y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}} L_{3x}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}} . \qquad \text{(A2)}$$

For the pin-hole fixturing scheme shown in Figure 2-2 (b), fixture locator deviation can be alternatively represented as

$$\mathbf{u}_{k,j}^{{}^{0}F} = \begin{bmatrix} \triangle L_{1z}^{{}^{0}F_{k,j}} & \triangle L_{2z}^{{}^{0}F_{k,j}} & \triangle L_{3z}^{{}^{0}F_{k,j}} & \triangle P_{1x}^{{}^{0}F_{k,j}} & \triangle P_{2x}^{{}^{0}F_{k,j}} & \triangle P_{1y}^{{}^{0}F_{k,j}} \end{bmatrix}^{T}$$

and the coefficient matrix is defined as

$$\mathbf{T}_{k,j}^{{}^{0}F} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ (L_{3y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}} L_{3x}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{3x}^{{}^{0}F_{k,j}} L_{1y}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{2y}^{{}^{0}F_{k,j}} L_{1x}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}} L_{2x}^{{}^{0}F_{k,j}}) \big/ C_{11} & 0 & 0 & 0 \\ (L_{2x}^{{}^{0}F_{k,j}} - L_{3x}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{3x}^{{}^{0}F_{k,j}} - L_{1x}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{1x}^{{}^{0}F_{k,j}} - L_{2x}^{{}^{0}F_{k,j}}) \big/ C_{11} & 0 & 0 & 0 \\ (L_{2y}^{{}^{0}F_{k,j}} - L_{3y}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{3y}^{{}^{0}F_{k,j}} - L_{1y}^{{}^{0}F_{k,j}}) \big/ C_{11} & (L_{1y}^{{}^{0}F_{k,j}} - L_{2y}^{{}^{0}F_{k,j}}) \big/ C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \big/ P_{2y}^{{}^{0}F_{k,j}} & 1 \big/ P_{2y}^{{}^{0}F_{k,j}} & 0 \end{bmatrix} \qquad \text{(A3)}$$

Appendix II: Derivation of Coefficient Matrices $\mathbf{T}_{1,k,j}$ through $\mathbf{T}_{6,k,j}$

Given the six datum features defined in Section 2.2.1, matrices $\mathbf{T}_1$ through $\mathbf{T}_6$ can be derived by investigating the six datum points where fixture locating pins and datum features touch with each other. It is assumed that datum points q_1, q_2 and q_3 are the datum points touching the three primary datum features, $p_{1,k,j}$, $p_{2,k,j}$, and $p_{3,k,j}$, respectively; q_4 and q_5, touch with the secondary datum features, $s_{1,k,j}$ and $s_{2,k,j}$, respectively, and q_6 touch with the tertiary datum feature $t_{k,j}$. The nominal coordinates of these six points in $F_{k,j}$ are denoted as $\mathbf{q}_1^{F_{k,j}}$, $\mathbf{q}_2^{F_{k,j}}$, $\mathbf{q}_3^{F_{k,j}}$, $\mathbf{q}_4^{F_{k,j}}$, $\mathbf{q}_5^{F_{k,j}}$ and $\mathbf{q}_6^{F_{k,j}}$, respectively. Denoting $\tilde{\mathbf{q}} = \begin{bmatrix} \mathbf{q}^{\mathrm{T}} & 1 \end{bmatrix}^{\mathrm{T}}$, the coordinates of point q_1 w.r.t. datum feature, $\tilde{\mathbf{q}}_1^{p_{1,k,j}}$, can be achieved by performing homogeneous transformation twice, i.e., from $F_{k,j}$ to RCS and from RCS to LCS of $p_{1,k,j}$. With the same strategy, following transformation relationships can be obtained:

