On the Role of Negotiation in Revenue Management and Supply Chain

by

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To My Family

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TABLE OF CONTENTS

LIST OF FIGURES

Figure

LIST OF TABLES

Table

LIST OF APPENDICES

Appendix

CHAPTER 1

Introduction

For many products such as cars, electronic appliances and furniture, the price that a customer pays is negotiated from the posted price. In such transactions, negotiation plays an important role in determining whether a customer purchases, and if so, at what price. We know all too well how common negotiation is when buying a car or other "big-ticket" items such as furniture and home appliances. Perhaps surprisingly, negotiation is becoming a more acceptable practice even at retailers selling small-ticket items. For example, two reporters from *The Wall Street Journal* who negotiated at 50 retail stores during the holiday season of 2001, were successful in getting a discount off the posted price at 18 of these stores for products ranging from a personal digital assistant ($20) to a fitness machine ($50) (Agins and Collins, 2001). The stores at which they got discounts range from Eddie Bauer to Sunglass Hut, from Kenneth Cole to Salvatore Ferragamo.

A retailer may want to adopt negotiation because negotiation allows retailers to price discriminate among customers with heterogenous willingness-to-pay, compared to traditional posted pricing strategy. However, negotiating the price with customers often comes at a cost, which possibly offsets the benefit generated from price discrimination. This becomes the key trade-off for retailers when considering whether to use

negotiation as a sales format or not. We have seen in many cases, different retailers use different sales formats when they sell the same product. In September 2007, Lithia Automotive Group, the eighth-largest dealer chain in the U.S., selling vehicles from all major manufacturers and brands (ranging from Porsche to GM to Toyota), announced that it would convert all of its 108 stores to haggle-free pricing within the next three years (Welch, 2007). Of course, many competing dealers will stick to the time-honored tradition of bargaining. Different sales format co-exist in other retail settings as well. For example, a store like Costco will sell home appliances or electronics at the posted prices without any room for a haggle, while consumers can successfully negotiate for such items at smaller discount stores such as Big George's in southeast Michigan. Furthermore, it has been recently reported that major retailers (including BestBuy, Home Depot, and Circuit City) are allowing their sales people to negotiate with customers. In fact, the cash registers at BestBuy are set up so that the final price can be reset at the time of check out (Richtel, 2008). All of these examples suggest that retailers actively decide whether to use negotiation or posted pricing.

In cases where a retailer has chosen to use negotiation, the starting point for negotiation will be the product's posted price or the sticker price chosen by the retailer. In many cases, retailers adjust the posted price during the selling season. For example, electronics stores such as Best Buy or Circuit City change the posted price of high-tech products since these products are outdated quickly. Likewise, department stores and "style goods" apparel retailers (such as Kenneth Cole and Sunglass Hut) frequently change the posted price during the course of the season. One reason these retailers turn to dynamic price adjustments is the difficulty of inventory replenishment during the selling season coupled with the drastic depreciation of product value

once the season is over. For example, it is well known that many apparel retailers face long replenishment lead times since most items are produced off-shore, which precludes the possibility of further replenishment during the short selling season of fashion items. (See, for example, Gallego and van Ryzin, 1994.) Therefore, adjusting price in response to sales helps these retailers improve their revenue from the limited inventory available. Such "dynamic pricing" practices may push the price up or down depending on the inventory level of the product and the remaining time in the selling season. Negotiation, on the other hand, tends to drive the price up, so that the retailer has more room to negotiate the price. Thus, when determining the price, the effects of dynamic pricing and negotiation may combine together or oppose one another. In Chapter 2 of this dissertation, we investigate the interaction between dynamic pricing and negotiation. We analyze if negotiation is more or less beneficial to the retailer in the presence of dynamic pricing, and how such benefits are influenced by inventory level and the length of the selling season.

Chapter 2 focuses on the effect of negotiation on the retailer, using a dynamic model of pricing decisions. However, the sales format that a retailer chooses not only determines the retailer's profits, but also influences the profits of the parties in a supply chain, in particular, the manufacturers who provide the goods sold by the retailer. In Chapter 3, we focus on the supply chain implications of the retailer's sales format choice. A retailer's decision to negotiate or not is driven by its self-interest, and the effect of this decision on a manufacturer's profit is unclear. In particular, the sales quantity, and therefore the quantity that the retailer orders from the manufacturer, are likely to be different between the two sales formats. Hence, the manufacturer would like to influence the retailer's sales format choice through the terms of trade. There are many ways in which the manufacturer can influence the retailer's deci-

sion, and even for a simple contract like the wholesale-price-only contract, it is not clear how the manufacturer's decision drives the retailer's choice. For example, as wholesale price increases, thereby increasing the manufacturer's unit profit margin, does the retailer come closer to using negotiation or posted pricing? In Chapter 3 of this dissertation, we answer such questions and highlight how the manufacturer's wholesale price promotes (or discourages) one sales format over another and what sales format and price choices may arise in equilibrium. The sales format choice – negotiation or posted pricing – may be even more critical in the presence of tight capacity constraints in the supply chain. Hence, Chapter 3 pays special attention to the effect of supply chain capacity on the equilibrium sales format, pricing and quantity decisions.

1.1 Overview of Chapter 2

Chapter 2 of the dissertation discusses how negotiation and revenue management interact so as to influence the retailer's revenue. A stochastic dynamic programming formulation is employed to embed a negotiation model in a more traditional dynamic pricing model. We consider a retailer who has limited inventories at the beginning of a relatively short selling season. In our model, the outcome of negotiation between the retailer and the customer depends on the retailer's inventory, the remaining time in the selling season and the price posted by the retailer.

This model produces a number of interesting analytical and numerical results. As one would expect, the optimal posted price of a negotiating seller includes a premium over that of the seller using take-it-or-leave-it pricing strategy. This price premium helps the seller extract more revenues from a bargainer with a high reservation price, and, surprisingly, peaks at moderate inventory levels. As the negotiating seller adds

a price premium, some customers will be worse off when buying from the negotiating seller (compared to buying from the seller using take-it-or-leave-it pricing). Nonetheless, we show that a group of bargainers with low reservation prices benefit from negotiation, in particular when the price premium is high. As expected, negotiation helps improve the seller's revenue, especially when inventory level is high. More surprisingly, we find that negotiation can act as a substitute or complement to dynamic pricing. For example, at moderate inventory levels, the benefit from dynamic pricing increases further when the seller can negotiate.

1.2 Overview of Chapter 3

In Chapter 3, we consider a supply chain has limited capacity and the retailer in the supply chain chooses one of two pricing regimes, posting a fixed price or negotiating, when selling to customers who are heterogeneous in their willingness-to-pay. The generalized Nash bargaining solution is employed to further explore the bargaining power of both the customer and the retailer, and determine the negotiation outcome where the customer and the retailer reach an agreement. We analyze the retailer's quantity and pricing regime decisions as well as the manufacturer's inducement of a pricing regime via the wholesale price. We pay special attention to the effect of capacity and negotiation cost on the equilibrium outcome. In addition, we analyze how the retailer's discretion to pick the sales format influences the manufacturer's profit.

We find that three types of equilibria may arise, depending on the cost of negotiation and the capacity of the supply chain: When the cost of negotiation is sufficiently low and the manufacturer's capacity is sufficiently high, the supply chain ends up at a negotiation equilibrium, which is the same as the equilibrium that would arise if

negotiation is the exogenous sales format. When the cost of negotiation is sufficiently high and the manufacturer's capacity is sufficiently low, the supply chain ends up at a posted pricing equilibrium, which again coincides with the equilibrium that would arise if posted pricing is the exogenous sales format. In between these two, at moderately high negotiation costs and capacity levels, the supply chain may settle at a different equilibrium. In this region, the manufacturer would prefer the retailer to use negotiation, but must offer the retailer a discounted wholesale price to induce such an outcome. This leads to a number of interesting observations: A retailer with a higher cost of negotiation may earn more in equilibrium than a retailer with a lower cost of negotiation, because the manufacturer concedes some profit margin to the high-cost retailer in order to induce it to use negotiation, when such a sacrifice is not needed when working with a low-cost retailer.

CHAPTER 2

Interaction between Negotiation and Revenue Management

2.1 Introduction

As discussed in Chapter 1, many retailers adjust posted prices dynamically, and some customers are able to negotiate discounts from these posted prices. Both dynamic adjustment of posted prices (hereafter, dynamic pricing) and negotiation are used by many retailers to increase the revenue, particularly when the inventory is limited. Although both strategies are used to improve the retailer's revenue, there are differences in how each one achieves this goal. Dynamic pricing adjusts the margin based on the inventory level relative to the remaining selling season: The posted price will increase when inventory is low, and will decrease when inventory is high. On the other hand, negotiation enables the retailer to extract more revenue from individual customers: The firm can set a high posted price, and those customers with high willingness-to-pay may buy the product after little or no negotiation, while others with low willingness-to-pay will buy at discounted prices after negotiating with the retailer. Therefore, the retailer chooses a high posted price in order to improve the range of price discrimination enabled by negotiation.

Depending on the inventory level, negotiation and dynamic pricing can drive the

posted price in the same or opposite direction. When there is little inventory of the product, the retailer's tendency to set high posted prices under negotiation will be reinforced by dynamic pricing. On the other hand, if the risk of excess inventory at the end of the season is significant, then dynamic pricing drives the posted price down to move the product faster, but such low posted prices reduce the retailer's ability to price discriminate via negotiation. These interactions among dynamic pricing, negotiation, and inventory motivate our research, which fills a gap in the literature by considering the joint use of dynamic pricing and negotiation.

In this chapter, we propose a model where negotiation and dynamic pricing take place together. We consider a seller who has limited inventory at the beginning of a relatively short selling season. We divide the season into periods, each of them short enough so that at most one customer can arrive in a period. The customer population is comprised of two types of consumers — price-takers and bargainers. Price-takers either buy at the posted price or quit without purchasing. On the other hand, bargainers initiate a negotiation in the hope of getting a discount from the seller. We assume that (as in most retail settings) negotiation typically happens over a short time span (sometimes a matter of minutes) within which the seller and the bargainer exchange a limited number of offers and counter-offers. At the end of the negotiation, one would expect to see many different outcomes: The bargainer may successfully negotiate a discount (the size of which may vary), end up buying at the posted price, or quit without purchasing.

We develop a negotiation model that assumes a limited number of exchanges while capturing the negotiation outcomes mentioned above: the bargainer makes an offer that is countered by the seller, which the bargainer either accepts or rejects. The bargainer's offer depends on the posted price (which affects the bargainer's beliefs

about the seller's valuation of the product). On the other hand, the seller's counter-offer depends on the bargainer's offer, the inventory, and time until the end of the selling season.

The remainder of the chapter is organized as follows. Section 2.2 provides a survey of the relevant literature. Section 2.3 outlines our model where a negotiation model is embedded into a dynamic pricing problem. In this section, we characterize the outcome of negotiation as a function of posted price, inventory level, and remaining time. In Section 2.4, under certain distributional assumptions on reservation prices, we derive analytical results regarding the optimal posted price. We then describe the results of our numerical study in Section 2.5. We conclude in Section 2.6. All proofs are provided in Appendix A.

2.2 Literature Review

There has been a significant volume of research in dynamic pricing of limited inventories in the last decade. Starting with Gallego and van Ryzin (1994), and Bitran and Mondschein (1997), this research focuses on products whose inventory cannot be replenished during their relatively short selling season, and the key question is how the seller should adjust the price of the product based on remaining time and inventory in order to maximize the total revenue over the selling season. For recent reviews of the literature, see Bitran and Caldentey (2003), and Elmaghraby and Keskinocak (2003). As the use of dynamic pricing has spread from airline and travel industries to the retail industry, researchers have studied many different dynamic pricing problems that correspond to specific business applications, such as dynamic pricing for multiple products (e.g., Zhang and Cooper, 2005, and Maglaras and Meissner, 2006), dynamic pricing in the presence of strategic consumers (e.g.,

Aviv and Pazgal, 2005, Elmaghraby, Gulcu and Keskinocak, 2006, Su, 2007, and Zhou, Fan and Cho, 2006), the use of dynamic pricing and discounting when making product offers to customers (e.g., Netessine, Savin and Xiao, 2006, and Aydin and Ziya, 2006) and dynamic pricing when the demand in each period is affected by prices over multiple periods (e.g., Popescu and Wu, 2006, and Ahn, Gumus and Kaminsky, 2007). However, the existing work on dynamic pricing has not considered retail situations where the customer can initiate a negotiation on the price of the product. Our contribution is to investigate the interaction between dynamic pricing and negotiation, and to analyze the effect of negotiation on the seller and the consumers in a setting where prices are adjusted dynamically.

In the majority of the existing work on dynamic pricing, a common assumption is that a customer, upon arrival, will observe the current price chosen by the seller, and if the customer purchases the product, she will buy at the posted price. Some of the more recent work on dynamic pricing makes alternative assumptions in this regard. For example, in most of the work regarding dynamic pricing in the presence of strategic consumers, the customer decides when and/or at what price to purchase. Nevertheless, while customers purchase at a price/time of their choice, the customer's choice is still limited to the prices posted by the seller in the course of the season. In contrast, in our model, the customer observes the posted price chosen by the seller and may make an offer to start the negotiation process. The posted price, inventory level of the seller and the time remaining in the selling season all influence the eventual outcome of the negotiation, i.e., whether the customer will buy and at what price.

Another research topic closely related to our model is bargaining, which has been studied extensively in economics. For a detailed review of the theory and

applications of bargaining, see Muthoo (1999). Two classic bargaining models in economics are the Nash bargaining solution and the Rubinstein model. Under the Nash bargaining solution, two parties bargaining over a surplus split the difference between the total surplus and the sum of their reservation utilities (also known as disagreement payoffs). The Nash bargaining solution does not specify an explicit bargaining procedure leading to this outcome. One interpretation is that two fully rational parties make simultaneous offers. The Rubinstein model, on the other hand, views bargaining as a series of alternating offers between two parties bargaining over a surplus. In its most basic form, the Rubinstein model assumes that the two parties have full information regarding each other's utilities and they make alternating offers with a fixed time interval between two successive offers to maximize discounted utility. This bargaining process leads to a unique subgame perfect equilibrium where the parties immediately settle at the very beginning of the bargaining process. The equilibrium of the Rubinstein model yields the Nash bargaining solution when the time interval between two offers approaches zero (or discount factor approaches one). There has been a wealth of further research in economics focusing on the bargaining between buyers and sellers. Among them, Chatterjee and Samuelson (1983) analyze a bargaining model where the buyer and seller have incomplete information about each other's valuations of the product and make simultaneous offers. Farrell and Gibbons (1989) consider the same model as Chatterjee and Samuelson with one difference: A party can use cheap talk prior to bargaining, which influences the other party's belief regarding the first party's valuation of the product.

While the bargaining models in economics are attractive and have many applications, they do not provide an appropriate framework for the retail environment we seek to model since they primarily concentrate on the outcome of one buyer and one

seller negotiating over one unit of an item. In our dynamic pricing problem, however, the seller with limited inventory will sell to multiple buyers who arrive at different times in the course of selling season. Thus, the seller's valuation changes over time as the remaining time, and inventory fluctuate during the selling season. In our model, the seller reacts to this fluctuation by adjusting the posted price, which precedes the bargaining between the seller and buyers. Furthermore, the posted price influences the buyer's belief on the seller's valuation of the product and the surplus over which the buyer and the seller are negotiating. Thus, existing bargaining models are not well-suited to articulate the effects of inventory, time and the posted price on bargaining. In this chapter, we propose an alternative bargaining model that is well-suited to our purpose of modeling negotiation in the presence of inventory considerations. This model is simple enough to be embedded into the dynamic pricing problem, but sophisticated enough to capture a spectrum of bargaining outcomes we observe in practice.

There are a few papers in economics that compare posted-price strategy with bargaining. One such paper is by Riley and Zeckhauser (1983) who show that posted-price strategy is superior to haggling if the seller incurs a cost for bringing a new potential buyer. Wang (1995), on the other hand, uses the Nash bargaining solution to model the outcome of bargaining, and finds that bargaining is always preferable to take-it-or-leave-it pricing if the cost of implementing bargaining is not too high. Both papers ignore the effect of limited inventory and finite selling season, thus the risk of excess inventory as well as the risk of shortage are ignored. Our model explicitly considers a seller with limited inventory and a finite selling season, thus capturing these important risks that the seller needs to bear.

There is some recent work that incorporates negotiation among supply chain

partners. For example, Wu (2004) uses the Rubinstein model to search for coordinating contracts in a supply chain with demand uncertainty. Gurnani and Shi (2006) use the Nash bargaining solution in order to model a contract design problem in a supply chain where the supplier's reliability is private information. Iyer and Villas-Boas (2003) consider bargaining issues in distribution channels. Dukes and Gal-Or (2003) consider the relationship between advertisers and media outlets, using the Nash bargaining solution to model the negotiation outcome. Nagarajan and Sosic (2008) review and extend the research in cooperative bargaining in supply chains.

There are also papers that examine bargaining as a pricing strategy. Desai and Purohit (2004) analyze how two competing retailers choose whether to use take-it-or-leave-it pricing or negotiation and analyze equilibrium outcome. Terwiesch, Savin and Hann (2005) analyze an online retailer that uses a negotiation process where customers name their own prices, and derive the retailer's optimal threshold price above which the retailer accepts all offers. None of this work models supply-side constraints, which we do through our focus on the limited inventory of the product.

2.3 Model Description and Negotiation Results

We consider a firm selling a limited inventory of a product over a predetermined selling season. We assume that the selling season is divided into T periods, each of which is short enough that at most one customer arrives in a given period, and we denote the probability that a customer arrives in a period by $\lambda \in [0, 1]$. A customer can be one of two types - a price-taker or a bargainer. Let q be the proportion of bargainers in the customer population. Facing two types of customers, the seller sets the posted price and negotiates with bargainers in each period to maximize his expected total revenue over the selling season.

2.3.1 Customer's Problem

Let r denote the reservation price of the customer (the maximum price that the customer is willing to pay for the product), unobservable to the firm. From the firm's perspective, an arriving customer's reservation price is a non-negative random variable R_c with a cumulative distribution function (cdf) $F(\cdot)$ and a probability density function (pdf) $f(\cdot)$. We assume that F is defined over the domain [0,b] for some $0 < b \leq \infty$. Define $\overline{F}(\cdot) := 1 - F(\cdot)$. Throughout the chapter, we define $x^+ := \max\{0, x\}$.

Upon arrival, all customers observe the posted price, but their subsequent behavior depends on their type. A price-taker buys the product if the posted price, p, is less than or equal to her reservation price, r, and quits otherwise. On the other hand, a bargainer observes the posted price and decides whether to negotiate or quit. We assume that the bargainer's offer is restricted to be within θ of the posted price, where $\theta > 0$ can be interpreted as the largest discount a customer will demand. Therefore, only bargainers with reservation price of $p - \theta$ or higher will proceed to negotiate; others will quit without making an offer. If θ is sufficiently large, most bargainers will choose to negotiate whether or not the posted price is high (e.g., Oriental rug store). On the other hand, if θ is small, many bargainers will choose to quit since they will not be able to negotiate the price down to the level they can afford (e.g., home appliances). If the bargainer decides to negotiate, she will make an offer, p_o, and the seller will respond with a counter-offer, p_c, that depends on the seller's inventory and time as well as the bargainer's offer as we will discuss later. Then, the customer either accepts the counter-offer (if $r \geq p_c$) or rejects (if $r < p_c$).

The inclusion of θ in our model reflects the fact that bargainers will not make unrealistically low offers. Although we assume that all bargainers have the same

$\theta > 0$, note that the price-takers in our model can be seen as customers with $\theta = 0$. Therefore, in effect, our model allows two heterogeneous types of customers with different bargaining skills. One could extend the model to allow for n types of bargainers, each with a different $\theta_i, i = 1, \ldots, n$, but many of our insights will remain the same.

How does a bargainer decide what offer to make? One could model this decision in different ways, depending on how much the bargainer knows about the seller's problem. If the bargainer knows all the relevant information about the seller's problem, which includes the seller's inventory and remaining time as we will see later, then the bargaining problem reduces to a sequential game between the buyer and the seller. In such a game, a subgame perfect equilibrium would be the bargainer making the minimum possible offer that will be acceptable to the seller. In fact, any subgame perfect equilibrium for such a sequential game will result in a final settlement price equal to the seller's marginal value of one unit of inventory. In most practical cases, however, the bargainer is not likely to know all the information pertinent to the seller's marginal value of one unit of inventory, such as the seller's inventory, the arrival rate of customers, the seller's belief on the customers' reservation prices, etc. In the absence of such information, the best signal that bargainers have about the seller's valuation of the product is the posted price; at the very least, bargainers know that the seller is willing to sell the product at the posted price, p. Thus, we assume that the bargainer believes that her probability of acquiring the product is a function of the posted price p and the offer p_o, and is given by cdf $G(p_o|p)$ with a corresponding pdf $g(p_o|p)$.

Ideally, the bargainer would make an offer p_o to maximize the expected surplus she will obtain as a result of the counter-offer from the seller. However, the bargainer

cannot perfectly predict how the seller will choose its counter-offer, since the counter-offer is a function of the seller's private information such as inventory level, arrival rate of customers, etc. Therefore, we assume that the bargainer maximizes her maximum expected surplus, which is the surplus the bargainer draws if the seller is willing to sell the product at the bargainer's offer. In other words, the objective function of a bargainer with reservation price r is

$$S(p_o, r) = (r - p_o)G(p_o|p). \tag{2.1}$$

Thus, the bargainer's maximization problem is

$$\max_{\{p_o|p-\theta \le p_o \le p\}} S(p_o, r) \tag{2.2}$$

The trade-off that the bargainer faces is as follows: The larger the offer, the larger the bargainer's probability of acquiring the product, but the smaller the maximum surplus she can obtain from the acquisition. The problem is further complicated by the lower bound of $p - \theta$ on the offer. To provide regularity to the bargainer's objective function, we will need the following assumption:

Assumption 1. *$G(\cdot|p)$ is strictly increasing and log-concave over the domain $[0, p]$.*[1] *In addition, $G(x|p) = 0$ for $x \le 0$ and $G(x|p) = 1$ for $x \ge p$.*

The following lemma states that $S(p_o, r)$ is a well-behaved function of the offer, p_o. The proofs of this and all other results are relegated to the appendix.

Lemma 2.3.1. *The objective function of a bargainer with reservation price r, $S(p_o, r)$, has the following properties:*

(a) $S(p_o, r)$ is unimodal in the customer's offer, p_o, for $p_o \in [0, p]$.

(b) $S(p_o, r) = 0$ for $p_o \le 0$ and $S(p_o, r)$ is strictly decreasing in p_o for $p_o \ge p$.

[1] In fact, the lower bound of the domain, 0, can be replaced by some $a > 0$ as long as $a \le \arg\max p\bar{F}(p)$, the myopic optimal price.

Given a posted price p, let $p_o^*(p,r)$ denote the optimal offer of a bargainer with reservation price r, i.e., $p_o^*(p,r)$ is the solution to the optimization problem given by (2.2). For a given posted price p, denote the unconstrained optimizer of the function $S(p_o,r)$ by

$$\overline{p}_o(r) := \sup\{x : S(x,r) \geq S(p_o,r), \forall p_o\}$$

Define function $\rho(\cdot)$ implicitly as

$$\rho(x) = \min\{r \in [0,b] : S(x,r) \geq S(p_o,r), \forall p_o\}, \tag{2.3}$$

i.e., $\rho(x)$ is the reservation price of the bargainer for whom $p_o = x$ is an optimizer of $S(p_o,r)$, the bargainer's unconstrained objective function. (We use min operator in case there exist a group of bargainers with different reservation prices for all of whom x is an optimizer.) Note that, with this definition, $\rho(x) = 0$ for any $x \leq 0$ (since $G(x|p) = 0$ for all $x \leq 0$, any bargainer with $r > 0$ could do better by making a strictly positive offer). With the help of the definitions made so far, we characterize the optimal offer of a bargainer in the following lemma.

Lemma 2.3.2. *Given a posted price p, the optimal offer of a bargainer with reservation price r is*

$$p_o^*(p,r) = \begin{cases} (p-\theta)^+ & \text{if } p-\theta \leq r \leq \rho(p-\theta); \\ \overline{p}_o(r) \in (p-\theta, p) & \text{if } \rho(p-\theta) < r \leq \rho(p); \\ \overline{p}_o(r) = p & \text{if } r > \rho(p). \end{cases} \tag{2.4}$$

In essence, customers with low reservation prices will ask for the largest discount, θ, and customers with higher reservation prices will make offers that depend on their reservation prices, those with the highest reservation prices offering to pay the posted

price p. One may wonder if a bargainer will ever offer to pay the posted price, p, in practice. Under Assumption 1, such a possibility exists if the arbitrarily chosen posted price is small but the reservation price of the bargainer is very high. To avoid this possibility, one could impose a reasonable technical assumption on $g(\cdot|p)$ such as $g(p|p) = 0$, which guarantees that all bargainers will make an offer strictly below p. Furthermore, in Section 2.4, we work with a uniformly distributed $G(\cdot|p)$, and we find that a bargainer never offers to pay the posted price when the posted price is chosen optimally.

2.3.2 The Firm's Revenue Maximization Problem

At the beginning of period t, $t = 1, \ldots, T$, the firm, given y units in inventory, sets the posted price, p. Since only price-takers with reservation prices greater than or equal to the posted price p will purchase the product, it follows that the expected revenue accrued from the price-taker in the current period is simply $p\overline{F}(p)$. On the other hand, the revenue accrued from the bargainer and the chance that the bargainer buys the product in the current period are determined by the outcome of negotiation. Let $K_t(p, y)$ denote the firm's expected revenue in period t, given that a bargainer has arrived and the firm has y units in inventory. In addition, let $B_t(p, y)$ denote the probability that a bargainer will buy the product at the end of the negotiation. The firm's problem of setting the posted price is given by the following optimality

equations:

$$
\begin{aligned}
V_t(y) &= \max_p \left\{ \begin{array}{l} \lambda q \left[K_t(p,y) + B_t(p,y) V_{t-1}(y-1)\right] \\ \\ +\lambda(1-q)\left[p\overline{F}(p) + \overline{F}(p) V_{t-1}(y-1)\right] \\ \\ +\left[1 - \lambda(qB_t(p,y) + (1-q)\overline{F}(p))\right] V_{t-1}(y) \end{array} \right\} \\
&\qquad \text{for } y > 0, t = 1, \ldots, T \qquad (2.5) \\
V_0(y) &= 0 \text{ for } y \geq 0, \text{ and } V_t(0) = 0 \text{ for } t = 1, \ldots, T
\end{aligned}
$$

For $y \geq 1$ and $t = 1, \ldots, T$, the optimality equation described in (2.5) can be rewritten as follows,

$$
\begin{aligned}
V_t(y) &= \max_p J_t(p,y) \\
\text{where} \quad & J_t(p,y) \\
&= V_{t-1}(y) + \lambda q \left[K_t(p,y) - B_t(p,y)(V_{t-1}(y) - V_{t-1}(y-1))\right] \\
&\quad +\lambda(1-q)\overline{F}(p)\left[p - (V_{t-1}(y) - V_{t-1}(y-1))\right] \qquad (2.6)
\end{aligned}
$$

Let $p_t^*(y)$ denote the optimal solution to the maximization problem in (2.6), i.e., $p_t^*(y)$ is the optimal posted price in period t with y units of inventory. Notice that $V_{t-1}(y) - V_{t-1}(y-1)$ represents the benefit from keeping an extra unit of inventory for period $t-1$ (i.e., the marginal value of inventory). Throughout the chapter, we let $\Delta_{t-1}(y) = V_{t-1}(y) - V_{t-1}(y-1)$ and we refer to it as the marginal value of inventory. To determine $p_t^*(y)$ and the resultant outcome of negotiation, it suffices to consider the case $p \geq \Delta_{t-1}(y)$ as stated in the following lemma.

Lemma 2.3.3. *The optimal posted price of the seller with y units of inventory and t periods to go must be greater than or equal to $\Delta_{t-1}(y)$.*

2.3.3 Seller's Counter-offer Problem

After seeing the customer's offer, the seller makes a counter-offer. Suppose in period t, with y units in inventory, a customer arrives and offers to pay p_o. By virtue of Lemma 2.3.2, there are two cases to consider: $p_o = p - \theta$ or $p_o > p - \theta$.

Case 1. $p_o = p - \theta$: In this case, the seller knows that the customer's reservation price is between $p-\theta$ and $\rho(p-\theta)$ and updates its belief on the bargainer's reservation price accordingly. Note that any counter-offer greater than $\rho(p-\theta)$ will be rejected by the bargainer. If the bargainer buys, the seller's revenue-to-go from next period onward is $V_{t-1}(y-1)$. On the other hand, if the negotiation breaks down, the seller's revenue-to-go is simply $V_{t-1}(y)$. Therefore, the expected profit of the seller charging counter-offer price p_c in period t with y units of inventory when facing the bargainer's offer $p_o = p - \theta$ is

$$Z_t(p_c, y) = \begin{cases} \frac{F(\rho(p-\theta))-F(p_c)}{F(\rho(p-\theta))-F(p-\theta)}(p_c + V_{t-1}(y-1)) \\ \quad + \left(\frac{F(p_c)-F(p-\theta)}{F(\rho(p-\theta))-F(p-\theta)}\right) V_{t-1}(y) & \text{if } p-\theta \le p_c \le \rho(p-\theta); \\ 0 & \text{if } p_c > \rho(p-\theta). \end{cases} \tag{2.7}$$

Obviously, the seller will never choose a counter-offer below the bargainer's offer $p_o = p - \theta$, i.e., $p_c \ge p - \theta$. Likewise, the seller is not allowed to make a counter-offer that exceeds the posted price since such a business practice would be unacceptable. Thus, the seller's optimization problem is given by

$$\max_{\{p_c | p-\theta \le p_c \le \min[p, \rho(p-\theta)]\}} Z_t(p_c, y) \tag{2.8}$$

We make the following assumption to guarantee that the function $Z_t(p_c, y)$ is well-behaved.

Assumption 2. *The cdf of R_c, $F(\cdot)$, is a strictly increasing function with an increasing failure rate.*

Under Assumption 2, the following result holds:

Lemma 2.3.4. *The seller's objective function, $Z_t(p_c, y)$, is unimodal in the counter-offer, p_c, for $p_c \in [p-\theta, \rho(p-\theta)]$.*

Let $\overline{p}_{ct}(y)$ be the solution to the optimization problem in (2.8), i.e.,

$$\overline{p}_{ct}(y) = \underset{\{p_c | p-\theta \le p_c \le \min[p, \rho(p-\theta)]\}}{\arg\max} Z_t(p_c, y).$$

Hence, if a customer offers $p_o = p - \theta$, the seller will set its counter-offer to $\overline{p}_{ct}(y)$, provided that selling to the bargainer at that price is better than keeping an extra unit of inventory for the next period, i.e., if $\overline{p}_{ct}(y) > \Delta_{t-1}(y)$. Otherwise, if $\overline{p}_{ct}(y) \le \Delta_{t-1}(y)$, then the seller would set its counter-offer equal to the marginal value of inventory, $\Delta_{t-1}(y)$. Thus, the seller's counter-offer is $\max[\overline{p}_{ct}(y), \Delta_{t-1}(y)]$.

Case 2. $p_o > p - \theta$: In this case, it must be that $p_o = \overline{p}_o(r)$ by Lemma 2.3.2. Therefore, the seller deduces that the bargainer's reservation price is at least $\rho(p_o)$, and will set its counter-offer equal to $\min\{\rho(p_o), p\}$, provided that selling to the bargainer is better than keeping an extra unit of inventory, i.e., $\min\{\rho(p_o), p\} > \Delta_{t-1}(y)$. Thus, the seller's counter-offer is equal to $\max\{\min\{\rho(p_o), p\}, \Delta_{t-1}(y)\}$.

The following lemma summarizes the optimal solution to the firm's counter-offer problem.

Lemma 2.3.5. *Let $p^*_{ct}(p, p_o, y)$ denote the optimal counter-offer that the firm will make in period t with y units in inventory, provided that a customer offers to pay p_o*

and the posted price is p. Then:

$$p^*_{ct}(p, p_o, y) = \begin{cases} \max\{\overline{p}_{ct}(y), \Delta_{t-1}(y)\} & \text{if } p_o = p - \theta; \\ \max\{\min\{\rho(p_o), p\}, \Delta_{t-1}(y)\} & \text{if } p_o > p - \theta. \end{cases} \tag{2.9}$$

2.3.4 The Outcome of the Negotiation

Using the results established in previous subsections, we are able to characterize the behavior of a bargainer with reservation price r arriving in period t, i.e., whether the bargainer will buy and, if so, at what price, given that the seller has y units of inventory and its posted price is p. It suffices to consider only the cases where $p \geq \Delta_{t-1}(y)$ (by Lemma 2.3.3). When a bargainer arrives, the interactions of a bargainer and the seller follow one of four cases, shown in Figure 2.1. (The results summarized in Figure 2.1 are proven in Lemma 2.3.6.)

In all four cases, a bargainer whose reservation price is below $p - \theta$ quits without making an offer. All other bargainers make an offer; an individual bargainer's offer depends on her reservation price, r. In the first of four cases, after receiving an offer from the bargainer, the seller is unwilling to negotiate further and sets the counter-offer equal to the posted price. Thus, only bargainers with $r \geq p$ purchase and they do so at the posted price, p. In the second case, in response to an offer from the bargainer, the seller chooses one of two counter-offers, $\overline{p}_{ct}(y)$ or p. As a result, bargainers with high reservation prices end up buying at the posted price p, whereas bargainers with $r \in [p - \theta, \rho(p - \theta)]$ are split into two; some drop out of the negotiation and others purchase at $\overline{p}_{ct}(y)$. In the third case, again in response to the bargainer's offer, the seller may choose one of three counter-offers, $\overline{p}_{ct}(y)$, p or the bargainer's reservation price, r. As in the second case, bargainers with

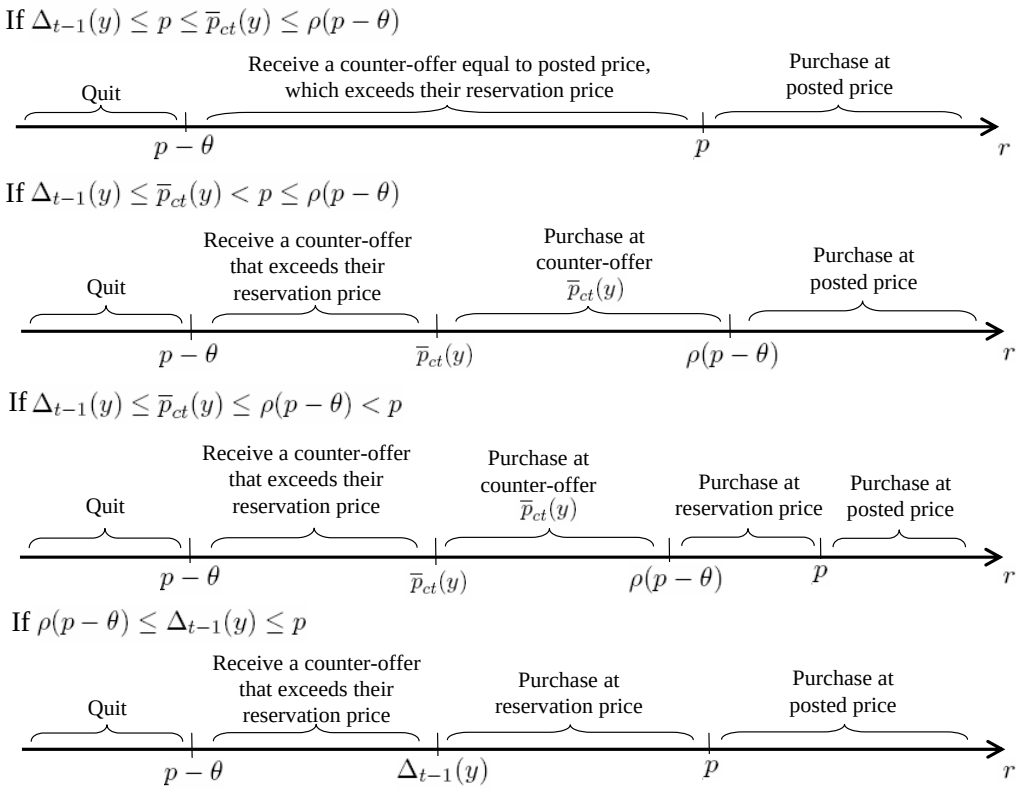

Figure 2.1: Four possible cases of negotiation outcome, shown as a function of the buyer's reservation price, r.

sufficiently high reservation prices end up buying at the posted price p. This time, only bargainers with $r \in [\overline{p}_{ct}(y), \rho(p-\theta)]$ purchase at $\overline{p}_{ct}(y)$ whereas bargainers with $r \in (\rho(p-\theta), p]$ pay their reservation price, r. As before, bargainers with $r \in [p-\theta, \overline{p}_{ct}(y))$ drop out of negotiation. In the fourth case, the seller's counter-offer will be the posted price p or the marginal value of inventory $\Delta_{t-1}(y)$, or the bargainer's reservation price. Once again, customers with sufficiently large reservation prices end up paying the posted price, p. Others either pay their reservation price or drop out of the negotiation.

Figure 2.2 shows, for a numerical example, the spectrum of bargainer behavior

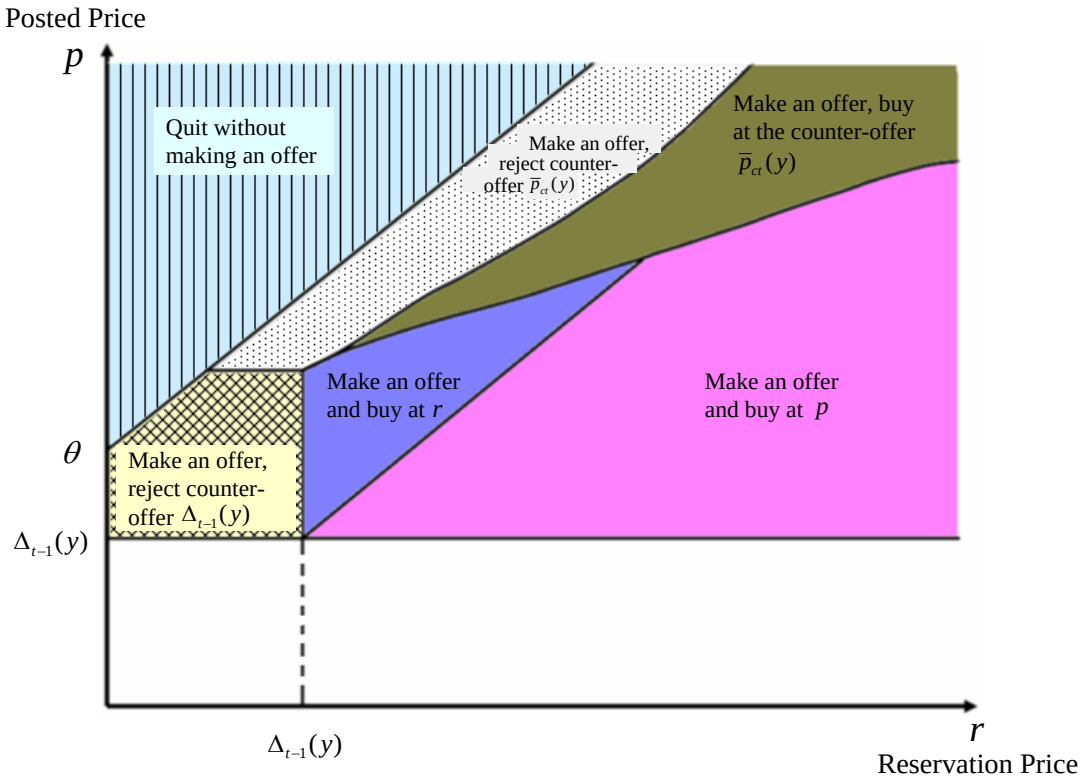

Figure 2.2: Different bargaining outcomes arise depending on posted price p and reservation price r. For this example, we use $F \sim Exp(0.02), G(x|p) = \frac{1-e^{-0.025x}}{1-e^{-0.025p}}, \lambda = 0.7, q = 0.8, t = 2, y = 1$, and $\theta = 20$.

as a function of the posted price p and the bargainer's reservation price r. As the example shows, our negotiation model captures many different kinds of outcomes that we would expect to see in practice.

Once we characterize the outcome of the negotiation as a function of p, r, t, and y, we can derive the seller's expected revenue from a bargainer, $K_t(p, y)$, and the probability that a bargainer will buy the product, $B_t(p, y)$, both of which are used in the optimality equation (2.5).

Lemma 2.3.6. *Suppose a bargainer arrives in period t when the seller with y units*

of inventory charges the posted price p.

(a) If $\Delta_{t-1}(y) \leq p \leq \overline{p}_{ct}(y) \leq \rho(p-\theta)$, then

$$\begin{aligned} K_t(p,y) &= p\overline{F}(p) \\ B_t(p,y) &= \overline{F}(p) \end{aligned}$$

(b) If $\Delta_{t-1}(y) \leq \overline{p}_{ct}(y) < p \leq \rho(p-\theta)$, then

$$\begin{aligned} K_t(p,y) &= \overline{p}_{ct}(y)\left(F(\rho(p-\theta)) - F(\overline{p}_{ct}(y))\right) + p\overline{F}(\rho(p-\theta)) \\ B_t(p,y) &= \overline{F}(\overline{p}_{ct}(y)) \end{aligned}$$

(c) If $\Delta_{t-1}(y) \leq \overline{p}_{ct}(y) \leq \rho(p-\theta) < p$, then

$$\begin{aligned} K_t(p,y) &= \overline{p}_{ct}(y)\left(F(\rho(p-\theta)) - F(\overline{p}_{ct}(y))\right) + \int_{\rho(p-\theta)}^{p} xf(x)dx + p\overline{F}(p) \\ B_t(p,y) &= \overline{F}(\overline{p}_{ct}(y)) \end{aligned}$$

(d) If $\rho(p-\theta) \leq \Delta_{t-1}(y) \leq p$, then

$$\begin{aligned} K_t(p,y) &= \int_{\Delta_{t-1}(y)}^{p} xf(x)dx + p\overline{F}(p) \\ B_t(p,y) &= \overline{F}(\Delta_{t-1}(y)) \end{aligned}$$

Lemma 2.3.6 enables us to embed the results of negotiation, $K_t(p,y)$ and $B_t(p,y)$, into the optimality equation and solve for the seller's optimal posted price at each period and inventory level. It is easy to construct numerical examples to demonstrate that the optimal posted price can lie in any one of the four cases of Lemma 2.3.6, depending on the marginal value of inventory and parameter values. Hence, none of the four cases can be ruled out as a potential optimal solution to the posted price problem. Since the revenue-to-go function $V_t(y)$ follows one of four cases depending on which case the optimal posted price lies in, it is difficult to prove additional structural results about the dynamic program to gain further managerial insights into the

problem. To this end, we impose additional assumptions on the customers' reservation price distribution and the seller's valuation distribution in the next section. Our numerical study in Section 2.5 demonstrates that the results of the next section hold under less-restrictive assumptions on the distributions as well.

2.4 Analysis

In this section we explore the seller's optimal pricing strategy as a function of inventory and remaining time and examine the effect of negotiation on the seller and customers (both price-takers and bargainers). We impose the following additional assumption to simplify the problem.

Assumption 3. *We assume that* $F(\cdot)$ *is uniform between* $(0, b)$ *and* $G(\cdot|p)$ *is uniform between* $(0, p)$.

One could pick non-zero lower bounds for the distributions F and G; such generalization only changes algebra without changing insights. In particular, if $F(\cdot) \sim U(a, b)$ and $G(\cdot|p) \sim U(a, p)$, then the lower-bound a imposes additional constraints on the seller's posted price and counter-offer. The remainder of the analysis follows the same reasoning. Likewise, if $F(\cdot) \sim U(a, b)$ and $G(\cdot|p) \sim U(c, p)$ with $a < c$, then one could simply ignore the bargainers whose reservation prices are between a and c (since those bargainers will not even make an offer), and the same insights will hold. On the other hand, the case with $a > c$ could be dealt with, but it is not a reasonable assumption in that it implies there are bargainers who believe the seller's valuation is less than the smallest reservation price among all customers. Furthermore, our numerical study, to be discussed in the next section, provides evidence that the insights remain the same when F and G follow non-uniform distributions.