$$\begin{cases} \mathbf{H}_R^{p_{1,k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_1^{F_{k,j}} = \tilde{\mathbf{q}}_1^{p_{1,k,j}} \\ \mathbf{H}_R^{p_{2,k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_2^{F_{k,j}} = \tilde{\mathbf{q}}_2^{p_{2,k,j}} \\ \mathbf{H}_R^{p_{3,k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_3^{F_{k,j}} = \tilde{\mathbf{q}}_3^{p_{3,k,j}} \\ \mathbf{H}_R^{s_{1,k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_4^{F_{k,j}} = \tilde{\mathbf{q}}_4^{s_{1,k,j}} \\ \mathbf{H}_R^{s_{2,k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_5^{F_{k,j}} = \tilde{\mathbf{q}}_5^{s_{2,k,j}} \\ \mathbf{H}_R^{t_{k,j}} \mathbf{H}_{F_{k,j}}^R \tilde{\mathbf{q}}_6^{F_{k,j}} = \tilde{\mathbf{q}}_6^{t_{k,j}} \end{cases}. \tag{A4}$$

When the parts are fixed, six locating pins are in touch with the six datum features. Therefore, points q_1 though q_6 will be right on the datum feature. Since the z direction is defined as the normal direction of the surface, the z coordinates of these six points, w.r.t corresponding datum feature, will be zeros, i.e., $\left[\tilde{\mathbf{q}}_i^{\bullet,k,j}\right]_{(3)} = 0$ for i=1,2,…,6 and $\bullet \in \{p_1, p_2, p_3, s_1, s_2, t\}$. $[\mathrm{v}]_{(3)}$ denotes the third (3rd) element of vector v. It is easy to show that q_1 touching $p_{1,k,j}$ leads to

$$\left[\mathbf{\Delta}_{p_{1,k,j}}^R \cdot {}^0\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} \cdot \tilde{\mathbf{q}}_1^{F_{k,j}}\right]_{(3)} = \left[\mathbf{\Delta}_{p_{1,k,j}}^R \cdot {}^0\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} \cdot \tilde{\mathbf{q}}_1^{F_{k,j}}\right]_{(3)}, \tag{A5}$$

where

$$\boldsymbol{\Delta}_{p_{1,k,j}}^{R} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{p_{1,k,j}}^{R} & \mathbf{d}_{p_{1,k,j}}^{R} \\ \mathbf{0} & 0 \end{bmatrix}, \text{ and}$$

$$^{0}\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} = \begin{bmatrix} ^{0}\mathbf{n}_{F_{k,j}}^{p_{1,k,j}} & ^{0}\mathbf{o}_{F_{k,j}}^{p_{1,k,j}} & ^{0}\mathbf{a}_{F_{k,j}}^{p_{1,k,j}} & ^{0}\mathbf{t}_{F_{k,j}}^{p_{1,k,j}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Eq. (A5) can be further manipulated as

$$\left[\boldsymbol{\Delta}_{p_{1,k,j}}^{R} \cdot {}^{0}\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} \cdot \tilde{\mathbf{q}}_{1}^{F_{k,j}}\right]_{(3)} = \left[\left[{}^{0}\mathbf{a}_{p_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \quad \left[\mathbf{q}_{1}^{F_{k,j}} \times {}^{0}\mathbf{a}_{p_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}}\right] \cdot \mathbf{x}_{F_{k,j}}^{R}, \text{ and} \tag{A6}$$

$$\begin{aligned} \left[\boldsymbol{\Delta}_{p_{1,k,j}}^{R} \cdot {}^{0}\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} \cdot \tilde{\mathbf{q}}_{1}^{F_{k,j}}\right]_{(3)} = \Big[& \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R} \times {}^{0}\mathbf{n}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} \quad \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R} \times {}^{0}\mathbf{o}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} \\ & \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R} \times {}^{0}\mathbf{a}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} \quad \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R} \times {}^{0}\mathbf{t}_{F_{k,j}}^{p_{1,k,j}} + \mathbf{d}_{p_{1,k,j}}^{R}\right]_{(3)} \Big] \cdot \tilde{\mathbf{q}}_{1}^{F_{k,j}}. \end{aligned} \tag{A7}$$

Same strategy can be used to derive the other five equations for q_2 though q_6, and the six equations can be combined together to form an equation system as defined in (A8).