Even with Assumption 3, the revenue-to-go function is still complex as discussed in the previous section (i.e., all four cases may arise). However, under this assumption, we find a closed form expression for the optimal posted price (and the resulting bargainer's offer and seller's counter-offer), which renders the problem analytically tractable.

The following lemma states the optimal posted price of the seller, $p_t^*(y)$. As the lemma shows, the optimal posted price depends critically on how the largest discount a customer will demand, θ, compares to the marginal value of inventory, $\Delta_{t-1}(y)$, in addition to the fraction of bargainers, q, and the range of reservation prices, $[0, b]$.

Lemma 2.4.1. *Suppose the seller has y units of inventory with t periods to go until the end of the season. Then:*

(a) If $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, then $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$.

(b) If $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$.

(c) If $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then $p_t^*(y) = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$.

Given the optimal posted price stated in Lemma 2.4.1, the bargainer's optimal offer will be as shown in the following lemma.

Lemma 2.4.2. *Suppose the seller has y units of inventory with t periods to go until the end of the season. Given the optimal posted price $p_t^*(y)$, all bargainers who do not quit will make an offer $p_o^*(p_t^*(y), r)$ strictly less than the posted price. In particular:*

(a) If $p_t^*(y) - \theta \leq r \leq 2(p_t^*(y) - \theta)$, then the optimal offer is $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$.

(b) If $2(p_t^*(y) - \theta) \leq r \leq b$, then the optimal offer $p_o^*(p_t^*(y), r) = \dfrac{r}{2}$.

Note from Lemma 2.4.2 that the bargainer's offer will be either the smallest possible offer she can make, $p_t^*(y) - \theta$ or half of her reservation price, $r/2$. One can verify from Lemmas 2.4.1 and 2.4.2 that, for any given r, the bargainer's optimal

offer, $p_o^*(p_t^*(y), r)$, is strictly less than the optimal posted price, $p_t^*(y)$. Following the optimal posted price and optimal bargainer's offer, we present the seller's optimal counter-offer $p_{ct}^*(p_t^*(y), p_o^*(p_t^*(y), r), y)$ in the following lemma. For brevity, we will use short-hand notation $p_{ct}^*(p_t^*, p_o^*, y)$ instead of $p_{ct}^*(p_t^*(y), p_o^*(p_t^*(y), r), y)$.

Lemma 2.4.3. *Suppose the seller has y units of inventory with t periods to go until the end of the season. Then:*

(a) If $\theta \leq \frac{\Delta_{t-1}(y)}{2}$, then $p_{ct}^*(p_t^*, p_o^*, y) = p_t^*(y)$ regardless of the bargainer's offer.

(b) If $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, then

$$p_{ct}^*(p_t^*, p_o^*, y) = \begin{cases} p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2} & \text{if } p_o^*(p_t^*(y), r) = p_t^*(y) - \theta; \\ p_t^*(y) & \text{if } p_o^*(p_t^*(y), r) > p_t^*(y) - \theta. \end{cases}$$

(c) If $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then

$$p_{ct}^*(p_t^*, p_o^*, y) = \begin{cases} p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2} & \text{if } p_o^*(p_t^*(y), r) = p_t^*(y) - \theta; \\ r & \text{if } p_o^*(p_t^*(y), r) > p_t^*(y) - \theta \text{ and } p_o^*(p_t^*(y), r) < \frac{p_t^*(y)}{2}; \\ p_t^*(y) & \text{if } p_o^*(p_t^*(y), r) > p_t^*(y) - \theta \text{ and } p_o^*(p_t^*(y), r) \geq \frac{p_t^*(y)}{2}. \end{cases}$$

(d) If $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then

$$p_{ct}^*(p_t^*, p_o^*, y) = \begin{cases} \Delta_{t-1}(y) & \text{if } p_t^*(y) - \theta \leq p_o^*(p_t^*(y), r) < \frac{\Delta_{t-1}(y)}{2}; \\ r & \text{if } \frac{\Delta_{t-1}(y)}{2} \leq p_o^*(p_t^*(y), r) < \frac{p_t^*(y)}{2}; \\ p_t^*(y) & \text{if } p_o^*(p_t^*(y), r) \geq \frac{p_t^*(y)}{2}. \end{cases}$$

The closed-form expressions stated in Lemmas 2.4.1 through 2.4.3 help us obtain a number of results regarding the effect of negotiation on the optimal posted price, the seller's expected revenue and the customer's surplus. Before we discuss these results, we first define the following auxiliary optimization problem of a seller with y units of inventory and t periods to go. This optimization problem represents the pricing problem faced by a seller using take-it-or-leave-it pricing in the current period

(i.e., forcing all customers to act as price-takers), and allowing negotiation from next period onward (i.e., the revenue-to-go from period $t-1$ onward is $V_{t-1}(\cdot)$ as defined by (2.5)):

$$\begin{aligned} & V_t^{TL}(y) \\ = & \max_p \left\{ \lambda \overline{F}(p) p + \lambda \overline{F}(p) V_{t-1}(y-1) + (1 - \lambda \overline{F}(p)) V_{t-1}(y) \right\}, y > 0, t = 1, \ldots, T \end{aligned} \tag{2.10}$$

Let $p_t^{TL}(y)$ denote the optimal solution to the optimization problem given by (2.10). It is not difficult to check that, under Assumption 3, we have $p_t^{TL}(y) = \dfrac{b + \Delta_{t-1}(y)}{2}$. The following proposition compares the optimal posted price, $p_t^*(y)$, with the take-it-or-leave-it price, $p_t^{TL}(y)$.

Proposition 2.4.1. *Suppose the seller has y units of inventory with t periods to go until the end of the season. Then the seller's optimal posted price, $p_t^*(y)$, is between $p_t^{TL}(y)$ and $p_t^{TL}(y) + \theta$.*

The proposition highlights the effect of negotiation on the posted price. If negotiation were not allowed in period t, the seller would charge the price $p_t^{TL}(y)$. Under negotiation, however, the seller adds a premium on top of $p_t^{TL}(y)$ with the intention of selling at a lower price to some customers. In other words, the premium allows the seller to price discriminate based on customers' willingness to pay. It is interesting to note that the premium is less than the largest discount a bargainer would demand, θ.

We now focus on how negotiation affects the seller's revenue. As stated in Proposition 2.4.1, the seller raises the posted price to price-discriminate among bargainers. The increased posted price results in loss of revenue from price-takers. Furthermore,

under negotiation, some bargainers may be able to negotiate down to a price below the take-it-or-leave-it price. Even so, one would expect that negotiation would improve the seller's expected revenue, given that the seller can always walk out of negotiation by repeating the posted price as its counter-offer. Indeed, the following proposition states that negotiation improves the seller's expected revenue.

Proposition 2.4.2. *Suppose the seller has y units in inventory and t periods to go and will use negotiation from period $t-1$ onward. The seller is better off by negotiating with posted price $p_t^*(y)$ in period t than using take-it-or-leave-it pricing with posted price $p_t^{TL}(y)$ in period t.*

It is not difficult to extend the result of Proposition 2.4.2 to the case where one seller uses negotiation throughout the selling season and the other uses take-it-or-leave-it pricing.

Corollary 2.4.1. *Suppose the seller has y units in inventory and t periods to go. The revenue of the seller across the t-period horizon is larger under negotiation than under take-it-or-leave-it pricing.*

In some cases, the posted price decision might be dictated by an outside party (e.g., manufacturer). We note that Corollary 2.4.1 extends to the case where the posted price p is determined exogenously, that is, the revenue of the seller across the selling horizon is larger when the seller is negotiating with the exogenous posted price p than using take-it-or-leave-it pricing at the same price. The result is not too surprising since the negotiating seller will settle for a price below p only if it improves the expected revenue-to-go (otherwise, the seller sticks to p as its counter-offer.)

We next turn our attention to the effect of negotiation on the customers. Since the seller is charging a premium under negotiation, it is clear that certain price-

takers who were able to afford the product under take-it-or-leave-it pricing will not be able to afford it under negotiation. However, it is not clear how the bargainers are affected. The following proposition characterizes which bargainers are better off due to negotiation.

Proposition 2.4.3. *No price-taker is better off under negotiation compared to take-it-or-leave-it pricing. As for a bargainer who purchases:*

(a) If $\theta \leq \frac{\Delta_{t-1}(y)}{2}$*, the bargainer ends up buying at* $p_t^{TL}(y)$*, thus she is neither worse nor better off under negotiation.*

(b) If $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$*, there exists a threshold reservation price such that the bargainer is better off if her reservation price is below the threshold and worse off otherwise.*

(c) If $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$*, then the bargainer is worse off under negotiation.*

Proposition 2.4.3(a) states that, if θ is too small, the seller reverts to take-it-or-leave-it pricing by setting both the posted price and counter-offer to $p_t^{TL}(y)$. In this case, the negotiation has no effect on the consumers. When θ is moderately large (as in Proposition 2.4.3(b)), some bargainers will be better off under negotiation compared to take-it-or-leave-it pricing while others are worse off. In this case, the seller takes advantage of price-takers or bargainers with high reservation prices, but yields to the bargainers with low reservation prices. The proof of the proposition reveals what is common across the bargainers who benefit from negotiation: Their reservation prices are large enough that they end up buying, but small enough that they successfully negotiate for the largest possible discount, θ. On the other hand, if θ is very large (as in proposition 2.4.3(c)), the seller charges a large premium on top of $p_t^{TL}(y)$ and sells only to customers with high reservation price. In this case, the seller does not sell to the bargainers who demand the largest possible discount, θ. For

the bargainer who purchases, her final purchase price is either her own reservation price or the posted price itself. Thus, no customer is better off.

We now investigate how the optimal posted price, $p_t^*(y)$, depends on the largest discount a customer would demand, θ, and the proportion of the bargainers in the customer population, q. To investigate the effect of θ and q, we check the comparative statics when θ or q changes in the current period only, while θ and q values for all other periods remain the same. The following proposition states our result:

Proposition 2.4.4. *The optimal posted price $p_t^*(y)$ is non-decreasing in the largest discount a customer would demand, θ, and in the proportion of the bargainers in the customer population, q, in the current period.*

From Lemma 2.4.1, we observe that when θ is very small (which is the case in Lemma 2.4.1(a)), the optimal posted price $p_t^*(y)$ is equal to $p_t^{TL}(y) = \frac{b+\Delta_{t-1}(y)}{2}$. Once θ is sufficiently high, the seller starts to charge a premium on top of $p_t^{TL}(y)$. Proposition 2.4.4 shows that the size of the premium depends not only on θ but also on the fraction of bargainers, q. If (i) θ is very small or (ii) θ is larger but the fraction of bargainers is small, then the seller needs to take into account the large revenue stream from price-takers and charges little or no premium in order not to turn away too many price-takers. As the fraction of bargainers increases, the seller puts less emphasis on the price-takers and starts to increase the premium to better price-discriminate among bargainers.

Of course, one would wonder what happens to the posted price when θ and q change across the entire planning horizon. As noted earlier, the seller's revenue in a period is the price collected from a purchaser, which is determined through the posted price, the corresponding buyer's offer, and the resulting seller's counter-offer, all of which are complicated functions of θ and q. Consequently, it is not easy to

extend Proposition 2.4.4 to the case where q or θ change across the entire horizon. We investigate this question through a numerical study in the next section.

At the heart of benefits from negotiation is the seller's ability to price-discriminate through a premium. As Proposition 2.4.4 indicates, the larger θ in the current period, the higher the price in the current period and, therefore, the larger the premium due to negotiation. This larger premium enables the seller to do a finer price discrimination. Therefore, one would expect the seller's revenue to increase in θ. Likewise, the more likely it is that the seller will encounter a bargainer in the current period, the higher the chances that the seller will be able to price-discriminate through negotiation. Thus, we expect that the seller's revenue increases in the fraction of bargainers, q. To investigate the effect of θ and q on the seller's expected revenue, we check the comparative statics when θ or q changes in the current period only. The following proposition states our result:

Proposition 2.4.5. *The optimal expected total revenue of the seller increases if the largest discount that a bargainer may demand, θ or the fraction of the bargainers, q increases in the current period.*

Our numerical results in the next section verify this result when q or θ changes across the entire horizon. Finally, we investigate how the optimal posted price is influenced by the stock level and time-to-go until the end of the horizon.

Proposition 2.4.6. *The optimal posted price $p_t^*(y)$ is non-increasing in the stock level, y, and non-decreasing in the number of remaining periods, t.*

Gallego and van Ryzin (1994) and Bitran and Mondschein (1997) show that the same behavior occurs in the case of take-it-or-leave-it pricing. In our model, where the seller charges a premium on top of the take-it-or-leave-it price, the same key

drivers (e.g., the risk of stock-out, the risk of having excess inventory) influence the optimal posted price, which leads to the same behavior.

2.5 Numerical Study

We conduct a numerical study to gain further managerial insights into the use of negotiation along with dynamic pricing. We identify the scenarios under which negotiation benefits the seller the most, and we investigate if and when dynamic pricing and negotiation reinforce each other. In addition, we explore the effect of negotiation on the posted price and we analyze how the gap between the posted price and the take-it-or-leave-it price depends on the inventory level and other problem parameters.

In our numerical study we consider several different combinations of parameter values. We use three different values for each of: probability that a customer arrives in a given period ($\lambda \in \{0.2, 0.5, 0.7\}$), the largest discount that a bargainer would demand ($\theta \in \{5, 20, 70\}$), and the proportion of bargainers ($q \in \{0.2, 0.5, 0.8\}$). We also consider three different pairs of $F(\cdot)$ and $G(\cdot|p)$ distributions:

i) $F(\cdot) \sim$ uniform over $[0, 200]$ and $G(\cdot|p) \sim$ uniform over $[0, p]$

ii) $F(\cdot) \sim$ exponential with mean 50 and $G(\cdot|p) = P(X|X \leq p)$ where $X \sim$ exponential with mean 40

iii) $F(\cdot) \sim$ Weibull with shape parameter 2 and scale parameter 50 and $G(\cdot|p) = P(X|X \leq p)$ where X is Weibull with shape parameter 2 and scale parameter 40.

For numerical convenience, we use $F(\cdot)$ and $G(\cdot|p)$ that come from the same family of distributions; our general model and results presented in Section 3.3 do not require such an assumption. Note that this parameter set results in 81 different combinations of λ, θ, q and $F(\cdot)$, $G(\cdot|P)$ distributions.

We first compare two sellers, one using negotiation and the other using take-it-or-leave-it pricing throughout the selling season. This comparison allows us to gain insights into the benefits of negotiation. We consider a 15-period selling season and we vary the starting inventory level from 1 to 15. For each starting inventory level and under all 81 combinations of λ, θ, q, and $F(\cdot)$, $G(\cdot|P)$ distributions described above, we solve the dynamic program associated with each seller and determine the seller's optimal expected revenue, resulting in 1,215 different problem instances. For each problem instance, we measure the percentage revenue improvement from negotiation and summarize the results in Table 2.1.

		F, G Uniform			F, G Exponential			F, G Weibull		
θ	q	Mean (Std)	Max	Min	Mean (Std)	Max	Min	Mean (Std)	Max	Min
5	0.2	0.03 (0.02)	0.05	0.00	0.25 (0.1)	0.31	0.01	0.93 (0.43)	1.24	0.04
5	0.5	0.08 (0.05)	0.13	0.00	0.63 (0.25)	0.79	0.04	2.31 (1.09)	3.11	0.11
5	0.8	0.13 (0.08)	0.20	0.00	1.02 (0.4)	1.27	0.06	3.7 (1.75)	4.99	0.18
20	0.2	0.58 (0.3)	0.80	0.02	2.23 (0.88)	2.77	0.14	4.52 (1.36)	5.34	0.56
20	0.5	1.44 (0.75)	2.00	0.04	5.53 (2.22)	6.93	0.35	13.28 (4.57)	16.11	1.35
20	0.8	2.3 (1.21)	3.20	0.07	8.78 (3.58)	11.09	0.56	23.39 (9.02)	29.68	2.10
70	0.2	6.93 (2.27)	8.35	0.72	15.12 (2.99)	16.67	4.25	8.41 (1.51)	9.37	4.06
70	0.5	17.35 (6.04)	21.30	1.65	43.3 (10.82)	49.67	9.49	26.25 (5.53)	29.95	11.41
70	0.8	27.64 (10.18)	34.63	2.43	71.19 (20.3)	84.37	13.47	56.03 (14.67)	66.86	20.59

Table 2.1: Summary Statistics for Percentage Revenue Improvement from Negotiation

We observe from Table 2.1 that the larger the largest discount asked by the customers, θ, the larger the revenue improvement due to negotiation. Likewise, the larger the proportion of bargainers, q, the larger the revenue improvement due to negotiation. Furthermore, we observe that the seller's benefit from negotiation tends to be larger under exponential and Weibull distributions for F and G, compared to uniform. Weibull and exponential reservation prices have heavier tails compared to

uniform, indicating a larger portion of customers with high reservation prices. The seller can extract more revenue out of those customers through negotiation, which results in the benefits from negotiation being higher under Weibull and exponential distributions. We observe in our numerical study that the percentage revenue improvement is larger when starting inventory level is higher. We will further explore the effect of starting inventory later in this section.

Table 2.2 summarizes the magnitude of percentage improvement for all 1,215 problem instances tested. In more than 70% of instances, the percentage revenue improvement is greater than 1%. In about 33% of instances, the revenue improvement is greater than 10%. These numbers suggest that, even when there is cost for implementing negotiation, the seller can realize significant benefit from negotiation.

% Improvement	$< 1\%$	$1 - 3\%$	$3 - 5\%$	$5 - 10\%$	$10 - 20\%$	$20 - 30\%$	$> 30\%$	Total
Number of cases	355	183	109	166	139	114	149	1215

Table 2.2: Frequency Table for Percentage Revenue Improvement

The effect of inventory level on the benefit of negotiation: Figure 2.3 illustrates the effect of inventory level on the negotiation outcome. As illustrated in the figure, the seller is able to do a finer price discrimination when inventory level is high. When the inventory level is low relative to the remaining selling horizon, the seller is not worried about leftover inventory, hence the optimal posted price is likely to be very high. In this case, a substantial portion of bargainers will quit (some will quit without even making an offer and others cannot afford the seller's counter-offer, thus drop out of negotiation) since the seller will not settle for a low price. On the other

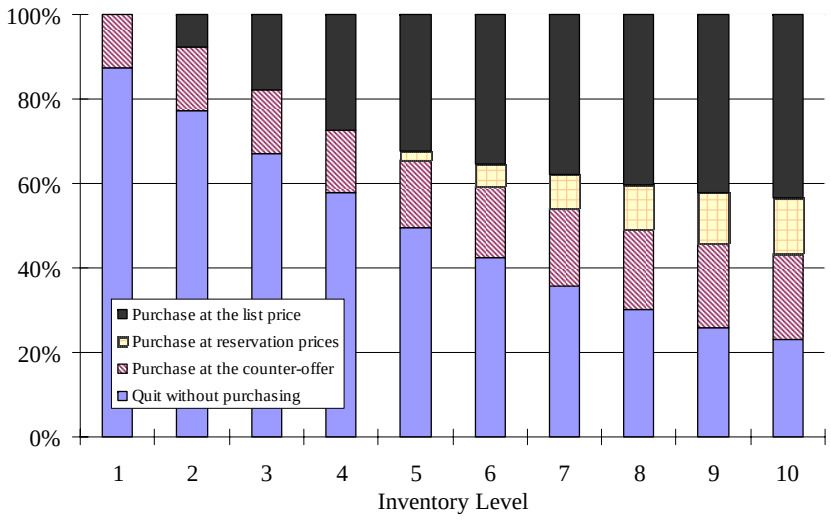

Figure 2.3: Negotiation outcomes with respect to the starting inventory level when $\lambda = 0.7$, $\theta = 70$, $q = 0.8$, $F(\cdot) \sim U[0, 200]$ and $G(\cdot|p) \sim U[0, p]$.

hand, when the inventory level is high, the negotiating seller can still charge a high posted price, but is willing to settle for less. Thus, fewer bargainers will quit and those who buy the product will pay a wide range of prices. For example, in Figure 2.3, when $y = 10$, some buy at the posted price (43.3% of bargainers), others buy at the reservation price (13.3%), and still others make the lowest possible offer and buy at the corresponding counter-offer (20.3%). In other words, when the inventory level is high, the seller is able to do a finer price discrimination, while at the same time increasing the chance of making a sale. Therefore, one would expect that the benefits from negotiation (i.e., the additional revenue a seller can realize by combining dynamic pricing with negotiation instead of using dynamic pricing only) will be higher when inventory level is high. In fact, Figure 2.4 shows that the percentage revenue improvement from negotiation increases in the inventory level. Furthermore, the larger the fraction of bargainers, the larger the revenue improvement is.

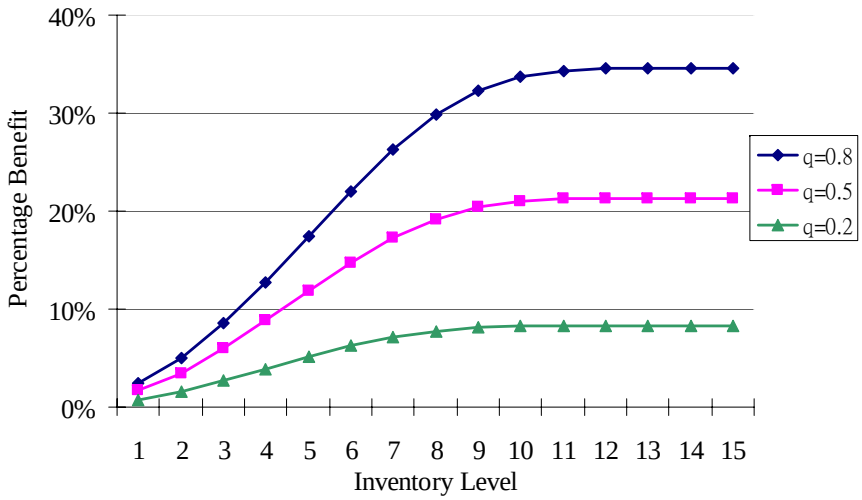

Figure 2.4: Percentage revenue improvement with respect to the starting inventory level when $\lambda = 0.7$, $\theta = 70$, $F(\cdot) \sim U[0, 200]$ and $G(\cdot|p) \sim U[0, p]$.

The effect of inventory level on the price premium: In Section 2.4, under Assumption 3, we proved that the posted price of a negotiating seller with y units of inventory in period t, $p_t^*(y)$ includes a premium on top of the price the same seller would choose if it did not use negotiation in period t, $p_t^{TL}(y)$. We now compare the posted price of the negotiating seller ($p_t^*(y)$) to the price that the seller would choose if the seller were using dynamically adjusted take-it-or-leave-it prices throughout the horizon. Figure 2.5 illustrates how the inventory level affects the price premium for the negotiating seller.

When inventory level is very low (compared to remaining selling horizon), the seller has many opportunities to sell. Thus, dynamic pricing dictates a high posted price regardless of whether negotiation is used or not, which overshadows the price premium that the negotiating seller would charge. On the other hand, if inventory level is very high, the seller is concerned about the possibility of excess inventory

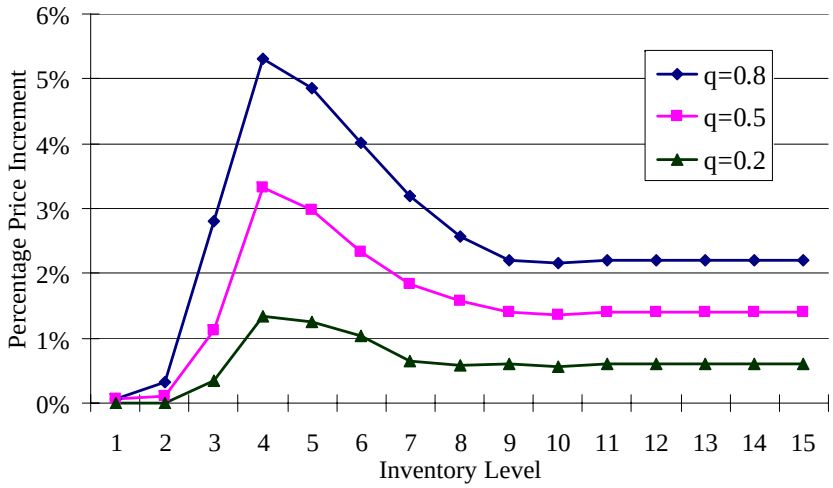

Figure 2.5: Percentage price premium under negotiation when $\lambda = 0.7$, $\theta = 5$, and $F(\cdot)$ exponential with mean 50, $G(\cdot|p) = P(X|X \leq p)$ where X is exponential with mean 40.

at the end of the selling season. In other words, the marginal value of inventory approaches zero and the premium caused by negotiation more or less stabilizes.

At moderate inventory levels, the price premium caused by negotiation is at its largest. At such inventory levels, the seller is neither pressured to move inventory quickly nor tempted to sell only to customers with very high reservation prices. Therefore, in the absence of negotiation, the seller would charge a moderate price. However, with negotiation, the seller can use a high price premium and start with a high posted price, with the intention to settle for lower prices that a bargainer may accept during negotiation.

Dynamic pricing or negotiation: Which one is better? Note that both negotiation and dynamic pricing are tools that a seller can use to improve its revenue. We next compare the benefits from each of these two strategies from the perspective of the seller who is using neither dynamic pricing nor negotiation in status quo.

Consider a seller who picks the the optimal static take-it-or-leave it price at the beginning of the selling season (i.e., the seller uses the same price throughout the selling season). Figure 2.6 illustrates, as a function of the seller's initial inventory, the revenue improvement the seller would realize by switching to dynamic pricing (without negotiation) or negotiation (without dynamic pricing). When inventory level is low, the seller would like to start with a very high price with the intention of reducing the price later in the season if the product is not selling well. Thus, the benefit from dynamic pricing exceeds the benefit from negotiation at low inventory levels. On the other hand, when inventory level is high, the seller's primary concern is to move inventory before the end of the season, in which case negotiation proves to be an effective tool, since the seller can still price discriminate without reducing the chances of making a sale. Hence, at high inventory levels, the benefit from negotiation exceeds the benefit from dynamic pricing.

Interaction between dynamic pricing and negotiation: We now examine how the benefit of negotiation depends on the seller's pricing strategy (dynamic vs. static.) To this end, consider two take-it-or-leave-it-pricing sellers: one using static pricing throughout the season and the other seller using dynamic pricing. We compare the revenue improvement (in percentage) from negotiation for these two sellers. This will enable us to identify whether or not dynamic pricing and negotiation reinforce each other.

We observe from Figure 2.7 that, when initial inventory is low, the static-pricing seller will realize larger benefits from negotiation than the dynamic-pricing seller. At low inventory levels, the seller caters only to customers with high reservation prices. Thus, both negotiation and dynamic pricing are used to raise the the price at which the product is sold, and if the seller is already using dynamic pricing, the additional

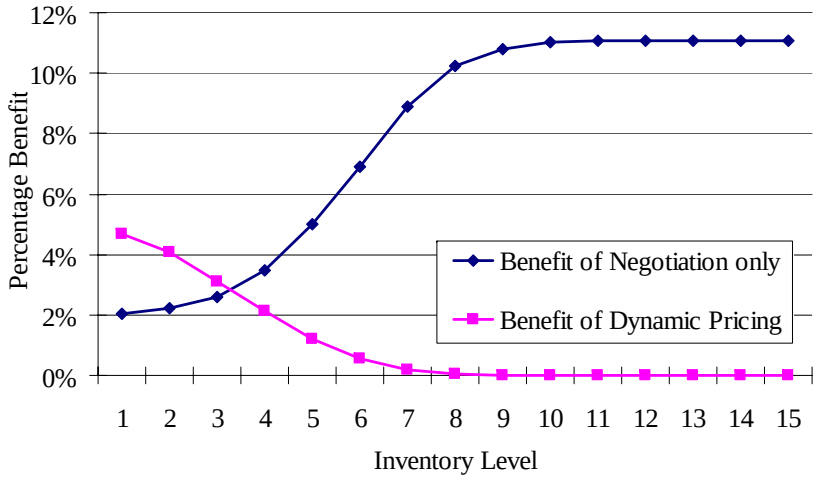

Figure 2.6: Consider the seller who currently uses static pricing with no negotiation. The figure shows the percentage revenue improvement when switching to dynamic pricing only and switching to negotiation only. Here, $\lambda = 0.7$, $\theta = 20$, $q = 0.8$, and $F(\cdot)$ exponential with mean 50, $G(\cdot|p) = P(X|X \leq p)$ where X is exponential with mean 40.

benefit from negotiation is small. In other words, dynamic pricing and negotiation act as substitutes at low inventory levels.

Interestingly, however, this is reversed at moderate inventory levels, that is, the benefit of adding negotiation is larger for the dynamic-pricing seller than the static-pricing seller. As we discussed before, at moderate inventory levels, the negotiating seller charges a high posted price under dynamic pricing with the intention to settle for lower prices during negotiation. The dynamic-pricing seller can do this, because if the product is not sold in the current period, the seller can always reduce the price in the next period. Under static pricing, however, if the seller started with a high posted price to take advantage of negotiation, then the seller would be stuck with that high price throughout the horizon, which curbs the seller's ability to exploit negotiation through the use of high list prices. Hence, at moderate inventory levels,

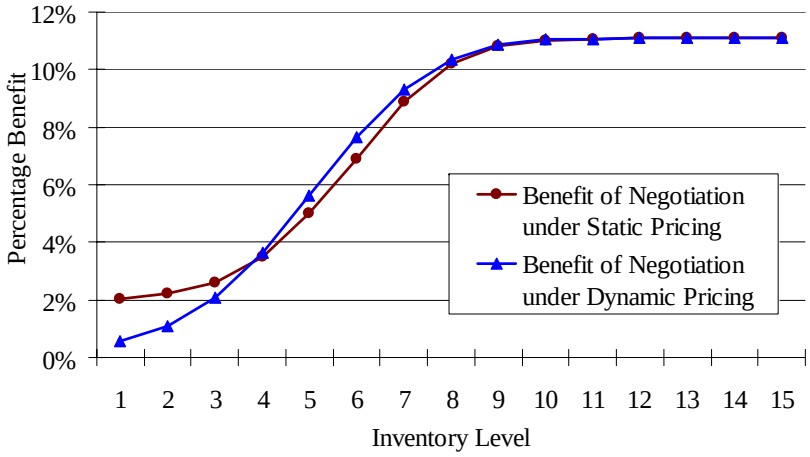

Figure 2.7: Consider the two take-it-or-leave-it pricing sellers, one using static pricing and one using dynamic pricing strategies. The figure illustrates the percentage revenue improvement that each seller can realize by adding negotiation. Here, $\lambda = 0.7$, $\theta = 20$, $q = 0.8$, and $F(\cdot)$ exponential with mean 50, $G(\cdot|p) = P(X|X \leq p)$ where X is exponential with mean 40.

negotiation is not as beneficial under static pricing as it is under dynamic pricing.

2.6 Summary

In this chapter we investigate the effect of negotiation on the dynamic pricing of a seller with limited inventory. We have presented a negotiation model for the seller with limited inventory, and embedded the outcome of the negotiation into the corresponding dynamic pricing problem. Our negotiation model allows us to capture interactions among key drivers of the seller and the buyer's decisions: the seller's marginal value of inventory and the buyer's reservation price and type. We have demonstrated that our proposed model captures a spectrum of outcomes that may arise in practice while maintaining analytical tractability to draw insightful results. Our results suggest that negotiation is an effective tool that eases the tension between revenue per sale and the risk of excess inventory at the end of the selling season, which

is a key trade-off in dynamic pricing.

We show that a negotiating seller increases the posted price. The high posted price increases the revenue per sale without compromising the chance of making a sale as some customers can purchase at discounted prices. This is particularly helpful when the inventory level is moderate or high relative to the remaining selling season. In such cases, as negotiation mitigates the risk of excess inventory, the seller can raise the posted price substantially higher than the seller who does not negotiate, and the benefit of using negotiation increases in the inventory level. On the other hand, when inventory level is low, the risk of excess inventory is already low, thus the seller is primarily interested in selling at high prices. As a result, the additional benefit from negotiation is not as significant.

CHAPTER 3

The Effect of Negotiation on the Supply Chain

3.1 Introduction

As discussed in Chapter 1, retailers actively decide whether to use negotiation or posted pricing when selling to the end customers, and different retailers use different sales formats to sell the same product. Negotiation enables the retailer to extract larger revenue from the customers who are willing to pay more, but this enhanced ability to price discriminate often comes at a cost: The negotiation takes time and effort on the parts of the retailer and customers. For example, eliminating negotiation and selling at a fixed, posted price tends to reduce the need for additional sales managers at a dealership, which could result in significant savings given that sales managers make about $150,000 per year. In addition, it is reported that dealers who adopt haggle-free pricing experience a reduction of about $300 in per-car advertisement costs. Posted pricing could bring similar benefits for customers. Customers of Scion, the only division of Toyota that does not allow dealers to bargain, spend 45 minutes to close a deal, as opposed to the national average of four and a half hours (Welch, 2007). When choosing the sales format, the retailer has to weigh the cost of negotiation against the benefit from price discrimination enabled by negotiation. Of course, the retailer's sales format choice affects the profit of the manufacturer

who provides the goods sold by the retailer. The manufacturer can influence the retailer's decision, for example, through a simple contract like the wholesale-price-only contract. This chapter aims to highlight what sales format and price choices may arise in equilibrium, given that the manufacturer's power to set the wholesale price is pitched against the retailer's power to determine the sales format.

When analyzing the equilibrium sales format, we place particular emphasis on the effect of capacity available in the supply chain. The availability of a product tends to have significant effect on the discounts that retailers concede to bargainers. For example, according to Edmunds.com, customers who purchase Camry XLE in southeast Michigan are able to negotiate, on average, a discount of 6% from MSRP, while the Prius, which is in short supply, is sold at an average discount of only 1.3%. Since the availability of a product has a significant effect on the transaction prices, one expects it to have some influence over the equilibrium sales format. In this chapter, we model availability in the form of a capacity constraint on the supply chain, and we analyze the effect of supply chain capacity on the equilibrium.

The remainder of the chapter is organized as follows. Section 3.2 provides a survey of the relevant literature. Section 3.3 outlines our model. In Section 3.4, we analyze the problem without capacity constraints, which sets the stage for Section 3.5 where we analyze the supply chain with capacity constraints. We conclude in Section 3.6 with a summary. All proofs are provided in Appendix B.

3.2 Literature Review

Bargaining has been studied extensively in economics and there is a wealth of research about predicting the bargaining outcome under several different bargaining processes and information structures. See Muthoo (1999) for a review of bargaining

theory and applications in economics. One classic approach to modeling the outcome of bargaining is to use the Nash bargaining solution. The Nash bargaining solution is the outcome of a cooperative game where two parties maximize the product of their surpluses net of their disagreement utilities. As such, the Nash bargaining solution is a mutually beneficial agreement that splits the total surplus (net of disagreement payoffs) equally. This classic Nash bargaining solution can be extended to the case where the two parties have different bargaining powers, in which case the more powerful party grabs a larger portion of the total surplus. This generalized Nash bargaining solution is what we use to model the outcome of the negotiation between the retailer and the customer.

There is another stream of research that focuses on the comparison of bargaining and posted pricing. A subset of this research stream uses generalized Nash bargaining to model the outcome of negotiation. Among these are Wang (1995) who considers a seller offering an indivisible object, Bester (1993) who considers a group of competing sellers all of whom collectively use either posted pricing or negotiation, Roth, Woratschek, and Pastowski (2006) who consider a seller offering a customizable product. In addition, other researchers have addressed the question of posted pricing versus bargaining using embellished models of negotiation, such as alternating offers by the seller and the customer, or incomplete information. These include Riley and Zeckhauser (1983), Arnold and Lippman (1998), Adachi (1999), Desai and Purohit (2004). Terwiesch, Savin and Hann (2005) considers the effect of negotiation in online haggling. Unlike all of this earlier work where the seller(s) decide whether to use negotiation or posted pricing, we focus on a two-stage supply chain problem, in which the retailer's sales format choice is influenced by the manufacturer's wholesale price. In addition, we explicitly account for the effect of supply chain capacity,

which may distort the retailer and the manufacturer's profits.

There are several recent papers that analyze negotiation in the context of supply chain management. These include Nagarajan and Bassok (2002), Wu (2004), Iyer and Villas-Boas (2003), Gurnani and Shi (2006), and Lovejoy (2007). For a review on cooperative bargaining in supply chains, see Nagarajan and Sosic (2008). Most of this work models negotiation between a supplier(s) and a buyer(s) who then meets the end customer demand by selling at a posted price. In contrast, we examine the sales format choice of the retailer, who may use posted pricing or negotiation when selling to the end customers, and we analyze how this choice can be influenced by the manufacturer whose profit also depends on the retailer's sales format.

3.3 Model Description

We consider a supply chain comprised of one manufacturer and one retailer where the manufacturer produces an item at a unit cost of c and sells at a unit wholesale price, $w \geq c$. After the manufacturer determines the wholesale price, the retailer decides which of the two sales formats to adopt when selling to the end customers: posted pricing or negotiation. If the retailer decides to adopt posted pricing, the retailer must choose the take-it-or-leave-it price to be used. If the retailer decides to adopt negotiation, the retailer must choose the minimum price that the retailer is willing to accept. These specific pricing decisions determine the rules of transaction between the retailer and consumers, and drive aggregate demand. The retailer then orders from the manufacturer up to the supply chain capacity, Q. The supply chain capacity, Q, admits multiple interpretations: It could arise from the manufacturer's limited production capacity or the retailer's storage space or working capital constraints. We assume that the capacity, Q, is exogenously given.

We consider an infinitesimally-divisible consumer population in which the consumers are heterogeneous in their valuation of the item. Let a be the size of the consumer population and $\overline{F}(x) := 1 - F(x)$ represent the fraction of the consumer population who values the product at x or more. Then, $a\overline{F}(x)$ can be interpreted as the portion of the consumer population with valuation x or higher. In the remainder of the chapter, we refer to $F(x)$ as the valuation distribution and we denote its density by $f(x)$.

3.3.1 Posted Pricing

If the retailer decides to adopt posted pricing and picks posted price p, then only consumers with valuations p or higher will buy the product. Thus, the aggregate demand at price p is given by $D(p) := a\overline{F}(p)$. Many commonly-used demand functions are covered by this model. If the valuation distribution is uniform, then the aggregate demand is linear in price. If the valuation distribution is exponential, then the aggregate demand is log-linear. In addition, by picking an appropriate valuation distribution $F(\cdot)$, one can model the case where the aggregate demand is given by the logit demand function, which represents the aggregate demand of utility-maximizing consumers, choosing between two options – 'buy' and 'no buy,' where the utility of each option is drawn from a Gumbel distribution (for more on this, see Chapter 7.3 in Talluri and van Ryzin, 2005). Table 3.1 lists several specific examples of the valuation distributions, and corresponding aggregate demand functions, covered by our model.

Since the supply chain capacity is limited to Q, given posted price p and wholesale

Valuation distribution $F(p)$	Aggregate Demand $D(p)$
Uniform$[0, \frac{a}{b}]$: $F(p) = \frac{pb}{a}$	$a - bp$ (linear demand)
Exponential(λ): $F(p) = 1 - e^{-\lambda p}$	$ae^{-\lambda p}$ (log-linear demand)
Weibull(α,β): $F(p) = 1 - e^{-(\frac{p}{\beta})^{\alpha}}$	$ae^{-(\frac{p}{\beta})^{\alpha}}$
Difference of two Gumbel r.v.'s with scale parameter 1 and means α and 0: $F(p) = \frac{1}{1+e^{\alpha-p}}$	$\frac{ae^{\alpha-p}}{1+e^{\alpha-p}}$ (logit demand)

Table 3.1: Examples of valuation distributions and corresponding aggregate demand functions.

price w, the retailer's and manufacturer's profits are given by

$$\Pi_{\text{RP}}(p, w, Q) = (p - w)\min\{D(p), Q\} = (p - w)\min\{a\overline{F}(p), Q\}, \text{ and} \tag{3.1}$$

$$\Pi_{\text{MP}}(w, p, Q) = (w - c)\min\{D(p), Q\} = (w - c)\min\{a\overline{F}(p), Q\}. \tag{3.2}$$

If capacity level Q is low enough, the quantity sold could be bounded by the capacity level when the posted price is low. Let $\bar{p}(Q)$ be the *market-clearing price* at which the demand equals the capacity: $D(\bar{p}(Q)) = Q$ (if it exists). Note that the retailer will not set the price below $\bar{p}(Q)$. Had the retailer set a price below $\bar{p}(Q)$, the retailer could increase the per-unit profit margin without changing the quantity sold.

3.3.2 Negotiation

If the retailer decides to adopt negotiation, the retailer must determine the *cut-off price*, i.e., the minimum price at which it is willing to sell, denoted by $p_{\min}$. Observe that it is not necessarily in the retailer's best interest to sell to customers with low valuations. Hence, the retailer may price some customers out of the market by setting $p_{\min}$ high enough, and doing so could increase its profit.

Negotiation takes time and effort on the parts of both the retailer and the cus-

tomer, which are captured by negotiation costs in our model. Let c_r and c_b denote the cost of negotiation incurred by the retailer and consumer, respectively. Effectively, the retailer's cost, c_r, reduces the retailer's profit margin while the customer's cost, c_b, reduces the customer's willingness-to-pay. In addition, one of the two parties may have more say in shaping the outcome of the negotiation, for example, the retailer could gain more if the customer has few outside options and in a weak bargaining position. Our model allows such asymmetry in bargaining power.

We model the outcome of negotiation between a customer and the retailer through generalized Nash bargaining, under which the total surplus is split between the two parties according to their relative bargaining power. The outcome of negotiation is determined by the retailer's cut-off price, $p_{\min}$, the customer's valuation of the item, denoted by r and drawn from $F(\cdot)$, costs of negotiation, and each party's relative bargaining power. Generalized Nash bargaining solution models the relative bargaining power in the form of the parameter, $\beta \in (0, 1)$. Let β be the customer's relative bargaining power and $1 - \beta$ the retailer's bargaining power.

The retailer must choose the cut-off price, $p_{\min}$, so that it covers at least the wholesale price plus the retailer's cost of negotiation: $p_{\min} \geq w + c_r$. In our model, a customer also incurs cost to negotiate, given by c_b. Thus, only the consumers with valuation $p_{\min} + c_b$ and above will engage in negotiation and buy the item; the rest will choose not to buy. If the final price agreed by both parties is p_{N}, a consumer with valuation r will obtain a surplus of $r - p_{\text{N}} - c_b$. For the same final price, the retailer's (extra) surplus beyond $p_{\min}$ is $p_{\text{N}} - p_{\min}$. Following the generalized Nash bargaining solution (Muthoo 1999), a consumer with valuation $r \geq p_{\min} + c_b$ and a retailer with the cut-off price $p_{\min} \geq w + c_r$ will agree on a final price $p^*_{\text{N}}(p_{\min}, r)$ that

maximizes the following objective function.

$$\max_{p_{\text{N}} \in [p_{\min}, r]} \quad (r - p_{\text{N}} - c_b)^{\beta} \, (p_{\text{N}} - p_{\min})^{1-\beta}. \tag{3.3}$$

Note the significance of β, which represents the relative bargaining power of the consumers. If $\beta \to 1$, any consumer with valuation $p_{\min} + c_b$ and above has all the bargaining power and extracts the entire surplus after paying the final price of $p^*_{\text{N}}(p_{\min}, r) = p_{\min}$. On the other hand, if $\beta \to 0$, the retailer extracts the entire surplus by charging $p^*_{\text{N}}(p_{\min}, r) = r - c_b$ to a consumer with valuation r. For any $\beta \in (0, 1)$, the final price $p^*_{\text{N}}(p_{\min}, r)$ is a convex combination of $r - c_b$ and $p_{\min}$, which splits the surplus:

$$\begin{aligned} p^*_{\text{N}}(p_{\min}, r) &= \arg\max_{p_{\text{N}}} [r - p_{\text{N}} - c_b]^{\beta} \times [p_{\text{N}} - p_{\min}]^{1-\beta} \\ &= (1 - \beta)(r - c_b) + \beta p_{\min}. \end{aligned} \tag{3.4}$$

Given the cut-off price $p_{\min}$, the lowest valuation among the customers who buy is $p_{\min} + c_b$, which we will denote by $q_{\min}$ and refer to as the *cut-off valuation*,

$$q_{\min} := p_{\min} + c_b. \tag{3.5}$$

Thus, choosing $p_{\min}$ is equivalent to choosing $q_{\min}$, and (3.4) can be re-written as a function of $q_{\min}$:

$$p^*_{\text{N}}(q_{\min} - c_b, r) = (1 - \beta) r + \beta q_{\min} - c_b \tag{3.6}$$

If the capacity level Q is low enough and the retailer chooses the cut-off valuation $q_{\min}$ so low that $q_{\min} < \bar{p}(Q)$, then the demand $a\overline{F}(q_{\min})$ will exceed the capacity Q. If $q_{\min}$ were so low, the retailer could always increase the cut-off valuation slightly, which would increase the transaction price $p^*_{\text{N}}(q_{\min} - c_b, r)$ without decreasing the quantity sold, thereby improving the retailer's total profit. Hence, the retailer will

never choose $q_{\min} < \bar{p}(Q)$. By setting $q_{\min} = \bar{p}(Q)$, the retailer could set demand equal to the capacity, Q.

For any $q_{\min} \geq \max[w + c_r + c_b, \bar{p}(Q)]$, the retailer's and manufacturer's profits are given by

$$\begin{aligned}\Pi_{\text{RN}}(q_{\min}, w, Q) &= aE_r\left[(p^*_{\text{N}}(q_{\min} - c_b, r) - w - c_r)\mathbf{1}_{\{r \geq q_{\min}\}}\right] \\ &= aE_r\left[((1-\beta)r + \beta q_{\min} - w - c_r - c_b)\mathbf{1}_{\{r \geq q_{\min}\}}\right] \\ &= a\int_{q_{\min}}^{\infty}\left[(1-\beta)x + \beta q_{\min} - w - c_r - c_b\right] f(x)dx, \text{ and} \qquad (3.7)\end{aligned}$$

$$\Pi_{\text{MN}}(w, q_{\min}, Q) = (w - c)\min\{D(q_{\min}), Q\} = (w - c)a\overline{F}(q_{\min}). \qquad (3.8)$$

Note that the retailer's choice of $q_{\min}$ (equivalently, $p_{\min}$) affects not only the profit margin per unit sold, but also the portion of consumers who negotiate successfully and buy: The larger $q_{\min}$ is, the higher the price paid by consumers, but the smaller the fraction of consumers who buy. This trade-off plays a critical role when choosing the optimal cut-off valuation.

3.3.3 Sales Format and Pricing Decisions in the Supply Chain

We first describe the problem that the retailer faces. Given capacity Q, the retailer's best response to the manufacturer's wholesale price w consists of the sales format choice and an associated pricing decision. Let $\mathcal{I}_R$ be an indicator variable that represents the retailer's decision on the sales format: $\mathcal{I}_R = 1$ if the retailer chooses the posted pricing strategy and $\mathcal{I}_R = 0$ otherwise. Let $p^*(w, Q)$ be the maximizer of $\Pi_{\text{RP}}(p, w, Q)$ and $q^*_{\min}(w, Q)$ the maximizer of $\Pi_{\text{RN}}(q_{\min}, w, Q)$ for given wholesale price w and capacity Q. Note that $p^*(w, Q)$ is the price that the retailer will use under posted pricing and $q^*_{\min}(w, Q)$ is the cut-off valuation that the retailer will use under negotiation. Then, the retailer's best response will be either $(\mathcal{I}_R = 1, p^*(w, Q))$ or

$(\mathcal{I}_R = 0, q^*_{\min}(w,Q))$. Thus, for given wholesale price w and capacity Q, the retailer solves the following problem:

$$\max_{\mathcal{I}_R \in \{0,1\}} [\mathcal{I}_R \Pi_{\text{RP}}(p^*(w,Q),w,Q) + (1-\mathcal{I}_R)\Pi_{\text{RN}}(q^*_{\min}(w,Q),w,Q)] \tag{3.9}$$

We next turn to the manufacturer's problem. The manufacturer chooses its wholesale price anticipating the retailer's best response. Let $\mathcal{I}^*_R(w,Q)$ denote the retailer's optimal sales format choice for a given w and Q. The manufacturer is then solving the following problem:

$$\max_{w \geq c} [\mathcal{I}^*_R(w,Q)\Pi_{\text{MP}}(w,p^*(w,Q),Q) + (1-\mathcal{I}^*_R(w,Q))\Pi_{\text{MN}}(w,q^*_{\min}(w,Q),Q)] \tag{3.10}$$

Notice that in our model the retailer actively chooses one of two sales formats, posted pricing or negotiation. As we will demonstrate, the retailer's discretion to choose the sales format has a crucial effect on the equilibrium outcome.

3.4 No Capacity Constraint

As a benchmark, we first analyze the case when the supply chain has sufficient capacity to meet any demand. For example, if capacity Q is greater than or equal to the size of consumer population, a, then the capacity plays no role. In this section, we drop Q from the notation.

Throughout the chapter, we make the following technical assumptions on the valuation distribution, $F(\cdot)$ and its density $f(\cdot)$. The first assumption ensures that the retailer's profit functions are well-behaved while the second assumption does the same for the manufacturer's profit functions.

(A1) The valuation distribution, $F(\cdot)$, is strictly increasing over the domain of non-negative real numbers, and has an increasing failure rate.

(A2) The density $f(\cdot)$ is twice differentiable and satisfies the following condition:

$$f'(x)(2f'(x)\overline{F}(x) + f^2(x)) - f''(x)f(x)\overline{F}(x) \geq 0 \tag{3.11}$$

Note that these assumptions are satisfied by many valuation distributions including those listed in Table 3.1 except for uniform. Our assumption that $F(\cdot)$ has an unbounded domain is what rules out the uniform distribution. The end point of a bounded domain causes expositional complications in the proofs, which is why we make the assumption of unbounded domains. We should note that we have analyzed the uniform case separately, taking advantage of closed-form expressions for the optimal solutions and profit functions. All the results stated as **Proposition** and **Corollary** throughout the chapter continue to hold for the uniform case. Throughout the chapter, we use increasing/decreasing and positive/negative in the weak sense unless otherwise specified as being *strict*.

In preparation for characterizing the equilibrium of the game, we first analyze the structure of the profit functions for the manufacturer and retailer under each sales format.

3.4.1 Posted Pricing

Consider a supply chain with unlimited capacity where posted pricing is imposed exogenously. Let $\Pi^u_{\mathbf{RP}}(p, w)$ and $\Pi^u_{\mathbf{MP}}(w, p)$ denote the retailer's and the manufacturer's profit functions under posted pricing. (Formally, $\Pi^u_{\mathbf{RP}}(p, w)$ and $\Pi^u_{\mathbf{MP}}(w, p)$ are defined by equations (3.1) and (3.2) with $Q = \infty$, respectively.) The following lemma establishes the structural properties of the manufacturer's and retailer's profit functions under posted pricing.

Lemma 3.4.1. [Profit functions under posted pricing]

(*a*) *The retailer's profit,* $\Pi^u_{\mathbf{RP}}(p, w)$*, is strictly unimodal in the posted price,* p*.*

(*b*) *Let* $p^u(w)$ *denote the optimal posted price, that is, the maximizer of* $\Pi^u_{\mathbf{RP}}(p, w)$. *Then,* $p^u(w)$ *is convex and strictly increasing in the wholesale price,* w.

(*c*) *Given that the retailer chooses the posted price optimally, the manufacturer's profit,* $\Pi^u_{\mathbf{MP}}(w, p^u(w))$, *is strictly unimodal in* w.

We should point out that Assumption (A1) is needed for Lemma 3.4.1(a) while Assumption (A2) is needed for Lemma 3.4.1(b), which enables us to show that the manufacturer's profit function is well-behaved as described in Lemma 3.4.1(c). Let $w^u_{\mathbf{P}}$ be the maximizer of $\Pi^u_{\mathbf{MP}}(w, p^u(w))$. Note that $w^u_{\mathbf{P}}$ is the wholesale price that the manufacturer will use in a supply chain where posted pricing is the exogenously chosen sales format.

3.4.2 Negotiation

Consider a supply chain with unlimited capacity where negotiation is imposed exogenously. The next lemma establishes the structural properties of the retailer's and the manufacturer's profit functions under negotiation, $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ and $\Pi^u_{\mathbf{MN}}(w, q_{\min})$, which are defined by equations (3.7) and (3.8) with capacity Q sufficiently large.

Lemma 3.4.2. [Profit functions under negotiation]

(*a*) *The retailer's profit,* $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$, *is strictly unimodal in the retailer's cut-off valuation,* $q_{\min}$.

(*b*) *Let* $q^u_{\min}(w)$ *denote the optimal cut-off valuation, that is, the maximizer of* $\Pi^u_{\mathbf{RP}}(p, w)$. *Then* $q^u_{\min}(w)$ *is convex and strictly increasing in* w.

(*c*) *Given that the retailer chooses the cut-off valuation optimally, the manufacturer's profit,* $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$, *is strictly unimodal in* w.

Let $w^u_{\mathbf{N}}$ be the maximizer of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$. Note that $w^u_{\mathbf{N}}$ is the wholesale price that the manufacturer should use in a supply chain where negotiation is the exoge-

nously chosen sales format. Consequently, $q^u_{\min}(w^u_{\mathbf{N}})$ would be the cut-off valuation chosen by the retailer in such a supply chain. The next lemma states a useful property about how $w^u_{\mathbf{N}}$ and $q^u_{\min}(w^u_{\mathbf{N}})$ depend on the cost of negotiation for the retailer, c_r and cost of negotiation for the customer, c_b.

Lemma 3.4.3. *For all (c_r, c_b) such that $c_r + c_b = c_{\mathbf{T}}$, for some $c_{\mathbf{T}} \geq 0$, $w^u_{\mathbf{N}}$ and $q^u_{\min}(w^u_{\mathbf{N}})$ remain the same.*

Lemma 3.4.3 implies that in a supply chain where negotiation is exogenously chosen as the sales format, both the manufacturer wholesale price and the retailer's cut-off valuation depend only on the total negotiation cost, $c_r + c_b$. To see why, first observe that the retailer's margin per unit sold is the transaction price, $p^*_{\mathbf{N}}$, minus the wholesale price, w, and the retailer's cost of negotiation, c_r. In addition, notice from equation (3.6) that the transaction price, $p^*_{\mathbf{N}}$, is reduced by the customer's cost of negotiation, c_b, which implies that the retailer absorbs the customer's cost of negotiation as well. Thus, for every unit sold, the retailer's margin depends only on the total cost of negotiation, but not on how that cost is allocated between c_b and c_r, which explains the above lemma. In the remainder of this chapter, let $c_{\mathbf{T}} := c_r + c_b$ denote the total cost of negotiation.

3.4.3 Equilibrium Analysis

Recall that, in our model, the sales format is not exogenously chosen, but the retailer chooses the sales format to maximize its profit given the manufacturer's wholesale price. To characterize the equilibrium behavior in our model, we first characterize the retailer's best response, represented by the sales format choice and the corresponding pricing decision, as a function of the manufacturer's wholesale price w.

Proposition 3.4.1. [Retailer's best response]

(a) If the retailer (weakly) prefers posted pricing at $w = c$, then the retailer strictly prefers posted pricing for all $w > c$.

Otherwise (i.e., if the retailer strictly prefers negotiation at $w = c$):

(b) either the retailer strictly prefers negotiation at all $w > c$,

(c) or there exists a unique threshold $\widehat{w}^u_{\mathbf{R}} > c$ such that the retailer is indifferent between negotiation and posted pricing if $w = \widehat{w}^u_{\mathbf{R}}$, strictly prefers negotiation if $w < \widehat{w}^u_{\mathbf{R}}$, and strictly prefers posted pricing if $w > \widehat{w}^u_{\mathbf{R}}$.

Proposition 3.4.1 implies that once the retailer prefers posted pricing at a given wholesale price, then the retailer continues to prefer posted pricing at all higher wholesale prices. To understand why, we first rewrite the retailer's profit function given by equation (3.7):

$$\begin{aligned}\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) &= a\int_{q^u_{\min}(w)}^{\infty} [q^u_{\min}(w) + (1-\beta)(x - q^u_{\min}(w)) - w - c_{\mathbf{T}}]\ f(x)dx \\ &= a\overline{F}(q^u_{\min}(w))(q^u_{\min}(w) - w - c_{\mathbf{T}}) + a(1-\beta)E_r[(r - q^u_{\min}(w))^+]\end{aligned} \tag{3.12}$$

The first term in the equation above is equivalent to the expected profit under posted pricing when the posted price is $q^u_{\min}(w)$ and the wholesale price is $w + c_{\mathbf{T}}$, leaving the retailer a unit margin of $q^u_{\min}(w) - w - c_{\mathbf{T}}$. This term is always less than the profit that the retailer could obtain if it used posted pricing at the wholesale price w. Therefore, had this been the only revenue obtained by the negotiating retailer, the retailer would always be better off using posted pricing. Under negotiation, however, only the marginal customer (with valuation $q_{\min}$) yields a margin precisely equal to $q^u_{\min}(w) - w - c_{\mathbf{T}}$, and customers with higher valuations yield higher margins (see equation (3.6)). In fact, a customer with valuation $r > q^u_{\min}(w)$ leaves an additional

$(1-\beta)(r-q^u_{\min}(w))$ on top of what the marginal customer yields. We refer to this difference as the price premium. The second term represents the expected price premium the retailer collects under negotiation. If this term is sufficiently large, then the retailer would be better off under negotiation. Now, note that the second term in (3.12), which represents the expected premium, could be rewritten as

$$a(1-\beta)\overline{F}(q^u_{\min}(w))E_r[r-q^u_{\min}(w)|r\geq q^u_{\min}(w)].$$

As the wholesale price w and, hence, the cut-off valuation $q^u_{\min}(w)$ increase, the expected premium decreases under our assumption that the valuation distribution has IFR.[1] Therefore, as w increases, the benefit from negotiation decreases, which makes negotiation less attractive at higher wholesale prices, as indicated by Proposition 3.4.1.

It is interesting to note that Proposition 3.4.1 will not hold if the valuation distribution has decreasing failure rate (DFR). In such a case, the mean residual valuation could increase in w, which would make negotiation more attractive at higher wholesale prices. In fact, the following figure shows a counter-example to Proposition 3.4.1 for a DFR valuation distribution. Nonetheless, many commonly used aggregate demand functions, including those listed in Table 3.1, are based on valuation distributions with IFR.

Observe from Proposition 3.4.1 that there may exist a wholesale price $\widehat{w}^u_{\mathbf{R}}$ that makes the retailer indifferent between the two sales formats. The following proposition states that at such a wholesale price, the manufacturer prefers negotiation.

Proposition 3.4.2. *Suppose there exists $\widehat{w}^u_{\mathbf{R}} > c$ that makes the retailer indifferent between negotiation and posted pricing, as described in Proposition 3.4.1(c). At the*

[1]The technical property that allows this result is that a random variable with IFR distribution has a decreasing mean residual lifetime.

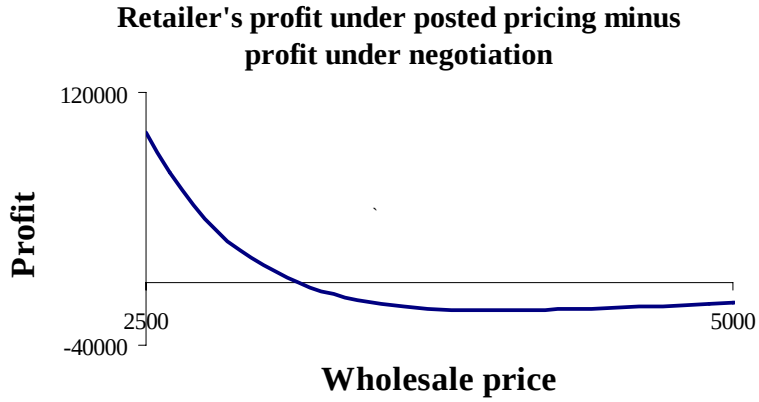

Figure 3.1: The figure illustrates the counter-example to Proposition 3.4.1. In this example, the retailer prefers posted pricing at lower wholesale prices and negotiation at higher wholesale prices. Here, $a = 200,000$, $\beta = 0.8$, $c_{\mathbf{T}} = 200$, and $F(\cdot)$ Weibull with shape parameter 0.5 and scale parameter 50.

wholesale price $\widehat{w}^u_{\mathbf{R}}$*, the manufacturer prefers negotiation. In other words,*

$$\Pi^u_{\mathbf{MN}}(\widehat{w}^u_{\mathbf{R}}, q^u_{\min}(\widehat{w}^u_{\mathbf{R}})) \geq \Pi^u_{\mathbf{MP}}(\widehat{w}^u_{\mathbf{R}}, p^u(\widehat{w}^u_{\mathbf{R}})).$$

Hence, negotiation is the Pareto-optimal sales format when the wholesale price is $\widehat{w}^u_{\mathbf{R}}$, and applying the Pareto-dominance criterion, we assume that the retailer chooses negotiation whenever $w = \widehat{w}^u_{\mathbf{R}}$.[2]

Based on the structure of the best response established in Propositions 3.4.1 and 3.4.2, the manufacturer's problem of selecting the wholesale price, stated in equation (3.10), can now be expressed as follows:

Manufacturer's Problem:

If the retailer prefers posted pricing for all $w \geq c$*, then:*

$$\max_{w \geq c} \ \Pi^u_{\mathbf{MP}}(w, p^u(w)) \tag{3.13}$$

[2]Such tie-breaking behavior on the part of the retailer can be easily induced by choosing the wholesale price $\widehat{w}^u_{\mathbf{R}} - \epsilon$ for arbitrarily small $\epsilon > 0$.

If the retailer prefers negotiation for all $w \geq c$, then:

$$\max_{w \geq c} \ \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \tag{3.14}$$

If the retailer prefers negotiation for $c \leq w \leq \widehat{w}^u_{\mathbf{R}}$ and posted pricing for $w > \widehat{w}^u_{\mathbf{R}}$ then:

$$\max \left[\max_{c \leq w \leq \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \ , \ \sup_{w > \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MP}}(w, p^u(w)) \right] \tag{3.15}$$

The manufacturer's problems in (3.13) and (3.14) correspond to Proposition 3.4.1(a) and (b), where the retailer's sales format choice cannot be influenced by the manufacturer's wholesale price. On the other hand, if Proposition 3.4.1(c) holds, with the retailer's discretion in mind, the manufacturer chooses the wholesale price to induce either negotiation (i.e., $c \leq w \leq \widehat{w}^u_{\mathbf{R}}$) or posted pricing (i.e., $w > \widehat{w}^u_{\mathbf{R}}$), whichever yields a larger profit for the manufacturer. The next proposition shows that the manufacturer will choose one of three wholesale prices, leading to one of three forms of equilibria:

Proposition 3.4.3. *The manufacturer chooses one of the following wholesale prices:*
(a) $w^u_{\mathbf{N}}$, the maximizer of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$, which leads the retailer to use negotiation with cut-off valuation $q^u_{\min}(w^u_{\mathbf{N}})$ in equilibrium, or
(b) $\widehat{w}^u_{\mathbf{R}}$, the threshold wholesale price, which leads the retailer to use negotiation with cut-off valuation $q^u_{\min}(\widehat{w}^u_{\mathbf{R}})$ in equilibrium, or
(c) $w^u_{\mathbf{P}}$, the maximizer of $\Pi^u_{\mathbf{MP}}(w, p^u(w))$, which leads the retailer to use posted pricing with price $p^u(w^u_{\mathbf{P}})$ in equilibrium.

The equilibrium outcome described in Proposition 3.4.3(a) is the same outcome that would arise if the supply chain were exogenously restricted to use negotiation. Likewise, the equilibrium outcome in Proposition 3.4.3(c) is the one that would arise

if the supply chain were exogenously restricted to use posted pricing. Proposition 3.4.3(b), on the other hand, shows that there is a different type of negotiation equilibrium that gives rise to a different wholesale price. This type of equilibrium is a consequence of the retailer's discretion over the sales format. It arises when the manufacturer would like to induce the retailer to choose negotiation, but cannot do so at the wholesale price $w^u_{\mathbf{N}}$. In such cases, the manufacturer offers a discounted wholesale price, $\widehat{w}^u_{\mathbf{R}}$, thereby sacrificing some of its profit margin in order to induce its preferred sales format. We refer to this form of equilibrium as *reconciliatory negotiation.*

The total cost of negotiation, $c_{\mathbf{T}}$, influences which of the three candidates arises as an equilibrium. The following proposition characterizes how the equilibrium changes with respect to $c_{\mathbf{T}}$.

Proposition 3.4.4. *There exist two thresholds, $\underline{c_{\mathbf{T}}}$ and $\overline{c_{\mathbf{T}}}$, $\underline{c_{\mathbf{T}}} \leq \overline{c_{\mathbf{T}}}$, such that*

(a) **[Negotiation]** *if $c_{\mathbf{T}} < \underline{c_{\mathbf{T}}}$, then the equilibrium sales format is negotiation with the wholesale price $w^u_{\mathbf{N}}$, resulting in the retailer's cut-off valuation $q^u_{\min}(w^u_{\mathbf{N}})$,*

(b) **[Reconciliatory Negotiation]** *if $\underline{c_{\mathbf{T}}} \leq c_{\mathbf{T}} < \overline{c_{\mathbf{T}}}$, then the equilibrium sales format is negotiation with the wholesale price $\widehat{w}^u_{\mathbf{R}}$, resulting in the retailer's cut-off valuation $q^u_{\min}(\widehat{w}^u_{\mathbf{R}})$, and*

(c) **[Posted Pricing]** *if $c_{\mathbf{T}} \geq \overline{c_{\mathbf{T}}}$, then the equilibrium sales format is posted price with the wholesale price $w^u_{\mathbf{P}}$, resulting in the posted price $p^u(w^u_{\mathbf{P}})$.*

The behavior described in Proposition 3.4.4 is illustrated in Figure 3.2. When the total cost of negotiation is sufficiently low (i.e., $c_{\mathbf{T}} < \underline{c_{\mathbf{T}}}$, with $\underline{c_{\mathbf{T}}} \approx 1.3$ in the figure), negotiation is preferred by both the retailer and the manufacturer. In such cases, the manufacturer can induce negotiation without giving up any of its profit margin. In contrast, as $c_{\mathbf{T}}$ increases, it becomes harder for the manufacturer to induce the retailer to adopt negotiation. Hence, in the middle region ($\underline{c_{\mathbf{T}}} \leq c_{\mathbf{T}} < \overline{c_{\mathbf{T}}}$,

from approximately 1.3 to 1.575 in the figure), the manufacturer finds it necessary to offer a reduced wholesale price to induce negotiation, resulting in a reconciliatory negotiation equilibrium. Finally, when c_{T} becomes sufficiently large (i.e., $c_{\mathrm{T}} \geq \overline{c_{\mathrm{T}}}$, beyond 1.575 in the figure), neither party is interested in negotiation, resulting in a posted pricing equilibrium.

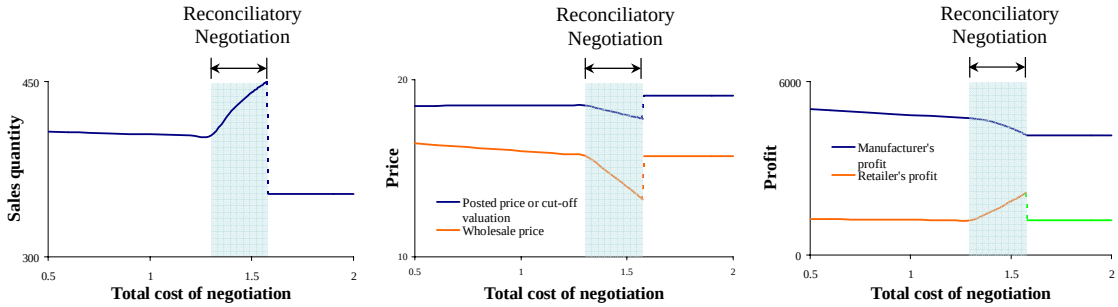

Figure 3.2: The figure illustrates the equilibrium sales quantity (left), the equilibrium wholesale price and equilibrium posted price or cut-off valuation (central), and the equilibrium manufacturer's and retailer's profits (right). Here, $a = 500$, $\beta = 0.3$, $c = 4$ and logit demand with $F(x) = \frac{1}{1+e^{20-x}}$.

It is rather surprising to note that the retailer's equilibrium profit and quantity sold may actually increase in the total cost of negotiation, c_{T}. In the region where negotiation is used, we observe from Figure 3.2 that the wholesale price continues to decrease as c_{T} increases, implying that the manufacturer is absorbing some of the increased cost of negotiation. This reduction in wholesale price becomes more pronounced in the reconciliatory negotiation region. In fact, our analysis shows that, in this region where the equilibrium wholesale price is $\widehat{w}^{u}_{\mathrm{R}}$, a unit increase in the total cost of negotiation triggers a wholesale price reduction of more than one unit (see Lemma B.1.2(b) in Appendix A). In other words, the manufacturer more than compensates the retailer for the increase in c_{T} so that negotiation remains to be the equilibrium sales format. This explains why the retailer's profit and quantity sold

increase in c_{T} under the reconciliatory negotiation regime.

3.5 Capacity Constraint

Building on our analysis of the unlimited capacity case, we now return to the original problem where the supply chain has a finite capacity, Q. In particular, we assume Q is smaller than the size of the consumer population, a. As in the previous section, we first characterize the retailer's and manufacturer's profit functions under each sales format.

3.5.1 Posted Pricing

Consider a supply chain with capacity Q, where posted pricing is imposed. If capacity Q is sufficiently low, the retailer will set the posted price to the market-clearing price, $\bar{p}(Q)$, (as described in Section 3.3.1) at which the demand equals the capacity Q: $a\overline{F}(\bar{p}(Q)) = Q$. Define the *market-clearing wholesale price* under posted pricing, $\overline{w}_{\text{P}}(Q)$, as follows:

$$p^u(\overline{w}_{\text{P}}(Q)) = \bar{p}(Q). \tag{3.16}$$

Observe that if the wholesale price is $\overline{w}_{\text{P}}(Q)$, the retailer in a supply chain with unlimited capacity finds it optimal to sell Q units under posted pricing. Notice the significance of $\overline{w}_{\text{P}}(Q)$. In the supply chain with capacity Q, if the wholesale price w is less than $\overline{w}_{\text{P}}(Q)$, then the retailer's optimal posted price, $p^*(w, Q)$, is the market-clearing price, $\bar{p}(Q)$ (since lowering the posted price any further will not increase sales quantity). On the other hand, if the wholesale price w exceeds the market-clearing wholesale price, $\overline{w}_{\text{P}}(Q)$, then the capacity is no longer binding, and the retailer's optimal posted price, $p^*(w, Q)$, is simply the price that is optimal in the supply chain with unlimited capacity, $p^u(w)$. Note that if Q is sufficiently large, there may

not exist $\overline{w}_{\text{P}}(Q) > c$. In other words, it may not be possible to induce the retailer to sell Q units. In such cases, we follow the convention of setting $\overline{w}_{\text{P}}(Q) = -\infty$. Based on these observations, when the capacity of the supply chain is Q, the retailer's optimal profit under posted pricing is

$$\Pi_{\text{RP}}(p^*(w,Q),w,Q) = \begin{cases} (\bar{p}(Q)-w)Q = \Pi_{\text{RP}}(\bar{p}(Q),w,Q) & \text{for } c \le w \le \overline{w}_{\text{P}}(Q), \\ a(p^u(w)-w)\overline{F}(p^u(w)) & \\ = \Pi^u_{\text{RP}}(p^u(w),w) & \text{for } w \ge \max\{c, \overline{w}_{\text{P}}(Q)\}. \end{cases} \tag{3.17}$$

Consequently, the manufacturer's profit under posted pricing is

$$\Pi_{\text{MP}}(w,p^*(w,Q),Q) = \begin{cases} (w-c)Q & \text{for } c \le w \le \overline{w}_{\text{P}}(Q), \\ a(w-c)\overline{F}(p^u(w)) & \\ = \Pi^u_{\text{MP}}(w,p^u(w)) & \text{for } w \ge \max\{c, \overline{w}_{\text{P}}(Q)\}. \end{cases} \tag{3.18}$$

Notice from (3.18) that the manufacturer would never set the wholesale price below $\overline{w}_{\text{P}}(Q)$, since a lower wholesale price would only decrease the unit profit margin, but the sales quantity would remain steady at Q. If the optimal wholesale price under posted pricing in a supply chain with unlimited capacity, w^u_{P}, is greater than $\overline{w}_{\text{P}}(Q)$, then w^u_{P} will result in a sales quantity less than the supply chain's capacity Q. In such a case, the manufacturer's optimal wholesale price is simply w^u_{P}. On the other hand, if $w^u_{\text{P}} \le \overline{w}_{\text{P}}(Q)$, then the manufacturer should set the wholesale price to $\overline{w}_{\text{P}}(Q)$. The following lemma formalizes this discussion on the manufacturer's optimal wholesale price under posted pricing in a supply chain with capacity Q, $w^*_{\text{P}}(Q)$.

Lemma 3.5.1. *For a given Q, $w^*_{\text{P}}(Q) = \max\{\overline{w}_{\text{P}}(Q), w^u_{\text{P}}\}$.*

3.5.2 Negotiation

Now consider a supply chain with capacity Q, where negotiation is imposed. If capacity Q is sufficiently low, the retailer will find it optimal to sell all Q units by setting the cut-off valuation exactly equal to $\bar{p}(Q)$ (as described in Section 3.3.2) at which the demand equals the capacity Q. Following the same line of logic used above, we define the market-clearing wholesale price under negotiation, $\overline{w}_{\text{N}}(Q)$ as follows:

$$q^u_{\min}(\overline{w}_{\text{N}}(Q)) = \bar{p}(Q) \tag{3.19}$$

When there does not exist $\overline{w}_{\text{N}}(Q) > c$ for a given Q, we follow the convention of setting $\overline{w}_{\text{N}}(Q) = -\infty$. Utilizing the definition of $\overline{w}_{\text{N}}(Q)$, the retailer's optimal profit under negotiation is

$$\Pi_{\text{RN}}(q^*_{\min}(w,Q),w,Q) = \begin{cases} \Pi_{\text{RN}}(\bar{p}(Q),w,Q) & \text{for } c \le w \le \overline{w}_{\text{N}}(Q), \\ \\ \Pi^u_{\text{RN}}(q^u_{\min}(w),w) & \text{for } w \ge \max\{c, \overline{w}_{\text{N}}(Q)\}. \end{cases} \tag{3.20}$$

Consequently, the manufacturer's profit under negotiation is

$$\Pi_{\text{MN}}(w,q^*_{\min}(w,Q),Q) = \begin{cases} (w-c)Q & \text{for } c \le w \le \overline{w}_{\text{N}}(Q), \\ \\ \Pi^u_{\text{MN}}(w,q^u_{\min}(w)) & \text{for } w \ge \max\{c, \overline{w}_{\text{N}}(Q)\}. \end{cases} \tag{3.21}$$

The following lemma describes the manufacturer's optimal wholesale price under negotiation in a supply chain with capacity Q, $w^*_{\text{N}}(Q)$.

Lemma 3.5.2. *For a given Q, $w^*_{\text{N}}(Q) = \max\{\overline{w}_{\text{N}}(Q), w^u_{\text{N}}\}$.*

3.5.3 Equilibrium Analysis

Recall that the retailer in our model chooses the sales format to maximize its profit given the manufacturer's wholesale price. In the previous section, we showed

that the retailer will prefer negotiation at low wholesale prices, and posted pricing at high wholesale prices. However, it is not obvious that this behavior will remain true in the presence of finite capacity. When the capacity is finite, there will be a range of wholesale prices over which the quantity sold hits the capacity ceiling under one sales format, but not the other. In fact, one can easily find examples where negotiation is bounded by capacity when posted pricing is not, and vice versa. At such wholesale prices, the retailer's preference is distorted by the capacity effect, which may tip the scales in favor of one or the other sales format. Despite such complications, we show that the same best response behavior holds true in the presence of finite capacity. That is, as stated in Proposition 3.4.1, three possibilities exist: (a) either the retailer prefers posted pricing at all wholesale prices, or (b) the retailer prefers negotiation at all wholesale prices, or (c) there exists a threshold wholesale price $\widehat{w}_{\mathbf{R}}(Q) > c$ below which the retailer strictly prefers negotiation and above which the retailer strictly prefers posted pricing. The proof is more involved due to the complications arising from the capacity constraint. We state and prove this result formally as Proposition B.2.1 in Appendix B. Furthermore, an equivalent of Proposition 3.4.2 holds for the limited capacity problem: If there exists $\widehat{w}_{\mathbf{R}}(Q) > c$ below which the retailer strictly prefers negotiation and above which it prefers posted pricing (while being indifferent at $\widehat{w}_{\mathbf{R}}(Q)$), then the manufacturer prefers negotiation at the wholesale price $\widehat{w}_{\mathbf{R}}(Q)$. Therefore, applying the Pareto-dominance criterion, we again assume that the retailer chooses to negotiate at the threshold wholesale price $\widehat{w}_{\mathbf{R}}(Q)$. This result is stated and proven formally as Proposition B.2.2 in Appendix B.

Given the three possible patterns of the retailer's preference as a function of the wholesale price, the manufacturer's problem to determine the wholesale price can now be expressed as follows:

Manufacturer's Problem:

If the retailer prefers posted pricing for all $w \geq c$, then:

$$\max_{w \geq c} \Pi_{\text{MP}}(w, p^*(w, Q), Q) \tag{3.22}$$

If the retailer prefers negotiation for all $w \geq c$, then:

$$\max_{w \geq c} \Pi_{\text{MN}}(w, q^*_{\min}(w, Q), Q) \tag{3.23}$$

If the retailer prefers negotiation for $c \leq w \leq \widehat{w}_{\text{R}}(Q)$ and posted pricing for $w > \widehat{w}_{\text{R}}(Q)$ then:

$$\max \left[\max_{c \leq w \leq \widehat{w}_{\text{R}}(Q)} \Pi_{\text{MN}}(w, q^*_{\min}(w, Q), Q) \ , \ \sup_{w > \widehat{w}_{\text{R}}(Q)} \Pi_{\text{MP}}(w, p^*(w, Q), Q) \right] \tag{3.24}$$

The next proposition describes the candidates for the manufacturer's optimal wholesale price and the resulting equilibria:

Proposition 3.5.1. *The manufacturer chooses one of the following wholesale prices:*
(a) $w^*_{\text{N}}(Q) = \max\{\overline{w}_N(Q), w^u_{\text{N}}\}$, the maximizer of $\Pi_{\text{MN}}(w, q^*_{\min}(w, Q), Q)$, which leads the retailer to use negotiation with cut-off valuation $q^*_{\min}(w^*_{\text{N}}(Q), Q)$ in equilibrium
(b) $\widehat{w}_{\text{R}}(Q)$, the threshold wholesale price, which leads the retailer to use negotiation with cut-off valuation $q^*_{\min}(\widehat{w}_{\text{R}}(Q), Q)$ in equilibrium, or
(c) $w^*_{\text{P}}(Q) = \max\{\overline{w}_{\text{P}}(Q), w^u_{\text{P}}\}$, the maximizer of $\Pi_{\text{MP}}(w, p^*(w, Q), Q)$, which leads the retailer to use posted pricing with price $p^*(w^*_{\text{P}}(Q), Q)$ in equilibrium.

The equilibrium wholesale price $\widehat{w}_{\text{R}}(Q)$ arises in cases where the manufacturer would like the retailer to use negotiation, but cannot induce that choice at the wholesale price $w^*_{\text{N}}(Q)$ that maximizes the profit function $\Pi_{\text{MN}}(w, q^*_{\min}(w, Q), Q)$. Hence, in such cases, the manufacturer offers the lower wholesale price $\widehat{w}_{\text{R}}(Q)$ in order to persuade the retailer to use negotiation, which gives rise to what we termed reconciliatory negotiation in Section 3.4. An analog of Proposition 3.4.4 holds in the

capacitated case as well: As the total cost of negotiation, c_{T}, increases, the supply chain moves from the negotiation equilibrium with wholesale price $w^*_{\mathrm{N}}(Q)$ to the reconciliatory negotiation equilibrium with wholesale price $\widehat{w}_{\mathrm{R}}(Q)$ and eventually to the posted pricing equilibrium with wholesale price $w^*_{\mathrm{P}}(Q)$. The result is stated and proven formally in Appendix B (see Proposition B.2.3).

Figure 3.3 illustrates the same example shown in Figure 3.2, but in a capacity-constrained supply chain with $Q = 406$. As in the case with unlimited capacity, the equilibrium moves from negotiation (c_{T} up to approximately 1.29) to reconciliatory negotiation (c_{T} between 1.29 and 1.42 approximately) to posted pricing (c_{T} above approximately 1.42). Observe that the quantity sold in equilibrium is bounded by capacity in two disjoint regions. The first region (c_{T} between 0.5 to 0.72) is when the total cost of negotiation is very low, in which the manufacturer prefers to exhaust the capacity and it induces this outcome by charging the market-clearing wholesale price, $\overline{w}_{\mathrm{N}}(Q)$. The second region ($c_{\mathrm{T}}$ between 1.31 to 1.42) spans a part of the region where the equilibrium is reconciliatory negotiation with wholesale price $\widehat{w}_{\mathrm{R}}(Q)$. As previously discussed, a unit increase in c_{T} decreases $\widehat{w}_{\mathrm{R}}(Q)$ by more than one unit, so it eventually becomes smaller than the market-clearing wholesale price, $\overline{w}_{\mathrm{N}}(Q)$. Notice if negotiation were exogenously imposed, the manufacturer would never pick a wholesale price below $\overline{w}_{\mathrm{N}}(Q)$. However, facing a retailer who has the discretion to choose the sales format, the manufacturer has to sacrifice some of its margin and offer $\widehat{w}_{\mathrm{R}}(Q)$ to induce negotiation.

Figure 3.4 illustrates an example where capacity is more severely constrained with $Q = 350$. In this case, regardless of c_{T} the capacity is exhausted. Even though the equilibrium is moving from negotiation to reconciliatory negotiation to posted pricing, the quantity sold remains at capacity.

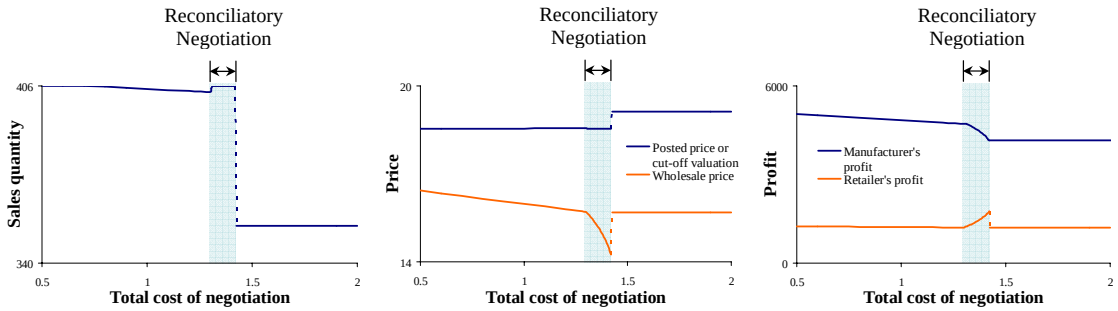

Figure 3.3: The figure illustrates the equilibrium sales quantity (left), the equilibrium wholesale price and equilibrium posted price or cut-off valuation (central), and the equilibrium manufacturer's and retailer's profits (right). Here, $a = 500$, $\beta = 0.3$, $c = 4$, $Q = 406$ and logit demand with $F(x) = \frac{1}{1+e^{20-x}}$.

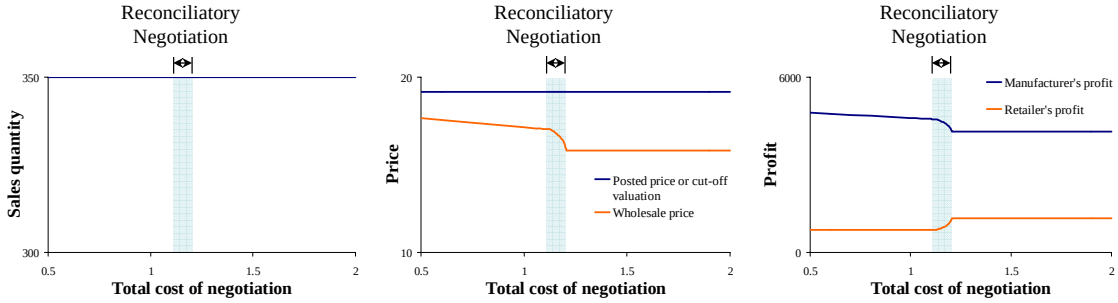

Figure 3.4: The figure illustrates the equilibrium sales quantity (left), the equilibrium wholesale price and equilibrium posted price or cut-off valuation (central), and the equilibrium manufacturer's and retailer's profits (right). Here, $a = 500$, $\beta = 0.3$, $c = 4$, $Q = 350$ and logit demand with $F(x) = \frac{1}{1+e^{20-x}}$.

Observe from Figures 3.2, 3.3, and 3.4 where the capacity progressively becomes tighter, different types of equilibria may arise at the same total cost of negotiation, $c_{\mathbf{T}}$ as capacity is changing. For example, the range of $c_{\mathbf{T}}$ values in which posted pricing is the equilibrium expands as the capacity becomes tighter: above 1.575 in Figure 3.2, above 1.42 in Figure 3.3, and above 1.215 in Figure 3.4. As the following proposition shows, the equilibrium sales format evolves from posted pricing to negotiation as the capacity becomes larger, giving rise to the reconciliatory negotiation equilibrium at

moderate capacity levels.

Proposition 3.5.2. *There exist two thresholds, $\underline{Q}$ and $\overline{Q}$, $0 \leq \underline{Q} \leq \overline{Q} \leq \infty$, such that*

(a) **[Posted Pricing]** *if $Q < \underline{Q}$, then the equilibrium sales format is posted pricing with the wholesale price $w^*_{\text{P}}(Q) = \max\{\overline{w}_{\text{P}}(Q), w^u_{\text{P}}\}$, resulting in the posted price $p^*(w^*_{\text{P}}(Q), Q)$,*

(b) **[Reconciliatory Negotiation]** *if $\underline{Q} \leq Q < \overline{Q}$, then the equilibrium sales format is negotiation with the wholesale price $\widehat{w}_{\text{R}}(Q)$, resulting in the retailer's cut-off valuation $q^*_{\min}(\widehat{w}_{\text{R}}(Q), Q)$, and*

(c) **[Negotiation]** *if $Q \geq \overline{Q}$, then the equilibrium sales format is negotiation with the wholesale price $w^*_{\text{N}}(Q) = \max\{\overline{w}_N(Q), w^u_{\text{N}}\}$, resulting in the retailer's cut-off valuation $q^*_{\min}(w^*_{\text{N}}(Q), Q)$.*

As the proposition shows, when capacity is tight, the equilibrium is posted pricing. To understand this behavior, first recall that the additional revenue from using negotiation (instead of posted pricing) arises from the premium collected from customers with high valuations. We have seen earlier that this additional revenue gets smaller as the cut-off valuation increases. In a setting with tight capacity, no matter what sales format is used, the retailer will sell only to customers with high valuations, resulting in a high posted price (if posted pricing is used) or high cut-off valuation (if negotiation is used). Hence, in such a setting, the additional revenue from negotiation will be small and may not cover the cost of negotiation. This is why posted pricing is preferred when capacity is tight. As capacity increases, the manufacturer would prefer if the supply chain used negotiation, but cannot induce it at the wholesale price $w^*_{\text{N}}(Q)$, which is the wholesale price the manufacturer would use if it could simply dictate the retailer to use negotiation. Hence, at moderate capac-

ity levels, the manufacturer induces reconciliatory negotiation through a discounted wholesale price. Finally, once the capacity is large enough, the retailer becomes increasingly willing to use negotiation, and the equilibrium becomes negotiation, where the wholesale price is $w_{\text{N}}^*(Q)$.