$$\begin{bmatrix} \left[{}^{0}\mathbf{a}_{p_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{1}^{F_{k,j}} \times {}^{0}\mathbf{a}_{p_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \\ \left[{}^{0}\mathbf{a}_{p_{2,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{2}^{F_{k,j}} \times {}^{0}\mathbf{a}_{p_{2,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \\ \left[{}^{0}\mathbf{a}_{p_{3,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{3}^{F_{k,j}} \times {}^{0}\mathbf{a}_{p_{3,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \\ \left[{}^{0}\mathbf{a}_{s_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{4}^{F_{k,j}} \times {}^{0}\mathbf{a}_{s_{1,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \\ \left[{}^{0}\mathbf{a}_{s_{2,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{5}^{F_{k,j}} \times {}^{0}\mathbf{a}_{s_{2,k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \\ \left[{}^{0}\mathbf{a}_{t_{k,j}}^{F_{k,j}}\right]^{\mathrm{T}} & \left[\mathbf{q}_{6}^{F_{k,j}} \times {}^{0}\mathbf{a}_{t_{k,j}}^{F_{k,j}}\right]^{\mathrm{T}} \end{bmatrix} \cdot \mathbf{x}_{F_{k,j}}^{R} =$$

$$
\begin{bmatrix}
\left[\left[\boldsymbol{\theta}_{p_{1,k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{p_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{1,k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{p_{1,k,j}}+\mathbf{d}_{p_{1,k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{1}^{F_{k,j}} \\
\left[\left[\boldsymbol{\theta}_{p_{2,k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{p_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{2,k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{p_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{2,k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{p_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{2,k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{p_{2,k,j}}+\mathbf{d}_{p_{2,k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{2}^{F_{k,j}} \\
\left[\left[\boldsymbol{\theta}_{p_{3,k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{p_{3,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{3,k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{p_{3,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{3,k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{p_{3,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{p_{3,k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{p_{3,k,j}}+\mathbf{d}_{p_{3,k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{3}^{F_{k,j}} \\
\left[\left[\boldsymbol{\theta}_{s_{1,k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{s_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{1,k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{s_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{1,k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{s_{1,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{1,k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{s_{1,k,j}}+\mathbf{d}_{s_{1,k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{4}^{F_{k,j}} \\
\left[\left[\boldsymbol{\theta}_{s_{2,k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{s_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{2,k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{s_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{2,k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{s_{2,k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{s_{2,k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{s_{2,k,j}}+\mathbf{d}_{s_{2,k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{5}^{F_{k,j}} \\
\left[\left[\boldsymbol{\theta}_{t_{k,j}}^{R}\times{}^{0}\mathbf{n}_{F_{k,j}}^{t_{k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{t_{k,j}}^{R}\times{}^{0}\mathbf{o}_{F_{k,j}}^{t_{k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{t_{k,j}}^{R}\times{}^{0}\mathbf{a}_{F_{k,j}}^{t_{k,j}}\right]_{(3)} & \left[\boldsymbol{\theta}_{t_{k,j}}^{R}\times{}^{0}\mathbf{t}_{F_{k,j}}^{t_{k,j}}+\mathbf{d}_{t_{k,j}}^{R}\right]_{(3)}\right]\cdot\tilde{\mathbf{q}}_{6}^{F_{k,j}}
\end{bmatrix}
$$

(A8)

By solving the equation system, we can express the DMV $\mathbf{x}_{F_{k,j}}^{R}$ as a linear combination of the deviations of datum features, as defined in Eq.(2-8). The fixture locating scheme ensure that all the six degree-of-freedom of the part/subassembly will be constrained. At the mean time, it also guarantees that the inverse of the matrix on the left hand side of Eq. (A8) exists. In practice, the primary, secondary and tertiary datum surfaces are often defined to be orthogonal to each other. This standardization, as illustrated in Figure 2-2, simplifies the derivation of $T_{1,k,j}$ through $T_{6,k,j}$. Given the nominal coordinates of q_1 through q_6 in $F_{k,j}$, $\mathbf{q}_1^{F_{k,j}} = \begin{bmatrix} q_{1x}^{F_{k,j}} & q_{1y}^{F_{k,j}} & 0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{q}_2^{F_{k,j}} = \begin{bmatrix} q_{2x}^{F_{k,j}} & q_{2y}^{F_{k,j}} & 0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{q}_3^{F_{k,j}} = \begin{bmatrix} q_{3x}^{F_{k,j}} & q_{3y}^{F_{k,j}} & 0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{q}_4^{F_{k,j}} = \begin{bmatrix} 0 & q_{4y}^{F_{k,j}} & q_{4z}^{F_{k,j}} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{q}_5^{F_{k,j}} = \begin{bmatrix} q_{5x}^{F_{k,j}} & q_{5y}^{F_{k,j}} & q_{5z}^{F_{k,j}} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{q}_6^{F_{k,j}} = \begin{bmatrix} q_{6x}^{F_{k,j}} & 0 & q_{6z}^{F_{k,j}} \end{bmatrix}^{\mathrm{T}}$, and nominal HTM for $F_{k,j}$ w.r.t. the LCS's of three PD features ($p_{1,k,j}$, $p_{2,k,j}$, and $p_{3,k,j}$),