The monotonic behavior of the equilibrium sales format with respect to capacity Q and total cost of negotiation c_{T} gives rise to the following corollary, which characterizes switching curves that separate different types of equilibria. Figure 3.5 illustrates three equilibrium regimes separated by two switching curves, stated in the corollary.

Corollary 3.5.1. *There exist two increasing switching curves, $\underline{Q}(c_{\text{T}})$ and $\overline{Q}(c_{\text{T}})$, $\underline{Q}(c_{\text{T}}) \leq \overline{Q}(c_{\text{T}})$, such that the equilibrium is posted pricing if $Q < \underline{Q}(c_{\text{T}})$, reconciliatory negotiation if $\underline{Q}(c_{\text{T}}) \leq Q < \overline{Q}(c_{\text{T}})$, and negotiation if $Q \geq \overline{Q}(c_{\text{T}})$.*

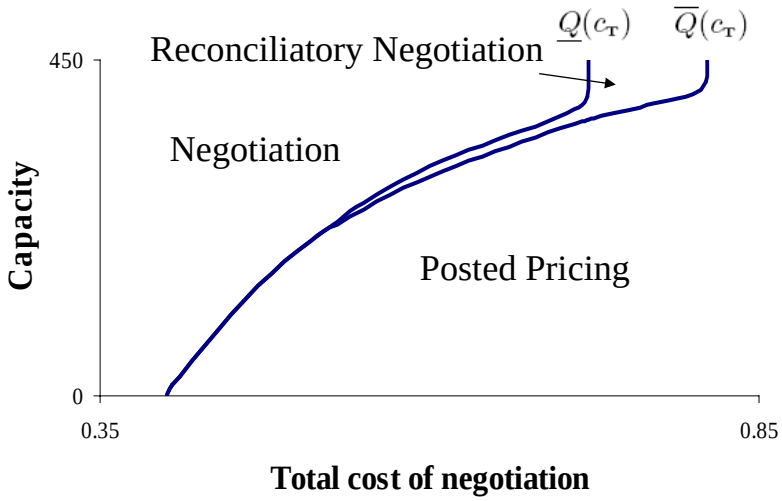

Figure 3.5: The figure illustrates three types of equilibria: Negotiation, Reconciliatory Negotiation and Posted Pricing. Here, $a = 500$, $\beta = 0.6$, $c = 4$, and logit demand with $F(x) = \frac{1}{1+e^{20-x}}$.

We now examine the effect of disparity in bargaining powers of the retailer and

customers. Figure 3.6 shows how the equilibrium outcome changes as the customer's relative bargaining power, β, increases. The behavior of equilibrium is similar to the behavior with respect to the total cost of negotiation, c_{T}: As β increases, the equilibrium sales format changes from negotiation to reconciliatory negotiation to posted pricing. At lower values of β, the retailer is able to extract much of the customer surplus, and the supply chain ends up using negotiation. As β increases, the retailer's ability to extract customer surplus is hampered, making the retailer more reluctant to choose negotiation. The manufacturer is willing to reduce the wholesale price to keep negotiation alive, and the discount is especially sharp at moderate values of β, resulting in reconciliatory negotiation. Once the depth of the discount needed to induce negotiation becomes too large, the manufacturer gives up on negotiation. The wholesale price increases and posted pricing becomes the equilibrium. Although this behavior can be intuitively explained, an analytical proof is difficult because of the highly non-linear dependence of the transaction price (and profit functions) on β.

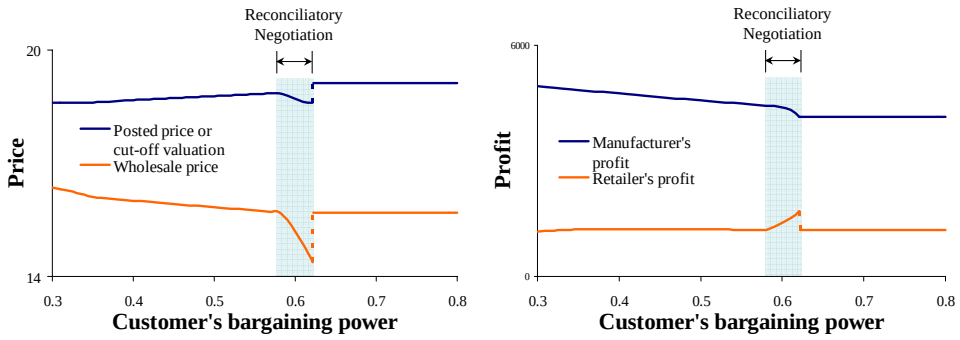

Figure 3.6: The figure illustrates the equilibrium wholesale price and equilibrium posted price or cut-off valuation (left) and the equilibrium manufacturer's and the retailer's profits (right). Here, $a = 500$, $c = 4$, $c_{\mathrm{T}} = 0.75$, $Q = 400$ and logit demand with $F(x) = \frac{1}{1+e^{20-x}}$.

3.6 Summary

In this chapter, we consider how the supply chain capacity influences the pricing and sales format decisions of the manufacturer and the retailer. We propose a model in which the supply chain has a limited supply and the retailer can choose either of two pricing regimes – negotiating or posting a fixed price when selling a product to heterogeneous customers with different willingness-to-pay. The generalized Nash bargaining solution is employed to characterize the outcome of negotiation between customers and the retailer. Costs of negotiation are incurred when customers and the retailer have reached an equilibrium transaction price. We consider unlimited capacity and limited capacity cases, and show how the capacity and the negotiation cost affect the retailer's pricing regime decisions as well as the manufacturer's inducement of a pricing regime through the wholesale price.

Our result shows there exist three types of equilibria depending on the negotiation cost and the capacity of the supply chain. When the negotiation cost is very low and the capacity of the manufacturer is very high, the supply chain ends up at a negotiation equilibrium, which is as same as that if negotiation were the exogenous sales format. When the negotiation cost is very high and the capacity of the manufacturer is very low, the supply chain ends up at a posted price equilibrium, which is again the same as that if posted pricing were the exogenous sales format. The third equilibrium, where the negotiation cost and the capacity level are sufficiently high, exists when the manufacturer prefers the retailer to use negotiation but it has to offer a lower wholesale price to benefit the retailer such that the retailer is willing to negotiate.

CHAPTER 4

Conclusions

One of the strategic decisions available to a retailer is the choice of sales format – whether to adopt negotiation or posted pricing. Much of operations management literature makes the implicit assumption that the retailer's sales format is exogenously fixed as posted pricing. This is a perfectly acceptable starting point as posted pricing is very common in practice. Nonetheless, many retailers actively choose between posted pricing and negotiation. This dissertation revisits two problem domains in operations management, assuming that a retailer can make an active choice about sales formats. Namely, we first consider a retailer's revenue management problem in the presence of inventory considerations, assuming that the retailer can negotiate. Second, we consider a capacity-constrained manufacturer's wholesale pricing problem, assuming that the retailer is free to choose between posted pricing and negotiation.

The traditional revenue management paradigm suggests that if a retailer has a limited supply of a product that can be sold only over a short selling season, then the retailer must adjust its prices over time (i.e., use dynamic pricing) in order to maximize the revenue to be collected over the selling season. Negotiation can be seen as yet another revenue management tool, in that negotiation allows a retailer to price

discriminate among customers. In this dissertation, a stochastic dynamic programming formulation is employed to embed a negotiation model in a more traditional dynamic pricing model. This model produces a number of interesting analytical and numerical results. As one would expect, the optimal posted price of a negotiating retailer includes a premium over that of a retailer using take-it-or-leave-it pricing strategy. This price premium helps the retailer extract more revenues from customers with high willingness-to-pay, and, surprisingly, peaks at moderate inventory levels (as opposed to low inventory levels). In addition, the results show that negotiation can act as a substitute or complement to dynamic pricing. For example, at moderate inventory levels, the benefit from dynamic pricing increases further when the seller can negotiate.

If negotiation is a viable sales format choice for the retailer, the manufacturer must take such retailer discretion into account. In our analysis, we find that the supply chain may settle in one of the three different types of equilibria. Two of these three are cases where the supply chain ends up doing what it would do even if the retailer had no discretion and the sales format were exogenously determined. The third equilibrium type, however, arises when the manufacturer wants to impose negotiation, but cannot induce the retailer to do so without sacrificing some of its profit margin. This is an equilibrium where the retailer benefits from its discretion over the sales format. We establish how the type of equilibrium outcome depends on the supply chain capacity and retailer's cost of negotiation. We find that the retailer benefits from its power to choose the sales format when negotiation costs and capacity levels are moderately high.

In our negotiation models, be it in the context of revenue management problem or the wholesale pricing problem, the retailer sets a minimum acceptable price and

serves only those customers who are willing to pay more. In this setting, customers pay different prices depending on their willingness-to-pay, and such price discrimination is exactly the reason why the retailer benefits from negotiation. With regard to the benefits from negotiation, a common theme emerges across the revenue management and wholesale pricing problems. In the revenue management problem, we find that the benefit of negotiation is larger when the retailer has more inventory at the beginning of the horizon. In the wholesale pricing problem, we find that benefit from negotiation is larger when the manufacturer has larger capacity. These results suggest that negotiation is a particularly viable tool when product availability is not constrained.

APPENDICES

APPENDIX A

A.1 Proofs of Lemmas in Section 2.3

Proof of Lemma 2.3.1

Proof of (a): We prove the unimodality of $S(p_o, r)$ in $p_o \in [0, p]$ by showing (i) $\frac{\partial S(p_o,r)}{\partial p_o}\Big|_{p_o=0} \geq 0$, (ii) $\frac{\partial^2 S(p_o,r)}{\partial p_o^2} < 0$ whenever $\frac{\partial S(p_o,r)}{\partial p_o} = 0$, and (iii) $S(p_o, r) \to (r - p)$ as $p_o \to p$.

First note that the first and second partial derivatives of $S(p_o, r)$ in p_o are

$$\frac{\partial S(p_o, r)}{\partial p_o} = -G(p_o|p) + (r - p_o)g(p_o|p) \text{ and} \tag{A.1}$$

$$\frac{\partial^2 S(p_o, r)}{\partial p_o^2} = (r - p_o)g'(p_o|p) - 2g(p_o|p). \tag{A.2}$$

Claim (i) follows from (A.1) and $G(0|p) = 0$ while claim (iii) follows from $G(p|p) = 1$. To show claim (ii), note from (A.1) and (A.2)

$$\frac{\partial^2 S(p_o, r)}{\partial p_o^2}\Big|_{\frac{\partial S(p_o,r)}{\partial p_o}=0} = \frac{1}{g(p_o|p)}(G(p_o|p)g'(p_o|p) - 2g^2(p_o|p)). \tag{A.3}$$

Since $G(p_o|p)$ is log-concave, $G(p_o|p)g'(p_o|p) - g^2(p_o|p) < 0$ at any p_o in $[0, p]$ and claim (ii) follows, concluding the proof of unimodality of $S(p_o, r)$ in $p_o \in [0, p]$.

Proof of (b): Note that $S(p_o, r) = 0$ for all $p_o \leq 0$ since $G(p_o|p) = 0$ for all $p_o \leq 0$. Furthermore, for all $p_o \geq p$, we have $S(p_o, r) = r - p_o$ (since $G(p_o|p) = 1$ for all

$p_o \geq p$) and $S(p_o, r)$ is strictly decreasing in p_o for $p_o \geq p$.

Proof of Lemma 2.3.2

We divide into two cases: $p \geq \theta$ and $p < \theta$.

First, consider the case with $p \geq \theta$. For bargainers with $\rho(p-\theta) < r \leq \rho(p)$, we have $p - \theta < \bar{p}_o(r) \leq p$, thus $p_o^*(p, r) = \bar{p}_o(r)$ is the optimal solution to (2.2). On the other hand, for bargainers with $r \in [p-\theta, \rho(p-\theta)]$, we have $\bar{p}_o(r) < p - \theta$, thus the optimal offer should lie on the boundary $p_o^*(p, r) = p - \theta$ (by the unimodality of $S(p_o, r)$ for $p_o \in [0, p]$). Likewise, for bargainers with $r > \rho(p)$, we have $\bar{p}_o(r) = p$, thus the optimal offer should lie on the boundary, that is, $p_o^*(p, r) = p$.

Next, consider the case $p < \theta$. Notice that $\rho(p-\theta) = 0$ in this case. Therefore, the only r that satisfies $p - \theta \leq r \leq \rho(p-\theta)$ is $r = 0$, at which the optimal offer is trivially $p_o^*(p, r) = 0$. For any bargainer with $0 < r \leq \rho(p)$, we have $p - \theta < 0 < \bar{p}_o(r) \leq p$, and $p_o^*(p, r) = \bar{p}_o(r)$ is the optimal solution to (2.2). If $r > \rho(p)$, we have $\bar{p}_o(r) = p$, and the optimal offer is on the boundary $p_o^*(p, r) = p$.

Proof of Lemma 2.3.3

Suppose that $p < \Delta_{t-1}(y)$. Note that any bargainer who purchases the product will pay p or less. Therefore, $K_t(p, y) \leq pB_t(p, y)$ and

$$\begin{aligned} J_t(p, y) &= \lambda q \left[K_t(p, y) - B_t(p, y)\Delta_{t-1}(y)\right] + \lambda(1-q)\overline{F}(p)\left[p - \Delta_{t-1}(y)\right] + V_{t-1}(y) \\ &\leq \lambda[qB_t(p, y) + (1-q)\overline{F}(p)](p - \Delta_{t-1}(y)) + V_{t-1}(y) < V_{t-1}(y). \end{aligned}$$

The seller would have been strictly better off by charging $p = \Delta_{t-1}(y)$ and setting the counter-offer to $\Delta_{t-1}(y)$ to all bargainers, since we would have $J_t(p, y) = V_{t-1}(y)$ in that case. Hence, setting $p < \Delta_{t-1}(y)$ cannot be optimal.

Proof of Lemma 2.3.4

We prove the unimodality of $Z_t(p_c, y)$ in p_c by showing (i) $\frac{\partial Z_t(p_c,y)}{\partial p_c}\Big|_{p_c=0} \geq 0$, (ii) $\frac{\partial^2 Z_t(p_c,y)}{\partial p_c^2} < 0$ whenever $\frac{\partial Z_t(p_c,y)}{\partial p_c} = 0$, and (iii) $Z_t(p_c, y) \to V_{t-1}(y)$ as $p_c \to \rho(p-\theta)$.

Using $\Delta_{t-1}(y) = V_{t-1}(y) - V_{t-1}(y-1)$, $y = 0, 1, \ldots$, we write the first and second derivatives of $Z_t(p_c, y)$ with respect to p_c as follows:

$$\frac{\partial Z_t(p_c, y)}{\partial p_c} = \frac{1}{F(\rho(p-\theta)) - F(p-\theta)}(F(\rho(p-\theta)) - F(p_c) + f(p_c)(\Delta_{t-1}(y) - p_c)), \tag{A.4}$$

$$\frac{\partial^2 Z_t(p_c, y)}{\partial p_c^2} = \frac{1}{F(\rho(p-\theta)) - F(p-\theta)}(f'(p_c)(\Delta_{t-1}(y) - p_c) - 2f(p_c)). \tag{A.5}$$

Claims (i) and (iii) easily follow from simple algebra. For claim (ii), we note from (A.4) and (A.5) that

$$\begin{aligned} &\frac{\partial^2 Z_t(p_c, y)}{\partial p_c^2}\Big|_{\frac{\partial Z_t(p_c,y)}{\partial p_c}=0} \\ = &\frac{1}{f(p_c)(F(\rho(p-\theta)) - F(p-\theta))}(-2f^2(p_c) - (F(\rho(p-\theta)) - F(p_c))f'(p_c)). \end{aligned} \tag{A.6}$$

Now consider two cases - $f'(p_c) \geq 0$ and $f'(p_c) < 0$. If $f'(p_c) \geq 0$, then (A.6) is negative, which is the desired result. Next, consider the case where $f'(p_c) < 0$. Note that, from Assumption 2 (i.e., F is strictly increasing and IFR), we have $f^2(p_c) + (1 - F(p_c))f'(p_c) > 0$. Then,

$$\begin{aligned} \frac{\partial^2 Z_t(p_c, y)}{\partial p_c^2}\Big|_{\frac{\partial Z_t(p_c,y)}{\partial p_c}=0} &= \frac{-2f^2(p_c) - (1 - F(p_c))f'(p_c) + (1 - F(\rho(p-\theta)))f'(p_c)}{f(p_c)(F(\rho(p-\theta)) - F(p-\theta))} \\ &< \frac{-f^2(p_c) - (1 - F(p_c))f'(p_c) + (1 - F(\rho(p-\theta)))f'(p_c)}{f(p_c)(F(\rho(p-\theta)) - F(p-\theta))} \\ &< 0. \end{aligned}$$

Proof of Lemma 2.3.5

It is not hard to see that the counter-offer will never be less than $\Delta_{t-1}(y)$. If $p_o = p-\theta$, then the optimal solution to the optimization problem in (2.8) is $\overline{p}_{ct}(y)$, since $Z_t(p_c, y)$ is unimodal in p_c. Likewise, if $p_o > p - \theta$, then the seller will set its counter-offer to the smaller of p or $\rho(p_o)$, provided that $\min\{\rho(p_o), p\}$ is larger than or equal to $\Delta_{t-1}(y)$.

Proof of Lemma 2.3.6

Throughout the proof, recall that a bargainer with reservation price r such that $r < p-\theta$ will quit without making an offer. We first deal with the case where $p > \theta$.

(a) Suppose $p \leq \overline{p}_{ct}(y) \leq \rho(p-\theta)$. We divide the remaining bargainers into two groups with respect to reservation price: (i) $r \in [p-\theta, \rho(p-\theta)]$, (ii) $r > \rho(p-\theta)$. In case (i), $p_o^*(p, r) = p - \theta$ by Lemma 2.3.2 and $p_{ct}^*(p, p-\theta, y) = p$ by Lemma 2.3.5, as a result of which the bargainer accepts the counter-offer only if $r \geq p$. In case (ii), $p_o^*(p, r) > p - \theta$ by Lemma 2.3.2 and $p_{ct}^*(p, p_o^*(p, r), y) = p$ by Lemma 2.3.5, as a result of which the bargainer will accept the offer. Thus, only bargainers with $r \geq p$ will purchase, and $K_t(p, y) = p\overline{F}(p)$ and $B_t(p, y) = \overline{F}(p)$.

(b) Suppose $\Delta_{t-1}(y) \leq \overline{p}_{ct}(y) < p \leq \rho(p-\theta)$. Again, consider two cases: (i) $r \in [p-\theta, \rho(p-\theta)]$, (ii) $r > \rho(p-\theta)$. In case (i), $p_o^*(p, r) = p-\theta$ by Lemma 2.3.2 and $p_{ct}^*(p, p-\theta, y) = \overline{p}_{ct}(y)$ by Lemma 2.3.5, as a result of which the bargainer accepts the counter-offer only if $r \geq \overline{p}_{ct}(y)$. In case (ii), $p_o^*(p, r) > p-\theta$ by Lemma 2.3.2 and $p_{ct}^*(p, p_o^*(p, r), y) = p$ by Lemma 2.3.5, as a result of which the bargainer will accept the offer. Thus, bargainers with $\overline{p}_{ct}(y) \leq r \leq \rho(p-\theta)$ will purchase at $\overline{p}_{ct}(y)$, and

bargainers with $r > \rho(p-\theta)$ will end up buying at p. The result follows.

(c) Suppose $\Delta_{t-1}(y) \leq \overline{p}_{ct}(y) \leq \rho(p-\theta) < p$. We divide remaining bargainers into three groups: (i) $r \in [p-\theta, \rho(p-\theta)]$, (ii) $\rho(p-\theta) < r \leq p$, and (iii) $r > p$. By Lemma 2.3.2, the bargainers in group (i) offer $p_o^*(p,r) = p-\theta$, to which the seller responds with the counter-offer $p_{ct}^*(p, p-\theta, y) = \overline{p}_{ct}(y)$ (by Lemma 2.3.5), as a result of which the bargainer accepts the counter-offer only if $r \geq \overline{p}_{ct}(y)$. For the bargainers in group (ii), $p_o^*(p,r) > p-\theta$ by Lemma 2.3.2 and $p_{ct}^*(p, p_o^*(p,r), y) = r$ by Lemma 2.3.5, as a result of which the bargainer will accept the seller's offer. For the bargainers in group (iii), the seller responds with $p_{ct}^*(p, p_o^*(p,r), y) = p$ by Lemma 2.3.5, thus the bargainer will buy at the posted price p. Thus, bargainers with $\overline{p}_{ct}(y) \leq r \leq \rho(p-\theta)$ will purchase at $\overline{p}_{ct}(y)$, and bargainers with $r > \rho(p-\theta)$ will end up buying at $\min[p, r]$. The result follows.

(d) Suppose $\rho(p-\theta) \leq \Delta_{t-1}(y) \leq p$. As in part (c), we consider three cases: (i) $r \in [p-\theta, \rho(p-\theta)]$, (ii) $\rho(p-\theta) < r \leq p$, and (iii) $r > p$. By Lemma 2.3.2, the bargainers in group (i) offer $p_o^*(p,r) = p-\theta$, to which the seller responds with the counter-offer $p_{ct}^*(p, p-\theta, y) = \Delta_{t-1}(y)$ (Lemma 2.3.5). Since $\rho(p-\theta) \leq \Delta_{t-1}(y)$, no bargainers in this group will purchase the product. Bargainers with $r \in (\rho(p-\theta), p]$ make an offer greater than $p-\theta$, to which the seller responds with the counter-offer $\max[\Delta_{t-1}(y), r]$. Thus, only those with $r \in [\Delta_{t-1}(y), p]$ purchase the product and they do so at their own reservation price. For the bargainers in group (iii), the seller responds with $p_{ct}^*(p, p_o^*(p,r), y) = p$ by Lemma 2.3.5, thus the bargainer will buy at the posted price p. The result follows.

As for the case where $p \leq \theta$, cases (a) and (b) of the lemma do not even arise. The proof of cases (c) and (d) are the same as before.

A.2 Proofs of Lemmas in Section 2.4

In this appendix, we prove Lemmas 2.4.1 through 2.4.3 stated in Section 2.4. The proofs utilize Lemmas A.2.1 through A.2.3, stated and proven at the end of Appendix A.2.

Proof of Lemma 2.4.1

By Lemma A.2.3, $\Delta_{t-1}(y) \leq b$. Therefore, we have

$$\frac{\Delta_{t-1}(y)}{2} \leq \frac{b + \Delta_{t-1}(y)}{4} \leq \frac{2b - q\Delta_{t-1}(y)}{2(2-q)}.$$

Hence, we divide the proof into four different cases: 1) $\theta \leq \frac{\Delta_{t-1}(y)}{2}$, 2) $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, 3) $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, 4) $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. For each of four cases, we apply the results of Lemmas A.2.1 and A.2.2, write the expected revenue-to-go as a function of the posted price and determine the optimal posted price for each case.

Case 1: $\theta \leq \frac{\Delta_{t-1}(y)}{2}$

We divide into sub-cases depending on the value of p: 1a) $\Delta_{t-1}(y) \leq p \leq \frac{b+\Delta_{t-1}(y)}{2}$, 1b) $\frac{b+\Delta_{t-1}(y)}{2} < p \leq \frac{b+\Delta_{t-1}(y)}{2} + \theta$, and 1c) $\frac{b+\Delta_{t-1}(y)}{2} + \theta < p \leq b$.

- Case 1a: $\Delta_{t-1}(y) \leq p \leq \frac{b+\Delta_{t-1}(y)}{2}$

  From Lemma A.2.1, bargainers with $r \in [p-\theta, 2(p-\theta)]$ choose $p_o^*(p,r) = p-\theta$ and bargainers with even higher reservation price (i.e., $2(p-\theta) < r < b$) choose $p_o(p,r) = \min[p, \frac{r}{2}]$. From Lemma A.2.2, the seller responds with a unilateral counter-offer $p_{ct}^*(p, p_o^*(p,r), y) = p$ regardless of the bargainer's offer. As a result, only bargainers with reservation price greater than p will buy and they will buy at the original posted price p. Thus, we have

  $$K_t(p,y) = p\frac{b-p}{b} \text{ and } B_t(p,y) = \frac{b-p}{b}.$$

Substituting these into equation (2.6), the expected revenue-to-go function for a given posted price p, $J_t(p, y)$ is

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q \left[\frac{b-p}{b} p - \frac{b-p}{b} \Delta_{t-1}(y) \right] + \lambda(1-q) \frac{b-p}{b} \left[p - \Delta_{t-1}(y) \right] \\ &= V_{t-1}(y) + \lambda \frac{b-p}{b} (p - \Delta_{t-1}(y)). \end{aligned}$$

Taking the derivative with respect to p, we observe

$$J_t'(p,y) = \lambda \frac{b - 2p + \Delta_{t-1}(y)}{b} \geq 0 \text{ for all } p \leq \tfrac{b+\Delta_{t-1}(y)}{2}.$$

Thus $J_t(p, y)$ is increasing in p up to $p = \frac{b+\Delta_{t-1}(y)}{2}$.

- Case 1b: $\frac{b+\Delta_{t-1}(y)}{2} < p \leq \frac{b+\Delta_{t-1}(y)}{2} + \theta$

First, note that $2(p-\theta) \geq b + \Delta_{t-1}(y) - 2\theta \geq b \geq \frac{b+\Delta_{t-1}(y)}{2} > 0$. Applying this to Lemma A.2.1, we notice that all bargainers with $r \in [p-\theta, b]$ choose $p_o^*(p, r) = p - \theta$. From Lemma A.2.2, the seller responds with a counter-offer $p_{ct}^*(p, p_o^*(p, r), y) = \frac{b+\Delta_{t-1}(y)}{2}$. As a result, bargainers with reservation price greater than $\frac{b+\Delta_{t-1}(y)}{2}$ will buy at price $\frac{b+\Delta_{t-1}(y)}{2}$. Thus, we have

$$K_t(p,y) = \frac{b - \Delta_{t-1}(y)}{2b} \frac{b + \Delta_{t-1}(y)}{2} \text{ and } B_t(p,y) = \frac{b - \Delta_{t-1}(y)}{2b}.$$

Substituting these into equation (2.6), we have

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q \left[\tfrac{b-\Delta_{t-1}(y)}{2b} \tfrac{b+\Delta_{t-1}(y)}{2} - \tfrac{b-\Delta_{t-1}(y)}{2b} \Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q) \frac{b-p}{b} \left[p - \Delta_{t-1}(y) \right] \\ &= V_{t-1}(y) + \lambda q \frac{(b - \Delta_{t-1}(y))^2}{4b} + \lambda(1-q) \frac{b-p}{b} \left[p - \Delta_{t-1}(y) \right]. \end{aligned}$$

Taking the derivative with respect to p, we observe

$$J_t'(p,y) = \lambda(1-q) \frac{b - 2p + \Delta_{t-1}(y)}{b} < 0 \text{ for all } p > \tfrac{b+\Delta_{t-1}(y)}{2}.$$

Thus $J_t(p, y)$ is decreasing in p from $p = \frac{b+\Delta_{t-1}(y)}{2}$ to $p = \frac{b+\Delta_{t-1}(y)}{2} + \theta$.

- Case 1c: $\frac{b+\Delta_{t-1}(y)}{2} + \theta < p \leq b$

  Here the posted price is even larger than Case 1b and, once again from Lemma A.2.1, all bargainers with $r \in [p-\theta, b]$ choose $p_o^*(p,r) = p - \theta$. From Lemma A.2.2, the seller responds with a counter-offer $p_{ct}^*(p, p_o^*(p,r), y) = p - \theta$. As a result, bargainers with reservation price greater than $p - \theta$ will buy at price $p - \theta$. Thus, we have

$$K_t(p,y) = \frac{b-p+\theta}{b}(p-\theta) \text{ and } B_t(p,y) = \frac{b-p+\theta}{b}.$$

  Substituting these into equation (2.6), we have

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q \left[\frac{b-p+\theta}{b}(p-\theta) - \frac{b-p+\theta}{b}\Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q)\frac{b-p}{b}\left[p - \Delta_{t-1}(y)\right]. \end{aligned}$$

  Taking the derivative with respect to p, we observe for all $p > \frac{b+\Delta_{t-1}(y)}{2} + \theta$

$$J_t'(p,y) = \lambda q \frac{b - 2(p-\theta) + \Delta_{t-1}(y)}{b} + \lambda(1-q)\frac{b - 2p + \Delta_{t-1}(y)}{b} < 0.$$

  Thus $J_t(p,y)$ is decreasing in $p, p > \frac{b+\Delta_{t-1}(y)}{2} + \theta$.

Combining three cases, it is easy to see that $J_t(p,y)$ is increasing in p up to $p = \frac{b+\Delta_{t-1}(y)}{2}$, then decreasing afterward. Thus, $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$.

Case 2: $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$

Similar to the previous case, we consider five different ranges of p: 2a) $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$, 2b) $\theta + \frac{\Delta_{t-1}(y)}{2} \leq p < 2\theta$, 2c) $2\theta \leq p < \frac{b}{2} + \theta$, 2d) $\frac{b}{2} + \theta \leq p \leq \frac{b+\Delta_{t-1}(y)}{2} + \theta$, and 2e) $\frac{b+\Delta_{t-1}(y)}{2} + \theta < p \leq b$.

- Case 2a: $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$

From Lemma A.2.1 and Lemma A.2.2, bargainers with $r \in [p-\theta, 2(p-\theta)]$ choose $p_o^*(p,r) = p-\theta$ to which the seller responds with the counter-offer $p_{ct}^*(p, p-\theta, y) = \Delta_{t-1}(y)$. Bargainers with even higher reservation price (i.e., $2(p-\theta) < r < b$) choose $p_o^*(p,r) = \min[p, \frac{r}{2}]$ to which the seller responds with the counter-offer $p_{ct}^*(p, p_o^*(p,r), y) = \max[\min[r,p], \Delta_{t-1}(y)]$. As a result, bargainers with reservation prices between $\Delta_{t-1}(y)$ and p end up buying at their reservation price and bargainers with $r > p$ will buy at the original posted price p. Thus, we have

$$K_t(p,y) = \frac{p^2 - \Delta_{t-1}^2(y)}{2b} + \frac{b-p}{b}p \text{ and } B_t(p,y) = \frac{b-\Delta_{t-1}(y)}{b}.$$

Substituting these into equation (2.6), the expected revenue-to-go function for a given posted price p, $J_t(p,y)$ is

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q \left[\frac{p^2 - \Delta_{t-1}^2(y)}{2b} + (\frac{b-p}{b})p - \frac{b-\Delta_{t-1}(y)}{b}\Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q)\frac{b-p}{b}\left[p - \Delta_{t-1}(y)\right]. \end{aligned}$$

Taking the derivative with respect to p, we observe

$$J_t'(p,y) = \lambda q \frac{b-p}{b} + \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b}.$$

Note that both terms are positive for $p < \theta + \frac{\Delta_{t-1}(y)}{2}$, thus $J_t(p,y)$ is increasing in p between $[\Delta_{t-1}(y), \theta + \frac{\Delta_{t-1}(y)}{2})$.

- Case 2b: $\theta + \frac{\Delta_{t-1}(y)}{2} \le p < 2\theta$

  Similar to Case 2a, bargainers with $r \in [p-\theta, 2(p-\theta)]$ choose $p_o^*(p,r) = p-\theta$ and bargainers with $2(p-\theta) < r < b$ choose $p_o^*(p,r) = \min[p, \frac{r}{2}]$. Applying Lemma A.2.2, the seller's counter-offers are $p_{ct}^*(p, p-\theta, y) = p - \theta + \frac{\Delta_{t-1}(y)}{2}$ to those with $p_o^*(p,r) = p-\theta$ and $p_{ct}^*(p, p_o^*(p,r), y) = \min[r,p]$ to those with $p_o^*(p,r) > p-\theta$.

As a result, bargainers with $r \in [p-\theta+\frac{\Delta_{t-1}(y)}{2}, 2(p-\theta)]$ buy at $p-\theta+\frac{\Delta_{t-1}(y)}{2}$, bargainers with $r \in (2(p-\theta), p]$ buy at their reservation price, and bargainers with even higher reservation price buy at the original posted price p. Thus:

$$K_t(p,y) = \frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2}) + \frac{p^2-4(p-\theta)^2}{2b} + (\frac{b-p}{b})p$$
$$B_t(p,y) = \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}.$$

Substituting these into equation (2.6), $J_t(p,y)$ and $J'_t(p,y)$ are

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) \\ &\quad +\lambda q\left[\frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2}) + \frac{p^2-4(p-\theta)^2}{2b}\right. \\ &\qquad \left. + (\frac{b-p}{b})p - \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}\Delta_{t-1}(y)\right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right] \text{ and} \\ J'_t(p,y) &= \lambda q\frac{b-3p+2\theta+\Delta_{t-1}(y)}{b} + \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b}. \end{aligned}$$

Note that both terms of $J'_t(p,y)$ are decreasing in p and positive at $p = 2\theta$ (since $b-4\theta+\Delta_{t-1}(y) > 0$ by assumption.) Thus, $J_t(p,y)$ is increasing in p between $[\theta+\frac{\Delta_{t-1}(y)}{2}, 2\theta)$.

- Case 2c: $2\theta \le p < \frac{b}{2}+\theta$

From Lemma A.2.1, bargainers with $r \in [p-\theta, 2(p-\theta)]$ choose $p^*_o(p,r) = p-\theta$ and bargainers with even higher reservation price (i.e., $p \le 2(p-\theta) < r < b$) choose $p^*_o(p,r) = \min[p, \frac{r}{2}]$. Facing these offers, the seller responds with the counter-offer $p^*_{ct}(p, p-\theta, y) = p-\theta+\frac{\Delta_{t-1}(y)}{2}$ to those with $p^*_o(p,r) = p-\theta$ and $p^*_{ct}(p, p^*_o(p,r), y) = p$ to those with $p^*_o(p,r) > p-\theta$. As a result, bargainers with $r \in [p-\theta+\frac{\Delta_{t-1}(y)}{2}, 2(p-\theta)]$ buy at $p-\theta+\frac{\Delta_{t-1}(y)}{2}$ and bargainers with

$r > 2(p-\theta)$ will buy at the original posted price p. Thus:

$$\begin{aligned} K_t(p,y) &= \frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2})+(\frac{b-2(p-\theta)}{b})p \text{ and} \\ B_t(p,y) &= \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}. \end{aligned}$$

Substituting these into equation (2.6), the expected revenue-to-go function for a given posted price p, $J_t(p,y)$ is

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) \\ &\quad +\lambda q\left[\frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2})+(\frac{b-2(p-\theta)}{b})p\right. \\ &\qquad \left. -\frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}\Delta_{t-1}(y)\right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right]. \end{aligned}$$

Since $J_t''(p,y) = -(2\lambda/b) < 0$, $J_t(p,y)$ is concave. Furthermore, Taking the derivative with respect to p, we observe

$$\begin{aligned} J_t'(p,y)|_{p=2\theta} &= \lambda\frac{b-4\theta+\Delta_{t-1}(y)}{b} \geq 0, \text{ and} \\ J_t'(p,y)|_{p=\frac{b}{2}+\theta} &= \lambda\frac{-2\theta+\Delta_{t-1}(y)}{b} < 0. \end{aligned}$$

Thus, $J_t(p,y)$ is maximized at the solution of the first order condition, $p = \frac{b+\Delta_{t-1}(y)}{2}$.

- Case 2d: $\frac{b}{2}+\theta \leq p \leq \frac{b+\Delta_{t-1}(y)}{2}+\theta$

From the fact that $2(p-\theta) \geq b$ and Lemma A.2.1, all bargainers with $r \in [p-\theta, b]$ choose $p_o^*(p,r) = p-\theta$, to which the seller responds with a unilateral counter-offer $p_{ct}^*(p, p_o^*(p,r), y) = \frac{b+\Delta_{t-1}(y)}{2}$ (from Lemma A.2.2). Thus, bargainers with $r \geq \frac{b+\Delta_{t-1}(y)}{2}$ buy at price $\frac{b+\Delta_{t-1}(y)}{2}$. Thus, we have

$$K_t(p,y) = \frac{b-\Delta_{t-1}(y)}{2b}\frac{b+\Delta_{t-1}(y)}{2} \text{ and } B_t(p,y) = \frac{b-\Delta_{t-1}(y)}{2b}.$$

Substituting these into equation (2.6), $J_t(p,y)$ and $J'(p,y)$ are

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q\left[\tfrac{b-\Delta_{t-1}(y)}{2b}\tfrac{b+\Delta_{t-1}(y)}{2} - \tfrac{b-\Delta_{t-1}(y)}{2b}\Delta_{t-1}(y)\right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right] \\ &= V_{t-1}(y) + \lambda q\frac{(b-\Delta_{t-1}(y))^2}{4b} + \lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right] \\ J_t'(p,y) &= \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b} < 0 \text{ for all } p \geq \tfrac{b}{2}+\theta. \end{aligned}$$

Thus $J_t(p,y)$ is decreasing in p from $p = \frac{b}{2}+\theta$ to $p = \frac{b+\Delta_{t-1}(y)}{2}+\theta$.

- Case 2e:$\frac{b+\Delta_{t-1}(y)}{2}+\theta < p \leq b$

  If the posted price increases even further, all bargainers with $r \in [p-\theta, b]$ choose $p_o^*(p,r) = p-\theta$ by Lemma A.2.1. Applying Lemma A.2.2, the seller accepts the buyer's offer (i.e., $p_{ct}^*(p, p-\theta, y) = p-\theta$). As a result, bargainers with reservation price greater than $p-\theta$ will buy at price $p-\theta$. Thus, we have

$$K_t(p,y) = \frac{b-p+\theta}{b}(p-\theta) \text{ and } B_t(p,y) = \frac{b-p+\theta}{b}.$$

  Substituting these into equation (2.6), we have

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q\left[\frac{b-p+\theta}{b}(p-\theta) - \frac{b-p+\theta}{b}\Delta_{t-1}(y)\right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right]. \end{aligned}$$

  Taking the derivative with respect to p, we observe for all $p > \frac{b+\Delta_{t-1}(y)}{2}+\theta$

$$J_t'(p,y) = \lambda q\frac{b-2(p-\theta)+\Delta_{t-1}(y)}{b} + \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b} < 0.$$

  Thus $J_t(p,y)$ is decreasing in p for $p > \frac{b+\Delta_{t-1}(y)}{2}+\theta$.

Combining five cases, it is easy to see that $J_t(p,y)$ is increasing in p upto $p = \frac{b+\Delta_{t-1}(y)}{2}$, then decreasing afterward. Hence, $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$.

Case 3: $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$

Once again, we consider five sub-cases depending on the value of p: 3a) $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$, 3b) $\theta + \frac{\Delta_{t-1}(y)}{2} \leq p < 2\theta$, 3c) $2\theta \leq p < \frac{b}{2} + \theta$, 3d) $\frac{b}{2} + \theta \leq p \leq \frac{b+\Delta_{t-1}(y)}{2} + \theta$, and 3e) $\frac{b+\Delta_{t-1}(y)}{2} + \theta < p \leq b$.

- Case 3a: $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$

  This case is identical to Case 2a, thus $J_t(p, y)$ is increasing in p between $p \in [\Delta_{t-1}(y), \theta + \frac{\Delta_{t-1}(y)}{2})$.

- Case 3b: $\theta + \frac{\Delta_{t-1}(y)}{2} \leq p < 2\theta$

  For a given p, the bargainers's offer and the seller's counter-offer are identical to those in Case 2b. Thus, we have

$$K_t(p, y) = \frac{p - \theta - \frac{\Delta_{t-1}(y)}{2}}{b}(p - \theta + \frac{\Delta_{t-1}(y)}{2}) + \frac{p^2 - 4(p-\theta)^2}{2b} + (\frac{b-p}{b})p$$
$$B_t(p, y) = \frac{b - p + \theta - \frac{\Delta_{t-1}(y)}{2}}{b}.$$

  Substituting these into equation (2.6), the expected revenue-to-go function for a given posted price p, $J_t(p, y)$ is

$$\begin{aligned} J_t(p, y) &= V_{t-1}(y) \\ &\quad + \lambda q \left[\frac{p - \theta - \frac{\Delta_{t-1}(y)}{2}}{b}(p - \theta + \frac{\Delta_{t-1}(y)}{2}) + \frac{p^2 - 4(p-\theta)^2}{2b} \right. \\ &\qquad \left. + (\frac{b-p}{b})p - \frac{b - p + \theta - \frac{\Delta_{t-1}(y)}{2}}{b}\Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q)\frac{b-p}{b}\left[p - \Delta_{t-1}(y)\right]. \end{aligned}$$

  Taking the derivative with respect to p, we observe

$$J_t'(p, y) = \lambda q \frac{b - 3p + 2\theta + \Delta_{t-1}(y)}{b} + \lambda(1-q)\frac{b - 2p + \Delta_{t-1}(y)}{b}.$$

Note that $J_t(p,y)$ is concave in p and the solution to the first order condition is $p^* = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$. The feasibility of p^* comes from the facts

$$p^* - \theta - \frac{\Delta_{t-1}(y)}{2} = \frac{2b - q\Delta_{t-1}(y) - 2\theta(2-q)}{2(2+q)} \geq 0 \text{ since } \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)},$$
$$p^* - 2\theta = \frac{b + \Delta_{t-1}(y) - 4\theta}{2+q} < 0 \text{ since } \theta > \frac{b+\Delta_{t-1}(y)}{4}.$$

Thus, $J_t(p,y)$ is maximized at $p^* = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$.

- Case 3c: $2\theta \leq p < \frac{b}{2} + \theta$

For a given p, the bargainers's offer and the seller's counter-offer are identical to those in Case 2c. Thus, we have

$$K_t(p,y) = \frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2}) + (\frac{b-2(p-\theta)}{b})p \text{ and } B_t(p,y) = \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}.$$

Substituting these into equation (2.6), the expected revenue-to-go function for a given posted price p, $J_t(p,y)$ is

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) \\ &\quad + \lambda q \left[\frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p - \theta + \frac{\Delta_{t-1}(y)}{2}) \right. \\ &\qquad \left. + (\frac{b-2(p-\theta)}{b})p - \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}\Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q)\frac{b-p}{b}\left[p - \Delta_{t-1}(y)\right]. \end{aligned}$$

Taking the derivative with respect to p, we observe

$$J_t'(p,y) = \lambda\frac{b - 2p + \Delta_{t-1}(y)}{b}.$$

If $p \geq 2\theta$, then $p > \frac{b+\Delta_{t-1}(y)}{2}$, then $J_t'(p,y) < 0$. Hence, $J_t(p,y)$ is decreasing in p between $[2\theta, \frac{b}{2} + \theta)$.

- Case 3d: $\frac{b}{2} + \theta \leq p \leq \frac{b+\Delta_{t-1}(y)}{2} + \theta$

This case is identical to Case 2d. Applying identical algebra, it is easy to see that $J_t(p,y)$ is decreasing in $p \in [\frac{b}{2}+\theta, \frac{b+\Delta_{t-1}(y)}{2}+\theta]$.

- Case 3e: $\frac{b+\Delta_{t-1}(y)}{2}+\theta < p \leq b$

  This case is identical to Case 2e, thus $J_t(p,y)$ is decreasing in $p \in (\frac{b+\Delta_{t-1}(y)}{2}, b]$.

Combining all five cases, we note that $J_t(p,y)$ is increasing in p up to $p = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$, then decreasing afterward. Thus, $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$.

Case 4: $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$

We divide into sub-cases depending on the value of p: 4a) $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$ and 4b) $\theta + \frac{\Delta_{t-1}(y)}{2} \leq p \leq b$.

- Case 4a: $\Delta_{t-1}(y) \leq p < \theta + \frac{\Delta_{t-1}(y)}{2}$

  From Lemma A.2.1, bargainers with $r \in [p-\theta, 2(p-\theta)]$ choose $p_o^*(p,r) = p-\theta$ to which the seller responds with the counter-offer $p_{ct}^*(p, p-\theta, y) = \Delta_{t-1}(y)$. Bargainers with even higher reservation price (i.e., $2(p-\theta) < r < b$) choose $p_o^*(p,r) = \min[p, \frac{r}{2}]$ to which the seller responds with the counter-offer $p_{ct}^*(p, p_o^*(p,r), y) = \max[\min[r,p], \Delta_{t-1}(y)]$. As a result, bargainers with reservation price between $\Delta_{t-1}(y)$ and p end up buying at their reservation price and bargainers with $r > p$ will buy at the original posted price p. This case is identical to Case 2a. Thus, we have

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q\left[\frac{p^2-\Delta_{t-1}^2(y)}{2b} + (\frac{b-p}{b})p - \frac{b-\Delta_{t-1}(y)}{b}\Delta_{t-1}(y)\right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right] \text{ and} \\ J_t'(p,y) &= \lambda q\frac{b-p}{b} + \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b}. \end{aligned}$$

Note that $J_t(p,y)$ is concave in p and the solution to the first order condition is $p^* = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$. The feasibility of p^* comes from the facts

$$p^* - \Delta_{t-1}(y) = \frac{b-\Delta_{t-1}(y)}{2-q} \geq 0$$
$$p^* - \theta - \frac{\Delta_{t-1}(y)}{2} = \frac{2b - q\Delta_{t-1}(y)}{2(2-q)} - \theta < 0.$$

Thus, $J_t(p,y)$ is maximized at $p^* = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$.

- Case 4b: $\theta + \frac{\Delta_{t-1}(y)}{2} \leq p \leq b$ For a given p, the bargainers's offer and the seller's counter-offer are identical to those in Case 2b. Thus, we have

$$\begin{aligned} J_t(y,p) &= V_{t-1}(y) \\ &\quad +\lambda q \left[\frac{p-\theta-\frac{\Delta_{t-1}(y)}{2}}{b}(p-\theta+\frac{\Delta_{t-1}(y)}{2}) + \frac{p^2-4(p-\theta)^2}{2b} \right. \\ &\qquad \left. +(\frac{b-p}{b})p - \frac{b-p+\theta-\frac{\Delta_{t-1}(y)}{2}}{b}\Delta_{t-1}(y) \right] \\ &\quad +\lambda(1-q)\frac{b-p}{b}\left[p-\Delta_{t-1}(y)\right], \text{ and} \\ J_t'(y,p) &= \lambda q \frac{b-3p+2\theta+\Delta_{t-1}(y)}{b} + \lambda(1-q)\frac{b-2p+\Delta_{t-1}(y)}{b}. \end{aligned}$$

Note that $J_t'(y,p)$ evaluated at $p = \theta + \frac{\Delta_{t-1}(y)}{2}$ is given by

$$\begin{aligned} J_t'(y,p)\big|_{p=\theta+\frac{\Delta_{t-1}(y)}{2}} &= \lambda q \frac{b-\theta-\frac{\Delta_{t-1}(y)}{2}}{b} + \lambda(1-q)\frac{b-2\theta}{b} \\ &= \frac{\lambda}{b}\left(q\theta + b - 2\theta - \frac{q\Delta_{t-1}(y)}{2} \right) < 0, \end{aligned}$$

where the inequality comes from $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Since $J_t'(y,p)$ is decreasing p, we conclude that $J_t(y,p)$ is decreasing in p for $p \geq \theta + \frac{\Delta_{t-1}(y)}{2}$.

Combining two cases, it is easy to see that $J_t(y,p)$ is increasing in p up to $p = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$, then decreasing afterward. Thus, $p_t^*(y) = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$.

Proof of Lemma 2.4.2

Note that, the optimal posted price described in Lemma 2.4.1, $p_t^*(y)$ is always greater than or equal to $\frac{b}{2}$. It is easy to see that $p_t^*(y) \geq \frac{b}{2}$ for the first and third cases of Lemma 2.4.1. For the second case, note that

$$p_t^*(t) - \frac{b}{2} = \frac{b + 2q\theta + \Delta_{t-1}(y)}{2+q} - \frac{b}{2} = \frac{4q\theta + 2\Delta_{t-1}(y) - bq}{2(2+q)}.$$

Since $\theta > \frac{b+\Delta_{t-1}(y)}{4}$, it follows that $p_t^*(y) \geq \frac{b}{2}$. Therefore, for any $r \in [0, b]$, we have $r \leq 2p_t^*(y)$. The result now follows by letting $p = p_t^*(y)$ in Lemma A.2.1.

Proof of Lemma 2.4.3

We consider four cases as in the proof of Lemma 2.4.1.

Case 1: $\theta \leq \frac{\Delta_{t-1}(y)}{2}$

In this case, the optimal posted price $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ by Lemma 2.4.1(a). Using simple algebra, it is easy to check $2(p_t^*(y) - \theta) \geq b$. Thus, from Lemma 2.4.2, all bargainers with $r \geq p_t^*(y) - \theta$ offer $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$. To this offer, the seller responds with the counter-offer $p_{ct}^*(p_t^*(y), p_t^*(y) - \theta, y) = p_t^*(y)$ by letting $p = p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ from Lemma A.2.2.

Case 2: $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$:

The optimal posted price is again $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ by Lemma 2.4.1(a). We first note that $p_t^*(y) \leq 2(p_t^*(y) - \theta) < b$ where the first inequality comes from $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$ and the second inequality from $\frac{\Delta_{t-1}(y)}{2} < \theta$. Hence, by Lemma 2.4.2, bargainers with $r \in [p_t^*(y) - \theta, 2(p_t^*(y) - \theta)]$ offer $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$ and bargainers with $r \in (2(p_t^*(y) - \theta), b]$ offer $p_o^*(p_t^*(y), r) = \frac{r}{2}$.

To determine the seller's optimal counter-offer to bargainers with $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$, we let $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ in Lemma A.2.2 and note that $\frac{\Delta_{t-1}(y)}{2} < \theta \leq$

$\frac{b+\Delta_{t-1}(y)}{4} \leq \frac{b}{2}$ implies that

$$\Delta_{t-1}(y) \leq p_t^*(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2} = \tfrac{b+\Delta_{t-1}(y)}{2} - \theta + \tfrac{\Delta_{t-1}(y)}{2} < p_t^*(y) = \tfrac{b+\Delta_{t-1}(y)}{2}.$$

Thus,

$$\begin{aligned} & p_{ct}^*(p_t^*(y), p_t^*(y) - \theta, y) \\ = \;& \max\left\{\min\{p_t^*(y), 2(p_t^*(y) - \theta), p_t^*(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2}\}, \Delta_{t-1}(y)\right\} \\ = \;& p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}. \end{aligned}$$

To determine the seller's optimal counter-offer to bargainers with $p_o^*(p_t^*(y), r) = \frac{r}{2}$, note that these offers are made by bargainers with $r \geq 2(p_t^*(y) - \theta)$. Also note that, in this case, $2(p_t^*(y) - \theta) \geq p_t^*(y)$. Thus, from Lemma A.2.2

$$p_{ct}^*(p_t^*(y), r/2, y) \;\; = \;\; \max\{\min\{r, p_t^*(y)\}, \Delta_{t-1}(y)\} = p_t^*(y).$$

Case 3: $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$:

The optimal posted price is $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$ by Lemma 2.4.1(b). We first note that $2(p_t^*(y)-\theta) < p_t^*(y) < b$ where the first inequality comes from $\theta > \frac{b+\Delta_{t-1}(y)}{4}$ and the second inequality from Lemma A.2.3. Hence, by Lemma 2.4.2, bargainers with $r \in [p_t^*(y) - \theta, 2(p_t^*(y) - \theta)]$ offer $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$ and bargainers with $r \in (2(p_t^*(y) - \theta), b]$ offer $p_o^*(p_t^*(y), r) = \frac{r}{2}$.

To determine the seller's optimal counter-offer to bargainers with $p_o^*(p_t^*(y), r) = p_t^*(y) - \theta$, we let $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$ in Lemma A.2.2 and note that $\theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$ implies that

$$\Delta_{t-1}(y) \leq p_t^*(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2} \leq 2(p_t^*(y) - \theta).$$

Thus,

$$\begin{aligned} & p^*_{ct}(p^*_t(y), p^*_t(y) - \theta, y) \\ = \;& \max\left\{\min\{p^*_t(y), 2(p^*_t(y) - \theta), p^*_t(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2}\}, \Delta_{t-1}(y)\right\} \\ = \;& p^*_t(y) - \theta + \frac{\Delta_{t-1}(y)}{2}. \end{aligned}$$

To determine the seller's optimal counter-offer to bargainers with $p^*_o(p^*_t(y), r) = \frac{r}{2}$, we first note that these offers are from bargainers with r such that $r \geq 2(p^*_t(y) - \theta)$. Furthermore, in this case, $\Delta_{t-1}(y) \leq 2(p^*_t(y) - \theta)$ as shown above. Hence, from Lemma A.2.2, we have

$$p^*_{ct}(p^*_t(y), r/2, y) \;=\; \max\{\min\{r, p^*_t(y)\}, \Delta_{t-1}(y)\} = \begin{cases} r & \text{if } r < p^*_t(y); \\ p^*_t(y) & \text{if } r \geq p^*_t(y). \end{cases}$$

Case 4: $\theta > \frac{2b - q\Delta_{t-1}(y)}{2(2-q)}$:

The optimal posted price is $p^*_t(y) = \frac{b + (1-q)\Delta_{t-1}(y)}{2-q}$ by Lemma 2.4.1(c). We first note that $2(p^*_t(y) - \theta) < p^*_t(y) < b$ where the first inequality comes from $\theta > \frac{2b - q\Delta_{t-1}(y)}{2(2-q)} > \frac{\Delta_{t-1}(y)}{2}$ and the second inequality from Lemma A.2.3. Hence, by Lemma 2.4.2, bargainers with $r \in [p^*_t(y) - \theta, 2(p^*_t(y) - \theta)]$ offer $p^*_o(p^*_t(y), r) = p^*_t(y) - \theta$ and bargainers with $r \in (2(p^*_t(y) - \theta), b]$ offer $p^*_o(p^*_t(y), r) = \frac{r}{2}$.

To determine the seller's optimal counter-offer to bargainers with $p^*_o(p^*_t(y), r) = p^*_t(y) - \theta$, we let $p^*_t(y) = \frac{b + (1-q)\Delta_{t-1}(y)}{2-q}$ in Lemma A.2.2 and note that $\theta > \frac{2b - q\Delta_{t-1}(y)}{2(2-q)}$ implies that

$$2(p^*_t(y) - \theta) < p^*_t(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2} < \Delta_{t-1}(y).$$

Thus,

$$\begin{aligned} & p^*_{ct}(p^*_t(y), p^*_t(y) - \theta, y) \\ = \;& \max\left\{\min\{p^*_t(y), 2(p^*_t(y) - \theta), p^*_t(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2}\}, \Delta_{t-1}(y)\right\} \\ = \;& \Delta_{t-1}(y). \end{aligned}$$

To determine the seller's optimal counter-offer to bargainers with $p^*_o(p^*_t(y), r) = \frac{r}{2}$, we first note that these offers are from bargainers with r such that $r \geq 2(p^*_t(y) - \theta)$. However, in this case, $2(p^*_t(y) - \theta) < \Delta_{t-1}(y)$ as shown above. As a result, the seller responds with the counter-offer $p^*_{ct}(p^*_t(y), r/2, y) = \Delta_{t-1}(y)$ if $r < \Delta_{t-1}(y)$, and $p^*_{ct}(p^*_t(y), r/2, y) = \min[r, p^*_t(y)]$ if $r \geq \Delta_{t-1}(y)$. Hence, from Lemma A.2.2, we have

$$\begin{aligned} & p^*_{ct}(p^*_t(y), r/2, y) \\ = \;& \max\{\min\{r, p^*_t(y)\}, \Delta_{t-1}(y)\} = \begin{cases} \Delta_{t-1}(y) & \text{if } r < \Delta_{t-1}(y); \\ r & \text{if } \Delta_{t-1}(y) \leq r < p^*_t(y); \\ p^*_t(y) & \text{if } r \geq p^*_t(y). \end{cases} \end{aligned}$$

Lemma A.2.1. *Under Assumption 3, given arbitrary posted price p, the optimal offer of a bargainer with reservation price r given is*

$$p^*_o(p, r) = \begin{cases} p - \theta & \text{if } p - \theta \leq r \leq 2(p - \theta); \\ \frac{r}{2} & \text{if } 2(p - \theta) < r \leq 2p; \\ p & \text{if } r > 2p. \end{cases} \tag{A.7}$$

Proof of Lemma A.2.1

Note that under Assumption 3, $S(p_o, r) = (r - p_o)\frac{p_o}{p}$, thus the unconstrained optimizer $\bar{p}_o(r)$ is $\frac{r}{2}$. Applying this to equation (2.3), we obtain $\rho(x) = 2x$. The result

now follows from Lemma 2.3.2.

Lemma A.2.2. *Let $p^*_{ct}(p, p_o, y)$ denote the optimal counter-offer when the seller has y units of inventory in period t given a customer offer p_o in response to an arbitrary posted price $p \geq \Delta_{t-1}(y)$. Then:*

$$p^*_{ct}(p, p_o, y) = \begin{cases} \max\left\{\min\{p, \max\{p-\theta, \frac{b+\Delta_{t-1}(y)}{2}\}\}, \Delta_{t-1}(y)\right\} & \text{if } p_o = p-\theta \text{ and } 2(p-\theta) > b; \\ \max\left\{\min\{p, 2(p-\theta), p-\theta+\frac{\Delta_{t-1}(y)}{2}\}, \Delta_{t-1}(y)\right\} & \text{if } p_o = p-\theta \text{ and } 2(p-\theta) \leq b; \\ \max\{\min\{2p_o, p\}, \Delta_{t-1}(y)\} & \text{if } p_o > p-\theta. \end{cases}$$

Proof of Lemma A.2.2

Throughout the proof, recall that $\rho(x) = 2x$ under Assumption 3. If $p_o > p - \theta$, the result follows from Lemma 2.3.5. Consider the case where $p_o = p - \theta$. Then, under Assumption 3, $Z_t(p_c, y)$ given by (2.7) reduces to

$$Z_t(p_c, y) = \begin{cases} \frac{\min\{b, 2(p-\theta)\}-p_c}{\min\{b, 2(p-\theta)\}-(p-\theta)}(p_c + V_{t-1}(y-1)) & \\ \qquad + \left(\frac{p_c-(p-\theta)}{\min\{b, 2(p-\theta)\}-(p-\theta)}\right) V_{t-1}(y) & \text{if } p_c \leq 2(p-\theta); \\ 0 & \text{if } p_c > 2(p-\theta). \end{cases}$$

We divide into two cases, (i) $2(p-\theta) > b$, and (ii) $2(p-\theta) \leq b$.

(i)If $2(p-\theta) > b$, then we have $\arg\max_{p_c}\{Z_t(p_c, y)\} = \frac{b+\Delta_{t-1}(y)}{2}$. Therefore, $\overline{p}_{ct}(y)$, defined as $\arg\max\{Z_t(p_c, y) | p-\theta \leq p_c \leq \min[p, \rho(p-\theta)]\}$, is given by $\min\{p, \max\{p-\theta, \frac{b+\Delta_{t-1}(y)}{2}\}\}$. (To verify this claim, recall that $Z_t(p_c, y)$ is unimodal in p_c by Lemma 2.3.4 and note that $b \geq \Delta_{t-1}(y)$ by Lemma A.2.3(b).) This observation along with Lemma 2.3.5 yields

$$p^*_{ct}(p, p_o, y) = \max\left\{\min\{p, \max\{p-\theta, \frac{b+\Delta_{t-1}(y)}{2}\}\}, \Delta_{t-1}(y)\right\}.$$

(ii)If $2(p-\theta) \leq b$, then we have $\arg\max_{p_c}\{Z_t(p_c, y)\} = p-\theta+\frac{\Delta_{t-1}(y)}{2}$. Therefore, $\overline{p}_{ct}(y) = \min\{p, 2(p-\theta), p-\theta+\frac{\Delta_{t-1}(y)}{2}\}$. This along with Lemma 2.3.5 yields

$$p^*_{ct}(p, p_o, y) = \max\left\{\min\{p, 2(p-\theta), p-\theta+\frac{\Delta_{t-1}(y)}{2}\}, \Delta_{t-1}(y)\right\}.$$

Lemma A.2.3. *Let $p^*_t(y)$ denote the optimal posted price when the firm has y units of inventory in period t and $F(\cdot)$ is uniform over $[0, b]$. Then:*

(a) $p^*_t(y) \leq b$.

(b) $\Delta_{t-1}(y) \leq b$ for any t and y.

Proof of Lemma A.2.3

Suppose $p > b$ in period t with y units in inventory. We will show that setting the posted price to b will not worsen the seller's expected revenue-to-go, which allows us to conclude that $p^*_t(y) \leq b$.

Consider first the case where $p \in (b, b+\theta]$. When $p \in (b, b+\theta]$, no price-taker will buy and, thus, the seller's expected revenue in period t from a price-taker is zero. From Lemmas A.2.1 and A.2.2, bargainers with $r \in [p-\theta, \min[2(p-\theta), b]]$ make an offer of $p-\theta$ and receive a counter-offer equal to some $\hat{p}_c$ where $\hat{p}_c \geq p-\theta$; bargainers with $r \in [\min[2(p-\theta), b], b]$ (if such an interval exists) make an offer of $r/2$ and receive a counter-offer of r. Therefore, $J_t(p, y)$ is given by

$$\begin{aligned} J_t(p,y) &= V_{t-1}(y) + \lambda q\left[K_t(p,y) - B_t(p,y)(V_{t-1}(y) - V_{t-1}(y-1))\right] \\ &= V_{t-1}(y) + \lambda q\left[\frac{\min[2(p-\theta),b]-\hat{p}_c}{b}\hat{p}_c + \int_{\min[2(p-\theta),b]}^{b} x\frac{1}{b}dx \right. \\ &\qquad \left. -\overline{F}(\hat{p}_c)(V_{t-1}(y) - V_{t-1}(y-1))\right]. \end{aligned}$$

Suppose now we set the posted price to b. No price-taker will buy and, thus, the seller's expected revenue in period t from a price-taker is zero. Once again, from Lemma A.2.1, bargainers with $r \in [b-\theta, \min[2(b-\theta), b]]$ make an offer of

$b-\theta$ and bargainers with $r \in [\min[2(b-\theta), b], b]$ (if such an interval exists) make an offer of $r/2$. Suppose the seller is using the following counter-offer strategy, which is not necessarily optimal: All bargainers who offer $b-\theta$ receive a counter-offer of $\hat{p}_c$; all bargainers with $r \in [\min[2(b-\theta), b], \min[2(p-\theta), b]]$ also receive a counter-offer of $\hat{p}_c$; bargainers with $r \in [\min[2(p-\theta), b], b]$ (if such an interval exists) receive a counter-offer of r. Note that under this counter-offer strategy, no bargainers with $r \in [b-\theta, p-\theta)$ will buy as $\hat{p}_c \geq p-\theta$. As a result, bargainers with $r \in [\hat{p}_c, \min[2(p-\theta), b]]$ will buy at $\hat{p}_c$ and bargainers with $r \in [\min[2(p-\theta), b], b]$ buy at r. Let $\tilde{J}_t(b, y)$ denote the expected revenue-to-go of a seller using this counter-offer strategy under posted price b. Then,

$$\begin{aligned}
\tilde{J}_t(b,y) &= V_{t-1}(y) \\
&\quad +\lambda q \left[\frac{\min[2(p-\theta), b] - \hat{p}_c}{b} \hat{p}_c + \int_{\min[2(p-\theta),b]}^{b} x \frac{1}{b} dx \right. \\
&\qquad \left. -\overline{F}(\hat{p}_c)(V_{t-1}(y) - V_{t-1}(y-1)) \right] \\
&= J_t(p,y).
\end{aligned}$$

Since the counter-offer strategy resulting in $\tilde{J}_t(b, y)$ is not necessarily optimal, we have $J_t(b, y) \geq \tilde{J}_t(b, y) = J_t(p, y)$. Thus, the seller can do at least as well with the posted price b as it does with the posted price p.

Now consider the case when $p > b + \theta$. In this case, neither a bargainer nor a price-taker will buy the product in period t. Thus, $J_t(p, y) = V_{t-1}(y)$. Using a logic similar to the previous case, it can be shown that $J_t(b, y) \geq J_t(p, y)$; the seller can do at least as well with posted price b as it does with posted price b..

Finally, part (b) follows from Lemma 2.3.3 and part (a) of the lemma; $\Delta_{t-1}(y) \leq p_t^*(y) \leq b$.

A.3 Proofs of Propositions 2.4.1 through 2.4.3, and Corollary 2.4.1 in Section 2.4

Here, we prove Propositions 2.4.1 through 2.4.3 and Corollary 2.4.1 in Section 2.4.

Proof of Proposition 2.4.1

One can check from equation (2.10) that $p_t^{TL}(y) = \frac{b+\Delta_{t-1}(y)}{2}$ under Assumption 3. We next compare $p_t^{TL}(y)$ with $p_t^*(y)$ given by Lemma 2.4.1.

(i)If $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, then $p_t^*(y) = p_t^{TL}(y)$ by Lemma 2.4.1(a).

(ii)If $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then, from Lemma 2.4.1(b), we have

$$p_t^*(y) - p_t^{TL}(y) = \frac{b + 2q\theta + \Delta_{t-1}(y)}{2+q} - \frac{b+\Delta_{t-1}(y)}{2} = \frac{q(4\theta - b - \Delta_{t-1}(y))}{2(2+q)} > 0$$

where the inequality follows from the condition $\theta > \frac{b+\Delta_{t-1}(y)}{4}$. Furthermore,

$$p_t^*(y) - p_t^{TL}(y) - \theta = \frac{2q\theta - bq - q\Delta_{t-1}(y) - 4\theta}{2(2+q)} \leq 0.$$

Hence, $p_t^{TL}(y) < p_t^*(y) \leq p_t^{TL}(y) + \theta$.

(iii)If $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then, from Lemma 2.4.1(c), we have

$$p_t^*(y) - p_t^{TL}(y) = \frac{b + (1-q)\Delta_{t-1}(y)}{2-q} - \frac{b+\Delta_{t-1}(y)}{2} = \frac{-q\Delta_{t-1}(y) + bq}{2(2-q)} \geq 0$$

where the inequality follows from the fact that $b \geq \Delta_{t-1}(y)$. Furthermore,

$$p_t^*(y) - p_t^{TL}(y) - \theta = \frac{-q\Delta_{t-1}(y) + bq}{2(2-q)} - \theta \leq 0$$

where the inequality follows from the condition that $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Hence, $p_t^{TL}(y) < p_t^*(y) \leq p_t^{TL}(y) + \theta$.

Proof of Proposition 2.4.2

Consider a policy where the seller uses a take-it-or-leave-it price p in period t with

y units of inventory and follows the optimal negotiation policy from period $t-1$ onward. We define the expected revenue of the seller using such policy as follows:

$$\tilde{J}_t(p,y) = \lambda\overline{F}(p)p + \lambda\overline{F}(p)V_{t-1}(y-1) + (1-\lambda\overline{F}(p))V_{t-1}(y).$$

Note that $V_t^{TL}(y) = \max_p \tilde{J}_t(p,y)$. Notice that $\tilde{J}_t(p,y)$ can be rewritten as

$$\begin{aligned}\tilde{J}_t(p,y) &= V_{t-1}(y) + \lambda q[p\bar{F}(p) - \bar{F}(p)(V_{t-1}(y) - V_{t-1}(y-1))] \\ &\quad +\lambda(1-q)[p\bar{F}(p) - \bar{F}(p)(V_{t-1}(y) - V_{t-1}(y-1))]\end{aligned}$$

Thus, $\tilde{J}_t(p,y)$ is also the expected revenue of a seller who sets a posted price p in period t with y units of inventory and whose counter-offer to any bargainer's offer in period t is simply the posted price p itself. This counter-offer is one of many counter-offers that the seller can choose whereas $K_t(p,y)$ and $B_t(p,y)$ correspond to the optimal counter-offer strategy for a given posted price p, thus

$$\begin{aligned}J_t(p,y) - \tilde{J}_t(p,y) &= \lambda q[K_t(p,y) - B_t(p,y)(V_{t-1}(y) - V_{t-1}(y-1))] \\ &\quad -\lambda q[p\bar{F}(p) - \bar{F}(p)(V_{t-1}(y) - V_{t-1}(y-1))] \quad \geq 0.\end{aligned}$$

Therefore, from the definitions of $p_t^*(y)$ (the optimal posted price under negotiation) and $p_t^{TL}(y)$ (the optimal posted price when negotiation is not allowed only in period t), we have

$$V_t(y) = J_t(p_t^*(y),y) \geq J_t(p_t^{TL}(y),y) \geq \tilde{J}_t(p_t^{TL}(y),y) = V_t^{TL}(y).$$

Proof of Corollary 2.4.1

For the purposes of this proof, define $V_t^R(y)$, the optimal expected revenue of the seller using take-it-or-leave-it pricing throughout the **remaining** t periods, i.e.,

$$\begin{aligned}V_t^R(y) &= \max_p \left\{\lambda\overline{F}(p)p + \lambda\overline{F}(p)V_{t-1}^R(y-1) + (1-\lambda\overline{F}(p))V_{t-1}^R(y)\right\}, \\ &\qquad\qquad y>0, t=1,\ldots,T \\ V_0^R(y) &= 0 \text{ for } y \geq 0, \text{ and } V_t^R(0) = 0 \text{ for } t=1,\ldots,T\end{aligned}$$

We would like to prove that $V_t(y) \geq V_t^R(y)$. The proof is by induction on t. The result holds trivially when $t = 0$. Suppose $V_k(y) \geq V_k^R(y)$, $k \leq t$ for some $t \geq 0$. We will prove that $V_{t+1}(y) \geq V_{t+1}^R(y)$. We first rearrange terms in $V_{t+1}(y)$:

$$\begin{aligned} V_{t+1}(y) &= V_t(y) + \max_p \left\{ \begin{array}{l} \lambda q \left[K_{t+1}(p,y) - B_{t+1}(p,y)(V_t(y) - V_t(y-1)) \right] \\ \\ +\lambda(1-q)\overline{F}(p) \left[p - (V_t(y) - V_t(y-1)) \right] \end{array} \right\} \\ &= \max_p \left\{ \begin{array}{l} \lambda \left(qK_{t+1}(p,y) + (1-q)\overline{F}(p)p \right) \\ \\ +\lambda V_t(y-1) \left[qB_{t+1}(p,y) + (1-q)\overline{F}(p) \right] \\ \\ +V_t(y) \left[1 - \lambda(qB_{t+1}(p,y) + (1-q)\overline{F}(p)) \right] \end{array} \right\}. \end{aligned}$$

From the induction hypothesis $V_t(y) \geq V_t^R(y)$, we have

$$\begin{aligned} V_{t+1}(y) &\geq \max_p \left\{ \begin{array}{l} \lambda \left(qK_{t+1}(p,y) + (1-q)\overline{F}(p)p \right) \\ \\ +\lambda V_t^R(y-1) \left[qB_{t+1}(p,y) + (1-q)\overline{F}(p) \right] \\ \\ +V_t^R(y) \left[1 - \lambda(qB_{t+1}(p,y) + (1-q)\overline{F}(p)) \right] \end{array} \right\} \\ &= V_t^R(y) + \max_p \left\{ \begin{array}{l} \lambda q \left[K_{t+1}(p,y) - B_{t+1}(p,y)(V_t^R(y) - V_t^R(y-1)) \right] \\ \\ +\lambda(1-q)\overline{F}(p) \left[p - (V_t^R(y) - V_t^R(y-1)) \right] \end{array} \right\} \end{aligned}$$

Note that $K_t(p,y)$ and $B_t(p,y)$ correspond to the optimal counter-offer strategy for given posted price p, thus the resultant expected revenue to go function is greater than or equal to the revenue under the policy where the seller's counter-offer is set

to the posted price p regardless of the bargainer's offer. Hence,

$$\begin{aligned} V_{t+1}(y) &\geq V_t^R(y) + \max_p \left\{ \begin{array}{l} \lambda q \overline{F}(p) \left[p - (V_t^R(y) - V_t^R(y-1))\right] \\ \\ +\lambda(1-q)\overline{F}(p) \left[p - (V_t^R(y) - V_t^R(y-1))\right] \end{array} \right\} \\ &= V_{t+1}^R(y) \end{aligned}$$

Proof of Proposition 2.4.3

Note that price-takers are not better-off since $p_t^{TL}(y) \leq p_t^*(y) \leq p_t^{TL}(y) + \theta$ by Proposition 2.4.1. Hence, we focus on the bargainers and compare the price that a bargainer pays with $p_t^{TL}(y) = \frac{b+\Delta_{t-1}(y)}{2}$.

(i) If $\theta \leq \frac{\Delta_{t-1}(y)}{2}$, then $p_t^*(y) = p_t^{TL}(y)$. Note that by Lemmas 2.4.2 and 2.4.3, bargainers with $r \in [p_t^*(y), b]$ pay $p_t^*(y) = p_t^{TL}(y)$. Hence, bargainers are not better off.

(ii) If $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, then $p_t^*(y) = p_t^{TL}(y)$. By Lemmas 2.4.2 and 2.4.3, bargainers with $r > 2(p_t^*(y) - \theta)$ pay $p_t^*(y) \geq p_t^{TL}(y)$ and thus are not better off; bargainers with $r \in [p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}, 2(p_t^*(y) - \theta)]$ pay $p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2} = p_t^{TL}(y) - \theta + \frac{\Delta_{t-1}(y)}{2} < p_t^{TL}(y)$ since $\theta > \frac{\Delta_{t-1}(y)}{2}$. Therefore, bargainers with $r \in [p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}, 2(p_t^*(y) - \theta)]$ are better off.

(iii) If $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$. By Lemmas 2.4.2 and 2.4.3, bargainers with $r > p_t^*(y)$ pay $p_t^*(y) \geq p_t^{TL}(y)$ and thus are not better off; bargainers with $r \in (2(p_t^*(y) - \theta), p_t^*(y)]$ pay r, and are not better off; bargainers with $r \in [p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}, 2(p_t^*(y) - \theta)]$ pay $p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}$. We observe

$$p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2} - p_t^{TL}(y) = \frac{2q\theta + 2\Delta_{t-1}(y) - 4\theta - bq}{2(2+q)} < 0$$

where the inequality follows from $\theta > \frac{b+\Delta_{t-1}(y)}{4} > \frac{\Delta_{t-1}(y)}{2}$. Hence, bargainers with $r \in [p_t^*(y) - \theta + \frac{\Delta_{t-1}(y)}{2}, 2(p_t^*(y) - \theta)]$ are better off.

(iv) If $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, then $p_t^*(y) = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$. By Lemmas 2.4.2 and 2.4.3, bargainers with $r > p_t^*(y)$ pay $p_t^*(y) \geq p_t^{TL}(y)$, and thus are not better off; bargainers with $r \in [\Delta_{t-1}(y), p_t^*(y)]$ pay r, and thus are not better off.

A.4 Proofs of Propositions 2.4.4 through 2.4.6 in Section 2.4

Here, we prove Propositions 2.4.4 through 2.4.6 in Section 2.4. The proof of Proposition 2.4.6 utilizes Lemma A.4.1, which is stated and proven at the end of Appendix A.4.

Proof of Proposition 2.4.4

In order to prove that $p_t^*(y)$ is non-decreasing in θ, we first use Lemma 2.4.1 and the following equalities to note that $p_t^*(y)$ is continuous in θ.

$$\frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}\bigg|_{\theta=\frac{b+\Delta_{t-1}(y)}{4}} = \frac{b+\Delta_{t-1}(y)}{2}, \text{ and}$$
$$\frac{b+(1-q)\Delta_{t-1}(y)}{2-q}\bigg|_{\theta=\frac{2b-q\Delta_{t-1}(y)}{2(2-q)}} = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}.$$

Now, note from Lemma 2.4.1 that $p_t^*(y)$ is constant with respect to θ when $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, increasing in θ when $\theta \in (\frac{b+\Delta_{t-1}(y)}{4}, \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}]$ and constant with respect to θ when $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Hence, $p_t^*(y)$ is non-decreasing in θ.

In order to prove that $p_t^*(y)$ is non-decreasing in q, we consider two cases:

Case 1: $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$: Note from Lemma 2.4.1 that if $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, then $p_t^*(y)$ is constant with respect to q and the result holds trivially.

Case 2: $\theta > \frac{b+\Delta_{t-1}(y)}{4}$: First, note that $\theta < \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$ if and only if $q > \frac{4\theta-2b}{2\theta-\Delta_{t-1}(y)}$. Hence, from Lemma 2.4.1, we can write:

$$p_t^*(y) = \begin{cases} \frac{b+(1-q)\Delta_{t-1}(y)}{2-q} & \text{if } q \leq \frac{4\theta-2b}{2\theta-\Delta_{t-1}(y)}; \\ \\ \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q} & \text{if } q > \frac{4\theta-2b}{2\theta-\Delta_{t-1}(y)}. \end{cases}$$

We note from the above equality that $p_t^*(y)$ is continuous in q (the fact comes from $\frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}\Big|_{q=\frac{4\theta-2b}{2\theta-\Delta_{t-1}(y)}} = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$). Furthermore, it is easy to check that $\frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$ is non-decreasing in q (by taking the derivative with respect to

q and noting that $b \geq \Delta_{t-1}(y)$ by Lemma A.2.3(b)). Similarly, one can check that $\frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$ is non-decreasing in q (by taking the derivative and noting that $\theta > \frac{b+\Delta_{t-1}(y)}{4}$). Hence, $p_t^*(y)$ is non-decreasing in q.

Proof of Proposition 2.4.5

We first prove the result for θ. From the proof of Lemma 2.4.1, it can be verified that the optimal expected revenue-to-go $V_t(y) = J_t(p_t^*(y), y)$ depends on θ and takes one of the following four forms.

(i) $0 \leq \theta \leq \frac{\Delta_{t-1}(y)}{2}$:

$$\begin{aligned} J_t(p_t^*(y), y) &= V_{t-1}(y) + \lambda q \left[\frac{b - p_t^*(y)}{b} p_t^*(y) - \frac{b - p_t^*(y)}{b} \Delta_{t-1}(y) \right] \\ &\quad + \lambda(1-q) \frac{b - p_t^*(y)}{b} \left[p_t^*(y) - \Delta_{t-1}(y) \right] \end{aligned}$$

(ii) $\frac{\Delta_{t-1}(y)}{2} < \theta \leq \frac{b+\Delta_{t-1}(y)}{4}$:

$$\begin{aligned} J_t(p_t^*(y), y) &= V_{t-1}(y) + \lambda(1-q) \tfrac{b-p_t^*(y)}{b} \left[p_t^*(y) - \Delta_{t-1}(y) \right] \\ &\quad + \lambda q \left[\tfrac{p_t^*(y) - \theta - \frac{\Delta_{t-1}(y)}{2}}{b} (p_t^*(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2}) \right. \\ &\quad \left. + (\tfrac{b - 2(p_t^*(y) - \theta)}{b}) p_t^*(y) - \tfrac{b - p_t^*(y) + \theta - \frac{\Delta_{t-1}(y)}{2}}{b} \Delta_{t-1}(y) \right] \end{aligned}$$

(iii) $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b - q\Delta_{t-1}(y)}{2(2-q)}$:

$$\begin{aligned} J_t(p_t^*(y), y) &= V_{t-1}(y) + \lambda(1-q) \tfrac{b-p_t^*(y)}{b} \left[p_t^*(y) - \Delta_{t-1}(y) \right] \\ &\quad + \lambda q \left[\begin{array}{l} \tfrac{p_t^*(y) - \theta - \frac{\Delta_{t-1}(y)}{2}}{b} (p_t^*(y) - \theta + \tfrac{\Delta_{t-1}(y)}{2}) + \tfrac{(p_t^*(y))^2 - 4(p_t^*(y) - \theta)^2}{2b} \\ \\ + (\tfrac{b - p_t^*(y)}{b}) p_t^*(y) - \tfrac{b - p_t^*(y) + \theta - \frac{\Delta_{t-1}(y)}{2}}{b} \Delta_{t-1}(y) \end{array} \right] \end{aligned}$$

(iv) $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$

$$\begin{aligned} J_t(p_t^*(y), y) &= V_{t-1}(y) + \lambda(1-q)\tfrac{b-p_t^*(y)}{b}\left[p_t^*(y) - \Delta_{t-1}(y)\right] \\ &\quad + \lambda q\left[\tfrac{(p_t^*(y))^2 - \Delta_{t-1}^2(y)}{2b} + (\tfrac{b-p_t^*(y)}{b})p_t^*(y) - \tfrac{b-\Delta_{t-1}(y)}{b}\Delta_{t-1}(y)\right] \end{aligned}$$

For each of the four cases, we substitute the corresponding $p_t^*(y)$ from Lemma 2.4.1 and take the derivative with respect to θ:

$$\frac{dJ_t(p_t^*(y), y)}{d\theta} = \begin{cases} 0 & \text{for } 0 \le \theta \le \frac{\Delta_{t-1}(y)}{2}; \\ \frac{\lambda q}{b}(2\theta - \Delta_{t-1}(y)) > 0 & \text{for } \frac{\Delta_{t-1}(y)}{2} < \theta \le \frac{b+\Delta_{t-1}(y)}{4}; \\ \frac{\lambda q}{b}\left(\frac{2b+2q\theta-4\theta-q\Delta_{t-1}(y)}{2+q}\right) \ge 0 & \text{for } \frac{b+\Delta_{t-1}(y)}{4} < \theta \le \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}; \\ 0 & \text{for } \theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}. \end{cases} \tag{A.8}$$

where the first and second inequalities follow from $\frac{\Delta_{t-1}(y)}{2} < \theta$ and $\theta \le \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, respectively. Therefore, $J_t(p_t^*(y), y)$ is non-decreasing in each of the four regions above. Finally, by substituting the values of θ at the boundaries of the four cases above, it can be shown that $J_t(p_t^*(y), y)$ is continuous in θ. Hence, $J_t(p_t^*(y), y)$ is non-decreasing in θ.

The result that $J_t(p_t^*(y), y)$ is non-decreasing in q can be shown following a similar logic, thus omitted.

Proof of Proposition 2.4.6

We first prove that $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$. We then use this fact and Lemma A.4.1 to conclude the proof. Throughout the proof, recall that $\Delta_{t-1}(y) \in [0, b]$. Consider three cases: $\theta \le \frac{b}{4}$, $\frac{b}{4} < \theta \le \frac{b}{2-q}$ and $\theta > \frac{b}{2-q}$.

Case 1: $\theta \leq \frac{b}{4}$: In this case, for all $\Delta_{t-1}(y) \in [0, b]$, we have $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$. Hence, $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ for all $\Delta_{t-1}(y) \in [0, b]$ by Lemma 2.4.1(a). It now follows that $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$.

Case 2: $\theta > \frac{b}{2-q}$: In this case, for all $\Delta_{t-1}(y) \in [0, b]$, we have $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Hence, $p_t^*(y) = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$ for all $\Delta_{t-1}(y) \in [0, b]$ by Lemma 2.4.1(c). It follows that $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$.

Case 3: $\frac{b}{4} < \theta \leq \frac{b}{2-q}$: Note that if $\Delta_{t-1}(y) = b$, then $\frac{b+\Delta_{t-1}(y)}{4} = \frac{2b-q\Delta_{t-1}(y)}{2(2-q)} = \frac{b}{2}$. This, along with the facts that $\frac{b+\Delta_{t-1}(y)}{4}$ is increasing in $\Delta_{t-1}(y)$ and $\frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$ is decreasing in $\Delta_{t-1}(y)$, implies that $\frac{b+\Delta_{t-1}(y)}{4} \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$ for all $\Delta_{t-1}(y) \in [0, b]$. Now, consider two cases:

Case 3(a) $\theta \geq \frac{b}{2}$: In this case, for all $\Delta_{t-1}(y) \in [0, \frac{2b-2(2-q)\theta}{q}]$, we have $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Hence, for all $\Delta_{t-1}(y) \in [0, \frac{2b-2(2-q)\theta}{q}]$, $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$ by Lemma 2.4.1(b), and non-decreasing in $\Delta_{t-1}(y)$ over that interval. In addition, for all $\Delta_{t-1}(y) \in (\frac{2b-2(2-q)\theta}{q}, b]$, we have $\theta > \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$, which implies, by Lemma 2.4.1(c), $p_t^*(y) = \frac{b+(1-q)\Delta_{t-1}(y)}{2-q}$ and $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$ over that interval. Furthermore, one can check from Lemma 2.4.1(b), (c) that $p_t^*(y)$ is continuous in $\Delta_{t-1}(y)$ at $\Delta_{t-1}(y) = \frac{2b-2(2-q)\theta}{q}$. Therefore, $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$ over the interval $\Delta_{t-1}(y) \in [0, b]$.

Case 3(b) $\theta < \frac{b}{2}$: In this case, for all $\Delta_{t-1}(y) \in [0, 4\theta - b)$, we have $\frac{b+\Delta_{t-1}(y)}{4} < \theta \leq \frac{2b-q\Delta_{t-1}(y)}{2(2-q)}$. Hence, for all $\Delta_{t-1}(y) \in [0, 4\theta - b)$, $p_t^*(y) = \frac{b+2q\theta+\Delta_{t-1}(y)}{2+q}$ by Lemma 2.4.1(b), and non-decreasing in $\Delta_{t-1}(y)$ over that interval. In addition, for all $\Delta_{t-1}(y) \in [4\theta - b, b]$, we have $\theta \leq \frac{b+\Delta_{t-1}(y)}{4}$, which implies, by Lemma 2.4.1(a), $p_t^*(y) = \frac{b+\Delta_{t-1}(y)}{2}$ and $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$ over that interval. Furthermore, one can check from Lemma 2.4.1(a), (b) that $p_t^*(y)$ is continuous in $\Delta_{t-1}(y)$ at $\Delta_{t-1}(y) = 4\theta - b$. Therefore, $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$ over the interval

$\Delta_{t-1}(y) \in [0, b]$.

Now that we have proven that $p_t^*(y)$ is non-decreasing in $\Delta_{t-1}(y)$, we conclude the proof by noting that $\Delta_{t-1}(y)$ is non-increasing in y and non-decreasing in t by Lemma A.4.1.

Lemma A.4.1. *The function $V_t(y)$ has the following three properties:*

$$H1(t,y): V_{t+1}(y+1) - V_{t+1}(y) \geq V_t(y+1) - V_t(y), y \geq 0, t = 1, \ldots, T.$$

$$H2(t,y): V_{t+1}(y) - V_t(y) \geq V_{t+2}(y) - V_{t+1}(y), y \geq 0, t = 1, \ldots, T.$$

$$H3(t,y): V_t(y+1) - V_t(y) \geq V_t(y+2) - V_t(y+1), y \geq 0, t = 1, \ldots, T.$$

Proof of Lemma A.4.1

Following Bitran and Mondschein (1993), we prove the result by induction on $t + y$. The three inequalities hold when $t + y = 0$. Suppose they hold for $t + y = m - 1$. We prove they hold when $t + y = m$ to complete the induction.

(i) $H1(t,y):\ V_{t+1}(y+1) - V_{t+1}(y) \geq V_t(y+1) - V_t(y)$.