$$
{}^{0}\mathbf{H}_{F_{k,j}}^{p_{1,k,j}} = \begin{bmatrix} 1 & 0 & 0 & {}^{0}t_{Fx_{k,j}}^{p_{1,k,j}} \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{p_{1,k,j}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{0}\mathbf{H}_{F_{k,j}}^{p_{2,k,j}} = \begin{bmatrix} 1 & 0 & 0 & {}^{0}t_{Fx_{k,j}}^{p_{2,k,j}} \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{p_{2,k,j}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^{0}\mathbf{H}_{F_{k,j}}^{p_{3,k,j}} = \begin{bmatrix} 1 & 0 & 0 & {}^{0}t_{Fx_{k,j}}^{p_{3,k,j}} \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{p_{3,k,j}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The coefficient matrices for modeling PD induced deviation are:

$$\mathbf{T}_{1,k,j}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{C_2}{C_1} & \frac{C_2}{C_1}\cdot(q_{1y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{1,k,j}}) & -\frac{C_2}{C_1}\cdot(q_{1x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{1,k,j}}) & 0 \\ 0 & 0 & \frac{C_3}{C_1} & \frac{C_3}{C_1}\cdot(q_{1y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{1,k,j}}) & -\frac{C_3}{C_1}\cdot(q_{1x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{1,k,j}}) & 0 \\ 0 & 0 & \frac{C_4}{C_1} & \frac{C_4}{C_1}\cdot(q_{1y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{1,k,j}}) & -\frac{C_4}{C_1}\cdot(q_{1x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{1,k,j}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{T}_{2,k,j}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{C_5}{C_1} & -\frac{C_5}{C_1}\cdot(q_{2y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{2,k,j}}) & \frac{C_5}{C_1}\cdot(q_{2x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{2,k,j}}) & 0 \\ 0 & 0 & -\frac{C_6}{C_1} & -\frac{C_6}{C_1}\cdot(q_{2y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{2,k,j}}) & \frac{C_6}{C_1}\cdot(q_{2x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{2,k,j}}) & 0 \\ 0 & 0 & -\frac{C_7}{C_1} & -\frac{C_7}{C_1}\cdot(q_{2y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{2,k,j}}) & \frac{C_7}{C_1}\cdot(q_{2x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{2,k,j}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{T}_{3,k,j}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{C_8}{C_1} & \frac{C_8}{C_1}\cdot(q_{3y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{3,k,j}}) & -\frac{C_8}{C_1}\cdot(q_{3x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{3,k,j}}) & 0 \\ 0 & 0 & \frac{C_9}{C_1} & \frac{C_9}{C_1}\cdot(q_{3y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{3,k,j}}) & -\frac{C_9}{C_1}\cdot(q_{3x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{3,k,j}}) & 0 \\ 0 & 0 & \frac{C_{10}}{C_1} & \frac{C_{10}}{C_1}\cdot(q_{3y}^{F_{k,j}}+{}^{0}t_{Fy_{k,j}}^{p_{3,k,j}}) & -\frac{C_{10}}{C_1}\cdot(q_{3x}^{F_{k,j}}+{}^{0}t_{Fx_{k,j}}^{p_{3,k,j}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $C_1 = q_{2x}^{F_{k,j}}\cdot q_{3y}^{F_{k,j}} - q_{2y}^{F_{k,j}}\cdot q_{3x}^{F_{k,j}} + q_{1y}^{F_{k,j}}\cdot q_{3x}^{F_{k,j}} - q_{1x}^{F_{k,j}}\cdot q_{3y}^{F_{k,j}} - q_{1y}^{F_{k,j}}\cdot q_{2x}^{F_{k,j}} + q_{1x}^{F_{k,j}}\cdot q_{2y}^{F_{k,j}}$, $C_2 = q_{2x}^{F_{k,j}}\cdot q_{3y}^{F_{k,j}} - q_{2y}^{F_{k,j}}\cdot q_{3x}^{F_{k,j}}$, $C_3 = q_{3x}^{F_{k,j}} - q_{2x}^{F_{k,j}}$, $C_4 = q_{3y}^{F_{k,j}} - q_{2y}^{F_{k,j}}$, $C_5 = q_{1x}^{F_{k,j}}\cdot q_{3y}^{F_{k,j}} - q_{1y}^{F_{k,j}}\cdot q_{3x}^{F_{k,j}}$, $C_6 = q_{3x}^{F_{k,j}} - q_{1x}^{F_{k,j}}$, $C_7 = q_{3y}^{F_{k,j}} - q_{1y}^{F_{k,j}}$, $C_8 = q_{1x}^{F_{k,j}}\cdot q_{2y}^{F_{k,j}} - q_{1y}^{F_{k,j}}\cdot q_{2x}^{F_{k,j}}$,