It is easy to show that the result holds at $y = 0$ for all $t = 1, \ldots, T$. Suppose $y > 0$. Note that

$$\begin{aligned} V_{t+1}(y) &= \lambda q \left[K_{t+1}(p_{t+1}^*(y), y) + B_{t+1}(p_{t+1}^*(y), y) V_t(y-1)\right] \\ &\quad + \lambda(1-q)\left[p_{t+1}^*(y)\overline{F}(p_{t+1}^*(y)) + \overline{F}(p_{t+1}^*(y)) V_t(y-1)\right] \\ &\quad + \left[1 - \lambda q B_{t+1}(p_{t+1}^*(y), y) - \lambda(1-q)\overline{F}(p_{t+1}^*(y))\right] V_t(y) \end{aligned}$$

Subtracting $V_t(y)$ from both sides, we get

$$\begin{aligned} & V_{t+1}(y) - V_t(y) \\ = \ & \lambda q K_{t+1}(p_{t+1}^*(y), y) + \lambda(1-q) p_{t+1}^*(y)\overline{F}(p_{t+1}^*(y)) \\ & + \left[\lambda q B_{t+1}(p_{t+1}^*(y), y) + \lambda(1-q)\overline{F}(p_{t+1}^*(y))\right](V_t(y-1) - V_t(y)). \end{aligned} \quad \text{(A.9)}$$

Similarly,

$$\begin{aligned}
& V_{t+1}(y+1) \\
= \; & \lambda q \left[K_{t+1}(p^*_{t+1}(y+1), y+1) + B_{t+1}(p^*_{t+1}(y+1), y+1) V_t(y)\right] \\
& + \lambda(1-q) \left[p^*_{t+1}(y+1)\overline{F}(p^*_{t+1}(y+1)) + \overline{F}(p^*_{t+1}(y+1)) V_t(y)\right] \\
& + \left[1 - \lambda q B_{t+1}(p^*_{t+1}(y+1), y+1) - \lambda(1-q)\overline{F}(p^*_{t+1}(y+1))\right] V_t(y+1).
\end{aligned}$$

Now, we note that a seller with $y+1$ units in inventory at period $t+1$ could set its posted price to the optimal posted price of a seller with y units in inventory at period $t+1$, i.e., $p^*_{t+1}(y)$, which is suboptimal. Also, note that a bargainer's offer remains the same as long as the posted price remains the same. Therefore, if a seller with $y+1$ units in inventory at period $t+1$ is using posted price $p^*_{t+1}(y)$, then it can also mimic the counter-offer strategy of a seller with y units in inventory at period $t+1$. Of course, doing so is suboptimal for the seller with $y+1$ units in inventory at period $t+1$. Therefore:

$$\begin{aligned}
V_{t+1}(y+1) \; \geq \; & \lambda q \left[K_{t+1}(p^*_{t+1}(y), y) + B_{t+1}(p^*_{t+1}(y), y) V_t(y)\right] \\
& + \lambda(1-q) \left[p^*_{t+1}(y)\overline{F}(p^*_{t+1}(y)) + \overline{F}(p^*_{t+1}(y)) V_t(y)\right] \\
& + \left[1 - \lambda q B_{t+1}(p^*_{t+1}(y), y) - \lambda(1-q)\overline{F}(p^*_{t+1}(y))\right] V_t(y+1)
\end{aligned}$$

Subtracting $V_t(y+1)$ from both sides of the above inequality, we obtain

$$\begin{aligned}
& V_{t+1}(y+1) - V_t(y+1) \\
\geq \; & \lambda q K_{t+1}(p^*_{t+1}(y), y) + \lambda(1-q) p^*_{t+1}(y)\overline{F}(p^*_{t+1}(y)) \\
& + \left[\lambda q B_{t+1}(p^*_{t+1}(y), y) + \lambda(1-q)\overline{F}(p^*_{t+1}(y))\right] (V_t(y) - V_t(y+1)).
\end{aligned} \tag{A.10}$$

From induction hypothesis $H3(t, y-1)$, we have

$$V_t(y) - V_t(y+1) \geq V_t(y-1) - V_t(y). \tag{A.11}$$

Thus, from equations (A.9)– (A.11), we have

$$H1(t,y): V_{t+1}(y+1) - V_t(y+1) \geq V_{t+1}(y) - V_t(y).$$

(ii) $H2(t,y): V_{t+1}(y) - V_t(y) \geq V_{t+2}(y) - V_{t+1}(y)$.

The case of $y = 0$ is trivial. Suppose $y > 0$. Note that

$$\begin{aligned} V_{t+2}(y) &= \lambda q \left[K_{t+2}(p^*_{t+2}(y), y) + B_{t+2}(p^*_{t+2}(y), y) V_{t+1}(y-1)\right] \\ &\quad + \lambda(1-q)\left[p^*_{t+2}(y)\overline{F}(p^*_{t+2}(y)) + \overline{F}(p^*_{t+2}(y)) V_{t+1}(y-1)\right] \\ &\quad + \left[1 - \lambda q B_{t+2}(p^*_{t+2}(y), y) - \lambda(1-q)\overline{F}(p^*_{t+2}(y))\right] V_{t+1}(y) \end{aligned}$$

Subtracting $V_{t+1}(y)$ from both sides, we get

$$\begin{aligned} & V_{t+2}(y) - V_{t+1}(y) \\ = \; & \lambda q K_{t+2}(p^*_{t+2}(y), y) + \lambda(1-q) p^*_{t+2}(y)\overline{F}(p^*_{t+2}(y)) \\ & + \left[\lambda q B_{t+2}(p^*_{t+2}(y), y) + \lambda(1-q)\overline{F}(p^*_{t+2}(y))\right](V_{t+1}(y-1) - V_{t+1}(y)). \end{aligned} \tag{A.12}$$

Now, we note that a seller with y units in inventory at period $t+1$ could mimic the optimal posted price and counter-offer strategy of the seller with y units in inventory at period $t+2$. Of course, doing so is suboptimal for the seller with with y units in inventory at period $t+1$. Therefore:

$$\begin{aligned} V_{t+1}(y) &\geq \lambda q \left[K_{t+2}(p^*_{t+2}(y), y) + B_{t+2}(p^*_{t+2}(y), y) V_t(y-1)\right] \\ &\quad + \lambda(1-q)\left[p^*_{t+2}(y)\overline{F}(p^*_{t+2}(y)) + \overline{F}(p^*_{t+2}(y)) V_t(y-1)\right] \\ &\quad + \left[1 - \lambda q B_{t+2}(p^*_{t+2}(y), y) - \lambda(1-q)\overline{F}(p^*_{t+2}(y))\right] V_t(y) \end{aligned}$$

Subtracting $V_t(y)$ from both sides, we get

$$\begin{aligned} & V_{t+1}(y) - V_t(y) \\ \geq\ & \lambda q K_{t+2}(p^*_{t+2}(y), y) + \lambda(1-q)p^*_{t+2}(y)\overline{F}(p^*_{t+2}(y)) \\ & + \left[\lambda q B_{t+2}(p^*_{t+2}(y), y) + \lambda(1-q)\overline{F}(p^*_{t+2}(y))\right](V_t(y-1) - V_t(y)). \end{aligned} \tag{A.13}$$

From induction hypothesis $H1(t, y-1)$, we have

$$V_t(y-1) - V_t(y) \geq V_{t+1}(y-1) - V_{t+1}(y). \tag{A.14}$$

Thus, from equations (A.12)– (A.14), we have

$$H2(t,y):\ V_{t+1}(y) - V_t(y) \geq V_{t+2}(y) - V_{t+1}(y).$$

(iii) $H3(t,y): V_t(y+1) - V_t(y) \geq V_t(y+2) - V_t(y+1)$.

Using arguments similar to those in parts (i) and (ii), we obtain

$$\begin{aligned} & V_t(y+2) - V_{t-1}(y+1) \\ =\ & \lambda q K_t(p^*_t(y+2), y+2) + \lambda(1-q)p^*_t(y+2)\overline{F}(p^*_t(y+2)) \\ & + \left[1 - \lambda q B_t(p^*_t(y+2), y+2) - \lambda(1-q)\overline{F}(p^*_t(y+2))\right] \times \\ & \qquad (V_{t-1}(y+2) - V_{t-1}(y+1)). \end{aligned} \tag{A.15}$$

and

$$\begin{aligned} & V_{t+1}(y+1) - V_t(y) \\ \geq\ & \lambda q K_t(p^*_t(y+2), y+2) + \lambda(1-q)p^*_t(y+2)\overline{F}(p^*_t(y+2)) \\ & + \left[1 - \lambda q B_t(p^*_t(y+2), y+2) - \lambda(1-q)\overline{F}(p^*_t(y+2))\right] \times \\ & \qquad (V_t(y+1) - V_t(y)). \end{aligned} \tag{A.16}$$

From $H1(t-1,y)$ and $H3(t-1,y)$, we have

$$V_t(y+1) - V_t(y) \geq V_{t-1}(y+1) - V_{t-1}(y) \geq V_{t-1}(y+2) - V_{t-1}(y+1). \quad \text{(A.17)}$$

Therefore, from equations (A.15) – (A.17), we obtain

$$V_{t+1}(y+1) - V_t(y) \geq V_t(y+2) - V_{t-1}(y+1). \quad \text{(A.18)}$$

From $H2(t-1,y+1)$, we have

$$V_t(y+1) - V_{t-1}(y+1) \geq V_{t+1}(y+1) - V_t(y+1). \quad \text{(A.19)}$$

Finally, adding (A.18) and (A.19), we obtain

$$H3(t,y) : V_t(y+1) - V_t(y) \geq V_t(y+2) - V_t(y+1).$$

APPENDIX B

B.1 Proofs of Lemmas and Propositions in Section 3.4

In this appendix, we prove the results stated in Section 3.4. The proofs utilize Lemmas B.1.1 and B.1.2, stated and proven at the end of Appendix B.1.

Proof of Lemma 3.4.1

Proof of (a): We prove the unimodality of $\Pi^u_{\mathbf{RP}}(p,w) = a(p-w)\overline{F}(p)$ in p by showing (1) $\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p}\Big|_{p=w} \geq 0$, (2) $\frac{\partial^2 \Pi^u_{\mathbf{RP}}(p,w)}{\partial p^2} < 0$ whenever $\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p} = 0$, and (3) $\Pi^u_{\mathbf{RP}}(p,w) \to 0$ as $p \to \infty$.

First note that the first and second partial derivatives of $\Pi^u_{\mathbf{RP}}(p,w)$ in p are

$$\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p} = a\overline{F}(p) - a(p-w)f(p) \text{ and} \tag{B.1}$$

$$\frac{\partial^2 \Pi^u_{\mathbf{RP}}(p,w)}{\partial p^2} = -2af(p) - a(p-w)f'(p). \tag{B.2}$$

Claim (1) follows from (B.1) while claim (3) follows from $\overline{F}(p) \to 0$ as $p \to \infty$. To show claim (2), note from (B.1) and (B.2)

$$\frac{\partial^2 \Pi^u_{\mathbf{RP}}(p,w)}{\partial p^2}\Big|_{\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p}=0} = -a\frac{2f(p)^2 + f'(p)\overline{F}(p)}{f(p)}. \tag{B.3}$$

Since F is IFR, $f'(\cdot)\overline{F}(\cdot)+f^2(\cdot)\geq 0$. Hence, claim (2) follows from (B.3), concluding the proof of unimodality of $\Pi^u_{\mathbf{RP}}(p,w)$ in p.

Proof of (b): From part (a), $p^u(w)$ satisfies $\overline{F}(p^u(w))-(p^u(w)-w)f(p^u(w))=0$. Implicit differentiation of this equality with respect to w yields

$$(2f(p^u(w))+(p^u(w)-w)f'(p^u(w)))\frac{dp^u(w)}{dw}-f(p^u(w))=0. \tag{B.4}$$

Substituting $p^u(w)-w=\frac{\overline{F}(w)}{f(w)}$ from (B.1) in (B.4), we obtain

$$\frac{dp^u(w)}{dw}=\frac{f^2(p^u(w))}{2f^2(p^u(w))+f'(p^u(w))\overline{F}(p^u(w))}. \tag{B.5}$$

Since F is IFR, we have $f'(\cdot)\overline{F}(\cdot)+f^2(\cdot)\geq 0$, which implies $\frac{dp^u(w)}{dw}>0$. Thus, $p^u(w)$ strictly increases in w.

To prove $p^u(w)$ is convex in w, we show $\frac{d^2p^u(w)}{dw^2}\geq 0$. Take the second derivative of (B.5) with respect to w, we obtain

$$\frac{d^2p^u(w)}{dw^2}=\frac{f(p^u(w))\frac{dp^u(w)}{dw}}{[2f^2(p^u(w))+f'(p^u(w))\overline{F}(p^u(w))]^2}\begin{bmatrix} f'(p^u(w))\Big(2f'(p^u(w))\overline{F}(p^u(w))+f^2(p^u(w))\Big) \\ -f''(p^u(w))f(p^u(w))\overline{F}(p^u(w)) \end{bmatrix}.$$

Since the term in the bracket is positive under Assumption (A2), we have $\frac{d^2p^u(w)}{dw^2}\geq 0$.

Proof of (c): We prove the unimodality of $\Pi^u_{\mathbf{MP}}(w,p^u(w))=a(w-c)\overline{F}(p^u(w))$ in w by showing (1) $\frac{d\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw}\Big|_{w=c}\geq 0$, (2)$\frac{d^2\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw^2}<0$ whenever $\frac{d\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw}=0$, and (3) $\Pi^u_{\mathbf{MP}}(w,p^u(w))\to 0$ as $w\to\infty$.

First note that the first and second partial derivatives of $\Pi^u_{\mathbf{MP}}(w,p^u(w))$ in w are

$$\frac{d\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw} = a\overline{F}(p^u(w))-a(w-c)f(p^u(w))\frac{dp^u(w)}{dw} \text{ and} \tag{B.6}$$

$$\begin{aligned}\frac{d^2\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw^2} &= -2af(p^u(w))\frac{dp^u(w)}{dw} \\ &\quad -a(w-c)\left[f(p^u(w))\frac{d^2p^u(w)}{dw^2}+f'(p^u(w))\left(\frac{dp^u(w)}{dw}\right)^2\right].\end{aligned} \tag{B.7}$$

Claim (1) follows from (B.6) while claim (3) follows from $\overline{F}(p^u(w)) \to 0$ as w and hence, $p^u(w)$ approach infinity. To show claim (2), note from (B.6) and (B.7)

$$\begin{aligned}
& \frac{d^2\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw^2}\Big|_{\frac{d\Pi^u_{\mathbf{MP}}(w,p^u(w))}{dw}=0} \\
= \; & -2af(p^u(w))\frac{dp^u(w)}{dw} \\
& -a\frac{\overline{F}(p^u(w))}{f(p^u(w))\frac{dp^u(w)}{dw}}\left[f(p^u(w))\frac{d^2p^u(w)}{dw^2}+f'(p^u(w))\left(\frac{dp^u(w)}{dw}\right)^2\right] \\
= \; & -a\frac{dp^u(w)}{dw}\left[f(p^u(w))+\frac{f'(p^u(w))\overline{F}(p^u(w))}{f(p^u(w))}\right] \\
& -af(p^u(w))\frac{dp^u(w)}{dw}-a\frac{\overline{F}(p^u(w))}{\frac{dp^u(w)}{dw}}\frac{d^2p^u(w)}{dw^2}. \qquad \text{(B.8)}
\end{aligned}$$

Since F is IFR, we have $f'(\cdot)\overline{F}(\cdot)+f^2(\cdot) \geq 0$ and the term in the bracket is positive. Since $p^u(w)$ increases in w and $\frac{d^2p^u(w)}{dw^2} \geq 0$ from part (b), all three terms are negative with the second term being strictly negative and ,thus, claim (2) follows, concluding the proof of unimodality of $\Pi^u_{\mathbf{MP}}(w,p^u(w))$ in w.

Proof of Lemma 3.4.2

Proof of (a): We prove the unimodality of $\Pi^u_{\mathbf{RN}}(q_{\min},w) = a\int_{q_{\min}}^{\infty}[(1-\beta)x+\beta q_{\min}-w-c_r-c_b]f(x)dx$ in $q_{\min}$ by showing (1) $\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}\Big|_{q_{\min}=w+c_r+c_b} \geq 0$, (2) $\frac{\partial^2\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}} < 0$ whenever $\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} = 0$, and (3) $\Pi^u_{\mathbf{RN}}(q_{\min},w) \to 0$ as $q_{\min} \to \infty$. First note that the first and second partial derivatives of $\Pi^u_{\mathbf{RN}}(q_{\min},w)$ in $q_{\min}$ are

$$\begin{aligned}
\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} &= a(-q_{\min}+w+c_r+c_b)f(q_{\min})+a\beta\overline{F}(q_{\min}) \text{ and} \qquad \text{(B.9)} \\
\frac{\partial^2\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}} &= -a(1+\beta)f(q_{\min})+a(-q_{\min}+w+c_r+c_b)f'(q_{\min}). \qquad \text{(B.10)}
\end{aligned}$$

Claim (1) follows from (B.9) while claim (3) follows from $\overline{F}(q_{\min}) \to 0$ as $q_{\min} \to \infty$. To show claim (2), note from (B.9) and (B.10)

$$\frac{\partial^2\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}}\Big|_{\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}=0} = -a\frac{(1+\beta)f^2(q_{\min})+\beta f'(q_{\min})\overline{F}(q_{\min})}{f(q_{\min})}. \qquad \text{(B.11)}$$

Since F is IFR, $f'(\cdot)\overline{F}(\cdot)+f^2(\cdot) \geq 0$. Hence claim (2) follows from (B.11), concluding the proof of unimodality of $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ in $q_{\min}$.

Proof of (b): From part (a), $q^u_{\min}(w)$ satisfies $(-q^u_{\min}(w)+w+c_r+c_b)f(q^u_{\min}(w))+\beta\overline{F}(q^u_{\min}(w))=0$. Implicit differentiation of this equality with respect to w yields

$$[(1+\beta)f(q^u_{\min}(w)) - (-q^u_{\min}(w)+w+c_r+c_b)\, f'(q^u_{\min}(w))]\frac{dq^u_{\min}(w)}{dw} = f(q^u_{\min}(w)).$$

Substituting $(-q^u_{\min}(w)+w+c_r+c_b) = \frac{-\beta\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))}$ from (B.9) in above, we obtain

$$\frac{dq^u_{\min}(w)}{dw} = \frac{f^2(q^u_{\min}(w))}{(1+\beta)f^2(q^u_{\min}(w)) + \beta f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))}, \tag{B.12}$$

Since F is IFR, we have $f'(\cdot)\overline{F}(\cdot)+f^2(\cdot)\geq 0$, which implies $\frac{dq^u_{\min}(w)}{dw} > 0$. Thus, $q^u_{\min}(w)$ strictly increases in w.

To prove $q^u_{\min}(w)$ is convex in w, we show $\frac{d^2q^u_{\min}(w)}{dw^2} \geq 0$. Take the second derivative of (B.12) with respect to w, we obtain

$$\begin{aligned}\frac{d^2q^u_{\min}(w)}{dw^2} \;=\; & \frac{\beta f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}}{[(1+\beta)f^2(q^u_{\min}(w)) + \beta f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))]^2} \times \\ & \left[\begin{array}{c} f'(q^u_{\min}(w))\big(2f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w)) + f^2(q^u_{\min}(w))\big) \\ -f''(q^u_{\min}(w))f(q^u_{\min}(w))\overline{F}(q^u_{\min}(w)) \end{array}\right].\end{aligned}$$

Since the term in the bracket is positive under Assumption (A2), we have $\frac{d^2q^u_{\min}(w)}{dw^2} \geq 0$.

Proof of (c): We prove the unimodality of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) = a(w-c)\overline{F}(q^u_{\min}(w))$ in w by showing (1) $\left.\frac{d\Pi^u_{\mathbf{MN}}(w,q^u_{\min}(w))}{dw}\right|_{w=c} \geq 0$, (2) $\frac{d^2\Pi^u_{\mathbf{MN}}(w,q^u_{\min}(w))}{dw^2} < 0$ whenever $\frac{d\Pi^u_{\mathbf{MN}}(w,q^u_{\min}(w))}{dw} = 0$, and (3) $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \to 0$ as $w \to \infty$.

First note that the first and second partial derivatives of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ in w are

$$\begin{aligned}
\frac{d\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))}{dw} &= a\overline{F}(q^u_{\min}(w)) - a(w-c)f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} \quad \text{and} \quad \text{(B.13)} \\
\frac{d^2\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))}{dw^2} &= -2af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} - a(w-c) \times \\
&\quad \left[f(q^u_{\min}(w))\frac{d^2q^u_{\min}(w)}{dw^2} + f'(q^u_{\min}(w)) \left(\frac{dq^u_{\min}(w)}{dw} \right)^2 \right]. \quad \text{(B.14)}
\end{aligned}$$

Claim (1) follows from (B.13) while claim (3) follows from $\overline{F}(q^u_{\min}(w)) \to 0$ as w, and hence $q^u_{\min}(w)$ approach infinity. To show claim (2), note from (B.13) and (B.14)

$$\begin{aligned}
& \left. \frac{d^2\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))}{dw^2} \right|_{\frac{d\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))}{dw}=0} \\
= \; & -2af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} \\
& -a\frac{\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}} \left[f(q^u_{\min}(w))\frac{d^2q^u_{\min}(w)}{dw^2} + f'(q^u_{\min}(w)) \left(\frac{dq^u_{\min}(w)}{dw} \right)^2 \right] \\
= \; & -a\frac{dq^u_{\min}(w)}{dw} \left[f(q^u_{\min}(w)) + \frac{f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))} \right] \\
& -af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} - a\frac{\overline{F}(q^u_{\min}(w))}{\frac{dq^u_{\min}(w)}{dw}}\frac{d^2q^u_{\min}(w)}{dw^2}. \quad \text{(B.15)}
\end{aligned}$$

Since F is IFR, $f'(\cdot)\overline{F}(\cdot) + f^2(\cdot) \geq 0$ and the term in the bracket is positive. Since $q^u_{\min}(w)$ increases in w and $\frac{d^2q^u_{\min}(w)}{dw^2} > 0$ from part (b), all three terms are negative with the second term being strictly negative and, thus, claim (2) follows, concluding the proof of unimodality of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ in w.

Proof of Lemma 3.4.3

Let $q^u_{\min}(w, c_r, c_b)$ and $w^u_{\mathbf{N}}(c_r, c_b)$ be the optimal cut-off valuation and the optimal wholesale price at a given w, c_r, c_b. Consider two pairs (c'_r, c'_b) and (c^o_r, c^o_b) such that $c'_r + c'_b = c^o_r + c^o_b = c_{\mathbf{T}}$ for some $c_{\mathbf{T}}$. From (B.9), for a given w, $q^u_{\min}(w, c'_r, c'_b) = q^u_{\min}(w, c^o_r, c^o_b)$. Also by substituting (B.12) into (B.13), we notice that the optimal

wholesale prices are the same for (c'_r, c'_b) and (c^o_r, c^o_b), i.e., $w^u_{\mathbf{N}}(c'_r, c'_b) = w^u_{\mathbf{N}}(c^o_r, c^o_b)$.

Proof of Proposition 3.4.1

Define $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$. We apply Lemma B.1.1 to prove the proposition. From Lemma B.1.1(a)(b), if $\Delta^u_{\mathbf{R}}(w)$ does not change sign, then one of two cases must be true:

(1) $\Delta^u_{\mathbf{R}}(w) < 0$ for all $w \geq c$ and the retailer prefers negotiation and chooses $q^u_{\min}(w)$ regardless of w (corresponding to part (b) of the proposition), or

(2) $\Delta^u_{\mathbf{R}}(w) \geq 0$ for all $w \geq c$ and the retailer (weakly) prefers posted pricing and chooses $p^u(w)$ regardless of w (corresponding to part (a) of the proposition).

On the other hand, if $\Delta^u_{\mathbf{R}}(w)$ changes sign, there exists a unique $\widehat{w}^u_{\mathbf{R}}$, $\Delta^u_{\mathbf{R}}(\widehat{w}^u_{\mathbf{R}}) = 0$, such that $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$ (retailer uses negotiation and chooses $q^u_{\min}(w)$), and $\Delta^u_{\mathbf{R}}(w) > 0$ for $w > \widehat{w}^u_{\mathbf{R}}$ (retailer uses posted pricing and chooses $p^u(w)$), which corresponds to part (c) of the proposition.

Proof of Proposition 3.4.2

Define $\Delta^u_{\mathbf{M}}(w) = \Pi^u_{\mathbf{MP}}(w, p^u(w)) - \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ and $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$. From the definition of $\widehat{w}^u_{\mathbf{R}}$, $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$ and $\Delta^u_{\mathbf{R}}(w) > 0$ for $w > \widehat{w}^u_{\mathbf{R}}$. Thus, $\Delta^u_{\mathbf{R}}(w)$ changes sign at $w = \widehat{w}^u_{\mathbf{R}}$. Then, from Lemma B.1.1(c), there exists a unique $\widehat{w}^u_{\mathbf{M}} \geq \widehat{w}^u_{\mathbf{R}}$ such that $\Delta^u_{\mathbf{M}}(w) \leq 0$ for $w \leq \widehat{w}^u_{\mathbf{M}}$ and $\Delta^u_{\mathbf{M}}(w) \geq 0$ for $w \geq \widehat{w}^u_{\mathbf{M}}$. That is, $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \geq \Pi^u_{\mathbf{MP}}(w, p^u(w))$ for $w \leq \widehat{w}^u_{\mathbf{M}}$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \leq \Pi^u_{\mathbf{MP}}(w, p^u(w))$ for $w \geq \widehat{w}^u_{\mathbf{M}}$. Thus, the result directly follows from $\widehat{w}^u_{\mathbf{R}} \leq \widehat{w}^u_{\mathbf{M}}$.

Proof of Proposition 3.4.3

First, observe from (3.14) that when the retailer chooses negotiation at all whole-

sale prices $w \geq c$, the manufacturer's optimal wholesale price is given by w^u_{N}, the maximizer of $\Pi^u_{\text{MN}}(w, q^u_{\min}(w))$, and the retailer picks the cut-off valuation $q^u_{\min}(w^u_{\text{N}})$. Likewise, from (3.13), we observe that when the retailer chooses posted pricing at all wholesale prices $w \geq c$, the manufacturer's optimal wholesale price is given by w^u_{P}, the maximizer of $\Pi^u_{\text{MP}}(w, p^u(w))$, and the retailer picks the price $p^u(w^u_{\text{P}})$.

We now focus on the case where there exists $\widehat{w}^u_{\text{R}}$ such that the retailer chooses negotiation when $w \leq \widehat{w}^u_{\text{R}}$ and posted pricing when $w > \widehat{w}^u_{\text{R}}$. For the purposes of this proof, temporarily define

$$G_{\text{N}} := \max_{c \leq w \leq \widehat{w}^u_{\text{R}}} \Pi^u_{\text{MN}}(w, q^u_{\min}(w)) \text{ and } w^o_{\text{N}} = \arg\max_{c \leq w \leq \widehat{w}^u_{\text{R}}} \Pi^u_{\text{MN}}(w, q^u_{\min}(w))$$

$$G_{\text{P}} := \sup_{w > \widehat{w}^u_{\text{R}}, w \geq c} \Pi^u_{\text{MP}}(w, p^u(w)) \text{ and } w^o_{\text{P}} = \arg\sup_{w > \widehat{w}^u_{\text{R}}, w \geq c} \Pi^u_{\text{MP}}(w, p^u(w))$$

With these definitions, observe that the manufacturer's problem of choosing the wholesale price, given by (3.15), reduces to picking the wholesale price w^o_{N} if $G_{\text{N}} \geq G_{\text{P}}$ or the wholesale price w^o_{P} if $G_{\text{N}} < G_{\text{P}}$. Consider two cases: (1) $G_{\text{N}} \geq G_{\text{P}}$ and (2) $G_{\text{N}} < G_{\text{P}}$.

(1) $G_{\text{N}} \geq G_{\text{P}}$

The manufacturer's optimal wholesale price is w^o_{N}. Lemma B.1.2(a) shows that the maximizer w^o_{N} is given by $\min\{\widehat{w}^u_{\text{R}}, w^u_{\text{N}}\}$. At wholesale price w^o_{N}, the retailer will choose negotiation. This case corresponds to parts (a) and (b) of the proposition.

(2) $G_{\text{N}} < G_{\text{P}}$

We will first prove that $w^u_{\text{P}} > \widehat{w}^u_{\text{R}}$. The proof is by contradiction. Suppose $G_{\text{N}} < G_{\text{P}}$, but $w^u_{\text{P}} \leq \widehat{w}^u_{\text{R}}$. In such a case, since $\Pi^u_{\text{MP}}(w, p^u(w))$ is unimodal in w, w^o_{P} is given by $\widehat{w}^u_{\text{R}}$, and hence, $G_{\text{P}} = \Pi^u_{\text{MP}}(\widehat{w}^u_{\text{R}}, p^u(\widehat{w}^u_{\text{R}}))$. We know that $G_{\text{P}} = \Pi^u_{\text{MP}}(\widehat{w}^u_{\text{R}}, p^u(\widehat{w}^u_{\text{R}})) \leq \Pi^u_{\text{MN}}(\widehat{w}^u_{\text{R}}, q^u_{\min}(\widehat{w}^u_{\text{R}})) \leq G_{\text{N}}$, where the first inequality is by Proposition 3.4.2 and the second comes from the definition of G_{N}. Hence, $G_{\text{P}} \leq G_{\text{N}}$, which is a contradiction

to the assumption that $G_{\mathbf{N}} < G_{\mathbf{P}}$.

Now that we have shown $w^u_{\mathbf{P}} > \widehat{w}^u_{\mathbf{R}}$, it follows from the unimodality of $\Pi^u_{\mathbf{MP}}(w, p^u(w))$ in w that $w^o_{\mathbf{P}} = w^u_{\mathbf{P}}$ (hence, sup can be replaced by max). At wholesale price $w^u_{\mathbf{P}}$, the retailer chooses posted pricing. This corresponds to part (c) of the proposition.

Proof of Proposition 3.4.4

We first show the existence of $\overline{c_{\mathbf{T}}}$. If $c_{\mathbf{T}} = 0$, notice from (3.12) that negotiation is better for the retailer regardless of the wholesale price. On the other hand, if $c_{\mathbf{T}} > E_r[r]$ (the total cost of negotiation is larger than the expected valuation), it can be shown from (3.12) that posted pricing is better for the retailer regardless of the wholesale price. Thus, at any given wholesale price, the retailer's sales format choice switches from negotiation to posted pricing at least once as $c_{\mathbf{T}}$ changes from 0 to $E_r[r]$, and $\overline{c_{\mathbf{T}}}$ exists.

Next, we will prove that if the equilibrium sales format is posted pricing at a given $c_{\mathbf{T}}$, then the equilibrium sales format is still posted pricing at higher $c_{\mathbf{T}}$. If this result holds, once the equilibrium sales format becomes posted pricing, it will never switch back to negotiation as $c_{\mathbf{T}}$ increases. We will then conclude that there exists a unique $\overline{c_{\mathbf{T}}}$ such that the equilibrium sales format is negotiation for $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$ and posted pricing for $c_{\mathbf{T}} \geq \overline{c_{\mathbf{T}}}$.

Suppose posted pricing is chosen in equilibrium at a given $c_{\mathbf{T}} = c^o_{\mathbf{T}}$. It must be that the equilibrium wholesale price is $w^u_{\mathbf{P}}$ (from Proposition 3.4.3(c)). We will divide the proof into two cases, depending on whether $\widehat{w}^u_{\mathbf{R}}$ exists at $c^o_{\mathbf{T}}$. The two cases are: (1) there exists $\widehat{w}^u_{\mathbf{R}} > c$, and (2) there does not exist $\widehat{w}^u_{\mathbf{R}}$ and the retailer chooses posted pricing for any $w \geq c$.

(1) $\widehat{w}^u_{\mathbf{R}} > c$

For the purposes of this proof, temporarily define, for a given $c_{\mathbf{T}}$:

$$G_{\mathbf{N}}(c_{\mathbf{T}}) = \max_{c \le w \le \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \text{ and } w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \arg \max_{c \le w \le \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$$

$$G_{\mathbf{P}}(c_{\mathbf{T}}) = \sup_{w > \widehat{w}^u_{\mathbf{R}}, w \ge c} \Pi^u_{\mathbf{MP}}(w, p^u(w)) \text{ and } w^o_{\mathbf{P}}(c_{\mathbf{T}}) = \arg \sup_{w > \widehat{w}^u_{\mathbf{R}}, w \ge c} \Pi^u_{\mathbf{MP}}(w, p^u(w))$$

With these definitions, the manufacturer's problem, given by (3.15), results in the optimal wholesale price $w^o_{\mathbf{N}}(c_{\mathbf{T}})$ if $G_{\mathbf{N}}(c_{\mathbf{T}}) \ge G_{\mathbf{P}}(c_{\mathbf{T}})$ or the optimal wholesale price $w^o_{\mathbf{P}}(c_{\mathbf{T}})$ if $G_{\mathbf{N}}(c_{\mathbf{T}}) < G_{\mathbf{P}}(c_{\mathbf{T}})$. Since the equilibrium sales format is posted pricing at $c^o_{\mathbf{T}}$, it must be that $G_{\mathbf{N}}(c^o_{\mathbf{T}}) < G_{\mathbf{P}}(c^o_{\mathbf{T}})$. Suppose we increase $c_{\mathbf{T}}$ marginally to $c^o_{\mathbf{T}} + \delta$ for some arbitrarily small $\delta > 0$. First, note from Lemma B.1.2(b) that $\widehat{w}^u_{\mathbf{R}}$ decreases in $c_{\mathbf{T}}$. Therefore, the feasible region of the optimization problem that determines $G_{\mathbf{P}}$ becomes larger when $c_{\mathbf{T}}$ increases. Furthermore, at a given w, $\Pi^u_{\mathbf{MP}}(w, p^u(w))$ is constant with respect to $c_{\mathbf{T}}$. Therefore, $G_{\mathbf{P}}$ is the optimal value of an objective function that itself does not depend on $c_{\mathbf{T}}$. Combining these two observations, we conclude that $G_{\mathbf{P}}(c^o_{\mathbf{T}} + \delta) \ge G_{\mathbf{P}}(c^o_{\mathbf{T}})$.

On the other hand, when $c_{\mathbf{T}}$ increases, Lemma B.1.2(e) shows that $G_{\mathbf{N}}(c_{\mathbf{T}})$ decreases. Hence, $G_{\mathbf{N}}(c^o_{\mathbf{T}} + \delta) \le G_{\mathbf{N}}(c^o_{\mathbf{T}})$. Therefore:

$$G_{\mathbf{N}}(c^o_{\mathbf{T}} + \delta) \le G_{\mathbf{N}}(c^o_{\mathbf{T}}) < G_{\mathbf{P}}(c^o_{\mathbf{T}}) \le G_{\mathbf{P}}(c^o_{\mathbf{T}} + \delta),$$

and the manufacturer will choose to induce posted pricing at $c^o_{\mathbf{T}} + \delta$.

(2) $\widehat{w}^u_{\mathbf{R}}$ does not exist

In this case, at $c^o_{\mathbf{T}}$, the retailer is choosing posted pricing for any $w \ge c$, that is, $\Pi^u_{\mathbf{RP}}(p^u(w), w) \ge \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ for $w \ge c$. We observe that $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ is constant with respect to $c_{\mathbf{T}}$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ decreases in $c_{\mathbf{T}}$. Hence, at $c_{\mathbf{T}} > c^o_{\mathbf{T}}$, we continue to have $\Pi^u_{\mathbf{RP}}(p^u(w), w) \ge \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ for $w \ge c$, and the retailer will choose posted pricing no matter what the wholesale price is.

Combining cases (1) and (2), we conclude that if the equilibrium sales format is posted pricing at a given $c_{\mathbf{T}}$, then the equilibrium sales format is still posted pricing at higher $c_{\mathbf{T}}$. Hence, there exists a unique $\overline{c_{\mathbf{T}}}$ such that the equilibrium sales format is negotiation for $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$ and posted pricing for $c_{\mathbf{T}} \geq \overline{c_{\mathbf{T}}}$.

It remains to show that $\underline{c_{\mathbf{T}}}$ exists and separates the regions where the equilibrium wholesale price is $w^u_{\mathbf{N}}$ versus $\widehat{w}^u_{\mathbf{R}}$. We know from the preceding discussion that there exists $\overline{c_{\mathbf{T}}}$ such that negotiation is the equilibrium for $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$ and posted pricing is the equilibrium for $c_{\mathbf{T}} \geq \overline{c_{\mathbf{T}}}$. Let us now focus on the region $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$. For any $c_{\mathbf{T}}$ in this region, we know from Proposition 3.4.3 that the equilibrium wholesale price must be either $\widehat{w}^u_{\mathbf{R}}$ or $w^u_{\mathbf{N}}$. Consider two cases:

(1) There does not exist $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$ such that the equilibrium wholesale price is $\widehat{w}^u_{\mathbf{R}}$

In this case, it must be that the equilibrium wholesale price is $w^u_{\mathbf{N}}$ for any $c_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$, in which case we have $\underline{c_{\mathbf{T}}} = \overline{c_{\mathbf{T}}}$.

(2)There exists $\tilde{c}_{\mathbf{T}} \in [0, \overline{c_{\mathbf{T}}})$ such that the equilibrium wholesale price is $\widehat{w}^u_{\mathbf{R}}$ at $\tilde{c}_{\mathbf{T}}$

From Lemma B.1.2(d), for any $c_{\mathbf{T}} \in [\tilde{c}_{\mathbf{T}}, \overline{c_{\mathbf{T}}})$, the manufacturer would choose $\widehat{w}^u_{\mathbf{R}}$. Hence, there exists $\underline{c_{\mathbf{T}}}$, given by the lowest such $\tilde{c}_{\mathbf{T}}$, and the equilibrium wholesale price is $\widehat{w}^u_{\mathbf{R}}$ for any $c_{\mathbf{T}} \in [\underline{c_{\mathbf{T}}}, \overline{c_{\mathbf{T}}})$.

Lemma B.1.1. *Given the wholesale price w, let $\Delta^u_{\mathbf{R}}(w)$ be the difference between the retailer's optimal profits under posted pricing and negotiation, that is, $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$, and $\Delta^u_{\mathbf{M}}(w)$ be the difference between the manufacturer's profits under posted pricing and negotiation, that is, $\Delta^u_{\mathbf{M}}(w) = \Pi^u_{\mathbf{MP}}(w, p^u(w)) - \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$. Then:*

(a) If $\Delta^u_{\mathbf{R}}(c) \geq 0$, then $\Delta^u_{\mathbf{R}}(w) \geq 0$ for all $w \geq c$

(*b*) *If* $\Delta^u_{\mathtt{R}}(c) < 0$*, then either:*

(i) $\Delta^u_{\mathtt{R}}(w) < 0$ *for all* $w \geq c$ *and* $\Delta^u_{\mathtt{R}}(w)$ *is strictly increasing in* w*, or*

(ii) $\Delta^u_{\mathtt{R}}(w)$ *is strictly unimodal and changes sign once. If* $\Delta^u_{\mathtt{R}}(w)$ *changes sign, it crosses zero at a unique* $w = \widehat{w}^u_{\mathtt{R}}$ *such that* $\Delta^u_{\mathtt{R}}(w) < 0$ *for* $w < \widehat{w}^u_{\mathtt{R}}$ *and* $\Delta^u_{\mathtt{R}}(w) \geq 0$ *for* $w \geq \widehat{w}^u_{\mathtt{R}}$*.*

(*c*) *If* $\Delta^u_{\mathtt{R}}(w)$ *changes sign at* $w = \widehat{w}^u_{\mathtt{R}}$ *, there must exist a unique* $\widehat{w}^u_{\mathtt{M}} \geq \widehat{w}^u_{\mathtt{R}}$ *such that* $\Delta^u_{\mathtt{M}}(w) \leq 0$ *for* $w \leq \widehat{w}^u_{\mathtt{M}}$*, and* $\Delta^u_{\mathtt{M}}(w) \geq 0$ *for* $w \geq \widehat{w}^u_{\mathtt{M}}$*.*

(*d*) $\Pi^u_{\mathtt{RP}}(p^u(w), w)$ *and* $\Pi^u_{\mathtt{RN}}(q^u_{\min}(w), w)$ *are convex decreasing in* w*.*

Proof of Lemma B.1.1

Proofs of (a) and (b): We prove the result by showing (1) $\Delta^u_{\mathtt{R}}(w) \to 0$ as $w \to \infty$, and (2) $\frac{d^2\Delta^u_{\mathtt{R}}(w)}{dw^2} < 0$ whenever $\frac{d\Delta^u_{\mathtt{R}}(w)}{dw} = 0$. Claim (2) implies that if a stationary point exists, it must be a maximum. The claim (2) thus implies that there exists at most one maximizer. (otherwise, there must be a minimizer between two local maxima, which contradicts the claim that all stationary points are local maxima.) If claims (1) and (2) hold, the behavior of the function $\Delta^u_{\mathtt{R}}(w)$ must follow either part (a) or part (b) of this lemma. Any other behavior would contradict (1) and/or (2), therefore cannot exist.

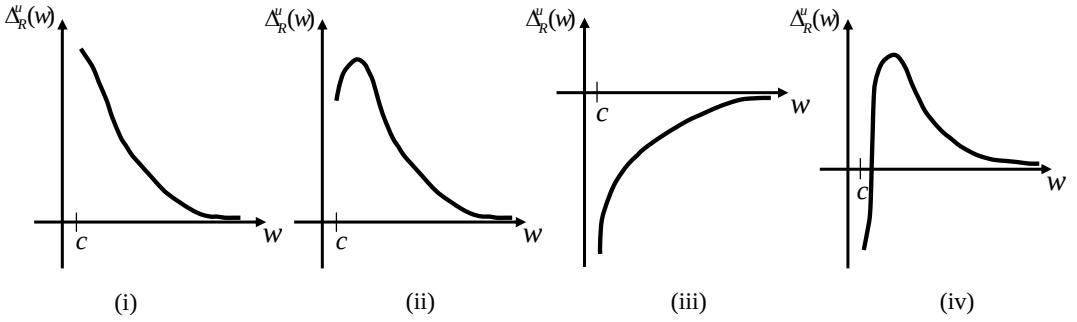

Figure B.1: The figure illustrates the possibilities discussed in parts (a) and (b) of Lemma B.1.1.

We now prove claims (1) and (2) hold. From the facts that $p^u(w) \to \infty$ and $q^u_{\min}(w) \to \infty$ as $w \to \infty$, it can be shown that $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ both approach zero as $w \to \infty$. Hence, as $w \to \infty$, $\Delta^u_{\mathbf{R}}(w)$ approaches zero, which proves (1).

To show (2), recall that $p^u(w)$ is a solution to $\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p} = 0$ and $q^u_{\min}(w)$ is a solution to $\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} = 0$, respectively. Applying the envelope theorem, we have

$$\frac{d\Pi^u_{\mathbf{RP}}(p^u(w), w)}{dw} = \left.\frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial w}\right|_{p=p^u(w)} = -a\overline{F}(p^u(w)), \text{ and} \tag{B.16}$$

$$\frac{d\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)}{dw} = \left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w)}{\partial w}\right|_{q_{\min}=q^u_{\min}(w)} = -a\overline{F}(q^u_{\min}(w)) \tag{B.17}$$

Therefore:

$$\frac{d\Delta^u_{\mathbf{R}}(w)}{dw} = -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)). \tag{B.18}$$

Let $\tilde{w}$ be a wholesale price such that $\frac{d\Delta^u_{\mathbf{R}}(\tilde{w})}{dw} = 0$. Thus, at $\tilde{w}$, we have $-a\overline{F}(p^u(\tilde{w})) = -a\overline{F}(q^u_{\min}(\tilde{w}))$ and, hence, $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$. Using the expressions for $\frac{dp^u(\tilde{w})}{dw}$ and $\frac{dq^u_{\min}(\tilde{w})}{dw}$, given by equations (B.5) and (B.12), we write:

$$\begin{aligned} \frac{dq^u_{\min}(\tilde{w})}{dw} &= \frac{f^2(q^u_{\min}(\tilde{w}))}{(1+\beta)f^2(q^u_{\min}(\tilde{w})) + \beta f'(q^u_{\min}(\tilde{w}))\overline{F}(q^u_{\min}(\tilde{w}))} \\ &> \frac{f^2(p^u(\tilde{w}))}{2f^2(p^u(\tilde{w})) + f'(p^u(\tilde{w}))\overline{F}(p^u(\tilde{w}))} = \frac{dp^u(\tilde{w})}{dw}, \end{aligned} \tag{B.19}$$

where the inequality follows from the facts that $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$, F is IFR and $0 < \beta < 1$.

Now, we can use (B.18) to write:

$$\begin{aligned} \left.\frac{d^2\Delta^u_{\mathbf{R}}(w)}{dw^2}\right|_{w=\tilde{w}} &= \left.\frac{d}{dw}\left[-a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w))\right]\right|_{w=\tilde{w}} \\ &= a\left(f(p^u(\tilde{w}))\frac{dp^u(\tilde{w})}{dw} - f(q^u_{\min}(\tilde{w}))\frac{dq^u_{\min}(\tilde{w})}{dw}\right) < 0, \end{aligned}$$

where the inequality is from $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$ and (B.19). Hence, (2) is proven, which concludes the proof of part (a) and (b).

Proof of (c): Our first goal is to prove that if $\Delta^u_{\mathbf{R}}(w)$ changes sign, then $\Delta^u_{\mathbf{M}}(w)$ changes sign exactly once by crossing zero from below. First, note that

$$\Delta^u_{\mathbf{M}}(w) = \Pi^u_{\mathbf{MP}}(w, p^u(w)) - \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) = a(w-c)\overline{F}(p^u(w)) - a(w-c)\overline{F}(q^u_{\min}(w)).$$

Hence, from (B.18), it follows that $\Delta^u_{\mathbf{M}}(w) = -(w-c)\frac{d\Delta^u_{\mathbf{R}}(w)}{dw}$. Therefore, it suffices to show that if $\Delta^u_{\mathbf{R}}(w)$ changes sign, then $\frac{d\Delta^u_{\mathbf{R}}(w)}{dw}$ changes sign exactly once by crossing zero from above. Suppose now $\Delta^u_{\mathbf{R}}(w)$ changes sign. From the discussion in parts (a) and (b), we know that we must be in case (ii) of part (b): $\Delta^u_{\mathbf{R}}(w)$ crosses zero from below and is unimodal with a peak at $w = \tilde{w}$ such that $\frac{d\Delta^u_{\mathbf{R}}(\tilde{w})}{dw} = 0$. Hence, $\frac{d\Delta^u_{\mathbf{R}}(w)}{dw}$ is positive for $w \leq \tilde{w}$ and negative for $w \geq \tilde{w}$. It now follows that $\Delta^u_{\mathbf{M}}(w)$ changes sign exactly once, and the point where it changes sign, $\widehat{w}^u_{\mathbf{M}}$, is given by $\tilde{w}$ such that $\frac{d\Delta^u_{\mathbf{R}}(\tilde{w})}{dw} = 0$. Furthermore, observe from Figure B.1 that, in case (ii) of part (b), the point at which $\Delta^u_{\mathbf{R}}(w)$ changes sign, $\widehat{w}^u_{\mathbf{R}}$, must come before $\widehat{w}^u_{\mathbf{M}} = \tilde{w}$.

Proof of (d): It immediately follows from (B.16) and (B.17) that both $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ are decreasing in w. Furthermore, from (B.16) and (B.17), we obtain:

$$\frac{d^2\Pi^u_{\mathbf{RP}}(p^u(w), w)}{dw^2} = af(p^u(w))\frac{dp^u(w)}{dw}, \text{ and}$$
$$\frac{d^2\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)}{dw^2} = af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}$$

Since both $p^u(w)$ and $q^u_{\min}(w)$ increase in w (by Lemma 3.4.1 and 3.4.2, respectively), both $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ are convex in w.

Lemma B.1.2. *Let $\Delta^u_{\mathbf{R}}(w)$ be the difference between the retailer's optimal profits under posted pricing and negotiation, that is, $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$. Suppose there exists $\widehat{w}^u_{\mathbf{R}}$ such that $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$ and $\Delta^u_{\mathbf{R}}(w) \geq 0$ for $w \geq \widehat{w}^u_{\mathbf{R}}$. Consider the following optimization problem:*

$$\max_{c \leq w \leq \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \tag{B.20}$$

Let $w^o_{\mathbf{N}}(c_{\mathbf{T}})$ denote the optimal solution to (B.20) *and $G_{\mathbf{N}}(c_{\mathbf{T}})$ be the optimal value of the objective function for a given $c_{\mathbf{T}}$. Then:*

(a) $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \min\{\widehat{w}^u_{\mathbf{R}}, w^u_{\mathbf{N}}\}$.

(b) $\widehat{w}^u_{\mathbf{R}}$ *decreases in* $c_{\mathbf{T}}$. *Furthermore,* $\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < -1$.

(c) $w^u_{\mathbf{N}}$ *decreases in* $c_{\mathbf{T}}$. *Furthermore,* $-1 \leq \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$.

(d) If $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \widehat{w}^u_{\mathbf{R}}$ *for some* $c_{\mathbf{T}} = c^o_{\mathbf{T}}$, *then* $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \widehat{w}^u_{\mathbf{R}}$ *for* $c_{\mathbf{T}} > c^o_{\mathbf{T}}$.

(e) $G_{\mathbf{N}}(c_{\mathbf{T}})$ *decreases in* $c_{\mathbf{T}}$.

Proof of Lemma B.1.2

Proof of (a): Recall that $w^u_{\mathbf{N}}$ is the unconstrained maximizer of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$. Since w must be chosen in $[c, \widehat{w}^u_{\mathbf{R}}]$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ is unimodal in w (by Lemma 3.4.2), the optimal solution to (B.20) is the minimum of $w^u_{\mathbf{N}}$ and $\widehat{w}^u_{\mathbf{R}}$.

Proof of (b): To express explicit dependence, we write $q^u_{\min}(w)$, $\widehat{w}^u_{\mathbf{R}}$, and $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ as $q^u_{\min}(w, c_{\mathbf{T}})$, $\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$, and $\Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})$, respectively. Recall that, by definition of $\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$:

$$\Pi^u_{\mathbf{RN}}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}) - \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})) = 0. \tag{B.21}$$

Implicit differentiation of (B.21) with respect to $c_{\mathbf{T}}$ yields:

$$\begin{aligned}
0 &= \frac{d\Pi^u_{\mathbf{RN}}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}}),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})}{dc_{\mathbf{T}}} - \frac{d\Pi^u_{\mathbf{RP}}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}))}{dc_{\mathbf{T}}} \\
&= \frac{dq^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})}{\partial q_{\min}}\right|_{q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}}),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} \\
&\quad + \frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})}{\partial w}\right|_{q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}}),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} + \left.\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})}{\partial c_{\mathbf{T}}}\right|_{q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}}),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} \\
&\quad - \frac{dp^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial p}\right|_{p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} - \frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial w}\right|_{p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})),\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} \qquad \text{(B.22)}
\end{aligned}$$

Note that the first and fourth terms of (B.22) are zero, since $q^u_{\min}$ and p^u satisfy the first-order conditions of $\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})$ and $\Pi^u_{\mathbf{RP}}(p,w)$, respectively. Also, recall that

$$\begin{aligned}
\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}}) &= a\int_{q_{\min}}^{\infty}\left[(1-\beta)x+\beta q_{\min}-w-c_{\mathbf{T}}\right]f(x)dx, \\
\Pi^u_{\mathbf{RP}}(p,w) &= a(p-w)\overline{F}(p).
\end{aligned}$$

Taking the partial derivatives of these profit functions, we obtain:

$$\frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})}{\partial w} = -a\overline{F}(q_{\min}), \quad \frac{\partial\Pi^u_{\mathbf{RN}}(q_{\min},w,c_{\mathbf{T}})}{\partial c_{\mathbf{T}}} = -a\overline{F}(q_{\min}), \quad \text{and}$$
$$\frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial w} = -a\overline{F}(p).$$

Substituting the above partial derivatives in (B.22) and rearranging the terms, we obtain:

$$\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}}(\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})) - \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))) + \overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})) = 0.$$

Hence:

$$\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} = -\frac{\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}}))}{\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})) - \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))}.$$

To show that $\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < -1$, it suffices to show $\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}),c_{\mathbf{T}})) > \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))$. Since $\Delta^u_{\mathbf{R}}(w)$ is changing sign at $w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$, it follows from Lemma B.1.1(a) that $\Delta^u_{\mathbf{R}}(w)$ must be strictly increasing in w at $w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$. (See (iv) of Figure B.1.)

Using this fact, we obtain from (B.18) that $\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}}), c_{\mathrm{T}})) > \overline{F}(p^u(\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}})))$, which concludes the proof of (b).

Proof of (c): In preparation for the proof, we will first derive a few useful expressions. First, substituting the expression for $\frac{dq^u_{\min}(w)}{dw}$, given by (B.12), into the manufacturer's first-order condition, (B.13), and recalling that w^u_{N} is the solution to the manufacturer's first-order condition, we get the following identity:

$$\frac{\overline{F}(q^u_{\min}(w^u_{\mathrm{N}}))}{f(q^u_{\min}(w^u_{\mathrm{N}}))} - (w^u_{\mathrm{N}} - c)\frac{f^2(q^u_{\min}(w^u_{\mathrm{N}}))}{(1+\beta)f^2(q^u_{\min}(w^u_{\mathrm{N}})) + \beta f'(q^u_{\min}(w^u_{\mathrm{N}}))\overline{F}(q^u_{\min}(w^u_{\mathrm{N}}))} = 0. \tag{B.23}$$

Let $\phi(x) := \frac{f^2(x)}{(1+\beta)f^2(x)+\beta f'(x)\overline{F}(x)}$. As an aside, note that

$$\frac{d\phi(x)}{dx} = \frac{\beta f(x)[f'(x)(2f'(x)\overline{F}(x) + f^2(x)) - f''(x)f(x)\overline{F}(x)]}{[(1+\beta)f^2(x) + \beta f'(x)\overline{F}(x)]^2}. \tag{B.24}$$

We observe from (B.24) that $\phi(x)$ increases in x (since the numerator is non-negative by Assumption (A2)). Using our definition of $\phi(x)$, we can rewrite (B.23) as

$$\frac{\overline{F}(q^u_{\min}(w^u_{\mathrm{N}}))}{f(q^u_{\min}(w^u_{\mathrm{N}}))} - (w^u_{\mathrm{N}} - c)\phi(q^u_{\min}(w^u_{\mathrm{N}})) = 0. \tag{B.25}$$

Now we are ready to prove the result. Here, to make explicit the dependence on c_{T}, we write $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ instead of w^u_{N}. In addition, for notational convenience, we write $q^u_{\min}(c_{\mathrm{T}})$ to denote $q^u_{\min}(w^u_{\mathrm{N}}(c_{\mathrm{T}}))$. With these notational changes, (B.25) can be written as:

$$\frac{\overline{F}(q^u_{\min}(c_{\mathrm{T}}))}{f(q^u_{\min}(c_{\mathrm{T}}))} - (w^u_{\mathrm{N}}(c_{\mathrm{T}}) - c)\phi(q^u_{\min}(c_{\mathrm{T}})) = 0. \tag{B.26}$$

We first show that $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \times \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$, that is, when c_{T} increases, $q^u_{\min}(c_{\mathrm{T}})$ and $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ cannot both strictly increase or strictly decrease. We prove this by contradiction.

Suppose now both $q^u_{\min}(c_{\mathbf{T}})$ and $w^u_{\mathbf{N}}(c_{\mathbf{T}})$ strictly increase in $c_{\mathbf{T}}$. In such a case, if $c_{\mathbf{T}}$ increases, then $\frac{\overline{F}(q^u_{\min}(c_{\mathbf{T}}))}{f(q^u_{\min}(c_{\mathbf{T}}))}$ decreases (because F is IFR). Furthermore, $\phi(q^u_{\min}(c_{\mathbf{T}}))$ increases (because $\phi(x)$ is increasing in x, as observed earlier). Hence, the left-hand side of (B.26) is strictly decreasing in $c_{\mathbf{T}}$, which yields a contradiction since (B.26) must hold as an equality for any $c_{\mathbf{T}}$.

Next, suppose that both $q^u_{\min}(c_{\mathbf{T}})$ and $w^u_{\mathbf{N}}(c_{\mathbf{T}})$ strictly decrease in $c_{\mathbf{T}}$. Once again, we will obtain a contradiction under this supposition. In such a case, if $c_{\mathbf{T}}$ increases, then $\frac{\overline{F}(q^u_{\min}(c_{\mathbf{T}}))}{f(q^u_{\min}(c_{\mathbf{T}}))}$ increases (because F is IFR). Furthermore, $\phi(q^u_{\min}(c_{\mathbf{T}}))$ decreases (because $\phi(x)$ is increasing in x, as observed earlier). Hence, the left-hand side of (B.26) is strictly increasing in $c_{\mathbf{T}}$, which again yields a contradiction since (B.26) must hold as an equality for any $c_{\mathbf{T}}$.

It is now proven that $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \times \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$. Next, we will utilize this result to show that $-1 \leq \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$. Implicit differentiation of the retailer's first-order condition, given by (B.9), with respect to $c_{\mathbf{T}}$ yields

$$\frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} = \left[1 + \frac{\beta\big(f'(q^u_{\min}(c_{\mathbf{T}}))\overline{F}(q^u_{\min}(c_{\mathbf{T}})) + f^2(q^u_{\min}(c_{\mathbf{T}}))\big)}{f^2(q^u_{\min}(c_{\mathbf{T}}))}\right]\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} - 1, \quad \text{(B.27)}$$

where the term in the brackets is positive, because F is IFR and, hence, $f'(\cdot)\overline{F}(\cdot) + f^2(\cdot) \geq 0$. If $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < 0$, then it must be that $\frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < 0$ and we get a contradiction to $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \times \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$. Thus, it must be that $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \geq 0$. It now follows that $\frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$ (since $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \times \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \leq 0$). Furthermore, from (B.27), we observe that $\frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \geq -1$ (since the term in brackets is positive and $\frac{dq^u_{\min}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \geq 0$). This concludes the proof of part (c).

Proof of (d): From part (a), we have $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \min\{\widehat{w}^u_{\mathbf{R}}, w^u_{\mathbf{N}}\}$. Hence, if $w^o_{\mathbf{N}}(c^o_{\mathbf{T}}) = \widehat{w}^u_{\mathbf{R}}$, it must be that $\widehat{w}^u_{\mathbf{R}} \leq w^u_{\mathbf{N}}$ at $c^o_{\mathbf{T}}$. From parts (b) and (c), we know that $\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} <$

$\frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}}$. Hence, if c_{T} increases, $\widehat{w}^u_{\mathrm{R}}$ continues to be less than or equal to w^u_{N}, and $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \widehat{w}^u_{\mathrm{R}}$ continues to hold for $c_{\mathrm{T}} > c^o_{\mathrm{T}}$.

Proof of (e): We will show that for $c^o_{\mathrm{T}} < c'_{\mathrm{T}}$, $G_{\mathrm{N}}(c^o_{\mathrm{T}}) \geq G_{\mathrm{N}}(c'_{\mathrm{T}})$. In this proof, we will write $q^u_{\min}(w, c_{\mathrm{T}})$, $\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}})$ and $\Pi^u_{\mathrm{MN}}(w, q_{\min}, c_{\mathrm{T}})$ instead of, respectively, $q^u_{\min}(w)$, $\widehat{w}^u_{\mathrm{R}}$ and $\Pi^u_{\mathrm{MN}}(w, q_{\min})$, to make the dependence on c_{T} explicit. It is not difficult to check that $\Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w, c_{\mathrm{T}}), c_{\mathrm{T}})$ is decreasing in c_{T}. Hence:

$$G_{\mathrm{N}}(c'_{\mathrm{T}}) = \Pi^u_{\mathrm{MN}}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), q^u_{\min}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), c'_{\mathrm{T}}), c'_{\mathrm{T}}) \leq \Pi^u_{\mathrm{MN}}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), q^u_{\min}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), c^o_{\mathrm{T}}), c^o_{\mathrm{T}}). \tag{B.28}$$

Furthermore, note that when $c_{\mathrm{T}} = c^o_{\mathrm{T}}$, $w = w^o_{\mathrm{N}}(c'_{\mathrm{T}})$ is a feasible solution for the optimization problem in (B.20). To see why, note that $\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}})$ decreases in c_{T}. Hence, $\widehat{w}^u_{\mathrm{R}}(c^o_{\mathrm{T}}) \geq \widehat{w}^u_{\mathrm{R}}(c'_{\mathrm{T}})$. It then follows that $w^o_{\mathrm{N}}(c'_{\mathrm{T}})$, which is feasible for the problem in (B.20) when $c_{\mathrm{T}} = c'_{\mathrm{T}}$, is also feasible when $c_{\mathrm{T}} = c^o_{\mathrm{T}}$. Therefore:

$$G_{\mathrm{N}}(c^o_{\mathrm{T}}) = \Pi^u_{\mathrm{MN}}(w^o_{\mathrm{N}}(c^o_{\mathrm{T}}), q^u_{\min}(w^o_{\mathrm{N}}(c^o_{\mathrm{T}}), c^o_{\mathrm{T}}), c^o_{\mathrm{T}}) \geq \Pi^u_{\mathrm{MN}}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), q^u_{\min}(w^o_{\mathrm{N}}(c'_{\mathrm{T}}), c^o_{\mathrm{T}}), c^o_{\mathrm{T}}). \tag{B.29}$$

Combining (B.28) and (B.29), we obtain $G_{\mathrm{N}}(c^o_{\mathrm{T}}) \geq G_{\mathrm{N}}(c'_{\mathrm{T}})$.

B.2 Proofs of Lemmas and Propositions in Section 3.5

In this appendix, we prove the results stated in Section 3.5. The proofs utilize Lemmas B.2.1 through B.2.3 and Propositions B.2.1 through B.2.3, stated and proven at the end of Appendix B.2.

Proofs of Lemmas 3.5.1 and 3.5.2

Notice from (3.18) that the maximizer of $\Pi_{\text{MP}}(w, p^*(w,Q), Q)$, denoted by $w^*_{\text{P}}(Q)$, cannot be strictly less than $\overline{w}_{\text{P}}(Q)$ (since $\Pi_{\text{MP}}(w, p^*(w,Q), Q)$ is linearly increasing in w for $w \in [c, \overline{w}_{\text{P}}(Q)]$). Now, for $w \geq \overline{w}_{\text{P}}(Q)$, $\Pi_{\text{MP}}(w, p^*(w,Q), Q)$ is equal to $\Pi^u_{\text{MP}}(w, p^u(w))$, which itself is unimodal and peaks at w^u_{P} by Lemma 3.4.1. Therefore, $w^*_{\text{P}}(Q)$ is given by w^u_{P} or $\overline{w}_{\text{P}}(Q)$, whichever is larger. The same line of arguments proves Lemma 3.5.2 as well.

Proof of Proposition 3.5.1

We omit the proof of this proposition. The proof is almost identical to that of Proposition 3.4.3, once we replace $q^u_{\min}(w)$, $p^u(w)$, w^u_{N} and w^u_{P} in the earlier proof with $q^*_{\min}(w,Q) = \max\{q^u_{\min}(w), \bar{p}(Q)\}$, $p^*(w,Q) = \max\{p^u(w), \bar{p}(Q)\}$, $w^*_{\text{N}}(Q) = \max\{\overline{w}_{\text{N}}(Q), w^u_{\text{N}}\}$ and $w^*_{\text{P}}(Q) = \max\{\overline{w}_{\text{P}}(Q), w^u_{\text{P}}\}$ here.

Proof of Proposition 3.5.2

The proof proceeds in two parts, the first part showing the existence of $\underline{Q}$ and the second part showing the existence of $\overline{Q}$.

Part 1: The existence of $\underline{Q}$

We first show that if the equilibrium sales format is negotiation at a given Q, then the equilibrium sales format is still negotiation for a larger Q. This allows us to conclude that if the equilibrium sales format is negotiation at some $Q \geq 0$, then the smallest such Q yields $\underline{Q}$. Otherwise, if the equilibrium sales format is posted pricing for all $Q < \infty$, then $\underline{Q} = \infty$.

Suppose that negotiation is the equilibrium sales format at Q^o. Necessarily, the equilibrium wholesale price must be the solution to either problem (3.23) or problem (3.24) defined in Section 3.5. We consider two cases separately.

Suppose that the equilibrium wholesale price is the solution to problem (3.23) at capacity Q^o. This is the case when the retailer prefers negotiation for all $w \geq c$, in other words,

$$\Delta_R(w, Q^o) = \Pi_{\text{RP}}(p^*(w, Q^o), w, Q^o) - \Pi_{\text{RN}}(q^*_{\min}(w, Q^o), w, Q^o) \leq 0 \text{ for all } w \geq c.$$

For any $Q > Q^o$, we show that the retailer will continue to prefer negotiation for all $w \geq c$, that is, $\Delta_R(w, Q) \leq 0$ for all $w \geq c$. If this is true, the equilibrium sales format is also negotiation at $Q > Q^o$. The proof is by contradiction. Suppose that there exists $Q > Q^o$ and $w' \geq c$ such that $\Delta_R(w', Q) > 0$. Recall from Proposition B.2.1 that if the retailer prefers posted pricing at some wholesale price w', it continues to prefer posted pricing at a higher wholesale price. Furthermore, note that the sales quantity under both formats decrease and converge to zero as w increases. Hence, there must exist a $w'' \geq w'$ such that $\Delta_R(w'', Q) > 0$ and both formats sell strictly less than Q^o. However, notice that at the wholesale price w'', quantities sold under both formats will be less than Q^o for any $Q \geq Q^o$. Therefore, $\Delta_R(w'', Q^o) = \Delta_R(w'', Q) > 0$. This contradicts the fact that $\Delta_R(w, Q^o) \leq 0$ for all $w \geq c$.

Now suppose that the equilibrium wholesale price is the solution to problem

(3.24) at capacity Q^o. For the sake of exposition, we temporarily define the following functions, which correspond to the optimal solutions to the sub-problems in problem (3.24).

$$G_{\mathbf{N}}(Q) = \max_{c \le w \le \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w,Q), Q) \quad \text{and}$$

$$G_{\mathbf{P}}(Q) = \sup_{w > \widehat{w}_{\mathbf{R}}(Q), w \ge c} \Pi_{\mathbf{MP}}(w, p^*(w,Q), Q).$$

Also define

$$w^o_{\mathbf{N}}(Q) = \arg \max_{c \le w \le \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w,Q), Q) \quad \text{and}$$

$$w^o_{\mathbf{P}}(Q) = \arg \sup_{w > \widehat{w}_{\mathbf{R}}(Q), w \ge c} \Pi_{\mathbf{MP}}(w, p^*(w,Q), Q).$$

Since the equilibrium sales format is negotiation at Q^o, it must be that $G_{\mathbf{N}}(Q^o) \ge G_{\mathbf{P}}(Q^o)$. We first note that, for a capacity Q' such that $Q' > Q^o$, the equilibrium wholesale price is the solution to problem (3.24) at capacity Q'. To see why this is true, note from Lemma B.2.3(b) that $\widehat{w}_{\mathbf{R}}(Q)$ is increasing in Q. Therefore, if there exists a $\widehat{w}_{\mathbf{R}}(Q) > c$ for $Q = Q^o$ (that is, if there exists a feasible wholesale price below which the retailer strictly prefers negotiation and above which the retailer strictly prefers posted pricing), then there must exist a $\widehat{w}_{\mathbf{R}}(Q) > c$ for $Q = Q'$ as well. We will conclude the proof by showing that for $Q' > Q^o$, we have $G_{\mathbf{N}}(Q') \ge G_{\mathbf{P}}(Q')$, which implies that the equilibrium sales format will be negotiation at capacity Q'.

We next state and prove a claim that will help us complete the proof:

Claim: Whenever there exists $\widehat{w}_{\mathbf{R}}(Q) > c$*:*

$$w^o_{\mathbf{P}}(Q) = \max\{w^*_{\mathbf{P}}(Q), \widehat{w}_{\mathbf{R}}(Q)\} = \max\{w^u_{\mathbf{P}}, \widehat{w}_{\mathbf{R}}(Q)\}$$

We now prove this claim. Recall that $w^*_{\mathbf{P}}(Q) = \max\{w^u_{\mathbf{P}}, \overline{w}_{\mathbf{P}}(Q)\}$ is the maximizer of $\Pi_{\mathbf{MP}}(w, p^*(w,Q), Q)$. Observe from (3.18) that $\Pi_{\mathbf{MP}}(w, p^*(w,Q), Q)$ is unimodal

in w^1. This establishes the first equality, i.e., $w^o_{\mathrm{P}}(Q) = \max\{w^*_{\mathrm{P}}(Q), \widehat{w}_{\mathrm{R}}(Q)\}$. The second equality follows directly if there does not exist a $\overline{w}_{\mathrm{P}}(Q) \geq c$ (i.e., there does not exist a feasible wholesale price at which posted pricing is constrained by the capacity), since $w^*_{\mathrm{P}}(Q) = w^u_{\mathrm{P}}$ in that case. If there exists a $\overline{w}_{\mathrm{P}}(Q) \geq c$, then $\widehat{w}_{\mathrm{R}}(Q) > \overline{w}_{\mathrm{P}}(Q)$ by Lemma B.2.1(c) and (d) together, which allows us to conclude $w^o_{\mathrm{P}}(Q) = \max\{w^u_{\mathrm{P}}, \widehat{w}_{\mathrm{R}}(Q)\}$.

Based on the above claim, we consider two cases : (1) $w^o_{\mathrm{P}}(Q') = w^u_{\mathrm{P}}$, and (2) $w^o_{\mathrm{P}}(Q') = \widehat{w}_{\mathrm{R}}(Q')$.

(1) $w^o_{\mathrm{P}}(Q') = w^u_{\mathrm{P}}$

Since $\widehat{w}_{\mathrm{R}}(Q)$ is increasing in Q (by Lemma B.2.3(b)) and w^u_{P} does not depend on Q, the above claim implies that $w^o_{\mathrm{P}}(Q^o) = w^u_{\mathrm{P}}$ (because $w^o_{\mathrm{P}}(Q') = w^u_{\mathrm{P}}$ in the current case). Hence, $G_{\mathrm{P}}(Q') = G_{\mathrm{P}}(Q^o)$. On the other hand, $G_{\mathrm{N}}(Q)$ increases in Q by Lemma B.2.3(d). Therefore, given that $G_{\mathrm{N}}(Q^o) \geq G_{\mathrm{P}}(Q^o)$ (since negotiation is the equilibrium at capacity Q^o),

$$G_{\mathrm{N}}(Q') \geq G_{\mathrm{N}}(Q^o) \geq G_{\mathrm{P}}(Q^o) = G_{\mathrm{P}}(Q').$$

(2) $w^o_{\mathrm{P}}(Q') = \widehat{w}_{\mathrm{R}}(Q')$

In this case,

$$\begin{aligned} G_{\mathrm{N}}(Q') &\geq \Pi_{\mathrm{MN}}(\widehat{w}_{\mathrm{R}}(Q'), q^*_{\min}(\widehat{w}_{\mathrm{R}}(Q'), Q'), Q') \\ &\geq \Pi_{\mathrm{MP}}(\widehat{w}_{\mathrm{R}}(Q'), p^*(\widehat{w}_{\mathrm{R}}(Q'), Q'), Q') = G_{\mathrm{P}}(Q'). \end{aligned}$$

where the first inequality is by definition of $G_{\mathrm{N}}(Q)$ and the second inequality is by Proposition B.2.2.

[1] To see why, note from (3.18) that $\Pi_{\mathrm{MP}}(w, p^*(w,Q), Q) = (w-c)Q$ for $w \leq \overline{w}_{\mathrm{P}}(Q)$ and $\Pi_{\mathrm{MP}}(w, p^*(w,Q), Q) = \Pi^u_{\mathrm{MP}}(w, p^u(w))$ for $w \geq \overline{w}_{\mathrm{P}}(Q)$, and $\Pi^u_{\mathrm{MP}}(w, p^u(w))$ itself is unimodal as shown in Lemma 3.4.1(c).

Cases (1) and (2) together conclude that $G_{\mathrm{N}}(Q') \geq G_{\mathrm{P}}(Q')$, which concludes Part 1.

Part 2: The existence of $\overline{Q}$

Suppose now $\underline{Q} < \infty$ so that there exists a range of capacities at which negotiation is the equilibrium. We now show that $\overline{Q}$ exists and separates the regions where the equilibrium wholesale price is $w^*_{\mathrm{N}}(Q)$ versus $\widehat{w}_{\mathrm{R}}(Q)$. Let us now focus on the region $Q \geq \underline{Q}$. For any Q in this region, we know from Proposition 3.5.1 that the equilibrium wholesale price must be either $\widehat{w}_{\mathrm{R}}(Q)$ or $w^*_{\mathrm{N}}(Q)$. Note that if there exists $\tilde{Q} \geq \underline{Q}$ such that the equilibrium wholesale price is $w^*_{\mathrm{N}}(Q)$ at $\tilde{Q}$, then from Lemma B.2.3(c), the manufacturer would choose $w^*_{\mathrm{N}}(Q)$ for any $Q > \tilde{Q}$. Hence, there exists $\overline{Q}$, given by the smallest such $\tilde{Q}$, and the equilibrium wholesale price is $w^*_{\mathrm{N}}(Q)$ for any $Q \geq \overline{Q}$. We observe that it is possible that $\overline{Q} = \underline{Q}$, in which case the equilibrium wholesale price is never $\widehat{w}_{\mathrm{R}}(Q)$. On the other hand, if there does not exist $Q \geq \underline{Q}$ such that the equilibrium wholesale price is $w^*_{\mathrm{N}}(Q)$, then it must be that the equilibrium wholesale price is $\widehat{w}_{\mathrm{R}}(Q)$ for any $Q \geq \underline{Q}$.

Lemma B.2.1. *For a given wholesale price w and capacity Q, let $\Delta_{\mathrm{R}}(w, Q)$ be the difference between the retailer's optimal profits under posted pricing and negotiation, that is, $\Delta_{\mathrm{R}}(w, Q) = \Pi_{\mathrm{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathrm{RN}}(q^*_{\min}(w, Q), w, Q)$. Then,*

(a) If $\Delta_{\mathrm{R}}(c, Q) \geq 0$, then $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for all $w \geq c$

(b) If $\Delta_{\mathrm{R}}(c, Q) < 0$, then either:

(i) $\Delta_{\mathrm{R}}(w, Q) < 0$ for all $w \geq c$, or

(ii) There exists a unique $\widehat{w}_{\mathrm{R}}(Q)$ such that $\Delta_{\mathrm{R}}(w, Q) < 0$ for $w < \widehat{w}_{\mathrm{R}}(Q)$ and $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for $w \geq \widehat{w}_{\mathrm{R}}(Q)$. In other words, $\Delta_{\mathrm{R}}(w, Q)$ crosses zero only

once at $\widehat{w}_{\mathrm{R}}(Q) > c$.

(*c*) *For a given* Q*, suppose that there exists* $\overline{w}_{\mathrm{P}}(Q) \geq c$ *but there does not exists* $\overline{w}_{\mathrm{N}}(Q) \geq c$*. Then,* $\Delta_{\mathrm{R}}(w, Q) \geq 0$ *for all* $w \geq c$.

(*d*) *For a given* Q*, suppose that there exist* $\overline{w}_{\mathrm{P}}(Q) \geq c$*,* $\overline{w}_{\mathrm{N}}(Q) \geq c$*,* $\widehat{w}_{\mathrm{R}}(Q) > c$*. It must be that* $\overline{w}_{\mathrm{N}}(Q) > \overline{w}_{\mathrm{P}}(Q)$ *and* $\widehat{w}_{\mathrm{R}}(Q) > \overline{w}_{\mathrm{P}}(Q)$.

Proof of Lemma B.2.1

In this proof, we omit Q whenever the dependence is obvious. For example, we write $p^*(w)$ instead of $p^*(w, Q)$ and $q^*_{\min}(w)$ instead of $q^*_{\min}(w, Q)$.

Proofs of (a) and (b): We consider four cases depending on whether there exist $\overline{w}_{\mathrm{N}}(Q) \geq c$ and/or $\overline{w}_{\mathrm{P}}(Q) \geq c$, that is, whether there exists a feasible wholesale price (i.e. greater than or equal to c) at which the quantity sold under negotiation and/or posted pricing is not bounded by capacity. These four cases are: (1) neither $\overline{w}_{\mathrm{N}}(Q)$ nor $\overline{w}_{\mathrm{P}}(Q)$ exists, (2) both $\overline{w}_{\mathrm{N}}(Q)$ and $\overline{w}_{\mathrm{P}}(Q)$ exist, (3) only $\overline{w}_{\mathrm{P}}(Q)$ exists, and (4) only $\overline{w}_{\mathrm{N}}(Q)$ exists.

Case (1) In case (1), the retailer and manufacturer's profits are never bounded by capacity. Hence, the problem collapses to the uncapacitated one, for which Lemma B.1.1(a)(b) shows the desired result.

Case (2) We will divide the proof of case (2) into three mutually exclusive subcases: (2.a) $\overline{w}_{\mathrm{P}} \geq \overline{w}_{\mathrm{N}}$, (2.b) $\overline{w}_{\mathrm{P}} < \overline{w}_{\mathrm{N}}$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) \geq 0$, and (2.c) $\overline{w}_{\mathrm{P}} < \overline{w}_{\mathrm{N}}$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) < 0$. As we will prove next, in subcases (2.a) and (2.b), part (a) of this lemma holds. In subcase (2.c), part (b) of this lemma holds.

(2.a) $\overline{w}_{\mathrm{P}} \geq \overline{w}_{\mathrm{N}}$

Since $\overline{w}_{\mathrm{P}} \geq \overline{w}_{\mathrm{N}} \geq c$, applying equations (3.17) and (3.20), we have

$$\begin{aligned}\Delta_{\mathrm{R}}(w,Q) &= \Pi_{\mathrm{RP}}(p^*(w),w) - \Pi_{\mathrm{RN}}(q^*_{\min}(w),w) \\ &= \begin{cases} \Pi_{\mathrm{RP}}(\bar{p},w) - \Pi_{\mathrm{RN}}(\bar{p},w) & \text{for } w \in [c,\overline{w}_{\mathrm{N}}], \\ \Pi_{\mathrm{RP}}(\bar{p},w) - \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w) & \text{for } w \in [\overline{w}_{\mathrm{N}},\overline{w}_{\mathrm{P}}], \\ \Pi^u_{\mathrm{RP}}(p^u(w),w) - \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w) & \text{for } w \geq \overline{w}_{\mathrm{P}}. \end{cases}\end{aligned} \tag{B.30}$$

From the definitions of $\overline{w}_{\mathrm{N}}$, $\overline{w}_{\mathrm{P}}$ and $\bar{p}$, it can be shown that $\Delta_{\mathrm{R}}(w,Q)$ is continuous and differentiable in w. To help with the proof, we substitute from (3.17) and (3.20) into (B.30), and take the derivative to obtain

$$\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = \begin{cases} 0 & \text{for } w \in [c,\overline{w}_{\mathrm{N}}], \\ -Q + a\overline{F}(q^u_{\min}(w)) & \text{for } w \in [\overline{w}_{\mathrm{N}},\overline{w}_{\mathrm{P}}], \\ -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\mathrm{P}}. \end{cases} \tag{B.31}$$

Notice that $a\overline{F}(q^u_{\min}(w)) \leq Q$ for $w \in [\overline{w}_{\mathrm{N}},\overline{w}_{\mathrm{P}}]$ (since $q^u_{\min}(w) \geq q^u_{\min}(\overline{w}_{\mathrm{N}}) = \bar{p}(Q)$ for $w \geq \overline{w}_{\mathrm{N}}$). Therefore, we observe from (B.31) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} \leq 0$ for $w \in [c,\overline{w}_{\mathrm{P}}]$.

First, we show that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for any $w \in [c,\overline{w}_{\mathrm{P}}]$. We prove this by contradiction. Suppose there exists some $w^o \leq \overline{w}_{\mathrm{P}}$ such that $\Delta_{\mathrm{R}}(w^o,Q) < 0$. Since $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} \leq 0$ for $w \in [c,\overline{w}_{\mathrm{P}}]$, it must be that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}},Q) < 0$. Notice from (B.30) that, for $w \geq \overline{w}_{\mathrm{P}}$, capacity is no longer binding and the retailer's profits under both sales formats are given by the profits in the uncapacitated problem: $\Delta_{\mathrm{R}}(w,Q) = \Delta^u_{\mathrm{R}}(w)$ for $w \geq \overline{w}_{\mathrm{P}}$. Combining the facts above, we must have

$$\Delta^u_{\mathrm{R}}(\overline{w}_{\mathrm{P}}) = \Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}},Q) < 0 \text{ and } \left.\frac{d\Delta^u_{\mathrm{R}}(w)}{dw}\right|_{w=\overline{w}_{\mathrm{P}}} = \left.\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw}\right|_{w=\overline{w}_{\mathrm{P}}} \leq 0.$$

However, this contradicts Lemma B.1.1(b) as the function $\Delta^u_{\text{R}}(w)$ cannot be (weakly) decreasing at a w where it is strictly negative. Therefore, $\Delta_{\text{R}}(w, Q) \geq 0$ for any $w \in [c, \overline{w}_{\text{P}}]$.

It remains to show that $\Delta_{\text{R}}(w, Q) \geq 0$ for $w > \overline{w}_{\text{P}}$. Recall that for $w \geq \overline{w}_{\text{P}}$, $\Delta_{\text{R}}(w, Q) = \Delta^u_{\text{R}}(w)$ and we have shown above that $\Delta^u_{\text{R}}(\overline{w}_{\text{P}}) \geq 0$. Lemma B.1.1(a) and (b) together imply that once $\Delta^u_{\text{R}}(w)$ is positive for some w, $\Delta^u_{\text{R}}(w)$ remains positive for any larger w. Therefore, $\Delta_{\text{R}}(w, Q) = \Delta^u_{\text{R}}(w) \geq 0$ for $w > \overline{w}_{\text{P}}$.

(2.b) $\overline{w}_{\text{P}} < \overline{w}_{\text{N}}$ and $\Delta_{\text{R}}(\overline{w}_{\text{P}}, Q) \geq 0$

Since $\overline{w}_{\text{P}} < \overline{w}_{\text{N}}$, applying equations (3.17) and (3.20), we have

$$\begin{aligned}\Delta_{\text{R}}(w, Q) &= \Pi_{\text{RP}}(p^*(w), w) - \Pi_{\text{RN}}(q^*_{\min}(w), w) \\ &= \begin{cases} \Pi_{\text{RP}}(\bar{p}, w) - \Pi_{\text{RN}}(\bar{p}, w) & \text{for } w \in [c, \overline{w}_{\text{P}}], \\ \Pi^u_{\text{RP}}(p^u(w), w) - \Pi_{\text{RN}}(\bar{p}, w) & \text{for } w \in [\overline{w}_{\text{P}}, \overline{w}_{\text{N}}], \\ \Pi^u_{\text{RP}}(p^u(w), w) - \Pi^u_{\text{RN}}(q^u_{\min}(w), w) & \text{for } w \geq \overline{w}_{\text{N}}. \end{cases}\end{aligned} \tag{B.32}$$

From the definitions of $\overline{w}_{\text{N}}$, $\overline{w}_{\text{P}}$ and $\bar{p}$, it can be shown that $\Delta_{\text{R}}(w, Q)$ is differentiable in w. To help with the proof, we substitute from (3.17) and (3.20) into (B.32), and take the derivative to obtain

$$\frac{d\Delta_{\text{R}}(w, Q)}{dw} = \begin{cases} 0 & \text{for } w \in [c, \overline{w}_{\text{P}}], \\ -a\overline{F}(p^u(w)) + Q & \text{for } w \in [\overline{w}_{\text{P}}, \overline{w}_{\text{N}}], \\ -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\text{N}}. \end{cases} \tag{B.33}$$

First, observe from (B.33) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathrm{P}}]$. Hence, given our assumption that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) \geq 0$, it follows that $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for $w \in [c, \overline{w}_{\mathrm{P}}]$.

Second, notice that $a\overline{F}(p^u(w)) \leq Q$ for $w \in [\overline{w}_{\mathrm{P}}, \overline{w}_{\mathrm{N}}]$ (since $p^u(w) \geq p^u(\overline{w}_{\mathrm{P}}) = \bar{p}(Q)$ for $w \geq \overline{w}_{\mathrm{P}}$). Therefore, we observe from (B.33) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} \geq 0$ for $w \in [c, \overline{w}_{\mathrm{N}}]$. Since $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) \geq 0$ by assumption, it follows that $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for all $w \in [c, \overline{w}_{\mathrm{N}}]$.

It remains to show that $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for $w > \overline{w}_{\mathrm{N}}$. Notice from (B.32) that for $w \geq \overline{w}_{\mathrm{N}}$, $\Delta_{\mathrm{R}}(w, Q) = \Delta^u_{\mathrm{R}}(w)$. We have shown above that $\Delta^u_{\mathrm{R}}(\overline{w}_{\mathrm{N}}) \geq 0$. Lemma B.1.1(a) and (b) together imply that once $\Delta^u_{\mathrm{R}}(w)$ is positive for some w, $\Delta^u_{\mathrm{R}}(w)$ remains positive for any larger w. Therefore, $\Delta_{\mathrm{R}}(w, Q) = \Delta^u_{\mathrm{R}}(w) \geq 0$ for $w > \overline{w}_{\mathrm{N}}$.

(2.c) $\overline{w}_{\mathrm{P}} < \overline{w}_{\mathrm{N}}$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) < 0$

Since $\overline{w}_{\mathrm{P}} < \overline{w}_{\mathrm{N}}$, $\Delta_{\mathrm{R}}(w, Q)$ and $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw}$ are given by (B.32) and (B.33), respectively. Observe that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathrm{P}}]$. Hence, given our assumption that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) < 0$, it must be that $\Delta_{\mathrm{R}}(w, Q) < 0$ for $w \in [c, \overline{w}_{\mathrm{P}}]$. Next, we consider the behavior of $\Delta_{\mathrm{R}}(w, Q)$ for $w > \overline{w}_{\mathrm{P}}$ by examining two subcases: (2.c.i) $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}, Q) \geq 0$, and (2.c.ii) $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}, Q) < 0$.

(2.c.i) $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}, Q) \geq 0$

For $w \in (\overline{w}_{\mathrm{P}}, \overline{w}_{\mathrm{N}}]$, observe from (B.33) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} > 0$ (since $p^u(w) > p^u(\overline{w}_{\mathrm{P}}) = \bar{p}(Q)$ for $w > \overline{w}_{\mathrm{P}}$). Combining this observation with the facts that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) < 0$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}, Q) \geq 0$, it must be that $\Delta_{\mathrm{R}}(w, Q)$ crosses zero only once for some $w \in (\overline{w}_{\mathrm{P}}, \overline{w}_{\mathrm{N}}]$.

As for $w \geq \overline{w}_{\mathrm{N}}$, observe from (B.32) that $\Delta_{\mathrm{R}}(w, Q) = \Delta^u_{\mathrm{R}}(w)$ when $w \geq \overline{w}_{\mathrm{N}}$.

By Lemma B.1.1(a) and (b) together, once $\Delta^u_{\mathbf{R}}(w)$ crosses zero at some w, it stays strictly positive for larger w. Therefore, given our assumption that $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}, Q) \geq 0$, it follows that $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w) \geq 0$ for $w \geq \overline{w}_{\mathbf{N}}$.

(2.c.ii) $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}, Q) < 0$

Recall that $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathbf{P}}]$ and $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} \geq 0$ for $w \in [\overline{w}_{\mathbf{P}}, \overline{w}_{\mathbf{N}}]$ from (B.33). Therefore, given the assumption that $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}, Q) < 0$, it must be that $\Delta_{\mathbf{R}}(w, Q) < 0$ for $w \in [c, \overline{w}_{\mathbf{N}}]$.

For $w \geq \overline{w}_{\mathbf{N}}$, recall that $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$. Since $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}, Q) = \Delta^u_{\mathbf{R}}(\overline{w}_{\mathbf{N}}) < 0$, the behavior of $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$ must follow the case in Lemma B.1.1(b) for $w \geq \overline{w}_{\mathbf{N}}$.