$C_9 = q_{2x}^{F_{k,j}} - q_{1x}^{F_{k,j}}$, and $C_{10} = q_{2y}^{F_{k,j}} - q_{1y}^{F_{k,j}}$. Given the nominal HTM for $F_{k,j}$ w.r.t. the LCS's of SD features ($s_{1,k,j}$, $s_{2,k,j}$) and that of TD feature $t_{k,j}$,

$$
{}^{0}\mathbf{H}_{F_{k,j}}^{s_{1,k,j}} = \begin{bmatrix} 0 & 0 & 1 & {}^{0}t_{Fx_{k,j}}^{s_{1,k,j}} \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{s_{1,k,j}} \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{0}\mathbf{H}_{F_{k,j}}^{s_{2,k,j}} = \begin{bmatrix} 0 & 0 & 1 & {}^{0}t_{Fx_{k,j}}^{s_{2,k,j}} \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}} \\ -1 & 0 & 0 & {}^{0}t_{Fz_{k,j}}^{s_{2,k,j}} \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{0}\mathbf{H}_{F_{k,j}}^{t_{k,j}} = \begin{bmatrix} 1 & 0 & 0 & {}^{0}t_{Fx_{k,j}}^{t_{k,j}} \\ 0 & 0 & 1 & {}^{0}t_{Fy_{k,j}}^{t_{k,j}} \\ 0 & -1 & 0 & {}^{0}t_{Fz_{k,j}}^{t_{k,j}} \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

the coefficient matrices for modeling SD and TD induced deviation are:

$$
\mathbf{T}_{4,k,j} = \frac{1}{q_{4y}^{F_{k,j}} - q_{5y}^{F_{k,j}}} \begin{bmatrix} 0 & 0 & q_{5y}^{F_{k,j}} & q_{5y}^{F_{k,j}}(q_{4y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{1,k,j}}) & -q_{5y}^{F_{k,j}}(q_{4x}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{s_{1,k,j}}) & 0 \\ 0 & 0 & -q_{6x}^{F_{k,j}} & -q_{6x}^{F_{k,j}}(q_{4y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{1,k,j}}) & q_{6x}^{F_{k,j}}(q_{4x}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{s_{1,k,j}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_{4y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{1,k,j}} & -q_{4z}^{F_{k,j}} - {}^{0}t_{Fx_{k,j}}^{s_{1,k,j}} & 0 \end{bmatrix}
$$