Case (3) Consider now the case where $\overline{w}_{\mathbf{P}} \geq c$ exists, but $\overline{w}_{\mathbf{N}} \geq c$ does not exist. The quantity sold under negotiation is not bounded by capacity for any $w \geq c$. Hence, $\Pi_{\mathbf{RN}}(q^*_{\min}(w), w) = \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ for all $w \geq c$. Given this fact and applying equation (3.17), we have

$$\begin{aligned}\Delta_{\mathbf{R}}(w, Q) &= \Pi_{\mathbf{RP}}(p^*(w), w) - \Pi_{\mathbf{RN}}(q^*_{\min}(w), w) \\ &= \begin{cases} \Pi_{\mathbf{RP}}(\bar{p}, w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \in [c, \overline{w}_{\mathbf{P}}], \\ \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \geq \overline{w}_{\mathbf{P}}. \end{cases}\end{aligned} \tag{B.34}$$

Notice that (B.34) is a special case of (B.30). Therefore, case (3) collapses to case (2.a), and $\Delta_{\mathbf{R}}(w, Q) \geq 0$ for all $w \geq c$.

Case (4) Consider now the case where $\overline{w}_{\mathbf{N}} \geq c$ exists, but $\overline{w}_{\mathbf{P}} \geq c$ does not exist. The quantity sold under posted pricing is not bounded by capacity for any $w \geq c$. Hence, $\Pi_{\mathbf{RP}}(p^*(w), w) = \Pi^u_{\mathbf{RP}}(p^u(w), w)$ for all $w \geq c$. Given this fact

and applying equation (3.17), we have

$$\begin{aligned}\Delta_{\mathrm{R}}(w,Q) &= \Pi_{\mathrm{RP}}(p^*(w),w) - \Pi_{\mathrm{RN}}(q^*_{\min}(w),w)\\ &= \begin{cases} \Pi^u_{\mathrm{RP}}(p^u(w),w) - \Pi_{\mathrm{RN}}(\bar{p},w) & \text{for } w \in [c,\overline{w}_{\mathrm{N}}],\\ \Pi^u_{\mathrm{RP}}(p^u(w),w) - \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w) & \text{for } w \geq \overline{w}_{\mathrm{N}}. \end{cases}\end{aligned} \tag{B.35}$$

Depending on whether $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}},Q) \geq 0$ or $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}},Q) < 0$, the result is the same as in case (2.c.i) or (2.c.ii), respectively.

Proof of (c): The result follows immediately from the discussion of case (3) above, as that discussion shows that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for all $w \geq c$ when only $\overline{w}_{\mathrm{P}} \geq c$ exists.

Proof of (d): If $\overline{w}_{\mathrm{N}} \geq c$ and $\overline{w}_{\mathrm{P}} \geq c$ both exist, the only case where $\widehat{w}_{\mathrm{R}}$ exists is case (2.c) discussed in the proof of parts (a) and (b): $\overline{w}_{\mathrm{P}} < \overline{w}_{\mathrm{N}}$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}},Q) < 0$. In the proof of case (2.c), we have shown that $\Delta_{\mathrm{R}}(w,Q) < 0$ for $w \in [c,\overline{w}_{\mathrm{P}}]$. Hence, the wholesale price at which $\Delta_{\mathrm{R}}(w,Q) = 0$ must be strictly greater than $\overline{w}_{\mathrm{P}}$, that is, $\widehat{w}_{\mathrm{R}} > \overline{w}_{\mathrm{P}}$.

Lemma B.2.2. *Define* $\Delta_{\mathrm{R}}(w,Q) = \Pi_{\mathrm{RP}}(p^*(w,Q),w,Q) - \Pi_{\mathrm{RN}}(q^*_{\min}(w,Q),w,Q)$. *Suppose there exists a unique* $\widehat{w}_{\mathrm{R}}(Q)$ *such that* $\Delta_{\mathrm{R}}(w,Q) < 0$ *for* $w < \widehat{w}_{\mathrm{R}}(Q)$ *and* $\Delta_{\mathrm{R}}(w,Q) \geq 0$ *for* $w \geq \widehat{w}_{\mathrm{R}}(Q)$. *Consider the following optimization problem:*

$$\max_{c \leq w \leq \widehat{w}_{\mathrm{R}}(Q)} \Pi_{\mathrm{MN}}(w, q^*_{\min}(w,Q), Q) \tag{B.36}$$

Let $w^o_{\mathrm{N}}(c_{\mathrm{T}})$ *denote the optimal solution to* (B.36) *and* $G_{\mathrm{N}}(c_{\mathrm{T}})$ *be the optimal value of the objective function for a given* c_{T}. *Then,*

(*a*) $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \min\{\widehat{w}_{\mathrm{R}}(Q), w^*_{\mathrm{N}}(Q)\}$.

(*b*) $\overline{w}_{\mathrm{N}}(Q)$ *decreases in* c_{T}. *Furthermore,* $\frac{d\overline{w}_{\mathrm{N}}(Q)}{dc_{\mathrm{T}}} = -1$.

(*c*) $\widehat{w}_{\mathrm{R}}(Q)$ *decreases in* c_{T}. *Furthermore,* $\frac{d\widehat{w}_{\mathrm{R}}(Q)}{dc_{\mathrm{T}}} < -1$.

(*d*) *If* $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \widehat{w}_{\mathrm{R}}(Q)$ *for some* $c_{\mathrm{T}} = c^o_{\mathrm{T}}$, *then* $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \widehat{w}_{\mathrm{R}}(Q)$ *for* $c_{\mathrm{T}} > c^o_{\mathrm{T}}$.

(*e*) $G_{\mathrm{N}}(c_{\mathrm{T}})$ *decreases in* c_{T}.

Proof of Lemma B.2.2

The proof of Lemma B.2.2 is similar to that of Lemma B.1.2 and mostly algebraic, therefore omitted.

Lemma B.2.3.

(*a*) *Suppose there exists a* $\overline{w}_{\mathrm{N}}(Q) > c$ *at a given* Q. *Then,* $\frac{d\overline{w}_{\mathrm{N}}(Q)}{dQ} \leq 0$.

(*b*) *Define* $\Delta_{\mathrm{R}}(w, Q) = \Pi_{\mathrm{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathrm{RN}}(q^*_{\min}(w, Q), w, Q)$. *Suppose there exists a unique* $\widehat{w}_{\mathrm{R}}(Q)$ *such that* $\Delta_{\mathrm{R}}(w, Q) < 0$ *for* $w < \widehat{w}_{\mathrm{R}}(Q)$ *and* $\Delta_{\mathrm{R}}(w, Q) \geq 0$ *for* $w \geq \widehat{w}_{\mathrm{R}}(Q)$. *Then,* $\frac{d\widehat{w}_{\mathrm{R}}(Q)}{dQ} \geq 0$.

Consider now the following optimization problem:

$$\max_{c \leq w \leq \widehat{w}_{\mathrm{R}}(Q)} \Pi_{\mathrm{MN}}(w, q^*_{\min}(w, Q), Q) \tag{B.37}$$

Let $w^o_{\mathrm{N}}(Q)$ *denote the optimal solution to* (B.37) *and* $G_{\mathrm{N}}(Q)$ *be the optimal value of the objective function for a given* Q. *Then,*

(*c*) *Suppose, for some* $Q = Q^o$, $w^o_{\mathrm{N}}(Q^o) = w^*_{\mathrm{N}}(Q^o)$. *Then,* $w^o_{\mathrm{N}}(Q) = w^*_{\mathrm{N}}(Q)$ *for* $Q > Q^o$.

(*d*) $G_{\mathrm{N}}(Q)$ *increases in* Q.

Proof of Lemma B.2.3

Proof of (a): For a given capacity Q, the market-clearing wholesale price under negotiation, $\overline{w}_{\mathrm{N}}(Q)$, is defined so that even a retailer with unlimited capacity will find it optimal to sell exactly Q units. In other words, $q^u_{\min}(\overline{w}_{\mathrm{N}}(Q)) = \bar{p}(Q)$ (see (3.19) and the preceding discussion). Hence, $q^u_{\min}(\overline{w}_{\mathrm{N}}(Q))$ and $\overline{w}_{\mathrm{N}}(Q)$ satisfy the first-order

condition of the retailer's profit function under negotiation, $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$. Using the expression for $\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}$ from (B.9) and the fact that $q^u_{\min}(w)$ satisfies the first-order condition for $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$:

$$\left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w)}{\partial q_{\min}}\right|_{q_{\min}=q^u_{\min}(w)} = a(-q^u_{\min}(w) + w + c_{\mathbf{T}})f(q^u_{\min}(w)) + a\beta\overline{F}(q^u_{\min}(w)) = 0.$$

Substituting $w = \overline{w}_{\mathbf{N}}(Q)$ and $q^u_{\min}(\overline{w}_{\mathbf{N}}(Q)) = \bar{p}(Q)$ in the above equation, we obtain the following identity:

$$\begin{aligned} &\big(-\bar{p}(Q) + \overline{w}_{\mathbf{N}}(Q) + c_{\mathbf{T}}\big)f(\bar{p}(Q)) + \beta\overline{F}(\bar{p}(Q)) = 0, \text{ or,} \\ &-\beta\frac{\overline{F}(\bar{p}(Q))}{f(\bar{p}(Q))} + \bar{p}(Q) = \overline{w}_{\mathbf{N}}(Q) + c_{\mathbf{T}}. \end{aligned} \tag{B.38}$$

When Q increases, $\bar{p}(Q)$ decreases (since $a\overline{F}(\bar{p}(Q)) = Q$) and, thus, $\beta\frac{\overline{F}(\bar{p}(Q))}{f(\bar{p}(Q))}$ increases due to F being IFR. Therefore, the left-hand side of the above identity decreases in Q. Hence, $\overline{w}_{\mathbf{N}}(Q)$ must decrease in Q.

Proof of (b): We consider four cases depending on whether there exist $\overline{w}_{\mathbf{N}}(Q) \geq c$ and/or $\overline{w}_{\mathbf{P}}(Q) \geq c$, that is whether there exists a feasible wholesale price (i.e. greater than or equal to c) at which the quantity sold under negotiation and/or posted pricing is not bounded by capacity. Four cases are: (1) both $\overline{w}_{\mathbf{N}}(Q)$ and $\overline{w}_{\mathbf{P}}(Q)$ exist, (2) only $\overline{w}_{\mathbf{N}}(Q)$ exists, (3) only $\overline{w}_{\mathbf{P}}(Q)$ exists, and (4) neither of them exists.

(1) both $\overline{w}_{\mathbf{P}}(Q)$ and $\overline{w}_{\mathbf{N}}(Q)$ exist

Note from Lemma B.2.1(d) that if $\widehat{w}_{\mathbf{R}}(Q)$ exists, it must be that $\overline{w}_{\mathbf{P}}(Q) < \min\{\widehat{w}_{\mathbf{R}}(Q), \overline{w}_{\mathbf{N}}(Q)\}$. Therefore, there are two possible subcases: (1.a) $\overline{w}_{\mathbf{P}}(Q) < \widehat{w}_{\mathbf{R}}(Q) < \overline{w}_{\mathbf{N}}(Q)$, and (1.b) $\overline{w}_{\mathbf{P}}(Q) < \overline{w}_{\mathbf{N}}(Q) \leq \widehat{w}_{\mathbf{R}}(Q)$.

Consider the first subcase (1.a). By definition, $\widehat{w}_{\mathbf{R}}(Q)$ satisfies $\Delta_{\mathbf{R}}(\widehat{w}_{\mathbf{R}}(Q), Q) = 0$. Observe from (3.17) and (3.20) that

$\Pi_{\mathbf{RP}}(p^*(\widehat{w}_{\mathbf{R}}(Q),Q),\widehat{w}_{\mathbf{R}}(Q),Q) = \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)),\widehat{w}_{\mathbf{R}}(Q))$ (since $\overline{w}_{\mathbf{P}}(Q) < \widehat{w}_{\mathbf{R}}(Q)$), and

$\Pi_{\mathbf{RN}}(q^*_{\min}(\widehat{w}_{\mathbf{R}}(Q),Q),\widehat{w}_{\mathbf{R}}(Q),Q) = \Pi_{\mathbf{RN}}(\bar{p}(Q),\widehat{w}_{\mathbf{R}}(Q),Q)$ (since $\widehat{w}_{\mathbf{R}}(Q) < \overline{w}_{\mathbf{N}}(Q)$).

Therefore, at $w = \widehat{w}_{\mathbf{R}}(Q)$, the following identity must be satisfied:

$$\Pi_{\mathbf{RN}}(\bar{p}(Q),\widehat{w}_{\mathbf{R}}(Q),Q) - \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)),\widehat{w}_{\mathbf{R}}(Q)) = 0.$$

Implicit differentiation of the above identity with respect to Q yields:

$$\begin{aligned} 0 &= \frac{d\Pi_{\mathbf{RN}}(\bar{p}(Q),\widehat{w}_{\mathbf{R}}(Q),Q)}{dQ} - \frac{d\Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)),\widehat{w}_{\mathbf{R}}(Q))}{dQ} \\ &= \frac{d\bar{p}(Q)}{dQ}\left.\frac{\partial\Pi_{\mathbf{RN}}(q_{\min},w,Q)}{\partial q_{\min}}\right|_{q_{\min}=\bar{p}(Q),w=\widehat{w}_{\mathbf{R}}(Q)} + \frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ}\left.\frac{\partial\Pi_{\mathbf{RN}}(q_{\min},w,Q)}{\partial w}\right|_{q_{\min}=\bar{p}(Q),w=\widehat{w}_{\mathbf{R}}(Q)} \\ &\quad -\frac{dp^u(\widehat{w}_{\mathbf{R}}(Q))}{dQ}\left.\frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial p}\right|_{p=p^u(\widehat{w}_{\mathbf{R}}(Q)),w=\widehat{w}_{\mathbf{R}}(Q)} - \frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ}\left.\frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial w}\right|_{p=p^u(\widehat{w}_{\mathbf{R}}(Q)),w=\widehat{w}_{\mathbf{R}}(Q)} \end{aligned} \tag{B.39}$$

Note that the third term on the right-hand side of (B.39) is zero since p^u satisfies the first-order condition of $\Pi^u_{\mathbf{RP}}(p,w)$. Recall that

$$\begin{aligned} \Pi_{\mathbf{RN}}(q_{\min},w,Q) &= a\int_{q_{\min}}^{\infty}[(1-\beta)x + \beta q_{\min} - w - c_{\mathbf{T}}]f(x)dx, \\ \Pi^u_{\mathbf{RP}}(p,w) &= a(p-w)\overline{F}(p). \end{aligned}$$

Take the partial derivatives of these functions, we obtain

$$\begin{aligned} \frac{\partial\Pi_{\mathbf{RN}}(q_{\min},w,Q)}{\partial q_{\min}} &= a(-q_{\min}+w+c_{\mathbf{T}})f(q_{\min}) + a\beta\overline{F}(q_{\min}), \\ \frac{\partial\Pi_{\mathbf{RN}}(q_{\min},w,Q)}{\partial w} &= -a\overline{F}(q_{\min}), \text{ and} \\ \frac{\partial\Pi^u_{\mathbf{RP}}(p,w)}{\partial w} &= -a\overline{F}(p). \end{aligned}$$

Substituting the partial derivatives above in (B.39) and rearranging the terms, we obtain:

$$\begin{aligned} &\frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ}\big(\overline{F}(\bar{p}(Q)) - \overline{F}(p^u(\widehat{w}_{\mathbf{R}}(Q)))\big) \\ =\ &\frac{d\bar{p}(Q)}{dQ}\big[\big(-\bar{p}(Q)+\widehat{w}_{\mathbf{R}}(Q)+c_{\mathbf{T}}\big)f(\bar{p}(Q)) + \beta\overline{F}(\bar{p}(Q))\big] \end{aligned} \tag{B.40}$$

Note from (B.38) that $\big(-\bar{p}(Q)+\overline{w}_{\mathrm{N}}(Q)+c_{\mathrm{T}}\big)f(\bar{p}(Q))+\beta\overline{F}(\bar{p}(Q))=0$. Since $\widehat{w}_{\mathrm{R}}(Q)<\overline{w}_{\mathrm{N}}(Q)$ in subcase (1.a), it follows that

$$\big(-\bar{p}(Q)+\widehat{w}_{\mathrm{R}}(Q)+c_{\mathrm{T}}\big)f(\bar{p}(Q))+\beta\overline{F}(\bar{p}(Q))<0.$$

Furthermore, $\frac{d\bar{p}(Q)}{dQ}<0$ since $\bar{p}(Q)$ is such that $a\overline{F}(\bar{p}(Q))=Q$. Hence, the right-hand side of (B.40) is positive. We then consider the left-hand side of (B.40). Note that, since $\overline{w}_{\mathrm{P}}(Q)<\widehat{w}_{\mathrm{R}}(Q)$ in subcase (1.a), it follows that $p^u(\widehat{w}_{\mathrm{R}}(Q))>p^u(\overline{w}_{\mathrm{P}}(Q))=\bar{p}(Q)$, where the equality is by definition of $\overline{w}_{\mathrm{P}}(Q)$. Hence, $\overline{F}\big(\bar{p}(Q)\big)>\overline{F}\big(p^u(\widehat{w}_{\mathrm{R}}(Q))\big)$. Since the right-hand side of (B.40) is positive, we now conclude $\frac{d\widehat{w}_{\mathrm{R}}(Q)}{dQ}\geq 0$.

Subcase (1.b) can be proven similarly by implicit differentiation of the same identity.

(2) only $\overline{w}_{\mathrm{N}}(Q)$ exists

We consider two separate subcases: (2.a) $\widehat{w}_{\mathrm{R}}(Q)<\overline{w}_{\mathrm{N}}(Q)$ and (2.b) $\overline{w}_{\mathrm{N}}(Q)\leq\widehat{w}_{\mathrm{R}}(Q)$. Note that the analysis of (2.a) is similar to case (1.a), and (2.b) is similar to case (1.b).

(3) only $\overline{w}_{\mathrm{P}}(Q)$ exists

Note that if $\overline{w}_{\mathrm{P}}(Q)$ exists and $\overline{w}_{\mathrm{N}}(Q)$ does not exist, Lemma B.2.1(c) shows that $\widehat{w}_{\mathrm{R}}(Q)$ does not exist. Therefore, this case cannot occur when there exists $\widehat{w}_{\mathrm{R}}(Q)$ at given Q.

(4) both $\overline{w}_{\mathrm{P}}(Q)$ and $\overline{w}_{\mathrm{N}}(Q)$ do not exist

The analysis is similar to case (1.b).

Proof of (c): Pick two capacity levels Q^o and Q' such that $Q^o<Q'$. We consider three cases depending on whether there exists a feasible wholesale price at which the quantity sold under negotiation will be capacity-constrained at each capacity level,

Q^o and Q': (1) both $\overline{w}_{\text{N}}(Q^o) \geq c$ and $\overline{w}_{\text{N}}(Q') \geq c$ exist, (2) neither of them exists, and (3) $\overline{w}_{\text{N}}(Q^o) \geq c$ exists, but $\overline{w}_{\text{N}}(Q') \geq c$ does not exist. (The case that $\overline{w}_{\text{N}}(Q^o) \geq c$ does not exist and $\overline{w}_{\text{N}}(Q') \geq c$ exists cannot occur since $\overline{w}_{\text{N}}(Q)$ decreases in Q, which is proven in part (a) of this lemma.)

(1) $\overline{w}_{\text{N}}(Q^o) \geq c$ and $\overline{w}_{\text{N}}(Q') \geq c$

Note from Lemma B.2.2(a) that $w^o_{\text{N}}(Q) = \min\{\widehat{w}_{\text{R}}(Q), w^*_{\text{N}}(Q)\}$ and from Lemma 3.5.2 that $w^*_{\text{N}}(Q) = \max\{\overline{w}_{\text{N}}(Q), w^u_{\text{N}}\}$. Therefore, given that $w^o_{\text{N}}(Q^o) = w^*_{\text{N}}(Q^o)$, it must be that $\widehat{w}_{\text{R}}(Q^o) \geq \max\{\overline{w}_{\text{N}}(Q^o), w^u_{\text{N}}\}$. Observe that w^u_{N} is constant with respect to Q and, from part (a) of this lemma, $\overline{w}_{\text{N}}(Q)$ decreases when Q increases. Also observe from part (b) of this lemma, $\widehat{w}_{\text{R}}(Q)$ increases as Q increases. Therefore,

$$\widehat{w}_{\text{R}}(Q') \geq \widehat{w}_{\text{R}}(Q^o) \geq \max\{\overline{w}_{\text{N}}(Q^o), w^u_{\text{N}}\} \geq \max\{\overline{w}_{\text{N}}(Q'), w^u_{\text{N}}\},$$

and $w^o_{\text{N}}(Q') = w^*_{\text{N}}(Q')$.

(2) neither of them exists

In this case, there does not exist a feasible wholesale price at which the quantity sold under negotiation is capacity-constrained at either Q^o or Q'. Hence, $w^*_{\text{N}}(Q^o) = w^u_{\text{N}}$ and $w^*_{\text{N}}(Q') = w^u_{\text{N}}$. It follows that

$$w^o_{\text{N}}(Q^o) = \min\{\widehat{w}_{\text{R}}(Q^o), w^u_{\text{N}}\} \text{ and } w^o_{\text{N}}(Q') = \min\{\widehat{w}_{\text{R}}(Q'), w^u_{\text{N}}\}.$$

Then, the result follows from the facts that w^u_{N} is constant with respect to Q, $\widehat{w}_{\text{R}}(Q') \geq \widehat{w}_{\text{R}}(Q^o)$ (from part (b) of this lemma) and $w^o_{\text{N}}(Q^o) = w^u_{\text{N}}$.

(3) only $\overline{w}_{\text{N}}(Q^o) \geq c$ exists

In this case, $w^*_{\text{N}}(Q') = w^u_{\text{N}}$. The result follows from the following set of inequalities:

$$\widehat{w}_{\text{R}}(Q') \geq \widehat{w}_{\text{R}}(Q^o) \geq \max\{\overline{w}_{\text{N}}(Q^o), w^u_{\text{N}}\} \geq w^u_{\text{N}}$$

where the first inequality comes from part (b) of this lemma and the second inequality comes from the fact that $w_{\text{N}}^o(Q^o) = w_{\text{N}}^*(Q^o) = \max\{\overline{w}_{\text{N}}(Q^o), w_{\text{N}}^u\}$.

Proof of (d): It is easy to check that $\Pi_{\text{MN}}(w, q_{\min}^*(w,Q), Q)$ increases in Q. Therefore, if $Q^o < Q'$, then

$$\begin{aligned} G_{\text{N}}(Q^o) &= \Pi_{\text{MN}}(w_{\text{N}}^o(Q^o), q_{\min}^*(w_{\text{N}}^o(Q^o), Q^o), Q^o) \\ &\leq \Pi_{\text{MN}}(w_{\text{N}}^o(Q^o), q_{\min}^*(w_{\text{N}}^o(Q^o), Q'), Q'). \end{aligned} \tag{B.41}$$

Furthermore, since $\widehat{w}_{\text{R}}(Q)$ increases in Q, $w_{\text{N}}^o(Q^o)$ must be feasible for the optimization problem (B.37) at $Q = Q' > Q^o$. Therefore,

$$\begin{aligned} G_{\text{N}}(Q') &= \Pi_{\text{MN}}(w_{\text{N}}^o(Q'), q_{\min}^*(w_{\text{N}}^o(Q'), Q'), Q') \\ &\geq \Pi_{\text{MN}}(w_{\text{N}}^o(Q^o), q_{\min}^*(w_{\text{N}}^o(Q^o), Q'), Q'). \end{aligned} \tag{B.42}$$

Combining (B.41) and (B.42), we obtain $G_{\text{N}}(Q') \geq G_{\text{N}}(Q^o)$.

Proposition B.2.1. [Retailer's best response]

(a) If the retailer (weakly) prefers posted pricing at $w = c$, then the retailer strictly prefers posted pricing for all $w > c$.

Otherwise (i.e., if the retailer strictly prefers negotiation at $w = c$):

(b) either the retailer strictly prefers negotiation at all $w > c$,

(c) or there exists a unique threshold $\widehat{w}_{\text{R}}(Q) > c$ such that the retailer is indifferent between negotiation and posted pricing if $w = \widehat{w}_{\text{R}}(Q)$, strictly prefers negotiation if $w < \widehat{w}_{\text{R}}(Q)$, and strictly prefers posted pricing if $w > \widehat{w}_{\text{R}}(Q)$.

Proof of Proposition B.2.1

Define $\Delta_{\text{R}}(w,Q) = \Pi_{\text{RP}}(p^*(w,Q),w,Q) - \Pi_{\text{RN}}(q^*_{\min}(w,Q),w,Q)$ to be the difference between the retailer's optimal profits under the two sales formats at a given wholesale price, w. Lemma B.2.1(a)(b) proves that either (1) $\Delta_{\text{R}}(w,Q) \geq 0$ for all $w \geq c$, or (2) $\Delta_{\text{R}}(w,Q) < 0$ for all $w \geq c$,or (3) if $\Delta_{\text{R}}(w,Q)$ crosses zero for some w, it does so only once and from below. These three possibilities correspond to the three possible best response patterns listed in the proposition.

Proposition B.2.2. *Suppose there exists $\widehat{w}_{\text{R}}(Q) > c$ that makes the retailer indifferent between negotiation and posted pricing, as described in Proposition B.2.1(c). At the wholesale price $\widehat{w}_{\text{R}}(Q)$, the manufacturer prefers negotiation. In other words,*

$$\Pi_{\text{MN}}(\widehat{w}_{\text{R}}(Q), q^*_{\min}(\widehat{w}_{\text{R}}(Q),Q),Q) \geq \Pi_{\text{MP}}(\widehat{w}_{\text{R}}(Q), p^*(\widehat{w}_{\text{R}}(Q),Q),Q).$$

Proof of Proposition B.2.2

For ease of exposition, we prove the results when there exist market-clearing wholesale prices, $\overline{w}_{\text{P}}(Q) \geq c$ and $\overline{w}_{\text{N}}(Q) \geq c$ for a given Q. Notice that if neither $\overline{w}_{\text{P}}(Q) \geq c$ nor $\overline{w}_{\text{N}}(Q) \geq c$ exists, then there would be no feasible wholesale price ($w \geq c$) that makes the capacity binding under either sales format and, hence, the problem reverts to the unlimited capacity version, for which the result has already been established in Section 3.4. If only one of $\overline{w}_{\text{P}}(Q) \geq c$ or $\overline{w}_{\text{N}}(Q) \geq c$ exists, then there would be no feasible wholesale price that makes capacity binding under one of the sales formats, and the result would follow as a special case of the proof we are providing here.

For this proof, we omit Q whenever the dependence is obvious. Define $\Delta_{\text{M}}(w,Q) = \Pi_{\text{MP}}(w,p^*(w)) - \Pi_{\text{MN}}(w,q^*_{\min}(w))$ to be the difference between the manufacturer's profits under the two sales formats, and define $\widehat{w}_{\text{M}}$ be such that $\Delta_{\text{M}}(w,Q) \leq 0$ for

$w \leq \widehat{w}_{\text{M}}$ and $\Delta_{\text{M}}(w, Q) \geq 0$ for $w \geq \widehat{w}_{\text{M}}$. Lemma B.2.1(d) shows that, if $\widehat{w}_{\text{R}}$ exists, it must be that $\overline{w}_{\text{N}} > \overline{w}_{\text{P}}$ and $\widehat{w}_{\text{R}} > \overline{w}_{\text{P}}$. Since $\overline{w}_{\text{N}} > \overline{w}_{\text{P}}$, applying equations (3.18) and (3.21), we have

$$\begin{aligned}
\Delta_{\text{M}}(w, Q) &= \Pi_{\text{MP}}(w, p^*(w)) - \Pi_{\text{MN}}(w, q^*_{\min}(w)) \\
&= \begin{cases} 0 & \text{for } w \in [c, \overline{w}_{\text{P}}], \\ a(w-c)\overline{F}(p^u(w)) - (w-c)Q \ \leq 0 & \text{for } w \in [\overline{w}_{\text{P}}, \overline{w}_{\text{N}}], \\ a(w-c)\overline{F}(p^u(w)) - a(w-c)\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\text{N}}. \end{cases}
\end{aligned} \tag{B.43}$$

Notice from above that $\Delta_{\text{M}}(w, Q) \leq 0$ for $w \leq \overline{w}_{\text{N}}$. We next analyze the behavior of $\Delta_{\text{M}}(w, Q)$ for $w > \overline{w}_{\text{N}}$. Observe that the sales quantity under neither format will be bounded by the capacity if $w \geq \overline{w}_{\text{N}}$. Since capacity Q plays no role under both sales formats when $w \geq \overline{w}_{\text{N}}$, $\Delta_{\text{M}}(w, Q)$ is equal to $\Delta^u_{\text{M}}(w)$ and $\Delta_{\text{R}}(w, Q)$ is equal to $\Delta^u_{\text{R}}(w)$ for $w \geq \overline{w}_{\text{N}}$.

From (B.43), $\Delta^u_{\text{M}}(w) = a(w-c)\left(\overline{F}(p^u(w)) - \overline{F}(q^u_{\min}(w))\right)$ and from (B.18) that $\frac{d\Delta^u_{\text{R}}(w)}{dw} = -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w))$. Then, we have

$$\Delta^u_{\text{M}}(w) = -(w-c)\frac{d\Delta^u_{\text{R}}(w)}{dw} \text{ for } w \geq \overline{w}_{\text{N}} \tag{B.44}$$

Since $\Delta_{\text{M}}(\overline{w}_{\text{N}}, Q) \leq 0$, it follows from (B.44) that $\frac{d\Delta^u_{\text{R}}(w)}{dw} \geq 0$ at $w = \overline{w}_{\text{N}}$.

To show that $\widehat{w}_{\text{M}}$ exists and that $\widehat{w}_{\text{M}} \geq \widehat{w}_{\text{R}}$, we examine two cases: (1) $\widehat{w}_{\text{R}} \leq \overline{w}_{\text{N}}$ and (2) $\widehat{w}_{\text{R}} > \overline{w}_{\text{N}}$ separately. Note that, by the definition of $\widehat{w}_{\text{R}}$, $\widehat{w}_{\text{R}} \leq \overline{w}_{\text{N}}$ is equivalent to $\Delta_{\text{R}}(\overline{w}_{\text{N}}, Q) \geq 0$, and $\widehat{w}_{\text{R}} > \overline{w}_{\text{N}}$ to $\Delta_{\text{R}}(\overline{w}_{\text{N}}, Q) < 0$.

(1) $\widehat{w}_{\text{R}} \leq \overline{w}_{\text{N}}$ (equivalently, $\Delta_{\text{R}}(\overline{w}_{\text{N}}, Q) \geq 0$)

Since $\Delta^u_{\text{R}}(w)$ is positive and increasing at $w = \overline{w}_{\text{N}}$, it now follows from Lemma B.1.1(b) that the function $\Delta^u_{\text{R}}(w)$ is unimodal and peaks at some $w^o \geq \overline{w}_{\text{N}}$. This

implies that $\frac{d\Delta^u_{\mathrm{R}}(w)}{dw}$ changes sign from positive to negative at $w = w^o \geq \overline{w}_{\mathrm{N}}$, which in turn implies that $\Delta_{\mathrm{M}}(w, Q)$ changes sign from negative to positive at $w = w^o \geq \overline{w}_{\mathrm{N}}$ (see (B.44)). Hence, $\widehat{w}_{\mathrm{M}} = w^o$ and $\widehat{w}_{\mathrm{M}} \geq \widehat{w}_{\mathrm{R}}$.

(2) $\widehat{w}_{\mathrm{R}} > \overline{w}_{\mathrm{N}}$ (equivalently, $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}, Q) < 0$)

Recall that $\Delta_{\mathrm{M}}(w, Q)$ is equal to $\Delta^u_{\mathrm{M}}(w)$ and $\Delta_{\mathrm{R}}(w, Q)$ is equal to $\Delta^u_{\mathrm{R}}(w)$ for $w \geq \overline{w}_{\mathrm{N}}$. Since $\widehat{w}_{\mathrm{R}} > \overline{w}_{\mathrm{N}}$, $\Delta^u_{\mathrm{R}}(w)$ changes sign at $w = \widehat{w}_{\mathrm{R}}$. The result directly follows from Lemma B.1.1(c).

Cases (1) and (2) together conclude that, if $\widehat{w}_{\mathrm{R}}$ exists, then there must exist $\widehat{w}_{\mathrm{M}} \geq \widehat{w}_{\mathrm{R}}$ and the result directly follows.

Proposition B.2.3. *There exist two thresholds, $\underline{c_{\mathrm{T}}}$ and $\overline{c_{\mathrm{T}}}$, $\underline{c_{\mathrm{T}}} \leq \overline{c_{\mathrm{T}}}$, such that*

(a) **[Negotiation]** *if $c_{\mathrm{T}} < \underline{c_{\mathrm{T}}}$, then the equilibrium sales format is negotiation with the wholesale price $w^*_{\mathrm{N}}(Q) = \max\{\overline{w}_N(Q), w^u_{\mathrm{N}}\}$, resulting in the retailer's cut-off valuation $q^*_{\min}(w^*_{\mathrm{N}}(Q), Q)$,*

(b) **[Reconciliatory Negotiation]** *if $\underline{c_{\mathrm{T}}} \leq c_{\mathrm{T}} < \overline{c_{\mathrm{T}}}$, then the equilibrium sales format is negotiation with the wholesale price $\widehat{w}_{\mathrm{R}}(Q)$, resulting in the retailer's cut-off valuation $q^*_{\min}(\widehat{w}_{\mathrm{R}}(Q), Q)$, and*

(c) **[Posted Pricing]** *if $c_{\mathrm{T}} \geq \overline{c_{\mathrm{T}}}$, then the equilibrium sales format is posted price with the wholesale price $w^*_{\mathrm{P}}(Q) = \max\{\overline{w}_{\mathrm{P}}(Q), w^u_{\mathrm{P}}\}$, resulting in the posted price $p^*(w^*_{\mathrm{P}}(Q), Q)$.*

Proof of Proposition B.2.3

We omit the proof of this proposition. The proof is almost identical to that of Proposition 3.4.4, once we replace $q^u_{\min}(w)$, $p^u(w)$, w^u_{N} and w^u_{P} in the earlier proof with $q^*_{\min}(w, Q)$, $p^*(w, Q)$, $w^*_{\mathrm{N}}(Q)$ and $w^*_{\mathrm{P}}(Q)$ here. The proof utilizes Lemma B.2.2

which is a counterpart for Lemma B.1.2 used in the proof of Proposition 3.4.4.

B.3 Proof of Assumption A2 in Section 3.4

In this section of Appendix, we prove that Assumption A2 is satisfied for several widely used reservation price distribution. Recall Assumption A2:

(A2) The density $f(\cdot)$ is twice differentiable and satisfies

$$f'(x)\big(2f'(x)\overline{F}(x)+f^2(x)\big) \;-\; f''(x)f(x)\overline{F}(x)\geq 0 \tag{B.45}$$

Uniform Distribution, $U[0,\frac{a}{b}]$:

Note that $F(x)=\frac{bx}{a}$, $f(x)=\frac{b}{a}$ and $f'(x)=f''(x)=0$ for $x\in[0,\frac{a}{b}]$. Therefore, $f'(x)(2f'(x)\overline{F}(x)+f^2(x))-f''(x)f(x)\overline{F}(x)=0$ and Assumption A2 holds trivially.

Exponential (λ):

Note that $F(x)=1-e^{-\lambda x}$, $f(x)=\lambda e^{-\lambda x}$, $f'(x)=-\lambda^2e^{-\lambda x}$ and $f''(x)=\lambda^3e^{-\lambda x}$ for all $x\geq 0$. Then,

$$\begin{aligned} & f'(x)\big(2f'(x)\overline{F}(x)+f^2(x)\big)-f''(x)f(x)\overline{F}(x) \\ = \;& -\lambda^2e^{-\lambda x}[-2\lambda^2(e^{-\lambda x})^2+\lambda^2(e^{-\lambda x})^2]-\lambda^4(e^{-\lambda x})^3=0, \end{aligned}$$

thus Assumption A2 holds.

Weibull (α,β) with $\alpha\geq 1$:

Note that $F(x)=1-e^{-(\frac{x}{\beta})^\alpha}$, $f(x)=\frac{\alpha}{\beta^\alpha}x^{\alpha-1}e^{-(\frac{x}{\beta})^\alpha}$, $f'(x)=\frac{\alpha}{\beta^\alpha}e^{-(\frac{x}{\beta})^\alpha}z$, and $f''(x)=\frac{\alpha}{\beta^\alpha}e^{-(\frac{x}{\beta})^\alpha}[-\frac{\alpha}{\beta^\alpha}x^{\alpha-1}z+(\alpha-1)(\alpha-2)x^{\alpha-3}-2\frac{\alpha}{\beta^\alpha}(\alpha-1)x^{2\alpha-3}]$ where $z=(\alpha-1)x^{\alpha-2}-x^{2\alpha-2}\frac{\alpha}{\beta^\alpha}$. We have

$$\begin{aligned} & f'(x)(2f'(x)\overline{F}(x)+f^2(x))-f''(x)f(x)\overline{F}(x) \\ = \;& (e^{-(\frac{x}{\beta})^\alpha})^3\left[2\frac{\alpha^2}{\beta^{2\alpha}}z^2+2\frac{\alpha^3}{\beta^{3\alpha}}x^{2\alpha-2}z-(\alpha-1)(\alpha-2)\frac{\alpha^2}{\beta^{2\alpha}}x^{2\alpha-4}+2(\alpha-1)\frac{\alpha^3}{\beta^{3\alpha}}x^{3\alpha-4}\right] \\ = \;& \alpha(\alpha-1)\frac{\alpha^2}{\beta^{2\alpha}}x^{2\alpha-4}\left(e^{-(\frac{x}{\beta})^\alpha}\right)^3\geq 0, \end{aligned}$$

thus, Assumption A2 holds.

Gumbel

Note that $F(x) = \frac{1}{1+e^{\alpha-x}}$, $f(x) = \frac{e^{\alpha-x}}{(1+e^{\alpha-x})^2}$, $f'(x) = \frac{e^{\alpha-x}(e^{\alpha-x}-1)}{(1+e^{\alpha-x})^3}$, and $f''(x) = \frac{-4(e^{\alpha-x})^2+(e^{\alpha-x})^3+e^{\alpha-x}}{(1+e^{\alpha-x})^4}$ for $x \geq 0$. We have

$$\begin{aligned} & f'(x)(2f'(x)\overline{F}(x) + f^2(x)) - f''(x)f(x)\overline{F}(x) \\ = & \frac{(e^{\alpha-x})^3}{(1+e^{\alpha-x})^7}\left[(e^{\alpha-x}-1)(2e^{\alpha-x}-1) - (-4e^{\alpha-x} + (e^{\alpha-x})^2 + 1)\right] \\ = & \frac{(e^{\alpha-x})^4}{(1+e^{\alpha-x})^6} \geq 0, \end{aligned}$$

thus, Assumption A2 holds.

BIBLIOGRAPHY

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[1] Adachi, M. M. 1999. On the choice of pricing policies: Ex ante commitment and prisoners' dilemma. *European Economic Review* **43** 1647–1663.

[2] Agins, T., S. Collins. 2001. A Haggler's Christmas. *Wall Street Journal* (Nov 16) W1.

[3] Ahn, H., M. Gumus, P. Kaminsky. 2007. Pricing and manufacturing decisions when demand is a function of prices in multiple periods. *Operations Research* **55**(6) 1039–1057.

[4] Arnold, M. A., S. A. Lippman 1998. Posted prices versus bargaining in markets with asymmetric information. *Economic Inquiry* **36**(3) 450–457.

[5] Aviv, Y., A. Pazgal. 2005. Optimal pricing of seasonal products in the presence of forward-looking consumers. Working paper, Washington University, St. Louis, MO.

[6] Aydin, G., S. Ziya. 2006. Pricing promotional products under upselling. To appear in *Manufacturing and Service Operations Management*.

[7] Bester, H. 1993. Bargaining versus price competition in markets with quality uncertainty. *The American Economic Review* **83** (1) 278–288.

[8] Bitran, G. R., R. Caldentey. 2003. An overview of pricing models for revenue management. *Manufacturing and Service Operations Management* **5**(3) 203–229.

[9] Bitran, G. R., S. V. Mondschein. 1993. Pricing perishable products: An application to the retail industry. Working paper, MIT, Cambridge, MA.

[10] Bitran, G. R., S. V. Mondschein. 1997. Periodic pricing of seasonal products in retailing. *Management Science* **43**(1) 64–79.

[11] Chatterjee, K., W. Samuelson. 1983. Bargaining under incomplete information. *Operations Research* **31**(5) 835–851.

[12] Desai, P., D. Purohit. 2004. "Let me talk to my manager": Haggling in a competitive environment. *Marketing Science* **23**(2) 219–233.

[13] Dukes, A., E. Gal-Or. 2003. Negotiations and exclusivity contracts for advertising. *Marketing Science* **22**(2) 222–245.

[14] Elmaghraby, W., A. Gulcu, P. Keskinocak. 2006. Designing optimal preannounced markdowns in the presence of rational customers with multi-unit demands. Working paper, Georgia Institute of Technology, Atlanta, GA.

[15] Elmaghraby, W., P. Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science* **49**(10) 1287–1309.

[16] Farrell, J., R. Gibbons. 1989. Cheap talk can matter in bargaining. *Journal of Economic Theory* **48** 221–237.

[17] Gallego, G., G. van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Science* **40**(8) 999–1020.

[18] Gurnani, H., M. Shi. 2006. A bargaining model for a first-time interaction under asymmetric beliefs of supply reliability. *Management Science* **52**(6) 865–880.

[19] Iyer, G., J. M. Villas-Boas. 2003. A bargaining theory of distribution channels. *Journal of Marketing Research* **40** 80–100.

[20] Lovejoy, W. S. 2007. Bargaining chains. Working paper, Ross School of Business, University of Michigan, Ann Arbor, MI.

[21] Maglaras, C., J. Meissner. 2006. Dynamic pricing strategies for multi-product revenue management problems. *Manufacturing and Service Operations Management* **8**(2) 136–148.

[22] Muthoo, A. 1999. *Bargaining theory with applications*. Cambridge University Press, Cambridge, UK.

[23] Nagarajan, M., G. Sosic. 2008. Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research.* **187**(3) 719–745.

[24] Nagarajan, M., Y. Bassok. 2002. A bargaining framework in supply chains (The assembly problem). Working paper, Marshall School of Business, University of Southern California, Los Angeles, CA.

[25] Netessine, S., S. Savin, W. Xiao. 2006. Revenue management through dynamic cross-selling in e-commerce retailing. *Operations Research* **54**(5) 893–913.

[26] Popescu, I., Y. Wu. 2006. Dynamic pricing strategies with reference effects. To appear in *Operations Research*.

[27] Richtel, M. 2008. Even at Megastores, Hagglers Find No Price Is Set in Stone. *New York Times* (Mar 23).

[28] Riley, J., R. Zeckhauser. 1983. Optimal selling strategies: When to haggle, when to hold firm. *The Quarterly Journal of Economics* **98**(2) 267–289.

[29] Roth, S., H. Woratschek, S. Pastowski. 2006. Negotiating prices for customized services. *Journal of Service Research* **8**(4) 316–329.

[30] Su, X. 2007. Inter-temporal pricing with strategic customer behavior. *Management Science* **53**(5) 726–741.

[31] Talluri, K., G. van Ryzin. 2005. *The Theory and Practice of Revenue Management*. Kluwer, New York.

[32] Terwiesch, C., S. Savin, I. Hann. 2005. Online haggling at a name-your-own-price retailer: Theory and application. *Management Science* **51**(3) 339–351.

[33] Wang, R. 1995. Bargaining versus posted-price selling. *European Economic Review* **39** 1747–1764.

[34] Welch, D. 2007. Haggling Starts to Go the Way of the Tail Fin. *BusinessWeek* (Oct 29).

[35] Wu, D. 2004. Supply chain intermediation: A bargaining theoretic framework. D. Simchi-Levi et al. ed. *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-business Era*, Kluwer Academic Publishers.

[36] Zhang, D., W. L. Cooper. 2005. Revenue management for parallel flights with customer-choice behavior. *Operations Research* **53**(3) 415–431.

[37] Zhou, Y., M. Fan, M. Cho. 2006. On the threshold purchasing behavior of customers facing dynamically priced perishable products. Working paper, University of Washington, Seattle, WA.