$$
\mathbf{T}_{5,k,j} = \frac{1}{q_{4y}^{F_{k,j}} - q_{5y}^{F_{k,j}}} \begin{bmatrix} 0 & 0 & -q_{4y}^{F_{k,j}} & q_{4y}^{F_{k,j}}(q_{5y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}}) & q_{4y}^{F_{k,j}}(q_{5z}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{s_{2,k,j}}) & 0 \\ 0 & 0 & q_{6x}^{F_{k,j}} & q_{6x}^{F_{k,j}}(q_{5y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}}) & -q_{6x}^{F_{k,j}}(q_{5z}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{s_{2,k,j}}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -q_{5y}^{F_{k,j}} - {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}} & q_{5z}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{s_{2,k,j}} & 0 \end{bmatrix}
$$

$$
\mathbf{T}_{6,k,j} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -q_{6y}^{F_{k,j}} - {}^{0}t_{Fy_{k,j}}^{t_{k,j}} & q_{6x}^{F_{k,j}} + {}^{0}t_{Fx_{k,j}}^{t_{k,j}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$

For pin-hole fixturing scheme shown in Figure 2-2 (b), the nominal HTM for $F_{k,j}$ w.r.t. the LCS's of SD features ($s_{1,k,j}$, $s_{2,k,j}$) and that of TD feature $t_{k,j}$, are

$$ {}^{0}\mathbf{H}_{F_{k,j}}^{s_{1,k,j}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{0}\mathbf{H}_{F_{k,j}}^{s_{2,k,j}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}} \\ -1 & 0 & 0 & {}^{0}t_{Fz_{k,j}}^{s_{2,k,j}} \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{0}\mathbf{H}_{F_{k,j}}^{t_{k,j}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

and the coefficient matrices for modeling SD and TD induced deviation will be

$$ \mathbf{T}_{4,k,j}^{*} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1\big/ q_{5y}^{F_{k,j}} & 0 & 0 & 0 \end{bmatrix}, $$

$$ \mathbf{T}_{5,k,j}^{*} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1\big/ q_{5y}^{F_{k,j}} & \left(q_{5y}^{F_{k,j}} + {}^{0}t_{Fy_{k,j}}^{s_{2,k,j}}\right)\big/ q_{5y}^{F_{k,j}} & 0 & 0 \end{bmatrix}, $$

$$ \mathbf{T}_{6,k,j}^{*} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. $$

Appendix III: Derivation of State Transition Equation

This appendix section shows the procedure for deriving Eq.(2-23): Substituting Eq. (2-21) into Eq. (2-22), we have

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{A}_k(4) \cdot \mathbf{x}^R_{F_k} + {}^O\mathbf{u}_k, \tag{A9}$$

Further plugging in Eq. (2-19), we have

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{A}_k(4) \cdot \left[{}^D\mathbf{x}^R_{F_k} + {}^F\mathbf{x}^R_{F_k} \right] + {}^O\mathbf{u}_k, \tag{A10}$$

Substituting Eq. (2-13) and Eq. (2-17) into (A10), we have

$$\begin{aligned} \mathbf{x}_k = \mathbf{x}_{k-1} &+ \mathbf{A}_k(4)\mathbf{A}_k(2)\mathbf{A}_k(1)\,{}^P\mathbf{x}^R_k \\ &+ \mathbf{A}_k(4)\mathbf{A}_k(2)\,{}^S\mathbf{u}_k + \mathbf{A}_k(4)\mathbf{A}_k(3)\,{}^F\mathbf{u}_k + {}^O\mathbf{u}_k, \end{aligned} \tag{A11}$$

And finally by substituting ${}^P\mathbf{x}^R_k$ by Eq. (2-12), we can achieve

$$\begin{aligned} \mathbf{x}_k = \mathbf{x}_{k-1} &+ \mathbf{A}_k(4)\mathbf{A}_k(2)\mathbf{A}_k(1)\mathbf{A}_k(0)\mathbf{x}_{k-1} \\ &+ \mathbf{A}_k(4)\mathbf{A}_k(2)\,{}^S\mathbf{u}_k + \mathbf{A}_k(4)\mathbf{A}_k(3)\,{}^F\mathbf{u}_k + {}^O\mathbf{u}_k, \end{aligned} \tag{A12}$$

This leads to Eq. (2-23).